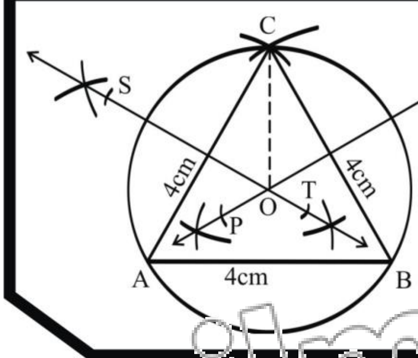


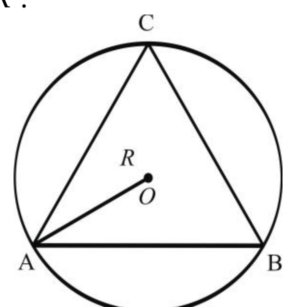
UNIT 13
PRACTICAL GEOMETRY - CIRCLE

Geometry (LHR 2014) **(K.B)**

The word geometry is derived from two Greek words Geo (earth) and Matron (measurement). Therefore the word geometry means measurement of earth. It deals with the shape, size and position of geometrical figures.

Circumcircle **(K.B)**

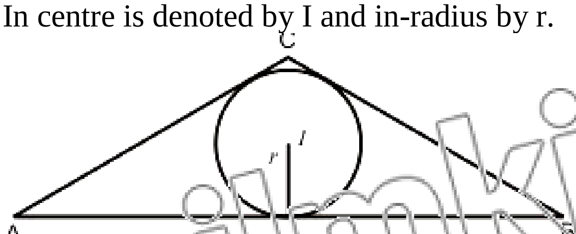
A circle passing through the vertices of a triangle is called circumcircle. Its radius is circumradius and centre is circumcentre. Circumcentre is denoted by '*O*' and circum radius by '*R*'.



Inscribed Circle **(K.B)**

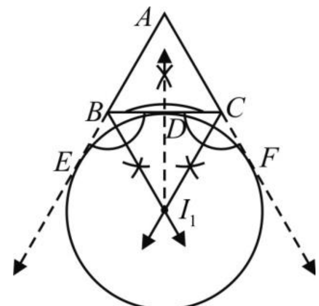
(FSD 2015, D.G.K 2015)

A circle which touches the three sides of a triangle internally is known as incircle. Its radius is in-radius and its centre is in-centre. In centre is denoted by I and in-radius by r.



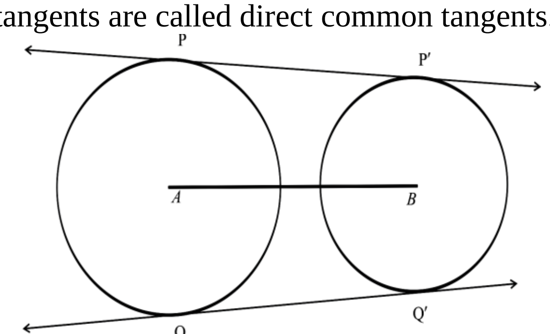
Escribed Circle (LHR 2015) **(K.B)**

A circle touching one side of a triangle externally and two produced sides internally is called escribed circle or e-circle. Its centre is represented by I_1, I_2, I_3 and radius by r_1, r_2 and r_3 and depending upon the vertex opposite to it.



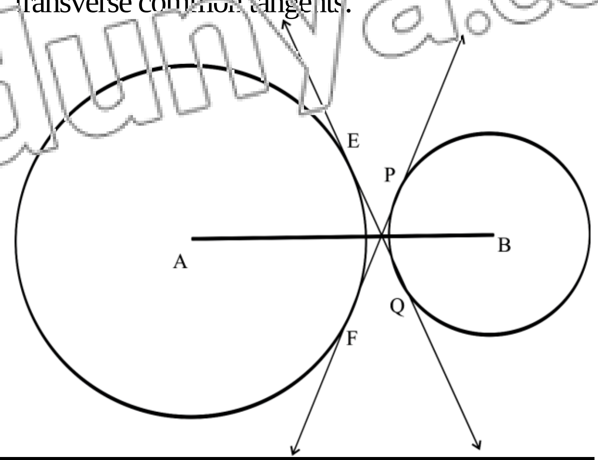
Direct Common Tangent **(K.B)**

If points of contact of common tangents to the two circles lie on the same side of line segment joining their centres then such tangents are called direct common tangents.



Transverse common tangents **(K.B)**

If the points of contact of common tangents to the two circles lie on opposite sides of line segment joining their centres, then such tangents are called transverse common tangents.



Note

Greeks Mathematicians contributed a lot, in particular "Euclid's Elements" have been taught.

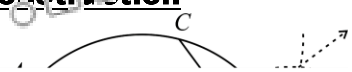
Construction of a Circle **(K.B)**

13.1(i) To locate the centre of a given circle

Given:

A circle

Steps of Construction



1. Draw two chords $\overline{AB}$ and $\overline{CD}$.
2. Draw EFG as perpendicular bisector of chord $\overline{AB}$
3. Draw $\overleftrightarrow{PQR}$ as perpendicular bisect or of chord $\overline{CD}$.

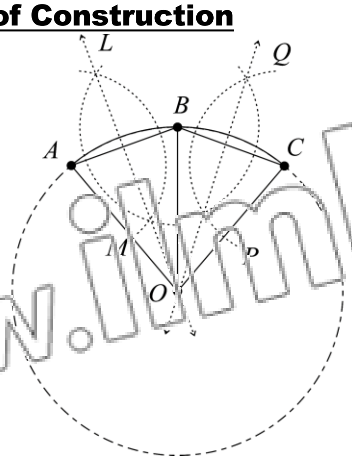
Perpendicular bisectors $\overleftrightarrow{EFG}$ and $\overleftrightarrow{PQR}$ intersect each other at O. O is the centre of circle.

13.1(ii) Draw a circle passing through three given non-collinear points:

Given:

Three non-collinear points A, B and C.

Steps of Construction



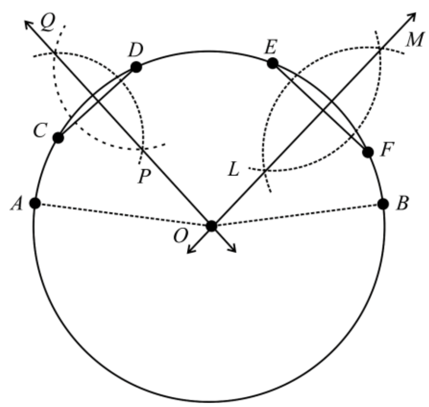
1. Join A with B and B with C.
2. Draw $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ right bisectors of $\overline{AB}$ and $\overline{BC}$ respectively. $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ intersect at point O.
3. Draws a circle with radius $\overline{OA} = \overline{OB} = \overline{OC}$ having centre at O, which is the required circle.

Page # 223 **(K.B)**

13.1(iii-a) To complete the circle by finding the centre when a part of a circumference is given

Given:

AB is Part of circumference of a circle



Steps of Construction

1. Let C, D, E and F be the four points on the given arc AB.
2. Draw chord $\overline{CD}$ and $\overline{EF}$.
3. Draw $\overleftrightarrow{PQ}$ as perpendicular bisector of $\overline{CD}$ and $\overleftrightarrow{LM}$ as perpendicular bisector of $\overline{EF}$.
4. $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ intersect at O.

$\therefore$ O is equidistant from points A, B, C, D, E and F.

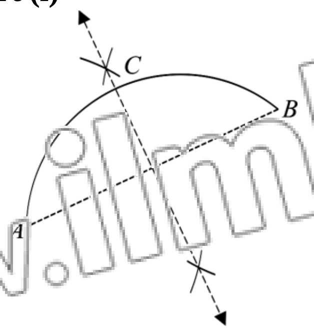
Complete the circle with centre O and radius $\left(\overline{OA} = \overline{OB} = \overline{OC} = \overline{OD} = \overline{OE} = \overline{OF}\right)$.

This will pass through all the points A, B, C, D, E and F on the given part of the circumference.

Exercise # 13.1

Q.1 Divide an arc of any length

(i) Part (i) **(A.B)**



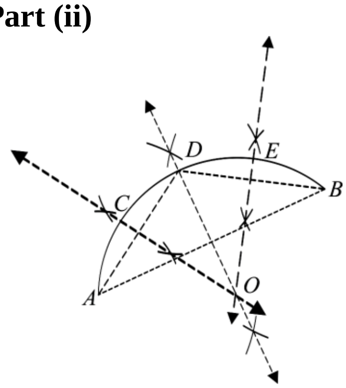
Steps of Construction

(i) Join A to B

(ii) Draw right bisector of $\overline{AB}$, which cuts AB at point C.

Thus $AC \cong BC$

(ii) Part (ii) **(A.B)**

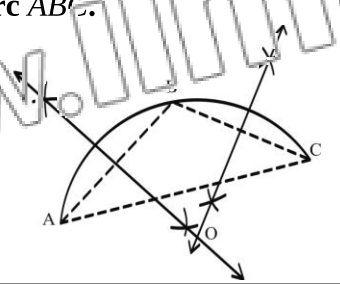


Steps of Construction

1. Join A to B
2. Draw right bisector of $\overline{AB}$, we get point D on AB.
3. Draw right bisectors of $\overline{AD}$ and $\overline{BD}$, passing through points C and E on AD and BD.

