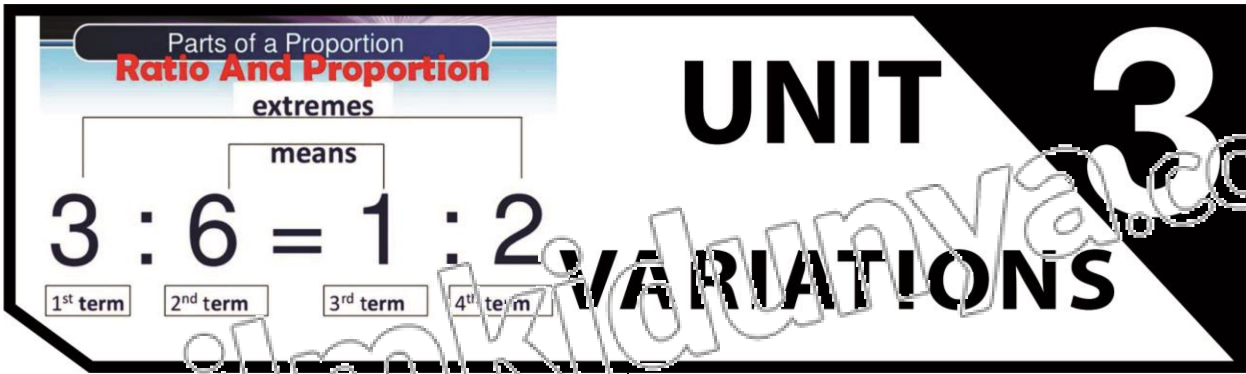

Ratio **(K.B)**

(FSD 2014, MTN 2015, SGD 2015, RWP 2014, 15, D.G.K 2015)

Ratio is a relation between two quantities of same kind. It tells what part one quantity of an other or how much time one quantity of other.

For any two quantities a and b of the same kind, it is represented as, $a:b=\frac{a}{b}$, where $b\neq 0$.

For example, if a hockey team wins 4 games and losses 5, then the ratio of the games won to lost is $4:5$ or $\frac{4}{5}$.

Antecedent **(K.B + U.B)**

In ratio $a:b$, the first term a is called antecedent.

For example: In 2:3, 2 is antecedent.

Consequent **(K.B + U.B)**

In the ratio $a:b$, the second term b is called consequent.

For example: In 2:3, 3 is consequent.

Proportion **(SGD 2017)** **(K.B + U.B)**

The statement of equality of two ratios is called proportion. If two ratios $a:b$ and $c:d$ are equal then, we can write it as $a:b::c:d$ or $a:b=c:d$, where quantities a and d are called extremes, while b and c are called means proportions.

Note **(K.B + U.B)**

- Ratio has no unit.
- The order of elements in ratio is important. i.e. $a:b\neq b:a$
- Product of extremes = Product of means

Example 1: (Page # 50) **(K.B + U.B)**

Find the ratio of

(i) 200*gm to* 700*gm*

(ii) 1*km to* 600*m*

Solution:

(i) Ratio of 200gm to 700gm **(A.B)**

$$200:700=\frac{200}{700}=\frac{2}{7}=2:7$$

Where, 2:7 is the simplest (lowest) form of the ratio 200:700.

(ii) Ratio of 1 *km* to 600 *m* **(A.B)**

Since 1*km*=1000*m*

Then $1km:600m=1000m:600m$

$$=1000:600$$
$$=10:6$$
$$=5:3$$

Example 2: (Page # 50) **(A.B)**

Find a, if the ratios $a+3:7+a$ and 4:5 are equal.

Solution:

Since the ratios $a+3:7+a$ and 4:5 are equal.

$$\frac{a+3}{7+a}=\frac{4}{5} \quad \therefore \text{ in fraction form}$$
$$5(a+3)=4(7+a)$$
$$5a+15=28+4a$$
$$5a-4a=28-15$$
$$a=13$$

Thus, given ratios will be equal if $a = 13$.

Example 3: (Page # 51) **(A.B)**

If 2 is added in each number of the ratio 3:4, we get a new ratio 5:6. Find the numbers.

Solution:

Ratio of two numbers is 3:4.

Multiply each number of the ratio with x. Then the numbers be $3x$, $4x$.

Now according to the given condition

$$\frac{3x+2}{4x+2}=\frac{5}{6}$$

$$6(3x+2)=5(4x+2) \Rightarrow 18x+12=20x+10$$
$$18x-20x=10-12 \Rightarrow -2x=-2 \Rightarrow x=1$$

Therefore

$$3x=3(1)=3$$
$$4x=4(1)=4$$

Thus, the required numbers are 3, 4

Example 4: (Page # 51) **(A.B)**

Find the ratio $3a+4b:5a+7b$ **if** $a:b=5:8$

Solution:

Give that $a:b=5:8$ or $\frac{a}{b}=\frac{5}{8}$

Now $3a+4b:5a+7b=\frac{3a+4b}{5a+7b}$

(Dividing numerator and denominator by b)

$$=\frac{\frac{3a+4b}{b}}{\frac{5a+7b}{b}}=\frac{3\left(\frac{a}{b}\right)+4\left(\frac{b}{b}\right)}{5\left(\frac{a}{b}\right)+7\left(\frac{b}{b}\right)}$$

$$=\frac{3\left(\frac{5}{8}\right)+4(1)}{5\left(\frac{5}{8}\right)+7(1)} \quad \left(\because \frac{a}{b}=\frac{5}{8}\right)$$

$$=\frac{\frac{15}{8}+4}{\frac{25}{8}+7}=\frac{\frac{15+32}{8}}{\frac{25+56}{8}}$$

$$=\frac{47}{81}$$

Hence, $3a+4b:5a+7b=47:81$

Example 6: (Page # 52) **(LHR 2014)** **(A.B)**

Find the cost of 15kg of sugar, if 7kg of sugar costs 560 rupees.

Solution:

Let the cost of 15kg of sugar be x-rupees.

Then in proportion form

15kg : 7kg :: Rs. x : Rs. 560

$15:7=x:560$

Product of extremes = Product of means

$$15\times560=7x$$
$$7x=15\times560$$
$$x=\frac{15\times560}{7}=15(80)=1200$$

Thus, Cost of 15 kg sugar is Rs.1200.

Exercise 3.1

Q.1 Expressing as a ratio and as a fraction.

Solution:

(i) Here **(GRW 2016, FSD 2017)** **(A.B)**

a = Rs.750

b = Rs.1250

Now,

$$a:b=750:1250 \quad (\div by 10)$$
$$=75:125 \quad (\div by 5)$$
$$=15:25 \quad (\div by 5)$$
$$\Rightarrow a:b=3:5$$

$\therefore$ **The fractional form of this expression is** $\frac{3}{5}$

(ii) Here **(A.B)**

a = 450 cm

b = 3m = 3 × 100cm = 300 cm

Now,

$$a:b=450:300 \quad (\div by 10)$$
$$=45:30 \quad (\div by 5)$$
$$\Rightarrow a:b=3:2 \quad (\div by 15)$$

$\therefore$ **The fractional form of this expression is** $\frac{3}{2}$.

(iii) Here **(A.B)**

$a=4kg=4\times1000gm=4000gm$

$b=2kg.750gm=[(2\times1000)+750]gm$

$=2750gm$

Now

$$a:b=4000:2750$$
$$=400:275 \quad (\div by 10)$$
$$=80:55 \quad (\div by 5)$$
$$\Rightarrow a:b=16:11 \quad (\div by 5)$$

The fractional form of this expression is $\frac{16}{11}$

(iv) a = 27 min 30 sec = (27 × 60 + 30) sec
= (1620 + 30) sec
= 1650 sec

b = 1 hour = 1 × 60 × 60 sec
= 3600 sec

$\Rightarrow a:b$ = 1650 sec : 3600 sec
= 1650 : 3600
= 165 : 360 $(\div by 10)$
= 33 : 72 $(\div by 5)$
= 11 : 24 $(\div by 3)$

The fractional form of this expression is $\frac{11}{24}$

(v) Here **(A.B)**

(SWL 2015, BWP 2016, D.G.K 2014)

$a = 75^o, b = 225^o$

Now, $a:b = 75:225 \quad (\div 25)$

$= 3:9 \quad (\div 3)$

$\Rightarrow a:b = 1:3$

The fractional form of this expression is $\frac{1}{3}$.

Q.2 In a class of 60 students, 25 students are girls and remaining students are boys. Computer the ratio of

Given **(A.B + K.B)**

Total students in the class = 60

Number of girls in the class = 25

Number of boys in the class = 60 – 25 = 35

Required

(i) Ratio of boys to total students

(ii) Ratio of boy to girls.

Solution:

(i) **(A.B)**

Now,

Boys: Total students = 35: 60
= 7: 12 $(\div 5)$

(ii) Here, **(A.B)**

Boys: Girls = 35: 25
= 7: 5 $(\div by 5)$

Q.3 If $3(4x-5y) = 2x-7y$, find the ratios $x:y$. **(A.B)**

Given

$3(4x-5y) = 2x-7y$

Required **(LHR 2015)**

$x:y = ?$ **(MTN 2016)**

Solution:

Here

$3(4x-5y) = 2x-7y$

$12x-15y = 2x-7y$

$12x-2x = 15y-7y$

$10x = 8y$

$\frac{x}{y} = \frac{8}{10}$

$\frac{x}{y} = \frac{4}{5}$

$\Rightarrow x:y = 4:5$

Result:

$x:y = 4:5$

Q.4 Find the value of p, if the ratios $2p+5:3p+4$ and $3:4$ are equal. Find value of 'p' **(A.B)**

(GRW 2015, SWL 2016, RWP 2015, 17)

Solution:

According to given condition.

$2p+5:3p+4 = 3:4$

$\frac{2p+5}{3p+4} = \frac{3}{4}$

By cross multiplication

$4(2p+5) = 3(3p+4)$

$8p+20 = 9p+12$

$8p-9p = 12-20$

$-p = -8$

$\Rightarrow p = 8$

Result:

$p = 8$

Q.5 If the ratios $3x+1:6+4x$ and $2:5$ are equal. Find the value of x.

Solution: **(D.G.K 2015)** **(A.B + K.B)**

Here

$3x+1:6+4x = 2:5$

$\Rightarrow \frac{3x+1}{6+4x} = \frac{2}{5}$

By cross multiplication, we get

$$5(3x+1)=2(6+4x)$$
$$15x+5=12+8x$$
$$15x-8x=12-5$$
$$7x=7$$
$$x=\frac{7}{7}$$
$$x=1$$

Result:

$x=1$

Q.6 Two numbers are in the ratio $5:8$. If 9 is added to each number, we get a new ratio $8:11$. Find the numbers. (FSD 2016) (A.B + K.B)

Solution:

Ratio between two numbers $=5:8$

Let required numbers are $5x, 8x$

According to given condition

$$5x+9:8x+9=8:11$$
$$\frac{5x+9}{8x+9}=\frac{8}{11}$$

By cross multiplication

$$11(5x+9)=8(8x+9)$$
$$55x+99=64x+72$$
$$55x-64x=72-99$$
$$-9x=-27$$
$$x=\frac{-27}{-9}$$
$$x=3$$

Now

$$5x=5(3)=15$$

And

$$8x=8(3)=24$$

Result:

$\therefore$ Required numbers are 15 and 24

Q.7 If 10 is added in each number of the ratio $4:13$, we get a new ratio $1:2$. What are the numbers? (A.B + K.B)

Solution:

Ratio between two numbers = 4 : 13

Let, the two numbers be $4x$ & $13x$.

According to the given condition;

$$4x+10:13x+10=1:2$$
$$\frac{4x+10}{13x+10}=\frac{1}{2}$$
$$2(4x+10)=1(13x+10)$$
$$8x+20=13x+10$$
$$20-10=13x-8x$$
$$10=5x$$
$$\frac{10}{5}=x$$
$$x=2$$
$$\Rightarrow \therefore 4x=4(2)=8$$
$$\Rightarrow 13x=13(2)=26$$

Result:

$\therefore$ Required two numbers are 8 and 26.

Q.8 Find the cost of 8kg of mangoes, if 5kg of mangoes cost Rs. 250.

Solution: (A.B + K.B)

Weight of mangoes = 5 kg

Cost of mangoes of 5kg = Rs.250

Now, weight of mangoes = 8kg

Here

weight: weight$::cost:cost$

$$5:8::250:x$$

Product of extreme = product of means

$$5x=250\times 8$$
$$5x=2000$$
$$x=\frac{2000}{5}$$
$$x=400$$

Result:

$\therefore$Cost of 8kg of mangoes is Rs. 400.

Q.9 If $a:b=7:6$, find the value of $3a+5b:7b-5a$. (A.B + K.B)

Solution: (FSD 2016)

Here

$$a:b=7:6$$
$$\Rightarrow \frac{a}{b}=\frac{7}{6}$$

or $a=\frac{7}{6}b$

Consider

$$3a+5b:7b-5a=3\left(\frac{7}{6}b\right)+5b:7b-5\left(\frac{7}{6}b\right)$$
$$=\frac{21b+30b}{6}:\frac{42b-35b}{6}$$

$$= \frac{51b}{6} : \frac{7b}{6}$$
$$= 51\,b : 7b \qquad (\times\ by\, 6)$$
$$\Rightarrow 3a + 5b : 7b - 5a = 51 : 7 \qquad (\div\ by\, b)$$

Result:
$3a + 5b : 7b - 5a = 51 : 7$

Q.10 Complete the following:

(i) **(A.B + K.B)**

Given Data:
$\frac{24}{7} = \frac{6}{x}$

Required
$4x = ?$

Solution:
Consider,
$\frac{24}{7} = \frac{6}{x}$
By cross multiplication, we get;
$24(x) = 6(7)$
$24x = 6(7)$
$\frac{24x}{6} = 7$
$4x = 7$

Result:
$4x = 7$

(ii) **(A.B + K.B)**

Given Data:
$\frac{5a}{3x} = \frac{15b}{y}$

Required
$ay = ?$

Solution:
Consider,
$\frac{5a}{3x} = \frac{15b}{y}$
By cross multiplication, we get
$5a(y) = 15b(3x)$
$5ay = 45bx$
$ay = \frac{45}{5}bx$
$ay = 9bx$

Result
$ay = 9bx$

(iii) **(SWL 2014)** **(A.B + K.B)**

Given Data:
$\frac{9pq}{2lm} = \frac{18p}{5m}$

Required
$5q = ?$

Solution:
Consider,
$\frac{9pq}{2lm} = \frac{18p}{5m}$
By cross multiplication, we get
$9pq(5m) = 18p(2lm)$
$45mpq = 36mpl$
$\frac{45mpq}{36mp} = l$
$\frac{5q}{4} = l$
$5q = 4l$

Result:
$5q = 4l$

Q.11 Find x in the following proportions.

