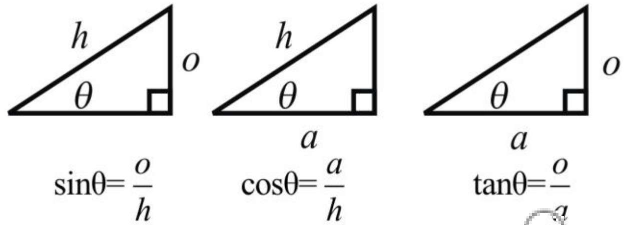
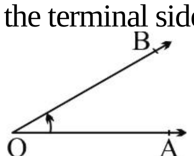


UNIT 7
INTRODUCTION TO TRIGONOMETRY

Angle **(K.B)**
(LHR 2015, SWL 2015, MTN 2016)

Two non-collinear rays with a common end point form an angle. The rays are called arms of the angle and the common point is called vertex of the angle. The original position of the ray is called initial side and the final position of the ray is called the terminal side of the angle.



Sexagesimal System
(D.G.K 2015) **(K.B)**

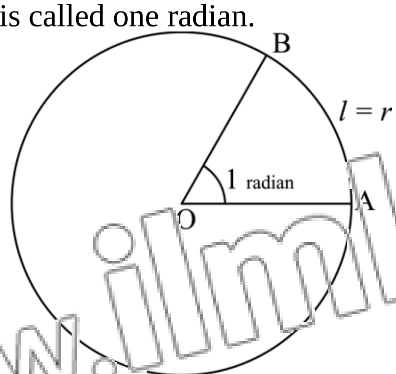
System of measurement of an angle in degrees, minutes and seconds is called sexagesimal system.

Degree
(RWP 2015, BWP 2015, SGD 2015) **(K.B)**

If we divide a circle/ circumference into 360 equal parts/arcs then central angle of one arc is called degree it is denoted by 1°.

Radians **(GRW 2014, FSD 2015)** **(K.B)**

The angle subtended at the centre of the circle by an arc, whose length is equal to its radius is called one radian.



$m\angle AOB = 1$ radians if $m\overline{OA} = mAB$

Circular System **(K.B)**

System of measurement of an angle in radians is called circular system.

Relationship between Radians and Degrees
(K.B)

$180° = \pi$ radians

$\Rightarrow 1° = \dfrac{\pi}{180}$ radians

$\Rightarrow x° = \dfrac{x\pi}{180}$ radians

And

$\Rightarrow$ 1 radian $= \dfrac{180°}{\pi}$

$\Rightarrow x$ radians $= \dfrac{x(180°)}{\pi}$

Note **(K.B)**

- 1 radians $= \dfrac{180°}{\pi} \approx 57.295795° \approx 57°17'45''$
- $1° = \dfrac{\pi}{180}$ radians ≈ 0.0175 radians

Important Formulae **(K.B)**

$x° = \dfrac{x\pi}{180}$ radians

x radians $= \dfrac{x(180°)}{\pi}$

$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$

Example 2 (Page # 147) **(K.B)**

Convert 12°23′35″ to decimal degrees correct to three decimal places.

Solution:

$$12°23'35'' = \left(12 + \frac{23}{60} + \frac{35}{3600}\right)^o$$
$$\approx (12 + 0.3833 + 0.00972)^o$$
$$\approx 12.393^o$$

Example 3 (Page # 148) **(A.B)**

Convert $45.36°$ to $D°M'S''$ form.

Solution:

$$45.36° = 45° + .36 \times 60'$$
$$= 45^{O} + 21.6'$$
$$= 45^{O} + 21' + .6 \times 60''$$
$$= 45^{O} + 21' + 36'' = 45^{O}21'36''$$

Example 4 (Page # 149) **(A.B)**

Convert $124°22'$ into radian measure.

