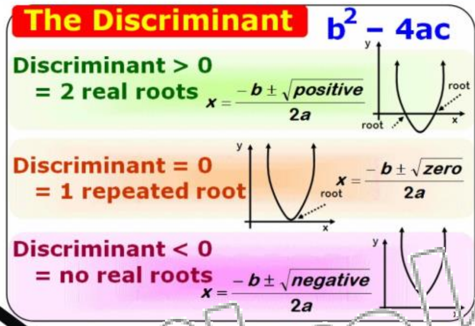


# UNIT 2

# THEORY OF QUADRATIC EQUATIONS

**Discriminant** (RWP 2018) **(U.B + K.B)**

"For a standard quadratic equation $ax^2+bx+c=0$, the value of the expression $b^2-4ac$ is called discriminant."

It is used to find the nature of roots without solving the equation.

**Nature or Characteristics of the Roots** **(U.B + K.B)**

Nature of a quadratic equation $ax^2+bx+c=0$, when $a,b,c\in Q$ and $a\neq 0$ as:

**(i)** If $b^2-4ac=0$, then the roots are rational (real) and equal.

**(ii)** If $b^2-4ac<0$, then the roots are complex conjugate or imaginary.

**(iii)** If $b^2-4ac>0$, and is a perfect square, then the roots are rational (real) and unequal.

**(iv)** If $b^2-4ac>0$, and is not a perfect square, the roots are irrational (real) and unequal.

**Note** **(K.B)**

If given polynomial expression is a perfect square then discriminant is 0.

**Example 2: (Page # 19)**

**Using discriminant, find the nature of the roots of the following equation and verify the result by solving the equation.**

$x^2-5x+5=0$

**(LHR 2015, GRW 2016, 17, SWL 2017 RWP 2015, D.G.K 2017)**

**Solution:**

$x^2-5x+5=0$

Here $a=1,\ b=-5,\ c=5$

Discriminant $= b^2-4ac$

$=(-5)^2-4(1)(5)$

$=25-20=5$

As discriminant > 0 but not perfect square, Roots are irrational (real) and unequal.

**Verification:**

Solving the equation by using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(5)}}{2(1)}$$

$$x=\frac{5\pm\sqrt{5}}{2}$$

Evidently, Roots are irrational (real) and unequal.

**Example 2: (Page # 21)**

Find $k$, if the roots of the equation $(k+3)x^2-2(k+1)x-(k+1)=0$ are equal, if $k\neq -3$ **(A.B)**

**Solution:**

$(k+3)x^2-2(k+1)x-(k+1)=0$

Here

$a=k+3,\ b=-2(k+1),\ c=-(k+1)$

As roots are equal, discriminant is zero

$\Rightarrow \text{Disc}=b^2-4ac=0$

$[-2(k+1)]^2-4(k+3)[-(k+1)]=0$

$4(k+1)^2+4(k+3)(k+1)=0$

$4(k+1)\left[(k+1)+(k+3)\right]=0$

$4(k+1)\left(2k+4\right)=0$

Either

$k+1=0$ or $2k+4=0 \quad \because 4\neq 0$

$k=-1$ or $2k=-4$

$k=-2$

**Thus, roots will be equal if** $k=-1,-2$

## Exercise 2.1

**Q.1 Find the discriminant of the following given quadratic equations:**

**Solution:**

**(i)** $2x^2+3x-1=0$ **(A.B)**

**(GRW 2017, FSD 2016, MTN 2014, D.G.K 2016)**

**By comparing given equation with** $ax^2+bx+c=0$**, we get**

$a=2,\ b=3,\ c=-1$

Disc $= b^2-4ac$

$= (3)^2-4(2)(-1)$

$= 9+8$

$= 17$

**(ii)** $6x^2-8x+3=0$ **(A.B)**

**(LHR 2016, SWL 2016, D.G.K 2015, 17)**

**By comparing given equation with** $ax^2+bx+c=0$**, we get**

$a=6,\ b=-8,\ c=3$

Disc $= b^2-4ac$

$= (-8)^2-4(6)(3)$

$= 64-72$

$= -8$

**(iii)** $9x^2-30x+25=0$ **(A.B)**

**(LHR 2017, MTN 2015)**

**By comparing given equation with** $ax^2+bx+c=0$**, we get**

$a=9,\ b=-30,\ c=25$

Disc $= (-30)^2-4(9)(25)$

$= 900-900$

$= 0$

**(iv)** $4x^2-7x-2=0$ **(A.B)**

**(GRW 2014, SGD 2017, MTN 2016)**

**By comparing given equation with** $ax^2+bx+c=0$**, we get**

$a=4,\ b=-7,\ c=-2$

Disc $= b^2-4ac$

$=(-7)^2-4(4)(-2)$

$= 49+32$

$= 81$

**Q.2 Find the nature of the roots of the following given quadratic equations and verify the result by solving the equations.**

**Solution:**

**(i)** $x^2-23x+120=0$ **(A.B)**

**By comparing given equation with** $ax^2+bx+c=0$**, we get**

$a=1, b=-23, c=120$

Disc= $b^2-4ac$

$= (-23)^2-4(1)(120)$

$= 529-480$

$= 49$

$= 7^2$

Since disc > 0 and perfect square, roots are rational (real) and unequal.

**Verification:**

$x^2-23x+120=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-23)\pm\sqrt{(-23)^2-4(1)(120)}}{2(1)}$$

$$=\frac{23\pm\sqrt{7^2}}{2}$$

$$x=\frac{23\pm 7}{2}$$

Either

$x=\dfrac{23-7}{2}$ or $x=\dfrac{23+7}{2}$

$=\dfrac{16}{2}$ or $=\dfrac{30}{2}$

$x=8$ $x=15$

Hence roots are rational and unequal.

**(ii)** $2x^2+3x+7=0$ **(A.B)**

**By comparing given equation with** $ax^2+bx+c=0$**, we get**

$a=2, b=3, c=7$

Disc $= b^2-4ac$

$= (3)^2-4(2)(7)$

$= 9-56$

$= -47$

Since disc < 0, roots are complex and imaginary.

**Verification:**

$$2x^2+3x+7=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-3\pm\sqrt{(3)^2-4(2)(7)}}{2(2)}$$

$$=\frac{-3\pm\sqrt{9-56}}{4}$$

$$=\frac{-3\pm\sqrt{-47}}{4}$$

$$=\frac{-3\pm\sqrt{47}i}{4}$$

Hence roots are complex/imaginary and unequal.

**(iii)** $16x^2-24x+9=0$ **(A.B)**

**By comparing given equation with** $ax^2+bx+c=0$**, we get**

$a=16,\ b=-24,\ c=9$

Disc $= b^2-4ac$

$= (-24)^2-4(16)(9)$

$= 576-576$

$= 0$

Since disc = 0, roots are rational (real) and equal.

**Verification:**

$$16x^2-24x+9=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-24)\pm\sqrt{(-24)^2-4(16)(9)}}{2(16)}$$

$$x=\frac{24\pm\sqrt{0}}{32}$$

$$x=\frac{24\pm0}{32}$$

Either

$x=\frac{24+0}{32}$ or $x=\frac{24-0}{32}$

$x=\frac{24}{32}$ $\qquad$ $x=\frac{24}{32}$

Hence roots are rational and equal.

**(iv)** $3x^2+7x-13=0$ **(A.B)**

**By comparing given equation with** $ax^2+bx+c=0$**, we get**

$a=3, b=7, c=-13$

Disc $= b^2-4ac$

$= (7)^2-4(3)(-13)$

$= 49 + 156$

$= 205$

Since disc > 0, but not a perfect sq. roots are irrational and unequal.

**Verification:**

$$3x^2+7x-13=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-7\pm\sqrt{(7)^2-4(3)(-13)}}{2(3)}$$

$$=\frac{-7\pm\sqrt{49+56}}{6}$$

$$=\frac{-7\pm\sqrt{205}}{6}$$

Either

$x=\frac{-7-\sqrt{205}}{6}$ or $x=\frac{-7+\sqrt{205}}{6}$

Hence roots are irrational and unequal.

**Q.3 For what value of $k$, the expression $k^2x^2+2(k+1)x+4$ is perfect square.** **(A.B + K.B)**

**Solution:**

$$k^2x^2+2(k+1)x+4=0$$

**By comparing given equation with** $ax^2+bx+c=0$**, we get**

$a=k^2,\ b=2(k+1),\ c=4$

Discriminant $= b^2-4ac$

$=[2(k+1)]^2-4(k^2)(4)$

$=4(k+1)^2-16k^2$

$=4(k^2+2k+1)-16k^2$

$=4k^2+8k+4-16k^2$

$=4+8k-12k^2$

$=4(1+2k-3k^2)$

As expression is a perfect square, the discriminant = 0

$\Rightarrow 4\left(1+2k-3k^2\right)=0$

$1+2k-3k^2=0 \quad \because\ 4\neq 0$

$1+3k-k-3k^2=0$

$1\left(1+3k\right)-k\left(1+3k\right)=0$

$\left(1+3k\right)\left(1-k\right)=0$

Either

$1+3k=0$ or $1-k=0$

$3k=-1$ $\quad k=1$

$k=-\frac{1}{3}$

**Result**

$k=1,\ -\frac{1}{3}$

**Q.4 Find the value of *k*, if the roots of the following equations are equal.**

**(A.B + K.B)**

**Solution:**

**(i)** $(2k-1)x^2+3kx+3=0$

Here $a=2k-1, b=3k, c=3$

Disc $= b^2-4ac$

$=(3k)^2-4(2k-1)(3)$

$=9k^2-24k+12$

Since roots are equal,

Disc = 0

$\Rightarrow 9k^2-24k+12=0$

$3(3k^2-8k+4)=0$

$3k^2-8k+4=0 \quad \because\ 3\neq 0$

$3k^2-6k-2k+4=0$

$3k(k-2)-2(k-2)=0$

$(k-2)(3k-2)=0$

Either

$k-2=0$ or $3k-2=0$

$\Rightarrow k=2$ or $3k=2$

$k=\frac{2}{3}$

**Result:**

$k=2,\ \frac{2}{3}$

**(ii)** $x^2+2(k+2)x+(3k+4)=0$

Here $a=1, b=2(k+2), c=3k+4$

Disc $= b^2-4ac$

$=\left[2(k+2)\right]^2-4(1)(3k+4)$

$=4(k+2)^2-4(3k+4)$

$=4(k^2+4k+4)-4(3k+4)$

$=4\left[k^2+4k+4-3k-4\right]$

$=4\left[k^2+k\right]$

$=4k(k+1)$

Since roots are equal, disc = 0

$4k\left(k+1\right)=0$

Either

$4k=0$ *or* $k+1=0$

$k=0$ *or* $k=-1$

**Result:**

$k=0,-1$

**(iii)** $(3k+2)x^2-5(k+1)x+(2k+3)=0$

**(A.B + K.B)**

Here

$a=3k+2, b=-5\left(k+1\right), c=2k+3$

Disc $= b^2-4ac$

$=\left[-5(k+1)\right]^2 - 4\ (3k+2)\ (2k+3)$

$=25\left(k+1\right)^2-4\left(6k^2+9k+4k+6\right)$

$=25\left(k^2+2k+1\right)-4\left(6k^2+13k+6\right)$

$=25k^2+50k+25-24k^2-52k-24$

$=k^2-2k+1$

Since roots are equal, disc = 0

$\Rightarrow k^2-2k+1=0$

$\left(k-1\right)^2=0$

Taking square root on both sides

$k-1=0$

$k=1$

**Result:**

$k=1$

**Q.5 Show that the equation $x^2+(mx+c)^2=a^2$ has equal roots, if $c^2=a^2(1+m^2)$** **(A.B + K.B)**

**Proof** **(FSD 2016)**

$x^2+(mx+c)^2=a^2$

$x^2+m^2x^2+2mcx+c^2=a^2$

$(1+m^2)x^2+2mcx+c^2-a^2=0$

Here

$A=1+m^2, B=2mc, C=c^2-a^2$

Disc $=B^2-4AC$

$=(2mc)^2-4(1+m^2)(c^2-a^2)$

$=4m^2c^2-4(c^2-a^2+m^2c^2-a^2m^2)$

$=4m^2c^2-4c^2+4a^2-4m^2c^2+4a^2m^2$

$=-4c^2+4a^2+4a^2m^2$

Disc $=-4\left[a^2(1+m^2)\right]+4a^2+4m^2$

$\left(\because c^2=a^2(1+m^2)\right)$

$=-4\left[a^2+a^2m^2\right]+4a^2+4a^2m^2$

$=-4a^2-4a^2m^2+4a^2-4a^2m^2$

$=0$

Disc=0

$\therefore$ Roots are equal

Hence roots are equal, if $c^2=a^2(1+m^2)$

**Proved**

**Q.6 Find the condition that the roots of the equation $(mx+c)^2-4ax=0$ are equal.** **(A.B + K.B + U.B)**

**Solution:**

$(mx+c)^2-4ax=0$

$m^2x^2+2mcx-4ax+c^2=0$

$m^2x^2+(2mc-4a)x+c^2=0$

Here $A=m^2, B=2mc-4a, C=c^2$

Disc $=B^2-4AC$

$=(2mc-4a)^2-4(m^2)(c^2)$

$=4m^2c^2-16amc+16a^2-4m^2c^2$

$=-16amc+16a^2$

$=-16a(mc-a)$

Since roots are equal, disc = 0

$\Rightarrow -16a(mc-a)=0$

Either

$mc-a=0$ or $-16a=0$

$a=mc$ $\quad a=0$

Roots of the given equation are equal

If $a=mc$ or $a=0$

**Q.7 If the roots of the equation. $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ are equal, then $a=0$ or $a^3+b^3+c^3=3abc$.** **(A.B + K.B + U.B)**

**Proof**

$(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$

Here

$A=c^2-ab, B=-2(a^2-bc), C=b^2-ac$

Disc = $B^2-4AC$

$=\left[-2(a^2-bc)\right]^2-4(c^2-ab)(b^2-ac)$

$=4(a^2-bc)^2-4(b^2c^2-ac^3-ab^3+a^2bc)$

$=4(a^4-2a^2bc+b^2c^2)-4(b^2c^2-ac^3-ab^3+a^2bc)$

$=4(a^4-2a^2bc+b^2c^2-b^2c^2+ac^3+ab^3-a^2bc)$

$=4(a^4+ab^3+ac^3-3a^2bc)$

$=4a(a^3+b^3+c^3-3abc)$

Since roots are equal, disc = 0

$\Rightarrow 4a(a^3+b^3+c^3-abc)=0$

Either

$4a=0$ *or* $a^3+b^3+c^3-3abc=0$

$a=0$ $\quad a^3+b^3+c^3=3abc$

Roots are equal if either

$a=0$ *or* $a^3+b^3+c^3=3abc$

**Q.8 Show that the roots of the following equations are rational.**

**Proof.** **(A.B + K.B + U.B)**

**(i)** $a(b-c)x^2+b(c-a)x+c(a-b)=0$

Here

$A=a(b-c),\ B=b(c-a),\ C=c(a-b)$

Discriminant = $B^2-4AC$

$=\left[b(c-a)\right]^2-4\left[a(b-c)\right]\left[c(a-b)\right]$

$=b^2(c-a)^2-4ac(b-c)(a-b)$

$$= b^2\left(c^2 - 2ac + a^2\right) - 4ac\left(ab - b^2 - ac + bc\right)$$
$$= b^2c^2 - 2ab^2c + a^2b^2 - 4a^2bc + 4ab^2c + 4a^2c^2 - 4abc^2$$
$$= b^2c^2 + a^2b^2 + 4a^2c^2 + 2ab^2c - 4a^2bc - 4abc^2$$
$$= (bc)^2 + (ab)^2 + (-2ac)^2 + 2(bc)(ab)$$
$$+2(ab)(-2ac) + 2(-2ac)(bc)$$
$$\therefore\ a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$$
$$= (bc + ab - 2ac)^2$$

As discriminate > 0 and perfect square, roots are rational.

**(ii)** $(a+2b)x^2 + 2(a+b+c)x + (a+2c) = 0$

Here

$A = a + 2b, \mathrm{B} = 2(a+b+c)$ , $\mathrm{C} = a + 2c$

Disc= $B^2 - 4AC$

$$= [2(a + b + c)]^2 - 4(a + 2b)(a + 2c)$$
$$= 4(a+b+c)^2 - 4\left(a^2 + 2ac + 2ab + 4bc\right)$$
$$= 4\left(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca\right)$$
$$-4\left(a^2 + 2ac + 2ab + 4bc\right)$$
$$= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$
$$-a^2 - 2ac - 2ab - 4bc)$$
$$= 4\left(b^2 + c^2 - 2bc\right)$$
$$= 4(b-c)^2$$
$$= \left[2(b-c)\right]^2 > 0$$

Since disc is perfect square roots are rational.

**Q.9 For all values of k, prove that the roots of the equation.**

$x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0, k \neq 0$ **are real.**

**Proof:** **(A.B + U.B)**

Here

$a = 1,\ b = -2\left(k + \frac{1}{k}\right),\ c = 3$

Discriminat = $b^2 - 4ac$

$$= \left[-2\left(k + \frac{1}{k}\right)\right]^2 - 4(1)(3)$$
$$= 4\left(k + \frac{1}{k}\right)^2 - 12$$
$$= 4\left(k^2 + \frac{1}{k^2} + 2\right) - 12$$
$$= 4\left[\left(k^2 + \frac{1}{k^2} + 2\right) - 3\right]$$
$$= 4\left[k^2 + \frac{1}{k^2} - 1\right]$$
$$= 4\left[k^2 + \frac{1}{k^2} - 2 + 1\right]$$
$$= 4\left[\left(k - \frac{1}{k}\right)^2 + 1\right]$$
$$> 0$$

As disc. > 0, roots are real.

**Q.10 Show that the roots of the equation** $(b-c)x^2 + (c-a)x + (a-b) = 0$ **are real.** **(A.B + U.B + K.B)**

**Proof:**

$(b-c)x^2 + (c-a)x + (a-b) = 0$

Here $A = b - c, \mathrm{B} = c - a, \mathrm{C} = a - b$

Disc = $B^2 - 4AC$

$$= (c-a)^2 - 4(b-c)(a-b)$$
$$= c^2 - 2ac + a^2 - 4\left(ab - b^2 - ac + bc\right)$$
$$= c^2 - 2ac + a^2 - 4ab + 4b^2 + 4ac - 4bc$$
$$= c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc$$
$$= (c)^2 + (a)^2 + (2b)^2 + 2(c)(a) + 2(a)(-2b) + 2(-2b)(c)$$
$$\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$$
$$= (c + a - 2b)^2 > 0$$

Hence roots are real.