(i) **(A.B + K.B)**
(GRW 2014, SWL 2016, MTN 2017, RWP 2017)

Given Data:
$3x - 2 : 4 :: 2x + 3 : 7$ **Required**
$x = ?$

Solution:
$3x - 2 : 4 :: 2x + 3 : 7$
Product of extremes = product of means
$7(3x - 2) = 4(2x + 3)$
$21x - 14 = 8x + 12$
$21x - 8x = 12 + 14$
$13x = 26$
$x = \frac{26}{13}$
$x = 2$

Result:
$x = 2$

(ii) $\frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$ **(A.B + K.B)**

Product of extremes = Product of means
$$\frac{\cancel{3}}{5}\left(\frac{2x}{\cancel{3}}\right) = \frac{\cancel{7}}{5}\left(\frac{3x-1}{\cancel{7}}\right)$$

$$\frac{2x}{\cancel{5}}=\frac{3x-1}{\cancel{5}}$$
$$2x=3x-1$$
$$1=3x-2x$$
$$\Rightarrow x=1$$

(iii) $\frac{x-3}{2}:\frac{5}{x-1}::\frac{x-1}{3}:\frac{4}{x+4}$

(GRW 2014) **(A.B + K.B)**

Product of extremes = Product of means

$$\left(\frac{x-3}{2}\right)\left(\frac{4}{x+4}\right)=\left(\frac{5}{x-1}\right)\left(\frac{x-1}{3}\right)$$
$$(x-3)\left(\frac{2}{x+4}\right)=\frac{5}{3}$$

By cross multiplication

$$6(x-3)=5(x+4)$$
$$6x-5x=20+18$$
$$x=38$$

(iv) Here **(A.B + K.B)**

$$p^2+pq+q^2:x::\frac{p^3-q^3}{p+q}:(p-q)^2$$

$\therefore$ Product of extremes = Product of means

$$(p^2+pq+q^2)(p-q)^2=x\left(\frac{p^3-q^3}{p+q}\right)$$
$$x\left(\frac{p^3-q^3}{p+q}\right)=(p^2+pq+q^2)(p-q)^2$$
$$x=(p^2+pq+q^2)(p-q)^2\times\frac{(p+q)}{p^3-q^3}$$
$$=\frac{(p^2+pq+q^2)(p-q)(p-q)(p+q)}{(p-q)(p^2+pq+q^2)}$$
$$=(p-q)(p+q)$$
$$\Rightarrow x=p^2-q^2$$

(v) Here (BWP 2014) **(A.B + K.B)**

$$8-x:11-x::16-x:25-x$$

$\therefore$ Product of extremes = Product of means

$$(8-x)(25-x)=(11-x)(16-x)$$
$$200-8x-25x+\cancel{x^2}=176-11x-16x+\cancel{x^2}$$
$$200-33x=176-27x$$
$$27x-33x=176-200$$
$$-6x=-24$$
$$x=\frac{-24}{-6}$$
$$\Rightarrow x=4$$

Variation **(K.B)**

Change in one quantity due to change in other quantity(s) is called variation.

Types of Variations **(K.B)**

There are two types of variations.

(i) Direct variation

(ii) Inverse variation

Direct Variation **(K.B)**

(LHR 2014, 16, 17, GRW 2014, 16, 17, BWP 2015, 16, SWL 2016, 17, SGD 2017)

A relation of two quantities, such that increase in one quantity causes the increase in the other or decrease in one quantity causes the decrease in other is called direct variation.

For any two quantities x and y, it is written as: $y\propto x$ or $y=kx$ where $k\neq 0$ is constant of proportionality.

For example: radius and circumference or faster the speed longer the distance etc.

Example 1: (Page # 53) **(A.B)**

Find the relation between distance d of a body falling from rest varies directly as the square of the time t, neglecting air resistance. Find k, if d=16 feet for t=1 sec. Also derive a relation between d and t.

Solution:

Since d is the distance of the body falling from rest in time t.

Then under the given condition

$$d\propto t^2$$

i.e., $d=kt^2$

Since d=16 feet and t=1 sec

Then equation (i) becomes

$$16=k(1)^2$$

i.e., $k=16$

put in eq. (i) $d=16t^2$

Which is a relationship between the distance d and time t.

Example 3: (Page # 54) **(K.B + A.B)**

Give that A varies directly as the square of r and $A=\frac{1782}{7}cm^2$, when $r=9cm$. If $r=14cm$, then find A.

Solution:

Since A varies directly as square of r

$\therefore \quad a \propto r^2$

or $\quad A = kr^2$

$$\frac{1782}{7}=k(9)^2$$

$$\frac{1782}{7\times 81}=k$$

or $\quad k=\frac{22}{7}$

Put $k=\frac{22}{7}$ and $r=14cm$ in eq. (i)

$$A=\frac{22}{7}(14)^2=\frac{22}{7}\times 14\times 14$$

$$=616cm^2$$

Inverse Variation: **(K.B)**

(LHR 2014, GRW 2017, SWL 2015, 16)

"A relation between two quantities, such that increase in one quantity causes the decrease in the other or vice versa is called inverse variation". For any two quantities x and y, it is written as: $y \propto \frac{1}{x}$ or $y=\frac{k}{x}$ or $yx=k$ where $k\neq 0$ is constant of proportionality.

For example:

Increase in worker will decrease the days or Increase the speed will decrease the time.

Example 2: (Page # 55) **(K.B + A.B)**

If y varies inversely as x^2 and $y=16$, when $x=5$, so find x, when $y=100$.

Solution:

Since y varies inversely as x^2, therefore

$$y\propto\frac{1}{x^2} \Rightarrow y=\frac{k}{x^2}$$

$$k=x^2y \qquad \rightarrow (i)$$

Put $x=5$ and $y=16$ in (i)

$$k=(5)^2\times 16$$

$$k=400$$

Now put $k=400$ and $y=100$ in (i)

$$400=100x^2 \quad or \quad x^2=\frac{400}{100}=4$$

$$\Rightarrow x=\pm 2$$

Exercise 3.2

Q.1

Given Data:

y **varies directly as** x

$y=8$ when $x=2$

Required

(i) y in terms of x

(ii) $y=?$ when $x=5$

(iii) $x=?$ when $y=28$

Solution:

Here $y \propto x$

$y=kx \rightarrow (\text{i})$

For value of k

Put $y=8$, $x=2$

$(8)=k(2)$

$\Rightarrow k=4$

(i) **y in terms of** x: **(A.B + K.B)**

Put $k=4$ in equation (i)

$y=4x$

(ii) **For value of** y: **(A.B + K.B)**

Put $x=5$ and $k=4$, we get

$y=(4)(5)$

$y=20$

(iii) **For value of** x: **(A.B + K.B)**

Put k and y in the equation (i), we get

$28=4x$

$\Rightarrow x=7$

Result

(i) y = $4x$

(ii) y = 20 when $x=5$

(iii) x = 7 when $y=28$

Q.2

Given Data:

$y \propto x$

y = 7 when x = 3

Required

(i) y in term of x

(ii) **(a)** $x = ?$ when $y = 35$ **(A.B)**

(b) $y = ?$ when $x = 18$ **(A.B)**

Solution:

Here $x \propto y$ or $y \propto x$

$y = kx \rightarrow (\text{i})$

For value of k

Put $y = 7, x = 3$

$7 = k(3)$

$\Rightarrow k = \frac{7}{3}$

(i) y **in terms of** x **(A.B)**

$y = kx$

$\Rightarrow y = \frac{7}{3}x$

(ii) **For values of** x**:** **(A.B)**

Put y = 35 in equation (i), we get

$35 = \frac{7}{3}x$

$\frac{35 \times 3}{7} = x$

$\Rightarrow x = 15$

For value of y

Put x = 18 in equation (i), we get;

$y = \frac{7}{3}(18)$

y = 7(6)

y = 42

Result

(i) $y = \frac{7}{3}x$

(ii) **(a)** $x = 15$

(b) $y = 42$

Q.3 **(A.B + U.B)**

Given Data:

$R \propto T$

$R = 5$ when $T = 8$

Required

Equation connecting R and T.

$R = ?$ when $T = 64$

$T = ?$ when $R = 20$

Solution:

Here, $R \propto T$

$R = kT$

For value of k

Put R=5 and T=8

5 = 8(k)

$\frac{5}{8} = k$

The equation connecting R and T is:

$R = \frac{5}{8}T \rightarrow (\text{i})$

For value of R

By putting T = 64 in equation (i), we get;

$R = \frac{5}{8}(64)$

$R = 5(8)$

$R = 40$

For value of T

By putting $R = 20$ in equation (i), we get;

$20 = \frac{5}{8}T$

$20 \times \frac{8}{5} = T$

$\Rightarrow T = 32$

Result

$R = \frac{5}{8}T$

$R = 40$ when $T = 64$

$T = 32$ when $R = 20$

Q.4 **(A.B + U.B)**

Given Data:

$R \propto T^2$

$R = 8$ when $T = 3$

Required

$R = ?$ when $T = 6$

Solution:

$R \propto T^2$

$R = kT^2 \rightarrow (\text{i})$

For value of k

Put $R = 8, T = 3$ in equation (i), we get

$8 = k(3)^2$

$8 = k(9)$

$\frac{8}{9} = k$

$\Rightarrow k = \frac{8}{9}$

$\therefore R = \frac{8}{9}T^2 \rightarrow$ (ii)

For value of R

Put $T = 6$ in equation (ii), we get

R = $\frac{8}{9}(6)^2$

R = $\frac{8}{9}(36)$

$R = 8(4)$

$R = 32$

Result:

$R = 32$ when $T = 6$

Q.5 **(A.B)**

Given Data:

$V \propto R^3$

$V = 5$ when $R = 3$

Required data:

$R = ?$ when $V = 625$

Solution:

Here, $V \propto R^3$

$V = kR^3 \rightarrow$ (i)

For value of k

Put $V = 5$ and $R = 3$ in equation (i), we get

$5 = k(3)^3$

$5 = 27k$

$k = \frac{5}{27}$

$V = \frac{5}{27}R^3$

For value of R

$625 = \frac{5}{27}R^3$

$\frac{625 \times 27}{5} = R^3$

$125 \times 27 = R^3$

$3375 = R^3$

Taking cube root on both sides

$\Rightarrow \sqrt[3]{R^3} = \sqrt[3]{3375}$

$R = 15$

Result:

$R = 15$ when $V = 625$

Q.6 **(A.B)**

Given Data:

$w \propto u^3$

$w = 81$ when $u = 3$

Required data:

$w = ?$ when $u = 5$

Solution

Here, $w \propto u^3$

$w = ku^3 \rightarrow$ (i)

For value of k

Put $w = 81, u = 3$ in equation (i), we get

$(81) = k(3)^3$

$81 = k(27)$

$\frac{81}{27} = R$

$k = 3$

$\therefore w = 3u^3 \rightarrow$ (ii)

Put $u = 5$ in equation (ii), we get

$w = 3(5)^3$

$w = 3(125)$

$w = 375$

Result

$w = 375$ when $u = 5$

Q.7 **(A.B)**

Given Data:

$y \propto \frac{1}{x}$

$y = 7$ when $x = 2$

Required data:

$y = ?$ when $x = 126$

Solution:

Here, $y \propto \frac{1}{x}$

$y = k\frac{1}{x} \rightarrow$ (i)

For value of k

Put $y=7$ and $x=2$ in equation (i), we get

$$7=\frac{k}{2}$$
$$\Rightarrow k=7(2)$$
$$k=14$$
$$\therefore y=\frac{14}{x}\rightarrow \text{(ii)}$$

For value of y

Put $x=126$ in equation (ii), we get

$$y=\frac{14}{126}$$
$$y=\frac{1}{9}$$

Result:

$$y=\frac{1}{9} \text{ when } x=126$$

Q.8 **(A.B + U.B)**

Given Data:

$$y\propto\frac{1}{x}$$
$y=4$ when $x=3$

Required data:

$x=?$ when $y=24$

Solution:

Here, $y\propto\frac{1}{x}$

$$y=\frac{k}{x}\rightarrow \text{(i)}$$

For value of k

Put $y=4$ and $x=3$ in equation (i), we get

$$4=\frac{k}{3}$$
$$\Rightarrow k=12$$
$$\therefore y=\frac{12}{x}\rightarrow \text{(ii)}$$

For value of x

Put $y=24$ in equation (ii), we get

$$24=\frac{12}{x}$$
$$\Rightarrow x=\frac{12}{24}$$
$$x=\frac{1}{2}$$

Result:

$$x=\frac{1}{2} \text{ when } y=24$$

Q.9 **(A.B + U.B)**

Given Data:

$$w\propto\frac{1}{z}$$
$w=5$ when $z=7$

Required data:

$w=?$ when $z=\frac{175}{4}$

Solution:

Here, $w\propto\frac{1}{z}$

$$w=\frac{k}{z}\rightarrow \text{(i)}$$

For value of k

Put $w=5$ and $z=7$ in equation (i), we get

$$5=\frac{k}{7}$$
$$\Rightarrow k=35$$
$$\therefore w=\frac{35}{z}\rightarrow \text{(ii)}$$

For value of w

Put $z=\frac{175}{4}$ in equation (ii), we get

$$w=\frac{35}{\left(\frac{175}{4}\right)}$$
$$w=35\div\frac{175}{4}$$
$$w=35\times\frac{4}{175}$$
$$w=\frac{4}{5}$$

Result:

$$w=\frac{4}{5} \text{ when } z=\frac{175}{4}$$

Q.10 **(A.B + U.B)**

Given Data:

$A \propto \frac{1}{r^2}$

$A = 2$ when $r = 3$

Required data:

$r = ?$ when $A = 72$

Solution:

$A \propto \frac{1}{r^2}$

$A = \frac{k}{r^2}$

For value of k

$A = \frac{k}{r^2}$

Put $A = 2$ and $r = 3$

$2 = \frac{k}{(3)^2}$

$18 = k$

Or $k = 18$

$A = \frac{18}{k}$

For value of r

$A = \frac{k}{r^2}$

Put $A = 72$ and $k = 18$

$72 = \frac{18}{r^2}$

$r^2 = \frac{18}{72}$

$r^2 = \frac{1}{4}$

Taking square root

$r = \pm\frac{1}{2}$

Result:

$r = \pm\frac{1}{2}$ when $A = 72$

Q.11 **(A.B + U.B)**

Given Data:

$a \propto \frac{1}{b^2}$

$a = 3$ when $b = 4$

Required data:

$a = ?$ when $b = 8$

Solution:

$a \propto \frac{1}{b^2}$

$a = \frac{k}{b^2} \rightarrow$ (i)