Solution:

$$124°22' = \left(124 + \frac{22}{60}\right)^{O} = 124.3666°$$
$$= 124.3666 \times \frac{\pi}{180} \text{ radians}$$
$$\approx 2.171 \text{ radians}$$

Example 5 (Page # 149) **(A.B)**

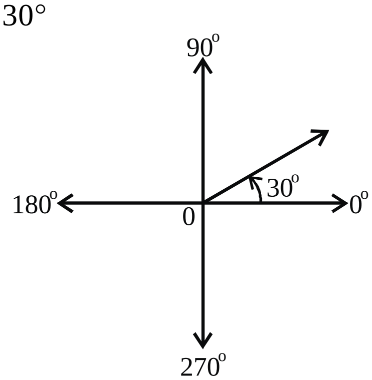
Express $\frac{2\pi}{3}$ radians into degrees.

$$\frac{2\pi}{3}\text{radians} = \frac{2\pi}{3} \times \frac{180^{O}}{\pi} \quad \because 1\text{rad} = \left(\frac{180}{\pi}\right)^{o}$$
$$= 120^{O}$$

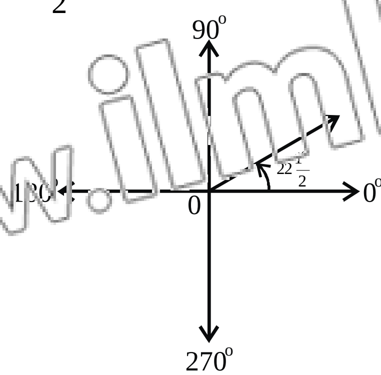
Exercise 7.1

Q.1 Locate the following angles:

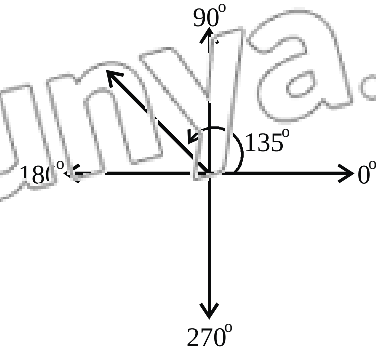
(i) $30°$ **(A.B)**



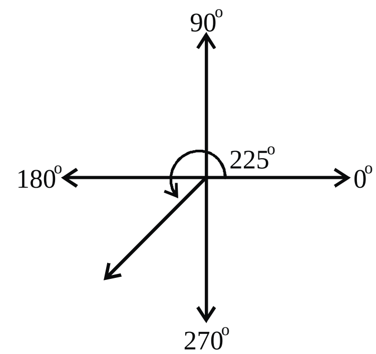
(ii) $22\frac{1^{o}}{2}$ **(A.B)**



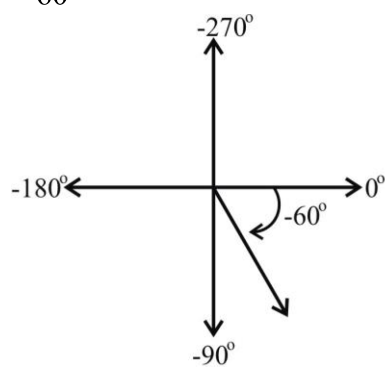
(iii) 135^{o} **(A.B)**



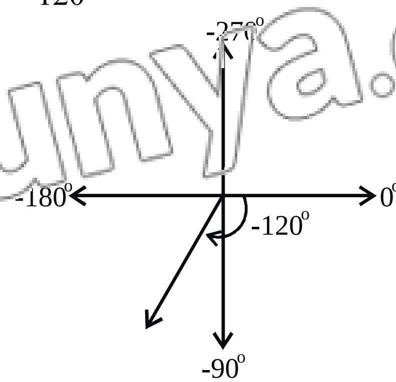
(iv) 225^{o} **(A.B)**



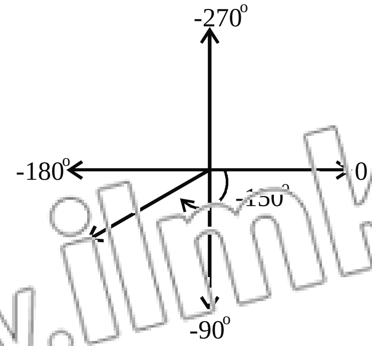
(v) -60^{o} **(A.B)**



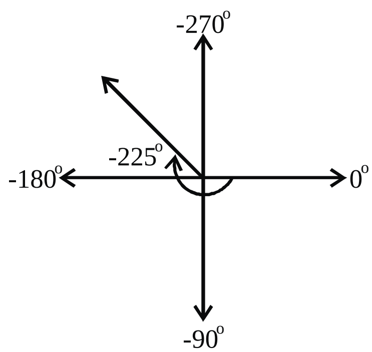
(vi) -120^{o} **(A.B)**



(vii) -150^o **(A.B)**



(viii) -225^o **(A.B)**



Q.2 Express the following sexagesimal measures of angles in decimal form:

