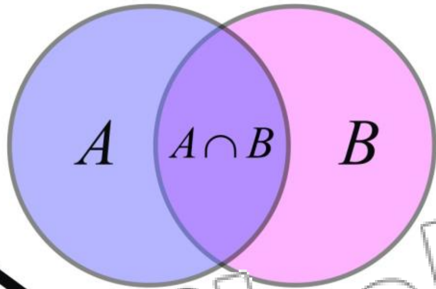


UNIT 5

SETS AND FUNCTIONS

Set **(K.B + U.B)**

A well define collection of distinct objects is called Set. A set is represented by capital English alphabets.

For example: A= Set of integers,

$B=\{1,2,3,...,10\}$ etc.

Presentation of Sets **(K.B + U.B)**

A set is presented by 3 ways

(i) Tabular form

(ii) Descriptive Form

(iii) Set builder notation

Some Important Sets **(K.B + U.B)**

N = The set of natural numbers={1,2,3,…}

W = The set of whole numbers={0,1,2,3,…}

Z = The set of all integers= $\{0,\pm1,\pm2,\pm3,...\}$

E = The set of all even integers

= $\{0,\pm2,\pm4,\pm6,...\}$

O = The set of all odd integers

$=\{\pm1,\pm3,\pm5,...\}$

P = The set of prime numbers={2,3,5,7,…}

Q = The set of rational numbers

$=\left\{x/x=\dfrac{m}{n}, \text{where } m,n\in Z\wedge n\neq 0\right\}$

Q $'$**=** The set of irrational numbers

$=\left\{x/x\neq\dfrac{m}{n}, \text{where } m,n\in Z\wedge n\neq 0\right\}$

R = The set of real numbers = $Q\cup Q'$

Empty Set or Null Set **(K.B + U.B)**

A set having no element in it is called an empty set. It is represented by ϕ (phi) or { }.

Singleton Set **(K.B + U.B)**

A set having only one element in it is called singleton set.

For example: {a}, {3} etc.

Subset **(K.B + U.B)**

(LHR 2016, D.G.K 2016)

"If A and B are two sets, such that every element of set A is present in set B, then set A is called subset of set B". It is represented as A⊆B.

For Example

If A = {1,2,3}, B = {1,2,3,4,5}

Then A ⊆ B.

Note **(K.B + U.B)**

- Number of all possible subsets = 2^n
- Number of all possible proper subsets = 2^n-1.
- Number of improper subsets = 1

Proper Subset **(K.B + U.B)**

"If A and B are two sets, such that every element of set A is present in set B and there is at least one element in set B which is not in set A, then set A is called proper subset of set B". It is represented as A⊂B.

For example:

If A = {1,2,3} and B={1,2,3,4,5}

Then set A ⊂ B.

Improper Subset **(K.B + U.B)**

If A and B are two sets, such that every element of set A is present in set B and there is no more element in set B which is not in set A, then set A is called improper subset of set B.

For example: $A=\{1,2,3\}$, $B=\{3,2,1\}$

Then A is improper subset of set B.

Equal Sets **(K.B + U.B)**

If A and B are two sets, such that A ⊆ B and B ⊆ A then A = B

For example

If A = {1,2,3}, B = {1,2,3}

Then A = B

Power Set **(K.B + U.B)**

A set consisting of all possible subsets of a set A is called power set of set A represented as P(A).

For example:

If A = {1,2} then P(A)={ϕ, {1}, {2}, {1,2}}

Note **(K.B + U.B)**

- Formula to find number of elements in the power set = 2^n

Disjoint Sets **(K.B + U.B)**

Two sets having no common elements are called disjoint sets.

i.e. If A∩B=ϕ then A and B are called disjoint sets.

For example

If $A=\{1,2\}$ $B=\{a,b\}$, then A and B are disjoint sets.

Overlapping Sets **(K.B + U.B)**

Two sets are called over lapping sets if they have:

- At least one element is common
- Neither set is subset of other.

For example

If A = {1,2,3}, B={2,4,6,8,10}

Then A and B are over lapping sets.

Operation on Sets

Union of Two Sets

(BWP 2014) **(K.B + U.B)**

The union of two sets A and B written as $A\cup B$ (read as A union B) is a set consisting of all the elements which are either in A or in B or in both.

$A\cup B=\{x\,|\,x\in A \text{ or } x\in B \text{ or } x\in A \text{ and } B \text{ both}\}$

For example

If $A=\{1,2,3\}, B=\{2,4,6,8,10\}$

Then $A\cup B=\{1,2,3,4,5,8,10\}$

Intersection of Two Sets

(LHR 2015) **(K.B + U.B)**

If A and B are two sets then a set consisting of common elements of set A and B is called A intersection B, represented by $A\cap B$.

$A\cap B=\{x\,|\,x\in A \text{ and } x\in B\}$

For example:

If $A=\{1,2,3\}, B=\{2,4,6,8,10\}$

Then $A\cap B=\{2\}$

Difference of Two Sets **(K.B + U.B)**

If A and B are two sets then their difference written as $A-B$ or $A\backslash B$ is a set consisting of all the elements of A which are not in B.

$A-B=\{x\,|\,x\in A \text{ and } x\notin B\}$

$B-A=\{x\,|\,x\in B \text{ and } x\notin A\}$

For example:

If $A=\{1,2,3\}, B=\{2,4,6,8,10\}$

Then $A-B=\{1,3\}$

Universal Set **(K.B + U.B)**

A set consists of all the elements of the sets under consideration is called universal set. It is represented by capital English alphabet U or X.

For example:

U= Set of integers, A = {1,2,3}, B={2,4,6,8,10},then U is universal set.

Complement of a Set **(K.B + U.B)**

(GRW 2014, SWL 2015, BWP 2016)

If U is a universal set and A is its subset then $U-A$ is called complement of A, represented as $A^/$ or A^c.

For example:

If U= {1,2,3,4,5}, A = {1,2,3}

$A^/ = U-A$

$=\{1,2,3,4,5\}-\{1,2,3\}$

$=\{4,5\}$

Exercise 5.1

Q.1 **(A.B)**

Given

$X=\{1,4,7,9\}$

$Y=\{2,4,5,9\}$

To Find

(i) X∪Y

(ii) X∩Y

(iii) Y∪X

(iv) Y∩X

Solution:

(i) $X\cup Y=\{1,4,7,9\}\cup\{2,4,5,9\}$

$=\{1,2,4,5,7,9\}$

(ii) $X \cap Y = \{1,4,7,9\} \cap \{2,4,5,9\}$

$= \{4,9\}$

(iii) $Y \cup X = \{2,4,5,9\} \cup \{1,4,7,9\}$

$= \{1,2,4,5,7,9\}$

(iv) $Y \cap X = \{2,4,5,9\} \cap \{1,4,7,9\}$

$= \{4,9\}$

Q.2 **(A.B)**

Given:

X = set of prime numbers less than or equal to 17.

Y = set of first 12 natural number

To Find:

(i) $X \cup Y$

(ii) $Y \cup X$

(iii) $X \cap Y$

(iv) $Y \cap X$

Solution:

Here

X = Set of prime numbers less than or equal to 17.

$= \{2,3,5,7,11,13,17\}$

Y = Set of first 12 natural numbers

$= \{1,2,3,\ldots,12\}$

(i) $X \cup Y = \{2,3,5,7,11,13,17\} \cap \{1,2,3,...,12\}$

$= \{1, 2, 3, \ldots, 12, 13, 17\}$

(ii) $Y \cup X = \{1,2,3,\ldots,12\} \cup \{2,3,5,7,11,13,17\}$

$= \{1, 2, 3 \ldots 12, 13, 17\}$

(iii) $X \cap Y = \{2,3,5,7,11,13,17\} \cap \{1,2,3,...,12\}$

$= \{2, 3, 5, 7, 11\}$

(iv) $Y \cap X = \{1,2,3,...,12\} \cap \{2,3,5,7,11,13,17\}$

$= \{2, 3, 5, 7, 11\}$

Q.3 **(A.B)**

Given

$X = \phi, \quad Y = Z^+, \quad T = O^+$

To Find

(i) $X \cup Y$

(ii) $X \cup T$

(iii) $Y \cup T$

(iv) $X \cap Y$

(v) $X \cap T$

(vi) $Y \cap T$

Solution:

(i) $X \cup Y = \phi \cup Z^+$

$= Z^+$

(ii) $X \cup T = \phi \cup O^+$

$= O^+$

(iii) $Y \cup T = Z^+ \cup O^+$

$= Z^+$

(iv) $X \cap Y = \phi \cap Z^+$

$= \phi$

(v) $X \cap T = \phi \cap O^+$

$= \phi$

(vi) $Y \cap T = Z^+ \cap O^+$

$= O^+$

Q.4 **Given** **(A.B)**

$U = \{x | x \in N \wedge 3 < x \leq 25\}$

$X = \{x \mid x \text{ is prime} \wedge 8 < x < 25\}$

$Y = \{x \mid x \in W \wedge 4 \leq x \leq 17\}$

To Find

(i) $(X \cup Y)'$ (ii) $X' \cap Y'$

(ii) $(X \cap Y)'$ (iv) $X' \cup Y'$

Solution: Here

$U = \{x / x \in N \wedge 3 < x \leq 25\}$

$= \{4,5,6,\ldots,25\}$

$X = \{x \mid x \text{ is prime} \wedge 8 < x < 25\}$

$= \{11,13,17,19,23\}$

$Y = \{x \mid x \in W \wedge 4 \leq x \leq 17\}$

$= \{4,5,6,7,\ldots,17\}$

(i) $X \cup Y = \{11,13,17,19,23\} \cup \{4,5,6,...,17\}$

$= \{4,5,6,....,17,19,23\}$

$(X \cup Y)' = U - (X \cup Y)$

$= \{4,5,6,...,25\} - \{4,5,6,...,17,19,23\}$

$= \{18,20,21,22,24,25\}$

(ii) $X' = U - X$

$= \{4,5,6,.....,25\} - \{11,13,17,19,23\}$

$= \{4,5,6,7,8,9,10,12,14,15,16,18,20,21,22,24,25\}$

$Y' = U - Y$

$= \{4,5,6,.....,25\} - \{4,5,6,.....,17\}$

$= \{18,19,20,21,22,23,24,25\}$

$X' \cap Y' = \{4,5,6,....,10,12,14,15,16,18,20,21,22,24,25\}$

$\cap \{18,19,20,.....,25\}$

$= \{18, 20, 21, 22, 24, 25\}$

(iii) $X \cap Y = \{11,13,17,19,23\} \cap \{4,5,6,...,17\}$
$= \{11,13,17\}$

$(X \cap Y)' = U - (X \cap Y)$
$= \{4, 5, 6, ..., 25\} - \{11, 13, 17\}$
$= \{4, 5, 6, ..., 10, 12, 14, 15, 16, 18, 19, 20, ..., 25\}$