For value of k

Put the value $a = 3$ and $b = 4$ in eq (i), we get

$3 = \frac{k}{(4)^2}$

$3(16) = k$

$\Rightarrow k = 48$

$\therefore a = \frac{48}{b^2} \rightarrow$ (ii)

For value of *a*

Put $b = 8$ in equation (ii), we get

$a = \frac{48}{8^2}$

$a = \frac{48}{64}$

$a = \frac{3}{4}$

Result:

$a = \frac{3}{4}$ when $b = 8$

Q.12 **(A.B + U.B)**

Given data:

$V \propto \frac{1}{r^3}$

$V = 5$ when $r = 3$

Required data:

$V = ?$ when $r = 6$

$r = ?$ when $V = 320$

Solution:

$V \propto \frac{1}{r^3}$

$V = \frac{k}{r^3} \rightarrow$ (i)

For value of k

Put $V = 5$ and $r = 3$ in equation (i), we get

$5 = \frac{k}{3^3}$

$5(27) = k$

k = 135

$\therefore V = \frac{135}{r^3} \rightarrow \text{(ii)}$

For value of *V*

Put $r = 6$ in equation (ii), we get

$V = \frac{135}{6^3}$

$V = \frac{135}{216}$

$V = \frac{5}{8}$

For value of *r*

Put $V = 320$ in equation (ii), we get

$320 = \frac{135}{r^3}$

$r^3 = \frac{27}{64}$

Taking cube root on both sides, we get

$\sqrt[3]{r^3} = \sqrt[3]{\frac{27}{64}}$

$r = \frac{3}{4}$

Result

$V = \frac{5}{8}$ when $r = 6$

$r = \frac{3}{4}$ when $V = 320$

Q.13 **(SGD 2014)** **(A.B + U.B)**

Given

$m \propto \frac{1}{n^3}$ and m = 2 when n = 4

Required data:

(i) m = ? When n = 6

(ii) n = ? when m = 432

Solution

$m \propto \frac{1}{n^3}$

$m = k \times \frac{1}{n^3}$

$m = \frac{k}{n^3} \rightarrow (i)$

For value of *k*

Put $m = 2,\ n = 4$ in equation (*i*)

$2 = \frac{k}{(4)^3}$

$2 \times 64 = k$

$k = 2 \times 64$

$k = 128$

$m = \frac{128}{n^3}$

For value of *m*

Put $k = 128,\ n = 6$

$m = \frac{128}{(6)^3} \quad \because n = 6$

$m = \frac{128}{216}$

$m = \frac{16}{27}$

For value of *n*

$m = \frac{k}{n^3}$

$432 = \frac{128}{n^3} \quad \because m = 432$

$432\left(n^3\right) = 128$

$n^3 = \frac{128}{432}$

$n^3 = \frac{64}{216}$

$n^3 = \frac{32}{108}$

$n^3 = \frac{16}{54}$

$n^3 = \frac{8}{27}$

Taking cube root on both sides

$n = \frac{2}{3}$

Result:

$m = \frac{16}{27}$ when $n = 6$

$n = \frac{2}{3}$ when $m = 432$

Finding 3rd, 4th, Mean and Continued Proportion

Third Proportional (K.B + U.B)

For three quantities a, b and c are related as, $a:b::b:c$, then c is called the third proportional.

i.e. $c=\dfrac{b^2}{a}$

Example 1: (Page # 56) **(A.B)**

Find a third proportional of $x+y$ and x^2-y^2

Solution:

Let c be the third proportional,

Then $x+y:x^2-y^2::x^2-y^2:c$

$$c(x+y)=\left(x^2-y^2\right)\left(x^2-y^2\right)$$

$$c=\frac{\left(x^2-y^2\right)\left(x^2-y^2\right)}{x+y}$$

$$=\frac{\left(x^2-y^2\right)(x-y)(x+y)}{x+y}$$

$$=\left(x^2-y^2\right)(x-y)$$

$$\Rightarrow c=(x+y)(x-y)^2$$

Mean Proportional (K.B)

If three quantities a, b and c are related as $a:b::b:c$, then b is called mean proportional. i.e. $b^2=ac$

Continued Proportion (K.B)

If three quantities a, b and c are related as $a:b::b:c$ where a is first, b is mean and c is the third proportional, then a, b and c are in continued proportion.

i.e. $b^2=ac$

Example 4: (Page # 57) **(A.B)**

(GRW 2016, FSD 2015, 17, SGD 2015)

Find p, if 12, p and 3 are in continued proportion.

Solution:

Since 12, p and 3 are in continued proportion.

$\therefore \quad 12:p::p:3$

$\because$ Product of extremes = product of means

$$p.p=(12)(3)$$

$$\Rightarrow p^2=36$$

Thus, $p=\pm 6$

Example 3: (Page # 57) **(A.B)**

Find the mean proportional of $9p^6q^4$ and r^8

Solution:

Let m be the mean proportional.

Then $9p^6q^4:m::m:r^8$

Or $m.m=9p^6q^4\left(r^8\right)$

$$m^2=9p^6q^4r^8$$

$$m=\pm\sqrt{9p^6q^4r^8}$$

$$\Rightarrow m=\pm 3p^3q^2r^4$$

Fourth Proportional (K.B + U.B)

If four quantities a, b, c and d are related as $a:b::c:d$, then d is called fourth proportional.

i.e. $d=\dfrac{bc}{a}$

Example 2: (Page # 57) **(K.B)**

Find fourth proportional of a^3-b^3, $a+b$, and a^2+ab+b^2

Solution:

Let x be the fourth proportional,

Then

$$\left(a^3-b^3\right):(a+b)::\left(a^2+ab+b^2\right):x$$

$\because$ Product of extremes = Product of means

$$x\left(a^3-b^3\right)=(a+b)\left(a^2+ab+b^2\right)$$

$$x=\frac{(a+b)\left(a^2+ab+b^2\right)}{a^3-b^3}$$

$$=\frac{(a+b)\left(a^2+ab+b^2\right)}{(a-b)\left(a^2+ab+b^2\right)}$$

$$x=\frac{a+b}{a-b}$$

Exercise 3.3

Q.1 Find a third proportional to

(i) 6, 12 **(A.B)**

(SGD 2017, RWP 2017, D.G.K 2016, 17)

Let, the third proportional = a

According to the given condition;

$6:12::12:a$

$\because$ Product of means = product of extremes

$12(12)=6a$

$144=6a$

$\frac{144}{6}=a$

$\Rightarrow a=24$

$\therefore$ **Third proportional is** 24

(ii) $a^3, 3a^2$ **(A.B)**

(MTN 2014, 16, RWP 2017, D.G.K 2014)

Let, third proportional = x

According to the given condition;

$a^3 : 3a^2 :: 3a^2 : x$

$\because$ Product of extremes = Product of means

$$(a^3)x=(3a^2)(3a^2)$$

$$a^3x=9a^4$$

$$x=\frac{9a^4}{a^3}$$

$$x=9a$$

$\therefore$ **Third proportional is** $9a$

(iii) $a^2-b^2, a-b$ **(A.B)** **(LHR 2015, GRW 2014, 16, SWL 2016, BWP 2015)**

Let, third proportional = x

According to the given condition;

$a^2-b^2 : a-b :: a-b : x$

$\because$ Product of extremes = product of means

$$(a^2-b^2)(x)=(a-b)^2$$

$$(a-b)(a+b)(x)=(a-b)^2$$

$$(a+b)x=a-b$$

$$x=\frac{a-b}{a+b}$$

$\therefore$ **Third proportional is** $\frac{a-b}{a+b}$

(iv) $(x-y)^2, (x^3-y^3)$ **(A.B)**

(FSD 2015, GRW 2016)

Let, third proportional = a

According to the given condition,

$(x-y)^2 : (x^3-y^3) :: (x^3-y^3) : a$

$\because$ Product of extreme = product of mean

$$(x-y)^2 a=(x^3-y^3)(x^3-y^3)$$

$$a(x-y)(x-y)$$

$$=(x-y)(x^2+xy+y^2)(x-y)(x^2+xy+y^2)$$

$$a=(x^2+xy+y^2)^2$$

$\therefore$ **Third proportional is** $(x^2+xy+y^2)^2$

(v) $(x+y)^2, x^2-xy-2y^2$ **(A.B)**

Let third proportional = a

According to given condition

$(x+y)^2 : x^2-xy-2y^2 :: x^2-xy-2y^2 : a$

$\because$ Product of extremes = Product of means

$$a(x+y)^2=(x^2-xy-2y^2)(x^2-xy-2y^2)$$

$$a(x+y)^2=(x^2-xy-2y^2)^2$$

$$a(x+y)^2=(x^2-2xy+xy-2y^2)^2$$

$$a(x+y)^2=[x(x-2y)+y(x-2y)]^2$$

$$=[(x-2y)(x+5)]^2$$

$$a(x+y)^2=(x-2y)^2(x+y)^2$$

$$a=(x-2y)^2$$

$\therefore$ **Third proportional** $=(x-2y)^2$

(vi) $\frac{P^2-q^2}{P^3+q^3}, \frac{P-q}{P^2-Pq+q^2}$ **(A.B)**

Let, the third proportional = a

According to the given condition;

$$\frac{p^2-q^2}{p^3+q^3} : \left(\frac{p-q}{p^2-pq+q^2}\right) :: \left(\frac{p-q}{P^2-pq+q^2}\right) : a$$

$\because$ Product of extremes = Product of means

$$a\left(\frac{p^2-q^2}{p^3+q^3}\right)=\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right)$$

$$a=\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p^3+q^3}{p^2-q^2}\right)$$

$$a=\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{p-q}{p^2-pq+q^2}\right)\left(\frac{(p+q)(p^2-pq+q^2)}{(p-q)(p+q)}\right)$$

$$a=\frac{p-q}{p^2-pq+q^2}$$

$\therefore$ **Third proportional is** $\frac{p-q}{p^2-pq+q^2}$

Q.2 Find a fourth proportional to

(i) $5, 8, 15$ **(A.B)**

(LHR 2014, GRW 2017, BWP 2016)

Let, the fourth proportional = x

According to the given condition:

$5 : 8 :: 15 : x$

$\because$ Product of mean = product of extreme

$(5)x = 8(15)$

$5x = 120$

$x = \frac{120}{5}$

$x = 24$

$\therefore$ **Fourth proportional is** 24

(ii) $4x^4, 2x^3, 18x^5$ **(A.B)**

(SWL 2015, BWP 2016)

Let, the fourth proportional = a

According to the given condition:

$4x^4 : 2x^3 :: 18x^5 : a$

$\because$ Product of means = product of extremes

$(18x^5)(2x^3) = (4x^4)(a)$

$\frac{36(x^8)}{4x^4} = a$

$\frac{18x^{8-4}}{2} = a$ $\quad 9x^4 = a$

$\Rightarrow \quad a = 9x^4$

$\therefore$ **Fourth proportional is** $9x^4$

(iii) $15a^5b^6, 10a^2b^5, 21a^3b^3$ **(A.B)**

(FSD 2017, D.G.K 2015, 17)

Let, the third proportional = x

According to the given condition;

$15a^5b^6 : 10a^2b^5 :: 21a^3b^3 : x$

$\because$ Product of extremes = product of means

$(15a^5b^6)(x) = (10a^2b^5)(21a^3b^3)$

$x = \frac{10 \times 21 \times a^5 \times b^8}{15a^5b^6}$

$x = \frac{2 \times 21 \times b^2}{3}$

$x = 2 \times 7 \times b^2$

$x = 14b^2$

$\therefore$ **Fourth proportional is** $14b^2$

(iv) $x^2 - 11x + 24, x - 3, 5x^4 - 40x^3$ **(A.B)**

Let, Fourth proportional is a

According to the given condition:

$x^2 - 11x + 24 : (x-3) :: 5x^4 - 40x^3 : a$

$\because$ Product of extremes = Product of means

$(x^2 - 11x + 24)(a) = (x-3)(5x^4 - 40x^3)$

$(x^2 - 8x - 3x + 24)a = (x-3)(5x^4 - 40x^3)$

$[x(x-8) - 3(x-8)]a = 5x^3(x-3)(x-8)$

$(x-3)(x-8)a = 5x^3(x-3)(x-8)$

$a = 5x^3$

$\therefore$ **Fourth proportional is** $5x^3$

(v) $p^3 + q^3, p^2 - q^2, p^2 - pq + q^2$

Let, Fourth proportional = a

According to given condition:

$p^3 + q^3 : p^2 - q^2 :: p^2 - pq + q^2 : a$

$\because$ Product of extremes = Product of means

$(p^3 + q^3)a = (p^2 - q^2)(p^2 - pq + q^2)$

$(p+q)(p^2 - pq + q^2)a$

$= (p+q)(p-q)(p^2 - pq + q^2)$

$a = p - q$

$\therefore$ **Fourth proportional** $= p - q$

(vi) $(p^2 - q^2)(p^2 + pq + q^2), p^3 + q^3, p^3 - q^3$

(A.B)

Let, the fourth proportional = x

According to the given condition;

$(p^2 - q^2)(p^2 + pq + q^2) : p^3 + q^3 :: p^3 - q^3 : x$

$\because$ Product of extremes = Product of means

$(p^2 - q^2)(p^2 + pq + q^2)(x) = (p^3 + q^3)(p^3 - q^3)$

$(p+q)(p-q)(p^2 + pq + q^2)(x)$

$= (p+q)(p^2 - pq + q^2)(p-q)(p^2 + pq + q^2)$

$x = p^2 - pq + q^2$

$\therefore$ **Fourth proportional is** $p^2 - pq + q^2$

Q.3 Find a mean proportional between

(i) $20, 45$ **(A.B)**

(LHR 2016, GRW 2014, D.G.K 2016)

Let, the mean proportional = x

According to the given condition;

$20 : x :: x : 45$

$\because$ Product of means = Product of extremes

$(x)(x) = (20)(45)$

$x^2 = 900$

Taking square root on both sides

$\sqrt{x^2} = \sqrt{900}$

$x = \pm 30$

$\therefore$ **The mean proportional is** ± 30

(ii) $20x^3y^3, 5x^7y$ **(A.B)**

Let, mean proportional = a
According to given condition
$20x^3y^3 : a :: a : 5x^7y$

$\because$ Product of extremes = Product of means

$\left(20x^3y^5\right)\left(5x^7y\right) = a.a$

$100x^{10}y^6 = a^2$

$a^2 = 100x^{10}y^6$

Taking Sq. root on both sides

$\sqrt{a^2} = \sqrt{100x^{10}y^6}$

$\Rightarrow a = \pm 10x^5y^3$

$\therefore$ **Mean proportional** = $\pm 10x^5y^3$

(iii) $15p^4qv^3,\ 135q^5r^7$ **(A.B)**

Let, the mean proportional = a
According to the given condition,
$15p^4qv^3 : a :: a : 135q^5r^7$

$\because$ Product of means = Product of extremes

$a^2 = \left(15p^4qr^3\right)\left(135q^5r^7\right)$

$a^2 = 2025p^4q^6r^{10}$

Taking square root on both sides,

$\sqrt{a^2} = \sqrt{2025p^4q^6r^{10}}$

$a = \pm 45p^2q^3r^5$

$\therefore$ **The mean proportional is** $\pm 45p^2q^3r^5$

(iv) $x^2 - y^2 = \dfrac{x-y}{x+y}$ **(GRW 2015)** **(A.B)**

Let, the mean proportional = a
According to given condition;

$x^2 - y^2 : a :: a : \dfrac{x-y}{x+y}$

$\because$ Product of means = Product of extremes

$a^2 = \left(x^2 - y^2\right)\left(\dfrac{x-y}{x+y}\right)$

$a^2 = (x-y)(x+y)\left(\dfrac{x-y}{x+y}\right)$

$a^2 = (x-y)^2$

Taking square root on both sides;

$\sqrt{a^2} = \sqrt{(x-y)^2}$

$a = \pm(x-y)$

$\therefore$ **The mean proportional is** $\pm(x-y)$

Q.4 Find the value of the letter involved in the following continued proportions.

