$$\frac{1}{x^2-1}=\frac{A}{x+1}+\frac{B}{x-1}$$

UNIT 4

PARTIAL FRACTIONS

Fraction **(K.B)**

(MTN 2017, BWP 2014, 15, 17, RWP 2016, D.G.K 2015, 17)

The quotient of two numbers or algebraic expressions is called a fraction. The quotient is indicated by a bar $(-)$. The dividend is written on the top of the bar and divisor below the bar.

For example: $\frac{2}{3}, \frac{x^2+4}{x-2}$ where $x \neq 2$

Note **(K.B + U.B)**

If $x=2$ in second example then the fraction is not defined because $x=2$ makes the denominator zero.

Rational Fraction **(K.B)**

(LHR 2014, 16, GRW 2016, FSD 2015, SGD 2015, 16, MTN 2015, D.G.K 2016)

An expression of the form $\frac{N(x)}{D(x)}$, where N(x) and D(x) are polynomials in x with real coefficients is called a rational fraction. The polynomial $D(x) \neq 0$

For example $\frac{x^2+4}{x-2}$ where $x \neq 2$

Types of Fractions **(K.B + U.B)**

There are two types of fractions.

(i) Proper Fraction

(ii) Improper Fraction

Proper Fraction **(K.B)**

A rational fraction $\frac{N(x)}{D(x)}$, where D(x) $\neq 0$ is called proper fraction, if degree of the polynomial N(x) is less than degree of the polynomial D(x)

For example: $\frac{2}{x+1}, \frac{5x-3}{x^2+4}$ etc.

Improper Fraction **(U.B + K.B)**

(LHR 2014, 15, GRW 2014, 17, FSD 2015, SGD 2017, RWP 2017, MTN 2015)

A rational fraction $\frac{N(x)}{D(x)}$, where D(x) $\neq 0$ is called an improper fraction, if degree of the polynomial N(x) is greater than or equal to degree of the polynomial D(x).

For example: $\frac{5x}{x+2}, \frac{6x^4}{x^3+1}$ etc.

Identity **(K.B)**

(GRWP 2014, 15, 17, RWP 2016, SGD 2016, D.G.K 2015, 17)

An identity is an equation, which is satisfied by all the values of the variables involved

For example: $(x+3)^2 = x^2+6x+9$,

$2(x+1) = 2x+2$ etc.

Conditional Equation **(K.B)**

An equation which is true for some specific value(s) of the variable involved.

For example: $x+2=3$ is true only for $x=1$.

Partial Fraction **(K.B)**

(LHR 2014, 16, 17, GRW 2015, FSD 2015, 17, RWP 2015, 16, BWP 2015,)

Decomposition of resultant fraction into its components or into different fractions is called partial fraction.

Note **(K.B + U.B)**

General method applicable to resolve all rational fractions of the form $\frac{N(x)}{D(x)}$, is as follows:

- The numerator $N(x)$ must be of lower degree than the denominator $D(x)$.
- Make substitutions constant accordingly.
- Multiply both sides by L.C.M.
- Arrange terms on both sides by decreasing order.
- Make the equations and solve to find constants.

Resultant Fraction **(K.B)**

Sum of two or more than two proper fractions in the form of a single fraction is called the resultant fraction.

For example:

$$\frac{1}{x-1}-\frac{2}{x+1}=\frac{-x+3}{(x-1)(x+1)}$$ is resultant fraction.

Example 2: (Page # 73) **(A.B)**

Resolve $\frac{1}{3+x-2x^2}$ into partial fractions.

Solution:

$\frac{1}{3+x-2x^2}$ can be written as for convenience $\frac{-1}{2x^2-x-2}$

The denominator

$$D(x)=2x^2-x-3=2x^2-3x+2x-3$$
$$=x(2x-3)+1(2x-3)=(x+1)(2x-3)$$

Let,

$$\frac{-1}{2x^2-x-3}=\frac{-1}{(x+1)(2x-3)}=\frac{A}{x+1}+\frac{B}{2x-3}\rightarrow(i)$$

multiplying both the sides by $(x+1)(2x-3)$, we get

$$-1=A(2x-3)+B(x+1)\rightarrow(ii)$$

Put $x+1=0 \Rightarrow x=-1$ in equation (ii)

$$-1=A[2(-1)-3]$$
$$-1=-5A$$
$$\Rightarrow A=\frac{1}{5}$$

Put $2x-3=0 \Rightarrow x=\frac{3}{2}$ in equation (ii)

$$-1=B\left(\frac{3}{2}+1\right)$$
$$-1=\frac{5}{2}B$$
$$\Rightarrow B=-\frac{2}{5}$$

Thus, $$\frac{1}{3+x-2x^2}=\frac{1}{5(x+1)}-\frac{2}{5(2x-3)}$$

Note

There are two methods to resolve into partial fraction:

(i) Zero Method

(ii) Equating coefficient

Exercise 4.1

Resolve into partial fractions.

Q.1 $\frac{7x-9}{(x+1)(x-3)}$ **(FSD 2015)** **(A.B)**

Solution:

Let $$\frac{7x-9}{(x+1)(x-3)}=\frac{A}{x+1}+\frac{B}{x-3}\rightarrow(i)$$

Multiplying with $(x+1)(x-3)$

$$\frac{7x-9}{\cancel{(x+1)}\cancel{(x-3)}}\times\cancel{(x+1)}\cancel{(x-3)}$$
$$=\frac{A}{\cancel{x+1}}\times\cancel{(x+1)}(x-3)+\frac{B}{\cancel{x-3}}\times(x+1)\cancel{(x-3)}$$
$$\Rightarrow 7x-9=A(x-3)+B(x+1)\rightarrow(ii)$$

Put $x+1=0\Rightarrow x=-1 in eq.(ii)$

$$7(-1)-9=A(-1-3)+B(0)$$
$$-7-9=-4(A)+0$$
$$-16=-4A$$

Or $$A=\frac{-16}{-4}$$

A = 4

Put $x-3=0$ *or* $x=3$ *in eq.*(ii)

$$7(3)-9=A(o)+B(3+1)$$

21-9 = 0 + B (4)

12 = 4B

Or 4B = 12

B = $\frac{12}{4}$

B = 3

Putting the values in equation. (i)

$$\Rightarrow\frac{7x-9}{(x+1)(x-3)}=\frac{4}{x+1}+\frac{3}{x-3}$$

Q.2 $\frac{x-11}{(x-4)(x+3)}$ **(A.B)**

Solution:

Let $\frac{x-11}{(x-4)(x+3)}=\frac{A}{x-4}+\frac{B}{x+3}\rightarrow(i)$

Multiplying equation (i) by (x-4) (x+3)

$$\frac{x-11}{\cancel{(x-4)}\cancel{(x+3)}}\times\cancel{(x-4)}\cancel{(x+3)}$$

$$=\frac{A}{\cancel{(x-4)}}\times\cancel{(x-4)}(x+3)+\frac{B}{\cancel{(x+3)}}(x-4)\cancel{(x+3)}$$

$$x-11=A(x+3)+B(x-4)\rightarrow(ii)$$

Put $x-4=0\Rightarrow x=4\ in\ eq.(ii)$

$$4-11=A(4+3)+B(0)$$
$$-7=A(7)+0$$
$$-7=7A$$

Or $7A=-7$

$$A=\frac{-7}{7}$$
$$\Rightarrow A=-1$$

Put x+3=0 or x = -3 in eq. (ii)

$$-3-11=A(-3+3)+B(-3-4)$$
$$-14=A(0)+B(-7)$$
$$-14=0+(-7B)$$

Or $-7B=-14$

$$B=\frac{-14}{-7}$$
$$\Rightarrow B=2$$

Putting the values of A and B in equation (i)

$$\therefore\frac{x-11}{(x-4)(x+3)}=\frac{-1}{x-4}+\frac{2}{x+3}$$

Q.3 $\frac{3x-1}{x^2-1}$ **(A.B)**

(GRWP 2017, SWL 2014, BWP 2017, D.G.K 2014)

Solution:

$$\frac{3x-1}{x^2-1}=\frac{3x-1}{(x+1)(x-1)}$$

Let $\frac{3x-1}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}$

Multiplying by (x+1) (x-1)

$$\frac{3x-1}{\cancel{(x+1)}\cancel{(x-1)}}\times\cancel{(x+1)}\cancel{(x-1)}$$

$$=\frac{A}{\cancel{(x+1)}}\cancel{(x+1)}(x-1)+\frac{B}{\cancel{(x-1)}}(x+1)\cancel{(x-1)}$$

$$3x-1=A(x-1)+B(x+1)\rightarrow(ii)$$

Put $x-1=0\Rightarrow x=1\text{in eq}(ii)$

$$3(1)-1=A(1-1)+B(1+1)$$
$$3-1=0+2B$$
$$2=2B$$

Or $2B=2$

$$B=\frac{2}{2}$$
$$B=1$$

Put $x+1=0\Rightarrow x=-1 in eq(ii)$

$$3(-1)-1=A(-1-1)+B(0)$$
$$-3-1=A(-2)$$
$$-4=-2A$$
$$-2A=-4$$
$$A=\frac{-4}{-2}$$
$$A=2$$

Now putting the values in eq (i)

$$\Rightarrow\frac{3x-1}{x^2-1}=\frac{2}{x+1}+\frac{1}{x-1}$$

Q.4 $\frac{x-5}{x^2+2x-3}$ **(A.B)**

(FSD 2015, MTN 2016, SGD 2015)

Solution:

$$\frac{x-5}{x^2+2x-3}=\frac{x-5}{x^2-x+3x-3}$$
$$=\frac{x-5}{x(x-1)+3(x-1)}$$
$$=\frac{x-5}{(x-1)(x+3)}$$

Let $\frac{x-5}{(x-1)(x+3)}=\frac{A}{x-1}+\frac{B}{x+3}\rightarrow(i)$

Multiplying by $(x-1)(x+3)$,we get

$$x-5=A(x+3)+B(x-1)\rightarrow(ii)$$

Put $x-1=0\Rightarrow x=1\ in\ eq\ (ii)$

$$1-5=A(1+3)+B(1-1)$$
$$-4=A(4)+B(0)$$
$$-4=4A$$
$$4A=-4$$
$$A=\frac{-4}{4}$$
A = -1