(iv) $X' \cup Y'$
$= \{4,5,6,...,10,12,14,15,16,18,20,21,22,24,25\}$
$\cup \{18, 19, 20, 21, 22, 23, 24, 25\}$
$= \{4, 5, 6, ..., 10, 12, 14, 15, 16, 18, 19, 20, ..., 25\}$

Q.5 Given **(A.B)**
$X = \{2,4,6,.....,20\}$ **(LHR 2014)**
$Y = \{4,8,12,...,24\}$ **(FSD 2015)**

To Find
(i) X – Y
(ii) Y – X

Solution:
(i) $X - Y = \{2,4,6,.....,20\} - \{4,8,12,....,24\}$
$= \{2, 6, 10, 14, 18\}$
(ii) $Y - X = \{4,8,12,.....,24\} - \{2,4,6,.....,20\}$
$= \{24\}$

Q.6 Given **(BWP 2014)** **(A.B)**
$A = N, B = W$ **(LHR 2015)**

To Find **(D.G.K 2014)**
(i) A – B
(ii) B – A

Solution:
(i) $A - B = N - W = \{1, 2, 3\ldots\} - \{0, 1, 2, 3, \ldots\}$
$= \phi$
(ii) $B - A = W - N = \{0, 1, 2, 3, ...\} - \{1, 2, 3, ...\}$
$= \{0\}$

Properties of Union and Intersection of Sets **(K.B)**

- **Commutative property of Union**
  $A \cup B = B \cup A$
- **Commutative property of intersection**
  $A \cap B = B \cap A$
- **Associative property of union**
  $A \cup (B \cup C) = (A \cup B) \cup C$
- **Associative property of intersection**
  $A \cap (B \cap C) = (A \cap B) \cap C$
- **Distributive property of union over intersection**
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- **Distributive property of intersection over union**
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- **De-Morgan's laws**
  $(A \cup B)' = A' \cap B'$
  $(A \cap B)' = A' \cup B'$

Commutative Property of Union

(K.B + U.B)

For any two sets A and B, prove that $A \cup B = B \cup A$.

Proof

Let $x \in A \cup B$
$\Rightarrow x \in A$ or $x \in B$ (by definition of union of sets)
$\Rightarrow x \in B$ or $x \in A$
$\Rightarrow x \in (B \cup A)$
$\Rightarrow (A \cup B) \subseteq (B \cup A) \rightarrow$ (i)

Now let $y \in (B \cup A)$
$\Rightarrow y \in B$ or $y \in A$ (by definition of union of sets)
$\Rightarrow y \in A$ or $y \in B$
$\Rightarrow y \in (A \cup B)$
$\Rightarrow (B \cup A) \subseteq (A \cup B) \rightarrow$ (ii)

From (i) and (ii), we have $A \cup B = B \cup A$.
(by definition of equal sets)

Commutative Property of Intersection

(MTN 2014) **(K.B + U.B)**

For any two sets A and B, prove that $A \cap B = B \cap A$

Proof

Let $x \in (A \cap B)$
$\Rightarrow x \in A$ and $x \in B$
(by definition of intersection of sets)
$\Rightarrow x \in B$ and $x \in A$
$\Rightarrow x \in (B \cap A)$
$\therefore (A \cap B) \subseteq (B \cap A) \rightarrow$ (i)

Now let $y \in (B \cap A)$
$\Rightarrow y \in B$ and $y \in A$
(by definition of intersection of sets)
$\Rightarrow y \in A$ and $y \in B$
$\Rightarrow y \in (A \cap B)$

Therefore, $(B \cap A) \subseteq (A \cap B) \rightarrow$ (ii)

From (i) and (ii), we have $A\cap B=B\cap A$ (by definition of equal sets)

Associative Property of Union

(K.B + U.B)

For any three sets A, B and C, prove that $(A\cup B)\cup C=A\cup(B\cup C)$

Proof

Let $x\in(A\cup B)\cup C$

$\Rightarrow x\in(A\cup B)$ or $x\in C$

$\Rightarrow (x\in A$ or $x\in B)$ or $x\in c$

(Associative property)

$\Rightarrow x\in A$ or $(x\in B$ or $x\in C)$

$\Rightarrow x\in A$ or $x\in(B\cup C)$

$\Rightarrow x\in A\cup(B\cup C)$

$\Rightarrow (A\cup B)\cup C\subseteq A\cup(B\cup C)\rightarrow$ (i)

Now let $y\in A\cup(B\cup C)$

$\Rightarrow y\in A$ or $y\in(B\cup C)$

$\Rightarrow y\in A$ or $(y\in B$ or $y\in C)$

$\Rightarrow y\in(A$ or $y\in B)$ or $y\in C$

$\Rightarrow y\in(A\cup B)$ or $y\in C$

$\Rightarrow y\in(A\cup B)\cup C$

$\therefore A\cup(B\cup C)\subseteq(A\cup B)\cup C\rightarrow$ (ii)

From (i) and (ii), we have $(A\cup B)\cup C=A\cup(B\cup C)$

Associative Property of Intersection

(K.B + U.B)

For any three sets A, B and C, prove that $(A\cap B)\cap C=A\cap(B\cap C)$

Proof

Let $x\in(A\cap B)\cap C$

$\Rightarrow x\in(A\cap B)$ and $x\in C$

$\Rightarrow (x\in A$ and $x\in B)$ and $x\in C$

$\Rightarrow x\in A$ and $(x\in B$ and $x\in C)$

$\Rightarrow x\in A$ and $x\in(B\cap C)$

$\Rightarrow x\in A\cap(B\cap C)$

$\Rightarrow (A\cap B)\cap C\subseteq A\cap(B\cap C)\rightarrow$ (i)

Now let $y\in A\cap(B\cap C)$

$\Rightarrow y\in A$ and $y\in(B\cap C)$

$\Rightarrow y\in A$ and $(y\in B$ and $y\in C)$

$\Rightarrow (y\in A$ and $y\in B)$ and $y\in C$

$\Rightarrow y\in(A\cap B)$ and $y\in C$

$\Rightarrow y\in(A\cap B)\cap C$

$\therefore A\cap(B\cap C)\subseteq(A\cap B)\cap C\rightarrow$ (ii)

From (i) and (ii), we have $(A\cap B)\cap C=A\cap(B\cap C)$

Distributive Property of Union over Intersection

(K.B + U.B)

For any three sets A, B and C, prove that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Proof

Let $x\in A\cup(B\cap C)$

$\Rightarrow x\in A$ or $x\in(B\cap C)$

$\Rightarrow x\in A$ or $(x\in B$ and $x\in C)$

$\Rightarrow (x\in A$ or $x\in B)$ and $(x\in A$ or $x\in C)$

$\Rightarrow x\in(A\cup B)$ and $x\in(A\cup C)$

$\Rightarrow x\in(A\cup B)\cap(A\cup C)$

Therefore

$A\cup(B\cap C)\subseteq(A\cup B)\cap(A\cup C)\rightarrow$ (i)

Similarly, now let y $\in(A\cup B)\cap(A\cup C)$

$\Rightarrow y\in(A\cup B)$ and $y\in(A\cup C)$

$\Rightarrow (y\in A$ or $y\in B)$ and $(y\in A$ or $y\in C)$

$\Rightarrow y\in A$ or $(y\in B$ and $y\in C)$

$\Rightarrow y\in A$ or $y\in(B\cap C)$

$\Rightarrow y\in A\cup(B\cap C)$

$\Rightarrow (A\cup B)\cap(A\cup C)\subseteq A\cup(B\cap C)\rightarrow$ (ii)

From (i) and (ii), we have

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Distributive Property of Intersection over Union

(K.B + U.B)

For any three sets A, B and C, prove that

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Proof

Let $x\in A\cap(B\cup C)$

$\Rightarrow x\in A \text{ and } x\in (B\cup C)$

$\Rightarrow x\in A \text{ and } (x\in B \text{ or } x\in C)$

$\Rightarrow (x\in A \text{ and } x\in B)$

or $(x\in A \text{ and } x\in C)$

$\Rightarrow x\in(A\cap B) \text{ or } x\in(A\cap C)$

$\Rightarrow x\in(A\cap B)\cup(A\cap C)$

$A\cap(B\cup C)\subseteq(A\cap B)\cup(A\cap C)\rightarrow(\text{i})$

Now let

$\Rightarrow y\in(A\cap B)\cup(A\cap C)$

$\Rightarrow y\in(A\cap B) \text{ or } y\in(A\cap C)$

or $(y\in A \text{ and } y\in C)$

$\Rightarrow (y\in A \text{ and } y\in B)$

$\Rightarrow y\in A \text{ and } (y\in B \text{ or } y\in C)$

$\Rightarrow y\in A \text{ and } y\in (B\cup C)$

$y\in A\cap(B\cup C)$

$(A\cap B)\cup(A\cap C)\subseteq A\cap(B\cup C)\rightarrow(\text{ii})$

From (i) and (ii), we have

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

De-Morgan's Laws **(K.B + U.B)**

(LHR 2016, MTN 2016, SGD 2015, BWP 2016, D.G.K 2016)

For any two sets A and B belonging from universal set U.