(i) $5, p, 45$ **(A.B)**

According to given condition
$5 : p :: p : 45$

$\because$ Product of means = product of extremes

$p^2 = 5 \times 45$
$p^2 = 225$
Talking square root
$p = \pm 15$

Result
$p = \pm 15$

(ii) $8, x, 18$ **(A.B)**

According to condition
$8 : x :: x : 18$

$\because$ Product of means = Product of extremes

$x^2 = 8 \times 18$
$x^2 = 144$
Taking square root on both sides
$x = \pm\sqrt{144}$
$\Rightarrow x = \pm 12$

Result:
$x = \pm 12$

(iii) $12, 3p-6, 27$ **(A.B)**

According to given condition
$12 : 3p-6 :: 3p-6 : 27$

$\because$ Product of extremes = Product of means

$12 \times 27 = (3p-6)(3p-6)$
$324 = (3p-6)^2$
$(3p-6)^2 = 324$
Taking Sq. root on both sides
$3p - 6 = \pm 18$

Either

$3p - 6 = -18$ or $3p - 6 = 18$
$3p = 6 - 18$ $3p = 6 + 18$
$3p = -12$ $3p = 24$
$p = \dfrac{-12}{3}$ $p = \dfrac{24}{3}$
$p = -4$ $p = 8$

Result:
$p = -4, 8$

(iv) $7, m-3, 28$ According to given condition: **(A.B)**

(GRWP 2015, 17, FSD 2017, MTN 2017, BWP 2015)

$7 : m-3 :: m-3 : 28$

$\because$ Product of means = Product of extremes

$(m-3)^2 = 7 \times 28$

$(m-3)^2 = 196$

Taking square root on both sides

$m-3 = \pm 14$

Either

$m-3=-14$ or $m-3=14$

$m=-14+3$ $\quad m=14+3$

$\Rightarrow m=-11$ $\quad m=17$

Result:

$m=-11, 17$

Theorems on Proportions

(K.B + U.B + A.B)

(LHR 2014, SWL 2016, BWP 2015, 16, MTN 2015)

❖ **Invertedo Theorem.**

If $a:b=c:d$ then $b:a=d:c$

❖ **Alternendo Theorem.**

If $a:b=c:d$ then $a:c=b:d$

❖ **Componendo Theorem**

If $a:b=c:d$ then

(i) $a+b:b=c+d:d$

(ii) $a:a+b=c:c+d$

❖ **Dividendo Theorem**

If $a:b=c:d$ then

(i) $a-b:b=c-d:d$

(ii) $a:a-b=c:c-d$

❖ **Componendo–Dividendo Theorem**

If $a:b=c:d$ then

(i) $a+b:a-b=c+d:c-d$

(ii) $a-b:a+b=c-d:c+d$

Example 5: (Page # 60) **(A.B)**

Given:

$m:n=p:q$

To Prove:

$3m+7n:3m-7n=3p+7q:3p-7q$

Solution:

$m:n=p:q$

$$\frac{m}{n}=\frac{p}{q}$$

Multiplying both sides by $\frac{3}{7}$, we get

$$\frac{3m}{7n}=\frac{3p}{7q}$$

Then using componendo-dividendo theorem

$$\frac{3m+7h}{3m-7n}=\frac{3p+7q}{3p-7q}$$

Thus, $3m+7n:3m-7n=3p+7q:3p-7q$

Example 7: (Page # 60) **(A.B)**

Using theorem of componendo-dividendo, find the value of

$$\frac{m+3p}{m-3p}+\frac{m+2p}{m-2p}, \text{ if } m=\frac{6pq}{p+q}$$

Solution:

Since $m=\frac{6pq}{p+q}$

Or $m=\frac{(3p)(2q)}{p+q} \rightarrow (i)$

$$\therefore \quad \frac{m}{3p}=\frac{2q}{p+q}$$

By componendo-dividendo theorem

$$\frac{m+3p}{m-3p}=\frac{2q+(p+q)}{2q-(p+q)}=\frac{2q+p+q}{2q-p-q}$$

$$\frac{m+3p}{m-3p}=\frac{p+3q}{q-p} \rightarrow (ii)$$

Again from eq. (i), we have

$$\frac{m}{2q}=\frac{3p}{q-p}$$

By componendo-dividendo theorem

$$\frac{m+3p}{m-3p}=\frac{2q+(p+q)}{2q-(p+q)}=\frac{2q+p+q}{2q-p-q}$$

$$\frac{m+2p}{m-2p}=\frac{4p+q}{2p-q} \rightarrow (iii)$$

Adding (ii) and (iii)

$$\frac{m+3p}{m-3p}+\frac{m+2q}{m-2q}=\frac{p+3q}{q-p}+\frac{4p+q}{2p-q}$$

$$=-\frac{p+3q}{p-q}+\frac{4p+q}{2p-q}$$

$$=\frac{-(p+3q)(2p-q)+(p-q)(4p+q)}{(p-q)(2p-q)}$$

$$=\frac{-2p^2-5pq+3q^2+4p^2-3pq-q^2}{(p-q)(2p-q)}$$

$$=\frac{-2p^2-8pq+2q^2}{(p-q)(2p-q)}=\frac{-2(p^2-4pq-q^2)}{(p-q)(2p-q)}$$

Example 8: (Page # 61) **(A.B)**

Using theorem of componendo-dividendo, solve the equation

$$\frac{\sqrt{x+3}+\sqrt{x-3}}{\sqrt{x+3}-\sqrt{x-3}}=\frac{4}{3}$$

Solution:

Given equation is $\frac{\sqrt{x+3}+\sqrt{x-3}}{\sqrt{x+3}-\sqrt{x-3}}=\frac{4}{3}$

By componendo-dividendo theorem

$$\frac{\sqrt{x+3}+\sqrt{x-3}+\sqrt{x+3}-\sqrt{x-3}}{\sqrt{x+3}+\sqrt{x-3}-\sqrt{x+3}+\sqrt{x-3}}=\frac{4+3}{4-3}$$

$$\frac{2\sqrt{x+3}}{2\sqrt{x-3}}=\frac{7}{1}\Rightarrow\sqrt{\frac{x+3}{x-3}}=7$$

Squaring both sides

$$\frac{x+3}{x-3}=49$$

$$x+3=49(x-3)$$

$$x+3=49x-147$$

$$x-49x=-147-3$$

$$-48x=-150$$

$$\Rightarrow x=\frac{150}{48}=\frac{25}{8}$$

Therefore, Solution Set $=\left\{\frac{25}{8}\right\}$

Exercise 3.4

Q.1

(i) Given **(A.B)**

$$\frac{4a+5b}{4a-5b}=\frac{4c+5d}{4c-5d}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{4a+5b}{4a-5b}=\frac{4c+5d}{4c-5d}$$

By applying componendo-dividedo property

$$\frac{(4a+5b)+(4a-5b)}{(4a+5b)-(4a-5b)}=\frac{(4c+5d)+(4c-5d)}{(4c+5d)-(4c-5d)}$$

$$\frac{8a}{10b}=\frac{8c}{10d}$$

(by using cancellation prop)

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Proved

(ii) Given **(A.B)**

$$\frac{2a+9b}{2a-9b}=\frac{2c+9d}{2c-9d}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{2a+9b}{2a-9b}=\frac{2c+9d}{2c-9d}$$

By applying componendo-dividendo prop

$$\frac{(2a+9b)+(2a-9b)}{(2a+9b)-(2a-9b)}=\frac{(2c+9d)+(2c-9d)}{(2c+9d)-(2c-9d)}$$

$$\frac{4a}{18b}=\frac{4c}{18d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Proved

(iii) Given **(A.B)**

$$\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}$$

By applying componendo-dividendo prop

$$\frac{(ac^2+bd^2)+(ac^2-bd^2)}{(ac^2+bd^2)-(ac^2-bd^2)}=\frac{(c^3+d^3)+(c^3-d^3)}{(c^3+d^3)-(c^3-d^3)}$$

$$\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}=\frac{c^3+d^3+c^3-d^3}{c^3+d^3-c^3+d^3}$$

$$\frac{2ac^2}{2bd^2}=\frac{c^3}{d^3}$$

$$\frac{ac^2}{bd^2}=\frac{c^3}{d^3}$$

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:\text{d}$$

Hence Proved

(iv) Given **(A.B)**

$$\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$$

By applying componendo-dividendo prop

$$\frac{\left(a^2c+b^2d\right)+\left(a^2c-b^2d\right)}{\left(a^2c+b^2d\right)-\left(a^2c-b^2d\right)}=\frac{\left(ac^2+bd^2\right)+\left(ac^2-bd^2\right)}{\left(ac^2+bd^2\right)-\left(ac^2-bd^2\right)}$$

$$\frac{a^2c+b^2d+a^2c-b^2d}{a^2c+b^2d-a^2c+b^2d}=\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}$$

$$\frac{2a^2c}{2b^2d}=\frac{2ac^2}{2bd^2}$$

$$\frac{a^2c}{b^2d}=\frac{ac^2}{bd^2}$$

Multiplying by $\frac{bd}{ac}$ on both sides

$$\frac{a^2c}{b^2d}\times\frac{bd}{ac}=\frac{ac^2}{bd^2}\times\frac{bd}{ac}$$

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence Proved

(v) Given **(A.B)**

$Pa+pq:pq-qb=pc+qd:pc-qd$

To prove

a : $b=c:d$

Proof

Here

$$Pa+pq:pq-qb=pc+qd:pc-qd$$

$$\frac{pa+qb}{pa-qb}=\frac{pc+qd}{pc-qd}$$

By applying componendo-dividendo prop

$$\frac{(pa+qb)+(pa-qb)}{(pa+qb)-(pa-qb)}=\frac{(pc+qd)+(pc-qd)}{(pc+qd)-(pc-qd)}$$

$$\frac{pa+qb+pa-qb}{pa+qb-pa+qb}=\frac{pc+qd+pc-qd}{pc+qd-pc+qd}$$

$$\frac{2pa}{2qb}=\frac{2pc}{2qd}$$

$$\frac{pa}{qb}=\frac{pc}{qd}$$

Multiply by $\frac{q}{p}$

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence Proved

(vi) Given **(A.B)**

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

By applying componendo-dividendo prop

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)}=\frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{a+b+\not c+\not d+a+b-\not c-\not d}{\not a+\not b+c+d-\not a-\not b+c+d}=\frac{a-b+\not c-\not d+a-b-\not c+\not d}{\not a-\not b+c-d-\not a+\not b+c-d}$$

$$\frac{2a+2b}{2c+2d}=\frac{2a-2b}{2c-2d}$$

By applying alternendo property

$$\frac{2a+2b}{2a-2b}=\frac{2c+2d}{2c-2d}$$

Again applying componendo-dividendo prop

$$\frac{(2a+2b)+(2a-2b)}{(2a+2b)-(2a-2b)}=\frac{(2c+2d)+(2c-2d)}{(2c+2d)-(2c-2d)}$$

$$\frac{2a+2\not{b}+2a-2\not{b}}{2\not{a}+2b-2\not{a}+2b}=\frac{2c+2\not{d}+2c-2\not{d}}{2\not{c}+2d-2\not{c}+2d}$$

$$\frac{4a}{4b}=\frac{4c}{4d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence Proved

(vii) Given **(A.B)**

$$\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$$

By applying cmponendo-dividendo prop

$$\frac{(2a+3b+2c+3d)+(2a+3b-2c-3d)}{(2a+3b+2c+3d)-(2a+3b-2c-3d)}$$

$$=\frac{(2a-3b+2c-3d)+(2a-3b-2c+3d)}{(2a-3b+2c-3d)-(2a-3b-2c+3d)}$$

$$\frac{2a+3b+2c+3d+2a+3b-2c-3d}{2a+3b+2c+3d-2a-3b+2c+3d}$$

$$=\frac{2a-3b+2c-3d+2a-3b-2c+3d}{2a-3b+2c-3d-2a+3b+2c-3d}$$

$$\frac{4a+9b}{4c+9d}=\frac{4a-9b}{4c-9d}$$

By applying alternendo property

$$\frac{4a+9b}{4a-9b}=\frac{4c+9d}{4c-9d}$$

Again applying componendo dividendo prop

$$\frac{(4a+9b)+(4a-9b)}{(4a+9b)-(4a-9b)}=\frac{(4c+9d)+(4c-9d)}{(4c+9d)-(4c-9d)}$$

$$\frac{4a+9b+4a-9b}{4a+9b-4a+9b}=\frac{4c+9d+4c-9d}{4c+9d-4c+9d}$$

$$\frac{8a}{18b}=\frac{8c}{18d}$$

Multiply by $\frac{18}{8}$ on both sides

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence Proved

(viii) Given **(A.B)**

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

To prove

$a:b=c:d$

Proof

Here

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

By applying componendo-dividendo prop

$$\frac{(a^2+b^2)+(a^2-b^2)}{(a^2+b^2)-(a^2-b^2)}=\frac{(ac+bd)+(ac-bd)}{(ac+bd)-(ac-bd)}$$

$$\frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}=\frac{ac+bd+ac-bd}{ac+bd-ac+bd}$$

$$\frac{2a^2}{2b^2}=\frac{2ac}{2bd}$$

$$\frac{a^2}{b^2}=\frac{ac}{bd}$$

(Multiply both sides by $\frac{b}{a}$)

$$\frac{a}{b}=\frac{c}{d}$$

$$\Rightarrow a:b=c:d$$

Hence Proved

Q.2 Using theorem of compoenendo-dividendo, find the value of

(i) Given **(A.B)**

$$x=\frac{4yz}{y+z}$$

Required

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=?$$

Solution:

Here

$$x=\frac{4yz}{y+z}$$

Dividing by 2y

$$\frac{x}{2y}=\frac{4yz}{2y(y+z)}$$

$$\frac{x}{2y}=\frac{2z}{y+z}$$

By applying componendo dividendo prop

$$\frac{x+2y}{x-2y}=\frac{2z+(y+z)}{2z-(y+z)}$$

$$=\frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y}=\frac{3z+y}{z-y}\rightarrow\text{(i)}$$

Again consider

$$x=\frac{4yz}{y+z}$$

Dividing by 2z

$$\frac{x}{2z}=\frac{4yz}{2z(y+z)}$$

$$=\frac{2y}{y+z}$$

By applying compoenendo dividendo prop

$$\frac{x+2z}{x-2z}=\frac{2y+(y+z)}{2y-(y+z)}$$

$$=\frac{2y+y+z}{2y-y-z}$$

$$=\frac{3y+z}{y-z}\rightarrow\text{(ii)}$$

Adding equation (i) and (ii)

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{3z+y}{z-y}+\frac{3y+z}{y-z}$$

$$=\frac{3z+y}{z-y}+\frac{3y+z}{-(z-y)}$$

$$=\frac{3z+y}{z-y}-\frac{3y+z}{z-y}$$

$$=\frac{3z+y-(3y+z)}{z-y}$$

$$=\frac{3z+y-3y-z}{z-y}$$

$$=\frac{2z-2y}{z-y}$$

$$=\frac{2(z-y)}{(z-y)}$$

$$\Rightarrow\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=2$$

(ii) Given: **(A.B)**

$$m=\frac{10np}{n+p}$$

Required

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=?$$

Solution:

Here

$$m=\frac{10np}{n+p}$$

Dividing by 5n

$$\frac{m}{5n}=\frac{10np}{(5n)(n+p)}$$

$$\frac{m}{5n}=\frac{2p}{n+p}$$

By applying componendo dividendo property

$$\frac{m+5n}{m-5n}=\frac{2p+(n+p)}{2p-(n+p)}$$

$$\frac{m+5n}{m-5n}=\frac{2p+n+p}{2p-n-p}$$

$$\frac{m+5n}{m-5n}=\frac{3p+n}{p-n}\rightarrow\text{(i)}$$

Again consider

$$m=\frac{10np}{n+p}$$

Dividing by 5p

$$\frac{m}{5p}=\frac{10np}{5p(n+p)}$$

$$\frac{m}{5p}=\frac{2n}{n+p}$$

Applying componendo – devidendo prop

$$\frac{m+5p}{m-5p}=\frac{2n+(n+p)}{2n-(n+p)}$$

$$\frac{m+5p}{m-5p}=\frac{2n+n+p}{2n-n-p}$$

$$\frac{m+5p}{m-5p}=\frac{3n+p}{n-p}\rightarrow(\text{ii})$$

Adding (i) and (ii)

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=\frac{3p+n}{p-n}+\frac{3n+p}{n-p}$$

$$=\frac{3p+n}{p-n}+\frac{3n+p}{-(p-n)}$$

$$=\frac{3p+n}{p-n}-\frac{3n+p}{p-n}$$

$$=\frac{3p+n-(3n+p)}{p-n}$$

$$=\frac{3p+n-3n-p}{p-n}$$

$$=\frac{2p-2n}{p-n}$$

$$=\frac{2(p-n)}{p-n}$$

$$=2$$

Hence

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=2$$

(iii) Given **(A.B)**

$$x=\frac{12ab}{a-b}$$

Required

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=?$$

Solution

Here

$$x=\frac{12ab}{a-b}$$

Dividing by $6a$

$$\frac{x}{6a}=\frac{2b}{a-b}$$

Applying componendo-devidendo prop

$$\frac{x+6a}{x-6a}=\frac{2b+(a-b)}{2b-(a-b)}$$

$$\frac{x+6a}{x-6a}=\frac{2b+a-b}{2b-a+b}$$

$$\frac{x+6a}{x-6a}=\frac{b+a}{3b-a}$$

By invertendo theorem

$$\frac{x-6a}{x+6a}=\frac{3b-a}{a+b}\rightarrow(\text{i})$$

Again consider

$$x=\frac{12ab}{a-b}$$

Dividing by 6b

$$\frac{x}{6b}=\frac{12ab}{6b(a-b)}$$

$$\frac{x}{6b}=\frac{2a}{a-b}$$

Applying componendo-Dividendo prop

$$\frac{x+6b}{x-6b}=\frac{2a+(a-b)}{2a-(a-b)}$$

$$\frac{x+6b}{x-6b}=\frac{2a+a-b}{2a-a+b}$$

$$\frac{x+6b}{x-6b}=\frac{3a-b}{a+b}\rightarrow(\text{ii})$$

Subtracting (i) and (ii)

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=\frac{3b-a}{a+b}-\frac{3a-b}{a+b}$$

$$=\frac{3b-a-(3a-b)}{a+b}$$

$$=\frac{4b-4a}{a+b}$$

$$=\frac{4(b-a)}{a+b}$$

Hence

$$\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}=\frac{4(b-a)}{a+b}$$

(iv) **Given** **(A.B)**

$$x=\frac{3yz}{y-z}$$

Required

$$\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}$$

Solution:

Here

$$x=\frac{3yz}{y-z}$$

Divided by 3y

$$\frac{x}{3y}=\frac{3yz}{3y(y-z)}$$

$$\frac{x}{3y}=\frac{z}{y-z}$$

By applying componendo dividendo prop

$$\frac{x+3y}{x-3y}=\frac{z+(y-z)}{z-(y-z)}$$

$$=\frac{z+y-z}{z-y+z}$$

$$=\frac{y}{2z-y}$$

Applying invertendo prop

$$\frac{x-3y}{x+3y}=\frac{2z-y}{y}\rightarrow\text{(i)}$$

Again consider

$$x=\frac{3yz}{y-z}$$

Divided by 3z

$$\frac{x}{3z}=\frac{3yz}{3z(y-z)}$$

$$=\frac{y}{y-z}$$

By applying componendo-dividendo prop

$$\frac{x+3z}{x-3z}=\frac{y+(y-z)}{y-(y-z)}$$

$$=\frac{y+y-z}{y-y+z}$$

$$=\frac{2y-z}{z}\rightarrow\text{(i)}$$

Sub.equation (i) and (ii)

$$\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}=\frac{2z-y}{y}-\frac{2y-z}{z}$$

$$=\frac{z(2z-y)-y(2y-z)}{yz}$$

$$=\frac{2z^2-yz-2y^2+yz}{yz}$$

$$=\frac{2z^2-2y^2}{yz}$$

$$\Rightarrow\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}=\frac{2(z^2-y^2)}{yz}$$

(v) **Given** **(A.B)**

$$s=\frac{6pq}{p-q}$$

To find value of

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=?$$

Solution:

Here

$$s=\frac{6pq}{p-q}$$

Divided by $3p$

$$\frac{s}{3p}=\frac{6pq}{3p(p-q)}$$

$$\frac{s}{3p}=\frac{2q}{p-q}$$

By applying componedo-dividendo prop (ii)

$$\frac{s-3p}{s+3p}=\frac{2q-(p-q)}{2q+(p-q)}$$

$$\frac{s-3p}{s+3p}=\frac{2q-p+q}{2q+p-q}$$

$$=\frac{3q-p}{p+q}\rightarrow\text{(i)}$$

Again consider

$$s=\frac{6pq}{p-q}$$

Divided by $3q$

$$\frac{s}{3q}=\frac{6pq}{3q(p-q)}$$

$$=\frac{2p}{p-q}$$

By applying componendo-dividendo prop

$$\frac{s+3q}{s-3q}=\frac{2p+(p-q)}{2p-(p-q)}$$

$$=\frac{2p+p-q}{2p-p+q}$$

$$=\frac{3p-q}{p+q}\rightarrow\text{(ii)}$$

Adding equation (i) and (ii)

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{3q-p}{p+q}+\frac{3p-q}{p+q}$$

$$=\frac{3q-p+3p-q}{p+q}$$

$$=\frac{2p+2q}{p+q}$$

$$=2\frac{(p+q)}{p+q}$$

$$\Rightarrow\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=2$$

(vi) **(FSD 2016)** **(A.B)**

Solution:

Here

$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$$

By applying invertendo property

$$\frac{(x-2)^2+(x-4)^2}{(x-2)^2-(x-4)^2}=\frac{13}{12}$$

By applying componendo-dividendo prop

$$\frac{\left[(x-2)^2+(x-4)^2\right]+\left[(x-2)^2-(x-4)^2\right]}{\left[(x-2)^2+(x-4)^2\right]-\left[(x-2)^2-(x-4)^2\right]}=\frac{13+12}{13-12}$$

$$\frac{(x-2)^2+\cancel{(x-4)^2}+(x-2)^2-\cancel{(x-4)^2}}{\cancel{(x-2)^2}+(x-4)^2-\cancel{(x-2)^2}+(x-4)^2}=\frac{25}{1}$$

$$\frac{2(x-2)^2}{2(x-4)^2}=25$$

$$\left(\frac{x-2}{x-4}\right)^2=25$$

Taking square root on both sides

$$\sqrt{\left(\frac{x-2}{x-4}\right)^2}=\sqrt{25}$$

$$\frac{x-2}{x-4}=\pm5$$

Either

$\frac{x-2}{x-4}=5$	or $\frac{x-2}{x-4}=-5$
$x-2=5(x-4)$	$x-2=-5(x-4)$
$x-2=5x-20$	$x-2=-5x+20$
$x-5x=-20+2$	$x+5x=20+2$
$-4x=-18$	$6x=22$
$x=\frac{18}{4}$	$x=\frac{11}{3}$
$x=\frac{9}{2}$	$x=\frac{11}{3}$

$\therefore$ **Solution Set =** $\left\{\frac{11}{3},\frac{9}{2}\right\}$

(vii) **(SGD 2015)** **(A.B)**

Solution:

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=\frac{2}{1}$$

By applying conponendo-dividendo prop

$$\frac{\left(\sqrt{x^2+2}+\sqrt{x^2-2}\right)+\left(\sqrt{x^2+2}-\sqrt{x^2-2}\right)}{\left(\sqrt{x^2+2}+\sqrt{x^2-2}\right)-\left(\sqrt{x^2+2}-\sqrt{x^2-2}\right)}=\frac{2+1}{2-1}$$

$$\frac{\sqrt{x^2+2}+\cancel{\sqrt{x^2-2}}+\sqrt{x^2+2}-\cancel{\sqrt{x^2-2}}}{\cancel{\sqrt{x^2+2}}+\sqrt{x^2-2}-\cancel{\sqrt{x^2+2}}+\sqrt{x^2-2}}=\frac{3}{1}$$

$$\frac{2\sqrt{x^2+2}}{2\sqrt{x^2-2}}=3$$

$$\sqrt{\frac{x^2+2}{x^2-2}}=3$$

Squaring on both sides

$$\frac{x^2+2}{x^2-2}=9$$
$$x^2+2=9\left(x^2-2\right)$$
$$x^2+2=9x^2-18$$
$$x^2-9x^2=-18-2$$
$$-8x^2=-20$$
$$x^2=\frac{-20}{-8}$$
$$\Rightarrow x^2=\frac{5}{2}$$

Taking square root on both sides

$$x=\pm\sqrt{\frac{5}{2}}$$

$\therefore$ **Solution Set=** $\left\{\pm\sqrt{\frac{5}{2}}\right\}$

(viii) **(A.B)**

Solution:

$$\frac{\sqrt{x^2+8P^2}-\sqrt{x^2-P^2}}{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}}=\frac{1}{3}$$

By applying Invertendo Property

$$\frac{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}}{\sqrt{x^2+8P^2}-\sqrt{x^2-P^2}}=\frac{3}{1}$$

By applying componendo-devidendo prop

$$\frac{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}+\left(\sqrt{x^2+8P^2}-\sqrt{x^2-P^2}\right)}{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}-\left(\sqrt{x^2+8P^2}-\sqrt{x^2-P^2}\right)}=\frac{3+1}{3-1}$$

$$\frac{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}+\sqrt{x^2+8P^2}-\sqrt{x^2-P^2}}{\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}-\sqrt{x^2+8P^2}+\sqrt{x^2-P^2}}=\frac{4}{2}$$

$$\frac{2\sqrt{x^2+8P^2}}{2\sqrt{x^2-P^2}}=2$$

Squaring on both sides

$$\left(\sqrt{\frac{x^2+8P^2}{x^2-P^2}}\right)^2=(2)^2$$

$$\frac{x^2+8P^2}{x^2-P^2}=4$$
$$x^2+8P^2=4x^2-4P^2$$
$$x^2-4x^2=-4P^2-8P^2$$
$$-3x^2=-12P^2$$
$$3x^2=12P^2$$
$$x^2=\frac{12P^2}{3}$$
$$x^2=4P^2$$

Taking square root on both sides

$$\sqrt{x^2}=\sqrt{4P^2}$$
$$x=\pm2P$$

Either

$x=2P$ or $x=-2P$

$\therefore$ **Solution Set** $=\{2P,-2P\}$

(ix) $\dfrac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\dfrac{13}{14}$ **(A.B)**

Solution:

$$\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\frac{13}{14}$$

By applying invertendo property

$$\frac{(x+5)^3+(x-3)^3}{(x+5)^3-(x-3)^3}=\frac{14}{13}$$

Now by applying componendo- dividendo property

$$\frac{\left[(x+5)^3+(x-3)^3\right]+\left[(x+5)^3-(x-3)^3\right]}{\left[(x+5)^3+(x-3)^3\right]-\left[(x+5)^3-(x-3)^3\right]}=\frac{14+13}{14-13}$$

$$\frac{(x+5)^3+(x-3)^3+(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3-(x+5)^3+(x-3)^3}=\frac{27}{1}$$

$$\frac{2(x+5)^3}{2(x-3)^3}=27$$

$$\left(\frac{x+5}{x-3}\right)^3=27$$

Taking cube root on both sides

$$\sqrt[3]{\left(\frac{x+5}{x-3}\right)^3}=\sqrt[3]{27}$$
$$\frac{x+5}{x-3}=3$$
$$x+5=3(x-3)$$
$$x+5=3x-9$$
$$x-3x=-9-5$$
$$-2x=-14$$
$$x=\frac{-14}{-2}$$
$$x=7$$

$\therefore$ **Solution Set** $=\{7\}$

Joint Variation **(K.B)**

A combination of direct and inverse variations of one or more than one variables forms joint variation.