Put $x+3=0\Rightarrow x=-3\,in\,eq\,(ii)$

$$-3-5=A(-3+3)+B(-3-1)$$
$$-8=-4B$$
$$-4B=-8$$
$$B=\frac{-8}{-4}$$
B=2

Now putting values in eq (i)

$$\frac{x-5}{(x-1)(x+3)}=\frac{-1}{x-1}+\frac{2}{x+3}$$
$$\Rightarrow\frac{x-5}{x^2+2x-3}=\frac{-1}{x-1}+\frac{2}{x+3}$$

Q.5 $\dfrac{3x+3}{(x-1)(x+2)}$ **(A.B)**

(GRW 2015, 16, FSD 2016, 17, BWP 2015, 16, SGD 2015)

Solution:

Let $\dfrac{(3x+3)}{(x-1)(x+2)}=\dfrac{A}{x-1}+\dfrac{B}{x+2}\rightarrow(i)$

Multiplying equation (i) by (x-1) (x+2)

$$\frac{3x+3}{(x-1)(x+2)}\times(x-1)(x+2)$$
$$=\frac{A}{x-1}\times(x-1)(x+2)+\frac{B}{x+2}\times(x-1)(x+2)$$
$$3x+3=A(x+2)+B(x-1)\rightarrow(ii)$$

Put $x-1=0\;or\;x=1\,in\,eq\,(ii)$

$$3(1)+3=A(1+2)+B(1-1)$$
$$3+3=3(A)+0$$
$$6=3A$$
$$3A=6$$
$$A=\frac{6}{3}$$
A = 2

Put $x+2=0\Rightarrow x=-2\,in\,eq\,(ii)$

$$3(-2)+3=A(-2+2)+B(-2-1)$$
$$-6+3=A(0)+B(-3)$$
$$-3=0-3B$$
$$3B=3$$
$$B=\frac{3}{3}$$
B = 1

Now putting values in eq (i)

$$\Rightarrow\frac{3x+3}{(x-1)(x+2)}=\frac{2}{x-1}+\frac{1}{x+2}$$

Q.6 $\dfrac{(7x-25)}{(x-4)(x-3)}$ **(A.B)**

(RWP 2016, SGD 2017, D.G.K 2014, 17)

Solution:

Let $\dfrac{(7x-25)}{(x-4)(x-3)}=\dfrac{A}{x-4}+\dfrac{B}{x-3}\rightarrow(i)$

Multiplying by (x-4) (x-3)

$$7x-25=A(x-3)+B(x-4)\rightarrow(ii)$$

Put $x-4=0\;or\;x=4\,in\,eq\,(ii)$

$$7(4)-25=A(4-3)+B(4-4)$$
$$28-25=A(1)+B(0)$$
$$3=A$$

Or A = 3

Put $x-3=0\;or\;x=3\,in\,eq\,(ii)$

$$7(3)-25=A(0)+B(3-4)$$
$$21-25=B(-1)$$
$$-B=-4$$
B = 4

Now putting values in equation (ii)

$$\Rightarrow\frac{7x-25}{(x-4)(x-3)}=\frac{3}{x-4}+\frac{4}{x-3}$$

Q.7 $\dfrac{x^2+2x+1}{(x-2)(x+3)}$ **(A.B + K.B)**

Solution:

$$\frac{x^2+2x+1}{(x-2)(x+3)}=\frac{x^2+2x+1}{x^2+x-6}\text{ (improper)}$$
$$\because x^2+x-6=x^2+3x-2x-6$$
$$=x(x+3)-2(x+3)$$
$$=(x+3)(x-2)$$

$$\begin{array}{r} 1 \\ x^2+x-6\overline{\big)\; x^2+2x+1} \\ -\underline{x^2 \pm x \mp 6} \\ x+7 \end{array}$$

$$\frac{x^2+2x+1}{x^2+x-6} = 1+\frac{x+7}{x^2+x-6}$$

$$= 1+\frac{x+7}{(x+3)(x-2)} \rightarrow (i)$$

Consider

$$\frac{x+7}{(x+3)(x-2)} = \frac{A}{x+3}+\frac{B}{x-2} \quad \rightarrow (\text{ii})$$

Multiplying by $(x+3)(x-2)$

$$x+7 = A(x-2)+B(x+3) \rightarrow (\text{iii})$$

Put $x+3=0$ or $x=-3$ in eq (iii)

$$-3+7 = A(-3-2)+B(-3+3)$$

$$4 = A(-5)+B(0)$$

$$4 = -5A$$

$$A = \frac{4}{-5}$$

$$A = -\frac{4}{5}$$

Put $x-2=0$ or $x=2$ in equation (ii)

$$2+7 = A(2-2)+B(2+3)$$

$$9 = A(0)+B(5)$$

$$9 = 5B$$

$$5B = 9$$

$$B = \frac{9}{5}$$

Putting the values of A and B in equation (ii)

$$\frac{x+7}{(x+3)(x-2)} = \frac{9/5}{x-2}+\frac{-4/5}{x+3}$$

$$= \frac{9}{5(x-2)}-\frac{4}{5(x+3)}$$

Now putting the values in equation (i)

$$\Rightarrow \frac{x^2+2x+1}{(x-2)(x+3)} = 1+\frac{9}{5(x-2)}-\frac{4}{5(x+3)}$$

Q.8 $\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$ **(A.B + K.B)**

Solution:

$\dfrac{6x^3+5x^2-7}{3x^2-2x-1}$ (improper fraction)

$$\begin{array}{r} 2x+3 \\ 3x^2-2x-1\overline{\big)\,6x^3+5x^2-7} \\ \underline{\pm 6x^3 \mp 4x^2 \quad \mp 2x} \\ 9x^2+2x-7 \\ \underline{\pm 9x^2 \mp 6x \mp 3} \\ 8x-4 \end{array}$$

$$\Rightarrow \frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x+3+\frac{8x-4}{3x^2-2x-1}$$

$$= 2x+3+\frac{8x-4}{3x^2-3x+x-1}$$

$$= 2x+3+\frac{8x-4}{3x(x-1)+1(x-1)}$$

$$= 2x+3+\frac{8x-4}{(x-1)(3x+1)} \rightarrow (i)$$

Consider

$$\frac{8x-4}{(x-1)(3x+1)} = \frac{A}{x-1}+\frac{B}{3x+1} \rightarrow (\text{ii})$$

Multiplying by $(x-1)(3x+1)$

$$8x-4 = A(3x+1)+B(x-1) \rightarrow (\text{iii})$$

Put $x-1=0$ or $x=1$ in equation (iii)

$$8-4 = A(3+1)+B(0)$$

$$4 = A(4)+0$$

$$4 = 4A$$

$$\Rightarrow A = 1$$

Put $3x+1=0$ or $x=-\frac{1}{3}$ in equation (iii)

$$8\left(-\frac{1}{3}\right)-4 = A\left[3\left(-\frac{1}{3}\right)+1\right]+B\left(-\frac{1}{3}-1\right)$$

$$\frac{-8-12}{3} = A(0)+B\frac{(-1-3)}{3}$$

$$-\frac{20}{3} = 0+B\left(-\frac{4}{3}\right)$$

$$-\frac{20}{3} = -\frac{4}{3}B$$
$$-\frac{20}{3}\left(-\frac{3}{4}\right) = B$$
$$5 = B$$
Or $B = 5$

Now putting values in equation (ii)
$$\frac{8x-4}{(x-1)(3x+1)} = \frac{1}{x-1} + \frac{5}{3x+1}$$
Putting the values in equation (i)
$$\Rightarrow \frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x+3+\frac{1}{x-1}+\frac{5}{3x+1}$$

Example: (Page # 79) **(A.B)**

Resolve $\frac{1}{(x-1)^2(x-2)}$ into partial fractions.

Solution:

Let, $\frac{1}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$

Multiplying both sides by $(x-1)^2(x-2)$, we get
$$1 = A(x-1)(x-2)+B(x-2)+C(x-1)^2$$
$$A(x^2-3x+2)+B(x-2)+C(x^2-2x+1) = 1 \rightarrow (i)$$
Since (i) is an identity and is true for all values of x

Put $x-1=0$ or $x=1$ in (i), we get

$B(1-2)=1 \quad \Rightarrow \quad -B=1$ or $B=-1$

Put $x-2=0$ or $x=2$ in (i), we get

$C(2-1)^2=1 \quad \Rightarrow \quad C=1$

Equating coefficients of x^2 on both sides of (i)

$A+C=0 \quad \Rightarrow \quad A=-C$ so $A=-1$

Hence required partial fractions are
$$\frac{-1}{x-1} - \frac{1}{(x-1)^2} + \frac{1}{(x-2)}$$
Thus
$$\frac{1}{(x-1)^2(x-2)} = \frac{1}{x+2} - \frac{1}{(x-1)} - \frac{1}{(x-1)^2}$$

Exercise 4.2

Resolve into partial fractions.

Q.1 $\frac{x^2-3x+1}{(x-1)^2(x-2)}$ **(A.B)**

Solution:

Let $\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} \rightarrow \text{(i)}$

Multiplying by $(x-1)^2(x-2)$
$$x^2-3x+1 = A(x-1)(x-2)+B(x-2)+C(x-1)^2 \rightarrow \text{(ii)}$$
$$x^2-3x+1 = A(x^2-3x+2)+B(x-2)+C(x^2-2x+1)$$
$$= Ax^2-3Ax+2A+Bx-2B+Cx^2-2Cx+C$$
$$= Ax^2+Cx^2-3Ax+Bx-2Cx+2A+C-2B$$
$$\Rightarrow x^2-3x+1 = (A+C)x^2+(-3A+B-2C)x + (2A-2B+C)$$
By comparing coefficients of alike powers of x

$1 = A+C \rightarrow$ (iii)

$-3 = -3A+B-2C \rightarrow$ (iv)

$1 = 2A-2B+C \rightarrow$ (v)

Put $x-1=0 \Rightarrow x=1$ in equation (ii)
$$(1)^2-3(1)+1 = A(1-1)(1-2)+B(1-2)+C(0)^2$$
$$1-3+1 = 0+B(-1)+0$$
$$-1 = -1B$$
$$B = 1$$
Put $B = x-2=0 \Rightarrow x=2$ in equation (ii)
$$(2)^2-3(2)+1 = A(2-1)(2-2)+B(2-2)+C(2-1)^2$$
$$4-6+1 = A(1)(0)+B(0)+C(1)^2$$
$$-1 = C$$
Or $C = -1$