$(A\cup B)'=A'\cap B'\rightarrow(\text{i})$

$(A\cap B)'=A'\cup B'\rightarrow(\text{ii})$

(i) Proof

Let $x\in(A\cup B)'$

$\Rightarrow x\notin(A\cup B)$ (by definition of complement of set)

$\Rightarrow x\notin A \text{ and } x\notin B$

$\Rightarrow x\in A' \text{ and } x\in B'$

$\Rightarrow x\in A'\cap B'$ (by definition of intersection of sets)

$\Rightarrow (A\cup B)'\subseteq A'\cap B'\rightarrow(\text{i})$

Let $y\in A'\cap B'$

$\Rightarrow y\in A' \text{ and } y\in B'$

$\Rightarrow y\notin A \text{ and } y\notin B$

$\Rightarrow y\notin(A\cup B)$ (by definition of union of set)

$y\in(A\cup B)'$

$A'\cap B'\subseteq(A\cup B)'\rightarrow(\text{ii})$

Using (i) and (ii), we have

$(A\cup B)'=A'\cap B'$

(ii) Proof

Let $x\in(A\cap B)'$

$\Rightarrow x\notin A\cap B$ (by definition of complement of set)

$\Rightarrow x\notin A \text{ or } x\notin B$

$\Rightarrow x\in A' \text{ or } x\in B'$

$\Rightarrow x\in A'\cup B'$ (by definition of union of set)

$\Rightarrow (A\cap B)'\subseteq A'\cup B'\rightarrow(\text{i})$

Let $y\in A'\cup B'$

$\Rightarrow y\in A' \text{ or } y\in B'$

$\Rightarrow y\notin A \text{ or } y\notin B$

$\Rightarrow y\notin A\cap B$ (by definition of intersection of sets)

$\Rightarrow y\in(A\cap B)'$

$\Rightarrow A'\cup B'\subseteq(A\cap B)'\rightarrow(\text{ii})$

From (i) and (ii), we have proved that

$(A\cap B)'=A'\cup B'$

Exercise 5.2

Q.1 Given $X=\{1,3,5,7,....,19\}$ **(A.B)**

$Y=\{0,2,4,6,....,20\}$

$Z=\{2,3,5,7,11,13,17,19,23\}$

To Find

(i) $X\cup(Y\cup Z)$

(ii) $(X\cup Y)\cup Z$

(iii) $X\cap(Y\cap Z)$

(iv) $(X\cap Y)\cap Z$

(v) $X\cup(Y\cap Z)$

(vi) $(X\cup Y)\cap(X\cup Z)$

(vii) $X\cap(Y\cup Z)$

(viii) $(X\cap Y)\cup(X\cap Z)$

Solution:

(i) $X\cup(Y\cup Z)$ **(RWP 2015)** **(A.B)**

$=\{1,3,5,7,.....,19\}\cup$

$(\{0,2,4,6,....,20\}\cup\{2,3,5,7,11,13,17,19,23\})$

$=\{1,3,5,....,19\}\cup$

$\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$=\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,23\}$

(ii) $(X\cup Y)\cup Z$ **(A.B)**

$X\cup Y=\{1,3,5,7,.....,19\}\cup\{0,2,4,6,....,20\}$

$=\{0,1,2,3,....,20\}$

$(X\cup Y)\cup Z$

$=\{0,1,2,3,.....,20\}\cup\{2,3,5,7,11,13,17,19,23\}$

$=\{0,1,2,3,.....,20,23\}$

(iii) $X\cap(Y\cap Z)$ **(A.B)**

$Y\cap Z=\{0,2,4,6,....,20\}\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{2\}$

$X\cap(Y\cap Z)=\{1,3,5,7,.....,19\}\cap\{2\}$

$=\{\}$

(iv) $(X\cap Y)\cap Z$ **(A.B)**

$X\cap Y=\{1,3,5,7,....,19\}\cap\{0,2,4,6,....,20\}$

$=\{\}$

$(X\cap Y)\cap Z=\{\}\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{\}$

(v) $X\cup(Y\cap Z)$ **(A.B)**

$Y\cap Z=\{0,2,4,6,....,20\}\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{2\}$

$X\cup(Y\cap Z)=\{1,3,5,7,....,19\}\cup\{2\}$

$=\{1,2,3,5,7,....,19\}$

(vi) $(X\cup Y)\cap(X\cup Z)$ **(A.B)**

$X\cup Y=\{1,3,5,....,19\}\cup\{0,2,4,6,....,20\}$

$=\{0,1,2,3,....,20\}$

$X\cup Z=\{1,3,5,...,19\}\cup\{2,3,5,7,11,13,17,19,23\}$

$=\{1,2,3,5,7,....,19,23\}$

$(X\cup Y)\cap(X\cup Z)=\{0,1,2,3,...,20\}$

$\cap\{1,2,3,5,7,.....,19,23\}$

$=\{1,2,3,5,7,....,19\}$

(vii) $X\cap(Y\cup Z)$ **(A.B)**

$Y\cup Z=\{0,2,4,6,...,20\}$

$\cup\{2,3,5,7,11,13,17,19,23\}$

$=\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$X\cap(Y\cup Z)=\{1,3,5,7,....,19\}\cap$

$\{0,2,3,4,5,6,7,8,10,11,12,13,14,16,17,18,19,20,23\}$

$=\{3,5,7,11,13,17,19\}$

(viii) $(X\cap Y)\cup(X\cap Z)$ **(A.B)**

$X\cap Y=\{1,3,5,7,....,19\}\cap\{0,2,4,6,....,20\}$

$=\{\}$

$X\cap Z=\{1,3,5,7,....,19\}$

$\cap\{2,3,5,7,11,13,17,19,23\}$

$=\{3,5,7,11,13,17,19\}$

$(X\cap Y)\cup(X\cap Z)=\{\}\cup\{3,5,7,11,13,17,19\}$

$=\{3,5,7,11,13,17,19\}$

Q.2 **Given** $A=\{1,2,3,4,5,6\}$ **(A.B + K.B)**

$B=\{2,4,6,8\}$

$C=\{1,4,8\}$

To Prove

(i) $A\cap B=B\cap A$

(ii) $A\cup B=B\cup A$

(iii) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

(iv) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Proof

(i) $A\cap B=B\cap A$ **(A.B + K.B)**

L.H.S = $A\cap B$

$=\{1,2,3,4,5,6\}\cap\{2,4,6,8\}$

$=\{2,4,6\}\rightarrow(i)$

R.H.S = $B\cap A$

$=\{2,4,6,8\}\cap\{1,2,3,4,5,6\}$

$=\{2,4,6\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cap B=B\cap A$

Hence Proved

(ii) $A\cup B=B\cup A$ **(A.B + K.B)**

Proof

L.H.S = $A\cup B$

$=\{1,2,3,4,5,6\}\cup\{2,4,6,8\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(i)$

R.H.S = $B\cup A$

$=\{2,4,6,8\}\cup\{1,2,3,4,5,6\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cup B=B\cup A$

Hence Proved

(iii) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

(A.B + K.B)

Proof

L.H.S = $A\cap(B\cup C)$

$B\cup C=\{2,4,6,8\}\cup\{1,4,8\}$

$=\{1,2,4,6,8\}$

$A\cap(B\cup C)$

$=\{1,2,3,4,5,6\}\cap\{1,2,4,6,8\}$

$=\{1,2,4,6\}\rightarrow(i)$

R.H.S $=(A\cap B)\cup(A\cap C)$

$A\cap B=\{1,2,3,4,5,6\}\cap\{2,4,6,8\}$

$=\{2,4,6\}$

$A\cap C=\{1,2,3,4,5,6\}\cap\{1,4,8\}$

$=\{1,4\}$

$(A\cap B)\cup(A\cap C)=\{2,4,6\}\cup\{1,4\}$

$=\{1,2,4,6\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Hence Proved

(iv) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(A.B + K.B)

Proof

L.H.S $=A\cup(B\cap C)$

$B\cap C=\{2,4,6,8\}\cap\{1,4,8\}$

$=\{4,8\}$

L.H.S $=A\cup(B\cap C)$

$=\{1,2,3,4,5,6\}\cup\{4,8\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(i)$

R.H.S $=(A\cup B)\cap(A\cup C)$

$A\cup B=\{1,2,3,4,5,6\}\cup\{2,4,6,8\}$

$=\{1,2,3,4,5,6,8\}$

$A\cup C=\{1,2,3,4,5,6\}\cup\{1,4,8\}$

$=\{1,2,3,4,5,6,8\}$

R.H.S $=(A\cup B)\cap(A\cup C)$

$=\{1,2,3,4,5,6,8\}\cap\{1,2,3,4,5,6,8\}$

$=\{1,2,3,4,5,6,8\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Hence Proved

Q.3 **Given** $U=\{1,2,3,....,10\}$

(A.B + K.B)

$A=\{1,3,5,7,9\}$

$B=\{2,3,5,7\}$

To Prove

(i) $(A\cap B)'=A'\cup B'$

(ii) $(A\cup B)'=A'\cap B'$

(i) $(A\cap B)'=A'\cup B'$

Proof

L.H.S = $(A\cap B)'$

$A\cap B=\{1,3,5,7,9\}\cap\{2,3,5,7\}$

$=\{3,5,7\}$

$(A\cap B)'=U-(A\cap B)$

$=\{1,2,3,....,10\}-\{3,5,7\}$

$=\{1,2,4,6,8,9,10\}\rightarrow(i)$

R.H.S = $A'\cup B'$

$A'=U-A$

$=\{1,2,3,...,10\}-\{1,3,5,7,9\}$

$=\{2,4,6,8,10\}$

$B'=U-B$

$=\{1,2,3,....,10\}-\{2,3,5,7\}$

$=\{1,4,6,8,9,10\}$

$A'\cup B'=\{2,4,6,8,10\}\cup\{1,4,6,8,9,10\}$

$=\{1,2,4,6,8,9,10\}\rightarrow(ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cap B)'=A'\cup B'$

Hence Proved

(ii) $(A \cup B)' = A' \cap B'$ **(A.B + K.B)**

Proof

L.H.S $= (A \cup B)'$

$A \cup B = \{1,3,5,7,9\} \cup \{2,3,5,7\}$

$= \{1,2,3,5,7,9\}$

L.H.S $= (A \cup B)' = U - (A \cup B)$

$= \{1,2,3,\ldots,10\} - \{1,2,3,5,7,9\}$

$= \{4,6,8,10\} \rightarrow (\text{i})$

R.H.S $= A' \cap B'$

$= \{2,4,6,8,10\} \cap \{1,4,6,8,9,10\}$

$= \{4,6,8,10\} \rightarrow (\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A \cup B)' = A' \cap B'$

Hence Proved

Q.4 Given **(A.B + K.B)**

$U = \{1,2,3,\ldots,20\}$

$X = \{1,3,7,9,15,18,20\}$

$Y = \{1,3,5,\ldots,17\}$

To Prove

(i) $X - Y = X \cap Y'$

(ii) $Y - X = Y \cap X'$

Proof

(i) $X - Y = X \cap Y'$ **(A.B + K.B)**

L.H.S = X – Y

$= \{1,3,7,9,15,18,20\} - \{1,3,5,\ldots,17\}$

$= \{18,20\} \rightarrow (\text{i})$

$Y' = U - Y$

$= \{1,2,3,\ldots,20\} - \{1,3,5,\ldots,17\}$

$= \{2,4,6,\ldots,18,19,20\}$

R.H.S $= X \cap Y'$

$= \{1,3,7,9,15,18,20\} \cap \{2,4,6,\ldots,18,19,20\}$

$= \{18,20\} \rightarrow (\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$X - Y = X \cap Y'$

Hence Proved

(ii) $Y - X = Y \cap X'$ **(A.B + K.B)**

Proof

L.H.S = Y – X

$= \{1,3,5,\ldots,17\} - \{1,3,7,9,15,18,20\}$

$= \{5,11,13,17\} \rightarrow (\text{i})$

R.H.S $= Y \cap X'$

$X' = U - X$

$= \{1,2,3,\ldots,20\} - \{1,3,7,9,15,18,20\}$

$= \{2,4,5,6,8,10,11,12,13,14,16,17,19\}$

$Y \cap X' = \{1,3,5,\ldots,17\}$

$\cap \{2,4,5,6,8,10,11,12,13,14,16,17,19\}$

$= \{5,11,13,17\} \rightarrow (\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$Y - X = Y \cap X'$

Hence Proved

Venn Diagram **(K.B)**

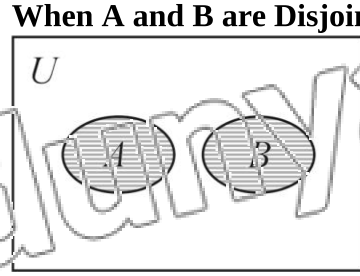
British mathematician John Venn introduced Venn Diagrams.