(i) $45^o30'$ **(A.B)**

$$= 45^o + \left(\frac{30}{60}\right)^o$$

$= 45^o + 0.5^o$

$= 45.5^o$

(ii) $60^o30'30''$ **(A.B)**

$$= 60^o + \left(\frac{30}{60}\right)^o + \left(\frac{30}{3600}\right)^o$$

$= 60^o + 0.5^o + 0.0083^\circ$

$= 60.5083^o$

(iii) $125^o22'50''$ **(A.B)**

$$= 125^o + \left(\frac{22}{60}\right)^o + \left(\frac{50}{3600}\right)^o$$

$= 125^o + (0.367)^o + (0.0139)^o$

$= 125.3809^o$

Q.3 Express the following into $D^oM'S''$ form. **(SWL 2015, BWP 2016)**

(i) 47.36^o **(A.B)**

$$= 47^o + (0.36 \times 60)' \quad \because 1^o = 60'$$

$= 47^o + 21.6'$

$= 47^o + 21' + (0.6 \times 60)''$

$= 47^o + 21' + 36''$

$= 47^o 21' 36''$

(ii) 125.45^o **(A.B)**

$$= 125^o + (0.45 \times 60)'$$

$= 125^o + 27'$

$= 125^o 27'$

(iii) 225.75^o **(A.B)**

$$= 225^o + (0.75 \times 60)'$$

$= 225^o + 45'$

$= 225^o 45'$

(iv) -22.5° **(A.B)**

$$= -\left(22^\circ + (0.5 \times 60)'\right)$$

$$= -(22^\circ + 30')$$

$$= -22^\circ 30'$$

(v) -67.58° **(A.B)**

$$= -\left(67^\circ + (0.58 \times 60)'\right)$$

$$= -(67^\circ + 34.8')$$

$$= -(67^\circ + 34' + (0.8'60)'')$$

$$= -(67^\circ + 34' + 48'')$$

$$= -(67^\circ 34' 48'')$$

(vi) 315.18^o **(A.B)**

$$= 315^o + (0.18 \times 60)'$$

$= 315^o + 10.8'$

$= 315^o + 10' + (0.8 \times 60)''$

$= 315^o + 10' + 48''$

$= 315^o 10' 48''$

Q.4 Express the following angles into radians.

(i) 30^o **(A.B)**

$$= 30 \times \frac{\pi}{180} \text{ radians}$$

$$= \frac{\pi}{6} \text{ radians}$$

(ii) 60^o **(A.B)**

$$= 60\times\frac{\pi}{180}\text{ radians}$$

$$=\frac{\pi}{3}\text{ radians}$$

(iii) 135^o **(BWP 2014, D.G.K 2016)** **(A.B)**

$$= 135\times\frac{\pi}{180}\text{ radians}$$

$$=\frac{3\pi}{4}\text{ radians}$$

(iv) 225^o **(A.B)**

$$= 225\times\frac{\pi}{180}\text{ radians}$$

$$=\frac{5\pi}{4}\text{ radians}$$

(v) -150° **(BWP 2014, D.G.K 2016)** **(A.B)**

$$=-150\times\frac{\pi}{180}\text{ radians}$$

$$=-\frac{5\pi}{6}\text{ radians}$$

(vi) -225° **(A.B)**

$$=-225\times\frac{\pi}{180}\text{ radians}$$

$$=\frac{-5\pi}{4}\text{ radians}$$

(vii) 300^o **(SGD 2015)** **(A.B)**

$$= 300\times\frac{\pi}{180}\text{ radians}$$

$$=\frac{5\pi}{3}\text{ radians}$$

(viii) 315^o **(A.B)**

$$= 315\times\frac{\pi}{180}\text{ radians}$$

$$=\frac{7\pi}{4}\text{ radians}$$

Q.5 Convert each of the following to degrees

(i) $\frac{3\pi}{4}$

(LHR 2015, MTN 2016, GRW 2016) **(A.B)**

$$=\left(\frac{3\pi}{4}\times\frac{180}{\pi}\right)^o \quad \because 1\text{radians}=\frac{180}{\pi}\deg ree$$