Put $C=-1$ in equation (iii)
$$1 = A+(-1)$$
$$1+1 = A$$
Or A = 2

Now putting values in equation (i)
$$\frac{x^2-3x+1}{(x-1)^2(x-2)} = \frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{-1}{(x-2)}$$
$$= \frac{2}{x-1} + \frac{1}{(x-1)^2} - \frac{1}{(x-2)}$$

Q.2 $\dfrac{x^2+7x+11}{(x+2)^2(x+3)}$ **(A.B)**

(LHR 2016, SGD 2016, RWP 2015)

Solution: $\dfrac{x^2+7x+11}{(x+2)^2(x+3)}$

Consider

$$\frac{x^2+7x+11}{(x+2)^2(x+3)}=\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{x+3}\rightarrow(\text{i})$$

Multiplying by $(x+2)^2(x+3)$

$$x^2+7x+11=A(x+2)(x+3)+B(x+3)+C(x+2)^2\rightarrow(\text{ii})$$

$$=A(x^2+5x+6)+B(x+3)+C(x^2+4x+4)$$

$$=Ax^2+5Ax+6A+Bx+3B+Cx^2+4Cx+4C$$

$$=Ax^2+Cx^2+5Ax+Bx+4Cx+6A+3B+4C$$

$$=(A+C)x^2+(5A+B+4C)x+(6A+3B+4C)$$

By comparing coefficients of alike powers of $'x'$

$1=A+C$ __________ (iii)

$7=5A+B+4C$ __________ (iv)

$11=6A+3B+4C$ __________ (v)

Put $x+2\Rightarrow x=-2$ in equation _______ (ii)

$$(-2)^2+7(-2)+11=A(-2+2)(-2+3)$$

$$+B(-2+3)+C(-2+2)^2$$

$$4-14+11=A(0)(1)+B(1)+C(0)^2\;-10+11=B$$

$$1=B$$

Or $B=1$

Put $x+3=0\Rightarrow x=-3$ in equation (ii)

$$(-3)^2+7(-3)+11=A(-3+2)(-3+3)$$

$$+B(-3+3)+C(-3+2)^2$$

$$9-21+11=A(-1)(0)+B(0)+C(-1)^2$$

$$-1=0+0+C$$

$$C=-1$$

Put C = -1 in equation (iii)

$$1=A+(-1)$$

$$1+1=A$$

Or A = 2

Now putting the values of A,B,C in (i)

$$\frac{x^2+7x+11}{(x+2)^2(x+3)}=\frac{2}{x+2}+\frac{1}{(x+2)^2}+\frac{-1}{x+3}$$

$$=\frac{2}{x+2}+\frac{1}{(x+2)^2}-\frac{1}{x+3}$$

Q.3 $\dfrac{9}{(x-1)(x+2)^2}$ **(A.B)**

(GWL 2014, RWP 2017, SGD 2015)

Solution:

Let $\dfrac{9}{(x-1)(x+2)^2}=\dfrac{A}{x-1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}\rightarrow(\text{i})$

Multiplying by $(x-1)(x+2)^2$

$$9=A(x+2)^2+B(x-1)(x+2)+C(x-1)\rightarrow(\text{ii})$$

$$9=A(x^2+4x+4)+B(x^2+x-2)+C(x-1)$$

$$0x^2+0x+9=Ax^2+4Ax+4A+Bx^2+Bx-2B+Cx-C$$

$$0x^2+0x+9=Ax^2+Bx^2+4Ax+Bx$$
$$+Cx+4A-2B-C$$

$$0x^2+0x+9=(A+B)x^2+(4A+B+C)x$$
$$+(4A-2B-C)$$

By comparing coefficients of alike powers of $'x'$

$0=A+B\rightarrow$ (iii)

$0=4A+B+C\rightarrow$ (iv)

$9=4A-2B-C\rightarrow$ (v)

Put $x-1=0\Rightarrow x=1$ in equation (ii)

$$9=A(1+2)^2+B(1-1)(1+2)+C(1-1)$$

$$9=A(3)^2+B(0)+C(0)$$

$$9=9A+0+0$$

$$9=9A$$

Or A = 1

Put $x+2=0\Rightarrow x=-2$ in equation (ii)

$$9=A(-2+2)+B(-2-1)(-2+2)+C(-2-1)$$

$$9=A(0)+B(-3)(0)+C(-3)$$

$$9=0+0-3C$$

$$C=-\frac{9}{3}$$

$$C=-3$$

Put A = 1 in equation (iii)

0 = 1 + B

Now putting the values in equation (i)

$$\Rightarrow\frac{9}{(x-1)(x+2)^2}=\frac{1}{x-1}+\frac{-1}{x+2}+\frac{-3}{(x+2)^2}$$

$$=\frac{1}{x-1}-\frac{1}{x+2}-\frac{3}{(x+2)^2}$$

Q.4 $\dfrac{x^4+1}{x^2(x-1)}$ **(K.B + A.B)**

Solution:

$$\frac{x^4+1}{x^2(x-1)}=\frac{x^4+1}{x^3-x^2} \quad \text{(improper fraction)}$$

$$\begin{array}{r|l} & x+1 \\ x^3-x^2 & x^4+1 \\ & \pm x^4 \mp x^3 \\ \hline & x^3+1 \\ & \pm x^3 \mp x^2 \\ \hline & x^2+1 \end{array}$$

$$\frac{x^4+1}{x^2(x-1)}=x+1+\frac{x^2+1}{x^3-x^2}$$

$$=x+1+\frac{x^2+1}{x^2(x-1)}\rightarrow \text{(i)}$$

Let $\dfrac{x^2+1}{x^2(x-1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x-1}\rightarrow$ (ii)

$x^2+1=Ax(x-1)+B(x-1)+C(x^2)$

$x^2+1=Ax^2-Ax+Bx-B+Cx^2$

$x^2+0x+1=Ax^2+Cx^2+Bx-Ax-B$

$=(A+C)x^2+(B-A)x-B$

By comparing coefficients of alike powers of 'x'

$1=A+C\rightarrow$ (iii)

$0=B-A\rightarrow$ (iv)

$1=-B\rightarrow$ (v)

From equation (v)

$B=-1$

Put in equation (iv)

$0=-1-A$

A = -1

Put in equation (iii)

$1=-1+C$

$1+1=C$

$C=2$

Now putting the values in equation (ii)

$$\frac{x^2+1}{x^2(x-1)}=\frac{-1}{x}+\frac{-1}{x^2}+\frac{2}{x-1}$$

$$=-\frac{1}{x}-\frac{1}{x^2}+\frac{2}{x-1}$$

Now putting the values in equation (i)

$$\Rightarrow\frac{x^4+1}{x^2(x-1)}=x+1-\frac{1}{x}-\frac{1}{x^2}+\frac{2}{x-1}$$

Q.5 $\dfrac{7x+4}{(3x+2)(x+1)^2}$ **(A.B)**

Solution:

Let $\dfrac{7x+4}{(3x+2)(x+1)^2}=\dfrac{A}{3x+2}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}\rightarrow$ (i)

Multiplying by $(3x+2)(x+1)^2$

$7x+4=A(x+1)^2+B(x+1)(3x+2)+C(3x+2)\rightarrow$ (ii)

$0x^2+7x+4=A(x^2+2x+1)$

$+B(3x^2+3x+2)+3Cx+2C$

$=Ax^2+2Ax+A+3Bx^2+3Bx+2B+3Cx+2C$

$=Ax^2+3Bx^2+2Ax+3Bx+3Cx+A+2B+2C$

$0x^2+7x+4=(A+3B)x^2+(2A+3B+3C)x$

$+(A+2B+2C)$

By Comparing coefficients of alike powers of x

$A=?$

$0=A+3B\rightarrow$ (iii)

$7=2A+3B+3C\rightarrow$ (iv)

$4=A+2B+2C\rightarrow$ (v)

Put $x+1\Rightarrow 0$, $x=-1$ in equation (ii)

$7(-1)+4=A(-1+1)^2+B(-1+1)+C[3(-1)+2]$

$-7+4=0+0+C(-3+2)$

$-3=(-1)C$

Or $C=3$

Put $A=-6$ in equation (iii)

$0=-6+3B$

$6=3B$

Or $B=2$

Putting all the values in equation (i)

$$\frac{7x+4}{(3x+2)(x+1)^2}=\frac{-6}{3x+2}+\frac{2}{x+1}+\frac{3}{(x+1)^2}$$

Q.6 $\dfrac{1}{(x-1)^2(x+1)}$ **(A.B)**

Solution:

Let $\dfrac{1}{(x-1)^2(x+1)}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x+1}\rightarrow$ (i)

Multiplication by $(x-1)^2(x+1)$

$1=A(x-1)(x+1)+B(x+1)+C(x-1)^2\rightarrow$ (ii)

$0x^2+0x+1=A(x-1)(x+1)+B(x+1)+C(x-1)^2$

$0x^2+0x+1=A(x^2-1)+B(x+1)+C(x^2-2x+1)$

$=Ax^2+Cx^2+Bx-2Cx-A+B+C$

$=(A+C)x^2+(B-2C)x+(-A+B+C)$

Comparing coefficients of powers $'x'$

$0=A+C$ __________ (iii)

$0=B-2C$ __________ (iv)

$1=-A+B+C$ __________ (v)

Put $x-1=0\Rightarrow x=1$ in equation (ii)

$1=A(0)(1+1)+B(1+1)+C(0)^2$

$1=0+B(2)+C(0)$

$1=2B$

Or $B=\dfrac{1}{2}$

Put $B=\dfrac{1}{2}$ in equation (iv)

$0=\dfrac{1}{2}-2C$

$2C=\dfrac{1}{2}$

$C=\dfrac{1}{4}$

Put $C=\dfrac{1}{4}$ in equation (iii)

$0=A+\dfrac{1}{4}$

$A=-\dfrac{1}{4}$

Now putting the values in equation (i)

$$\frac{1}{(x-1)^2(x+1)}=\frac{-1/4}{x-1}+\frac{1/2}{(x-1)^2}+\frac{1/4}{x+1}$$

$$\frac{1}{(x-1)^2(x+1)}=\frac{-1}{4(x-1)}+\frac{1}{2(x-1)^2}+\frac{1}{4(x+1)}$$

Q.7 $\dfrac{3x^2+15x+16}{(x+2)^2}$ **(A.B)**

Solution:

$\dfrac{3x^2+15x+16}{(x+2)^2}=\dfrac{3x^2+15x+16}{x^2+4x+4}$

(improper fraction)

$$\begin{array}{r|l} & \quad 3 \\ x^2+4x+4 & 3x^2+15x+16 \\ & \underline{\pm 3x^2 \pm 12x \pm 12} \\ & \quad 3x+4 \end{array}$$

$\dfrac{3x^2+15x+16}{(x+2)^2}=3+\dfrac{3x+4}{x^2+4x+4}$ _____ (i)

Let $\dfrac{3x+4}{(x+2)^2}=\dfrac{A}{x+2}+\dfrac{B}{(x+2)^2}$ ______ (ii)

Multiplying by $(x+2)^2$

$3x+4=A(x+2)+B$

$3x+4=Ax+2A+B$

By comparing coefficients of powers of $'x'$

3 = A __________ (iii)

$4=2A+B$ __________ (iv)

From equation (iii)

A = 3

Put in equation (iv)

$4=2(3)+B$

$4=6+B$

$4-6=B$

$B=-2$

Now putting values in equation (ii)

$\dfrac{3x+4}{(x+2)^2}=\dfrac{3}{x+2}+\dfrac{-2}{(x+2)^2}$

$=\dfrac{3}{x+2}-\dfrac{2}{(x+2)^2}$

Now putting the values in equation (i)

$\Rightarrow\dfrac{3x^2+15x+16}{(x+2)^2}=3+\dfrac{3}{x+2}-\dfrac{2}{(x+2)^2}$

Q.8 $\frac{1}{(x^2-1)(x+1)}$ **(K.B + A.B)**

Solution:

$$\frac{1}{(x^2-1)(x+1)}=\frac{1}{(x+1)(x-1)(x+1)}$$

$$=\frac{1}{(x+1)^2(x-1)}$$

Let $\frac{1}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}\rightarrow$(i)

Multiplication by $(x-1)(x+1)^2$

$1=A(x+1)+B(x-1)(x+1)+C(x-1)\rightarrow$(ii)

Put $x-1=0\Rightarrow x=1$ in equation (ii)

$1=A(1+1)^2+B(0)+C(0)$

$1=A(2)^2$

$1=4A$

$\Rightarrow A=\frac{1}{4}$

Put $x+1=0\Rightarrow x=-1$ in equation (ii)

$1=A(-1+1)+B(-1-1)(-1+1)+C(-1-1)$

$1=A(0)+B(0)+C(-2)$

$1=-2C$

$\Rightarrow C=-\frac{1}{2}$

Equation (ii)

$0x^2+0x+1=A(x+1)^2+B(x-1)(x+1)+C(x-1)$

$=A(x^2+2x+1)+B(x^2-1)+Cx-C$

$=Ax^2+2Ax+A+Bx^2-B+Cx-C$

$=Ax^2+Bx^2+2Ax+Cx+A-B-C$

$=(A+B)x^2+(2A+C)x+(A-B-C)$

By comparing coefficients of alike powers of x,

$0=A+B$ __________ (iii)

$0=2A+C$ __________ (iv)

$1=A-B-C$ __________ (v)

Put $A=\frac{1}{4}$ in equation (iii)

$0=\frac{1}{4}+B$

$B=-\frac{1}{4}$

Now putting values in equation (i)

$$\frac{1}{(x-1)(x+1)^2}=\frac{1/4}{x-1}+\frac{-1/4}{x+1}+\frac{-1/2}{(x+1)^2}$$

$$\Rightarrow\frac{1}{(x-1)(x+1)^2}=\frac{1}{4(x-1)}-\frac{1}{4(x+1)}-\frac{1}{2(x+1)^2}$$

Example: (Page # 80) **(A.B)**

Resolve $\frac{11x+3}{(x-3)(x^2+9)}$ into partial fractions.

Solution:

Let, $\frac{11x+3}{(x-3)(x^2+9)}=\frac{A}{(x-3)}+\frac{Bx+C}{x^2+9}$

Multiplying both the sides by $(x-3)(x^2+9)$ on both sides

$\Rightarrow\quad 11x+3=A(x^2+9)+(Bx+C)(x-3)$

$11x+3=A(x^2+9)+B(x^2-3x)+C(x-3)\rightarrow(i)$

Since (i) is an identity, we have on substituting $x=3$

$33+3=A(9+9)$

$\Rightarrow\ 18A=36$

$\Rightarrow\quad A=2$

Comparing the coefficients of x^2 and x on both sides of (i), we get

$A+B=0$

$\Rightarrow B=-2$

$-3B+C=11$

$\Rightarrow-3(-2)+C=11$

$\Rightarrow C=5$

Therefore, the partial fractions are

$\frac{2}{x-3}+\frac{-2x+5}{x^2+9}$

Thus, $\frac{11x+3}{(x-3)(x^2+9)}=\frac{2}{x-3}+\frac{-2x+5}{x^2+9}$

Exercise 4.3

Resolve into partial fractions.

Q.1 $\dfrac{3x-11}{(x+3)(x^2+1)}$ **(A.B)**

Solution:

Let $\dfrac{3x-11}{(x+3)(x^2+1)}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+1}\rightarrow(\text{i})$

Multiplying by $(x+3)(x^2+1)$

$3x-11=A(x^2+1)+(Bx+C)(x+3)\rightarrow(\text{ii})$

$3x-11=Ax^2+A+Bx^2+3Bx+3C+Cx$

$3x-11=Ax^2+Bx^2+3Bx+Cx+A+3C$

$0x^2+3x-11=(A+B)x^2+(3B+C)x+(A+3C)$

By comparing co-efficient of alike powers of x

$0=A+B$ ________ (iii)

$3=3B+C$ ________ (iv)

$-11=A+3C$ ________ (v)

Put $x+3=0\Rightarrow x=-3$ in equation (ii)

$3(-3)-11=A\left[(-3)^2+1\right]+(B(-3)+C)(-3+3)$

$-9-11=A(9+1)+(Bx+C)(0)$

$-20=A(10)+0$

Or A = − 2

Put in equation (iii)

$0=-2+B$

$B=2$

Put in equation (iv)

$3=3(2)+C$

$3=6+C$

$3-6=C$

$-3=C$

$C=-3$

Now putting in equation (i)

$\dfrac{3x-11}{(x+3)(x^2+1)}=\dfrac{-2}{x+3}+\dfrac{2x-3}{x^2+1}$

Or $\dfrac{3x-11}{(x+3)(x^2+1)}=\dfrac{2x-3}{x^2+1}-\dfrac{2}{x+3}$

Q.2 $\dfrac{3x+7}{(x^2+1)(x+3)}$ **(A.B)**

(SWL 2015, MTN 2015)

Solution:

Let $\dfrac{3x+7}{(x^2+1)(x+3)}=\dfrac{Ax+B}{x^2+1}+\dfrac{C}{x+3}\rightarrow(\text{i})$

Multiplying by $(x^2+1)(x+3)$

$3x+7=(Ax+B)(x+3)+C(x^2+1)\rightarrow(\text{ii})$

$0x^2+3x+7=Ax^2+3Ax+Bx+3B+Cx^2+C$

$=Ax^2+Cx^2+Bx+3Ax+3B+C$

$=(A+C)x^2+(B+3A)x+(3B+C)$

By comparing coefficients of alike powers of x

$0=A+C$ ________ (iii)

$3=3A+B$ ________ (iv)

$7=3B+C$ ________ (v)

Put $x+3=0\ \Rightarrow\ x=-3$ in equation (ii)

$3(-3)+7=(A(-3)+B)(0)+C\left[(-3)^2+1\right]$

$-9+7=C(9+1)$

$-2=0+C(10)$

$-\dfrac{2}{10}=C$

$\Rightarrow C=-\dfrac{1}{5}$

Put in equation (iii)

$0=A-\dfrac{1}{5}$

Or $A=\dfrac{1}{5}$

Put in equation (iv)

$3=3\left(\dfrac{1}{5}\right)+B$

$3=\dfrac{3}{5}+B$

$3-\dfrac{3}{5}=B$

$\dfrac{15-3}{5}=B$

$\dfrac{12}{5}=B$

$$\Rightarrow B=\frac{12}{5}$$

Now putting values in equation (i)

$$\frac{3x+7}{\left(x^2+1\right)\left(x+3\right)}=\frac{\frac{1}{5}x+\frac{12}{5}}{x^2+1}+\frac{-\frac{1}{5}}{x+3}$$

$$=\frac{\frac{1}{5}\left(x+12\right)}{x^2+1}-\frac{1}{5\left(x+3\right)}$$

$$\Rightarrow\frac{3x+7}{\left(x^2+1\right)\left(x+3\right)}=\frac{x+12}{5\left(x^2+1\right)}-\frac{1}{5\left(x+3\right)}$$

Q.3 $\dfrac{1}{\left(x+1\right)\left(x^2+1\right)}$ **(A.B)**

Solution:

Let $\dfrac{1}{\left(x+1\right)\left(x^2+1\right)}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}$ __(i)

Multiplying by $\left(x+1\right)\left(x^2+1\right)$

$$1=A\left(x^2+1\right)+\left(Bx+C\right)\left(x+1\right) \quad \text{(ii)}$$

$$=Ax^2+A+Bx^2+Bx+C+Cx$$

$$=Ax^2+Bx^2+Bx+Cx+A+C$$

$$0x^2+0x+1=\left(A+B\right)x^2+\left(B+C\right)x+\left(A+C\right)$$

By comparing coefficients of alike powers of x

$0=A+B$ __________ (iii)

$0-B+C$ __________ (iv)

$1=A+C$ __________ (v)

Put $x+1=0\rightarrow x=-1$ in equation (ii)

$$1=A\left[\left(-1\right)^2+1\right]+\left(B\left(-1\right)+C\right)\left(0\right)$$

$$1=A\left(1+1\right)+0$$

$$1=A\left(2\right)$$

Or $A=\dfrac{1}{2}$

Put in equation (iii)

$$0=\frac{1}{2}+B$$

$$B=-\frac{1}{2}$$

Put in equation (iv)

$$0=-\frac{1}{2}+C$$

$$\Rightarrow C=\frac{1}{2}$$

$$\frac{1}{\left(x+1\right)\left(x^2+1\right)}=\frac{\frac{1}{2}}{2\left(x+1\right)}+\frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1}$$

$$\frac{1}{\left(x+1\right)\left(x^2+1\right)}=\frac{1}{2\left(x+1\right)}+\frac{-\frac{1}{2}\left(x-1\right)}{x^2+1}$$

$$\Rightarrow\frac{1}{\left(x+1\right)\left(x^2+1\right)}=\frac{1}{2\left(x+1\right)}-\frac{x-1}{2\left(x^2+1\right)}$$