Thus $AC \cong CD \cong DE \cong EB$

Q.2 Practically find the centre of an arc ABC. **(A.B)**

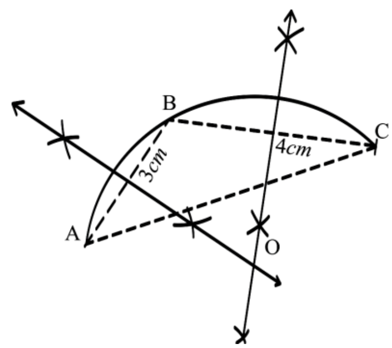


Steps of Construction

1. Mark two chords $\overline{AB}$ and $\overline{BC}$
2. Draw right bisectors of $\overline{AB}$ and $\overline{BC}$, they intersect each other at point O. Thus O is Centre of the arc.

Q.3

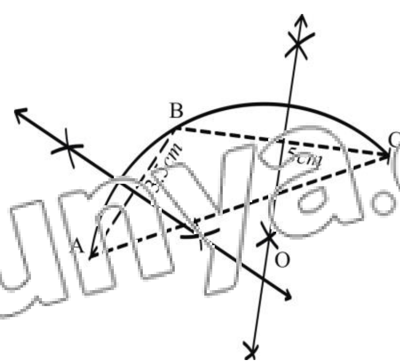
(i) If $\left|\overline{AB}\right| = 3cm$ and $\left|\overline{BC}\right| = 4cm$ are the lengths of two chords of an arc, then locate the centre of the arc **(A.B)**



Steps of Construction

1. Draw two chords $\overline{AB} = 3cm$ and $\overline{BC} = 4cm$.
2. Draw right bisectors of $\overline{AB}$ and $\overline{BC}$.
3. Both cut each other at point 'O'.
   Thus O is centre of the arc.

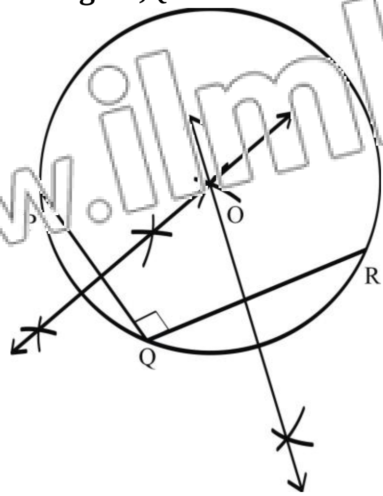
(ii) If $\left|\overline{AB}\right| = 3.5cm$ and $\left|\overline{BC}\right| = 5cm$ are the lengths of two chords of an arc, then locate the centre of the arc. **(A.B)**



Steps of Construction

1. Draw two chords $\overline{AB} = 3.5cm$ and $\overline{BC} = 5cm$.
2. Draw right bisectors of $\overline{AB}$ and $\overline{BC}$.
3. Both cut each other at point 'O'.
   Thus O is centre of the arc.

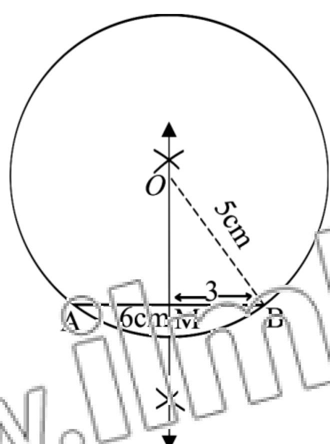
Q.4 For an arc draw two perpendicular bisectors of the chords $\overline{PQ}$ and $\overline{QR}$ of this arc, construct a circle through P,Q and R. **(A.B)**



Steps of Construction

1. Draw two chords $\overline{PQ}$ and $\overline{QR}$,s.t $\overline{PQ} \perp \overline{QR}$.
2. Draw right bisectors of $\overline{PQ}$ and $\overline{QR}$. They intersect each other at point O.
3. With centre O, draw a circle of radius $\overline{OP}$.

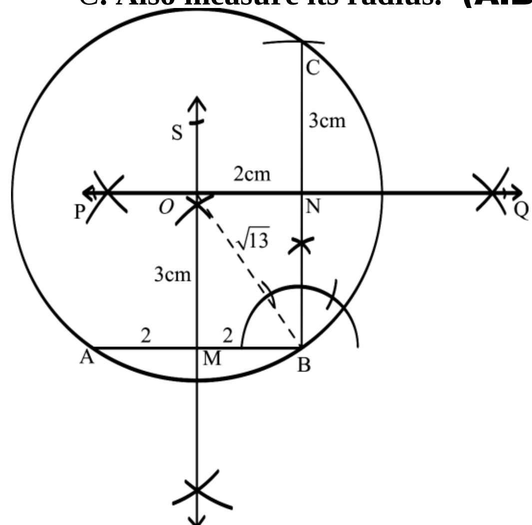
Q.5 Describe a circle of radius $5cm$ passing through points A and B, $6cm$ apart. Also find distance from the centre to the line segment AB. **(A.B)**



Steps of Construction

1. Draw a circle of radius $5cm$.
2. Take a point A on circle and draw an arc of radius $6cm$ to get point B.
3. Draw right bisector of $\overline{AB}$, passing through point 'M'.
4. Measure O to M. It is $4cm$.

Q.6 If $|\overline{AB}| = 4cm$ and $|\overline{BC}| = 6cm$, such that $\overline{AB}$ is perpendicular to $\overline{BC}$ construct a circle through A, B and C. Also measure its radius. **(A.B)**

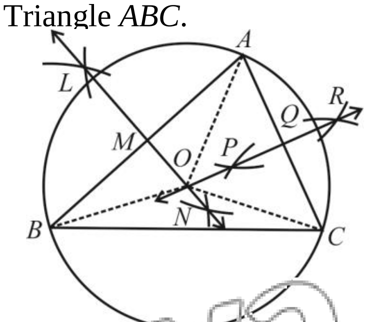


CIRCLES ATTACHED TO POLYGONS

13.2(i) Circumscribe a circle about a given triangle. **(A.B)**

Given:

Triangle ABC.



Steps of Construction

1. Draw $\overleftrightarrow{LMN}$ as perpendicular bisector of side $\overline{AB}$.
2. Draw $\overleftrightarrow{PQR}$ as perpendicular bisector of side $\overline{AC}$.
3. $\overleftrightarrow{LN}$ and $\overleftrightarrow{PR}$ intersect at point O.
4. With centre O and radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$, draw a circle.

This circle will pass through A, B and C whereas O is the circumcentre of the circumscribed circle.

Remember:

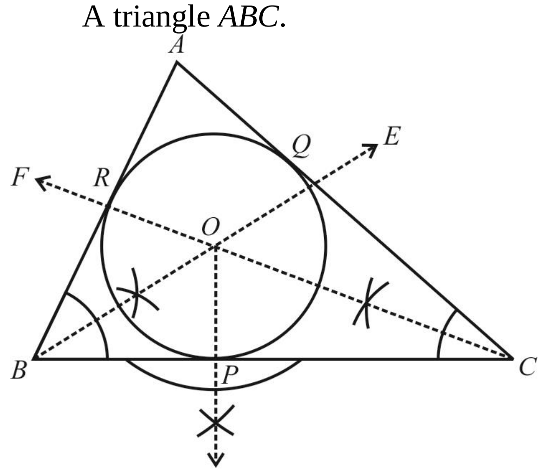
The circle passing through the vertices of triangle ABC is known as circumcircle, its radius as circumradius and centre as circumcentre.

Page # 225 **(A.B)**

13.2(ii) Inscribe a circle in a given triangle:

Given:

A triangle ABC.



Steps of Construction

1. Draw $\overleftrightarrow{BE}$ and $\overleftrightarrow{CF}$ to bisect the angles ABC and ACB respectively. Rays $\overleftrightarrow{BE}$ and $\overleftrightarrow{CF}$ intersect each other at point O.
2. O is the centre of the inscribed circle.
3. From O draw $\overleftrightarrow{OP}$ perpendicular to $\overline{BC}$. With centre O and radius $\overline{OP}$ draw a circle. This circle is the inscribed circle of triangle ABC.

Remember:

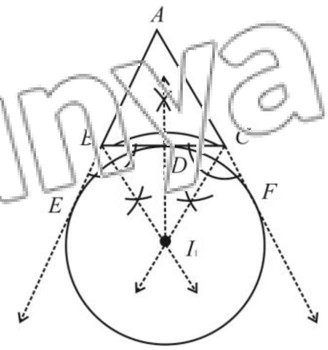
A circle which touches the three sides of a triangle internally is known as incircle, its radius as in-radius and centre as in-centre.

Page # 225 **(A.B)**

13.2(iii) Escribe a circle to given triangle:

Given:

A triangle ABC.



Steps of Construction

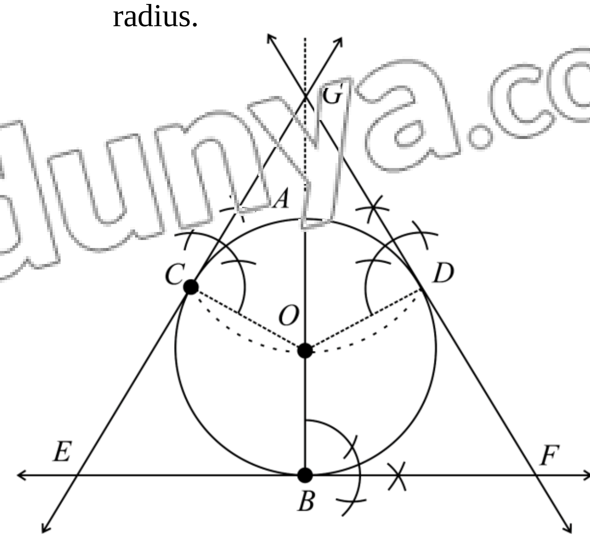
1. Produce the sides $\overline{AB}$ and $\overline{AC}$ of ΔABC.
2. Draw bisectors of exterior angles ABC and ACB. There bisectors of exterior angles meet at I_1.
3. From I_1 draw perpendicular on side $\overline{BC}$ of ΔABC. Which $\overline{I_1D}$ intersect $\overline{BC}$ at $D. I_1D$ is the radius of the escribed circle with centre at I_1.
4. Draw the circle with radius $\overline{I_1D}$ and centre at I_1 that will touch the side BC of the ΔABC externally and the produced sides AB and AC.