$$=3\times 45^\circ$$

$$=135^o$$

(ii) $\frac{5\pi}{6}$ **(A.B)**

(SWL 2014, SGD 2016, MTN 2015, 16)

$$=\left(\frac{5\pi}{6}\times\frac{180}{\pi}\right)^o$$

$$=150^o$$

(iii) $\frac{7\pi}{8}$ **(A.B)**

$$=\left(\frac{7\pi}{8}\times\frac{180}{\pi}\right)^o$$

$$=\frac{315^o}{2}$$

$$=157.5^o$$

(iv) $\frac{13\pi}{16}$ **(A.B)**

$$=\left(\frac{13\pi}{16}\times\frac{180}{\pi}\right)^o$$

$$=\left(\frac{585}{4}\right)^o$$

$$=146.25^o$$

(v) 3 **(A.B)**

$$=\left(3\times\frac{180}{\pi}\right)^o$$

$$=171.89^o$$

(vi) 4.5 **(A.B)**

$$=\left(4.5\times\frac{180}{\pi}\right)^o=257.83^o$$

(vii) $-\frac{7\pi}{8}$ **(A.B)**

(GRW 2017, RWP 2015, D.G.K 2017)

$$=-\left(\frac{7\pi}{8}\times\frac{180}{\pi}\right)^o=-157.5^\circ$$

(viii) $-\frac{13\pi}{16}$ **(A.B)**

$$=-\left(\frac{13\pi}{16}\times\frac{180}{\pi}\right)^\circ=-146.25^\circ$$

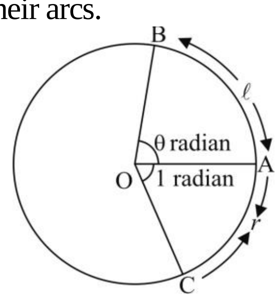
Relation between Length of Arc and Radius of a Circle (K.B)

Statement:

Establish the rule $l = r\theta$, where l is the length of an arc, r is radius of a circle and θ is the central angle measured in radians.

Proof:

Let an arc AB denoted by l, subtends a central angle θ radians. Consider an other arc AC whose length is equal to its radius then its central angle will be equal to 1 radian. In plane geometry, measure of central angles of the arcs of a circles are proportional to the lengths of their arcs.



$$\therefore \frac{m\angle AOB}{m\angle AOC} = \frac{mAB}{mAC}$$

$$\frac{\theta \text{ radians}}{1 \text{ radion}} = \frac{l}{r}$$

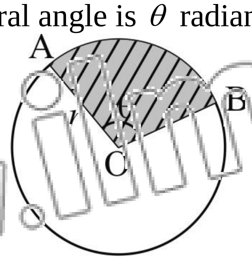
Or $r\theta = l$

Or $l = r\theta$

Area of Sector (K.B)

$$A = \frac{1}{2}r^2\theta$$

Find area of sector of a circle whose radius is r and central angle is θ radians.



Consider a sector AOB, whose central angle is θ radians. In plane geometry

$$\frac{\text{Area of Sector}}{\text{Area of Circle}} = \frac{\text{Angle of the Sector}}{\text{Angle of the Circle}}$$

$$\therefore \frac{\text{Area of Sector}}{\pi r^2} = \frac{\theta}{2\pi}$$

Area of sector AOB= $\dfrac{\theta}{2\pi} \times \pi r^2$

Area of sector AOB = $\dfrac{1}{2}r^2\theta$

Example 2: (Page # 151) **(A.B)**

Find the distance travelled by a cyclist moving on a circle of radius 15m, if he makes 3.5 revolutions.

Solution:

1 revolution = 2π radians

3.5 revolutions = $3.5 \times 2\pi$ radians

$= 7\pi$ radians

Radius of circle = 15m

Distance travelled = l = ?

Formula

$l = r\theta$

Putting the values

$l = 7\pi \times 15m$

$= 105\pi\, m = 329.87m$

$\therefore$ Distance travelled by cyclist = 329.87m

Example 3: (Page # 152) **(A.B)**

(LHR 2014, D.G.K 2014)

Find the area of sector of a circle of radius 16cm if the angle at the centre is 60°.