"A graphical method to represent sets in which Universal Set is represented by a rectangle and its subsets by a closed shaped (i.e. circle or oval) in it, is called Venn Diagram".

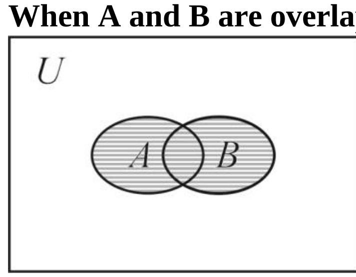
Operations on Sets Through Venn Diagrams **(K.B)**

Union of Two Sets

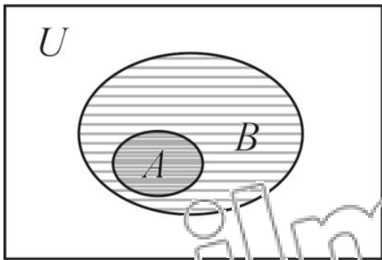
When A and B are Disjoint **(K.B)**



When A and B are overlapping **(K.B)**

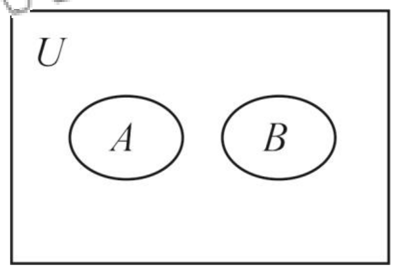


When $A \subseteq B$ **(K.B)**

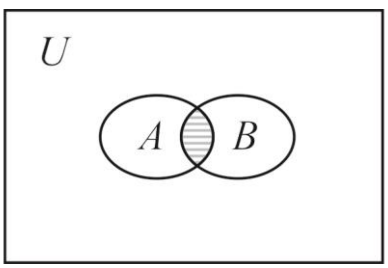


Intersection of Two Sets **(K.B)**

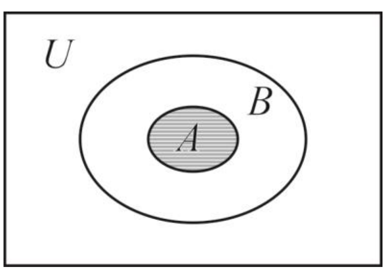
When A and B are Disjoint



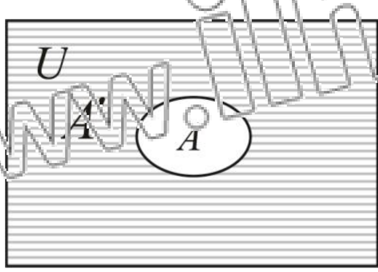
When A and B are overlapping **(K.B)**



When $A \subseteq B$



Compliment of a Set **(K.B)**



Exercise 5.3

Q.1 **Given** $U = \{1,2,3,4,...,10\}$

$A = \{1,3,5,7,9\}$ **(K.B)**

$B = \{1,4,7,10\}$ **(A.B)**

(LHR 2017, GRW 2016, FSD 2017, SWL 2017, RWP 2016, BWP 2016, D.G.K 2016)

To Prove

(i) $A - B = A \cap B'$

(ii) $B - A = B \cap A'$

(iii) $(A \cup B)' = A' \cap B'$

(iv) $(A \cap B)' = A' \cup B'$

(v) $(A - B)' = A' \cup B$

(vi) $(B - A)' = B' \cup A$

Proof

(i) $A - B = A \cap B'$

L.H.S = A – B

$= \{1,3,5,7,9\} - \{1,4,7,10\}$

$= \{3,5,9\} \rightarrow (\text{i})$

R.H.S = $A \cap B'$

$B' = U - B$

$= \{1,2,3,....,10\} - \{1,4,7,10\}$

$= \{2,3,5,6,8,9\}$

$A \cap B' = \{1,3,5,7,9\} \cap \{2,3,5,6,8,9\}$

$= \{3,5,9\} \rightarrow (\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$A - B = A \cap B'$

Hence Proved

(ii) $B - A = B \cap A'$ **(K.B + A.B)**

L.H.S = B – A

$= \{1,4,7,10\} - \{1,3,5,7,9\}$

$= \{4,10\} \rightarrow (\text{i})$

R.H.S = $B \cap A'$

$A' = U - A = \{1,2,3,...,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$B \cap A' = \{1,4,7,10\} \cap \{2,4,6,8,10\}$

$= \{4,10\} \rightarrow (\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$B - A = B \cap A'$

Hence Proved

(iii) $(A\cup B)' = A'\cap B'$ **(K.B + A.B)**

$A\cup B = \{1,3,5,7,9\}\cup\{1,4,7,10\}$

$= \{1,3,4,5,7,9,10\}$

$\text{L.H.S} = (A\cup B)' = U - (A\cup B)$

$= \{1,2,3,\ldots,10\} - \{1,3,4,5,7,9,10\}$

$= \{2,6,8\} \rightarrow (i)$

$\text{R.H.S} = A'\cap B'$

$A' = U - A = \{1,2,3\ldots,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$B' = U - B = \{1,2,3,\ldots,10\} - \{1,4,7,10\}$

$= \{2,3,5,6,8,9\}$

$\text{R.H.S} = A'\cap B'$

$= \{2,4,6,8,10\}\cap\{2,3,5,6,8,9\}$

$= \{2,6,8\} \rightarrow (ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cup B)' = A'\cap B'$

Hence Proved

(iv) $(A\cap B)' = A'\cup B'$ **(K.B + A.B)**

Proof

$A\cap B = \{1,3,5,7,9\}\cap\{1,4,7,10\}$

$= \{1,7\}$

$\text{L.H.S} = (A\cap B)' = U - (A\cap B)$

$= \{1,2,3,\ldots,10\} - \{1,7\}$

$= \{2,3,4,5,6,8,9,10\} \rightarrow (i)$

$\text{R.H.S} = A'\cup B'$

$= \{2,4,6,8,10\}\cup\{2,3,5,6,8,9\}$

$= \{2,3,4,5,6,8,9,10\} \rightarrow (ii)$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cap B)' = A'\cup B'$

Hence Proved

(v) $(A-B)' = A'\cup B$ **(K.B + A.B)**

Proof

$\text{L.H.S} = (A-B)'$

$A-B = \{1,3,5,7,9\} - \{1,4,7,10\}$

$= \{3,5,9\}$

$\text{L.H.S} = (A-B)' = U - (A-B)$

$= \{1,2,3,\ldots,10\} - \{3,5,9\}$

$= \{1,2,4,6,7,8,10\} \rightarrow (i)$

$\text{R.H.S} = A'\cup B$

$A' = U - A$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$A'\cup B = \{2,4,6,8,10\}\cup\{1,4,7,10\}$

$= \{1,2,4,6,7,8,10\} \rightarrow (ii)$

From (i) and (ii), we get

L.H.S=R.H.S

$(A-B)' = A'\cup B$

Hence Proved

(vi) $(B-A)' = B'\cup A$ **(K.B + A.B)**

(FSD 2015)

Proof

$B-A = \{1,4,7,10\} - \{1,3,5,7,9\}$

$B-A = \{4,10\}$

$\text{L.H.S} = (B-A)' = U - (B-A)$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{4,10\}$

$= \{1,2,3,5,6,7,8,9\} \rightarrow (i)$

$\text{R.H.S} = B'\cup A$

$B' = U - B$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,4,7,10\}$

$= \{2,3,5,6,8,9\}$

$B'\cup A = \{2,3,5,6,8,9\}\cup\{1,3,5,7,9\}$

$= \{1,2,3,5,6,7,8,9\} \rightarrow (ii)$

From (i) and (ii) we get

L.H.S=R.H.S

$(B-A)' = B'\cup A$

Hence Proved

Q.2 **If** $U=\{1,2,3,4,5,6,7,8,9,10\}$

$A=\{1,3,5,7,9\}$ **(K.B + A.B)**

$B=\{1,4,7,10\}$

$C=\{1,5,8,10\}$

Then Verify

(i) $(A\cup B)\cup C=A\cup(B\cup C)$

(ii) $(A\cap B)\cap C=A\cap(B\cap C)$

(iii) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(iv) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

(i) $(A\cup B)\cup C=A\cup(B\cup C)$

(K.B + A.B)

Proof

$A\cup B=\{1,3,5,7,9\}\cup\{1,4,7,10\}$

$=\{1,3,4,5,7,9,10\}$

$\text{L.H.S}=(A\cup B)\cup C$

$=\{1,3,4,5,7,9,10\}\cup\{1,5,8,10\}$

$=\{1,3,4,5,7,8,9,10\}\rightarrow(\text{i})$

$B\cup C=\{1,4,7,10\}\cup\{1,5,8,10\}$

$=\{1,4,5,7,8,10\}$

$\text{R.H.S}=A\cup(B\cup C)$

$=\{1,3,5,7,9\}\cup\{1,4,5,7,8,10\}$

$=\{1,3,4,5,7,8,9,10\}\rightarrow(\text{ii})$

From (i) and (ii), we get

L.H.S=R.H.S

$(A\cup B)\cup C=A\cup(B\cup C)$

Hence proved

(ii) $(A\cap B)\cap C=A\cap(B\cap C)$

(K.B + A.B)

Proof

$\text{L.H.S}=(A\cap B)\cap C$

$=(\{1,3,5,7,9\}\cap\{1,4,7,10\})\cap\{1,5,8,10\}$

$=\{1,7\}\cap\{1,5,8,10\}$

$=\{1\}\rightarrow(\text{i})$

$\text{R.H.S}=A\cap(B\cap C)$

$=\{1,3,5,7,9\}\cap(\{1,4,7,10\}\cap\{1,5,8,10\})$

$=\{1,3,5,7,9\}\cap\{1,10\}$

$=\{1\}\rightarrow(\text{ii})$

From (i) and (ii), we get

L.H.S=R.H.S

$(A\cap B)\cap C=A\cap(B\cap C)$

Hence proved

(iii) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

(K.B + A.B)