Page # 226 **(A.B)**

13.2(iv) Circumscribe an equilateral triangle about a given circle

Given:

A circle with centre O of reasonable radius.



Steps of Construction

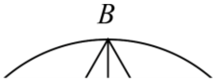
1. Draw $\overline{AB}$, the diameter of the circle for locating.
2. Draw an arc of radius $m\overline{OA}$ with centre at A for locating points C and D on the circle.
3. Join O to the points C and D.
4. Draw tangents to the circle at points B, C and D.
5. These tangents intersect at points E, F and G.

Page 226 **(A.B)**

13.2(v) Inscribe an equilateral triangle in a given circle.

Given:

A circle with centre O.



Steps of Construction

1. Draw any diameter $\overline{AB}$ of the circle.
2. Draw an arc of radius $\overline{OA}$ from point A. The arc cuts the circle at points C and D.
3. Join the points B, C and D to form straight line segments $\overline{BC}$, $\overline{CD}$ and $\overline{BD}$.
   Triangle BCD is the required inscribed equilateral triangle.

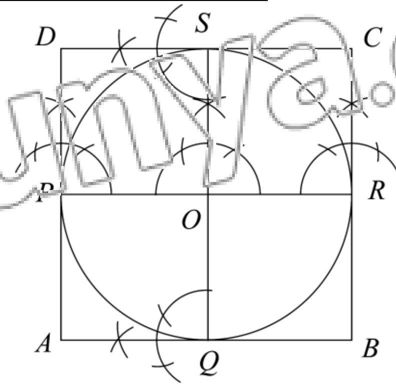
Page # 226 **(A.B)**

13.2(vi) Circumscribe a square about a given circle.

Given:

A circle with centre O.

Steps of Construction

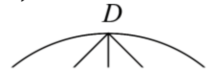


1. Draw two diameter $\overline{PR}$ and $\overline{QS}$ of the circle which bisect each other at right angle.
2. At points P, Q, R and S draw tangents to the circle.
3. Produce the tangents to meet each other at A, B, C and D. $ABCD$ is the required circumscribed square.

13.2(vii) Inscribe a square in a given circle

Given:

A circle, with centre at O.



Steps of Construction

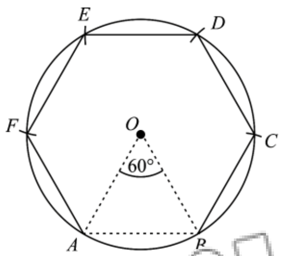
1. Through O draw two diameters $\overline{AC}$ and $\overline{BD}$ which bisect each other at right angle.
2. Join A with B, B with C, C with D, and D with A. $ABCD$ is the required square inscribed in the circle.

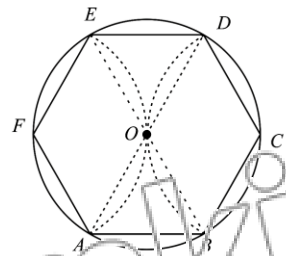
Page # 228 **(A.B)**

13.2(ix) Inscribe a regular hexagon in a given circle:

Given:

A circle, with centre at O.





Steps of Construction

1. Take any point A on the circle and point with O.
2. From point A, draw an arc of radius $\overline{OA}$ which intersects the circle at point B and F.
3. Join O and A with points B and F.
4. ΔOAB and $x\Delta OAF$ are equilateral triangles therefore $\angle AOB$ and $\angle AOF$ are of measure 600 *i.e.*, $m\overline{OA} = m\overline{AB} = m\overline{AF}$
5. Produce $\overline{FO}$ to meet the circle at C. Join B to C. Since in $\angle BOC = 60$ therefore $m\overline{BC} = m\overline{OA}$.
6. From C and F, draw arcs of radius OA, which intersect the circle at points O and E.
7. Join C to D, D to E and E to F ultimately.

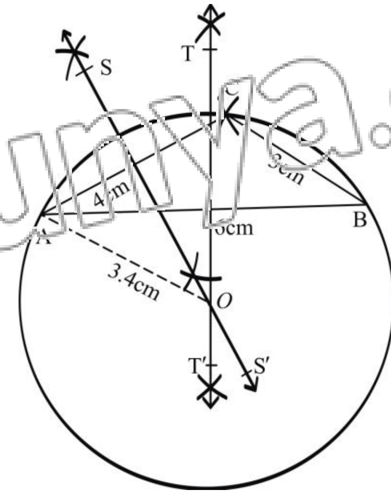
We have

$$m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = m\overline{OE} = m\overline{OF}$$

Thus the figure $ABCDEF$ is a regular hexagon inscribed in the circle.

Exercise 13.2

Q.1 Circumscribe a circle about a triangle ABC with sides $|\overline{AB}| = 6cm, |\overline{BC}| = 3cm$ and $|\overline{CA}| = 4cm$ Also measure its circum radius. **(A.B)**



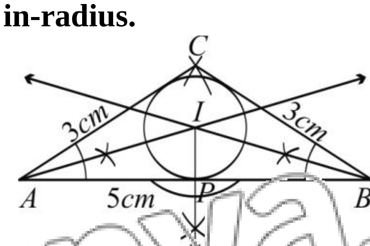
Steps of Construction

1. Construct ΔABC with given information.
2. Draw right bisectors of $\overline{AC}$ and $\overline{AB}$. Both cut each other at point O.
3. With centre 'O', draw a circle of radius $\overline{OA}$.

Thus required circumscribed circle is formed.

Radius of the circle is 3.4cm.

Q.2 Inscribe a circle in a triangle ABC with sides $|\overline{AB}| = 5cm, |\overline{BC}| = 3cm$ and $|\overline{CA}| = 3cm$. Also measure its in-radius. **(A.B)**

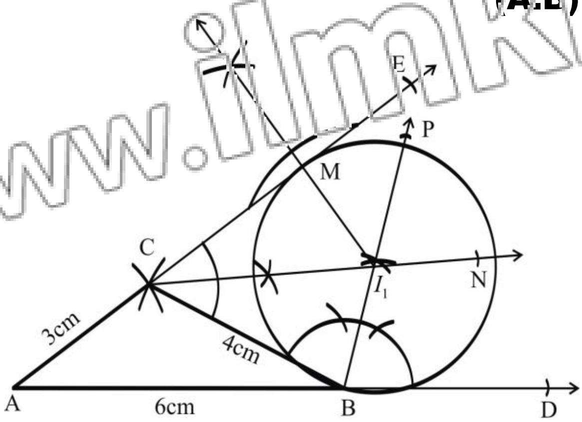


Steps of Construction

1. Construct ΔABC with given information.
2. Draw bisectors of $\angle A$ and $\angle B$, both cut each other at point I.
3. From point I, draw $\overline{IP} \perp \overline{AB}$.
4. With centre I, draw a circle of radius $\overline{IP}$.

Thus required inscribed circle is formed.

Q.3 Escribe a circle opposite to vertex A to a triangle ABC with sides $|\overline{AB}| = 6cm, |\overline{BC}| = 4cm$ and $|\overline{CA}| = 3cm$. Find its radius also. **(A.B)**

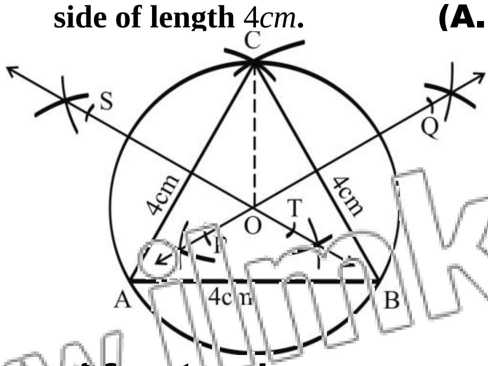


Steps of Construction

1. Construct ΔABC with given information.
2. Draw external bisectors of $\angle B$ and $\angle C$, both cut each other at point I_1.
3. From point I_1, draw $\overline{I_1M} \perp \overline{AC}$ produced.
4. With centre I_1, draw a circle of radius I_1M.

Thus required escribed circle is formed.

Q.4 Circumscribe a circle about an equilateral triangle ABC with each side of length $4cm$. **(A.B)**

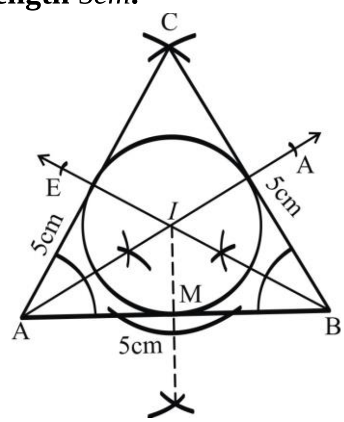


Steps of Construction

1. Construct ΔABC with given information.
2. Draw right bisectors of $\overline{AC}$ and $\overline{BC}$. Both cut each other at point O.
3. With centre 'O', draw a circle of radius $\overline{OC}$.