Solution:

Radius of circle = r = 16cm

Angle of sector = θ = 60°

$= 60 \times \dfrac{\pi}{180} = \dfrac{\pi}{3}$ radians

Formula

Area of sector = $A = \dfrac{1}{2}r^2\theta$

Putting the values

$$A = \frac{1}{2}(16)^2 \frac{\pi}{3}\ cm^2$$

$\Rightarrow A = 143.1cm^2$

Exercise 7.2

Q.1

(i) Find θ, when *l* = 2cm, *r* = 3.5cm
(LHR 2015, GRW 2016, BWP 2014, MTN 2015, 16, 17, SGD 2015) **(A.B)**

Solution:

We know that

$l = r\theta$

or $\theta = \frac{l}{r}$

Putting the values

$= \frac{2\text{cm}}{3.5\text{cm}}$

θ = 0.57radians

(ii) Given: **(A.B)**

l = 4.5m, *r* = 2.5m

(FSD 2014, SGD 2014, 16, MTN 2016, D.G.K 2015, 17)

Required:

θ = ?

Solution:

We know that

$\theta = \frac{l}{r}$

Putting the values

$\theta = \frac{4.5}{2.5}$

$\Rightarrow \theta = 1.8$ radians

Q.2

(i) Given $\theta = 180^o, r = 4.9cm$ **(A.B)**
(LHR 2014, GRW 2014, FSD 2015, SWL 2016, D.G.K 2016)

Required:

$l = ?$

Solution:

Here

θ = 180°

$= 180 \times \frac{\pi}{180}$ radians $\because 1^o = \frac{\pi}{180}$ rad

= 3.14 radians

We know that

$l = r\theta$

Putting the values

= 4.9 cm × 3.14

$\Rightarrow l = 15.39$cm

(ii) Find *l*, when θ=60° 30′,r = 15mm
(LHR 2014, SWL 2016, BWP 2016, MTN 2015)

Solution:

Here

θ = 60° 30′

$= 60^o + \left(\frac{30}{60}\right)^o$

= 60.5°

$= 60.5 \times \frac{\pi}{180}$ radians

= 1.0559 radians

We know that

$l = r\theta$

putting the values

= (15mm)(1.0559)

$\Rightarrow l = 15.84mm$

Q.3 **(A.B)**

(i) Find *r*, when $l = 4cm, \theta = \frac{1}{4}$ **radian**

Solution:

We know that

$r = \frac{l}{\theta}$

Putting the values

$r = \frac{4cm}{\frac{1}{4}}$

$\Rightarrow r = 16cm$

(ii) Given: $l = 52cm$, $\theta = 45^o$ **(A.B)**
(LHR 2015, 17, GRW 2017, SWL 2014, 17, RWP 2010, 15, D.G.K 2017)

Required:

$r = ?$

Solution:

Here θ = 45°

$= 45 \times \frac{\pi}{180}$ radians $\because 1^o = \frac{\pi}{180}$ rad

= 0.785 radians

We know that

$r = \frac{l}{\theta}$

$= \frac{52}{0.785}cm$

$\Rightarrow r = 66.21cm$

Q.4 **Given:** *r* = 12m, θ = 1.5 radian
(SGD 2014) **(A.B)**

Required:

$l = ?$

Solution:

We know that

$l = r\theta$

Putting the values

l = (12m) (1.5)

$\Rightarrow l = 18m$

Q.5 Given: r = 10m, θ = angle formed by 3.5 revolutions **(A.B)**

Required:

$l = ?$

Solution:

Angle in one revolution = 2π radians

Angle in 3.5 revolution

$= (2\pi \times 3.5)$ radians

$= 7\pi$ radians

We know that

$l = r\theta$

Putting the values

$= (10m)(7\pi)$

$= 70\pi m$

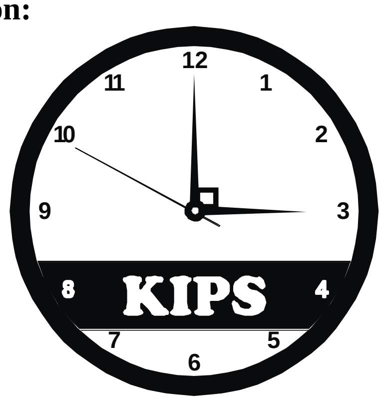
$= 220\,m$

Result:

Distance travelled by point $= 220\,m$

Q.6 What is the circular measure of the angle between the hands of the watch at 3 O' clock? (A.B)

Solution:

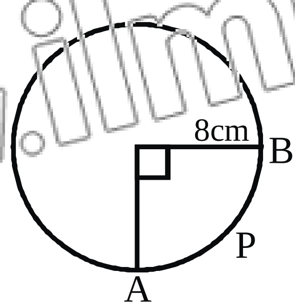


Circular measure of angle between hands of clock = $90 \times \frac{\pi}{180}$ radians

$= \frac{\pi}{2}$ radian

Q.7 What is the length of arc APB? (A.B)

Solution:



Here

θ = 90°

$= 90 \times \frac{\pi}{180}$ radians $\because 1^o = \frac{\pi}{180}$ rad

$= \frac{\pi}{2}$ radian

r = 8cm

We know that

l = rθ

Putting the values

$l = (8cm)\left(\frac{\pi}{2}\text{radian}\right)$

$= 4\pi$ radians

l = 12.57cm

Result:

Length of arc $APB = 12.57cm$

Q.8 In a circle of radius 12*cm*, how long an are subtends a central angle of 84°? (A.B)

Given:

r = 12cm, θ = 84°

Required:

$l = ?$

Solution:

Here

θ = 84°

$= 84 \times \frac{\pi}{180}$ radian $\because 1^o = \frac{\pi}{180}$ rad

= 1.466 radian

We know that

l = r θ

Putting the values

= (12cm) (1.466)

= 17.592cm

Result:

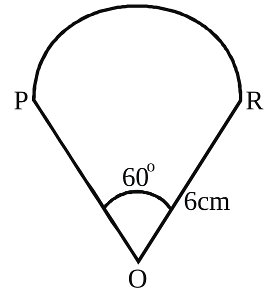
$\therefore$ Length of arc = 17.592cm

Q.9 Find the area of the sector OPR. (A.B)

(a) Given: θ = 60°, r = 6*cm*

Required:

Area of sector = A =?



Solution:

$\theta = 60^o$

$= 60 \times \frac{\pi}{180}$ radians $\because 1^o = \frac{\pi}{180}$ rad

$= \frac{\pi}{3}$ radians

We know that

$A = \frac{1}{2} r^2\theta$

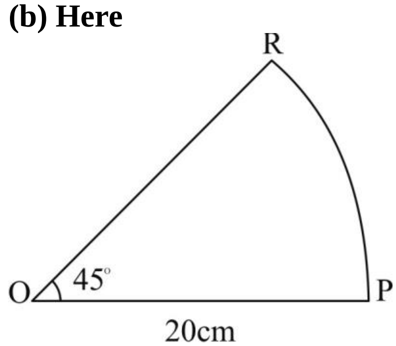
Putting the values

$= \frac{1}{2}(6cm)^2\left(\frac{\pi}{3}\right)$

$= \frac{1}{2} \times 36 \times \frac{\pi}{3} = 6\pi$

$\Rightarrow A = 18.85cm^2$

(b) Here



$\theta = 45^o$

$= 45 \times \frac{\pi}{180}$ radians $\because 1^o = \frac{\pi}{180}$ rad

$= \frac{\pi}{4}$ radians

We know that

$A = \frac{1}{2} r^2\theta$

Putting the values

$A = \frac{1}{2}(20cm)^2\left(\frac{\pi}{4}\right)$

$= \frac{1}{2} \times 400 \times \frac{\pi}{4} = 50\pi$

$\Rightarrow A = 157.08cm^2$

Q.10 Find the area of sector inside a central angle of 20° in a circle of radius 7m. (A.B)

Given:

$\theta = 20^o$, $r = 7m$

Area of sector = A = ?