Q.4 $\dfrac{9x-7}{\left(x+3\right)\left(x^2+1\right)}$ **(A.B)**

Solution:

Let $\dfrac{9x-7}{\left(x+3\right)\left(x^2+1\right)}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+1}$ __ (i)

Multiplication by $\left(x+3\right)\left(x^2+1\right)$

$$9x-7=A\left(x^2+1\right)+\left(Bx+C\right)\left(x+3\right) \quad \text{(ii)}$$

$$=Ax^2+A+Bx^2+Cx+3Bx+3C$$

$$=Ax^2+Bx^2+3Bx+Cx+A+3C$$

$$0x^2+9x-7=\left(A+B\right)x^2+\left(3B+C\right)x+\left(A+3C\right)$$

By comparing coefficients of alike powers of x

$0=A+B$ __________(iii)

$9=3B+C$ __________(iv)

$-7=A+3C$ __________(v)

Put $x+3=0\Rightarrow x=-3$ in equation (ii)

$$9\left(-3\right)-7=A\left[\left(-3\right)^2+1\right]+\left[B\left(-3\right)+C\right]\left(-3+3\right)$$

$$-27-7=A\left(9+1\right)+0$$

$$-34=A\left(10\right)$$

$$A=-\frac{34}{10}$$

$$A=-\frac{17}{5}$$

Put in equation (iii)

$$0=-\frac{17}{5}+B$$

$$B=\frac{17}{5}$$

Put in equation (iv)

$$9=3\left[\frac{17}{5}\right]+C$$

$$9=\frac{51}{5}+C$$

$$9-\frac{51}{5}=C$$

$$\frac{45-51}{5}=C$$

$$-\frac{6}{5}=C$$

$$C=-\frac{6}{5}$$

Now putting the values in equation (i)

$$\frac{9x-7}{(x+3)(x^2+1)}=-\frac{\frac{17}{5}}{(x+3)}+\frac{\frac{17}{5}x+\left(-\frac{6}{5}\right)}{x^2+1}$$

$$=-\frac{17}{5(x+3)}+\frac{\frac{1}{5}(17x-6)}{x^2+1}$$

$$=-\frac{17}{5(x+3)}+\frac{17x-6}{5(x^2+1)}$$

$$\Rightarrow \frac{9x-7}{(x+3)(x^2+1)}=\frac{17x-6}{5(x^2+1)}-\frac{17}{5(x+3)}$$

Q.5 $\dfrac{3x+7}{(x+3)(x^2+4)}$ **(A.B)**

(SWL 2015, MTN 2015)

Solution:

Let $\dfrac{3x+7}{(x+3)(x^2+4)}=\dfrac{A}{x+3}+\dfrac{Bx+C}{x^2+4}$ ____ (i)

Multiplying by $(x+3)(x^2+4)$

$$3x+7=A(x^2+4)+(Bx+C)(x+3) \text{ ____ (ii)}$$

$$=Ax^2+4A+Bx^2+3Bx+Cx+3C$$

$$=Ax^2+Bx^2+3Bx+Cx+4A+3C$$

$$0x^2+3x+7=(A+B)x^2+(3B+C)x+(4A+3C)$$

By comparing coefficients of alike powers of x,

$$0=A+B \text{ ________ (iii)}$$

$$3=3B+C \text{ ________ (iv)}$$

$$7=4A+3C \text{ ________ (v)}$$

Put $x+3=0 \Rightarrow x=-3$ in equation (ii)

$$3(-3)+7=A\left[(-3)^2+4\right]+\left(B(-3)+C\right)(-3+3)$$

$$-9+7=A(9+4)+0$$

$$-2=A(13)$$

$$A=-\frac{2}{13}$$

Put in equation (iii)

$$0=-\frac{2}{13}B$$

$$B=\frac{2}{13}$$

Put in equation (iv)

$$3=3\left[\frac{2}{13}\right]+C$$

$$3=\frac{6}{13}+C$$

$$3-\frac{6}{13}+C$$

$$\frac{39-6}{13}=C$$

$$\frac{33}{13}=C$$

$$C=\frac{33}{13}$$

Now putting the values in equation (i)

$$\frac{3x+7}{(x+3)(x^2+4)}=\frac{-\frac{2}{13}}{13(x+3)}+\frac{\frac{2}{13}x+\frac{33}{13}}{x^2+4}$$

$$=-\frac{2}{13(x+3)}+\frac{\frac{1}{13}(2x+33)}{x^2+4}$$

$$\Rightarrow \frac{3x+7}{(x+3)(x^2+4)}=\frac{2x+33}{13(x^2+4)}-\frac{2}{13(x+3)}$$

Q.6 $\dfrac{x^2}{(x+2)(x^2+4)}$ **(A.B)**

Solution:

Let $\dfrac{x^2}{(x+2)(x^2+4)} = \dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+4}$ ___ (i)

Multiplying by $(x+2)(x^2+4)$

$x^2 = A(x^2+4) + (Bx+C)(x+2)$ ___ (ii)

$= Ax^2 + 4A + Bx^2 + 2Bx + Cx + 2C$

$= Ax^2 + Bx^2 + 2Bx + Cx + 4A + 2C$

$x^2 + 0x + 0 = (A+B)x^2 + (2B+C)x + (4A+2C)$

By comparing coefficients of alike powers of x

$1 = A + B$ __________ (iii)

$0 = 2B + C$ __________ (iv)

$0 = 4A + 2C$ __________ (v)

Put $x+2=0 \Rightarrow x=-2$ in equation (ii)

$(-2)^2 = A\left[(-2)^2+4\right] + \left[B(-2)+C\right](0)$

$4 = A(4+4)+0$

$4 = A(8)$

$4 = 8A$

$A = \dfrac{1}{2}$

Put in equation (iii)

$1 = \dfrac{1}{2} + B$

$1 - \dfrac{1}{2} = B$

$\dfrac{2-1}{2} = B$

$B = \dfrac{1}{2}$

Put in equation (iv)

$0 = 2\left(\dfrac{1}{2}\right) + C$

$0 = 1 + C$

$C = -1$

Now putting values of A,B,C in equation (i)

$\dfrac{x^2}{(x+2)(x^2+4)} = \dfrac{\frac{1}{2}}{(x+2)} + \dfrac{\frac{1}{2}x + (-1)}{x^2+4}$

$= \dfrac{1}{2(x+2)} + \dfrac{\frac{1}{2}(x-2)}{(x^2+4)}$

$\Rightarrow \dfrac{x^2}{(x+2)(x^2+4)} = \dfrac{1}{2(x+2)} + \dfrac{x-2}{2(x^2+4)}$

Q.7 $\dfrac{1}{x^3+1}$ **(K.B + A.B)**

Solution:

$\dfrac{1}{x^3+1} = \dfrac{1}{(x+1)(x^2-x+1)}$

Let $\dfrac{1}{(x+1)(x^2-x+1)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2-x+1}$ ___ (i)

Multiplying by $(x+1)(x^2-x+1)$

$1 = A(x^2-x+1) + (Bx+C)(x+1)$ ____ (ii)

$1 = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C$

$1 = Ax^2 + Bx^2 + Bx + Cx - Ax + A + C$

$0x^2 + 0x + 1 = (A+B)x^2 + (B+C-A)x + (A+C)$

By comparing coefficients of alike power of x

$0 = A + B$ __________ (iii)

$0 = B + C - A$ __________ (iv)

$1 = A + C$ __________ (v)

Put $x+1=0 \Rightarrow x=-1$ in equation (ii)

$1 = A\left[(-1)^2 - (-1) + 1\right] + \left[B(-1)+C\right](-1+1)$

$1 = A(1+1+1) + (Bx+C)(0)$

$1 = A(3) + 0$

$A = \dfrac{1}{3}$

Put in equation (iii)

$0 = \dfrac{1}{3} + B$

$B = -\dfrac{1}{3}$

Put $A = \dfrac{1}{3}$ and $B = -\dfrac{1}{3}$ in equation (iv)

$$0=-\frac{1}{3}+C-\frac{1}{3}$$

$$C=\frac{1}{3}+\frac{1}{3}$$

$$C=\frac{2}{3}$$

Now putting values in equation (i)

$$\frac{1}{(x+1)(x^2-x+1)}=\frac{\frac{1}{3}}{(x+1)}+\frac{-\frac{1}{3}x+\frac{2}{3}}{x^2-x+1}$$

$$=\frac{1}{3(x+1)}+\frac{-\frac{1}{3}(x-2)}{x^2-x+1}$$

$$\Rightarrow\frac{1}{(x+1)(x^2-x+1)}=\frac{1}{3(x+1)}-\frac{x-2}{3(x^2-x+1)}$$

Q.8 $\frac{x^2+1}{x^3+1}$ **(K.B + A.B)**

Solution:

$$\frac{x^2+1}{x^3+1}=\frac{x^2+1}{(x+1)(x^2-x+1)}$$

Let $\frac{x^2+1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}\rightarrow$(i)

Multiplying by $(x+1)(x^2-x+1)$

$$x^2+1=A(x^2-x+1)+(Bx+C)(x+1)\_\_\text{ (ii)}$$

$$=Ax^2-Ax+A+Bx^2+Bx+Cx+C$$

$$=Ax^2+Bx^2-Ax+Bx+Cx+A+C$$

$$x^2+0x+1=(A+B)x^2+(B+C-A)x+(A+C)$$

By comparing coefficients of power of x

$1=A+B$ __________ (iii)

$0=B+C-A$ __________ (iv)

$1=A+C$ __________ (v)

Put $x+1=0\Rightarrow x=-1$ in equation (ii)

$$(-1)^2+(1)=A\left[(-1)^2-(-1)+1\right]+\left[B(-1)+C\right](0)$$

$$1+1=A(1+1+1)+0$$

$$2=A(3)$$

$$\frac{2}{3}=A$$

$$A=\frac{2}{3}$$

Put in equation (iii)

$$1=\frac{2}{3}+B$$

$$1-\frac{2}{3}=B$$

$$\frac{3-2}{3}=B$$

$$\frac{1}{3}+B$$

$$B=\frac{1}{3}$$

Put $P=\frac{2}{3}$ in equation (v)

$$1=\frac{2}{3}+C$$

$$1-\frac{2}{3}+C$$

$$C=\frac{1}{3}$$

Putting the values in equation (i)

$$\frac{x^2+1}{x^3+1}=\frac{\frac{2}{3}}{3(x+1)}+\frac{\frac{1x}{3}+\frac{1}{3}}{x^2-x+1}$$

$$=\frac{2}{3(x+1)}+\frac{\frac{1}{3}(x+1)}{x^2-x+1}$$

$$\Rightarrow\frac{x^2+1}{x^3+1}=\frac{2}{3(x+1)}+\frac{x+1}{3(x^2-x+1)}$$

Example: (Page # 81) **(K.B + A.B)**

Resolve $\frac{x^3-2x^2-2}{(x^2+1)^2}$ into partial fractions

Solution:

$\frac{x^3-2x^2-2}{(x^2+1)^2}$ is a proper fraction as degree of numerator is less than the degree of denominator.