Thus required circumscribed circle is formed.

Q.5 Inscribe a circle in an equilateral triangle ABC with each side of length $5cm$. **(A.B)**



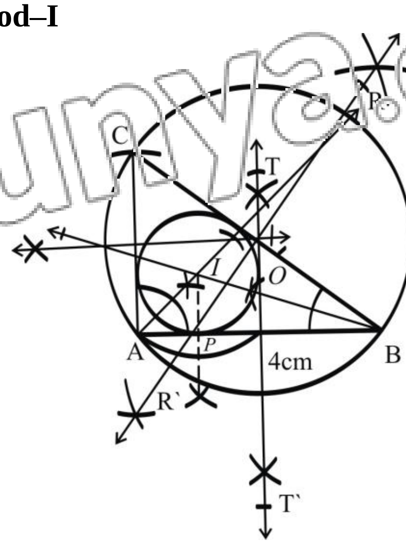
Steps of Construction

1. Construct ΔABC with given information.
2. Draw bisectors of $\angle A$ and $\angle B$, both cut each other at point I.
3. From point I, draw $\overline{IM} \perp \overline{AB}$.
4. With centre I, draw a circle of radius $\overline{IM}$.

Thus required inscribed circle is formed.

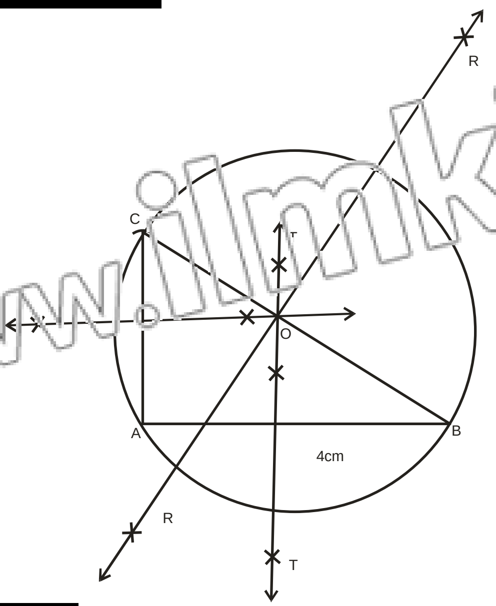
Q.6 Circumscribe and inscribe circles with regard to a right angle triangle with sides, $3cm$, $4cm$ and $5cm$. **(A.B)**

Method–I

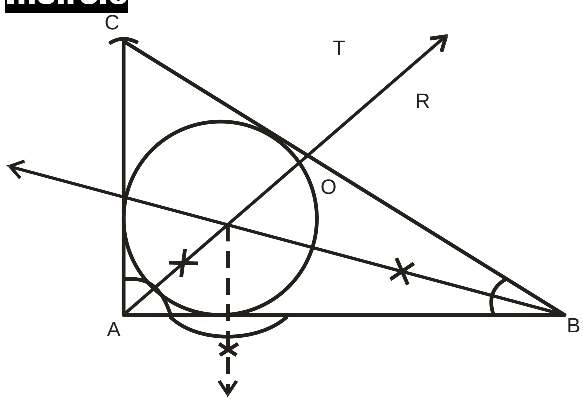


Method–II

Circumcircle



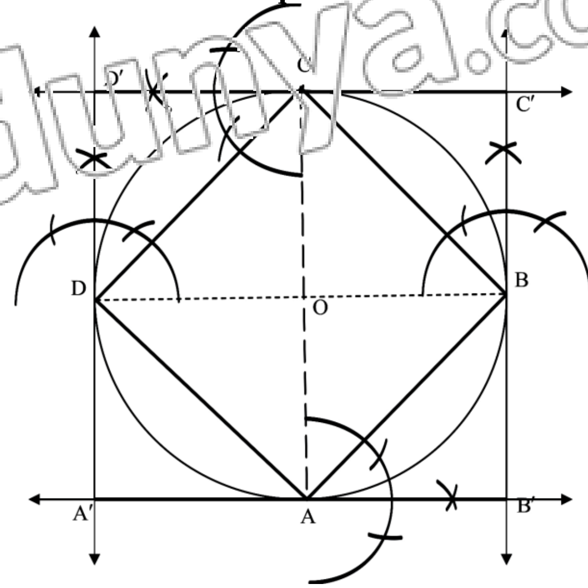
Incircle



Steps of Construction

1. Construct ΔABC with given information.
2. Draw right bisectors of $\overline{AC}$ and $\overline{AB}$. Both cut each other at point *O*.
3. With centre '*O*', draw a circle of radius $\overline{OA}$.
   Thus required circumscribed circle is formed.
4. Draw bisectors of $\angle A$ and $\angle B$, both cut each other at point *I*.
5. From point *I*, draw $\overline{IP} \perp \overline{AB}$.
6. With centre I, draw a circle of radius $\overline{IP}$.
   Thus required inscribed circle is formed.

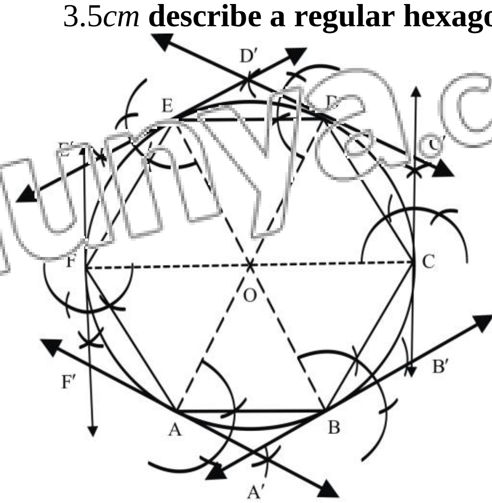
Q.7 In and about a circle of radius 4*cm* **describe a square.**



Steps of Construction

1. Draw a circle of radius 4*cm*.
2. Draw two diameters $\overline{AC}$ and $\overline{BD}$ which bisect each other at right angle.
3. Join *B* to *C*, *C* to *D*, *D* to *A* and *A* to *B*. Thus required inscribed square is formed.
4. At point *A*,*B*,*C* and *D* draw tangents, which intersect each other at points B', C', D' and A'.
   Thus circumscribed square is formed.

Q.8 In and about a circle of radius 3.5*cm* **describe a regular hexagon.**



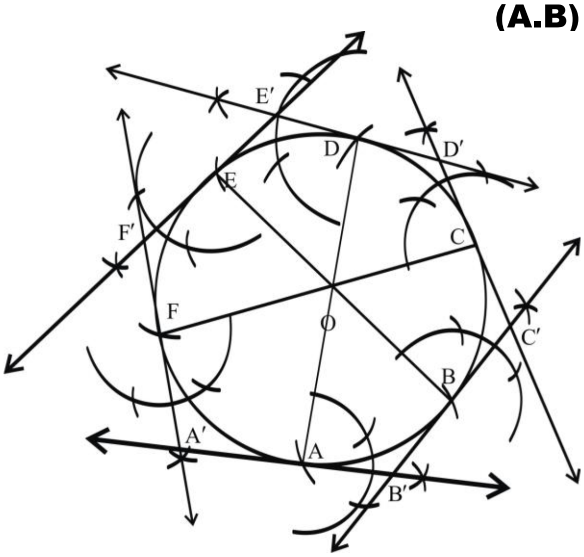
Steps of Construction

1. Draw any diameter $\overline{AD}$.
2. From point A draw an arc of radius AO (the radius of the circle), which cuts the circle at points B and F.
3. Join B with O and extend it to meet the circle at E.
4. Join F with O and extend it to meet the circle at C
5. Draw tangents to the circle at points A,B,C,D,E and F intersecting one another at points A',B',C',D',E' and F' respectively.

Thus $A'B'C'D'E'F'$ is the circumscribed regular hexagon.

Q.9 Circumscribe a regular hexagon about a circle of radius 3*cm*.

(A.B)



Steps of Construction

1. Draw any diameter $\overline{AD}$.
2. From point A draw an arc of radius AO (the radius of the circle), which cuts the circle at points B and F.
3. Join B with O and extend it to meet the circle at E.
4. Join F with O and extend it to meet the circle at C
5. Draw tangents to the circle at points A,B,C,D,E and F intersecting one another at points A',B',C',D',E' and F' respectively.

Thus $A'B'C'D'E'F'$ is the circumscribed regular hexagon.

13.3 TANGENT TO THE CIRCLE

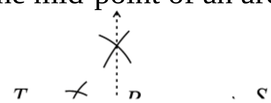
13.3(i) To draw a tangent to a given arc without using the centre through a given point *P*: **(A.B)**

Case (i)

When P is the middle point of the arc AB

Given:

P is the mid-point of an arc AB.



Steps of Construction

1. Join A and B, to form the chord $\overline{AB}$.
2. Draw the perpendicular bisector of chord $\overline{AB}$ which passes through mid-point P of AB and mid-point R of $\overline{AB}$.
3. At points P construct a right angle TPR.
4. Produce $\overrightarrow{PT}$ in the direction of P beyond point S. Thus $\overleftrightarrow{TP}$ is the required tangent to the arc AB at point P.

Page # 229 **(A.B)**

Case (ii)

When P is at end point of the arc

Given:

P is the end point of arc PQR.