Symbolically

If a variable *y* varies directly as *x* and varies inversely as *z*, then

$y \propto x$ and $y \propto \frac{1}{z}$

In joint variation,

$y \propto \frac{x}{z}$, or $y = \frac{kx}{z}$; $k \neq 0$

Problems Related to Joint Variation

Example 2: (Page # 64) **(K.B)**

p varies jointly was *q* and r^2 and inversely as *s* and t^2, *p=40*, when *q*=8, *r*=5, *s*=3, *t*=2. Find *p* in terms of *q, r, s* and *t*. Also find the value of *p* when *q*=-2, *r*=4, *s*=3 and *t*=-1.

Solution:

Given that $p \propto \frac{qr^2}{st^2}$

$$p=k\frac{qr^2}{st^2} \quad \text{(i)}$$

Put $p=40, q=8, r=5, s=3$ and $t=2$

$$40=k\frac{(8)(5)^2}{3(2)^2}$$
$$\frac{40\times3\times4}{8\times25}=k$$
$$k=\frac{12}{5}$$

Then eq. (i) becomes

$$p=\frac{12}{5}\frac{qr^2}{st^2}$$

Now for $q=-2, r=4, s=3$ and $t=-1$, we have

$$p=\frac{12}{5}\ \frac{(-2)(4)^2}{(3)(-1)^2}=-\frac{128}{5}$$

Exercise 3.5

Q.1 Given **(K.B + A.B)**

(FSD 2015, SWL 2015)

$s \propto u^2$ and $s \propto \frac{1}{v}$

$s = 7$ when $u=3, v=2$

To find

Value of *s* =? when *u* = 6, *v* = 10

Solution:

Here

$s \propto u^2$ and $s \propto \frac{1}{v}$

In joint variation

$$s \propto \frac{u^2}{v}$$
$$\Rightarrow s=\frac{ku^2}{v} \rightarrow \text{(i)}$$

For value of *k*

Put *s* = 7, *u* = 3 and *v* = 2 in equation (i)

$$7=\frac{k(3)^2}{2}$$
$$\frac{14}{9}=k$$
$$\therefore s=\frac{14u^2}{9v}$$

For value of *s*

Put $u=6, v=10$ and $k=\frac{14}{9}$ in equation (i)

$$s=\frac{14}{9}\times\frac{(6)^2}{10}$$
$$s=\frac{14}{9}\times\frac{36}{10}$$

$s = \frac{28}{5}$

Result

$s = \frac{28}{5}$ when u = 6, v = 10

Q.2 Given **(K.B + A.B)**

$w \propto xy^2z$

$w = 5$ when $x = 2, y = 3, z = 10$

To find

$w = ?$ when $x = 4, y = 7, z = 3$

Solution:

Here $w \propto xy^2z$

$w = kxy^2z \rightarrow$ (i)

For value of k

Put $w = 5, x = 2, y = 3$ and $z = 10$ in eq (i)

$5 = k(2)(3)^2(10)$

$5 = 180k$

$\frac{5}{180} = k$

$k = \frac{1}{36}$

$\therefore\ w = \frac{xy^2z}{36}$

For value of w

Put $x = 4, y = 7, z = 3$ and $k = \frac{1}{36}$ in eq (i)

$w = \frac{1}{36}(4)(7)^2(3)$

$\Rightarrow w = \frac{49}{3}$

Result

$w = \frac{49}{3}$ when $x = 4$, $y = 7$ and $z = 3$

Q.3 Given **(K.B + A.B)**

$y \propto x^3$ and $y \propto \frac{1}{z^2t}$

$y = 16$ when $x = 4$, $z = 2$, $t = 3$

To find

$y = ?$ when $x = 2, z = 3, t = 4$

Solution:

Here $y \propto x^3$, $y \propto \frac{1}{z^2t}$

In joint variation:

$y \propto \frac{x^3}{z^2t}$

$\Rightarrow y = \frac{kx^3}{z^2t} \rightarrow$ (i)

For value of k

Put $y = 16, x = 4, z = 2, t = 3$ in equation (i)

$16 = \frac{k(4)^3}{(2)^2(3)}$

$16 = \frac{k(64)}{12}$

$\frac{16 \times 12}{64} = k$

$3 = k$

$\Rightarrow\ k = 3$

$\therefore\ y = \frac{3x^3}{z^2t}$

For value of 'y'

Put $x = 2, z = 3, t = 4$ and $k = 3$ in eq (i)

$y = \frac{3(2)^3}{(3)^2(4)}$

$y = \frac{3(8)}{(9)(4)}$

$\Rightarrow y = \frac{2}{3}$

Result

$y = \frac{2}{3}$ when $x = 2$, $z = 3$ and $t = 4$

Q.4 Given **(K.B + A.B)**

$u \propto x^2$ and $u \propto \frac{1}{yz^3}$

$u = 2$ when $x = 8, y = 7, z = 2$

To find

$u = ?$ when $x = 6, y = 3, z = 2$

Solution:

Here

$u \propto x^2$ and $u \propto \frac{1}{yz^3}$

In joint variation:

$u \propto \frac{x^2}{yz^3}$

Or $u = \frac{kx^2}{yz^3} \to$ (i)

For value of *k*

Put $u = 2, x = 8, y = 7, z = 2$ in equation (i)

$2 = \frac{k(8)^2}{7(2)^3}$

$2 = \frac{k(64)}{7(8)}$

$2 \times \frac{7}{8} = k$

$\frac{7}{4} = k$

$\Rightarrow \quad k = \frac{7}{4}$

$\therefore u = \frac{7x^2}{4yz^3}$

For value of *u*

Put $k = \frac{7}{4}$, $x = 6$, $y = 3$ and $z = 2$

$u = \frac{7}{4} \cdot \frac{(6)^2}{3(2)^3}$

$u = \frac{7}{4} \times \frac{36}{3(8)}$

$u = \frac{21}{8}$

Result

$u = \frac{21}{8}$ when $x = 6, y = 3, z = 2$

Q.5 Given **(K.B + A.B)**

$v \propto xy^3$ and $v \propto \frac{1}{z^2}$

v = 27 when $x = 7$, y = 6, z = 7

To find

v = ? when $x = 6$, y = 2, z = 3

Solution:

Here

$V \propto xy^3$ and $v \propto \frac{1}{z^2}$

In joint variation

$v \propto \frac{xy^3}{z^2}$

$v = \frac{kxy^3}{z^2} \to$ (i)

For value of *k*

Put v = 27, $x = 7$, y = 6 and z = 7

$27 = \frac{k(7)(6)^3}{(7)^2}$

$27 = \frac{k(7)(216)}{49}$

$\frac{27 \times 7}{216} = k$

$\frac{7}{8} = k$

Or $k = \frac{7}{8}$

$\therefore \quad v = \frac{7xy^3}{8z^2}$

For value of *v*

Put $x = 6, y = 2, z = 3$ and $k = \frac{7}{8}$ in equation (i)

$v = \frac{7}{8} \frac{(6)(2)^3}{3^2}$

$v = \frac{7}{8} \times \frac{6 \times 8}{9}$

$v = \frac{14}{3}$

Result

$v = \frac{14}{3}$ when $x = 6, y = 2, z = 3$

Q.6 Given (D.G.K 2015) **(K.B + A.B)**

$w \propto \frac{1}{u^3}$

$w = 5$ when $u = 3$

To find

$w = ?$ when $u = 6$

Solution:

Here

$w \propto \frac{1}{u^3}$

$w = \frac{k}{u^3} \to$ (i)

For value of k

Put $w = 5, u = 3$ in equation (i)

$5 = \frac{k}{(3)^3}$

$5 \times 27 = k$

$135 = k$

$w = \frac{135}{u^3}$

For value of w

Put $k = 135$ and $u = 6$ in equation (i)

$w = \frac{135}{(6)^3}$

$= \frac{135}{216}$

$= \frac{45}{72}$

$= \frac{15}{24}$

$= \frac{5}{8}$

Result

$w = \frac{5}{8}$ when $u = 6$

K-Method **(K.B + A.B)**

If $a:b::c:d$ is a proportion, then

Let $\frac{a}{b} = \frac{c}{d} = k$

Or $\frac{a}{b} = k$, $\frac{c}{d} = k$

$a = bk, c = dk$

These equations are used to evaluate certain expression more easily.

Using these equations is called *K*-Method.

Example 3: (Page # 66) **(K.B + A.B)**

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then show that

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{ace}{bdf}$$

Solution:

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

Then $\frac{a}{b} = k, \frac{c}{d} = k$ and $\frac{e}{f} = k$

i.e., $a=bk$, $c=dk$ and $e=fk$

To prove:

$$\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{ace}{bdf}$$

Proof:

Now L.H.S $= \frac{a^3+c^3+e^3}{b^3+d^3+f^3}$

$= \frac{(bk)^3+(dk)^3+(fk)^3}{b^3+d^3+f^3}$

$= \frac{b^3k^3+d^3k^3+f^3k^3}{b^3+d^3+f^3}$

$= k^3\left(\frac{b^3+d^3+f^3}{b^3+d^3+f^3}\right) = k^3$

Also R.H.S $= \frac{ace}{bdf} = \frac{bk+dk+fk}{bdf}$

$= k^3\frac{bdf}{bdf} = k^3$

$\therefore$ L.H.S=R.H.S

i.e., $\frac{a^3+c^3+e^3}{b^3+d^3+f^3} = \frac{ace}{bdf}$

Hence proved

Exercise 3.6

Q.1 **(K.B + A.B)**

(FSD 2015, SGD 2015)

Given $a:b = c:d$

To prove

(i) $\frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$

(ii) $\frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$

(iii) $\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$

(iv) $a^6+b^6 : b^6+d^6 = a^3c^3 : b^3d^3$

(v) $p(a+b)+qb : p(c+d)+qd = a:c$

(vi) $a^2+b^2:\frac{a^3}{a+b}=c^2+d^2:\frac{c^3}{c+d}$

(vii) $\frac{a}{a-b}:\frac{a+b}{b}=\frac{c}{c-d}:\frac{c+d}{d}$

Proof:

Let $a:b=c:d=k$

$\Rightarrow \frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow \frac{a}{b}=k,\quad \frac{c}{d}=k$

$a=bk,\quad c=dk$

(i) $\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$ **(A.B)**

L.H.S = $\frac{4a-9b}{4a+9b}$

Putting the values of a

$=\frac{4bk-9b}{4bk+9b}$

$=\frac{b(4k-9)}{b(4k+9)}$

$=\frac{4k-9}{4k+9}\to$ (i)

R.H.S = $\frac{4c-9d}{4c+9d}$

Putting the value of 'c'

$=\frac{4dk-9d}{4dk+9d}$

$=\frac{d(4k-9)}{d(4k+9)}$

$=\frac{4k-9}{4k+9}\to$ (ii)

From equation (i) and (ii)

L.H.S = R.H.S

$\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$

Hence Proved

(ii) $\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}$ **(A.B)**

L.H.S = $\frac{6a-5b}{6a+5b}$

Putting the value of 'a'

$=\frac{6(bk)-5b}{6(bk)+5b}$

$=\frac{b(6k-5)}{b(6k+5)}=\frac{6k-5}{6k+5}\to$ (i)

R.H.S = $\frac{6c-5d}{6c+5d}$

Putting the value of 'c'

$=\frac{6dk-5d}{6dk+5d}$

$=\frac{d(6k-5)}{d(6k+5)}$

$=\frac{6k-5}{6k+5}\to$ (ii)

From equation (i) and (ii)

R.H.S = R.H.S

$\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}$

Hence Proved

(iii) $\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$ **(A.B)**

L.H.S = $\frac{a}{b}$

Putting the value of a

$=\frac{bk}{b}$

L.H.S $=k\to$ (i)

R.H.S = $\sqrt{\frac{a^2+c^2}{b^2+d^2}}$

Putting the values

$=\sqrt{\frac{(bk)^2+(dk)^2}{b^2+d^2}}$

$=\sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}$

$=\sqrt{\frac{k^2(b^2+d^2)}{b^2+d^2}}=\sqrt{k^2}$

R.H.S $=k$ ………. (ii)

From Equation (i) and (ii)

L.H.S = R.H.S

$$\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$$

Hence Proved

(iv) $a^6+b^6:b^6+d^6=a^3c^3:b^3d^3$ **(A.B)**

L.H.S = $a^6+c^6:b^6+d^6$

Putting the value of 'a' and 'c'

$=(bk)^6+(dk)^6:b^6+d^6$

$=b^6k^6+d^6k^6:b^6+d^6$

$=k^6(b^6+d^6):1(b^6+d^6)$

$=k^6 \rightarrow$ (i)

R.H.S $=a^3c^3:b^3d^3$

Putting the values of a and c

$=(bk)^3(dk)^3:b^3d^3$

$=b^3k^3d^3k^3:b^3d^3$

$=b^3d^3k^6:b^3d^3$

$=k^6 \rightarrow$ (ii)

From equation (i) and (ii)

L.H.S = R.H.S

$a^6+b^6:b^6+d^6=a^3c^3:b^3d^3$

Hence Proved

(v) $p(a+b)+qb:p(c+d)+qd=a:c$ **(A.B)**

L.H.S $=p(a+b)+qb:p(c+d)+qd$

Putting the value of a and c

$=p(bk+b)+qb:p(dk+d)+qd$

$=pb(k+1)+qb:pd(k+1)+qd$

$=b[p(k+1)+q]:d(p(k+1)+q]$

$=b:d \rightarrow$ (i)

R.H.S $=a:c$

Putting the value of a and c

$=bk:dk$

$=b:d \rightarrow$ (ii)

From equation (i) and (ii)

L.H.S = R.H.S

$p(a+b)+qb:p(c+d)+qd=a:c$

Hence Proved

(vi) $a^2+b^2:\frac{a^3}{a+b}=c^2+d^2:\frac{c^3}{c+d}$ **(A.B)**

L.H.S = $a^2+b^2:\frac{a^3}{a+b}$

Putting the values

$=(bk)^2+b^2:\frac{(bk)^3}{bk+b}$

$=b^2k^2+b^2:\frac{b^3k^3}{b(k+1)}$

$=b^2(k^2+1):\frac{b^2k^3}{k+1}$

$=\frac{b^2(k^2+1)}{b^2k^3/k+1}$

$=\frac{b^2(k^2+1)(k+1)}{b^2k^3}$

L.H.S $=\frac{(k^2+1)(k+1)}{k^3}$ … (i)

R.H.S = $c^2+d^2:\frac{c^3}{c+d}$

Putting the values

$=(dk)^2+d^2:\frac{(dk)^3}{dk+d}$

$=d^2k^2+d^2:\frac{d^3k^3}{d(k+1)}$

$=d^2(k^2+1):\frac{d^2k^3}{k+1}$

$=\frac{d^2(k^2+1)}{d^2k^3/k+1}$

$=\frac{d^2(k^2+1)}{d^2k^3}\times k+1$

R.H.S $=\frac{(k^2+1)(k+1)}{k^3}$ … (i)