Proof

$\text{L.H.S}=A\cup(B\cap C)$

$=\{1,3,5,7,9\}\cup(\{1,4,7,10\}\cap\{1,5,8,10\})$

$=\{1,3,5,7,9\}\cup\{1,10\}$

$=\{1,3,5,7,9,10\}\rightarrow(\text{i})$

$A\cup B=\{1,3,5,7,9\}\cup\{1,4,7,10\}$

$=\{1,3,4,5,7,9,10\}$

$A\cup C=\{1,3,5,7,9\}\cup\{1,5,8,10\}$

$=\{1,3,5,7,8,9,10\}$

$\text{R.H.S}=(A\cup B)\cap(A\cup C)$

$=\{1,3,4,5,7,9,10\}\cap\{1,3,5,7,8,9,10\}$

$=\{1,3,5,7,9,10\}\rightarrow(\text{ii})$

From (i) and (ii), we get

L.H.S=R.H.S

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Hence Proved

(iv) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

(K.B + A.B)

Proof

$B\cup C=\{1,4,5,7,8,10\}\cup\{1,5,8,10\}$

$=\{1,4,5,7,8,10\}$

$\text{L.H.S}=A\cap(B\cup C)$

$=\{1,3,5,7,9\}\cap\{1,4,5,7,8,10\}$

$=\{1,5,7\}\rightarrow(\text{i})$

$A\cap B=\{1,3,5,7,9\}\cap\{1,4,7,10\}$

$=\{1,7\}$

$A\cap C=\{1,3,5,7,9\}\cap\{1,5,8,10\}$

$=\{1,5\}$

$\text{R.H.S}=(A\cap B)\cup(A\cap C)$

$=\{1,7\}\cup\{1,5\}$

$= \{1,5,7\} \rightarrow (\text{ii})$

From (i) and (ii), we get

L.H.S=R.H.S

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Hence Proved

Q.3 Given $U = N, \quad A = \phi, \quad B = P$ **(K.B + A.B)**

(LHR 2016, GRW 2016, RWP 2015, 17, MTN 2016)

To prove:

De-Morgan's Laws

(i) $(A\cup B)' = A'\cap B'$

(ii) $(A\cap B)' = A'\cup B'$

(i) $(A\cup B)' = A'\cap B'$

Proof

$\text{L.H.S} = (A\cup B)'$

$A\cup B = \phi\cup P$

$= P$

$\text{L.H.S} = (A\cup B)' = U-(A\cup B)$

$= N - P \rightarrow (\text{i})$

$\text{R.H.S} = A'\cap B'$

$A' = U - A = N - \phi$

$= N$

$B' = U - B = N - P$

$\text{R.H.S} = A'\cap B' = N\cap(N-P)$

$= N - P \rightarrow (\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cup B)' = A'\cap B'$

Hence Proved

(ii) $(A\cap B)' = A'\cup B'$ **(K.B + A.B)**

Proof

$A\cap B = \phi\cap P$

$= \phi$

$\text{L.H.S} = (A\cap B)'$

$= U-(A\cap B)$

$= N - \phi$

$= N$

$\text{R.H.S} = A'\cup B'$

$= N\cup(N-P)$

$= N \rightarrow (\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cap B)' = A'\cup B'$

Hence Proved

Method-(II)

(i) $(A\cup B)' = A'\cap B'$ **(K.B + A.B)**

Proof

$A\cup B = \{\ \}\cup\{2,3,5,7.....\}$

$= \{2,3,5,7,\ldots.\}$

$\text{L.H.S} = (A\cup B)' = U-(A\cup B)$

$= \{1,2,3,....\}-\{2,3,5,7,....\}$

$= \{1,4,6,8,9,10,12,.....\} \rightarrow (\text{i})$

$A' = U - A$

$= \{1,2,3,....\}-\{\ \}$

$= \{1,2,3,4.....\}$

$B' = U - B$

$= \{1,2,3,4,.....\}-\{2,3,5,7....\}$

$= \{1,4,6,8,9,10,....\}$

Now

$\text{R.H.S} = A'\cap B' = \{1,2,3,...\}\cap\{1,4,6,8,9,10,...\}$

$= \{1,4,6,8,9,10,....\} \rightarrow (\text{ii})$

From equation (i) and (ii)

L.H.S = R.H.S

$(A\cup B)' = A'\cap B'$

Hence Proved

Q.4 If $U = \{1,2,3,....,10\}$ **(K.B + A.B)**

$A = \{1,3,5,7,9\}$

$B = \{2,3,4,5,8\}$

then prove the following question by venn diagram.

(i) $A - B = A\cap B'$

(ii) $B - A = B\cap A'$

(iii) $(A\cup B)' = A'\cap B'$

(iv) $(A\cap B)' = A'\cup B'$

(v) $(A-B)' = A' \cup B$

(vi) $(B-A)' = B' \cup A$

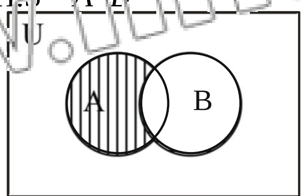
(Through Venn diagram)

(i) $A - B = A \cap B'$ **(K.B)**

Proof

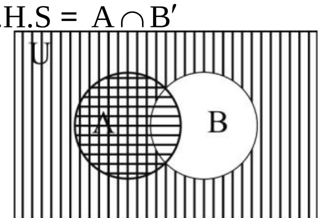
A and B are overlapping sets because $A \cap B = \{3,5\}$

L.H.S $= A - B$



B-A |||

R.H.S = $A \cap B'$



A ≡, #

B' |||, #

$A \cap B'$ #

Same regions represent that

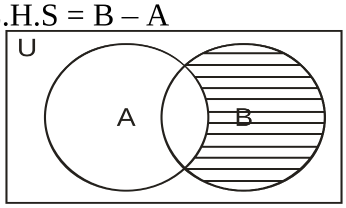
L.H.S = R.H.S

$A - B = A \cap B'$

Hence Proved

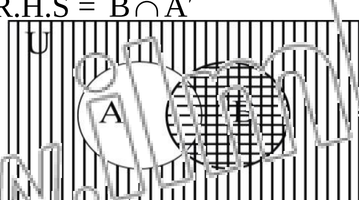
(ii) $B - A = B \cap A'$ **(K.B)**

L.H.S = B – A



B-A ≡

R.H.S = $B \cap A'$



B ≡, #

A' |||, #

$B \cap A'$ #

Same regions represent that

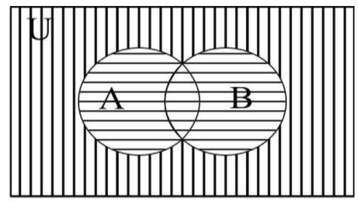
L.H.S = R.H.S

$B - A = B \cap A'$

Hence Proved

(iii) $(A \cup B)' = A' \cap B'$ **(K.B)**

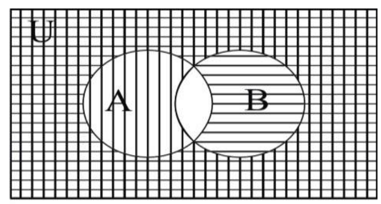
L.H.S = $(A \cup B)'$



$A \cup B$ ≡

$(A \cup B)'$ |||

R.H.S = $A' \cap B'$



A' ≡, #

B' |||, #

$A' \cap B'$ #

Same regions represent that

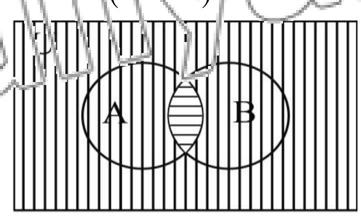
L.H.S = R.H.S

$(A \cup B)' = A' \cap B'$

Hence Proved

(iv) $(A \cap B)' = A' \cup B'$ **(K.B)**

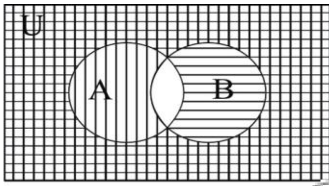
L.H.S = $(A \cap B)'$



$A \cap B$ ≡

$(A \cap B)'$ |||

R.H.S = $A' \cup B'$

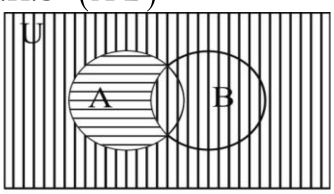


$A' \equiv, \#$
$B' \ |||, \#$
$A' \cup B' \equiv, |||, \#$

Same regions represent that
L.H.S = R.H.S
$(A \cap B)' = A' \cup B'$
Hence Proved

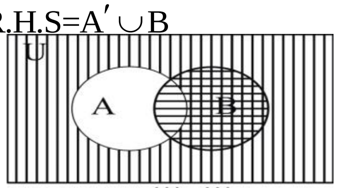
(v) $(A - B)' = A' \cup B$ **(K.B)**

L.H.S=$(A\text{-}B)'$



$A - B \ \equiv$
$(A - B)' \ |||$

R.H.S=$A' \cup B$

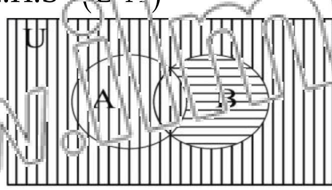


$A' \ |||, \#$
$B \ \equiv, \#$
$A' \cup B \equiv, |||, \#$

Same regions represent that
L.H.S = R.H.S
$(A - B)' = A' \cup B$
Hence Proved

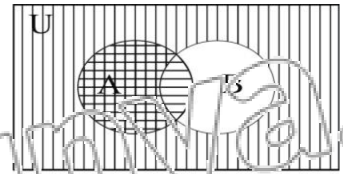
(vi) $(B - A)' = B' \cup A$ **(K.B)**

L.H.S=$(B\text{-}A)'$



$B - A \equiv$
$(B - A)' \ |||$

R.H.S=$B' \cup A$



$A \ \equiv, \#$
$B' \ |||, \#$
$B' \cup A \equiv, |||, \#$

Same regions represent that
L.H.S = R.H.S
$(B - A)' = B' \cup A$
Hence Proved

Ordered Pairs **(K.B)**

Any two numbers x and y, written in the form (x, y) is called an ordered pair. In an ordered pair (x, y), the order of numbers is important.
For example: $(3,2)$ is different from $(2,3)$.
Hence $(x, y) \neq (y, x)$ unless $x = y$.