Steps of Construction

1. Take a point A on the arc PQR.
2. Join the points A and P.
3. Draw perpendicular g at A which intersects the arc PQR at B
4. Join the points B and P.
5. Draw $\angle APD$ of measure equal to that of $\angle ABP$.
6. Now

$$\begin{aligned} m\angle BPD &= m\angle BPA + m\angle aPD \\ &= m\angle BPA + m\angle ABP \\ &= 90^\circ \end{aligned}$$

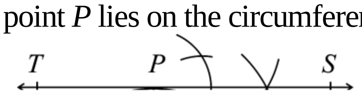
Thus $\overleftrightarrow{PD}$ is the required tangent.

Page # 230 **(A.B)**

13.3(ii-a) To draw a tangent to a circle from a given point *P* at a given point on the circumference:

Given:

A circle with the centre O and some point P lies on the circumference.



Steps of Construction

1. Join point P to the centre O, so that $\overline{OP}$ is the radius of the circle.
2. Draw a line TPS which is perpendicular to the radius $\overline{OP}$.

$\therefore$ $\overleftrightarrow{TPS}$ is the required tangent to the circle at given point P.

Page # 231 **(A.B)**

13.3(ii-b) To draw a tangent to a circle from a given point *P* which lies outside the circle:

Given:

A circle with centre O and some point P outside the circle.

Steps of Construction

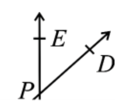
1. Join point P to the centre O.
2. Find M, the mid-point of $\overline{OP}$.
3. Construct a semi-circle on diameter $\overline{OP}$, with M as its centre. This semi-circle cuts the given circle at T.
4. Join P with T and produce $\overline{PT}$ on both sides, then $\overleftrightarrow{PT}$ is the required tangent.

Page # 231 **(A.B)**

13.3(iii) To draw two tangents to a circle meeting each other at a given angle:

Given:

A circle with centre O, $\angle MNS$ is a given angle.



Steps of Construction

1. Take a point A on the circumference of circle having centre O.
2. Join the points O and A.
3. Draw $\angle COA$ of measure equal to that of $\angle MNS$.

4. Produce $\overline{CO}$ to meet the circle at *B*.
5. $m\angle AOB = 180^\circ - m\angle COA$
6. Draw $\overleftrightarrow{AD}$ perpendicular to $\overline{OA}$.
7. Draw $\overleftrightarrow{BE}$ perpendicular to $\overline{OB}$.
9. $m\angle AOB + m\angle APB = 180^\circ$, that is, $m\angle AOB = 180^\circ - m\angle APB$
10. From step 5 and step 9, We have 180°

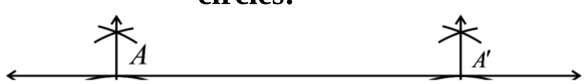
$-m\angle COA = 180^\circ - m\angle ABP \Rightarrow m\angle COA = m\angle APB$

$\Rightarrow m\angle APB = m\angle MNS \quad (\because m\angle COA = m\angle MNS)$

$\overleftrightarrow{AP}$ and $\overleftrightarrow{BP}$ are the required tangents meeting at the given $\angle MNS$.

Page # 232 **(A.B)**

13.3(iv-a) To draw direct or (external) common tangents to equal circles:



Given:

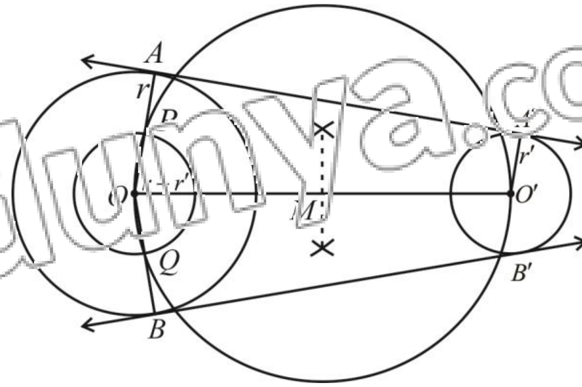
Two circles of equal radii with centres *O* and *O′* respectively.

Steps of Construction

1. Join the centres *O* and *O′*.
2. Draw diameter *AOB* of the first circle so that $\overline{AOB} \perp \overline{OO'}$.
3. Dow diameter $A'O'B'$ of the second circle so that $\overline{A'O'B'} \perp \overline{OO'}$
4. Draw $\overleftrightarrow{AA'}$ and $\overleftrightarrow{BB'}$ which are the required common tangents.

Page # 233 **(A.B)**

13.3(v-a) To Draw direct or (external) common tangents to (two) unequal circles:



Given: Two unequal circles with centres O, O' and radii $r.r'\,(r > r')$ respectively.

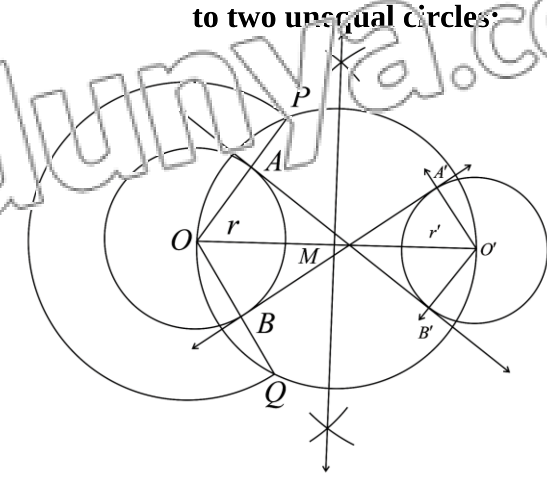
Steps of Construction

1. Join the centres *O* and *O′*.
2. On diameter $\overline{OO'}$, construct a new circle with centre *M*, the midpoint of $\overline{OO'}$.
3. Draw another circle with centre at *O* and radius $= r - r'$ cutting the circle with diameter $\overline{OO'}$ at *P* and *Q*.
4. Produce $\overline{OP}$ and $\overline{OQ}$ to meet the first circle at *A* and *B* respectively.
5. Draw $\overrightarrow{O'A'} \square \overline{OA}$ and $\overrightarrow{O'B'} \square \overline{OB}$
6. Join AA' and BB' which are the required direct common tangents.

Thus $\overleftrightarrow{AA'}$ and $\overleftrightarrow{BB'}$ are the required common tangents.

Page # 233 **(A.B)**

13.3(v-b) To draw to transverse or internal common tangents to two unequal circles:



Given:

Two unequal circles with centres O, O' and radii r, r' respectively.

Steps of Construction

1. Join the centres O and O' of the given circles.

2. Find mid-point M of $\overline{OO'}$.

3. On diameter OO', construct a new circle with centre M.

4. Draw another circle with centre at O and radius $= r + r$ intersecting the circle of diameter OO' at P and Q.

5. Join O with P and Q. $\overline{OP}$ and $\overline{OQ}$ meet the circle with radius r at A and B respectively.

6. Draw $\overrightarrow{O'B'} \parallel \overline{OA}$ and $\overrightarrow{O'A'} \parallel \overline{OB}$.

7. Join A with B' and A' with B. Thus $\overleftrightarrow{AB'}$ and $\overleftrightarrow{A'B}$ are the required transverse common tangents.

Page # 234 **(A.B)**

13.3(vi-a) To draw a tangent to two unequal touching circles:

Case–I

Given:

Two unequal touching circles with centres O and O'.

Steps of Construction

1. Join O with O' and produce $\overline{OO'}$ to meet the circles at the point A where these circles touch each other.

Fig. 1

2. Tangent is perpendicular to the line segment OA.

Draw perpendicular to $\overline{OA}$ at the point A which is the required tangent.

Page # 233 **(A.B)**

Case–II

Given:

Two unequal touching circles with centres O and O'

Steps of Construction

1. Join O with O'. $\overline{OO'}$ intersects the circles at the point B where these circles touch each other.

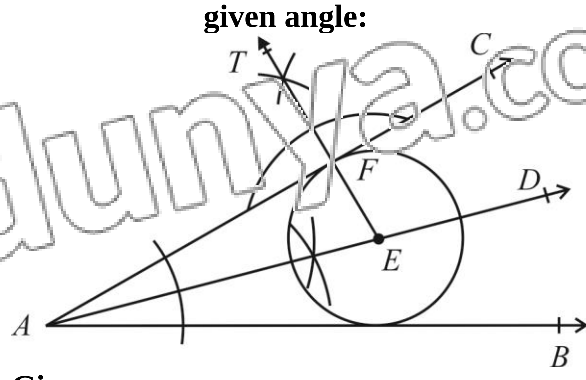
See Fig. 2

2. Tangent is perpendicular to line segment containing the centres of the circles.

3. Draw perpendicular to $\overline{OO'}$ at the point B which is the required tangent.

Page # 235 **(A.B)**

13.3 (vii-a) To draw a circle which touches both the arms of a given angle:



Given:

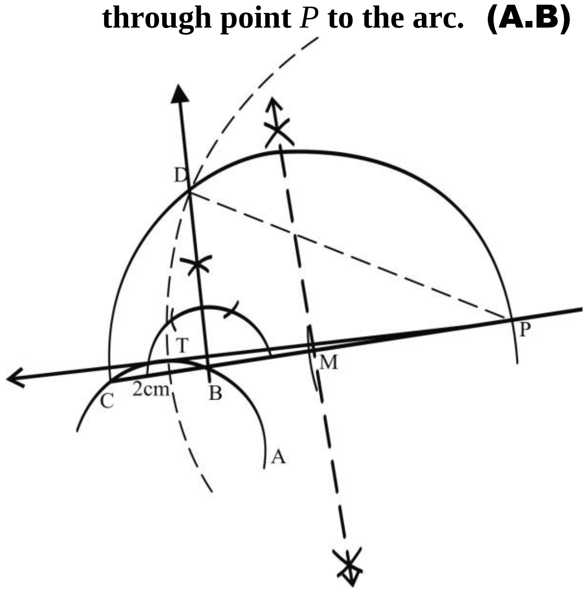
An angle $\angle BAC$.