From Equation (i) and (ii)

L.H.S = R.H.S

$a^2+b^2:\frac{a^3}{a+b}=c^2+d^2:\frac{c^3}{c+d}$

Hence Proved

(vii) $\frac{a}{a-b}:\frac{a+b}{b}=\frac{c}{c-d}:\frac{c+d}{d}$ **(A.B)**

L.H.S = $\frac{a}{a-b}:\frac{a+b}{b}$

Putting the value of 'a'

$= \dfrac{bk}{bk-b} : \dfrac{bk+b}{b}$

$= \dfrac{bk}{b(k-1)} : \dfrac{b(k+1)}{b}$

$= \dfrac{k}{k-1} : k+1 \rightarrow \text{(i)}$

R.H.S $= \dfrac{c}{c-d} : \dfrac{c+d}{d}$

Putting the value of 'c'

$= \dfrac{dk}{dk-d} : \dfrac{dk+d}{d}$

$= \dfrac{dk}{d(k-1)} : \dfrac{d(k+1)}{d}$

$= \dfrac{k}{k-1} : k+1 \rightarrow \text{(ii)}$

From equation (i) and (ii)

L.H.S = R.H.S

$$\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$$

Hence Proved

Q.2 **If** $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} \left(a, \text{b}, \text{c}, \text{d}, \text{e}, \text{f} \neq 0\right)$ **then show that** **(A.B)**

(i) $\dfrac{a}{b} = \sqrt{\dfrac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

(ii) $\dfrac{ac+ce+ea}{bd+df+fb} = \left[\dfrac{ace}{bdf}\right]^{\frac{1}{2}}$

(iii) $\dfrac{ac}{bd} + \dfrac{ce}{df} + \dfrac{ea}{fb} = \dfrac{a^2}{b^2} + \dfrac{c^2}{d^2} + \dfrac{e^2}{f^2}$

Proof: Let $\dfrac{a}{b} = \dfrac{c}{d} = \dfrac{e}{f} = k$

$\Rightarrow \dfrac{a}{b} = k, \dfrac{c}{d} = k, \dfrac{e}{f} = k$

$\Rightarrow a = bk, c = dk, e = fk$

(i) $\dfrac{a}{b} = \sqrt{\dfrac{a^2+c^2+e^2}{b^2+d^2+f^2}}$ **(A.B)**

L.H.S $= \dfrac{a}{b}$

Putting the value of 'a'

$= \dfrac{bk}{b}$

$= k \longrightarrow \text{(i)}$

R.H.S $= \sqrt{\dfrac{a^2+c^2+e^2}{b^2+d^2+f^2}}$

Putting the values of a, c and e

$= \sqrt{\dfrac{(bk)^2+(dk)^2+(fk)^2}{b^2+d^2+f^2}}$

$= \sqrt{\dfrac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$

$= \sqrt{\dfrac{k^2\left(b^2+d^2+f^2\right)}{\left(b^2+d^2+f^2\right)}}$

$= \sqrt{k^2}$

$= k \longrightarrow \text{(i)}$

From equation (i) and (ii)

L.H.S = R.H.S

$$\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$$

Hence Proved

(ii) $\dfrac{ac+ce+ea}{bd+df+fb} = \left[\dfrac{ace}{bdf}\right]^{\frac{1}{2}}$ **(A.B)**

(FSD 2016, MTN 2015)

L.H.S $= \dfrac{ac+ce+ea}{bd+df+fb}$

Putting the values

$= \dfrac{(bk)(dk)+(dk)(fk)+(fk)(bk)}{bd+df+fb}$

L.H.S $= \dfrac{bdk^2+dfk^2+fbk^2}{bd+df+fb}$

$= \dfrac{k^2(bd+df+fb)}{(bd+df+fb)}$

L.H.S $= k^2$ …… (i)

R.H.S $= \left[\dfrac{ace}{bdf}\right]^{2/3}$

Putting the values

$$= \left[\frac{(bk)(dk)(fk)}{bdf}\right]^{2/3}$$

$$= \left[\frac{bdfk^3}{bdf}\right]^{2/3}$$

$$= \left(k^3\right)^{2/3}$$

R.H.S = k^2 ….. (ii)

From Eq. (i) and (ii)

L.H.S = R.H.S

$$\frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf}\right]^{\frac{1}{2}}$$

Hence Proved

(iii) $\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb} = \frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$ **(A.B)**

L.H.S = $\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb}$

$$= \frac{bk.dk}{bd}+\frac{dk.fk}{df}+\frac{fk.bk}{fb}$$

$$= \frac{bdk^2}{bd}+\frac{dfk^2}{df}+\frac{fbk^2}{fb}$$

$$= k^2+k^2+k^2$$

$$= 3k^2 \longrightarrow \text{(i)}$$

R.H.S = $\frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$

Putting the value of a, c and e

$$= \frac{(bk)^2}{b^2}+\frac{(dk)^2}{d^2}+\frac{(fk)^2}{f^2}$$

$$= \frac{b^2k^2}{b^2}+\frac{d^2k^2}{d^2}+\frac{f^2k^2}{f^2}$$

$$= k^2+k^2+k^2$$

$$= 3k^2 \longrightarrow \text{(ii)}$$

From equation (i) and (ii)

L.H.S = R.H.S

$$\frac{ac}{bd}+\frac{ce}{df}+\frac{ea}{fb} = \frac{a^2}{b^2}+\frac{c^2}{d^2}+\frac{e^2}{f^2}$$

Hence Proved

Real Life Problems Based on Variation

Example 2 (Page # 68) **(A.B + U.B)**

The current in a wire varies directly as the electromotive force E and inversely as the resistance R. If $I=32$ amperes, when $E=128$ volts and $R=8$ ohms. Find I, when $E=150$ volts and $R=18$ ohms.

Solution:

In joint variation, we have $I \propto \frac{E}{R}$,

$$I = \frac{kE}{R} \rightarrow (i)$$

For $I=32, E=128$ and $R=8$,

$$32 = \frac{k(128)}{8}$$

$$\Rightarrow \frac{32\times 8}{128} = k$$

$$\Rightarrow k = 2$$

Put in equation (i)

$$I = \frac{2E}{R}$$

Now for $E=150$ and $R=18$

$$I = \frac{2(150)}{18} = \frac{50}{3} \text{ amp.}$$

Exercise 3.7

Q.1 The surface area A of a cube varies directly as the square of the length l of an edge and $A=27$ square units when $l=3$ units. Find A when l = 4 units (ii) l when $A=12$sq. units. **(A.B + K.B)**

Given

A ∝ l^2

A = 27 square unit when l = 3 units

To find

A = ? when l = 4 units

l = ? when A = 12 square unit

Solution:

Here

A ∝ l^2

A = kl^2 $\longrightarrow$(i)

Put A = 27, l = 3

27 = k$(3)^2$

$27 = 9k$
$3 = k$
$k = 3$
For value of *A*
Put k = 3, $l = 4$ in equation (i)
$A = 3(4)^2$
$A = 48$
For value of *l*
Put k = 3, A = 12 square unit in equation (i)
$12 = 3l^2$
$4 = l^2$
Or $l^2 = 4$
Taking square root on b/s
$l^2 = \pm 2$
$\Rightarrow l = 2$ (Length is always positive)
Result:
A = 48 square unit when $l = 4$ units
$l = 2$ unit when A = 12 square units

Q.2 The surface area *S* of the sphere varies directly as the square of radius *r*, and $S = 16\pi$ when $r = 2$. Find *r* when $S = 36\pi$.
(A.B K.B)

Given
$S \propto r^2$
$S = 16\pi$ when $r = 2$
To Find
r = ? when $s = 36\pi$
Solution:
Here $S \propto r^2$
$S = kr^2 \longrightarrow (i)$
For value of *k*
Put $S = 16\pi$, $r = 2$ in equation (i)
$16\pi = k(2)^2$
$16\pi = 4k$
$4\pi = k$
Or $k = 4\pi$
$S = \pi r^2$
For value of *r*
Put $k = 4\pi$, $S = 36\pi$ in equation (i)
$36\pi = 4\pi r^2$
$9 = r^2$
Taking square root on both sides
$r = \pm 3$
$\Rightarrow r = 3$ (Length is positive)
Result:
$r = 3$ when $S = 36\pi$

Q.3 In Hook's law the force *F* applied to stretch a spring varies directly as the amount of elongation *S* and $F = 32lb$ when $S = 1.6$ in. Find (i) *S* when $F = 50lb$ (ii) *F* when $S = 0.8$ in
(K.B +A.B)

Given
$F \propto S$
F = 32 lb when S = 1.6 m.
To find
S = ? when F = 50 lb
F = ? when S = 0.8 in
Solution:
Here $F \propto S$
$F = kS \rightarrow (i)$
For value of *k*
Put F = 32 and S = 1.6 in equation (i)
$32 = k(1.6)$
$20 = k$ Or $k = 20$
$F = 20S$
For value of *S*
Put k = 20 and F = 50
$50 = 20S$
$\frac{5}{2} = S$
Or $S = \frac{5}{2}$
For value of *F*
Put k = 20 and S = 0.8 in equation (i)
$F = 20(0.8)$
$F = 16$
Result:
$S = \frac{5}{2}$ in when F = 50 lb

Q.4 F = 16 lb when S = 0.8 inThe intensity *I* of light from a given source varies inversely as the square of the distance *d* from it. If the intensity is 20 candlepower at a distance of 12ft. From the source, in *d* the intensity at a point 8ft. from the source. **(K.B +A.B)**
Given
$I \propto \frac{1}{d^2}$
I = 20 candle power when d = 12ft

To find

I = ? when d = 8ft

Solution:

Here

$I \propto \frac{1}{d^2}$

$I = \frac{k}{d^2} \rightarrow$ (i)

For value of *k*

Put I = 20 and d = 12 in equation (i)

$20 = \frac{k}{(12)^2}$

$20 \times 144 = k$

Or $2880 = k$

$I = \frac{2880}{d^2}$

For value of *I*

Put $k = 2880$ and $d = 8$ in equation (i)

$I = \frac{2880}{(8)^2}$

$I = \frac{2880}{64}$

I = 45

Result

I = 45 candle power when d = 8ft.

Q.5 The pressure *P* in a body of fluid varies directly as the depth *d*. If the pressure exerted on the bottom of a tank by a column of fluid 5ft. high is 2.25 lb/sq. in, how deep must the fluid be to exert a pressure of 9 lb/sq. in?

(K.B +A.B)

Given

$P \propto d$

P = 2.25 lb/sq when d = 5ft

To Find

d = ? when P = 9 lb/sq

Solution:

Here

$P \propto d$

$P = kd \longrightarrow$ (i)

For value of *k*

Put P = 2.25 and d = 5

2.25 = k(5)

Or k=0.45

$P = 0.45d$

For value of *d*

Put k = 0.45 and P = 9

9 = 0.45 d

$\frac{9}{0.45} = d$

20 = d

Or d = 20

Result

d = 20ft when P = 9 lb/sq.

Q.6 Labour costs *c* varies jointly as the number of workers *n* and the average number of days *d*, if the cost of 800 workers for 13 days is Rs. 286000, then find the labour cost of 600workers for 18 days

(K.B +A.B)

Given

$c \propto nd$

c = Rs. 286000 when $n = 800$ workers, $d = 13$ days

To find

c = ? when n = 600 workers, d = 18 days

Solution:

Here

$c \propto nd$

$c = knd \longrightarrow$ (i)

For value of *k*

Put c = 286000, n = 800 and d = 13 in eq (i)

286000 = k(800)(13)

$\frac{286000}{800 \times 13} = k$

$\Rightarrow \frac{55}{2} = k$

$c = \frac{55}{2} nd$

For value of *c*

Put $k = \frac{55}{2}$, n = 600 and 18 in equation (i)

$c = \frac{55}{2} \times 600 \times 18$

c = 297000

Result:

c = Rs.29700 when n = 600 workers and $d = 18$ days

Q.7 The supporting load *c* of a pillar varies as the fourth power of its diameter *d* and inversely as the square of its length l. A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4 inch pillar must be to support a load of 28 tons?

(K.B +A.B)

Given

$c \propto d^4$ and $c \propto \frac{1}{l^2}$

c = 63 tons when d = 6 inches and l = 30 feet

To Find

l = ? when d = 4 inches c = 28 tons

Solution:

Here

$c \propto d^4$ and $c \propto \frac{1}{l^2}$

In joint variation:

$$c \propto \frac{d^4}{l^2}$$

$$\Rightarrow c = k\frac{d^4}{l^2}$$

For value of *k*

Put c = 63, d = 6 and l = 30 in equation (i)

$$63 = k\frac{(6)^4}{(30)^2}$$

$$63 = k\frac{1296}{900}$$

$$\frac{63 \times 900}{1296} = k$$

$$\frac{175}{4} = k$$

Or $k = \frac{175}{4}$

$$c = \frac{175 d^4}{4 l^2}$$

For value of *l*

Put $k = \frac{175}{4}$, d = 4, c = 28 in equation (i)

$$28 = \frac{175}{4}\frac{(4)^4}{l^2}$$

$$28 \times 4l^2 = 175 \times 256$$

$$l^2 = 400$$

Taking positive square root on both sides

$$\Rightarrow l = 20$$

Result

l = 20 feet when d = 4 inches and c = 28 tons

Q.8 The time *T* required for an elevator to lift a weight varies jointly as the weight *w* and the lifting depth *d* varies inversely as the power *p* of the motor. If 25 sec. are required for a 4-hp motor to lift 500 lb through 40 ft, what power is required to lift 800 lb, through 120 ft in 40 sec.?

(K.B +A.B)

Given

$T \propto wd$ and $T \propto \frac{1}{P}$

T = 25 sec when P = 4 hp, w = 500 lb

To Find

P = ? when c = 800 lb, d = 120 ft and T = 40 sec.

Solution:

Here

$T \propto wd$ and $T \propto \frac{1}{P}$

In joint variation:

$$T \propto \frac{wd}{P}$$

$$T = \frac{kwd}{P} \longrightarrow \text{(i)}$$

For value of *k*

Put $T = 25, P = 4, w = 500, d = 40$ in equation (i)

$$25 = k\frac{500 \times 40}{40}$$

$$\frac{25}{5000} = k$$

$$\frac{1}{200} = k$$

Or $k = \frac{1}{200}$

$$T = \frac{wd}{200P}$$

For value of *P*

Put $k = \frac{1}{200}$, w = 800, d = 120 and $T = 40$ in equation (i)

$$40 = \frac{1}{200}\frac{800 \times 120}{P}$$

$40P = 4 \times 120$

$P = 12$

Result:

P = 12 hp when w = 800 lb, d = 120 ft and $T = 40$ sec.