**Derivation of Cube Roots of Unity**

**(A.B + K.B)**

Let $x = \sqrt[3]{1}$

Taking cube on both sides

$$x^3 = 1$$
$$x^3 - 1 = 0$$
$$(x)^3 - (1)^3 = 0$$

$$(x-1)(x^2+x+1)=0$$
$$(x-1)(x^2+x+1)=0$$

Either

$x-1=0$ or $x^2+x+1=0$

$x=1$

Here $a=1, b=1, c=1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{1-4}}{2}$$
$$=\frac{-1\pm\sqrt{-3}}{2}=\frac{-1\pm\sqrt{3}i}{2} \quad \because \sqrt{-1}=i$$

Either

$$x=\frac{-1+\sqrt{3}i}{2}, \qquad x=\frac{-1-\sqrt{3}i}{2}$$
$$=\omega \qquad\qquad =\omega^2$$

$\therefore$ **Cube root of unity are** $1, \omega, \omega^2$

**Note** **(K.B)**

We can write anyone complex cube root as $\omega$ (Omega), then other will be $\omega^2$.

## Properties of Cube Root of Unity

**(i) Proving that each complex cube root of unity is the square of other.**

i.e. $$\left(\frac{-1+\sqrt{-3}}{2}\right)^2=\frac{-1-\sqrt{-3}}{2}$$

and $$\left(\frac{-1-\sqrt{-3}}{2}\right)^2=\frac{-1+\sqrt{-3}}{2}$$

**(LHR 2014, GRW 2017, FSD 2016, 17, SGD 2015, 16, BWP 2017, MTN 2017)**

**(K.B + U.B + A.B)**

**Proof:**

We have to prove

(i) $$\left(\frac{-1+\sqrt{-3}}{2}\right)^2=\frac{-1-\sqrt{-3}}{2}$$

Consider

$$\left(\frac{-1+\sqrt{-3}}{2}\right)^2$$
$$=\frac{(-1)^2+\left(\sqrt{-3}\right)^2+2(-1)\left(\sqrt{-3}\right)}{4}$$
$$=\frac{1+(-3)-2\sqrt{-3}}{4}$$
$$=\frac{1-3-2\sqrt{-3}}{4}$$
$$=\frac{-2-2\sqrt{-3}}{4}$$
$$=\frac{2\left(-1-\sqrt{-3}\right)}{4}$$
$$=\frac{-1-\sqrt{-3}}{2}$$

(ii) $$\left(\frac{-1-\sqrt{-3}}{2}\right)^2=\frac{-1+\sqrt{-3}}{2}$$

Now consider

$$\left(\frac{-1-\sqrt{-3}}{2}\right)^2$$
$$=\frac{(1)^2+\left(\sqrt{-3}\right)^2-2(-1)\left(-\sqrt{3}\right)}{4}$$
$$=\frac{1+(-3)+2\sqrt{-3}}{4}$$
$$=\frac{1-3+2\sqrt{-3}}{4}$$
$$=\frac{-2+2\sqrt{-3}}{4}$$
$$=\frac{2\left(-1+\sqrt{-3}\right)}{4}$$
$$=\frac{-1+\sqrt{-3}}{2}$$

Thus, each of the complex cube roots of unity is the square of the other.

**(ii) Proving that product of three cube root of unity is one.**

i.e. $1.\omega.\omega^2 = 1$ **(K.B + U.B)**

**Proof:**

$\text{L.H.S} = 1.\omega.\omega^2$

By putting the values

$$= (1)\left(\frac{-1+\sqrt{-3}}{2}\right)\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$$= \frac{(-1)^2 - \left(\sqrt{-3}\right)^2}{4}$$

$$= \frac{1-(-3)}{4}$$

$$= \frac{1+3}{4}$$

$$= \frac{4}{4}$$

= 1

= R.H.S

**Proved**

**(iii) Proving that sum of three cube roots of unit is zero.**

**i.e.** $1+\omega+\omega^2 = 0$

**(LHR 2016, GRW 2017, FSD 2015, 17, SGD 2015, 16, BWP 2016, RWP 2015)**

**Proof:**

L.H.S $= 1+\omega+\omega^2$

(By putting values)

$$= 1 + \frac{-1+\sqrt{-3}}{2} + \frac{-1-\sqrt{-3}}{2}$$

$$= \frac{2+(-1)+\sqrt{-3}-1-\sqrt{-3}}{2}$$

$$= \frac{2-1+\sqrt{-3}-1-\sqrt{-3}}{2}$$

$$= \frac{0}{2}$$

= 0

= R.H.S

**Proved**

**Important Results** **(K.B + U.B)**

**(i)** $1+\omega+\omega^2 = 0$

$\Rightarrow 1+\omega = -\omega^2$

$1+\omega^2 = -\omega$

$\omega+\omega^2 = -1$

**(ii)** $1.\omega.\omega^2 = 1$

$\Rightarrow \omega^3 = 1$

**(iii)** $1.\omega.\omega^2 = 1$

$\omega.\omega^2 = 1$

$\Rightarrow \omega = \frac{1}{\omega^2}$ or $\omega^2 = \frac{1}{\omega}$

**Note: Complex cube roots are reciprocal of each other.**

**(iv)** $\omega^4 = \omega^3.\omega = 1.\omega = \omega$

$\omega^5 = \omega^3.\omega^2 = 1.\omega^2 = \omega^2$

$\omega^6 = \left(\omega^3\right)^2 = (1)^2 = 1$ **and so on**

$$\omega^{-16} = \frac{1}{\omega^{16}} = \frac{1}{\omega^{15}\omega} = \frac{1}{(\omega^3)^5.\omega} = \frac{1}{1.\omega} = \frac{1}{\omega} = \omega^2$$

**Example 1: (Page # 25)**

**Evaluate:** $\left(-1+\sqrt{-3}\right)^8 + \left(-1-\sqrt{-3}\right)^8$ **(A.B)**

**Solution:**

$$\left(-1+\sqrt{-3}\right)^8 + \left(-1-\sqrt{-3}\right)^8$$

$$= \left[2\left(\frac{-1+\sqrt{-3}}{2}\right)\right]^8 + \left[2\left(\frac{-1-\sqrt{-3}}{2}\right)\right]^8$$

$$= (2\omega)^8 + \left(2\omega^2\right)^8$$

$$= 2^8\omega^8 + 2^8\omega^{16}$$

$$= 2^8\left(\omega^6.\omega^2 + \omega^{15}\omega\right)$$

$$= 256\left(\left(\omega^3\right)^2.\omega^2 + \left(\omega^3\right)^5\omega\right)$$

$$= 256\left((1)^2.\omega^2 + (1)^5\omega\right) \quad \because \omega^3 = 1$$

$$256\left(\omega^2+\omega\right)$$

$$= 256(-1) \quad \because 1+\omega+\omega^2 = 0$$

$$= -256$$

**Example 2: (Page # 25)** **(A.B)**

**To Prove**

$$x^3 - y^3 = (x-y)(x-\omega y)(x-\omega^2 y)$$

**Proof:**

$$\text{R.H.S} = (x-y)(x-\omega y)(x-\omega^2 y)$$
$$= (x-y)(x^2 - \omega xy - \omega^2 xy + \omega^3 y^2)$$
$$= (x-y)(x^2 - (\omega + \omega^2)xy + y^2)$$
$$= (x-y)(x^2 - (-1)xy + y^2) \because 1+\omega+\omega^2 = 0$$
$$= (x-y)(x^2 + xy + y^2)$$
$$= x^3 - y^3 = \text{L.H.S}$$

## Exercise 2.2

**Q.1 Find the cube roots of** $-1, 8, -27, 64$.

**(LHR 2015) (K.B + A.B)**

**Solution:**

**(i) Finding cube roots of $-1$**

Let $x = \sqrt[3]{-1}$

Taking cube on both sides

$$x^3 = -1$$
$$x^3 + 1 = 0$$
$$\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$(x+1)(x^2 - x + 1) = 0$$

Either

$x + 1 = 0 \rightarrow (i)$ or $x^2 - x + 1 = 0 \rightarrow (ii)$

Equation $(i) \Rightarrow$

$$x = -1$$

Equation $(ii) \Rightarrow$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{1 \pm \sqrt{1-4}}{2}$$
$$= \frac{1 \pm \sqrt{-3}}{2}$$

Either

$x = \frac{1-\sqrt{-3}}{2}$ or $x = \frac{1+\sqrt{-3}}{2}$

$x = -\left(\frac{-1+\sqrt{-3}}{2}\right)$ $\quad x = -\left(\frac{-1-\sqrt{-3}}{2}\right)$

$= -\omega$ $\quad = -\omega^2$

$\therefore$ **Cube roots of $-1$ are** $-1, -\omega, -\omega^2$

**(ii) Finding cube roots of 8**

**(K.B + A.B)**

Let $x = \sqrt[3]{8}$

Taking cube on both sides

$$x^3 = 8$$
$$x^3 - 8 = 0$$
$$x^3 - 2^3 = 0$$
$$\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$(x-2)(x^2 + 2x + 4) = 0$$

Either

$x - 2 = 0 \rightarrow (i)$ or $x^2 + 2x + 4 = 0 \rightarrow (ii)$

Equation $(i) \Rightarrow$

$$x = 2$$

Equation $(ii) \Rightarrow$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{4-16}}{2}$$
$$= \frac{-2 \pm \sqrt{-12}}{2}$$
$$= \frac{-2 \pm 2\sqrt{-3}}{2}$$

Either

$x = 2\left(\frac{-1+\sqrt{-3}}{2}\right)$ or $x = 2\left(\frac{-1-\sqrt{-3}}{2}\right)$

$= 2\omega$ $\quad = 2\omega^2$

$\therefore$ **Cube roots of 8 are 2,** $2\omega, 2\omega^2$

**(iii) Finding cube roots of –27**

**(SGD 2014) (K.B + A.B)**

Let

$$x = \sqrt[3]{-27}$$
$$x^3 + 27 = 0$$
$$\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$x^3 + 3^3 = 0$$
$$(x+3)(x^2 - 3x + 9) = 0$$

Either

$x + 3 = 0 \rightarrow (i)$ or $x^2 - 3x + 9 = 0 \rightarrow (ii)$

Equation $(i) \Rightarrow$

$$x = -3$$

Equation $(ii) \Rightarrow$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-36}}{2}$$

$$= \frac{3 \pm \sqrt{-27}}{2}$$

$$= \frac{3 \pm 3\sqrt{-3}}{2}$$

$$= -3\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$$

Either

$$x = -3\left(\frac{-1+\sqrt{-3}}{2}\right) \text{ or } x = -3\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$$x = -3\omega \qquad x = -3\omega^2$$

$\therefore$ **Cube roots of –27 are –3, –3 $\omega$, –3 $\omega^2$**

**(iv) Finding cube roots of 64.**

**(LHR 2015) (K.B + A.B)**

Let

$$x = \sqrt[3]{64}$$

Taking cube on both sides

$$x^3 = 64$$

$$x^3 - 64 = 0$$

$$x^3 - 4^3 = 0$$

$$\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$(x-4)(x^2 + 4x + 16) = 0$$

Either

$$x - 4 = 0 \rightarrow (i) \text{ or } x^2 + 4x + 16 = 0 \rightarrow (ii)$$

Equation $(i) \Rightarrow$

$$x = 4$$

Equation $(ii) \Rightarrow$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(16)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16-64}}{2}$$

$$x = \frac{-4 \pm \sqrt{-48}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{-3}}{2}$$

$$x = 4\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$$

Either

$$x = 4\left(\frac{-1+\sqrt{-3}}{2}\right) \text{ or } x = 4\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$$x = 4\omega \qquad \text{or} \qquad x = 4\omega^2$$

$\therefore$ **Cube roots of 64 are** $4, 4\omega, 4\omega^2$

**Q.2 Evaluate: (K.B + A.B)**

**(FSD 2017, BWP 2016, RWP 2015, MTN 2014, 15, 17, D.G.K 2015, 17)**

**(i)** $(1-\omega-\omega^2)^7$

**(ii)** $(1-3\omega-3\omega^2)^5$

**(iii)** $(9+4\omega+4\omega^2)^3$

**(iv)** $(2+2\omega-2\omega^2)(3-3\omega+3\omega^2)$

**(v)** $(-1+\sqrt{-3})^6 + (-1-\sqrt{-3})^6$

**(vi)** $\left(\frac{1+\sqrt{-3}}{2}\right)^9 + \left(\frac{1-\sqrt{-3}}{2}\right)^9$

**(vii)** $\omega^{37} + \omega^{38} - 5$

**(viii)** $\omega^{-13} + \omega^{-17}$

**Solution:**

**(i)** $(1-\omega-\omega^2)^7$

**(GRW 2014, 16, 17, FSD 2016, BWP 2017, MTN 2017)**

$$= \left[1-(\omega+\omega^2)\right]$$

$$= \left[1-(-1)\right]^7 \qquad \because \omega+\omega^2 = -1$$

$$= (1+1)^7$$

$$= 2^7$$

$$= 128$$

**(ii)** $(1-3\omega-3\omega^2)^5$

$$= \left[1-3(\omega+\omega^2)\right]^5$$

$$= \left[1-3(-1)\right]^5 \qquad \because \omega+\omega^2 = -1$$

$$= (1+3)^5$$

$$= (4)^5$$

$$= 1024$$

**(iii)** $(9 + 4\omega + 4\omega^2)^3$

**(GRW 2014, RWP 2017, FSD 2017, BWP 2016)**

$= \left[9+4\left(\omega+\omega^2\right)\right]^3$

$= \left[9+4(-1)\right]^3 \quad \because \omega+\omega^2 = -1$

$= (9-4)^3$

$= (5)^3$

$= 125$

**(iv)** $\left(2+2\omega-2\omega^2\right)\left(3-3\omega+3\omega^2\right)$

**(SWL 2017)**

$= 2\left(1+\omega-\omega^2\right)3\left(1-\omega+\omega^2\right)$

$= 6\left[\left(1+\omega\right)-\omega^2\right]\left[\left(1+\omega^2\right)-\omega\right]$

$\because 1+\omega+\omega^2 = 0$

$= 6\left(-\omega^2-\omega^2\right)\left(-\omega-\omega\right)$

$= 6\left(-2\omega^2\right)\left(-2\omega\right)$

$= 6(4\omega^3)$

$= 24\omega^3$

$= 24(1) \quad \because \omega^3 = 1$

$= 24$

**(v)** $\left(-1+\sqrt{-3}\right)^6+\left(-1-\sqrt{-3}\right)^6$

$\because \omega = \dfrac{-1+\sqrt{-3}}{2}, \omega^2 = \dfrac{-1-\sqrt{-3}}{2}$

$= \left(2\omega\right)^6+\left(2\omega^2\right)^6$

$= 2^6\omega^6+2^6\omega^{12}$

$= 2^6\left(\omega^6+\omega^{12}\right)$

$= 64\left[\left(\omega^3\right)^2+\left(\omega^3\right)^4\right]$

$= 64\left[\left(1\right)^2+\left(1\right)^4\right] \quad \because \omega^3 = 1$

$= 64(1+1)$

$= 64(2)$

$= 128$

**(vi)** $\left(\dfrac{1+\sqrt{-3}}{2}\right)^9+\left(\dfrac{1-\sqrt{-3}}{2}\right)^9$

$\omega = \dfrac{-1+\sqrt{-3}}{2}, \omega^2 = \dfrac{-1-\sqrt{-3}}{2}$

$= \left(\omega\right)^9+\left(\omega^2\right)^9$

$= \left(\omega\right)^9+\left(\omega\right)^{18}$

$= \left(\omega^3\right)^3+\left(\omega^3\right)^6$

$= \left(1\right)^3+\left(1\right)^6$

$= 1+1$

$= 2$

**(vii)** $\omega^{37}+\omega^{38}-5$

**(LHR 2015, SWL 2016, MTN 2015, D.G.K 2016, 17)**

$= \omega.\omega^{36}+\omega^2.\omega^{36}-5$

$= \left(\omega+\omega^2\right).\omega^{36}-5$

$= (-1)\left[\left(\omega^3\right)^{12}\right]-5$

$= -1\left[\left(1\right)^{12}\right]-5 \quad \because \omega^3 = 1$

$= -1(1)-5$

$= -1-5$

$= -6$

**(viii)** $\omega^{-13}+\omega^{-17}$

$= \dfrac{1}{\omega^{13}}+\dfrac{1}{\omega^{17}}$

$= \dfrac{1}{\omega.\omega^{12}}+\dfrac{1}{\omega^2.\omega^{15}}$

$= \dfrac{1}{\omega\left(\omega^3\right)^4}+\dfrac{1}{\omega^2.\left(\omega^3\right)^5}$

$= \dfrac{1}{\omega\left(1\right)^4}+\dfrac{1}{\omega^2\left(1\right)^5} \quad \because \omega^3 = 1$

$= \dfrac{1}{\omega(1)}+\dfrac{1}{\omega^2(1)}$

$= \dfrac{1}{\omega}+\dfrac{1}{\omega^2}$

$= \omega^2+\omega \quad \because \omega.\omega^2 = 1$

$= -1 \quad \because 1+\omega+\omega^2 = 0$

**Q.3 Prove that** **(K.B + U.B)**

$$x^3+y^3=(x+y)(x+\omega y)(x+\omega^2 y)$$

**(SGD 2015, BWP 2016)**

**Proof:**

**R.H.S**

$=(x+y)(x+\omega y)(x+\omega^2 y)$

$=(x+y)(x^2+\omega^2 xy+\omega xy+\omega^3 y^2)$

$=(x+y)\left[x^2+(\omega^2+\omega)xy+(1)y^2\right] \because \omega^3=1$

$=(x+y)\left[x^2+(-1)xy+y^2\right] \because 1+\omega+\omega^2=0$

$=(x+y)\left[x^2-xy+y^2\right]$

$=x^3+y^3$

= L.H.S

**Proved**

**Q.4 Prove that** $x^3+y^3+z^3-3xyz$

$=(x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$

**(K.B + U.B)**

**Proof:**

**R.H.S**

$=(x+y+z)(x+\omega y+\omega^2 z)(x+\omega^2 y+\omega z)$

$=(x+y+z)(x^2+\omega^2 xy+\omega xz+\omega xy+\omega^3 y^2$
$+\omega^2 yz+\omega^2 xz+\omega^4 yz+\omega^3 z^2)$

$=(x+y+z)(x^2+\omega^3 y^2+\omega^3 z^2+\omega^2 xy+\omega xy$
$+\omega^2 yz+\omega^4 yz+\omega xz+\omega^2 xz)$

$=(x+y+z)(x^2+(1)y^2+(1)z^2+(\omega^2+\omega)xy$
$+(\omega^2+\omega^4)yz+(\omega+\omega^2)xz) \quad \because \omega^3=1$