Here

$\theta = 20^o$

$= 20 \times \frac{\pi}{180}$ radians $\because 1^o = \frac{\pi}{180}$ rad

$= \frac{\pi}{9}$ radians

We know that

$A = \frac{1}{2} r^2\theta$

Putting the values

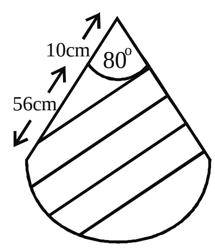
$A = \frac{1}{2}(7m)^2\left(\frac{\pi}{9}\right)$

$= \frac{1}{2} \times 49 \times \frac{\pi}{9}\ m^2$

$= \frac{49}{18} \times 3.14\ m^2$

$\Rightarrow A = 8.552m^2$

Q.11 Sehar is making a skirt. Each panel of this skirt is of the shape shown shaded in the diagram. How much material (cloth) is required for each panel? (A.B)



Given:

$\theta = 80^o, r_1 = 10cm$

$r_2 = 56cm + 10cm = 66cm$

Required:

Area of shaded region $= A = ?$

Solution:

Here

$\theta = 80°$

$= 80 \times \frac{\pi}{180}$ radians $\because 1^o = \frac{\pi}{180}$ rad

$= \frac{20\pi}{45}$ radians

Formula:

Area of small sector $= \frac{1}{2} r_1^2 \theta$

$= \frac{1}{2} \times (10)^2 \times \frac{20\pi}{45}$

$= \frac{1}{2} \times 100 \times \frac{20\pi}{45}$

$= 50 \times \frac{20\pi}{45}$

$= 69.841 cm^2$

Area of big sector $= \frac{1}{2} r_2^2 \times \theta$

$= \frac{1}{2}(66cm)^2 \times \frac{20\pi}{45}$

$\Rightarrow A = 3042.285 cm^2$

Area of shaded region =

Area of big sector − Area of small sector

$= 3042.285 - 69.841$

$= 2972.4 cm^2$

$\therefore 2972.4 cm^2$ material (cloth) is required for each panel.

Q.12 Find the area of sector with central angle of $\frac{\pi}{5}$ radian in a circle of radius 10cm. (A.B)

Given:

$\theta = \frac{\pi}{5}$ radians, r = 10cm

Required:

$A = ?$

Solution:

We know that

$A = \frac{1}{2} r^2 \theta$

Putting the values

$= \frac{1}{2}(10cm)^2 \left(\frac{\pi}{5}\right)$

$= \frac{1}{2} \times 100 \times \frac{\pi}{5} = 10\pi$

$\Rightarrow A = 31.416 cm^2$

Q.13 The area of the sector with a central angle θ in a circle of radius *2m* is 10 square metre. Find θ in radians. (A.B)

Given:

Area of Sector = $A = 10$ square m

Radius of circle $r = 2cm$

Required:

Central Angle $= \theta = ?$

Solution:

We know that

$A = \frac{1}{2} r^2 \theta$

Or $\theta = \frac{2a}{r^2}$

$= \frac{2 \times 10}{(2)^2}$

$= \frac{20}{4}$

= 5 radians

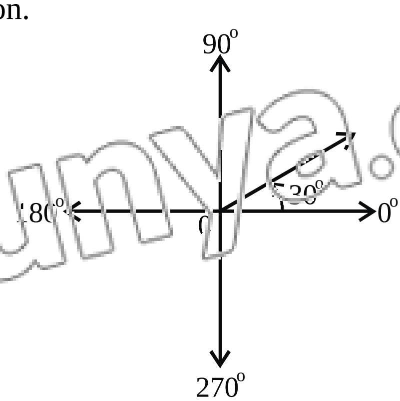
Coterminal Angles (K.B)

Two or more angles with the same initial and terminal sides are called coterminal angles. For example 30°, 390°, 750°, ….. are coterminal angles $(\theta + 360k), k \in Z$

Angle in Standard Position (K.B)

A general angle is said to be in standard position if its vertex is at the origin and its initial side is directed along the positive direction of the *x-axis* of a rectangular coordinate system.

For example, angle below is in standard position.