Steps of Construction

1. Draw $\overrightarrow{AD}$ bisecting $\angle BAC$.
2. Take any point E on $\overrightarrow{AD}$
3. Draw $\overrightarrow{ET}$ perpendicular to $\overrightarrow{AC}$ intersecting $\overrightarrow{AC}$ at the point F.
4. Draw a circle with centre E and radius $m\overline{EF}$.
   This circle touches both the arms of $\angle BAC$.

Exercise 13.3

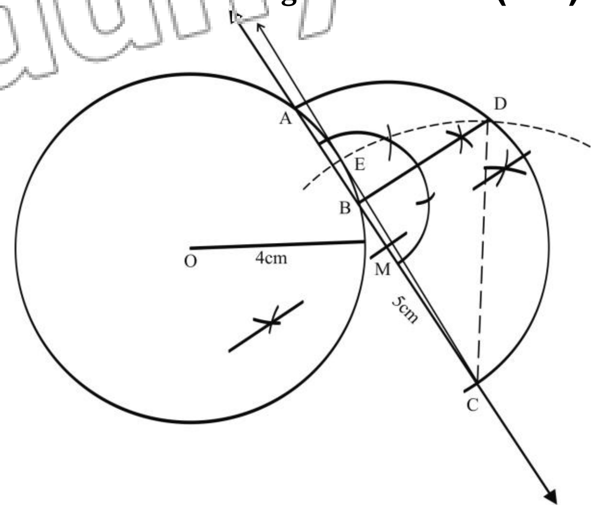
Q.1 **In an arc ABC the length of the chord $\left|\overline{BC}\right| = 2cm$. Draw a secant $\left|\overline{PBC}\right| = 8cm$, where P is the point outside the arc. Draw a tangent through point P to the arc. (A.B)**



Steps of Construction

1. Draw an arc.
2. Draw a secant line PBC, such that $mBC = 2cm$.
3. Find the mid-point M of $\overline{PC}$.
4. With centre M, draw a semi circle of radius PM.
5. Draw BD perpendicular to PC.
6. With centre P, draw an arc radius PD. It cut the circle at point T.
7. Join P to T produce it.
   Thus required tangent line is formed.

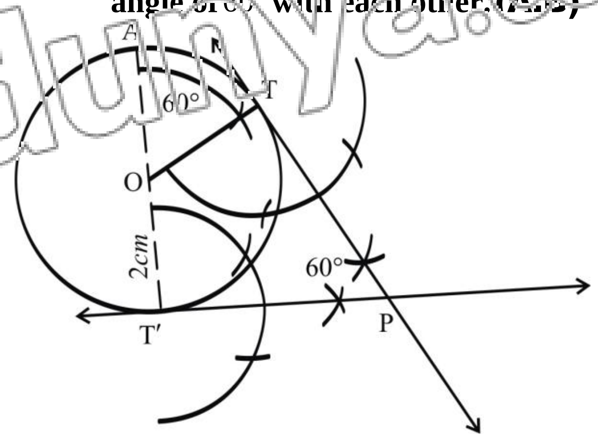
Q.2 **Construct a circle with diameter $8cm$. Indicate a point C, $5cm$ away from its circumference. Draw a tangent from point C to the circle without using its centre. (A.B)**



Steps of Construction

1. Draw a circle of radius $4cm$.
2. Take a point C, $5cm$ away from the circumference of circle.
3. Take a secant line CA.
4. Find the mid-point M of $\overline{CA}$.
5. With centre M, draw a semi circle of radius AM.
6. Draw BD perpendicular to AC.
7. With centre C, draw an arc of radius CD. It cut the circle at point E.
8. Join C to E produce it.
   Thus required tangent line is formed.

Q.3 **Construct a circle of radius $2cm$. Draw two tangents making an angle of $60°$ with each other. (A.B)**

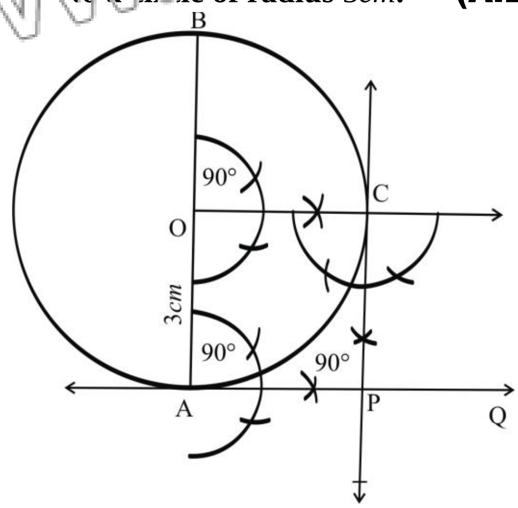


Steps of Construction

1. Draw a circle of radius $2cm$.
2. Draw diameter $\overline{AT'}$.
3. At point T', draw $\overleftrightarrow{PT'} \perp \overline{OT'}$.
4. At centre 'O', draw $\angle AOT = 60°$ to get radial segment $\overline{OT}$.
5. At point T, draw $\overleftrightarrow{PT} \perp \overline{OT}$

   Thus required tangents are formed.

Q.4 Draw two perpendicular tangents to a circle of radius $3cm$. **(A.B)**

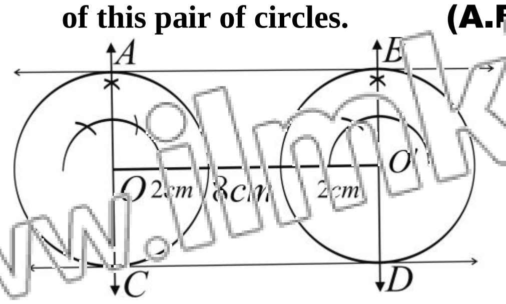


Steps of Construction

1. Draw a circle of radius $3cm$.
2. Draw diameter $\overline{AB}$.
3. At point A, draw $\overline{AP} \perp \overline{OB}$.
4. At centre 'O', draw $\angle AOC = 90°$ to get radial segment $\overline{OC}$.
5. At point C, draw $\overleftrightarrow{PC} \perp \overline{OC}$.

   Thus required tangents are formed.

Q.5 Two equal circles are at $8cm$ apart. Draw two direct common tangents of this pair of circles. **(A.B)**

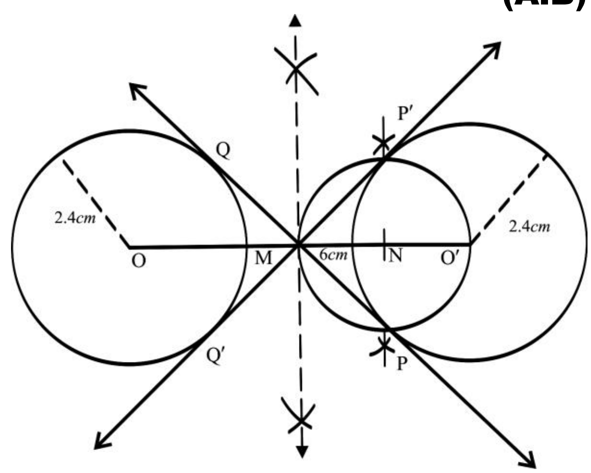


Steps of Construction

1. Draw a line segment $\overline{OO'}$ of measure $8cm$.
2. With centres O and O' draw circles of radii $2cm$.
3. At points O and O', draw perpendiculars $\overline{AC}$ and $\overline{BD}$, meeting the circles at points A, B, C and D.
4. Join A to B and C to D.

   Thus required tangents $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are formed.

Q.6 Draw two equal circles of each radius $2.4cm$. If the distance between their centres is $6cm$, then draw their transverse tangents. **(A.B)**

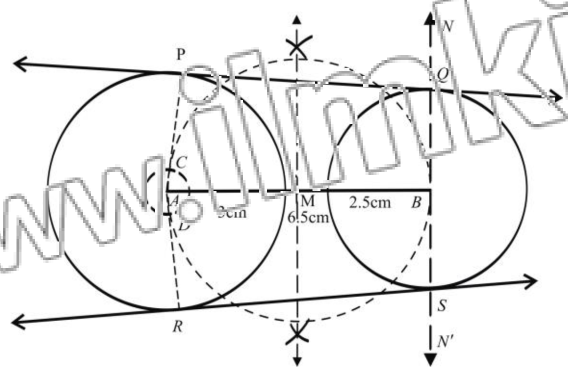


Steps of Construction

1. Draw a line segment $\overline{OO'} = 6cm$
2. Draw circles with centres as O and O' having radii $2.4cm$.
3. Find mid-point M of $\overline{OO'}$.
4. Find mid-point N of $\overline{MO'}$.
5. Taking point N as centre and radius equal to $m\overline{MN}$ draw a circle intersecting the circle with centre O' at points P and P'.
6. Draw a line through the points M and P touching the second circle at the point Q.
7. Draw a line through the points M and P' touching the second circle at the point Q'.

   Thus $\overleftrightarrow{PQ}$ are $\overleftrightarrow{P'Q'}$ the required transverse common tangents to the given circles.