$=(x+y+z)\left[x^2+y^2+z^2+(-1)xy+(\omega^2+\omega)yz+(-1)xz\right]$

$\because 1+\omega+\omega^2=0, \omega^4=\omega.\omega^3=\omega$

$=(x+y+z)(x^2+y^2+z^2-xy+(-1)yz-xz)$

$=(x+y+z)(x^2+y^2+z^2-xy-yz-xz)$

$=x^3+y^3+z^3-3xyz$

= L.H.S

**Proved**

**Q.5 Prove that**

$$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)...2n$$

**(K.B + A.B + U.B)**

**Proof:**

**L.H.S**

$=(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)...2n$ factors

$\because \omega^4=\omega\times\omega^3=\omega, \omega^8=\omega^2\times\omega^6=\omega^2\times(\omega^3)^2=\omega^2(1)=\omega^2$

$=(1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2)...2n$ factors

$=\left[(1+\omega)(1+\omega)(1+\omega).....n \text{ factors}\right]$

$\left[(1+\omega^2)(1+\omega^2)(1+\omega^2).....n \text{ factors}\right]$

$=(1+\omega)^n(1+\omega^2)^n$

$=\left[(1+\omega)(1+\omega^2)\right]^n$

$=(1+\omega+\omega^2+\omega^3)^n$

$=(0+1)^n \quad \because 1+\omega+\omega^2=0, \omega^3=1$

$=(1)^n$

$=1$

=R.H.S

## Relation between Roots and Co-efficient of a Quadratic Equation **(K.B + U.B)**

**Roots of standard quadratic equation** $ax^2+bx+c=0$ **are**

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$

**If** $\alpha=\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta=\dfrac{-b-\sqrt{b^2-4ac}}{2a}$

**Sum of roots**

$$S=\frac{-b+\sqrt{b^2-4ac}}{2a}+\frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-b+\sqrt{b^2-4ac}+(-b)-\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2b}{2a}$$

$$\Rightarrow S=-\frac{b}{a}$$

**Product of roots**

$$P=\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$\because (a+b)(a-b)=a^2-b^2$

$$=\frac{(-b)^2-\left(\sqrt{b^2-4ac}\right)^2}{(2a)^2}$$

$$=\frac{b^2-\left(b^2-4ac\right)}{4a^2}$$

$$=\frac{b^2-b^2+4ac}{4a^2}$$

$$=\frac{4ac}{4a^2}$$

$$\Rightarrow P=\frac{c}{a}$$

**Note** **(K.B + U.B)**

**(i)** $S=-\frac{b}{a}=-\frac{\text{Coefficient of } x}{\text{Coefficent of } x^2}$

**(ii)** $P=\frac{c}{a}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

## Quadratic Equation with Given Roots

**(K.B + U.B)**

A quadratic equation whose roots are given can be obtained by using formula

$x^2-Sx+P=0$ **Or**

$x^2-(\text{Sum of roots})x+\text{Product of roots}=0$

**Example 1: (Page # 26)** **(A.B)**

**Without solving, find the sum and product of roots of the equation**

$3x^2-5x+7=0$

**Solution:**

$3x^2-5x+7=0$

Here $a=3, b=-5, c=7$

Sum of roots $=S=-\frac{b}{a}$

$=-\frac{-5}{3}=\frac{5}{3}$

Product of roots $=P=\frac{c}{a}$

$=\frac{7}{3}$

## To find unknown values involved in a given Quadratic Equation

**Example 1: (Page # 27)** **(A.B)**

**Find the value of h, if the sum of roots is equal to 3-times the product of roots of the equation:**

$3x^2+(9-6h)x+5h=0$.

**Solution:**

$3x^2+(9-6h)x+5h=0$

Here $a=3, b=9-6h, c=5h$

Let $\alpha,\beta$ be the roots of given equation

Then $\alpha+\beta=-\frac{b}{a}=-\frac{9-6h}{3}=\frac{6h-9}{3}$

And $\alpha\beta=\frac{c}{a}=\frac{5h}{3}$

According to given condition

Sum of roots = 3(Product of roots)

$\alpha+\beta=3\alpha\beta$

$\frac{6h-9}{3}=3\left(\frac{5h}{3}\right)$

$6h-9=15h$

$6h-15h=9$

$-9h=9$

$h=-1$

## Exercise 2.3

**Q.1 Without solving, find the sum and the product of the roots of the following quadratic equations.**

**(i)** $x^2-5x+3=0$ **(MTN 2017)** **(A.B)**

Here $a=1, b=-5, c=3$

Sum of roots $=-\frac{b}{a}$

$=-\frac{(-5)}{1}$

$=5$

Product of roots $=\frac{c}{a}$

$=\frac{3}{1}$

$=3$

**(ii)** $3x^2+7x-11=0$ **(A.B)**

**(LHR 2017, SWL 2017, SGD 2016, D.G.K 2014)**

Here

$a=3,\ b=7,\ c=-11$

Sum of roots $=-\dfrac{b}{a}$

$=-\dfrac{7}{3}$

Product of roots $=\dfrac{c}{a}$

$=\dfrac{-11}{3}$

$=-\dfrac{11}{3}$

**(iii)** $px^2-qx+r=0$ **(A.B)**

**(LHR 2014, GRW 2014, SWL 2016, MTN 2017, SGD 2016)**

Here $a=p, b=-q, c=r$

Sum of roots $=-\dfrac{b}{a}$

$=-\dfrac{(-q)}{p}$

$=\dfrac{q}{p}$

Product of roots $=\dfrac{c}{a}$

$=\dfrac{r}{p}$

**(iv)** $(a+b)x^2-ax+b=0$ **(A.B)**

**(BWP 2014, 17)**

Here

$A=a+b,\ B=-a,\ C=b$

Sum of roots $=-\dfrac{B}{A}$

$=-\dfrac{-a}{a+b}$

$=\dfrac{a}{a+b}$

Product of roots $=\dfrac{C}{A}$

$=\dfrac{b}{a+b}$

**(v)** $(l+m)x^2+(m+n)x+n-1=0$

Here

$a=l+m,\ b=m+n,\ c=n-l$

Sum of roots $=S=-\dfrac{b}{a}$

$=-\dfrac{m+n}{l+m}$

Product of roots $=P=\dfrac{c}{a}$

$=\dfrac{n-l}{l+m}$

**(vi)** $7x^2-5mx+9n=0$ **(A.B)**

Here

$a=7,\ b=-5\text{m},\ c=9n$

Sum of roots $=-\dfrac{b}{a}$

$=-\dfrac{-5\text{m}}{7}$

$=\dfrac{5\text{m}}{7}$

Product of roots $=\dfrac{c}{a}$

$=\dfrac{9n}{7}$

**Q.2 Find the value of $k$, if**

**(i) Sum of the roots of the equation $2kx^2-3x+4k=0$ is twice the product of the roots.** **(A.B)**

**Solution:**

Let $\alpha, \beta$ be the roots of equation

$2kx^2-3x+4k=0$

Here

$a=2k,\ b=-3,\ c=4k$

$S=\alpha+\beta=-\dfrac{b}{a}$

$=\dfrac{-(-3)}{2k}$

$=\dfrac{3}{2k}$

$P=\alpha\beta=\dfrac{c}{a}$

$=\dfrac{4k}{2k}$

$=2$

According to given condition

$S = 2P$

$\frac{3}{2k} = 2\times 2$

$\frac{3}{2k} = \frac{4}{1}$

$3\times 1 = 4\times 2k$

$3 = 8k$

$\frac{3}{8} = k$

$k = \frac{3}{8}$

**(ii) Sum of the roots of the equation $x^2 + (3k-7)x + 5k = 0$ is $\frac{3}{2}$ times the product of the roots.** **(A.B)**

**Solution:**

$x^2 + (3k-7)x + 5k = 0$

Here, $a = 1, b = 3k-7, c = 5k$

Sum of roots = $S = \frac{-b}{a}$

$= -\frac{3k-7}{1}$

$= -3k+7$

Product of roots = P = $\frac{c}{a}$

$= \frac{5k}{1}$

$= 5k$

According to given condition:

$S = \frac{3}{2}P$

$-3k+7 = \frac{3}{2}(5k)$

$2(-3k+7) = 3(5k)$

$-6k + 14 = 15k$

$14 = 15k + 6k$

$14 = 21k$

$\frac{14}{21} = k$

$\frac{2}{3} = k$

Or $k = \frac{2}{3}$

**Q.3 Find *k*,**

**(i) If sum of squares of the roots of the equation $4kx^2 + 3kx - 8 = 0$ is 2.**

**Solution** **(FSD 2015)** **(A.B)**

$4kx^2 + 3kx - 8 = 0$

Here a = 4k, b = 3k, c = –8

Let α, β be the roots

Sum of roots = $\alpha + \beta = -\frac{b}{a}$

$= -\frac{3k}{4k}$

$\Rightarrow \alpha + \beta = -\frac{3}{4}$

Product of roots = $\alpha\beta = \frac{c}{a}$

$= \frac{-8}{4k}$

$\Rightarrow \alpha\beta = \frac{-2}{k}$

According to given condition

$\alpha^2 + \beta^2 = 2$

$(\alpha+\beta)^2 - 2\alpha\beta = 2$

Putting the values

$\left(-\frac{3}{4}\right)^2 - 2\left(-\frac{2}{k}\right) = 2$

$\frac{9}{16} + \frac{4}{k} = 2$

$\frac{4}{k} = 2 - \frac{9}{16}$

$\frac{4}{k} = \frac{32-9}{16}$

$\frac{4}{k} = \frac{23}{16}$

$\Rightarrow \frac{k}{4} = \frac{16}{23}$

$k = \frac{64}{23}$

**(ii) Sum of the squares of the roots of the equation $x^2-2kx+(2k+1)=0$ is 6.** **(A.B)**

**Solution:**

Here

$a=1, b=-2k, c=2k+1$

Let, $\alpha,\beta$ be the roots of given equation

Then

Sum of roots $=\alpha+\beta=-\frac{b}{a}$

$=-\left(\frac{-2k}{1}\right)$

= 2k

Product of roots $=\alpha\beta=\frac{c}{a}$

$=\frac{2k+1}{1}$

$=2k+1$

According to given condition

$\alpha^2+\beta^2=6$

Or $(\alpha+\beta)^2-2\alpha\beta=6$

Putting the values

$(2k)^2-2(2k+1)=6$

$4k^2-4k-2-6=0$

$4k^2-4k-8=0$

$4(k^2-k-2)=0$

Or $k^2-k-2=0$

$k^2-2k+k-2=0$

$k(k-2)+1(k-2)=0$

$(k-2)(k+1)=0$

Either

$k-2=0$ or $k=2$

$k+1=0$

$k=-1$

**Result**

$k=2,-1$

**Q.4 Find $p$, if**

**(i) The roots of the equation $x^2-x+p^2=0$ differ by unity.** **(FSD 2015) (A.B)**

Let $\alpha,\beta$ be the roots of given equation.

Here $a=1,\quad b=-1,\quad c=p^2$

Then $\alpha+\beta=-\frac{b}{a}$

$=-\left(\frac{-1}{1}\right)$

$=1$

$\alpha\beta=\frac{c}{a}$

$=\frac{p^2}{1}$

$=p^2$

According to given condition

$\alpha-\beta=1$

Taking square on both sides

$(\alpha-\beta)^2=1$

$\because (a+b)^2-(a-b)^2=4ab$

$(\alpha+\beta)^2-4\alpha\beta=1$

Putting the values

$(1)^2-4p^2=1$

$1-4p^2=1$

$1-1=4p^2$

$0=4p^2$

Or

$p^2=0$

By taking square root

$p=0$

**Result:**

$p=0$

**(ii) The roots of the equation $x^2+3x+P-2=0$ differ by 2.**

**Solution:** **(A.B)**

$x^2+3x+P-2=0$

Here $a=1, b=3, c=P-2$

Let roots of given equation are $\alpha,\beta$

Then sum of roots $=\alpha+\beta=-\dfrac{b}{a}$

$=-\dfrac{3}{1}$

$\Rightarrow \alpha+\beta=-3$

Product of roots $=\alpha\beta=\dfrac{c}{a}$

$=\dfrac{P-2}{1}$

According to given condition

$\alpha-\beta=2$

Taking square of both sides

$(\alpha-\beta)^2=4$

$(\alpha+\beta)^2-4\alpha\beta=4$

Putting the values

$(-3)^2-4(P-2)=4$

$9-4P+8=4$

$9+8-4=4P$

$13=4P$

Or $P=\dfrac{13}{4}$

**Result:** $P=\dfrac{13}{4}$

**Q.5 Find m, if**

**(i) The roots of the equation $x^2-7x+3m-5=0$ satisfy the relation $3\alpha+2\beta=4$** **(A.B)**

**Solution:**

$x^2-7x+3m-5=0$

Here $a=1, b=-7, c=3m-5$

Let $\alpha,\beta$ be the roots of given equation

Then sum of roots $=\alpha+\beta=-\dfrac{b}{a}$

$=-\dfrac{-7}{1}$

$\Rightarrow \alpha+\beta=7\rightarrow(i)$

Product of roots $=\alpha\beta=\dfrac{c}{a}$

$=\dfrac{3m-5}{1}$

$\Rightarrow \alpha\beta=3m-5\rightarrow(ii)$

According to given condition

$3\alpha+2\beta=4\rightarrow(iii)$

Multiply equation (i) by '2'

$2\alpha+2\beta=14\longrightarrow(iv)$

Sub. Equation (iii) & (iv)

$3\alpha+2\beta=4$

$\underline{-2\alpha\pm2\beta=-14}$

$\alpha \quad =-10$

Put in equation (i)

$\alpha+\beta=7$

$-10+\beta=7$

$\beta=7+10$

$\beta=17$

Putting the values of α and β in equation (ii)

$\alpha\beta=3m-5$

$-10(17)=3m-5$

$-170+5=3m$

$\dfrac{-165}{3}=m$

Or $m=-55$

**Result:**

$m=-55$

**(ii) The roots of the equation $x^2+7x+3m-5=0$ satisfy the relation $3\alpha-2\beta=4$** **(A.B)**

Let $\alpha,\beta$ be the roots of given equation

Here

$a=1,\quad b=7,\quad c=3m-5$

Then

$\alpha+\beta=-\dfrac{b}{a}$

$=\dfrac{-7}{1}$

$\alpha+\beta=-7\longrightarrow(i)$

$\alpha\beta=\dfrac{c}{a}$

$=\dfrac{3m-5}{1}$

$\alpha\beta=3m-5\longrightarrow(ii)$

Also given

$3\alpha-2\beta=4\longrightarrow(iii)$

Multiply equation (i) by 2

$2\alpha+2\beta=-14 \longrightarrow (iv)$

Adding equation (iii) and (iv)

$3\alpha-2\beta=4$

$\underline{2\alpha+2\beta=-14}$

$5\alpha = -10$

$\alpha=-\frac{10}{5}$

$\Rightarrow \alpha=-2$

Put in equation (i)

$\alpha+\beta=-7$

$-2+\beta=-7$

$\beta=-7+2$

$\beta=-5$

Now putting the values in equation (ii)

$\alpha\beta=3m-5$

$-2(-5)=3m-5$

$10+5=3m$

$15=3m$

$\frac{15}{3}=m$

$5=m$

**Result:**

$m=5$

**(iii)** $3x^2-2x+7m+2=0$ **(A.B)**

Here $a=3, b=-2, c=7m+2$

Let $\alpha,\beta$ be the roots of given equation

Then sum of roots = $\alpha+\beta = -\frac{b}{a}$

$= -\frac{-2}{3}$

$\Rightarrow \alpha+\beta=\frac{2}{3} \longrightarrow$ (i)

Product of roots = $\alpha\beta = \frac{c}{a}$

$= \frac{7m+2}{3} \longrightarrow$ (ii)

Also given

$7\alpha-3\beta=18 \longrightarrow$ (iii)

Multiply equation (i) by '3'

$3\alpha+3\beta=2 \longrightarrow$ (iv)

Adding equation (iii) and (iv)

$7\alpha-3\beta=18$

$\underline{3\alpha+3\beta=2}$

$10\alpha = 20$

$\Rightarrow \alpha=2$

Put in equation (i)

$\alpha+\beta=\frac{2}{3}$

$2+\beta=\frac{2}{3}$

$\beta=\frac{2}{3}-2$

$=\frac{2-6}{3}$

$\beta=\frac{-4}{3}$

Now putting the values of α and β in equation (ii)

$2\left(\frac{-4}{3}\right)=\frac{7m+2}{3}$

$-8 = 7m + 2$

$-8 - 2 = 7m$

$-10 = 7m$

$\frac{-10}{7}=m$

**Result:**

$m=\frac{-10}{7}$

**Q.6 Find m, if sum and product of the roots of the following equations is equal to a given number $\lambda$.** **(A.B)**

**(i)** $(2m+3)x^2+(7m-5)x+(3m-10)=0$

Here

$a=2m+3, b=7m-5$ and $c=3m-10$

Let $\alpha,\beta$ be the roots of given equation,

Then $\alpha+\beta = -\frac{b}{a}$

$=-\frac{7m-5}{2m+3}$

$=\frac{5-7m}{2m+3}$

And $\alpha\beta = \dfrac{c}{a}$

$= \dfrac{3m-10}{2m+3}$

According to given condition

$\alpha+\beta=\lambda=\alpha\beta$

(By using transitive property)

$\Rightarrow \alpha+\beta=\alpha\beta$

Putting the values

$\dfrac{5-7m}{2m+3}=\dfrac{3m-10}{2m+3}$

Or

$5-7m=3m-10$

(By using cancellation property)

$-7m-3m=-10-5$

$-10m=-15$

$m=\dfrac{-15}{-10}$

$\Rightarrow m=\dfrac{3}{2}$

**Result:** $m=\dfrac{3}{2}$

**(ii)** $4x^2-(3+5m)x-(9m-17)=0$

**(A.B)**

Here

$a=4, b=-(3+5m), c=-(9m-17)$

Let $\alpha, \beta$ be the roots

Then $\alpha+\beta = -\dfrac{b}{a}$

$= -\dfrac{-(3+5m)}{4}$

$= \dfrac{3+5m}{4}$

And $\alpha\beta = \dfrac{c}{a}$

$= \dfrac{-(9m-17)}{4}$

According to given condition:

$\alpha+\beta=\lambda$ and $\alpha\beta=\lambda$

$\Rightarrow \alpha+\beta=\alpha\beta$

$\because$ Transitive property of equality

Putting the values

$\dfrac{3+5m}{4}=\dfrac{-(9m-17)}{4}$

$\Rightarrow 3+5m=-9m+17$

$5m+9m=17-3$

$14m=14$

$\Rightarrow m=1$

**Result:** $m=1$

## Symmetric Function of the Roots of Quadratic Equation **(K.B + U.B)**

**(MTN 2014, FSD 2014)**

A function in which the roots involved are such that the value of the expression remains same, when roots are interchanged is called symmetric function. i.e. $f(\alpha,\beta)=f(\beta,\alpha)$

**Some symmetric functions are:**

$\alpha^2+\beta^2, \quad \alpha^3+\beta^3, \quad \dfrac{1}{\alpha}+\dfrac{1}{\beta}$

**Example: (Page # 30)** **(K.B + U.B)**

**Verify that**

$\alpha^2+\beta^2+2\alpha\beta$ **is Symmetric**

**Verification:**

Let $f(\alpha,\beta)=\alpha^2+\beta^2+2\alpha\beta \rightarrow (i)$

$f(\beta,\alpha)=(\beta)^2+(\alpha)^2+2\beta\alpha$

$=\beta^2+\alpha^2+2\alpha\beta$

$=\alpha^2+\beta^2+2\alpha\beta \rightarrow (ii)$

From equation (i) and (ii) we get

$f(\alpha,\beta)=f(\beta,\alpha)$

**Hence** $\alpha^2+\beta^2+2\alpha\beta$ **is symmetric**

**Example: (Page # 30)** **(A.B + U.B)**

**Find the value of** $\alpha^3+\beta^3+3\alpha\beta$**, if** $\alpha=2, \beta=1$**. Also find the value of** $\alpha^3+\beta^3+3\alpha\beta$ **if** $\alpha=1, \beta=2$**.**

**Solution:**

When $\alpha=2, \beta=1$

$\alpha^3+\beta^3+3\alpha\beta=(2)^3+(1)^3+3(2)(1)$

$=8+1+6=15$

When $\alpha=1, \beta=2$

$\alpha^3+\beta^3+3\alpha\beta=(1)^3+(2)^3+3(1)(2)$

$=1+8+6=15$

**Note**

Expression $\alpha^3+\beta^3+3\alpha\beta$ represents a symmetric function.