Cartesian Product **(K.B)**
(FSD 2015, D.G.K 2015, 17)

Cartesian product of two non-empty sets A and B denoted by $A \times B$ consists of all the ordered pairs (x, y) such that $x \in A$ and $y \in B$
i.e $A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$
For example:
If $A = \{1, 2\}, B = \{3, 4\}$
Then $A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

Exercise 5.4

Q.1 Given $A = \{a, b\}$ **(GRW 2014)** **(A.B)**
$B = \{c, d\}$ **(RWP 2015)**

To Find
(i) $A \times B$
(ii) $B \times A$

Solution:
(i) $A \times B = \{a,b\} \times \{c,d\}$
$= \{(a,c),(a,d),(b,c),(b,d)\}$
(ii) $B \times A = \{c,d\} \times \{a,b\}$
$= \{(c,a),(c,b),(d,a),(d,b)\}$

Q.2 **Given** A = $\{0,2,4\}$ **(A.B)**

B = $\{-1,3\}$

(FSD 2015, SWL 2017, BWP 2015)

To Find

$A\times B \quad B\times A \quad A\times A \quad B\times B$

Solution:

(i) $A\times B = \{0,2,4\}\times\{-1,3\}$

$=\{(0,-1),(0,3),(2,-1),(2,3),(4,-1),(4,3)\}$

(ii) $B\times A = \{-1,3\}\times\{0,2,4\}$

$=\{(-1,0),(-1,2),(-1,4),(3,0),(3,2),(3,4)\}$

(iii) $A\times A = \{0,2,4\}\times\{0,2,4\}$

$=\{(0,0),(0,2),(0,4),(2,0),(2,2),(2,4),(4,0),(4,2),(4,4)\}$

(iv) $B\times B = \{-1,3\}\times\{-1,3\}$

$=\{(-1,-1),(-1,3),(3,-1),(3,3)\}$

Q.3 **(A.B)**

(i) **Given** $(a-4,\ b-2)=(2,1)$

(GRW 2016, 17, FSD 2017, SWL 2015, SGD 2017, MTN 2016)

Required

Values of *a* and *b*

Solution:

Given that

$(a-4, b-2)=(2,1)$

By comparing, we get

a – 4 = 2 and b – 2 = 1

a = 2 + 4 b = 1 + 2

$\Rightarrow a = 6$, b = 3

(ii) **Given** $(2a+5,3)=(7,b-4)$ **(A.B)**

(SWL 2017, MTN 2017, RWP 2016, D.G.K 2015)

Required

Values of *a* and *b*

Solution:

Given that

$(2a+5,3)=(7,b-4)$

By comparing, we get

2a + 5 = 7 and 3 = b – 4

2a = 7 – 5 3 + 4 = b

2a = 2 7 = b

a = 1 b = 7

(iii) **Given** $(3-2a,b-1)=(a-7,2b+5)$

(GRW 2014) **(A.B)**

Required

Values of *a* and *b* =?

Solution:

Given that

$(3-2a,b-1)=(a-7,2b+5)$

By comparing, we get

$3-2a=a-7$ and $b-1=2b+5$

$-2a-a=-7-3$ $b-2b=5+1$

$-3a=-10$ $-b=6$

$a=\frac{-10}{-3}$ $b=-6$

$\Rightarrow a=\frac{10}{3}$

Q.4 **Given**

$X\times Y=\{(a,a),(b,a),(c,a),(d,a)\}$

(A.B)

Required

Set *X* and *Y*

Solution:

Given that

$X\times Y=\{(a,a),(b,a),(c,a),(d,a)\}$

$X=\{a,b,c,d\}$

$Y=\{a\}$

Q.5 **Given** $X=\{a,b,c\}$ **(A.B)**

$Y=\{d,e\}$

Required

Number of elements in

(i) X × Y

(ii) Y × X

(iii) X × X

Solution:

(i) $X\times Y$

$n(X)=3$

$n(Y)=2$

$n(X\times Y)=n(X)\times n(Y)$

$=3\times 2$

$=6$

Method-2 **(K.B)**

(i) Number of elements in $X\times Y = m\times n$

$=3\times 2$

$=6$

(ii) Number of elements in $Y\times X = m\times n$

$=2\times 3$

$= 6$

(iii) Number of elements in $X \times X = m \times n$
$= 3 \times 3$
$= 9$

Binary Relation (K.B)

(GRW 2017, RWP 2016, SWL 2016, MTN 2015)

If A and B are any two non-empty sets then a subset $R \subseteq A \times B$ is called a binary relation from set A into set B.

i.e $A = \{1, 2\}, B = \{a\}$

Then $A \times B = \{(1, a), (2, a)\}$ and

$R_1 = \{(1, a)\}$, $R_2 = \{(2, a)\}$

$R_3 = \{(1, a), (2, a)\}$, $R_4 = \phi$

Are all possible relations.

Note (K.B + U.B)

Formula to find number of binary relations is $2^{m \times n}$ where $m =$ number of elements in set A and $n =$ number of elements in set B.

Domain of Relation (K.B)

Domain of a relation denoted by Dom(R) is a set consisting of all the first elements of each ordered pair in the relation.

For example:

If $R = \{(0, 2), (2, 3), (3, 3), (3, 4)\}$

Then $Dom(R) = \{0, 2, 3\}$

Range of a Relation (K.B)

Range of a relation is a set containing all the second elements of each ordered pair of the relation.

For Example:

If $R = \{(0, 2), (2, 3), (3, 3), (3, 4)\}$

Then Range (R) = $\{2, 3, 4\}$

Function (K.B + U.B + A.B)

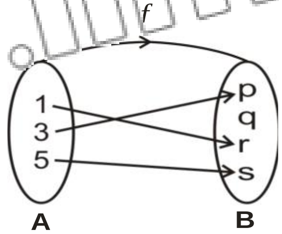
(LHR 2014)

Suppose A and B are two non-empty sets, then relation $f : A \to B$ is called a function if

(i) Dom $(f) = A$

(ii) Every $x \in A$ appears in one and only one ordered pair in f.

Example:



Types of Function (K.B + U.B)

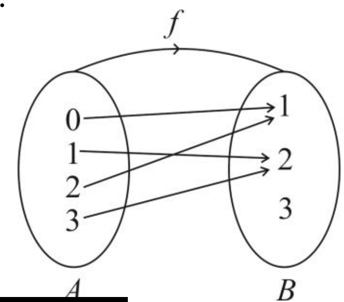
Into Function (A.B)

A function $f : A \to B$ is called an into function, if at least one element in set B is not an image of some element of set A i.e., Range of $f \subset B$.

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3\}$, then $f : A \to B$ such that

$f = \{(0, 1), (1, 2), (2, 1), (3, 2)\}$.

$\because$ Range$(f) = \{1, 2\} \subset B$. Thus f is an into function.



Onto Function (K.B + U.B + A.B)

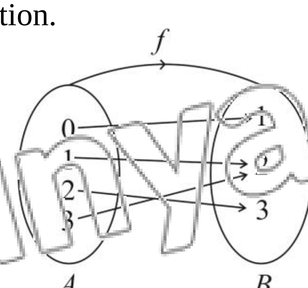
(SWL 2016, BWP 2014, 16, 17)

A function $f : A \to B$ is called an onto function, if every element of set B is an image of at least one element of set A i.e., Range of $f = B$.

For example, if $A = \{0, 1, 2, 3\}$ and $B = \{1, 2, 3\}$, then $f : A \to B$ such that

$f = \{(0, 1), (1, 2), (2, 3), (3, 2)\}$.

$\because$ Range $(f) = \{1, 2, 3\} = B$. Thus f is an onto function.



One-to-One Function

(LHR 2015) **(K.B + U.B + A.B)**

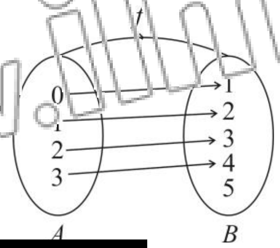
A function $f : A \to B$ is called one-one function if all distinct elements of A have distinct images in B, i.e., $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \in A$

or $x_1 \neq x_2 \in A \; \forall x_1 \in A \Rightarrow f(x_1) \neq f(x_2)$

For example, if $A=\{0,1,2,3\}$ and $B=\{1,2,3,4,5\}$, then we define a function $f:A\rightarrow B$ such that

$f=\{(0,1),(1,2),(2,3),(3,4)\}$

f is one-one function because no element in B is repeated.



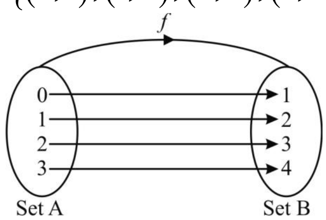
Bijective Function

(GRW 2014) **(K.B + U.B + A.B)**

A function $f:A\rightarrow B$ is called bijective function iff function f is one-one and onto.

For example, if $A=\{0,1,2,3,\}$ and $B=\{1,2,3,4\}$

Then $f=\{(0,1),(1,2),(2,3),(3,4)\}$



Note **(K.B)**

- Every function is ration but converse may not be true.
- Every function may not be one-one.
- Every function may not be onto.

Exercise 5.5

Q.1 Given (LHR 2014) **(A.B)**

$L=\{a,b,c\}$

$M=\{3,4\}$

To Find

Two binary relations of $L\times M$ and $M\times L$

Solution:

$L\times M=\{a,b,c\}\times\{3,4\}$

$=\{(a,3),(a,4),(b,3),(b,4),(c,3),(c,4)\}$

Two binary relations of $L\times M$ are:

$R_1=\{(a,3),(a,4)\}$

$R_2=\{(a,3),(b,3),(c,4)\}$

Now $M\times L=\{3,4\}\times\{a,b,c\}$

$=\{(3,a),(3,b),(3,c),(4,a),(4,b),(4,c)\}$

Two binary relations of $M\times L$ are:

$R_1=\{(3,a)\}$

$R_2=\{(3,a),(4,b)\}$

Q.2 Given **(A.B)**

(LHR 2015, GRW 2014)

$Y=\{-2,1,2\}$

Required:

Two binary relations of $Y\times Y$ and also find their domain and range.