Let, $\dfrac{x^3-2x^2-2}{\left(x^2+1\right)^2}=\dfrac{Ax+B}{x^2+1}+\dfrac{Cx+D}{\left(x^2+1\right)^2}$

Multiplying both the sides by $\left(x^2+1\right)^2$, we have

$x^3-2x^2-2=\left(Ax+B\right)\left(x^2+1\right)+Cx+D$

$x^2-2x^2-2=A\left(x^3+x\right)+B\left(x^2+1\right)+Cx+D\rightarrow(i)$

Equating the coefficients of and constant on the both sides of (i).

Coefficients of x^3: $A=1$

Coefficients of x^2: $B=-2$

Coefficients of x: $A+C=0 \Rightarrow C=-1$

Constants: $B+D=-2\rightarrow(ii)$

Putting the value of B in equation (ii)

$D=-2-B$

$=-2-(-2)$

$=-2+2=0$

$\Rightarrow D=0$

Thus,

$$\frac{x^2-2x^2-2}{\left(x^2+1\right)^2}=\frac{x-2}{x^2+1}+\frac{-x+0}{\left(x^2+1\right)^2}$$

$$\Rightarrow\frac{x^2-2x^2-2}{\left(x^2+1\right)^2}=\frac{x-2}{x^2+1}-\frac{x}{\left(x^2+1\right)^2}$$

Exercise 4.4

Resolve into partial fractions.

Q.1 $\dfrac{x^3}{\left(x^2+4\right)^2}$ **(K.B + A.B)**

Solutions:

Let $\dfrac{x^3}{\left(x^2+4\right)^2}=\dfrac{Ax+B}{x^2+4}+\dfrac{Cx+D}{\left(x^2+4\right)^2}$ ____ (i)

Multiplying by $\left(x^2+4\right)^2$

$x^3=\left(Ax+B\right)\left(x^2+4\right)+Cx+D$ ____ (ii)

$x^3=Ax^3+4Ax+Bx^2+4B+Cx+D$

x^3+0x^2+0x

$=Ax^3+Bx^2+\left(4A+C\right)x+4B+D$

By comparing coefficients of alike powers of 'x'

$1=A$ ____ (iii)

$0=B$ ____ (iv)

$0=4A+C$ ____ (v)

$0=4B+D$ ____ (vi)

From equation (iii) and (iv)

$A=1,\ B=0$

Put A = 1 in equation (v)

$0=4(1)+C$

$0=4+C$

$C=-4$

Put $B=0$ in equation (iv)

$0=4B+D$

$0=4(0)+D$

$D=0$

Putting the values in equation (i)

$$\frac{x^3}{\left(x^2+4\right)^2}=\frac{x+0}{x^2+4}+\frac{-4x+0}{\left(x^2+4\right)^2}$$

$$\Rightarrow\frac{x^3}{\left(x^2+4\right)^2}=\frac{x}{x^2+4}-\frac{4x}{\left(x^2+4\right)^2}$$

Q.2 $\dfrac{x^4+3x^2+x+1}{\left(x+1\right)\left(x^2+1\right)^2}$ **(K.B + A.B)**

Solution:

Let $\dfrac{x^4+3x^2+x+1}{\left(x+1\right)\left(x^2+1\right)^2}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{\left(x^2+1\right)^2}\rightarrow(\text{i})$

Multiplying by $\left(x+1\right)\left(x^2+1\right)^2$

$x^4+3x^2+x+1=A\left(x^2+1\right)^2$

$+\left(Bx+C\right)\left(x+1\right)\left(x^2+1\right)+\left(Dx+E\right)\left(x+1\right)\rightarrow(\text{ii})$

$=A\left(x^4+2x^2+1\right)+\left(Bx^2+Bx+Cx+C\right)\left(x^2+1\right)$

$+Dx^2+Dx+Ex+E$

$x^4+0x^3+3x^2+x+1$

$=Ax^4+Bx^4+Bx^3+Cx^3+2Ax^2+Bx^2+$

$Cx^2+Dx^2+Bx+Cx+Dx+Ex+A+C+E$

$=\left(A+B\right)x^4+\left(B+C\right)x^3+\left(2A+B+C+D\right)x^2$

$+\left(B+C+D+E\right)x+\left(A+C+E\right)$

By comparing coefficients of alike powers 'x'

$1 = A + B$ ___________ (iii)
$0 = B + C$ ___________ (iv)
$3 = 2A + B + C + D$ ___________ (v)
$1 = B + C + D + E$ ___________ (vi)
$1 = A + C + E$ ___________ (vii)

Put $x+1=0 \Rightarrow x=-1$ in equation (ii)

$$(-1)^4 + 3(-1)^2 + (-1) + 1 = A\left[(-1)^2+1\right]^2$$
$$+\left[B(-1)+C\right](0)+\left[D(-1)+E\right](0)$$
$$1+3(1)-1+1 = A(1+1)^2 + 0$$
$$4 = 4A$$
$$A = 1$$

Put in equation (iii)
$$1 + B = 1$$
$$B = 1-1$$
$$B = 0$$

Put in equation (iv)
$$0 + C = 0$$
$$C = 0$$
$$C = 0$$

Put $A=1, B=0$ and $C=0$ in equation (v)
$$2(1)+0+0+D=3$$
$$2+D=3$$
$$D=3-2$$
$$D=1$$

Put $A=1, C=0$ in equation (vii)
$$1+0+E=1$$
$$E=1-1$$
$$E=0$$

Now putting the values in equation (i)
$$\frac{x^4+3x^2+x+1}{(x+1)(x^2+1)} = \frac{1}{x+1} + \frac{0(x)+0}{x^2+1} + \frac{1(x)+0}{(x^2+1)^2}$$
$$\Rightarrow \frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2} = \frac{1}{x+1} + \frac{x}{(x^2+1)^2}$$

Q.3 $\dfrac{x^2}{(x+1)(x^2+1)^2}$ **(K.B + A.B)**

Solution:

Let $\dfrac{x^2}{(x+1)(x^2+1)^2} = \dfrac{A}{x+1} + \dfrac{Bx+C}{(x^2+1)} + \dfrac{Dx+E}{(x^2+1)^2} \rightarrow$ (i)

Multiplying by $(x+1)(x^2+1)^2$
$$x^2 = A(x^2+1)^2 + (Bx+C)(x+1)(x^2+1)$$
$$+(Dx+E)(x+1) \rightarrow \text{(ii)}$$
$$x^2 = A(x^4+1+2x^2) + (Bx+C)(x+1)(x^2+1)$$
$$+(Dx+E)(x+1)$$
$$x^2 = Ax^4 + 2Ax^2 + A + Bx^4 + Bx^2 + Bx^3 + Bx$$
$$+Cx^3 + C + Dx^2 + Dx + Ex + E$$
$$\Rightarrow 0x^4 + 0x^3 + x^2 + 0x + 0$$
$$= (A+B)x^4 + (B+C)x^3 + (2A+B+D)x^2$$
$$+(B+D+E)x + (A+C+E)$$

By comparing coefficients of alike powers of 'x'

$A + B = 0$ ___________ (iii)
$B + C = 0$ ___________ (iv)
$2A + B + D = 1$ ___________ (v)
$B + D + E = 0$ ___________ (vi)
$A + C + E = 0$ ___________ (vii)

Put $x+1=0 \Rightarrow x=-1$ in equation (ii)
$$(-1)^2 = A\left[(-1)^2+1\right]^2 + \left[B(-1)+C\right](0)$$
$$D = ?$$
$$+\left[D(-1)+E\right](0)$$
$$1 = A(1+1)^2 + 0 + 0$$
$$1 = A(2)^2$$
$$1 = 4A$$
$$A = \frac{1}{4}$$

Put in equation (iii)
$$\frac{1}{4} + B = 0$$
$$B = -\frac{1}{4}$$

Put in equation (iv)
$$-\frac{1}{4} + C = 0$$
$$C = \frac{1}{4}$$

Put $A = \dfrac{1}{4}$ and $C = \dfrac{1}{4}$ in equation (vii)

$$\frac{1}{4}+\frac{1}{4}+E=0$$

$$E=-\frac{1}{4}-\frac{1}{4}$$

$$=\frac{-2}{4}$$

$$E=-\frac{1}{2}$$

Now putting the values in equation (i)

$$\frac{x^2}{(x+1)(x^2+1)}=\frac{\frac{1}{4}}{4(x+1)}$$

$$+\frac{-\frac{1}{4}x+\frac{1}{4}}{x^2+1}+\frac{\frac{1}{2}x+\left(-\frac{1}{2}\right)}{(x^2+1)^2}$$

$$=\frac{1}{4(x+1)}+\frac{-\frac{1}{4}(x-1)}{x^2+1}+\frac{\frac{1}{2}(x-1)}{(x^2+1)^2}$$

$$\frac{x^2}{(x+1)(x^2+1)}=\frac{1}{4(x+1)}-\frac{x-1}{4(x^2+1)}+\frac{x-1}{2(x^2+1)^2}$$

Q.4 $\dfrac{x^2}{(x-1)(x^2+1)^2}$ **(K.B + A.B)**

Solution:

$$\frac{x^2}{(x-1)(x^2+1)^2}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{(x^2+1)^2}\rightarrow\text{(i)}$$