Q.9 The kinetic energy (*K.E.*) of a body varies jointly as the mass "*m*" of the body and the square of its velocity "*v*". If the kinetic energy is 4320 ft/lb when the mass is 45 lb and the velocity is 24 ft/sec. Determine the kinetic energy of a 3000 lb automobile travelling 44 ft/sec. **(K.B +A.B)**

Given

K.E $\propto$ mv^2

K.E = 4320 ft/lb when m = 45 lb and v = 24 ft/sec

To find

K.E = ? when m = 3000 lb, v = 44 ft/sec

Solution:

Here

K.E $\propto$ mv^2

K.E = kmv^2 $\longrightarrow$ (i)

For value of *k*

Put K.E = 4320, m = 45 and v = 24

4320 = k(45)(24)^2

$$\frac{4320}{45 \times 576} = k$$

$$k = \frac{1}{6}$$

$$K.E = \frac{1}{6}mv^2$$

For value of *K.E*

Put $k = \frac{1}{6}$, m = 3000, v = 44 in equation (i)

$$K.E = \frac{1}{6} \times 3000 \times (44)^2$$

K.E = 968000

Result:

K.E = 968000 *ft/lb* when m = 3000 *lb* and v = 44 *ft/sec*

Miscellaneous Exercise 3

Q.1 Multiple Choice Questions

Four possible answers are given for the following question. Tick (✓) the correct answer.

(1) **In a ratio** $a:b$**,** a **is called;** (SWL 2014, MTN 2015, D.G.K 2014, 15) **(K.B +A.B)**

(a) Relation **(b)** Antecedent
(c) Consequent **(d)** None of these

(2) **In a ratio** $x:y$**,** y **is called;** (LHR 2014, GRW 2014, RWP 2015) **(K.B +A.B)**

(a) Relation **(b)** Antecedent
(c) Consequent **(d)** None of these

(3) **In a proportion** $a:b::c:d$**,** a **and** d **are called;** (LHR 2015, MTN 2015) **(K.B +A.B)**

(a) Means **(b)** Extremes
(c) Third proportional **(d)** None of these

(4) **In a proportion** $a:b::c:d$**, b and** c **are called;** (LHR 2015) **(K.B +A.B)**

(a) Means **(b)** Extremes
(c) Fourth proportional **(d)** None of these

(5) **In continued proportion** $a:b=b:c, ac=b^2$**,** b **is said to be _____ proportional between** a **and** c**.** **(K.B +A.B)**

(a) Third **(b)** Fourth
(c) Mean **(d)** None of these

(6) **In continued proportion** $a:b=b:c$**,** c **is said to be ____ proportional between** a **and** b**.** **(K.B +A.B)**

(a) Third **(b)** Fourth
(c) Means **(d)** None of these

(7) **Find x in proportion** $4:x::5:15$ **(K.B +U.B)**

(a) $\frac{75}{4}$ **(b)** $\frac{4}{3}$
(c) $\frac{3}{4}$ **(d)** 12

(8) **If** $u \propto v^2$**, then** (LHR 2014, MTN 2015, D.G.K 2014) **(K.B +U.B)**

(a) $u=v^2$ **(b)** $u=kv^2$
(c) $uv^2=k$ **(d)** $uv^2=1$

(9) **If** $y^2 \propto \frac{1}{x^3}$**, then** (FSD 2015, SWL 2014, D.G.K 2015) **(K.B +A.B)**

(a) $y^2=\frac{k}{x^3}$ **(b)** $y^2=\frac{1}{x^3}$
(c) $y^2=x^2$ **(d)** $y^2=kx^3$

(10) **If** $\frac{u}{v}=\frac{v}{w}=k$**, then** (LHR 2014, D.G.K 2015) **(K.B +U.B)**

(a) $u=wk^2$ **(b)** $u=vk^2$
(c) $u=w^2k$ **(d)** $u=v^2k$

(11) The third proportional of x^2 and y^2 is; **(K.B +A.B)**

(GRW 2014, MTN 2015, D.G.K 2015)

(a) $\frac{y^2}{x^2}$ (b) x^2y^2

(c) $\frac{y^4}{x^2}$ (d) $\frac{y^2}{x^4}$

(12) The fourth proportional w of $x : y :: v : w$ is; (FSD 2014, 15, RWP 2014) **(K.B +U.B)**

(a) $\frac{xy}{v}$ (b) $\frac{vy}{x}$

(c) xyv (d) $\frac{x}{vy}$

(13) If $a:b = x:y$, then alternando property is; (SGD 2014) **(K.B +U.B)**

(a) $\frac{a}{x} = \frac{b}{y}$ (b) $\frac{a}{b} = \frac{x}{y}$

(c) $\frac{a+b}{x} = \frac{x+y}{y}$ (d) $\frac{a-b}{x} = \frac{x-y}{y}$

(14) If $a:b = x:y$, then invertendo property is; **(K.B +U.B)**

(a) $\frac{a}{x} = \frac{b}{y}$ (b) $\frac{a}{a-b} = \frac{x}{x-y}$

(c) $\frac{a+b}{b} = \frac{x+y}{y}$ (d) $\frac{b}{a} = \frac{y}{x}$

(15) If $\frac{a}{b} = \frac{c}{d}$, then componendo property is; **(K.B +U.B)**

(FSD 2014, SGD 2014, RWP 2015)

(a) $\frac{a}{a+b} = \frac{c}{c+d}$ (b) $\frac{a}{a-b} = \frac{c}{c-d}$

(c) $\frac{ad}{bc}$ (d) $\frac{a-b}{b} = \frac{c-d}{d}$

ANSWER KEY

1	b	6	a	11	c
2	c	7	d	12	b
3	b	8	b	13	a
4	a	9	a	14	d
5	c	10	a	15	a

Q.2 Write short answers of the following questions.

(i) Define ratio and give one example.

Ans. See definition Page # 87. **(K.B)**

(ii) Define proportion. **(K.B)**

Ans. See definition Page # 87.

(iii) Define direct variation. **(K.B)**

Ans. See definition Page # 92.

(iv) Define inverse variation. **(K.B)**

Ans. See definition Page # 93.

(v) State theorem of componendo-dividend. **(K.B)**

Ans. See theorem on proportion Pg # 103.

(vi) Given. **(A.B)**

$6 : x :: 3 : 5$

Required

Value of $x = ?$

Solution:

Here

$6 : x :: 3 : 5$

Product of extremes = product of means

$6 \times 5 = 3 \times x$

$\frac{30}{3} = x$

$\Rightarrow x = 10$

(vii) Given **(A.B)**

$x \propto y^2$

$x = 27$ when $y = 4$

To find

$y = ?$ when $x = 3$

Solution:

Here

$x \propto y^2$

$x = ky^2 \rightarrow (\text{i})$

For value of *k*

Put $x = 27$ and $y = 4$

$27 = k(4)^2$

$\frac{27}{16} = k$

Or $k = \frac{27}{16}$

$x = \frac{27}{16} y^2$

For value of *y*

Put $k = \frac{27}{16}$ and $x = 3$

$3 = \frac{27}{16} y^2$

$\frac{48}{27} = y^2$

$\frac{16}{9} = y^2$

$y^2 = \frac{16}{9}$

Taking square root

$\Rightarrow y = \pm \frac{4}{3}$

(viii) Given **(A.B)**

$u \propto \frac{1}{V}$

$u = 8, v = 3$

To find

$v = ?$ when $u = 12$

Solution:

Here

$u \propto \frac{1}{v}$

$u = \frac{k}{v} \rightarrow (\text{i})$

For value of *k*

Put $u = 8$ and $v = 3$ in equation (i)

$8 = \frac{k}{3}$

$\Rightarrow k = 24$

$u = \frac{24}{v}$

For value of *v*

Put $k = 24$ and $u = 12$ in equation (i)

$12 = \frac{24}{v}$

$v = 2$

Result

$v = 2$ when $u = 12$

(ix) Let fourth proportional = x **(A.B)**

(LHR 2014, GRW 2017, BWP 2016)

According to given condition

$8 : 7 :: 6 : x$

$\because$ Product of extremes = Product of means

$8x = 7 \times 6$

$x = \frac{42}{8}$

$x = \frac{21}{4}$

Result:

Fourth proportional = $x = \frac{21}{4}$

(x) Let mean proportional = x **(A.B)**
(GRW 2015, 17 FSD 2016, MTN 2015, 17, BWP 2015)

According to given condition

$16 : x :: x : 49$

$\because$ Product of means = product of extremes

$x^2 = 16 \times 49$

$x^2 = 784$

Taking square root on both sides

$x = 28$

Result:

Mean proportional = 28

(xi) Let third proportional = x **(A.B)**
(SWL 2014, SGD 2014, D.G.K 2016)

According to given condition

$28 : 4 :: 4 : x$

Product of extremes = product of means

$28x = 4 \times 4$

$x = \frac{16}{28}$

$\Rightarrow x = \frac{4}{7}$

Result:

Third proportional = $\frac{4}{7}$

(xii) Given **(A.B)**

$y \propto \frac{x^2}{z}$

$y = 28$ when $x = 7, z = 2$

Required

Value of $y = ?$

Solution:

Given that $y \propto \frac{x^2}{z}$

$y = k\frac{x^2}{z} \rightarrow \text{(i)}$

For value of k

Put $y = 28, x = 7, z = 2$ in equation (i)

$28 = k\frac{(7)^2}{2}$

$28 \times 2 = 49k$

$\frac{56}{49} = k$ or $k = \frac{8}{7}$

For value of y

Put $k = \frac{8}{7}$ in equation (i)

$y = \frac{8}{7}\frac{x^2}{z} = \frac{8x^2}{7z}$

(xiii) Given data: **(A.B)**

$z \propto xy$

$z = 36$ when $x = 2, y = 3$

Required

$z = ?$

Solution:

$z \propto xy$

$z = kxy \rightarrow \text{(i)}$

For value of k

Put $z = 36, x = 2, y = 3$ in equation (i)

$36 = k(2)(3)$

$\frac{36}{6} = k$

$6 = k$

For value of z

Put $k = 6$ in equation (i)

$z = 6xy$

(xiv) Given data **(A.B)**

$w \propto \frac{1}{v^2}$

$w = 2$ when $v = 3$

Required

Value of $w = ?$

Solution:

Here $w \propto \frac{1}{v^2}$

$w = k \times \frac{1}{v^2} \rightarrow \text{(i)}$

For value of k

Put $w = 2, v = 3$ in equation (i)

$2 = k \times \frac{1}{(3)^2}$

$2 \times 9 = k$

$18 = k$

For value of w

Put $k = 18$ in equation (i)

$w = 18 \times \frac{1}{v^2}$

$w = \frac{18}{v^2}$

Q.3 Fill in the blanks

(i) The simplest form of the ratio $\dfrac{(x+y)(x^2+xy+y^2)}{x^3-y^3}$ is_____. **(K.B)**

(ii) In a ratio $x:y$; x is called_____. **(K.B)**

(iii) In a ratio $a:b$; b is called_____. **(K.B)**

(iv) In a proportion $a:b::x:y$; a and y are called _____. **(K.B)**

(v) In a proportion $p:q::m:n$; q and m are called _____. **(K.B)**

(vi) In proportion $7:4::p:8$, $p=$_____. **(A.B)**

(vii) If $6:m::9:12$, then $m=$_____. **(A.B)**

(viii) If x and y varies directly, then $x=$_____. **(A.B)**

(ix) If v varies directly as u^3, then $u^3=$_____. **(A.B)**

(x) If w varies inversely as p^2, then $k=$_____. **(A.B)**

(xi) A third proportional of 12 and 4, is_____. **(A.B)**

(xii) The fourth proportional of $15, 6, 5$ is _____. **(A.B)**

(xiii) The mean proportional of $4m^2n^4$ and p^6 is_____. **(A.B)**

(xiv) The continued proportion of $4, m$ and 9 is_____. **(A.B)**

ANSWER KEY

(i) $\dfrac{x+y}{x-y}$

(ii) Antecedent

(iii) Consequent

(iv) Extremes

(v) Means

(vi) $p=14$

(vii) $m=8$

(viii) ky

(ix) $\dfrac{v}{k}$

(x) p^2w

(xi) $\dfrac{4}{3}$

(xii) 2

(xiii) $K=\pm 2mn^2p^3$

(xiv) $m=\pm 6$

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 The third proportional of x^2 and y^2 is:

(A) $\frac{y^2}{x^2}$ **(B)** x^2y^2

(C) $\frac{y^4}{x^2}$ **(D)** $\frac{y^2}{x^4}$

2 If 16, a and 4 are in continued proportion, then a is equal to:

(A) $\pm\sqrt{20}$ **(B)** ± 8

(C) ± 64 **(D)** $\pm\sqrt{12}$

3 Find x in proportion $4:x::5:15$

(A) $\frac{75}{4}$ **(B)** $\frac{4}{3}$

(C) $\frac{3}{4}$ **(D)** 12

4 In a ratio $x:y$**, "** y **" called**

(A) Relation **(B)** Antecedent

(C) Consequent **(D)** None

5 If $\frac{u}{v}=\frac{v}{\omega}=k$**, then**

(A) $u=\omega k^2$ **(B)** $u=vk^2$

(C) $u=\omega^2 k$ **(D)** $u=v^2 k$

6 If $a:b=x:y$**, then invertendo property is:**

(A) $\frac{a+b}{b}=\frac{x+y}{y}$ **(B)** $\frac{a}{a-b}=\frac{x}{x-y}$

(C) $\frac{a}{x}=\frac{b}{y}$ **(D)** $\frac{b}{a}=\frac{y}{x}$

7 As 'x' and 'y' varies inversely and $x=2$ **and** $k=6$ **then 'y' is:**

(A) 3 **(B)** 12

(C) 3/2 **(D)** 8

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Find the third proportional of: $a^2 - b^2, a - b$

(ii) If $a:b = c:d, (a,b,c,d \neq 0)$, then show that $\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$

(iii) If *y* varies inversely as x^2 and $y = 16$, when $x = 5$, so find *x*, when $y = 100$.

(iv) Find *a*, if the ratios $a+3 : 7+a$ and 4:5 are equal.

(v) If *y* varies jointly as x^2 and *z* and *y* = 6 when *x* = 4, *z* = 9. Write *y* as a function of *x* and *z* and determine the value of *y*, when *x* = –8 and *z* = 12.

Q.3 Answer the following Questions. **(4+4=8)**

(a) Solve the following by using componendo-dividendo property. $\frac{\sqrt{x+3}+\sqrt{x-3}}{\sqrt{x+3}-\sqrt{x-3}} = \frac{4}{3}$.

(b) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} (a,b,c,d,e,f \neq 0)$, then show that $\frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf}\right]^{2/3}$

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.