Solution:

$Y\times Y=\{-2,1,2\}\times\{-2,1,2\}$

$=\{(-2,-2),(-2,1),(-2,2),(1,-2),$
$(1,1),(1,2),(2,-2),(2,1),(2,2)\}$

Two Binary relations for $Y\times Y$ are:

$R_1=\{(-2,-2),(-2,1),(-2,2)\}$

$R_2=\{(-2,-2),(1,1)\}$

Domain and Range

Dom $(R_1)=\{-2\}$

Range $(R_1)=\{-2,1,2\}$

Dom $(R_2)=\{-2,1\}$

Range $(R_2)=\{-2,1\}$

Q.3 Given **(A.B)**

$L=\{a,b,c\}$

$M=\{d,e,f,g\}$

Required

Two binary relation for

(i) $L\times L$ (ii) $L\times M$ (iii) $M\times M$

Solution:

$L\times L=\{a,b,c\}\times\{a,b,c\}$

$=\{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\}$

$L\times M=\{a,b,c\}\times\{d,e,f,g\}$

$=\{(a,d),(a,e),(a,f),(a,g),(b,d),(b,e),$
$(b,f),(b,g),(c,d),(c,e),(c,f),(c,g)\}$

$M\times M=\{d,e,f,g\}\times\{d,e,f,g\}$

$= \{(d,d),(d,e),(d,f),(d,g),(e,d),(e,e),(e,f),(e,g),$
$(f,d),(f,e),(f,f),(f,g),(g,d),(g,e),(g,f),(g,g)\}$

Two binary relations for $L \times L$ are:

$R_1 = \{(a,a),(b,b),(c,c)\}$

$R_2 = \{(a,b)\}$

Two Binary Relations for $L \times M$ are:

$R_1 = \{(a,d),(a,e)\}$

$R_2 = \{(a,d),(b,e),(c,f)\}$

Two Binary Relations for $M \times M$ are:

$R_1 = \{(d,d)\}$

$R_2 = \{(d,e),(f,g)\}$

Q.4 If set M has 5 elements, then find the number of binary relations in M. **(A.B)**

Solution:

No of binary relations in $M = 2^{mxn}$
$= 2^{5\times5}$
$= 2^{25}$

Q.5 Given $L = \{x \mid x \in N \wedge x \le 5\}$
$M = \{y \mid y \in P \wedge y < 10\}$ **(A.B + K.B)**

To Find

(i) $R_1 = \{(x,y) \mid y < x\}$

(ii) $R_2 = \{(x,y) \mid y = x\}$

(iii) $R_3 = \{(x,y) \mid x + y = 6\}$

(iv) $R_4 = \{(x,y) \mid y - x = 2\}$

Also find the domain and range of each relation.

Solution:

Here

$L = \{x \mid x \in N \wedge x \le 5\} = \{1,2,3,4,5\}$

And $M = \{y \mid y \in P \wedge y < 10\} = \{2,3,5,7\}$

L x M $= \{1,2,3,4,5\}$ x $\{2,3,5,7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),$
$(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),$
$(4,5),(4,7),(5,2),(5,3),(5,5),(5,7)\}$

(i) Relation **(A.B + K.B)**

$R_1 = \{(x,y) \mid y < x\}$
$= \{(3,2),(4,2),(4,3),(5,2),(5,3)\}$

Domain and Range

Dom (R_1) $= \{3,4,5\}$

Range (R_1) $= \{2,3\}$

(ii) Relation **(A.B + K.B)**

$R_2 = \{(x,y) \mid y = x\}$
$= \{(2,2),(3,3),(5,5)\}$

Domain and Range

Dom (R_2) $= \{2,3,5\}$

Range (R_2) $= \{2,3,5\}$

(iii) Relation **(A.B + K.B)**

$R_3 = \{(x,y) \mid x + y = 6\}$
$= \{(1,5),(3,3),(4,2)\}$

Domain and Range of R_3

Dom (R_3) $= \{1,3,4\}$

Range (R_3) $= \{2,3,5\}$

(iv) Relation **(A.B + K.B)**

$R_4 = \{(x,y) \mid y - x = 2\}$
$= \{(1,3),(3,5),(5,7)\}$

Domain and Range of R_4

Dom (R_4) $= \{1,3,5\}$

Range (R_4) $= \{3,5,7\}$

Q.6 Indicate relations, into function, one-one function, onto function and bijective function from the following. Also find their domain and the range. (In each part $(i) \rightarrow (vi)$ the co-domain set is equal to the range set) **(A.B + K.B + U.B)**

(i) $R_1 = \{(1,1),(2,2),(3,3),(4,4)\}$

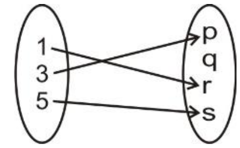
(ii) $R_2 = \{(1,2),(2,1),(3,4),(3,5)\}$

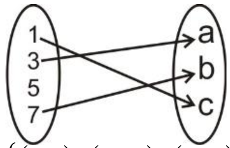
(iii) $R_3 = \{(b,a),(c,a),(d,a)\}$

(iv) $R_4 = \{(1,1),(2,3),(3,4),(4,3),(5,4)\}$

(v) $R_5 = \{(a,b),(b,a),(c,d),(d,e)\}$

(vi) $R_6 = \{(1,2),(2,3),(1,3),(3,4)\}$

(vii) $R_7 =$ 

(viii) R_8 = 

Solution:

(i) $\mathbf{R_1} = \{(1,1),(2,2),(3,3),(4,4)\}$

- Dom (R_1) $= \{1,2,3,4\}$

  There is no repetition in 1st element of any two ordered pairs, so R_1 is a function.

- Range (R_1) $= \{1,2,3,4\}$

  Also there is no repetition in 2nd element of any two ordered pairs. So R_1 is bijective function.

(ii) $\mathbf{R_2} = \{(1,2),(2,1),(3,4),(3,5)\}$

- Dom (R_2) $= \{1,2,3\}$

  There is a repetition in first element of last two ordered pairs. So R_2 is not a function.

(iii) $\mathbf{R_3} = \{(b,a),(c,a),(d,a)\}$

- Dom (R_3) $= \{b,c,d\}$

  There is no repetition in 1st element of any two pairs, so R_3 is a function.

- Range (R_3) $= \{a\}$

  There is a repetition in second element of all ordered pairs. So R_3 is on to function.

(iv) $\mathbf{R_4} = \{(1,1),(2,3),(3,4),(4,3),(5,4)\}$

- Dom (R_4) $= \{1,2,3,4,5\}$

  There is no repetition in 1st element of any two pairs, so R_4 is a function.

- Range (R_4) $= \{1,3,4\}$

  There is a repetition in second element of all ordered pairs. So R_4 is on to function.

(v) $\mathbf{R_5} = \{(a,b),(b,a),(c,d),(d,e)\}$

- Dom (R_5) $= \{a,b,c,d\}$

  There is no repetition in 1st element of any two ordered pairs, so R_5 is a function.

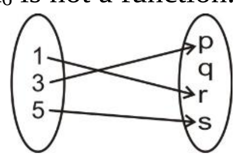
- Range (R_5) $= \{a,b,d,e\}$

  Also there is no repetition in 2nd element of any two ordered pairs. So R_5 is bijective function.

(vi) $\mathbf{R_6} = \{(1,2),(2,3),(1,3),(3,4)\}$

- Dom (R_6) $= \{1,2,3\}$

  There is a repetition in first element of two ordered pairs. So R_6 is not a function.

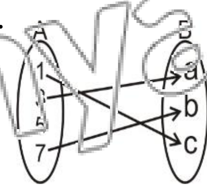
(vii) $\mathbf{R_7}$= 

- Dom (R_7) $= \{1,3,5\} = A$

  Also there is no repetition in 1st element of any two ordered pairs. So R_7 is a function.

- Range (R_7) $= \{p,r,s\} \subset B$

  Also there is no repetition in 2nd element of any two ordered pairs. So R_7 is one-to-one (injective) function.

(viii) $\mathbf{R_8}$ = 

- Dom (R_8) $= \{1,3,7\} \neq A$

  Therefore R_8 is not a function.

Miscellaneous Exercise 5

Q.1 Multiple choice questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(1) A collection of well-defined distinct objects is called; **(K.B)**
(a) Subset (b) Power set
(c) Set (d) None of these

(2) A set $Q = \left\{\frac{a}{b} \mid a, b \in Z \wedge b \neq 0\right\}$ **is called a set of;** **(K.B)**
(a) Whole numbers (b) Natural numbers
(c) Irrational numbers (d) Rational numbers

(3) The different number of ways to describe a set are; **(K.B)**
(a) 1 (b) 2
(c) 3 (d) 4

(4) A set with no element is called; **(K.B)**
(a) Subset (b) Empty set
(c) Singleton set (d) Super set

(5) The set $\{x \mid x \in W \wedge x \leq 101\}$ **is;** **(K.B)**
(a) Infinite set (b) Subset
(c) Null set (d) Finite set

(6) The set having only one element is called; **(K.B)**
(a) Null set (b) Power set
(c) Singleton set (d) Subset

(7) Power set of an empty set is; **(K.B)**
(a) ϕ (b) $\{a\}$
(c) $\{\phi, \{a\}\}$ (d) $\{\phi\}$

(8) The number of elements in power set $\{1, 2, 3\}$ **is;** **(K.B)**
(a) 4 (b) 6
(c) 8 (d) 9

(9) If $A \subseteq B$**, then** $A \cup B$ **is equal to;** **(K.B)**
(a) A (b) B
(c) ϕ (d) None of these

(10) If $A \subseteq B$ **and** $A \cap B$ **is equal to;** **(K.B)**
(a) A (b) B
(c) ϕ (d) None of these

(11) If $A \subseteq B$ **and** $A - B$ **is equal to;** **(K.B)**
(a) A (b) B
(c) ϕ (d) B-A

(12) $(A \cup B) \cup C$ **is equal to;** **(K.B)**
(a) $A \cap (B \cup C)$ (b) $(A \cup B) \cap C$
(c) $A \cup (B \cup C)$ (d) $A \cap (B \cap C)$

(13) $A\cup(B\cap C)$ **is equal to;** **(K.B)**

(a) $(A\cup B)\cap(A\cup B)$ **(b)** $(A\cup B)\cap C$

(c) $(A\cap B)\cup(A\cap C)$ **(d)** $A\cap(B\cap C)$

(14) **If** A **and** B **are disjoint sets, then** $A\cup B$ **is equal to;** **(K.B)**

(a) A **(b)** B

(c) ϕ **(d)** $B\cup A$

(15) **If number of elements in set** A **is** 3 **and in set** B **is** 4**, then number of elements in** $A\times B$ **is;** **(K.B)**

(a) 3 **(b)** 4

(c) 12 **(d)** 7

(16) **If number of elements in set** A **is** 3 **and in set** B **is** 2**, then number of binary relations in** $A\times B$ **is;** **(K.B)**

(a) 2^3 **(b)** 2^6

(c) 2^8 **(d)** 2^2

(17) **The domain of** $R=\{(0,2),(2,3),(3,3),(3,4)\}$ **is;** **(K.B)**

(a) $\{0,3,4\}$ **(b)** $\{0,2,3\}$

(c) $\{0,2,4\}$ **(d)** $\{2,3,4\}$

(18) **The range of** $R=\{(1,3),(2,2),(3,1),(4,4)\}$ **is;** **(K.B)**

(a) $\{1,2,4\}$ **(b)** $\{3,2,4\}$

(c) $\{1,2,3,4\}$ **(d)** $\{1,3,4\}$

(19) **point** $(-1,4)$ **lies in the quadrant;** **(K.B)**

(a) I **(b)** II

(c) III **(d)** IV

(20) **the relation** $\{(1,2),(2,3),(3,3),(3,4)\}$ **is;** **(K.B)**

(a) Onto function **(b)** Into function

(c) Not a function **(d)** One-One function

ANSWER KEY

i	c	vi	c	xi	c	xvi	b
ii	d	vii	d	xii	c	xvii	b
iii	c	viii	c	xiii	a	xviii	c
iv	b	ix	b	xiv	d	xix	b
v	d	x	a	xv	c	xx	c

Q.2 Write short answers of the following questions.