## Exercise 2.4

**Q.1 If α, β are roots of the equation $x^2+px+q=0$, then evaluate**

**(i)** $\alpha^2+\beta^2$

**(ii)** $\alpha^3\beta+\alpha\beta^3$

**(iii)** $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

**(i) Solution:** **(A.B + U.B)**

$x^2+px+q=0$

Here a = 1, b = p, c = q

Roots of given equation are $\alpha,\beta$

Then

$$\alpha+\beta=-\frac{b}{a}=-\frac{p}{1}=-p$$

$$\alpha\beta=\frac{c}{a}=\frac{q}{1}=q$$

**(i)** Now $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$$\alpha^2+\beta^2=(-p)^2-2(q)=p^2-2q$$

**(ii)** $\alpha^3\beta+\alpha\beta^3=\alpha\beta(\alpha^2+\beta^2)$ **(A.B)**

$$=\alpha\beta\left[(\alpha+\beta)^2-2\alpha\beta\right]$$

$$=q\left[(-p)^2-2(q)\right]$$

$$\Rightarrow \alpha^3\beta+\alpha\beta^3=q(p^2-2q)$$

**(iii)** $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$

**(LHR 2015) (K.B + U.B)**

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{(-p)^2-2(q)}{q}$$

$$\Rightarrow \frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{p^2-2q}{q}$$

**Q.2 If α, β are the roots of the equation $4x^2-5x+6=0$, then find the value of**

**(i)** $\frac{1}{\alpha}+\frac{1}{\beta}$ **(K.B + A.B)**

**(ii)** $\alpha^2\beta^2$ **(A.B + U.B)**

**(iii)** $\frac{1}{\alpha^2\beta}+\frac{1}{\alpha\beta^2}$ **(U.B + A.B)**

**(iv)** $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ **(K.B + U.B)**

**(LHR 2016, GRW 2014, SWL 2016, MTN 216, SGD 2015, D.G.K 2014)**

**Solution:**

$4x^2-5x+6=0$

Here $a=4, b=-5, c=6$

Since $\alpha,\beta$ be the roots of the given equation

Then

$$\alpha+\beta=-\frac{b}{a}=-\frac{(-5)}{4}=\frac{5}{4}$$

$$\alpha\beta=\frac{c}{a}=\frac{6}{4}=\frac{3}{2}$$

**(i)** $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$

$$=\frac{\alpha+\beta}{\alpha\beta}=\frac{\frac{5}{4}}{\frac{3}{2}}=\frac{5}{4}\times\frac{2}{3}$$

$$\Rightarrow \frac{1}{\alpha}+\frac{1}{\beta}=\frac{5}{6}$$

**(ii)** $\alpha^2\beta^2 = (\alpha\beta)^2$

$$= \left(\frac{3}{2}\right)^2$$

$$\Rightarrow \alpha^2\beta^2 = \frac{9}{4}$$

**(iii)** $\dfrac{1}{\alpha^2\beta} + \dfrac{1}{\alpha\beta^2} = \dfrac{\beta+\alpha}{\alpha^2\beta^2}$

$$= \frac{\alpha+\beta}{(\alpha\beta)^2}$$

$$= \frac{\frac{5}{4}}{\left(\frac{3}{2}\right)^2}$$

$$= \frac{\frac{5}{4}}{\frac{9}{4}}$$

$$= \frac{5}{4} \times \frac{4}{9}$$

$$\Rightarrow \frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \frac{5}{9}$$

**(iv)** $\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha} = \dfrac{\alpha^3+\beta^3}{\alpha\beta}$

$$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

Putting the values

$$= \frac{\left(\frac{5}{4}\right)^3 - 3\left(\frac{3}{2}\right)\left(\frac{5}{4}\right)}{\frac{3}{2}}$$

$$= \frac{\frac{125}{64} - \frac{45}{8}}{\frac{3}{2}}$$

$$= \frac{125-360}{64} \times \frac{2}{3}$$

$$= \frac{-235}{64} \times \frac{2}{3}$$

$$\Rightarrow \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = -\frac{235}{96}$$

**Q.3 If α, β are the roots of the equation $lx^2 + mx + n = 0$ $(l \neq 0)$, then find the values of**

**(i)** $\alpha^3\beta^2 + \alpha^2\beta^3$ **(A.B + U.B)**

**(ii)** $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ **(K.B + U.B)**

**Solution:**

$lx^2 + mx + n = 0$

Roots of given equation are $\alpha, \beta$

Then $\alpha + \beta = -\dfrac{b}{a} = -\dfrac{m}{l}$

$$\alpha\beta = \frac{c}{a}$$

$$= \frac{n}{l}$$

**(i)** Now $\alpha^3\beta^2 + \alpha^2\beta^2 = \alpha^2\beta^2(\alpha+\beta)$

$$= (\alpha\beta)^2(\alpha+\beta)$$

$$= \left(\frac{n}{l}\right)^2 \times \left(-\frac{m}{l}\right)$$

$$= \frac{n^2}{l^2} \times \left(-\frac{m}{l}\right)$$

$$\Rightarrow \alpha^3\beta^2 + \alpha^2\beta^2 = -\frac{mn^2}{l^3}$$

**(ii)** $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = \dfrac{\beta^2+\alpha^2}{\alpha^2\beta^2}$

**(FSD 2017, SWL 2017, BWP 2014, D.G.K 2017)**

$$= \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(-\frac{m}{l}\right)^2 - 2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2}$$

$$= \frac{\frac{m^2}{l^2} - \frac{2n}{l}}{\frac{n^2}{l^2}}$$

$$= \frac{m^2 - 2ln}{l^2} \times \frac{l^2}{n^2}$$

$$\Rightarrow \frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{m^2-2ln}{n^2}$$

## Exercise 2.5

**Q.1 Write the quadratic equations having following roots.**

(a) 1, 5 **(K.B + A.B)**
(b) 4, 9 **(K.B + A.B)**
(c) –2, 3 **(K.B + A.B)**
(d) 0, –3 **(K.B + A.B)**
(e) 2, –6 **(K.B + A.B)**
(f) –1, –7 **(K.B + A.B)**
(g) $1+i, 1-i$ **(K.B + A.B)**
(h) $3+\sqrt{2}, 3-\sqrt{2}$ **(K.B + A.B)**

**Solution:**

**(a)** Roots of required equation are 1, 5
Then sum of roots = S = 1 + 5 = 6
And product of roots = P = 1 × 5 = 5
∴ Required quadratic equation is:
$$x^2-Sx+P=0$$
$$x^2-6x+5=0$$

**(b)** **(FSD 2016, 17, RWP 2017, RWP 2017)**
Roots of required equation are 4, 9
Then sum of roots = S = 4 + 9 = 13
And product of roots $=P=4\times9=36$
∴ Required quadratic equation is:
$$x^2-Sx+P=0$$
$$x^2-13x+36=0$$

**(c)** **(LHR 2014, 16, GRW 2016, 17, SGD 2017, D.G.K 2017)**
Roots of required equation are –2, 3
Then sum of roots = S = –2 + 3 = 1
And product of roots = P = –2(3) = –6
∴ Required quadratic equation is:
$$x^2-Sx+P=0$$
$$x^2-1x+(-6)=0$$ **(K.B + A.B)**
$$x^2-x-6=0$$

**(d)** **(SGD 2014, RWP 2017)**
Roots of required equation are 0, –3
Then sum of roots = S = 0 + (–3) = –3
And product of roots = P = 0(–3) = 0
∴ Required quadratic equation is:
$$x^2-Sx+P=0$$
$$x^2-(-3)x+0=0$$ **(K.B + A.B)**
$$x^2+3x=0$$

**(e)** **(LHR 2014, 16, GRW 2016, 17, SGD 2017, D.G.K 2017)**
Roots of required equation are 2, –6
Sum of roots = S = 2 + (–6) = –4
Product of roots = P = 2(–6) = –12
∴ Required quadratic equation is:
$$x^2-Sx+P=2$$
$$x^2-(-4)x+(-12)=0$$
$$x^2+4x-12=0$$ **(K.B + A.B)**

**(f)** **(LHR 2015, 17, RWP 2016)**
Roots of required equation are –1, –7
Sum of roots = S = –1 + (–7) = –8
Product of roots = P = –1(–7) = 7
∴ Required quadratic equation is:
$$x^2-Sx+P=0$$
$$x^2-(-8)x+7=0$$ **(K.B + A.B)**
$$x^2+8x+7=0$$

**(g)** **(D.G.K 2014, SGD 2017)**
$1+i, 1-i$ **(K.B + A.B)**
Roots of the required equation are
$1+i,\ 1-i$
Sum of roots $=S=(1+i)+(1-i)$
$=1+i+1-i$
$=2$
Product of roots $=P=(1+i)(1-i)$
$=(1)^2-(i)^2$
$=1-i^2$
$=1-(-1)$
$=1+1$
$=2$
Required quadratic equation is
$$x^2-Sx+P=0$$
$$x^2-2x+2=0$$

**(h)** Roots of required equation are
$3+\sqrt{2}, 3-\sqrt{2}$ **(K.B + A.B)**
Sum of roots $=\left(3+\sqrt{2}\right)+\left(3-\sqrt{2}\right)$
$=3+\sqrt{2}+3-\sqrt{2}$
$=6$
Product of roots $=\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$

$= (3)^2 - (\sqrt{2})^2$
$= 9 - 2$
$= 7$

$\therefore$ Required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - 6x + 7 = 0$

**Q.2 If $\alpha, \beta$ are the roots of the equation $x^2 - 3x + 6 = 0$. Form equations whose roots are**

**(a)** $2\alpha+1,\ 2\beta+1$

**(b)** $\alpha^2, \beta^2$

**(c)** $\frac{1}{\alpha}, \frac{1}{\beta}$

**(d)** $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

**(e)** $\alpha+\beta,\ \frac{1}{\alpha}+\frac{1}{\beta}$

**Solution:** **(K.B + A.B)**

$x^2 - 3x + 6 = 0$

Here $a = 1, b = -3, c = 6$

Roots of given equations are $\alpha, \beta$

Then $\alpha + \beta = -\frac{b}{a}$

$= -\frac{-3}{1}$

$= 3$

$\alpha\beta = \frac{c}{a}$

$= \frac{6}{1}$

$= 6$

**(a)** Roots of required equation are $2\alpha+1,\ 2\beta+1$

(FSD 2015) **(K.B + A.B)**

Sum of roots $= S = (2\alpha+1) + (2\beta+1)$

$= 2\alpha + 1 + 2\beta + 1$

$= 2\alpha + 2\beta + 2$

$= 2(\alpha+\beta) + 2$

$= 2(3) + 2$

$= 6 + 2$

$= 8$

Product of roots $= P = (2\alpha+1)(2\beta+1)$

$= 4\alpha\beta + 2\alpha + 2\beta + 1$

$= 4(6) + 6 + 1$

$= 24 + 7 = 31$

$\therefore$ Required quadratic equation is:

$x^2 - Sx + P = 0$

$x^2 - 8x + 31 = 0$

**(b)** Roots of required equation are $\alpha^2, \beta^2$

Sum of roots $= S = \alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$

$= (3)^2 - 2(6)$

$= 9 - 12$

$= -3$ **(K.B + A.B)**

Product of roots $= P = \alpha^2\beta^2$

$= (\alpha\beta)^2$

$= (6)^2$

$= 36$

$\therefore$ Required quadratic equation is:

$x^2 - Sx + P = 0$

$x^2 - (-3)x + 36 = 0$

$x^2 + 3x + 36 = 0$

**(c)** Roots of required equation are $\frac{1}{\alpha}, \frac{1}{\beta}$

**(K.B + A.B)**

Sum of roots $= S = \frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{\beta+\alpha}{\alpha\beta}$

$= \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{3}{6}$

$= \frac{1}{2}$

Product of roots $= P = \frac{1}{\alpha}.\frac{1}{\beta}$

$= \frac{1}{\alpha\beta}$

$= \frac{1}{6}$

$\therefore$ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$$

Multiply by '6'

$$6x^2 - 3x + 1 = 0$$

**(d)** Roots of required equation are $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

**(K.B + A.B)**

$$S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(3)^2 - 2(6)}{6}$$

$$= \frac{9-12}{6} = \frac{-3}{6}$$

$$\Rightarrow S = -\frac{1}{2}$$

$$P = \frac{\alpha}{\beta} \times \frac{\beta}{\alpha}$$

$$\Rightarrow P = 1$$

Required quadratic equation is

$$x^2 - Sx + p = 0$$

Or $x^2 - \left(-\frac{1}{2}\right)x + 1 = 0$

$x^2 + \frac{1}{2}x + 1 = 0$ (Multiplying by 2)

$$2x^2 + x + 2 = 0$$

**(e)** **Roots of required equation,**

$\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$ **(K.B + A.B)**

$$S = (\alpha + \beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= (\alpha + \beta) + \left(\frac{\beta + \alpha}{\alpha\beta}\right)$$

$$= (\alpha + \beta) + \left(\frac{\alpha + \beta}{\alpha\beta}\right)$$

$$= 3 + \frac{3}{6} = \frac{18+3}{6}$$

$$= \frac{21}{6} = \frac{7}{2}$$

$$P = (\alpha + \beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= (\alpha + \beta)\left(\frac{\beta + \alpha}{\alpha\beta}\right)$$

$$= 3\left(\frac{3}{6}\right)$$

$$= (3)\left(\frac{1}{2}\right)$$

$$P = \frac{3}{2}$$

Required equation is

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Multiplying by (2)

$$2x^2 - 7x + 3 = 0$$

**Q.3** **If α, β are the roots of the equation $x^2 + px + q = 0$ form equations whose roots are:**

**(a)** $\alpha^2, \beta^2$ **(FSD 2015)**

**(b)** $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

**Solution:**

$$x^2 + px + q = 0$$

Here

$$a = 1, b = p, c = q$$

Roots of given equation are $\alpha, \beta$

Then $\alpha + \beta = -\frac{b}{a} = -p$

$$\alpha\beta = \frac{c}{a} = q$$

**(a)** Roots of required equation are $\alpha^2, \beta^2$

**(K.B + A.B)**

Sum of roots = S = $\alpha^2 + \beta^2$

$$= (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-P)^2 - 2q$$

$$= p^2 - 2q$$

Product of roots = P = $\alpha^2\beta^2$

$= (\alpha\beta)^2$

$= (q)^2$

$= q^2$

$\therefore$ Required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - \left(p^2 - 2q\right)x + q^2 = 0$

**(b)** Roots of required equation are $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$

**(K.B + A.B)**

Sum of roots = S = $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$

$= \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \dfrac{p^2 - 2q}{q}$

Product of roots = P = $\dfrac{\alpha}{\beta} \cdot \dfrac{\beta}{\alpha}$

$= 1$

$\therefore$ Required quadratic equation is:

$x^2 - Sx + P = 0$

$\Rightarrow x^2 - \dfrac{p^2 - 2q}{q}x + 1 = 0$

Multiply by 'q'

$qx^2 - \left(p^2 - 2q\right)x + q = 0$

## Synthetic Division (K.B)

It is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial.

It is a short cut of long division.

**Example 3: (Page # 35) (A.B)**

**Use synthetic division, divide the polynomial $P(x) = 5x^4 + x^3 - 3x$ by $x - 2$.**

**Solution:**

$P(x) = 5x^4 + x^3 + 0x^2 - 3x + 0$

Here $x - a = x - 2 \Rightarrow x = 2$

| | | | | | |
|---|---|---|---|---|---|
| 2 | 5 | 1 | 0 | - 3 | 0 |
| | ↓ | 10 | 22 | 44 | 82 |
| | 5 | 11 | 22 | 41 | 82 |

$\therefore Q(x) = 5x^3 + 11x^2 + 22x + 41$

$R = 82$

**Example 3: (Page # 36) (A.B)**

Using synthetic division, find the value of $h$. If the zero of polynomial $p(x) = 3x^2 + 4x - 7h$ is 1.

**Solution:**

$p(x) = 3x^2 + 4x - 7h$ and its zero is 1.

Then by the synthetic division.

| | | | |
|---|---|---|---|
| 1 | 3 | 4 | -7$h$ |
| | | 3 | 7 |
| | 3 | 7 | 7-7$h$ |

Remainder $= 7 - 7h$

Since 1 is the zero of the polynomial, therefore,

Remainder $= 0$, that is

$7 - 7h = 0$

$7 = 7h$

$\Rightarrow h = 1$

**Example 4: (Page # 36) (A.B)**

Using synthetic division, find the values of $l$ and $m$, if $x - 1$ and $x + 1$ are the factors of the polynomial

$P(x) = x^3 + 3lx^2 + mx - 1$

**Solution:**

Since $x - 1$ and $x + 1$ are the factors of $P(x) = x^3 + 3lx^2 + mx - 1$

Therefore, 1 and –1 are zeros of polynomial $P(x)$.