Multiplying by $(x-1)(x^2+1)^2$

$$x^2=A(x^2+1)^2+(Bx+C)(x-1)(x^2+1)$$
$$+(Dx+E)(x-1)\rightarrow\text{(ii)}$$

$$x^2=A(x^4+2x^2+1)+(Bx^2-Bx+Cx-C)(x^2+1)$$
$$+Dx^2-Dx+Ex-E$$

$$0x^4+0x^3+x^2+0x+0=Ax^4+2Ax^2+A+Bx^4$$
$$+Bx^2-Bx^3-Bx+Cx^3+Cx-Cx^2-C$$
$$+Dx^2-Dx+Ex-E$$
$$=(A+B)x^4+(-B+C)x^3+(2A+B-C+D)x^2$$
$$+(-B+C-D+E)x+(A-C-E)$$

By comparing coefficients of alike powers of x

$0=A+B$ ______ (iii)

$0=-B+C$ ______ (iv)

$1=2A+B-C+D$ ______ (v)

$0=-B+C-D+E$ ______ (vi)

$0=A-C-E$ ______ (vii)

Put $x-1=0\Rightarrow x=1$ in equation (ii)

$$(1)^2=A\left[(1)^2+1\right]^2$$
$$+(B(1)+C)(1-1)\left[(1)^3+1\right]+\left[D(1)+E\right](0)$$

$$1=A(1+1)^2+0+0$$

$$1=A(2)^2$$

$$1=4A$$

$$A=\frac{1}{4}$$

Put in equation (iii)

$$0=\frac{1}{4}+B$$

$$B=-\frac{1}{4}$$

Put in equation (iv)

$$0=-\left[-\frac{1}{4}\right]+C$$

$$0=\frac{1}{4}+C$$

$$\Rightarrow C=-\frac{1}{4}$$

Put $A=\frac{1}{4}$ and $C=-\frac{1}{4}$ in equation (vii)

$$0=\frac{1}{4}-\left[-\frac{1}{4}\right]-E$$

$$0=\frac{1}{4}+\frac{1}{4}-E$$

$$E=\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$$

$$\Rightarrow E=\frac{1}{2}$$

Put in equation (vi)

$$0=-\left[-\frac{1}{4}\right]+\left[-\frac{1}{4}\right]-D+\frac{1}{2}$$

$$0=\frac{1}{4}-\frac{1}{4}-D+\frac{1}{2}$$

$$\Rightarrow D=\frac{1}{2}$$

Put in equation (i)

$$\frac{x^2}{(x-1)(x^2+1)^2}=\frac{\frac{1}{4}}{x-1}+\frac{\left(-\frac{1}{4}\right)x-\frac{1}{4}}{x^2+1}+\frac{\frac{1}{2}x+\frac{1}{2}}{(x^2+1)^2}$$

$$=\frac{1}{4(x-1)}+\frac{-\frac{1}{4}(x+1)}{x^2+1}+\frac{\frac{1}{2}(x+1)}{(x^2+1)^2}$$

$$\Rightarrow\frac{x^2}{(x-1)(x^2+1)^2}=\frac{1}{4(x-1)}-\frac{x+1}{4(x^2+1)}+\frac{x+1}{2(x^2+1)^2}$$

Q.5 $\dfrac{x^4}{(x^2+2)^2}$ **(K.B + A.B)**

Solution:

$$\frac{x^4}{(x^2+2)^2}=\frac{x^4}{x^4+4x^2+4}$$

(improper fraction)

$$\begin{array}{r|l} & 1 \\ \hline x^4+4x^2+4 & x^4 \\ & \underline{\pm x^4 \pm 4x^2 \pm 4} \\ & -4x^2-4 \\ & -(4x^2+4) \end{array}$$

$$\frac{x^4}{(x^2+2)^2}=1-\frac{4x^2+4}{x^4+4x^2+4}$$

$$=1-\left(\frac{4x^2+4}{(x^2+2)^2}\right)\rightarrow(i)$$

Consider

$$\frac{4x^2+4}{(x^2+2)^2}=\frac{Ax+B}{x^2+2}+\frac{Cx+D}{(x^2+2)^2}\rightarrow(ii)$$

Multiply by $(x^2+2)^2$ on both sides

$$4x^2+4=(Ax+B)(x^2+2)+Cx+D$$

$$4x^2+4=Ax^3+2Ax+Bx^2+Cx+D+2B$$

$$0x^3+4x^2+0x+4=Ax^3+Bx^2+2Ax+Cx+D+2B$$

By comparing coefficients of alike powers of 'x'

$0=A$ __________ (iii)

$4=B$ __________ (iv)

$0=2A+C$ __________ (v)

$4=D+2B$ __________ (vi)

Equation (iii) $\Rightarrow$ A = 0

Equation (iv) $\Rightarrow$ B = 4

Put $A=0$ in equation (v)

$$0=2(0)+C$$

$$\Rightarrow C=0$$

Equation (vi)

$$4=D+2B$$

$$4=D+2(4)$$

$$4=D+8$$

$$D=4-8$$

$$D=-4$$

Putting the values in equation (ii)

$$\frac{4x^2+4}{(x^2+2)^2}=\frac{0x+4}{x^2+2}+\frac{0x-4}{(x^2+2)^2}$$

$$=\frac{4}{x^2+2}-\frac{4}{(x^2+2)^2}$$

Now putting the value in equation

$$\frac{x^4}{(x^2+2)^2} = 1 - \left(\frac{4}{x^2+2} - \frac{4}{(x^2+2)^2}\right)$$

$$\Rightarrow \frac{x^4}{(x^2+2)^2} = 1 - \frac{4}{x^2+2} + \frac{4}{(x^2+2)^2}$$

Q.6 $\dfrac{x^5}{(x^2+1)^2}$ **(K.B + A.B)**

Solution:

$$\frac{x^5}{(x^2+1)^2} = \frac{x^5}{x^4+2x^2+1}$$

(improper fraction)

$$\begin{array}{r|l} & x \\ \hline x^4+2x^2+1 & x^5 \\ & \underline{\pm x^5 \pm 2x^3 \pm x} \\ & -2x^3 - x \end{array}$$

$$\Rightarrow \frac{x^5}{x^4+2x^2+1} = x + \frac{-2x^3-x}{x^4+2x^2+1}$$

$$\frac{x^5}{(x^2+1)^2} = x - \frac{2x^3+x}{x^4+2x^2+1} \quad \text{—— (i)}$$

Let $\dfrac{2x^3+x}{(x^2+1)^2} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2}$ ——(ii)

Multiplying by $(x^2+1)^2$

$$2x^3 + x = (Ax+B)(x^2+1) + (Cx+D)$$

$$2x^3 + x = Ax^3 + Ax + Bx^2 + B + Cx + D$$

$$2x^3 + x = Ax^3 + Bx^2 + Ax + Cx + B + D$$

$$2x^3 + x = Ax^3 + Bx^2 + (A+C)x + (B+D)$$

By comparing coefficients of alike powers of x

$A = 2$ ———— (iii)

$B = 0$ ———— (iv)

$A + C = 1$ ———— (v)

$B + D = 0$ ———— (vi)

From equation (iii) and (iv)

$\Rightarrow A = 2$

$B = 0$

Put A = 2 in equation (v)

$2 + C = 1$

$C = 1 - 2$

$C = -1$

Put $B = 0$ in equation (vi)

$B + D = 0$

$0 + D = 0$

$D = 0$

Putting the values in equation (ii)

$$\frac{2x^3+x}{(x^2+1)^2} = \frac{2x}{x^2+1} - \frac{x}{(x^2+1)^2}$$

Put in equation (i)

$$\frac{x^5}{(x^2+1)^2} = x - \left[\frac{2x}{x^2+1} - \frac{x}{(x^2+1)^2}\right]$$

$$\Rightarrow \frac{x^5}{(x^2+1)^2} = x - \frac{2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$

Miscellaneous Exercise 4

Q.1 Multiple Choice Questions

Four possible answers are given for the following question. Tick (✓) the correct answer.

(i) **The identity** $(5x+4)^2 = 25x^2 + 40x + 16$ **is true for;** (SWL 2015) **(K.B + A.B)**

(a) One value of x **(b)** Two values of x
(c) All values of x **(d)** None of these

(ii) **A function of the form** $f(x) = \dfrac{N(x)}{D(x)}$**, with** $D(x) \neq 0$**, where** $N(x)$ **and** $D(x)$ **are polynomials in** x **is called;** **(K.B + A.B)**

(a) An identity **(b)** An equation
(c) A fraction **(d)** Algebraic relation

(iii) **A fraction in which the degree of numerator is greater or equal to the degree of denominator is called;** (GRW 2017, RWP 2015, 17, FSD 2015) **(K.B + A.B)**

(a) A proper fraction **(b)** An improper fraction
(c) An equation **(d)** Algebraic relation

(iv) **A fraction in which the degree of numerator is less than the degree of the denominator is called** (GRW 2014, 15, 16, FSD 2017) **(K.B + A.B)**

(a) An equation **(b)** An improper fraction
(c) An identity **(d)** A proper fraction

(v) $\dfrac{2x+1}{(x+1)(x-1)}$ **is;** **(K.B + A.B)**

(a) An improper fraction **(b)** An equation
(c) A proper fraction **(d)** None of these

(vi) $(x+3)^2 = x^2 + 6x + 9$ **is;** (LHR 2014, 15, 16, MTN 2015, SWL 2015, D.G.K 2015) **(K.B + A.B)**

(a) A linear equation **(b)** An equation
(c) An identity **(d)** None of these

(vii) $\dfrac{x^3+1}{(x-1)(x+2)}$ **is;** **(K.B + A.B)**

(a) A proper fraction **(b)** An improper fraction
(c) An identity **(d)** A constant term

(viii) **Partial fraction of** $\dfrac{x-2}{(x-1)(x+2)}$ **are of the form;** **(K.B + A.B)**

(a) $\dfrac{A}{x-1} + \dfrac{B}{x+2}$ **(b)** $\dfrac{Ax}{x-1} + \dfrac{B}{x+2}$
(c) $\dfrac{A}{x-1} + \dfrac{Bx+C}{x+2}$ **(d)** $\dfrac{Ax+B}{x-1} + \dfrac{C}{x+2}$

(ix) **Partial fraction of** $\dfrac{x+2}{(x+1)(x^2+2)}$ **are of the form;** (GRW 2014, RWP 2017) **(K.B + A.B)**

(a) $\dfrac{A}{x+1} + \dfrac{B}{x^2+2}$ **(b)** $\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+2}$
(c) $\dfrac{Ax+B}{x+1} + \dfrac{C}{x^2+2}$ **(d)** $\dfrac{A}{x+1} + \dfrac{Bx}{x^2+2}$

(x) **Partial fraction of $\dfrac{x^2+1}{(x+1)(x-1)}$ are of the form;** **(K.B + A.B)**

(a) $\dfrac{A}{x+1}+\dfrac{B}{x-1}$ (b) $1+\dfrac{A}{x+1}+\dfrac{Bx+C}{x-1}$

(c) $1+\dfrac{A}{x+1}+\dfrac{B}{x-1}$ (d) $1+\dfrac{Ax+B}{(x+1)}+\dfrac{C}{x-1}$

ANSWER KEY

i	ii	iii	iv	v	Vi	vii	viii	ix	x
c	c	b	d	c	c	b	a	b	c

Q.2 Write short answers of the following questions.