Q.7 Draw two circles with radii $2.5cm$ **and** $3cm$**. If their centres are** $6.5m$ **apart, then draw two direct common tangents.** **(A.B)**



Steps of Construction

1. Take a line segment $\overline{AB}$ of measure $6.5cm$.

2. Draw two circles of radii 3 and 2.5 with centres at A and B respectively.

3. Taking centre at A, draw a circle of radius $3 - 2.5 = 0.5cm$.

4. Bisect the line segment AB at point M.

5. Taking centre at M and radius mAM, draw a circle intersecting the circle of radius $0.5cm$ at C and D.

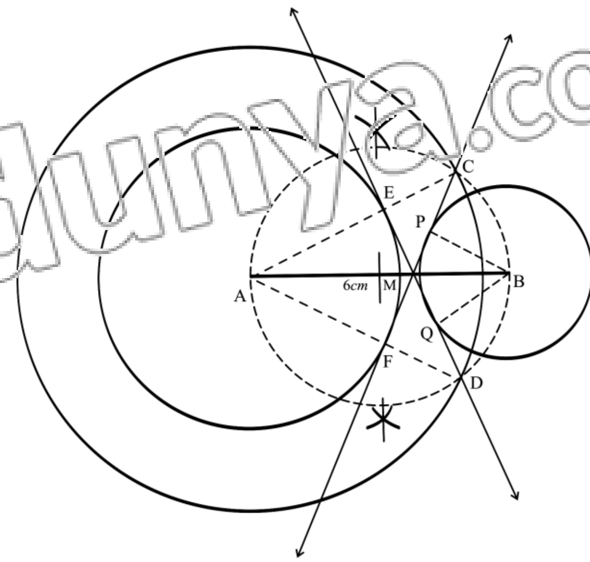
6. Join the point A with C and produce it to meet the circle with centre A at P. Also join A with D and produce it to meet the circle with centre A at R.

7. Draw $\overrightarrow{BQ}$ parallel to $\overline{AP}$.

8. Draw a line joining the P to Q and R to S.

Thus required direct common tangents are formed.

Q.8 Draw two circles with radii $3.5cm$ **and** $2cm$**. If their centres are** $6cm$ **apart, then draw two transverse common tangents.** **(A.B)**



Steps of Construction

1. Take a line segment $\overline{AB}$ of measure $6cm$.

2. Draw two circles of radii 3.5 and 2 with centres at A and B respectively.

3. Taking centre at A, draw a circle of radius $3.5 + 2 = 5.5cm$.

4. Bisect the line segment AB at point M.

5. Taking centre at M and radius mAM, draw a circle intersecting the circle of radius $5.5cm$ at C and D.

6. Join the point A with C. Also join A with D.

7. Draw $\overrightarrow{BQ}$ parallel to $\overline{AC}$.

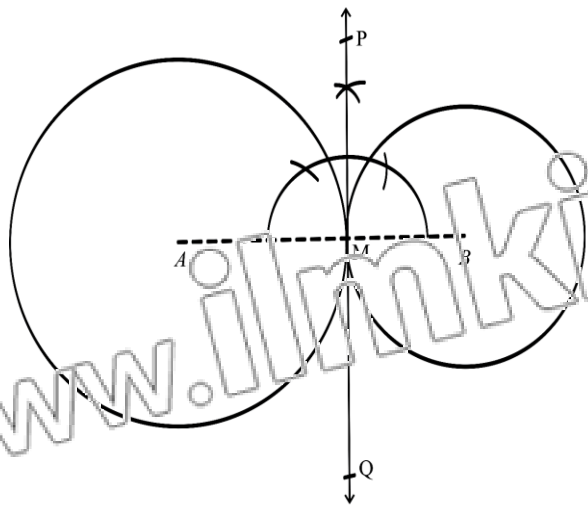
8. Draw a line joining the P to F and Q to E.

Thus required transverse common tangents are formed.

Q.9 Draw two common tangents to two touching circles of radii $2.5cm$ **and** $3.5cm$**.** **(A.B)**

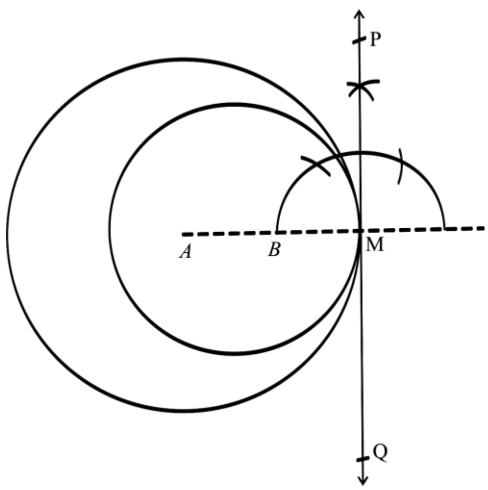
Case–I

Circles touch each other externally



Case–II

Circles touch each other internally



Steps of Construction

Case–I

(Circles touch each other externally)

1. Draw a line segment $\overline{AB}$ of measure $6cm$ (sum of radii).

2. With centre A and B, draw two circles of radius $3.5cm$ and $2.5cm$. They touch each other at point M.

3. At point M, draw $\overleftrightarrow{PQ} \perp \overline{AB}$.

Thus required common tangent is formed.

Case–II

(Circles touch each other internally)

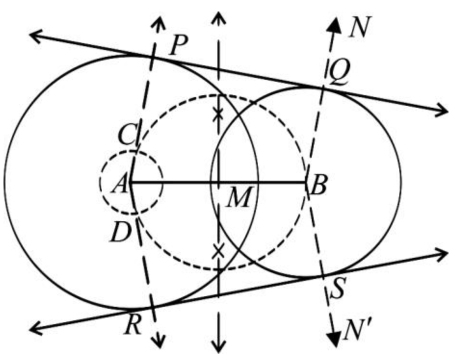
4. Draw a line segment $\overline{AB}$ of measure $1cm$ (difference of radii).

5. With centre A and B, draw two circles of radius $3.5cm$ and $2.5cm$. They touch each other at point M.

6. At point M, draw $\overleftrightarrow{PQ} \perp \overline{AM}$.

Thus required common tangent is formed.

Q.10 Draw two common tangents to two intersecting circle of radii $3cm$ and $4cm$. **(A.B)**



Steps of Construction

1. Take a line segment $\overline{AB}$ of measure $6cm$.

2. Draw two circles of radii 3 and 2.5 with centres at A and B respectively.

3. Taking centre at A, draw a circle of radius $3-2.5=0.5cm$.

4. Bisect the line segment AB at point M.

5. Taking centre at M and radius mAM, draw a circle intersecting the circle of radius 0.5cm at C and D.

6. Join the point A with C and produce it to meet the circle with centre A at P. Also join A with D and produce it to meet the circle with centre A at R.

7. Draw $\overrightarrow{BQ}$ parallel to AP.

8. Draw a line joining the P to Q and R to S.

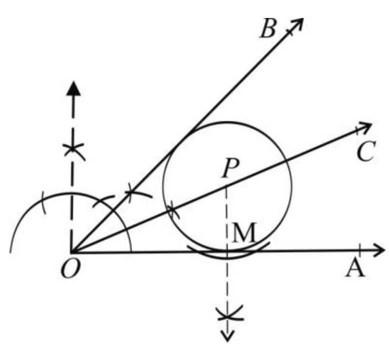
Thus required direct common tangents are formed.

Q.11 Draw circles which touches both the arms of angles **(A.B)**

(i) 45°

(ii) 60°

(i) 45°



Steps of Construction

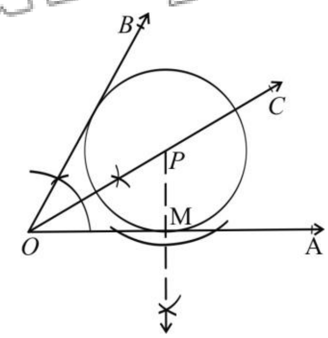
1. Draw an angle AOB of measure $45°$.

2. Draw $\overrightarrow{OC}$ the bisector of $\angle AOB$.

3. Take any point P on $\overrightarrow{OC}$. From point P draw $\overline{PM} \perp \overrightarrow{OA}$.

4. With centre P, draw a circle of radius $\overline{PM}$ which touches both arms of angle $45°$.

Thus required circle is formed. $60°$



Steps of Construction

5. Draw an angle AOB of measure $60°$.

6. Draw $\overrightarrow{OC}$ the bisector of $\angle AOB$.

7. Take any point P on $\overrightarrow{OC}$. From point P draw $\overline{PM} \perp \overrightarrow{OA}$.

8. With centre P, draw a circle of radius $\overline{PM}$ which touches both arms of angle $60°$.

Thus required circle is formed.

Miscellaneous Exercise 13

Q.1 Multiple choice questions

Four possible answers are given for the following question. Tick (✓) the correct answer.