Now by synthetic division

| | | | | |
|---|---|---|---|---|
| 1 | 1 | $3l$ | $m$ | $-1$ |
| | ↓ | $-1$ | $-3l+1$ | $3l+m+1$ |
| | 1 | $3l-1$ | $3l+m+1$ | $3l+m$ |

Since 1 is the zero of polynomial, therefore, remainder is zero, that is,

$3l + m = 0 \rightarrow (i)$

And

| | | | | |
|---|---|---|---|---|
| –1 | 1 | $3l$ | $m$ | $-1$ |
| | ↓ | $-1$ | $-3l+1$ | $3l-m-1$ |
| | 1 | $3l-1$ | $3l+m+1$ | $3l-m-2$ |

Since 1 is the zero of polynomial, therefore, remainder is zero, that is,

$3l - m - 2 = 0 \rightarrow (ii)$

Adding equations (i) and (ii), we get

$6l - 2 = 0$

$6l = 2 \Rightarrow l = \dfrac{2}{6} = \dfrac{1}{3}$

Put the value of $l$ in equation (i)

$3\left(\frac{1}{3}\right)+m=0$ or

$1+m=0 \Rightarrow m=-1$

Thus $l=\frac{1}{3}$ and $m=-1$

**Example 6: (Page # 38)** **(A.B)**

By synthetic division, solve the equation $x^4-49x^2+36x+252=0$ having roots $-2$ and 6.

**Solution:**

Since –2 and 6 are the roots of the given equation $x^4-49x^2+36x+252=0$.

Then by synthetic division, we get

| | | | | | |
|---|---|---|---|---|---|
| | 1 | 0 | –49 | 36 | 252 |
| –2 | ↓ | –2 | 4 | 90 | –252 |
| | 1 | –2 | –45 | 126 | 0 |
| 6 | | 6 | 24 | –126 | |
| | 1 | 4 | –21 | 0 | |

$\because$ The depressed equation is

$x^2+4x-21=0$

$x^2+7x-3x-21=0$

$x(x+7)-3(x+7)=0$

$(x+7)(x-3)=0$

Either $x+7=0$ or $x-3=0$

$x=-7$ or $x=3$

Thus $-2, 6-7$ and 3 are the roots of the given equation.

## Exercise 2.6

**Q.1 Use synthetic division to find the quotient and the remainder, when**

**(i)** $(x^2+7x-1)\div(x+1)$

**(ii)** $(4x^3-5x+15)\div(x+3)$

**(iii)** $(x^3+x^2-3x+2)\div(x-2)$

**Solution:**

**(i)** $P(x)=x^2+7x-1$

**(FSD 2016, 17) (A.B)**

| | | | |
|---|---|---|---|
| | 1 | 7 | –1 |
| –1 | ↓ | –1 | –6 |
| | 1 | 6 | –7 |

$\therefore Q(x) = x + 6$

R = –7

**(ii)** $P(x)=4x^3-5x+15$ **(A.B)**

**(SWL 2016, SGD 2017, MTN 2016)**

$= 4x^3+0x^2-5x+15$

| | | | | |
|---|---|---|---|---|
| | 4 | 0 | –5 | 15 |
| –3 | ↓ | –12 | 36 | –93 |
| | 4 | –12 | 31 | –78 |

$\therefore Q(x)=4x^2-12x+31$

R = –78

**(iii)** $P(x)=x^3+x^2-3x+2$ **(A.B)**

**(GRW 2017, FSD 2015, MTN 2017, D.G.K 2015)**

| | | | | |
|---|---|---|---|---|
| | 1 | 1 | –3 | 2 |
| 2 | ↓ | 2 | 6 | 6 |
| | 1 | 3 | 3 | 8 |

$\therefore Q(x)=x^2+3x+3$

R = 8

**Q.2 Find the value of h using synthetic division, if** **(A.B)**

**(i)** 3 is the zero of the polynomial $2x^3-3hx^2+9$

**(ii)** 1 is the zero of the polynomial $x^3-2hx^2+11$

**(iii)** –1 is the zero of the polynomial $2x^3+5hx-23$

**Solution:**

**(i)** $P(x)=2x^3-3hx^2+9$

**(SWL 2014) (A.B)**

$= 2x^3-3hx^2+0x+9$

| | | | | |
|---|---|---|---|---|
| | 2 | $-3h$ | 0 | 9 |
| 3 | ↓ | 6 | $-9h+18$ | $-27h+54$ |
| | 2 | $-3h+6$ | $-9h+18$ | $-27h+63$ |

Since 3 is the zero of given polynomial, $R=0$

$\Rightarrow$ –27h + 63 = 0

–27h = –63

$h = \frac{-63}{-27}$

$\Rightarrow$ $h = \frac{7}{3}$

**(ii)** $P(x) = x^3 - 2hx^2 + 11$ **(A.B)**

$= x^3 - 2hx^2 + 0x + 11$

| | | | | |
|---|---|---|---|---|
| | 1 | –2h | 0 | 11 |
| 1 | ↓ | 1 | –2h+1 | –2h+1 |
| | 1 | –2h+1 | –2h+1 | –2h+12 |

Since 1 is zero of given polynomial, R = 0

$\Rightarrow$ $-2h + 12 = 0$

$-2h = -12$

$h = \dfrac{-12}{-2}$

$h = 6$

**Result:**

h = 6

**(iii)** $P(x) = 2x^3 + 5hx - 23$ **(A.B)**

$= 2x^3 + 0x^2 + 5hx - 23$

| | | | | |
|---|---|---|---|---|
| | 2 | 0 | 5h | –23 |
| –1 | ↓ | –2 | 2 | –5h–2 |
| | 1 | –2h+1 | –2h+1 | –5h – 25 |

Since 1 is zero of given polynomial, R = 0

$\Rightarrow -5h - 25 = 0$

$-5h = 25$

$h = -\dfrac{25}{5} = -5$

**Result:**

$h = -5$

**Q.3** **Use synthetic division to find the values of *l* and m, if** **(A.B)**

**(i)** (*x* + 3) and (*x* – 2) are the factors of the polynomial $x^3 + 4x^2 + 2l + m$

**(ii)** (*x* – 1) and (*x* + 1) are the factors of the polynomial $x^3 - 3lx^2 + 2mx + 6$

**Solution:**

**(i)** $P(x) = x^3 + 4x^2 + 2lx + m$ **(A.B)**

| | | | | |
|---|---|---|---|---|
| | 1 | 4 | 2l | m |
| –3 | ↓ | –3 | –3 | –6l–9 |
| | 1 | 1 | 2l–3 | –6l + m + 9 |
| 2 | ↓ | 2 | 6 | |
| | 1 | 3 | 2l + 3 | |

Since *x* – 2 is a factor, R = 0

$\Rightarrow$ $2l + 3 = 0$

$2l = -3$

$\Rightarrow$ $l = \dfrac{-3}{2}$

Also $x + 3$ is a factor, R = 0

$\Rightarrow$ $-6l + m + 9 = 0$

$-6\left(\dfrac{-3}{2}\right) + m + 9 = 0 \quad \because l = \dfrac{-3}{2}$

$9 + m + 9 = 0$

$m + 18 = 0$

$\Rightarrow$ $m = -18$

**Result**

$l = \dfrac{-3}{2}$, $m = -18$

**(ii)** $P(x) = x^3 - 3lx^2 + 2mx + 6$ **(A.B)**

| | | | | |
|---|---|---|---|---|
| | 1 | –3l | 2m | 6 |
| 1 | ↓ | 1 | –3l+1 | –3l+2m+1 |
| | 1 | –3l +1 | –3l+2m+1 | –3l + 2m + 7 |
| -1 | ↓ | –1 | 3l | |
| | 1 | –3l | 2m + 1 | |

Since $x + 1$ is a factor, R = 0

$\Rightarrow$ $2m + 1 = 0$

$2m = -1$

$\Rightarrow$ $m = \dfrac{-1}{2}$

Also *x* – 1 is a factor, R = 0

$-3l + 2m + 7 = 0$

$-3l + 2\left(-\dfrac{1}{2}\right) + 7 = 0 \because m = -\dfrac{1}{2}$

$-3l - 1 + 7 = 0$

$-3l + 6 = 0$

$-3l = -6$

$\Rightarrow$ $l = 2$

**Result**

$l = 2$, $m = -\dfrac{1}{2}$

**Q.4** **Solve by using synthetic division, if**

**(i)** 2 is the root of the equation $x^3 - 28x + 48 = 0$

**(ii)** 3 is the root of the equation $2x^3 - 3x^2 - 11x + 6 = 0$

**(iii)** –1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$

**Solution:**

**(i)** $P(x) = x^3 - 28x + 48$ **(A.B)**

$= x^3 + 0x^2 - 28x + 48$

| | | | | |
|---|---|---|---|---|
| | 1 | 0 | –28 | 48 |
| 2 | ↓ | 2 | 4 | –48 |
| | 1 | 2 | –24 | 0 |

$\therefore$ Depressed equation is:

$x^2 + 2x - 24 = 0$

$x^2 + 6x - 4x - 24 = 0$

$x(x+6) - 4(x+6) = 0$

$(x+6)(x-4) = 0$

Either

$x + 6 = 0$ or $x - 4 = 0$

$x = -6$ $\quad$ $x = 4$

Thus 2, 4 and –6 are the roots of the given equation.

$\therefore$ Solution Set $= \{2, 4, -6\}$

**(ii)** $P(x) = 2x^3 - 3x^2 - 11x + 6$ **(A.B)**

| | | | | |
|---|---|---|---|---|
| | 2 | –3 | –11 | 6 |
| 3 | ↓ | 4 | 9 | –6 |
| | 2 | 3 | –2 | 0 |

$\therefore$ Depressed equation is:

$2x^2 + 3x - 2 = 0$

$2x^2 + 4x - x - 2 = 0$

$2x(x+2) - 1(x+2) = 0$

$(x+2)(2x-1) = 0$

Either

$x + 2 = 0$ Or $2x - 1 = 0$

$x = -2$ $\quad$ $2x = 1$

$x = \frac{1}{2}$

$\therefore$ **Solution Set** $= \left\{3, -2. \frac{1}{2}\right\}$

**(iii)** $P(x) = 4x^3 - x^2 - 11x - 6$ **(A.B)**

| | | | | |
|---|---|---|---|---|
| | 4 | –1 | –11 | –6 |
| –1 | ↓ | –4 | 5 | 6 |
| | 4 | –5 | –6 | 0 |

$\therefore$ Depressed equation is

$4x^2 - 5x - 6 = 0$

$4x^2 - 8x + 3x - 6 = 0$

$4x(x-2) + 3(x-2) = 0$

$(x-2)(4x+3) = 0$

Either

$x - 2 = 0$ or $4x + 3 = 0$

$x = 2$ $\quad$ $4x = -3$

$x = -\frac{3}{4}$

Thus $-1, 2, -\frac{3}{4}$ are the roots of the given equation.

$\therefore$ **Solution Set** $= \left\{-1, 2, -\frac{3}{4}\right\}$

**Q.5 Solve by using synthetic division, if**

**(i)** 1 and 3 are the roots of the equation $x^4 - 10x^2 + 9 = 0$

**(ii)** 3 and –4 are the roots of the equation $x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$

**Solution:** **(A.B)**

**(i)** $P(x) = x^4 + 0x^3 - 10x^2 + 0x + 9$

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 0 | –10 | 0 | 9 |
| | | 1 | 1 | –9 | –9 |
| 3 | 1 | 1 | –9 | –9 | 0 |
| | | 3 | 12 | 9 | |
| | 1 | 4 | 3 | 0 | |

$\therefore$ Depressed equation is

$x^2 + 4x + 3 = 0$

$x^2 + 3x + x + 3 = 0$

$x(x+3) + 1(x+3) = 0$

$(x+3)(x+1) = 0$

Either

$x + 3 = 0$ or $x + 1 = 0$

$x = -3$ $\quad$ $x = -1$

$\therefore$ **Solution Set** $= \{\pm 3, \pm 1\}$

**(ii)** $P(x) = x^4 + 2x^3 - 13x^2 - 14x + 24$

| | | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | –13 | –14 | 24 |
| 3 | ↓ | 3 | 15 | 6 | –24 |
| | 1 | 5 | 2 | –8 | 0 |
| –4 | ↓ | –4 | –4 | 8 | |
| | 1 | 1 | –2 | 0 | |

$\therefore$ Depressed equation is:

$x^2+2x-x-2=0$

$x(x+2)-1(x+2)=0$

$x(x+2)-1(x+2)=0$

$(x+2)(x-1)=0$

Either

$x+2=0$ or $x-1=0$

$x=-2$ $x=1$

Thus $-2, -4, 1$ and $3$ are the roots of the given equation.

$\therefore$ **Solution Set** $=\{-2,-4,1,3\}$

**Simultaneous Equation** **(K.B)**

A system of equations having a common solution is called a system of simultaneous equations.

For example $x+2y=3,\ 2x-y=1$

having same solution (1,1).

**Solution Set** **(K.B)**

The set of all the ordered pairs $(x,y)$, which satisfies the system of equations is called the solution set of the system.

**Ordered Pair** **(K.B)**

An ordered pair of real numbers $x$ and $y$ is a pair $(x,y)$ in which elements are written in specific order.

For example: $(x,y)$ is an ordered pair in which first elements (abscissa) is $x$ and second element (ordinate) is $y$.

**Note** **(K.B)**

$(x,y)\neq(x,y)$. For example $(2,3)$ and $(3,2)$ are two different ordered pairs.

**Example 1 (Page # 39)** **(A.B)**

Solve the system of equations

$3x+y=4$ and $3x^2+y^2=52$.

**Solution:**

The given equations are

$3x+y=4\rightarrow$ (i)

$3x^2+y^2=52\rightarrow$ (ii)

From equation (i) $y=4-3x\rightarrow$ (iii)

Put value of y in equation (ii)

$3x^2+(4-3x)^2=52$

$3x^2+16-24x+9x^2-52=0$

$12x^2-24x-36=0$

$12(x^2-2x-3)=0$

$x^2-2x-3=0\ \because 12\neq0$

By factorization

$x^2-3x+x-3=0$

$x(x-3)+1(x-3)=0$

$(x-3)(x+1)=0$

Either

$x-3=0$ or $x+1=0$

$x=3$ or $x=-1$

Put the values of $x$ is equation (iii)

| When $x=3$ | when $x=-1$ |
|---|---|
| $y=4-3x$ | $y=4-3x$ |
| $y=4-3(3)$ | $y=4-3(-1)$ |
| $=4-9$ | $=4+3$ |
| $y=-5$ | $y=7$ |

$\therefore$ ordered pairs are $(3,-5)$ and $(-1,7)$

Thus, the solution set is $\{(3,-5),(-1,7)\}$

**Example 2 (Page # 40)** **(A.B)**

Solve the equations

$x^2+y^2+2x=8$ and $(x-1)^2+(y+1)^2=8$

**Solution:**

The given equation are

$x^2+y^2+2x=8\rightarrow$ (i)

$(x-1)^2+(y+1)^2=8\rightarrow$ (ii)

From equation (ii), we get

$x^2-2x+1+y^2+2y+1=8$

Or $x^2+y^2-2x+2y=6\rightarrow(iii)$

Subtracting equation (iiii) from equation (i) we have

$4x-2y=2$ or $2x-y=1$

$\Rightarrow$ $y=2x-1$

Put the value of y in equation (ii)

$(x-1)^2+(2x-1+1)^2=8$
$x^2-2x+1+4x^2-8=0$
$5x^2-2x-7=0$
$5x^2-7x+5x-7=0$
$x(5x-7)+1(5x-7)=0$
$(5x-7)(x+1)=0$
Either
$5x-7=0$ or $x+1=0$
$5x=7$ or $x=-1$
$\Rightarrow x=\frac{7}{5}$

Now putting the values of $x$ in equation (iv), we have

When $x=\frac{7}{5}$ when $x=-1$

$y=2\left(\frac{7}{5}\right)-1$ $y=2(-1)-1$

$y=\frac{14}{5}-1=\frac{14-5}{5}=\frac{9}{5}$ $y=-3$

Thus, the solution set is

$\left\{(-1,-3),\left(\frac{7}{5},\frac{9}{5}\right)\right\}$

**Example 3 (Page # 41)** **(A.B)**

Solve the equations
$x^2+y^2=7$ and $2x^2+3y^2=18$

**Solution:**

Given equations are
$x^2+y^2=7$ (i)
$2x^2+3y^2=18$ (ii)
Multiply equation (i) with 3
$3x^2+3y^2=21$ (iii)
Subtracting equations (ii) from (iii)
$x^2=3 \Rightarrow x=\pm\sqrt{3}$
When $x=\sqrt{3}$ then from equation (i)
$x^2+y^2=7$ or
$3+y^2=7 \Rightarrow y^2=4 \Rightarrow y=\pm 2$
When $x=-\sqrt{3}$, then $y=\pm 2$
Thus, the required solution set is
$\left\{(\pm\sqrt{3},\pm 2)\right\}$.

**Example 4 (Page # 41)** **(A.B)**

Solve the equations
$x^2+y^2=20$ (i)
$6x^2+xy-y^2=0$ (ii)
The equation (ii) can be written as
$y^2-xy-6x^2=0$

$$\Rightarrow \quad y=\frac{-(-x)\pm\sqrt{(-x)^2-4\times 1\times(-6x^2)}}{2\times 1}$$

$$=\frac{x\pm\sqrt{x^2+24x^2}}{2}=\frac{x\pm\sqrt{25x^2}}{2}$$

$$=\frac{x\pm 5x}{2}$$

We have $y=\frac{x+5x}{2}=\frac{6x}{2}=3x$

Or $y=\frac{x-5x}{2}=\frac{-4x}{2}=-2x$

Substituting $y=3x$ in the equation (i), we get
$x^2+(3x)^2=20$
$x^2+9x^2=20$
$\Rightarrow 10x^2=20$
$\Rightarrow x^2=4 \Rightarrow x=\pm\sqrt{2}$
When $x=\sqrt{2}, y=3(\sqrt{2})=3\sqrt{2}$ and
when $x=-\sqrt{2}$, $y=3(-\sqrt{2})=-3\sqrt{2}$

Substituting $y=-2x$ in the equation (i), we have
$x^2+(-2x)^2=20$ or $x^2+4x^2=20$
$\Rightarrow 5x^2=20 \Rightarrow x^2=4 \Rightarrow x=\pm 2$
When $x=2 \Rightarrow y=-2(2)=-4$
when $x=-2 \Rightarrow y=-2(-2)=4$
Thus, the solution is
$\left\{(\sqrt{2},3\sqrt{2}),(-\sqrt{2},-3\sqrt{2}),(2,-4),(-2,4)\right\}$.