(i) Define a rational fraction. **(K.B)**

Ans:

Rational Fraction

An expression of the form $\dfrac{N(x)}{D(x)}$, where N(x) and D(x) are polynomials in x with real coefficients is called a rational fraction. The polynomial $D(x)\neq 0$

For example $\dfrac{x^2+4}{x-2}$ where $x\neq 2$

(ii) What is a proper fraction? **(K.B)**

Ans:

Proper Fraction

A rational fraction $\dfrac{N(x)}{D(x)}$, where D(x) ≠ 0 is called proper fraction, if degree of the polynomial N(x) is less than degree of the polynomial D(x)

For example: $\dfrac{2}{x+1},\dfrac{5x-3}{x^2+4}$ etc.

(iii) What is an improper fraction? **(K.B)**

Ans:

Improper Fraction

A rational fraction $\dfrac{N(x)}{D(x)}$, where D(x) ≠ 0 is called an improper fraction, if degree of the polynomial N(x) is greater than or equal to degree of the polynomial D(x).

For example: $\dfrac{5x}{x+2},\dfrac{6x^4}{x^3+1}$ etc.

(iv) What are partial fractions? **(K.B)**

Ans:

Partial Fraction

(LHR 2016, FSD 2016, BWP 2014, RWP 2016, 17, MTN 2016, 17, SWL 2017, SGD 2017)

Decomposition of resultant fraction into its components or into different fractions is called partial fraction.

(v) **How can we make partial fractions of $\dfrac{x-2}{(x+2)(x+3)}$?** **(K.B)**

Solution:

Let $\dfrac{x-2}{(x+2)(x+3)}=\dfrac{A}{x+2}+\dfrac{B}{x+3}$ $\rightarrow$(i)

Multiply by $(x+2)(x+3)$

$x-2=A(x+3)+B(x+2)$ $\rightarrow$(ii)

Put $x+2=0$ or $x=-2$ in equation (ii)

$-2-2=A(-2+3)+B(0)$

$-4=A(1)+0$

$\Rightarrow A=-4$

Put $x+3=0$ or $x=-3$ in equation (ii)

$-3-2=A(0)+B(-3+2)$

$-5=0+B(-1)$

$-5=-B$

$\Rightarrow B=5$

Now putting the values in equation (i)

$$\frac{x-2}{(x+2)(x+3)}=\frac{-4}{x+2}+\frac{5}{x+3}$$

(vi) Resolve $\frac{1}{x^2-1}$ into partial fractions.

(A.B + K.B)

Solution:

(LHR 2016, 17, GRW 2016, 17, SWL 2017, RWP 2017, BWP 2015, 16, 17, MTN 2017, SGD 2014, 15, 16, 17)

$$\frac{1}{x^2-1}=\frac{1}{(x+1)(x-1)}$$

Let $\frac{1}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1} \quad \to (\text{i})$

Multiply $(x+1)(x-1)$ on both sides

$1=A(x-1)+B(x+1) \quad \to (\text{ii})$

Put $x+1=0 \Rightarrow x=-1$ in equation (ii)

$1=A(-1-1)+B(0)$

$1=-2A+0$

$1=-2A$

$\Rightarrow A=\frac{-1}{2}$

Put $x-1=0 \Rightarrow x=1$ in equation (ii)

$1=A(0)+B(1+1)$

$1=0+2B$

$1=2B$

$\Rightarrow B=\frac{1}{2}$

Now putting the values in equation $\to (\text{i})$

$$\frac{1}{(x+1)(x-1)}=\frac{\frac{-1}{2}}{x+1}+\frac{\frac{1}{2}}{x-1}$$

$$\Rightarrow \frac{1}{x^2-1}=-\frac{1}{2(x+1)}+\frac{1}{2(x-1)}$$

(vii) Find partial fractions of $\frac{3}{(x+1)(x-1)}$.

(BWP 2015, SGD 2016)

Solution:

Let $\frac{3}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1} \quad \to (\text{i})$

Multiply by $(x+1)(x-1)$ on both sides

$3=A(x-1)+B(x+1) \quad \to (\text{ii})$

Put $x+1=0 \Rightarrow x=-1$ in equation (ii)

$3=A(-1-1)+B(0)$

$3=-2A+0$

$3=-2A$

$\frac{3}{-2}=A$

Or $A=\frac{-3}{2}$

Put $x-1=0$ or $x=1$ in equal (ii)

$3=A(0)+B(1+1)$

$3=0+2B$

$3=2B$

$\frac{3}{2}=B$

Or $B=\frac{3}{2}$

Now put the values in equation (i)

$$\frac{3}{(x+1)(x-1)}=\frac{\frac{-3}{2}}{x+1}+\frac{\frac{3}{2}}{x-1}$$

$$\frac{3}{(x+1)(x-1)}=-\frac{3}{2(x+1)}+\frac{3}{2(x-1)}$$

Or $\frac{3}{(x+1)(x-1)}=\frac{3}{2}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)$

(viii) Resolve $\frac{x}{(x-3)^2}$ into partial fractions.

(A.B + K.B)

(FSD 2015, RWP 2014, D.G.K 2014)

Solution:

Let $\frac{x}{(x-3)^2}=\frac{A}{x-3}+\frac{B}{(x-3)^2} \to (\text{i})$

Multiply by $(x-3)^2$ on both sides

$x=A(x-3)+B$

$x=Ax-3A+B$

$x+0=Ax-3A+B$

By comparing coefficient of alike powers of "x"

$1=A \quad \to (\text{ii})$

$0=-3A+B \quad \to (\text{iii})$

$A=1$ Put in equation (iii)

$0=-3(1)+B$

$0=-3+B$

Or $B=3$

Now putting the values in equation (i)

$$\frac{x}{(x-3)^2}=\frac{1}{x-3}+\frac{3}{(x-3)^2}$$

(ix) How we can make the partial fractions of $\frac{x}{(x+a)(x-a)}$?

(A.B + K.B + U.B)

Solution:

Let $\frac{x}{(x+a)(x-a)}=\frac{A}{x+a}+\frac{B}{x-a}$

Multiplying by L.C.M i.e. $(x+a)(x-a)$

$$(x+a)(x-a)\frac{x}{(x+a)(x-a)}$$

$$=(x+a)(x-a)\frac{A}{x+a}+(x+a)(x-a)\frac{B}{x-a}$$

$$x=A(x-a)+B(x+a)\rightarrow\text{(i)}$$

Put $x=-a$ in equation (i)

$$-a=A(-a-a)+B(-a+a)$$

$$-a=A(-2a)+B(0)$$

$$-a=-2aA$$

$$A=\frac{-a}{-2a}$$

$$A=\frac{1}{2}$$

Put $x=a$ in equation (i)

$$a=A(a-a)+B(a+a)$$

$$a=0+B(2a)$$

$$a=2aB$$

$$B=\frac{a}{2a}$$

$$B=\frac{1}{2}$$

$$\therefore\frac{x}{(x+a)(x-a)}=\frac{1}{2(x+a)}+\frac{1}{2(x-a)}$$

(x) Whether $(x+3)^2=x^2+6x+9$ is an identity? (A.B + K.B + U.B)

$$(x+3)^2=x^2+6x+9$$

Let $x=1$

$$(1+3)^2=(1)^2+6(1)+9$$

$$(4)^2=1+6+9$$

$$16=16 \qquad (True)$$

Let $x=25$

$$(2+3)^2=(2)^2+6(2)+9$$

$$(5)^2=4+12+9$$

$$25=25 \qquad (True)$$

Hence:

Given equation is an identity because it is true for all values of variable ‘x’

CUT HERE

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 **A function of the form** $f(x)=\frac{N(x)}{D(x)}$**, with** $D(x)\neq 0$**, where** $N(x)$ **and** $D(x)$ **are polynomials in** x **is called:**

(A) An identity **(B)** An equation
(C) A fraction **(D)** None of these

2 **Partial fractions of** $\frac{x+2}{(x+1)(x^2+2)}$ **are of the form:**

(A) $\frac{A}{x+1}+\frac{B}{x^2+2}$ **(B)** $\frac{A}{x+1}+\frac{Bx+C}{x^2+2}$
(C) $\frac{Ax+B}{x+1}+\frac{C}{x^2+2}$ **(D)** $\frac{A}{x+1}+\frac{Bx}{x^2+2}$

3 **An improper rational fraction can be reduced by division to a:**

(A) Rational fraction **(B)** Irrational fraction
(C) Polynomial and rational form **(D)** None of these

4 $(x+3)^2=x^2+6x+9$ **is:**

(A) An identity **(B)** An equation
(C) A linear equations **(D)** None

5 **A fraction in which the degree of the numerator is less than the degree of the denominator is called:**

(A) An improper fraction **(B)** An equation
(C) A proper fraction **(D)** None

6 **Partial fractions of** $\frac{1}{(x+1)(x^2-1)}$ **will be of the form:**

(A) $\frac{A}{x+1}+\frac{Bx+C}{x^2-1}$ **(B)** $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}$
(C) $\frac{A}{x+1}+\frac{B}{x+1}+\frac{C}{x-1}$ **(D)** None of these

7 **The degree of the expression** $4x^3+2x+5$ **is:**

(A) 4 **(B)** 5
(C) 2 **(D)** 3

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Define partial fractions.

(ii) What is difference between conditional equation and identity?

(iii) Resolve $\frac{x^2+1}{(x-1)(x+2)}$ into partial fractions

(iv) Define improper rational fraction.

(v) Convert into proper fraction: $\frac{3x^2-2x-1}{x^2-x+1}$.

Q.3 Answer the following Questions. **(4+4=8)**

(a) Resolve into partial fractions: $\frac{x^5}{\left(x^2+1\right)^2}$.

(b) Resolve into partial fractions $\frac{6x^3+5x^2-7}{3x^2-2x-1}$

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.