(i) The circumference of a circle is called **(K.B)**
(a) Chord **(b)** Segment **(c)** Boundary

(ii) A line intersecting a circle is called **(K.B)**
(a) Tangent **(b)** Secant **(c)** Chord

(iii) The portion of a circle between two radii and an arc is called **(K.B)**
(a) Sector **(b)** Segment **(c)** Chord

(iv) Angle inscribed in a semi-circle is **(K.B)**
(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$ **(c)** $\frac{\pi}{4}$

(v) The length of the diameter of a circle is how many times the radius of the circle **(K.B)**
(a) 1 **(b)** 2 **(c)** 3

(vi) The tangent and radius of a circle at the point of contact are **(K.B)**
(a) Parallel **(b)** Not perpendicular **(c)** Perpendicular

(vii) Circles having three points in common **(K.B)**
(a) Over lapping **(b)** Not coincide **(c)** Coincide

(viii) If two circles touch each other, their centres and point of contact are **(K.B)**
(a) Coincident **(b)** Non-collinear **(c)** Collinear

(ix) The measure of the external angle of a regular hexagon is **(GRW 2014)** **(K.B)**
(a) $\frac{\pi}{3}$ **(b)** $\frac{\pi}{4}$ **(c)** $\frac{\pi}{6}$

(x) If the incentre and circumcentre of a triangle coincide, the triangle is **(K.B)**
(a) An isosceles **(b)** A right triangle **(c)** An equilateral

(xi) The measure of the external angle of a regular octagon is **(K.B)**
(LHR 2015, FSD 2018)
(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{6}$ **(c)** $\frac{\pi}{8}$

(xii) Tangents drawn at the end points of the diameter of a circle are **(K.B)**
(a) Parallel **(b)** Perpendicular **(c)** Intersecting

(xiii) The length of two transverse tangents to a pair of circles are
(a) Unequal **(b)** Equal **(c)** Overlapping

(xiv) How many tangents can be drawn from a point outside the circle? **(K.B)**
(a) 1 **(b)** 2 **(c)** 3

(xv) If the distance between the centres of two circles is equal to the sum of their radii, then the circles will **(K.B)**
(a) Intersect **(b)** Do not intersect **(c)** Touch each other externally

(xvi) If the two circles touches externally, then the distance between their centers is equal to the **(K.B)**
(a) Difference of their radii **(b)** Sum of their radii
(c) Product of their radii

(xvii) How many common tangents can be drawn for two touching circles? **(K.B)**
(a) 2 **(b)** 3 **(c)** 4

(xviii) How many common tangents can be drawn for two disjoint circles **(K.B)**
(a) 2 **(b)** 3 **(c)** 4

ANSWER KEY

i	c	Iv	a	vii	d	x	c	xiii	b	xvi	b
ii	b	V	b	viii	c	xi	a	xiv	b	xvii	b
iii	a	Vi	c	ix	a	xii	a	xv	c	xviii	c

Q.2 Define and draw the following geometric figures:

(i) Define and draw the following geometric figures: **(A.B)**

(a) The segment of a circle.

Ans:

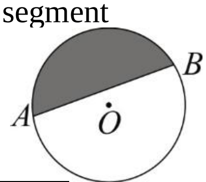
Segment of a Circle **(K.B)**

"A chord divides a circular region in two parts called segment of a circle".

In the given figure, $\overline{AB}$ divides the circle into two segments.

(i) Major segment

(ii) Minor segment



Minor Segment **(K.B)**

"Circular region bounded by a chord and minor arc is called minor segment". In the given figure, shaded part is minor segment.

Major Segment **(K.B)**

"Circular region bounded by a chord and major arc is called major segment". In the given figure, non shaded part is major segment.

(b) The tangent to a circle. **(K.B)**

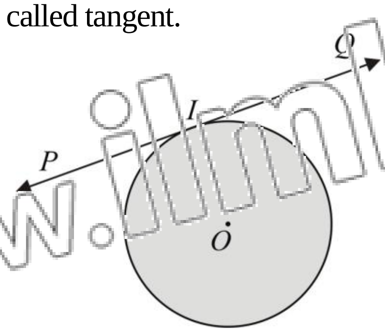
Ans:

Tangent Line

A line which has one point common with a circle is called tangent line.

Or

A straight line which touches the circumference at one point only is called tangent.



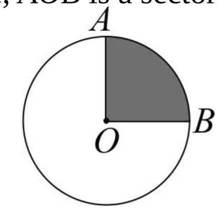
In the figure $\overleftrightarrow{PIQ}$ is tangent line.

(c) The sector of a circle. **(K.B)**

Ans:

Sector of a Circle

"Circular region bounded by an arc and its two corresponding radial segments is called sector in the given figure, AOB is a sector of the circle.



(d) The inscribed circle. **(K.B)**

Ans: See definition page # 304

(e) The circumscribed circle. **(K.B)**

Ans: See definition page # 304

(f) The escribed circle. **(K.B)**

Ans: See definition page # 304

(ii) The length of each side of a regular octagon is 3*cm*. Measure its perimeter. **(K.B)**

Ans: Length of each side of octagon $= 3cm$

Sides of a octagon = 8

Perimeter of Octagon = 8 × 3cm

$\therefore$ Perimeter $= n \times l,$

$= 24$cm

(iii) Write down the formula for finding the angle subtended by the side of a n-sided polygon at the centre of the circle. **(K.B)**

Ans: Formula for finding the angle subtended by the side of a n-sided polygon at the centre of the circle

$$= \frac{360^o}{n}$$

(iv) The length of the side of a regular pentagon is 5*cm* what is its perimeter? **(K.B)**

Ans: Length of each side of octagon $= 5cm$

Sides of a pentagon= 5

Perimeter of pentagon= 5 × 5cm

$\therefore$ Perimeter $= n \times l,$

$= 25cm$

Q.3 Fill in the blanks

(i) The boundary of a circle is called __________. **(K.B)**
(ii) The circumference of a circle is called __________ of the circle. **(K.B)**
(iii) The line joining the two points of circle is called __________. **(K.B)**
(iv) The point of intersection of perpendicular bisectors of two non-parallel chords of a circle is called the __________. **(K.B)**
(v) Circles having three points in common will __________. **(K.B)**
(vi) The distance of a point inside the circle from its centre is ______ than the radius. **(K.B)**
(vii) The distance of a point outside the circle from its centre is ______ than the radius. **(K.B)**
(viii) A circle has only ______ centre.
(ix) One and only one circle can be drawn through three ______ points. **(K.B)**
(x) Angle inscribed in a semi-circle is a ______ angle.
(xi) If two circles touch each other, the point of ______ and their ______ are collinear. **(K.B)**
(xii) If two circles touch each other, their point of contact and centres are __________. **(K.B)**
(xiii) From a point outside the circle ______ tangents can be drawn. **(K.B)**
(xiv) A tangent is ______ to the radius of a circle at its point of contact. **(K.B)**
(xv) The straight line drawn $\perp$ to the radius of a circle is called the ______ to the circle. **(K.B)**
(xvi) Two circles can not cut each other at more than ______ points. **(K.B)**
(xvii) The $\perp$-bisector of a chord of circle passes through the __________. **(K.B)**
(xviii) The length of two direct common tangents to two circles are______ to each other. **(K.B)**
(xix) The length of two transverse common tangents to two circles are ______ to each other. **(K.B)**
(xx) If the in-centre and circum-centre of a triangle coincide the triangle is__________.**(K.B)**
(xxi) Two intersecting circles are not __________. **(K.B)**
(xxii) The centre of an inscribed circle is called __________. **(K.B)**
(xxiii) The centre of a circumscribed circle is called __________. **(K.B)**
(xxiv) The radius of an inscribed circle is called __________. **(K.B)**
(xxv) The radius of a circumscribed circle is called __________. **(K.B)**

ANSWER KEY

(i) Circumference
(ii) Boundary
(iii) Chord
(iv) Centre
(v) Coincide
(vi) Less
(vii) Greater
(viii) One
(ix) Non-collinear
(x) Right
(xi) Contact, centres
(xii) Collinear
(xiii) Two
(xiv) Perpendicular
(xv) Tangent
(xvi) Two
(xvii) Centre
(xviii) Equal
(xix) Equal
(xx) Equilateral
(xxi) Concentric
(xxii) Incentre
(xxiii) Circumcentre
(xxiv) In-radius
(xxv) Circum-radius

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 A line intersecting a circle is called:
(A) Tangent **(B)** Secant
(C) Chord **(D)** Radius

2 Angle inscribed in a semi-circle is:
(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$ **(D)** $\frac{\pi}{6}$

3 If two circles touch each other, their centres and point of contact are:
(A) Coincident **(B)** Non-collinear
(C) Collinear **(D)** None of these

4 The measure of the external angle of a regular hexagon is:
(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{4}$
(C) $\frac{\pi}{6}$ **(D)** $\frac{\pi}{2}$

5 If the incentre and circumcentre of a triangle coincide, the triangle is:
(A) An isoscenes **(B)** A right triangle
(C) An equilateral **(D)** Scalene

6 If the distance between the centers of two circles is equal to the sum of their radii, then the circles will:
(A) Intersect **(B)** Do not intersect
(C) Touch each other externally **(D)** None of these

7 How many common tangents can be drawn for two touching circles?
(A) 2 **(B)** 3
(C) 4 **(D)** 1

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(1) The length of the side of a regular pentagon is 5cm what is its perimeter?

(2) If $|\overline{AB}|=3cm$ and $|\overline{BC}|=4cm$ are the lengths of two chords of an arc, then locate the centre of the arc.

(3) Draw circles which touches both the arms of angles 60°.

(4) Define and draw the following geometric figure of the inscribed circle.

(i) Draw two perpendicular tangents to a circle of radius 3cm.

Q.3 Answer the following Questions in detail. **(4+4=8)**

(a) Escribe a circle opposite to vertex A to a triangle ABC with sides $|AB|=6cm, |BC|=4cm, |CA|=3cm$. Find its radius also.

(b) Draw two circles with radii 3.5 cm and 2 cm of their centers are 6 cm apart then draw two transverse common tangents.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.