**Example 5 (Page # 42)** **(A.B)**

Solve the equations
$x^2+y^2=40$ and $3x^2-2xy-y^2=80$

**Solution:**

Given equations are
$x^2+y^2=40 \rightarrow$ (i)

$3x^2-2xy-y^2=80 \rightarrow (\text{ii})$

Multiplying equation (i) by 2, we have

$2x^2+2y^2=80 \rightarrow (\text{iii})$

Subtracting the equation (iii) from equation (ii), we get

$x^2-2xy-3y^2=0 \rightarrow (\text{iv})$

The equation (iv) can be written as

$x^2-3xy+xy-3y^2=0$

$x(x-3y)+y(x-3y)=0$

$(x-3y)(x+y)=0$

Either $x-3y$ or $x+y=0$

$\Rightarrow x=3y \rightarrow (v)$ or $x=-y \rightarrow (vi)$

Put in equation (i),

Equation (i) $\Rightarrow (3y)^2+y^2=40$

$10y^2=40$

$y^2=4$

$y=\pm 2$

$y=2$ $\quad$ $y=-2$

eq. $(v)\Rightarrow$

$x=3(2)$ $\quad$ $x=3(-2)$

$x=6$ $\quad$ $x=-6$

Equation (i) $\Rightarrow (-y)^2+y^2=40$

$2y^2=40$

$y^2=20$

$y=\pm 2\sqrt{5}$

$y=2\sqrt{5}$ $\quad$ $y=-2\sqrt{5}$

$eq.(vi)\Rightarrow$

$x=-\left(2\sqrt{5}\right)$ $\quad$ $x=-\left(-2\sqrt{5}\right)$

$x=-2\sqrt{5}$ $\quad$ $=2\sqrt{5}$

$\therefore$ The solution set is

$\left\{(6,2),(-6,-2),\left(2\sqrt{5},-2\sqrt{5}\right),\left(-2\sqrt{5},2\sqrt{5}\right)\right\}$

## Exercise 2.7

**Solve the following simultaneous equations.**

**Q.1** $x+y=5$ **(A.B)**

$x^2-2y-14=0$

**Solution:**

$x+y=5 \rightarrow (\text{i})$

$x^2-2y-14=0 \rightarrow (\text{ii})$

From equation (i)

$y=5-x \rightarrow (\text{iii})$

Put in equation (ii)

$x^2-2y-14=0$

$x^2-2(5-x)-14=0$

$x^2-10+2x-14=0$

$x^2+2x-24=0$

$x^2+6x-4x-24=0$

$x(x+6)-4(x+6)=0$

$(x+6)(x-4)=0$

Either

$x+6=0$ or $x-4=0$

$\Rightarrow$ $x=-6$ $\quad$ $x=4$

Put in equation (iii)

$y=5-x$ $\quad$ $y=5-x$

$=5-(-6)$ $\quad$ $=5-4$

$=5+6$ $\quad$ $=1$

$=11$

$\therefore$ **Solution Set** $=\{(-6,11),(4,1)\}$

**Q.2** $3x-2y=1$ **(A.B)**

$x^2+xy-y^2=1$

**Solution:**

$3x-2y=1 \rightarrow (\text{i})$

and $x^2+xy-y^2=1 \rightarrow (\text{ii})$

From equation (i)

$3x-2y=1$

$3x=2y+1$

$x=\dfrac{2y+1}{3} \rightarrow (\text{iii})$

Now put in equation (ii)

$x^2+xy-y^2=1$

$$\left(\frac{1+2y}{3}\right)^2+\left(\frac{1+2y}{3}\right)y-y^2=1$$

$$\frac{1+4y+4y^2}{9}+\frac{y+2y^2}{3}-y^2=1$$

Multiplying both sides by LCM

We get

$$1+4y+4y^2+3(y+2y^2)-9y^2=9$$

$$1+4y+4y^2+3y+6y^2-9y^2-9=0$$

$$y^2+7y-8=0$$

$$y^2+8y-y-8=0$$

$$\Rightarrow y(y+8)-1(y+8)=0$$

$$(y+8)(y-1)=0$$

Either

$y+8=0$ or $y-1=0$

$y=-8$ $y=1$

Putting values in equation (iii)

When $y=-8$

$$x=\frac{1+2y}{3}$$

$$x=\frac{1+2(-8)}{3}$$

$$x=\frac{1-16}{3}$$

$$x=\frac{-15}{3}$$

$x=-5$

When $y=1$

$$x=\frac{1+2y}{3}$$

$$x=\frac{1+2(1)}{3}$$

$$x=\frac{3}{3}$$

$x=1$

$\therefore$ **Solution Set** = $\{(-5,-8),(1,1)\}$

**Q.3** $x-y=7$ **(A.B)**

$$\frac{2}{x}-\frac{5}{y}=2$$

**Solution:**

$x-y=7\rightarrow$ (i)

$$\frac{2}{x}-\frac{5}{y}=2\rightarrow(\text{ii})$$

From equation (ii)

$$\frac{2}{x}-\frac{5}{y}=2$$

Multiple by $xy$

$$2y-5x=2xy$$

$$2y-5x-2xy=0$$

$$2y-2xy-5x=0\rightarrow(\text{iii})$$

From equation (i)

$$x=7+y\rightarrow(\text{iv})$$

Put in equation (iii)

$$2y-2(7+y)(y)-5(7+y)=0$$

$$2y-14y-2y^2-5y-35=0$$

$$-2y^2-17y-35=0$$

$$-(2y^2+17y+35)=0$$

$$\Rightarrow 2y^2+17y+35=0$$

$$2y^2+10y+7y+35=0$$

$$2y(y+5)+7(y+5)=0$$

$$(y+5)(2y+7)=0$$

Either

$y+5=0$ or $2y+7=0$

$y=-5$ $y=\frac{-7}{2}$

Now put in equation (i)

when $y=-5$

$$x-(-5)=7$$

$$x+5=7$$

$$x=7-5$$

$$x=2$$

when

$$y=\frac{-7}{2}$$

$$x-\left(\frac{-7}{2}\right)=7$$

$$x+\frac{7}{2}=7$$

$$x=7-\frac{7}{2}$$

$$x=\frac{14-7}{2}$$

$$x=\frac{7}{2}$$

$\therefore$ **Solution Set** = $\left\{(2,-5),\left(\frac{7}{2},\frac{-7}{2}\right)\right\}$

**Q.4** $x+y=a-b$ **(A.B)**

$$\frac{a}{x}-\frac{b}{y}=2$$

**Solution:**

$$x+y=a-b\rightarrow \text{(i)}$$

$$\frac{a}{x}-\frac{b}{y}=2\rightarrow \text{(ii)}$$

From equation (ii)

$$\frac{a}{x}-\frac{b}{y}=2$$

Multiply by '$xy$'

$$ay-bx=2xy\longrightarrow \text{(iii)}$$

From equation (i)

$$\Rightarrow x+y=a-b$$

$$y=a-b-x\rightarrow (\text{iv})$$

Put in equation (iii)

$$a(a-b-x)-bx=2x(a-b-x)$$

$$a^2-ab-ax-bx=2ax-2bx-2x^2$$

$$2x^2-ax-2ax-bx+2bx+a^2-ab=0$$

$$2x^2-3ax+bx+a^2-ab=0$$

$$2x^2-(3a-b)x+(a^2-ab)=0$$

Here $A=2, B=-(3a-b), C=a^2-ab$

Using quadratic formula

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{-[-(3a-b)]\pm\sqrt{[-(3a-b)]^2-4(2)(a^2-ab)}}{2(2)}$$

$$x=\frac{3a-b\pm\sqrt{(3a-b)^2-8(a^2-ab)}}{4}$$

$$=\frac{3a-b\pm\sqrt{9a^2-6ab+b^2-8a^2+8ab}}{4}$$

$$=\frac{3a-b\pm\sqrt{a^2+2ab+b^2}}{4}$$

$$=\frac{3a-b\pm\sqrt{(a+b)^2}}{4}$$

$$=\frac{3a-b\pm(a+b)}{4}$$

Either

$x=\frac{3a-b+a+b}{4}$ or $x=\frac{3a-b-a-b}{4}$

$=\frac{4a}{4}$ | $=\frac{2a-2b}{4}$

$x=a$ | $x=\frac{a-b}{2}$

Put in equation (iv)

$y=a-b-a$ | $y=a-b-\frac{a-b}{2}$

$y=-b$ | $y=\frac{2a-2b-(a-b)}{2}$

$y=\frac{a-b}{2}$

$\therefore$ **Solution Set** = $\left\{(a,-b),\left(\frac{a-b}{2},\frac{a-b}{2}\right)\right\}$

**Q.5** $x^2+(y-1)^2=10$ **(A.B)**

$$x^2+y^2+4x=1$$

**Solution:**

$$x^2+(y-1)^2=10\rightarrow \text{(i)}$$

$$x^2+y^2+4x=1\rightarrow \text{(ii)}$$

Equation $(\text{i})\Rightarrow$

$$x^2+y^2-2y+1=10$$

$$x^2+y^2-2y=9\rightarrow \text{(iii)}$$

Subtract equation (ii) and (iii)

$$x^2+y^2+4x \quad = 1$$
$$\underline{-x^2 \pm y^2 \quad \mp 2y = -9}$$
$$4x+2y=-8$$
$$2(2x+y)=-8$$
$$2x+y=-4$$
$$y=-4-2x \rightarrow \text{(iv)}$$

Put in equation (ii)

$$x^2+(-4-2x)^2+4x=1$$
$$x^2+\left[-(4+2x)\right]^2+4x=1$$
$$x^2+(4+2x)^2+4x=1$$
$$x^2+16+16x+4x^2+4x-1=0$$
$$5x^2+2x+15=0$$
$$5\left(x^2+4x+3\right)=0$$
$$\Rightarrow x^2+4x+3=0 \quad \because 5\neq 0$$
$$x^2+3x+x+3=0$$
$$x(x+3)+1(x+3)=0$$
$$(x+3)(x+1)=0$$

Either

$x+3=0$ or $x+1=0$

$x=-3$ $\quad$ $x=-1$

Put in equation (iv)

y = – 4 – 2 (–3) $\quad$ y = – 4 – 2 (–1)

y = – 4 + 6 $\quad$ y = – 4 + 2

y = 2 $\quad$ y = –2

$\therefore$ **Solution Set**= $\{(-3,\ 2),(-1,\ -2)\}$

**Q.6** $(x+1)^2+(y+1)^2=5$ **(A.B)**

$(x+2)^2+y^2-5$

**Solution:**

$$(x+1)^2+(y+1)^2=5 \rightarrow (i)$$
$$(x+2)^2+y^2=5 \rightarrow \text{(ii)}$$

From equation (i) $\Rightarrow$

$$x^2+2x+1+y^2+2y+1=5$$
$$x^2+y^2+2x+2y=3$$
$$x^2+y^2+2x+2y=3 \rightarrow \text{(iii)}$$
$$(x+2)^2+y^2=5$$
$$x^2+4x+4+y^2=5$$
$$x^2+y^2+4x=1 \rightarrow \text{(iv)}$$

Sub equation (iii) and (iv)

$$x^2+y^2+2x+2y=-3$$
$$\underline{-x^2 \pm y^2 \pm 4x \quad = -1}$$
$$-2x+2y=2$$
$$-x+y=1$$
$$y=x+1 \rightarrow \text{(v)}$$

Put in equation (iv)

$$x^2+(x+1)^2+4x=1$$
$$x^2+x^2+2x+1+4x-1$$
$$2x^2+6x=0$$
$$2x(x+3)=0$$

Either

$2x=0$ or $x+3=0$

$x=0$ $\quad$ $x=-3$

Put in equation (v)

When $x=0$

y = 1 + 0

y = 1

When $x=-3$

$y=1-3$

$y=-2$

$\therefore$ **Solution Set** $=\{(0,1),(-3,-2)\}$

**Q.7** $x^2+2y^2=22$ **(A.B)**

$5x^2+y^2=29$

**Solution:**

$$x^2+2y^2=22 \rightarrow (i)$$
$$5x^2+y^2=29 \rightarrow (ii)$$

Multiply equation (ii) by '2'

$$10x^2+2y^2=58 \longrightarrow \text{(iii)}$$

Subtract equation (i) and (iii)

$$10x^2+2y^2=58$$
$$\underline{-x^2 \pm 2y^2 = -22}$$
$$9x^2 \quad =36$$
$$x^2=4 \quad \text{(Div both sides by 9)}$$

Taking square root on both sides

$$x=\pm 2$$

Put $x^2 = 4$ in equation (ii)
$5(4) + y^2 = 29$
$20 + y^2 = 29$
$y^2 = 29 - 20$
$y^2 = 9$
Taking square root
$y = \pm 3$
$\therefore$ **Solution Set** $= \{(\pm 2, \pm 3)\}$

**Q.8** $4x^2 - 5y^2 = 6$ **(A.B)**
$3x^2 + y^2 = 14$

**Solution:**
$4x^2 - 5y^2 = 6 \rightarrow (i)$
$3x^2 + y^2 = 14 \rightarrow (ii)$
Multiply equation (ii) by 5
$15x^2 + 5y^2 = 70 \rightarrow (iii)$
Adding equation (i) and (iii)
$4x^2 - 5y^2 = 6$
$15x^2 + 5y^2 = 70$
$19x^2 = 76$
$x^2 = \frac{76}{19}$
$x^2 = 4$
Taking square root
$x = \pm 2$
Put $x^2 = 4$ in equation (ii)
$3(4) + y^2 = 14$
$y^2 = 14 - 12$
$y^2 = 2$
Taking square root
$y = \pm\sqrt{2}$
$\therefore$ **Solution Set** $= \{(\pm 2, \pm\sqrt{2})\}$

**Q.9** $7x^2 - 3y^2 = 4$ **(A.B)**
$2x^2 + 5y^2 = 7$

**Solution:**
$7x^2 - 3y^2 = 4 \rightarrow (i)$
$2x^2 + 5y^2 = 7 \rightarrow (ii)$
Multiply equation (i) by '5'
$35x^2 - 15y^2 = 20 \rightarrow (iii)$
Multiply equation (ii) by '3'
$6x^2 + 15y^2 = 21 \rightarrow (iv)$
Add equation (iii) and (iv)
$35x^2 - 15y^2 = 20$
$6x^2 + 15y^2 = 21$
$41x^2 = 41$
$x^2 = 1$
On taking square root, we get
$x = \pm 1$
Put $x^2 = 1$ in equation (ii)
$2x^2 + 5y^2 = 7$
$2(1) + 5y^2 = 7$
$2 + 5y^2 = 7$
$5y^2 = 5$
$y^2 = \frac{5}{5}$
$y^2 = 1$
On taking the square root, we get
$y = \pm 1$
$\therefore$ **Solution Set** $= \{(\pm 1, \pm 1)\}$

**Q.10** $3x^2 - y^2 = 3$ **(A.B)**
$x^2 + 4xy - 5y^2 = 0$

**Solution:**
$3x^2 - y^2 = 3 \rightarrow (i)$
$x^2 + 4xy - 5y^2 = 0 \rightarrow (ii)$
Equation (ii) $\Rightarrow$
$x^2 + 4xy - 5y^2 = 0$
$x^2 + 5xy - xy - 5y^2 = 0$
$x(x + 5y) - y(x + 5y) = 0$
$(x + 5y)(x - y) = 0$
Either
$x + 5y = 0$ or $x - y = 0$
$x = -5y \rightarrow (iii)$ $x = y \rightarrow (iv)$
Put $x = -5y$ in equation (i)
$x^2 + 2y^2 = 3$
$(-5y)^2 + 2y^2 = 3$
$25y^2 + 2y^2 = 3$
$27y^2 = 3$

$y^2 = \frac{3}{27}$

$y^2 = \frac{1}{9}$

On taking square root, we get

$y = \pm\frac{1}{3}$

Putting in equation (iii)

When

$y = \frac{1}{3}$

$x = -5\left(\frac{1}{3}\right)$

$x = -\frac{5}{3}$

When $y = -\frac{1}{3}$

$x = -5\left(-\frac{1}{3}\right)$

$x = \frac{5}{3}$

Put $x = y$ in equation (i)

$x^2 + 2y^2 = 3$

$y^2 + 2y^2 = 3$

$3y^2 = 3$

$y^2 = 1$

On taking square root, we get

$y = \pm 1$

Put in equation (iv)

When y = 1

$x = 1$

When $y = -1$

$x = -1$

$\therefore$ **Solution Set**

$= \left\{(1,1), (-1,-1), \left(-\frac{5}{3}, \frac{1}{3}\right), \left(\frac{5}{3}, -\frac{1}{3}\right)\right\}$

**Q.11** $3x^2 - y^2 = 26$ **(A.B)**

$3x^2 - 5xy - 12y^2 = 0$

**Solution:**

$3x^2 - y^2 = 26 \rightarrow$ (i)

$3x^2 - 5xy - 12y^2 = 0 \rightarrow$ (ii)

Equation (ii) $\Rightarrow$ $3x^2 - 5xy - 12y^2 = 0$

$3x^2 - 9xy + 4xy - 12y^2 = 0$

$3x(x - 3y) + 4y(x - 3y) = 0$

$(x - 3y)(3x + 4y) = 0$

Either

$x - 3y = 0 \rightarrow (a)$ or $3x + 4y = 0 \rightarrow (b)$

Equation $(a) \Rightarrow x = 3y \rightarrow$ (iii)

Put in equation (i)

$3(3y)^2 - y^2 = 26$

$3(9y^2) - y^2 = 26$

$27y^2 - y^2 = 26$

$26y^2 = 26$

$y^2 = 1$

$\Rightarrow$ $y = \pm 1$

Put in equation (iii)

When y = – 1

$x = 3(-1)$

$x = -3$

When y = 1

$x = 3(1)$

$x = 3$

Equation $(b) \Rightarrow 3x + 4y = 0$

$3x = -4y$

$x = \frac{-4}{3}y \rightarrow$ (iv)

Put in equation (i)

$3\left(\frac{-4}{3}y\right)^2 - y^2 = 26$

$\left(\frac{16}{3}y^2\right) - y^2 = 26$

$\frac{16y^2 - 3y^2}{3} = 26$

$13y^2 = 26 \times 3$

$y^2 = 6$

$\Rightarrow y=\pm\sqrt{6}$

Put in equation (iv)

When $y=-\sqrt{6}$

$x=\frac{-4}{3}\left(-\sqrt{6}\right)$

$x=\frac{4\sqrt{6}}{3}$

When $y=\sqrt{6}$

$x=\frac{-4}{3}\left(\sqrt{6}\right)$

$x=\frac{-4\sqrt{6}}{3}$

$\therefore$ **Solution Set**

$=\left\{(-3,-1),(3,1),\left(\frac{-4\sqrt{6}}{3},\sqrt{6}\right),\left(\frac{4\sqrt{6}}{3},-\sqrt{6}\right)\right\}$

**Q.12** $x^2+xy=5$ **(A.B)**

$y^2+xy=3$

**Solution:**

$x^2+xy=5\rightarrow(\text{i})$

$y^2+xy=3\rightarrow(\text{ii})$

Multiply equation (i) by '3' and equation (ii) '5'

$3x^2+3xy=15\rightarrow(\text{iii})$

$5y^2+5xy=15\rightarrow(\text{iv})$

Subtraction equation (iii) and (iv)

$3x^2+3xy=15$

$\underline{5y^2+5xy=15}$

$3x^2-2xy-5y^2=0\rightarrow(\text{v})$

Equation (v)

$3x^2-5xy+3xy-5y^2=0$

$x(3x-5y)+y(3x-5y)=0$

$(3x-5y)(x+y)=0$

Either

$3x-5y=0$ or $x+y=0$

$x=\frac{5y}{3}\rightarrow(\text{vi})$ $x=-y\rightarrow(\text{vii})$

Put $x=\frac{5y}{3}$ in equation (i)

$\left(\frac{5y}{3}\right)^2+\left(\frac{5y}{3}\right)y=5$

$\frac{25}{9}y^2+\frac{5y^2}{3}=5$

$\frac{25y^2+15y^2}{9}=5$

$25y^2+15y^2=45$

$40y^2=45$

$y^2=\frac{9}{8}$

On taking square root, we get.