(i) Define a subset and give one example. **(K.B + U.B)**

Answer

Subset

If A and B are two sets such that every element of set A is an element of set B, then set A is called subset of set B.

For example $A=\{1,3,5,7\}$ and $B=\{1,2,3,...,10\}$ then A is subset of B and represented by $A\subseteq B$.

(ii) Write all the subsets of the set {a, b}

Answer

Let $S=\{a,b\}$

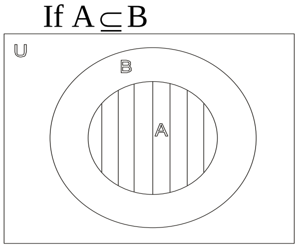
All possible subset of set S are:

$\phi,\{a\},\{b\},\{a,b\}$

(iii) Show $A\cap B$ by Venn diagram. When $A\subseteq B$. **(MTN 2014, SGD 2016)** **(K.B)**

Answer

If $A\subseteq B$

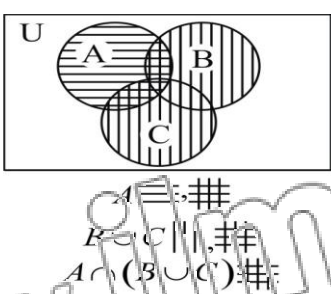


$A\cap B$ ⊞

(iv) Show by Venn diagram $A\cap(B\cup C)$ **(K.B + A.B)**

Answer

Let A, B and C are overlapping (General Case)



(v) Define intersection of two sets **(K.B)**

Answer

Intersection of Two Sets **(K.B)**

The intersection of two sets A and B, written as $A\cap B$ (read as 'A intersection B') is the set consisting of all the common elements of A and B. Thus

$A\cap B=\{x\mid x\in A \text{ and } x\in B\}$.

Clearly $x\in A\cap B\Rightarrow x\in A$ and $x\in B$

For example, if $A=\{a,b,c,d\}$ and $B=\{c,d,e,f\}$, then $A\cap B=\{c,d\}$

(vi) Define a function

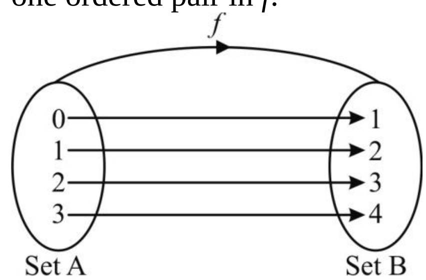
Answer

Function **(K.B)**

Suppose A and B are two non-empty sets, then relation $f:A\to B$ is called a function if

(i) Dom $f=A$

(ii) Every $x\in A$ appears in one and only one ordered pair in f.



(vii) Define one-one function

Answer

One – One Function **(K.B)**

A function $f:A\to B$ is called one-one function if all distinct elements of A have distinct images in B, i.e., $f(x_1)=f(x_2)$ $\Rightarrow x_1=x_2\in A$ or $x_1\neq x_2\in A$ $\forall x_1\in A$ $\Rightarrow f(x_1)\neq f(x_2)$.

For example, if $A=\{0,1,2,3\}$ and $B=\{1,2,3,4,5\}$, then we define a function $f:A\to B$ such that

$f=\{(0,1),(1,2),(2,3),(3,4)\}$

f is one-one function because no element in B is repeated.

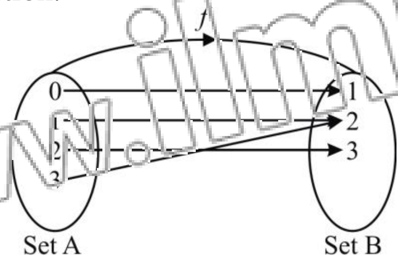
(viii) Define an Onto function or Surjective function.

Answer

Onto (Surjective) Function **(K.B)**

A function $f:A\to B$ is called an onto function, if every element of set B is an image of at least one element of set A i.e., Range of $f=B$.

For example, if $A=\{0,1,2,3\}$ and $B=\{1,2,3\}$, then $f:A\rightarrow B$ such that $f=\{(0,1),(1,2),(2,3),(3,2)\}$. Here Range $f=\{1,2,3\}=B$. Thus f so defined is an onto function.



(ix) Define a Bijective function

Answer

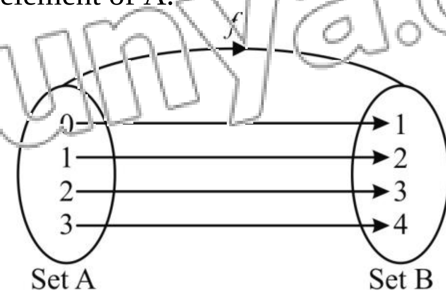
Bijective Function **(K.B)**

A function $f:A\rightarrow B$ is called bijective function iff function f is one-one and onto.

For example, if $A=\{0,1,2,3\}$ and $B=\{1,2,3,4\}$

Then $f=\{(0,1),(1,2),(2,3),(3,4)\}$

Evidently this function is one-one because distinct elements of A have distinct images in B. This is an onto function also because every element of B is the image of at least one element of A.



(x) Write De Morgan's Laws. **(K.B)**

Answer

De Morgan's Laws

For any two sets A and B belonging to universal set U,

(i) $(A\cap B)'=A'\cup B'$

(ii) $(A\cup B)'=A'\cap B'$ are called De Morgan's laws.

Q.3 Fill in the blanks

(i) If $A\subseteq B$, then $A\cup B=$ ______. **(K.B + A.B)**

(ii) If $A\cap B=\phi$ then A and B are ____. **(K.B + A.B)**

(iii) If $A\subseteq B$ and $B\subseteq A$ then _____. **(K.B + A.B)**

(iv) $A\cap(B\cup C)=$ _______. **(K.B + A.B)**

(v) $A\cup(B\cap C)=$ ______. **(K.B + A.B)**

(vi) The complement of U is _______. **(K.B + A.B)**

(vii) The complement of ϕ is _______. **(K.B + A.B)**

(viii) $A\cap A=$ ___________. **(K.B + A.B)**

(ix) $A\cup A^c=$ ___________. **(K.B + A.B)**

(x) The set $\{x\mid x\in A$ and $x\notin B\}=$ _____. **(K.B + A.B)**

(xi) The point $(-5,-7)$ lies in ______ quadrant. **(K.B + A.B)**

(xii) The point $(4,-6)$ lies in _______ quadrant. **(K.B + A.B)**

(xiii) The y co-ordinate of every point is ______ on-*x-axis*. **(K.B + A.B)**

(xiv) The x co-ordinate of every point is _____ on-*y-axis*. **(K.B + A.B)**

(xv) The domain of $\{(a,b),(b,c),(c,d)\}$ is ______. **(K.B + A.B)**

(xvi) The range of $\{(a,a),(b,b),(c,c)\}$ is _____. **(K.B + A.B)**

(xvii) Venn-diagram was first used by ___. **(K.B + A.B)**

(xviii) A subset of $A \times A$ is called the ____________ in *A*. **(K.B + A.B)**

(xix) If $f : A \to B$ and range of $f = B$, then *f* is an ___________ function. **(K.B + A.B)**

(xx) The relation $\{(a,b),(b,c),(a,d)\}$ is __________a function. **(K.B + A.B)**

ANSWER KEY

(i) B
(ii) Disjoint sets
(iii) $A = B$
(iv) $(A \cap B) \cup (A \cap C)$
(v) $(A \cup B) \cap (A \cup C)$
(vi) ϕ
(vii) U
(viii) ϕ
(ix) U
(x) $A - B$
(xi) IIIrd quadrant
(xii) IVth quadrant
(xiii) Zero
(xiv) Zero
(xv) $\{a,b,c\}$
(xvi) $\{a,b,c\}$
(xvii) John Venn
(xviii) Binary relation
(xix) Onto
(xx) Not

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 Power set of an empty set is:

(A) ϕ **(B)** $\{a\}$

(C) $\{0,\{a\}\}$ **(D)** $\{\phi\}$

2 $(A\cup B)\cup C$ **is equal to:**

(A) $A\cap(B\cup C)$ **(B)** $(A\cup B)\cap C$

(C) $A\cup(B\cup C)$ **(D)** $A\cap(B\cup C)$

3 The number of ways to describe a set are:

(A) 1 **(B)** 2

(C) 3 **(D)** 4

4 If $A\subseteq B$**, then** $A\cup B$ **is equal to:**

(A) A **(B)** B

(C) ϕ **(D)** U

5 The domain of $R=\{(0,2),(2,3),(3,3),(3,4)\}$ **is:**

(A) $\{0,3,4\}$ **(B)** $\{0,2,3\}$

(C) $\{0,2,4\}$ **(D)** $\{2,3,4\}$

6 Point $(-1,4)$ **lies in the quadrant:**

(A) I **(B)** II

(C) III **(D)** IV

7 If the number of elements in set A **is 3 and in set** B **is 4, then the number of elements in** $A\times B$ **is:**

(A) 3 **(B)** 4

(C) 12 **(D)** 7

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Define bijective function.

(ii) Show $A\cap B$ by Venn diagram when $\boldsymbol{A\subseteq B}$

(iii) Find "a" and "b" if $(3-2a,b-1)=(a-7,2b+5)$.

(iv) If $P=\{-2,-1\}$ then make two binary relations for $P\times P$.

(v) If $U=\{x\,|\,x\in N\wedge 3<x\le 25\}$, $X=\{x\,|\,x \text{ is prime}\wedge 8<x<25\}$ and $Y=\{x\,|\,x\in W\wedge 4\le x\le 17\}$. Find the $(X\cap Y)'$.

Q.3 Answer the following Questions. **(4+4=8)**

(a) If $U=\{1,2,3,...,10\}$, $A=\{1,3,5,7,9\}$ and $B=\{2,3,4,5,8\}$, then prove that $(A-B)'=A'\cup B$.

(b) If $L=\{x\,|\,x\in N\wedge x\le 5\}$, $M=\{y\,|\,y\in P\wedge y<10\}$, then make the following relations from L to M. $R=\{(x,y)\,|\,x+y=6\}$ Also write the domain and range of the relation.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.