$y=\pm\frac{3}{2\sqrt{2}}$

Put $y=\frac{3}{2\sqrt{2}}$ in equation (vi)

$x=\frac{5}{3}\times\frac{3}{2\sqrt{2}}$

$x=\frac{5}{2\sqrt{2}}$

Now put $y=\frac{-3}{2\sqrt{2}}$ in equation (vi)

$x=\frac{5}{3}\left(\frac{-3}{2\sqrt{2}}\right)$

$x=\frac{-5}{2\sqrt{2}}$

Now put $x=-y$ in equation (i)

$(-y)^2+(-y)(y)=5$

$y^2-y^2=5$

$0=5$

But $0\neq5$

$\therefore$ **Solution Set** $=\left\{\left(\frac{5}{2\sqrt{2}},\frac{3}{2\sqrt{2}}\right),\left(\frac{-5}{2\sqrt{2}},\frac{-3}{2\sqrt{2}}\right)\right\}$

**Alternate Method**

$x^2+xy=5\rightarrow(\text{i})$

$y^2+xy=3\rightarrow(\text{ii})$

$eq(i)\Rightarrow$

$x(x+y)=5\rightarrow(iii)$

$eq(ii)\Rightarrow$

$y(x+y)=3\rightarrow(iv)$

Dividing equation (*iii*) and (*iv*)

$$\frac{x(x+y)}{y(x+y)}=\frac{5}{3}$$

$$\frac{x}{y}=\frac{5}{3}$$

$$x=\frac{5y}{3}$$

Put $x=\frac{5y}{3}$ in equation (i)

$$\left(\frac{5y}{3}\right)^2+\left(\frac{5y}{3}\right)y=5$$

$$\frac{25}{9}y^2+\frac{5y^2}{3}=5$$

$$\frac{25y^2+15y^2}{9}=5$$

$$25y^2+15y^2=45$$

$$40y^2=45$$

$$y^2=\frac{9}{8}$$

On taking square root, we get.

$$y=\pm\frac{3}{2\sqrt{2}}$$

Put $y=\frac{3}{2\sqrt{2}}$ in equation (vi)

$$x=\frac{5}{3}\times\frac{3}{2\sqrt{2}}$$

$$x=\frac{5}{2\sqrt{2}}$$

Now put $y=\frac{-3}{2\sqrt{2}}$ in equation (vi)

$$x=\frac{5}{3}\left(\frac{-3}{2\sqrt{2}}\right)$$

$$x=\frac{-5}{2\sqrt{2}}$$

$\therefore$ **Solution Set** $=\left\{\left(\frac{5}{2\sqrt{2}},\frac{3}{2\sqrt{2}}\right),\left(\frac{-5}{2\sqrt{2}},\frac{-3}{2\sqrt{2}}\right)\right\}$

**Q.13** $x^2-2xy=7$ **(A.B)**

$xy+3y^2=2$

**Solution:**

$x^2-2xy=7\rightarrow$ (i)

$xy+3y^2=2\rightarrow$ (ii)

Multiply equation (i) by '2' and equation (ii) by '7'

$2x^2-4xy=14\rightarrow$ (iii)

$7xy+21y^2=14\rightarrow$ (iv)

Subtract equation (iii) and (iv)

$$2x^2-4xy=14$$

$$\underline{-7xy\pm21y^2=-14}$$

$$2x^2-11xy-21y^2=0$$

$$2x^2-14xy+3xy-21y^2=0$$

$$2x(x-7y)+3y(x-7y)=0$$

$$(x-7y)(2x+3y)=0$$

Either

$x-7y=0$ or $2x+3y=0$

| | |
|---|---|
| $x=7y\longrightarrow$(v) | $x=\frac{-3}{2}y\longrightarrow$(vi) |
| Put in equation (ii) | Put in equation (ii) |
| $(7y)y+3y^2=2$ | $\left(\frac{-3}{2}y\right)y+3y^2=2$ |
| $7y^2+3y^2=2$ | $\frac{-3y^2+6y^2}{2}=2$ |
| $10y^2=2$ | $3y^2=4$ |
| $5y^2=1$ | $y^2=\frac{4}{3}$ |
| $y^2=\frac{1}{5}$ | $y=\pm\frac{2}{\sqrt{3}}$ |
| $y=\pm\frac{1}{\sqrt{5}}$ | Put in equation (vi) |
| Put in equation (v) | when $y=\frac{-2}{\sqrt{3}}$ |
| when $y=-\frac{1}{\sqrt{5}}$ | $x=\frac{-3}{2}\left(-\frac{2}{\sqrt{3}}\right)$ |
| $x=7\left(-\frac{1}{\sqrt{5}}\right)$ | $x=\frac{3}{\sqrt{3}}$ |

$x = \frac{-7}{\sqrt{5}}$

when

$y = \frac{1}{\sqrt{5}}$

$x = 7\left(\frac{1}{\sqrt{5}}\right)$

$x = \frac{7}{\sqrt{5}}$

$x = \sqrt{3}$

when $y = \frac{2}{\sqrt{3}}$

$x = \frac{-3}{2}\left(\frac{2}{\sqrt{3}}\right)$

$x = -\sqrt{3}$

$\therefore$ **Solution Set**

$= \left\{\left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \left(\frac{-7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \left(-\sqrt{3}, \frac{2}{\sqrt{3}}\right), \left(\sqrt{3}, \frac{-2}{3}\right)\right\}$

## Problems Leading to Quadratic Equations (A.B + K.B + U.B)

**Example 1: (Page # 43)**

Three less than a certain number multiplied by 9 less than twice the number is 104. Find the number.

**Solution:**

Let the required number $= x$

Then, three less than the number $= x - 3$

And, 9 less than twice the number $= 2x - 9$

According to given condition

$(x-3)(2x-9) = 104$

$2x^2 - 9x - 6x + 27 - 104 = 0$

$2x^2 - 15x - 77 = 0$

$2x^2 - 22x + 7x - 77 = 0$

$2x(x-11) + 7(x-11) = 0$

$(x-11)(2x+7) = 0$

Either

$x - 11 = 0$ or $2x + 7 = 0$

$\Rightarrow x = 11$ or $x = -\frac{7}{2}$

**Result:**

Thus, required number is either 11 or $-\frac{7}{2}$

**Example 2: (Page # 44)** **(A.B)**

The length of rectangle is 4cm more than its breadth. If the area of rectangle is $45cm^2$. Find its sides.

**Solution:**

Let breadth of rectangle $= x$

Then, length of rectangle $= x + 4$

Area of rectangle $= 45cm^2$

According to given condition

$x(x+4) = 45$

$x^2 + 4x - 45 = 0$

$x^2 + 9x - 5x - 45 = 0$

$x(x+9) - 5(x+9) = 0$

$(x+9)(x-5) = 0$

Either

$x + 9 = 0$ or $x - 5 = 0$

$\Rightarrow x = -9$ or $x = 5$

(Neglecting –ve value)

$\therefore\ x + 4 = 5 + 4 = 9$

**Result:**

Thus, the breadth is 5cm and length is 9cm

## Exercise 2.8

**Q.1 The product of two positive consecutive numbers is 182. Find the numbers.** **(A.B)**

**Solution:**

Let two positive consecutive numbers are $x, x+1$

According to given condition:

$x(x+1) = 182$

$x^2 + x - 182 = 0$

$x^2 + 14x - 13x - 182 = 0$

$x(x+14) - 13(x+14) = 0$

$(x+14)(x-13) = 0$

Either

$x + 14 = 0$ or $x - 13 = 0$

$x = -14$ $\quad$ $x = 13$

(Ignore negative value)

Therefore,

$x = 13$

$\Rightarrow\ x + 1 = 13 + 1 = 14$

**Result:**

Thus, required Numbers are 13 and 14.

**Q.2 The sum of squares of three positive consecutive numbers is 77. Find the numbers. (A.B + K.B)**
**(SWL 2015)**

**Solution:**

Let three consecutive numbers are

$x, x+1, x+2$

According to given condition:

$x^2+(x+1)^2+(x+2)^2=77$

$x^2+x^2+2x+1+x^2+4x+4-77=0$

$3x^2+6x-72=0$

$3\left(x^2+2x-24\right)=0$

$x^2+2x-24=0$

$x^2+6x-4x-24=0$

$x(x+6)-4(x-6)=0$

$(x+6)(x-4)=0$

Either

$x+6=0$ or $x-4=0$

$x=-6$ $\quad$ $x=4$

(Ignore negative value)

Therefore, $x = 4$

$\Rightarrow \quad x+1=4+1=5$

& $\quad x+2=4+2=6$

**Result:**

Thus required numbers are 4, 5 and 6.

**Q.3 The sum of five times a number and the square of the numbers is 204. (A.B + K.B)**

**Solution:**

Let required number = $x$

Five times of the number $=5x$

According to given condition:

$x^2+5x=204$

$x^2+5x-204=0$

$x^2+17x-12x-204=0$

$x(x+17)-12(x+17)=0$

$(x+17)(x-12)=0$

Either

$x+17=0$ or $x-12=0$

$x=-17$ $\quad$ $x=12$

**Result:**

Thus required number is either –17 or 12.

**Q.4 The product of five less than three times a certain number and one less than four times the number is7. Find the number.**
**(A.B + K.B)**

**Solution:**

Let the required number is $x$

Five less than three times the number $=3x-5$

One less than four times a number $=4x-1$

According to given condition

$(3x-5)(4x-1)=7$

$12x^2-3x-20x+5-7=0$

$12x^2-23x-2=0$

$12x^2-24x+x-2=0$

$12x(x-2)+1(x-2)=0$

$12x(x-2)+1(x-2)=0$

$(x-2)(12x+1)=0$

Either

$x-2=0$ or $12x+1=0$

$x=2$ $\quad$ $12x=-1$

$x=\dfrac{-1}{12}$

**Result:**

Thus, required number is either 2 or $-\dfrac{1}{12}$.

**Q.5 The difference of a number and its reciprocal is $\frac{15}{4}$. Find the number.**
**(A.B + K.B)**

**Solution:**

Let required number is $x$

Reciprocal of the number $=\dfrac{1}{x}$

Difference of the numbers $=\dfrac{15}{4}$

According to given condition

$x-\dfrac{1}{x}=\dfrac{15}{4}$

$\dfrac{x^2-1}{x}=\dfrac{15}{4}$

By cross multiplication

$$4x^2 - 4 = 15x$$
$$4x^2 - 15x - 4 = 0$$
$$4x^2 - 16x + x - 4 = 0$$
$$4x(x-4) + 1(x-4) = 0$$
$$(x-4)(4x+1) = 0$$

Either

$x - 4 = 0$ or $4x + 1 = 0$

$x = 4$ $\quad 4x = -1$

$x = \frac{-1}{4}$

**Result:**

Thus, required number is either 4 or $\frac{-1}{4}$

**Q.6 The sum of a number of two digits of a positive integral number is 65 and the number is 9 times the sum of its digits. Find the number.** **(A.B + K.B)**

**Solution:**

Let unit's digit = $x$

And ten's digit = y

$\therefore$ Required number $= 10y + x$

According to given condition (I)

$x^2 + y^2 = 65 \longrightarrow$(i)

According to condition (II)

$10y + x = 9(x + y)$

$10y + x = 9x + 9y$

$10y - 9y = 9x - x$

$y = 8x \longrightarrow$(ii)

Put in equation (i)

$x^2 + (8x)^2 = 65$

$x^2 + 64x^2 = 65$

$65x^2 = 65$

$x^2 = 1$

Taking positive square root

$x = 1$

Put in equation (ii)

$y = 8(1)$

$y = 8$

Required number $= 10y + x$

$= 10(8) + 1$

$= 80 + 1$

$= 81$

**Result:**

Thus, required number is 81

**Q.7 The sum of the co-ordinates of a point is 9 and sum of their squares is 45. Find the co-ordinates of the point.** **(A.B + K.B)**

**Solution:**

Let required point is (x, y)

According to condition I

$x + y = 9 \longrightarrow$(i)

According to condition II

$x^2 + y^2 = 45 \longrightarrow$(ii)

Equation (i) $\Rightarrow$

$x = 9 - y \longrightarrow$(iii)

Put in equation (ii)

$(9 - y)^2 + y^2 = 45$

$81 - 18y + y^2 + y^2 = 45$

$2y^2 - 18y + 81 - 45 = 0$

$2y^2 - 18y + 36 = 0$

$2(y^2 - 9y + 18) = 0$

$y^2 - 9y + 18 = 0$

$y^2 - 6y - 3y + 18 = 0$

$y(y-6) - 3(y-6) = 0$

$(y-6)(y-3) = 0$

Either

$y - 6 = 0$ or $y - 3 = 0$

$y = 6$ $\quad y = 3$

Put in equation (iii)

$x = 9 - 6$ $\quad x = 9 - (3)$

$x = 3$ $\quad x = 9 - 3$

$x = 6$

**Result:**

Thus, required point is either

(3,6) or (6,3)

**Q.8 Find two integers whose sum is 9 and the difference of their squares is also 9. (A.B + K.B)**

**Solution:**

Let two integer are $x$ and y

According to condition I

$x + y = 9 \longrightarrow (i)$

According to condition (ii)

$x^2 - y^2 = 9 \longrightarrow (ii)$

Equation (i) $\Rightarrow y = 9 - x \longrightarrow (iii)$

Put in equation (ii)

$x^2 - (9-x)^2 = 9$

$x^2 - (81 - 18x + x^2) = 9$

$x^2 - 81 + 18x - x^2 = 9$

$18x = 9 + 81$

$\Rightarrow \quad x = 5$

Put in equation (iii)

y = 9 – 5 = 4

**Result:**

Thus, required numbers are 5 and 4.

**Q.9 Find two integers whose difference is 4 and whose squares differ by 72. (A.B + K.B)**

**Solution:**

Let the integers are $x$ and y

According to condition-I

$x - y = 4 \longrightarrow (i)$

According to condition-II

$x^2 - y^2 = 72 \longrightarrow (ii)$

Equation (i) $\Rightarrow x = y + 4 \longrightarrow (iii)$

Put in equation (ii)

$(y+4)^2 - y^2 = 72$

$y^2 + 8y + 16 - y^2 = 72$

8y = 72 – 16

8y = 56

$\Rightarrow$y = 7

Put in equation (iii)

$x = 7 + 4$

$x = 11$

**Result:**

Thus, required integers are 11 and 7.

**Q.10 Find the dimensions of a rectangle, whose perimeter is 80cm and its area is $375cm^2$ (K.B + A.B)**

**Solution:**

Let length of rectangle = $x$ cm

And width of rectangle = y cm

According to condition-I

$2(x + y) = 80$

$\because$ perimeter=2(length+ width)

$x + y = 40 \longrightarrow (i)$

$\because$ Area = length × width

According to condition-II

$xy = 375 \longrightarrow (ii)$

From equation (i)

$y = 40 - x \longrightarrow (iii)$

Put in (ii)

$x(40 - x) = 375$

$40x - x^2 - 375 = 0$

$-x^2 - 40 - 375 = 0$

$x^2 - 40x + 375 = 0$

$x^2 - 25x - 15x + 375 = 0$

$x(x-25) - 15(x-25) = 0$

$(x-25)(x-15) = 0$

Either $x - 25 = 0$ or $x - 15 = 0$

$x = 25$ $\quad x = 15$

Put in equation (iii)

y = 40 – 25 $\quad$ y = 40 – 15

y = 15 $\quad$ y = 25

**Result:**

Dimension of rectangle are either 25cm by 15cm or 15cm by 25cm.

## Miscellaneous Exercise 2

**Q.1 Multiple Choice Questions**

**Four possible answers are given for the following question. Tick (✓) the correct answer.**

**(i)** **If $\alpha, \beta$ are the roots of $3x^2+5x-2=0$, then $\alpha+\beta$ is;** **(K.B + A.B)**

**(LHR 2017, SWL 2014, MTN 2015, 17, SGD 2015, 17, RWP 2016, D.G.K 2016)**

**(a)** $\frac{5}{3}$ **(b)** $\frac{3}{5}$

**(c)** $\frac{-5}{3}$ **(d)** $\frac{-2}{3}$

**(ii)** **If $\alpha, \beta$ are the roots of $7x^2-x+4=0$, then $\alpha\beta$ is;** **(K.B + U.B)**

**(LHR 2014, GRW 2014, 15, FSD 2016, BWP 2016)**

**(a)** $\frac{-1}{7}$ **(b)** $\frac{4}{7}$

**(c)** $\frac{7}{4}$ **(d)** $\frac{-4}{7}$

**(iii)** **Roots of the equation $4x^2-5x+2=0$ are;** **(K.B + A.B)**

**(a)** Irrational **(b)** Imaginary

**(c)** Rational **(d)** None of these

**(iv)** **Cube roots of $-1$ are;** **(K.B + U.B)**

**(LHR 2017, GRW 2017, SWL 2017, MTN 2014, 17, SGD 2015, 16, D.G.K 2017)**

**(a)** $-1,-\omega,-\omega^2$ **(b)** $-1,\omega,-\omega^2$

**(c)** $-1,-\omega,\omega^2$ **(d)** $1,-\omega,-\omega^2$

**(v)** **Sum of the cube roots of unity is;** **(K.B + A.B)**

**(a)** 0 **(b)** 1

**(c)** $-1$ **(d)** 3

**(vi)** **Product of cube roots of unity is;** **(K.B + A.B)**

**(LHR 2016, GRW 2014, 16, SGD 2015, 17, BWP 2016, 17, RWP 2017)**

**(a)** 0 **(b)** 1

**(c)** $-1$ **(d)** 3

**(vii)** **If $b^2-4ac<0$, then the roots of $ax^2+bx+c=0$ are;** **(GRW 2014)** **(K.B + A.B)**

**(a)** Irrational **(b)** Rational

**(c)** Imaginary **(d)** None of these

**(viii)** **If $b^2-4ac>0$, but not a perfect square then roots of $ax^2+bx+c=0$ are;** **(K.B)**

**(LHR 2014, BWP 2017)**

**(a)** Imaginary **(b)** Rational

**(c)** Irrational **(d)** None of these

**(ix)** $\frac{1}{\alpha}+\frac{1}{\beta}$ **is equal to;** **(K.B + U.B)**

**(LHR 2014, 15, GRW 2016, FSD 2017, BWP 2017, RWP 2016, SGD 2017)**

**(a)** $\frac{1}{\alpha}$ **(b)** $\frac{1}{\alpha}-\frac{1}{\beta}$

**(c)** $\frac{\alpha-\beta}{\alpha\beta}$ **(d)** $\frac{\alpha+\beta}{\alpha\beta}$

**(x)** $\alpha^2+\beta^2$ **is equal to;** **(U.B + A.B)**

**(LHR 2014, 15, GRW 2014, 17, FSD 2016, BWP 2015, RWP 2016, 17)**

**(a)** $\alpha^2-\beta^2$ **(b)** $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

**(c)** $(\alpha+\beta)^2-2\alpha\beta$ **(d)** $\alpha+\beta$

**(xi)** **Two square roots of unity are;** **(U.B + A.B)**

**(LHR 2015, 16, GRW 2014, FSD 2015, 16, MTN 2016, SGD 2016, D.G.K 2015, 16, 17)**

**(a)** $1, -1$ **(b)** $1, \omega$

**(c)** $1, -\omega$ **(d)** $\omega, \omega^2$

**(xii)** **Roots of the equation** $4x^2-4x+1=0$ **are;** **(U.B + A.B)**

**(LHR 2015, GRW 2017, FSD 2016, BWP 2015, MTN 2017, SGD 2016)**

**(a)** Real equal **(b)** Real unequal

**(c)** Imaginary **(d)** Irrational

**(xiii)** **If** $\alpha, \beta$ **are the roots of** $px^2+qx+r=0$**, then sum of the roots** $2\alpha$ **and** $2\beta$ **is;** **(K.B)**

**(a)** $\frac{-q}{p}$ **(b)** $\frac{r}{p}$

**(c)** $\frac{-2q}{p}$ **(d)** $-\frac{q}{2p}$

**(xiv)** **If** $\alpha, \beta$ **are the roots of** $x^2-x-1=0$**, then product of the roots** $2\alpha$ **and** $2\beta$ **is;** **(U.B)**

**(FSD 2014, 17, BWP 2016, D.G.K 2015, 16, 17)**

**(a)** $-2$ **(b)** $2$

**(c)** $4$ **(d)** $-4$

**(xv)** **The nature of the roots of equation** $ax^2+bx+c=0$ **is determined by;** **(A.B)**

**(GRW 2016, SWL 2015, 2017, MTN 2015)**

**(a)** Sum of the roots **(b)** Product of the roots

**(c)** Synthetic division **(d)** Discriminant

**(xvi)** **The discriminant of** $ax^2+bx+c=0$ **is;** **(K.B + A.B)**

**(LHR 2016, FSD 2017, SWL 2016, 17, RWP 2014, 16, SGD 2016, MTN 2015, D.G.K 2016)**

**(a)** $b^2-4ac$ **(b)** $b^2+4ac$

**(c)** $-b^2+4ac$ **(d)** $-b^2-4ac$

## ANSWER KEY

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| (i) | c | (v) | a | (ix) | d | (xiii) | c |
| (ii) | b | (vi) | b | (x) | c | (xiv) | d |
| (iii) | b | (vii) | c | (xi) | a | (xv) | d |
| (iv) | a | (viii) | c | (xii) | a | (xvi) | a |

**Q.2**

**(i) Discuss the nature of the roots of the following equations.** **(A.B)**

**Solution:**

**(a)** $x^2+3x+5=0$

Here $a=1, b=3, c=5$

Disc $= b^2 - 4ac$
$= (3)^2 - 4(1)(5)$
$= 9 - 20$
$= -11$
$< 0$

∴ Roots are complex conjugate or imaginary.

**(b)** $2x^2-7x+3=0$ **(A.B)**

**(GRW 2016, SGD 2014, RWP 2017, D.G.K 2016)**

Here $a=2, b=-7, c=3$

Disc $= b^2 - 4ac$
$= (-7)^2 - 4(2)(3)$
$= 49 - 24$
$= 25$

Since disc >0 and perfect square roots are rational and unequal.

**(c)** $x^2+6x-1=0$ **(A.B)**

Here $a=1, b=6, c=-1$

Disc $= b^2 - 4ac$
$= (6)^2 - 4(1)(-1)$
$= 36 + 4$
$= 40$

Since Disc. >0 and not a perfect square roots are irrational and unequal.

**(d)** $16x^2-8x+1=0$ **(FSD 2017)** **(A.B)**

Here $a=16, b=-8, c=1$

Disc. $= b^2-4ac$
$= (-8)^2-4(16)(1)$
$= 64 - 64$
$= 0$

Since, Disc. = 0, roots are rational and equal.

**(ii) Find $\omega^2$, if $\omega=\dfrac{-1+\sqrt{-3}}{2}$** **(A.B)**

**(GRW 2017, SGD 2014, BWP 2010)**

**Solution:**

Here

$$\omega=\frac{-1+\sqrt{-3}}{2}$$

Square both sides

$$(\omega)^2=\left(\frac{-1+\sqrt{-3}}{2}\right)^2$$

$$\omega^2=\frac{(-1)^2+\left(\sqrt{-3}\right)^2+2(-1)\left(\sqrt{-3}\right)}{(2)^2}$$

$$\omega^2=\frac{1+(-3)-2\sqrt{-3}}{4}$$

$$\omega^2=\frac{-2-2\sqrt{-3}}{4}$$

$$\omega^2=2\left(\frac{-1-\sqrt{-3}}{4}\right)$$

$$\omega^2=\frac{-1-\sqrt{-3}}{2}$$

**(iii) Prove that the sum of all cube roots of unity is zero.** **(A.B + K.B)**

**Ans.** See property of cube roots Page # 45

**(iv) Find the product of complex cube roots of unity.** **(A.B + K.B)**

**Ans.** See property of cube roots Page # 44

**(v) Show that:**

$$x^3+y^3=(x+y)(x+\omega y)(x+\omega^2 y)$$

**(A.B + K.B)**

**Ans.** See Exe-2.2 Q.3 Page # 47

**(vi) Evaluate:** $\omega^{37}+\omega^{38}+1$ **(A.B + K.B)**

**Solution:**

$\omega^{37}+\omega^{38}+1$
$= \omega^{36}.\omega+\omega^{36}.\omega^2+1$
$= \left(\omega^3\right)^{12}.\omega+\left(\omega^3\right)^{12}.\omega^2+1$
$= (1)^{12}.\omega+(1)^{12}.\omega^2+1$
$= \omega + \omega^2 + 1$
$= 0$
$\therefore \omega^{37}+\omega^{38}+1=0$

**(vii) Evaluate** $\left(1-\omega+\omega^2\right)^6$ **(A.B + K.B)**

**Solution:**

$(1 - \omega + \omega^2)^6$
$= [1 + \omega^2 - \omega]^6$
$= (- \omega - \omega)^6$
$= (-2\omega)^6$
$= (-2)^6\omega^6$
$= 64(\omega^3)^2$
$= 64(1)^2$
$= 64(1)$
$= 64$

**(viii) If $\omega$ is cube root of unity, form an equation whose roots are $3\omega$ and $3\omega^2$. (A.B + K.B)**

**Solution:**

Roots of required equation are $3\omega$ and $3\omega^2$.

Sum of roots = S = $3\omega + 3\omega^2$

$= 3(\omega + \omega^2)$

$= 3(-1)$

$= -3$

Product of roots = P = $(3\omega)(3\omega^2)$

$= 9\omega^3$

$= 9(1)$

$= 9$

∴ Required quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - (-3)x + 9 = 0$$

$$x^2 + 3x + 9 = 0$$

**(ix) Use synthetic division, find the remainder and quotient when $(x^3 + 3x^2 + 2) \div (x - 2)$. (A.B)**

**Solution:**

$P(x) = x^3 + 3x^2 + 2$

$= x^3 + 3x^2 + 0x + 2$

| | | | | |
|---|---|---|---|---|
| | 1 | 3 | 0 | 2 |
| 2 | ↓ | 2 | 10 | 20 |
| | 1 | 5 | 10 | 22 |

∴ Remainder = 22

$Q(x) = x^2 + 5x + 10$

**(x) Use synthetic division, show that $x-2$ is the factor $x^3 + x^2 - 7x + 2$.**

**Solution: (A.B + K.B)**

$P(x) = x^3 + x^2 - 7x + 2$

| | | | | |
|---|---|---|---|---|
| | 1 | 1 | –7 | 2 |
| 2 | ↓ | 2 | 6 | –2 |
| | 1 | 3 | –1 | 0 |

Since remainder is zero, $x - 2$ is a factor of given polynomial.

**(xi) Find the sum and product of the roots of the equation $2Px^2 + 3qx - 4r = 0$ (A.B + K.B)**

**Solution:**

$2Px^2 + 3qx - 4r = 0$

Here $a = 2P, b = 3q, c = -4r$

Sum of roots $= \dfrac{-b}{a}$

$= -\dfrac{3q}{2P}$

Product of roots $= \dfrac{c}{a}$

$= \dfrac{-4r}{2P}$

$= -\dfrac{2r}{P}$

**(xii) Find $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ when α, β are of the roots of the equation $x^2 - 4x + 3 = 0$. (A.B + K.B)**

**Solution:**

$x^2 - 4x + 3 = 0$

Here $a = 1, b = -4, c = 3$

Let roots of given equation are α, β

Then sum of roots = $\alpha + \beta = -\dfrac{b}{a}$

$= -\dfrac{-4}{1}$

$= 4$

Product of roots = $\alpha\beta = \dfrac{c}{a}$

$\alpha\beta = \dfrac{3}{1}$

$\alpha\beta = 3$

Consider

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$$

$$= \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}$$

$$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{(4)^2 - 2(3)}{(3)^2}$$

$$= \frac{16 - 6}{9}$$

$$\Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{10}{9}$$

**(xiii) If α, β are the roots of $4x^2 - 3x + 6 = 0$, find (A.B)**

**(a)** $\alpha^2 + \beta^2$

**(b)** $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

**(c)** $\alpha - \beta$

**Solution:**

$4x^2 - 3x + 6 = 0$

Roots of given equation are α, β

$$\alpha + \beta = -\frac{-3}{4} = \frac{3}{4}$$

$$\alpha\beta = \frac{6}{4} = \frac{3}{2}$$

**(a)** $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ **(A.B)**

$$= \left(\frac{3}{4}\right)^2 - 2\left(\frac{3}{2}\right)$$

$$= \frac{9}{16} - 3$$

$$= \frac{9 - 48}{16}$$

$$\Rightarrow \alpha^2 + \beta^2 = \frac{-39}{16}$$

**(b)** $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$ **(A.B)**

$$= \frac{-39/16}{3/2}$$

$$= \frac{-39}{16} \times \frac{2}{3}$$

$$\Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{-13}{8}$$

**(c)** $\alpha - \beta = \sqrt{(\alpha - \beta)^2}$ **(A.B)**

$$= \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \sqrt{\left(\frac{3}{4}\right)^2 - 4\left(\frac{3}{2}\right)}$$

$$= \sqrt{\frac{9}{16} - 6}$$

$$= \sqrt{\frac{9 - 96}{16}}$$

$$= \sqrt{\frac{-87}{16}}$$

$$\Rightarrow \alpha - \beta = \frac{\sqrt{-87}}{4}$$

**(xiv) If α, β are the roots of $x^2 - 5x + 7 = 0$, find an equation whose roots are**

**(a)** $-\alpha, -\beta$

**(b)** $2\alpha, 2\beta$

**Solution:**

$x^2 - 5x + 7 = 0$

Roots of given equation are α, β

$$\alpha + \beta = -\frac{-5}{1} = 5$$

$$\alpha\beta = \frac{7}{1}$$

$\alpha\beta = 7$

**(a) Roots of required equation are $-\alpha, -\beta$** **(A.B)**

Sum of roots = S = $-\alpha + (-\beta)$

$= -(\alpha + \beta)$

$= -5$

Prod. Of roots = P = $(-\alpha)(-\beta)$

$= \alpha\beta$

$= 7$

∴ Required equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - (-5)x + 7 = 0$$

$$x^2 + 5x + 7 = 0$$

**(b) Roots of required equation are $2\alpha, 2\beta$** **(A.B)**

Sum of roots = S = $2\alpha + 2\beta$

$= 2(\alpha + \beta)$

$= 2(5)$

$= 10$

Prod. Of roots = P = $(2\alpha)(2\beta)$

$= 4\alpha\beta$

$= 4(7)$

$= 28$

∴ Required quadratic equation is:

$$x^2 - Sx + P = 0$$

$$x^2 - 10x + 28 = 0$$

**Q.3 Fill in the blanks**

**(i)** The discriminant of $ax^2+bx+c=0$ is ________. **(K.B)**

**(ii)** If $b^2-4ac=0$, then roots of $ax^2+bx+c=0$ are ________. **(K.B)**

**(iii)** If $b^2-4ac>0$, then the roots of $ax^2+bx+c=0$ are ________. **(K.B)**

**(iv)** If $b^2-4ac<0$, then the root of $ax^2+bx+c=0$ are ________. **(K.B)**

**(v)** If $b^2-4ac>0$ and perfect square, then the roots of $ax^2+bx+c=0$ are________. **(K.B)**

**(vi)** If $b^2-4ac>0$ and not a perfect square, then roots of $ax^2+bx+c=0$ are________. **(K.B)**

**(vii)** If $\alpha,\beta$ are the roots of $ax^2+bx+c=0$, then sum of the roots is________. **(K.B)**

**(viii)** If $\alpha,\beta$ are the roots of $ax^2+bx+c=0$, then product of the roots is________. **(K.B)**

**(ix)** If $\alpha,\beta$ are the roots of $7x^2-5x+3=0$, then sum of the roots is________. **(K.B)**

**(x)** If $\alpha,\beta$ are the roots of $5x^2+3x-9=0$, then product of the roots is________. **(K.B)**

**(xi)** For a quadratic equation $ax^2+bx+c=0$, $\frac{1}{\alpha\beta}$ is equal to ________. **(K.B)**

**(xii)** Cube roots of unity are________. **(K.B)**

**(xiii)** Under usual notation sum of the roots of unity is________. **(K.B)**

**(xiv)** If $1,\omega,\omega^2$ are the cube roots of unity, then $\omega^{-7}$ is equal to________. **(K.B)**

**(xv)** If $\alpha,\beta$ are the roots of the quadratic equation, then the quadratic equation is written as ________.

**(xvi)** If $2\omega$ and $2\omega^2$ are the roots of an equation, then equation is________.

## ANSWER KEY

**(i)** $b^2-4ac$

**(ii)** Equal

**(iii)** Real

**(iv)** Imaginary

**(v)** Rational

**(vi)** Irrational (real)

**(vii)** $-\frac{b}{a}$

**(viii)** $\frac{c}{a}$

**(ix)** $\frac{5}{7}$

**(x)** $\frac{-9}{5}$

**(xi)** $\frac{a}{c}$

**(xii)** $1,\omega,\omega^2$

**(xiii)** Zero

**(xiv)** $\omega^2$

**(xv)** $x^2-(\alpha+\beta)x+\alpha\beta=0$

**(xvi)** $x^2+2x+4=0$

CUT HERE

## SELF TEST

**Time: 40 min** **Marks: 25**

**Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer.** **(7×1=7)**

**1** $\omega^{-7} =$ ________ **is:**

**(A)** $\omega^2$ **(B)** $\omega$
**(C)** 1 **(D)** 0

**2 Which is not a symmetric function?**

**(A)** $\alpha^2 - \beta^2$ **(B)** $\alpha^2 + \beta^2$
**(C)** $\alpha^3 + \beta^3$ **(D)** $\frac{1}{\alpha} + \frac{1}{\beta}$

**3 If** $\frac{3}{2}, \frac{1}{2}$ **are the roots of a quadratic equation, then required quadratic equation is:**

**(A)** $2x^2 + 2x + 3 = 0$ **(B)** $4x^2 + 8x + 3 = 0$
**(C)** $x^2 + 4x + 3 = 0$ **(D)** $4x^2 - 8x + 3 = 0$

**4 If roots of a quadric equation** $x^2 + qx + p = 0$ **are the additive inverse of each other, then:**

**(A)** $p = 0, q = 0$ **(B)** $p = 0$
**(C)** $q = 0$ **(D)** $p = 1, q = 1$

**5 What will be the remainder if** $4x^3 - 5x + 15$ **is divided by** $x + 3$**?**

**(A)** 78 **(B)** 139
**(C)** –78 **(D)** 125

**6 Cube roots of** $-1$ **are:**

**(A)** $-1, -\omega, -\omega^2$ **(B)** $-1, \omega, -\omega^2$
**(C)** $-1, -\omega, \omega^2$ **(D)** $1, \omega, \omega^2$

**7 Roots of the equation** $4x^2 - 5x + 2 = 0$ **are:**

**(A)** Irrational **(B)** Imaginary
**(C)** Rational **(D)** None

**Q.2 Give Short Answers to following Questions.** **(5×2=10)**

**(i)** Evaluate: $(1 - 3\omega - 3\omega^2)^5$.

**(ii)** Prove that each complex cube root of unity is reciprocal of the other.

**(iii)** Show that the roots of the equation $(p+q)x^2 - px - q = 0$ are rational.

**(iv)** If $\omega$ is a cube root of unity, form an equation whose roots are $2\omega$ and $2\omega^2$.

**(v)** Use synthetic division to find the quotient and the remainder when the polynomial $x^4 - 10x^2 - 2x$ is divided by $x + 3$.

**Q.3 Answer the following Questions.** **(4+4=8)**

**(a)** Prove that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)$

**(b)** Find two integers whose sum is 9 and the difference of their squares is also 9.

**NOTE:** Parents or guardians can conduct this test in their supervision in order to check the skill of students.