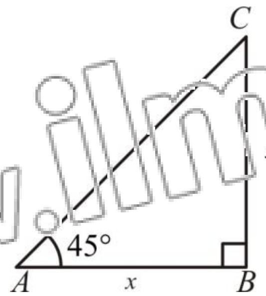
Quadrantal Angles (K.B)

If the terminal side of an angle in standard position falls on *x-axis* or *y-axis* then it is called a quadrental angle i.e. 0°, 90°, 180°, 270°, 360° are quadretal angles

Trigonometric Ratios of 45°

(K.B + U.B+ A.B)

Consider a right angled isosceles ΔABC, in which $m\overline{AB} = mBC = x$



By Pythagoras theorem,

$$m\overline{AC} = x^2 + x^2$$
$$= 2x^2$$
$$m\overline{AC} = \sqrt{2x^2}$$
$$= \sqrt{2}x$$

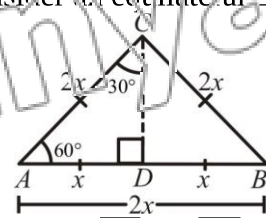
Now trigonometric Ratios are

$$\sin 45° = \frac{m\overline{BC}}{m\overline{AC}} = \frac{x}{\sqrt{2}x}$$
$$= \frac{1}{\sqrt{2}}$$
$$\cos 45° = \frac{m\overline{AB}}{m\overline{AC}}$$
$$= \frac{x}{\sqrt{2}x}$$
$$= \frac{1}{\sqrt{2}}$$
$$\tan 45° = \frac{m\overline{BC}}{m\overline{AB}} = \frac{x}{x}$$
$$= 1$$
$$\text{cosec}45° = \frac{1}{\sin 45°}$$
$$= \sqrt{2}$$
$$\sec 45° = \frac{1}{\cos 45°}$$
$$= \sqrt{2}$$
$$\cot 45° = \frac{1}{\tan 45°}$$
$$= 1$$

Trigonometric Ratios of 60° & 30°

(K.B + U.B+ A.B)

Consider an equilateral ΔABC,



In which $m\overline{AB} = m\overline{BC} = m\overline{AC} = 2x$

Draw $\overline{CD} \perp \overline{AB}$, then

$\overline{AD} = \overline{BD} = x$

From ΔCAD, using Pythagoras theorem

$$\left(m\overline{AD}\right)^2 + \left(m\overline{CD}\right)^2 = \left(m\overline{AC}\right)^2$$
$$x^2 + \left(m\overline{CD}\right)^2 = (2x)^2$$
$$\left(m\overline{CD}\right)^2 = 4x^2 - x^2$$
$$\left(m\overline{CD}\right)^2 = 3x^2$$
$$\Rightarrow m\overline{CD} = \sqrt{3x^2}$$
$$= \sqrt{3}x$$

Trigonometric ratios of 60° are:

$$\sin 60° = \frac{m\overline{CD}}{m\overline{AC}} = \frac{\sqrt{3}x}{2x}$$
$$= \frac{\sqrt{3}}{2}$$
$$\cos 60° = \frac{m\overline{AD}}{m\overline{AC}} = \frac{x}{2x}$$
$$= \frac{1}{2}$$
$$\tan 60° = \frac{m\overline{CD}}{m\overline{AD}}$$
$$= \frac{\sqrt{3}x}{x}$$
$$= \sqrt{3}$$
$$\text{cosec}60° = \frac{1}{\sin 60°}$$
$$= \frac{2}{\sqrt{3}}$$
$$\sec 60° = \frac{1}{\cos 60°}$$
$$= 2$$

$$\cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}}$$

Now Trigonometric Ratios of 30°

In ΔACD

$$\sin 30^\circ = \frac{m\overline{AD}}{m\overline{AC}} = \frac{x}{2x} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{m\overline{CD}}{m\overline{AC}} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$$

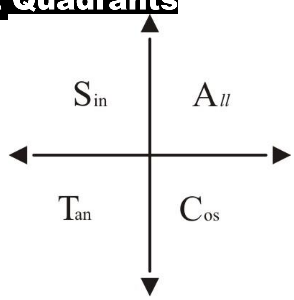
$$\tan 30^\circ = \frac{m\overline{AD}}{m\overline{CD}} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}}$$

$$\text{cosec}30^\circ = \frac{1}{\sin 30^\circ} = \frac{2}{\sqrt{3}}$$

$$\sec 30^o = \frac{1}{\cos 30^o} = \frac{2}{\sqrt{3}}$$

$$\cot 30^\circ = \frac{1}{\tan 30^\circ} = \sqrt{3}$$

Signs of Trigonometric Ratios in Different Quadrants (K.B)



(ASTC = After school to college)

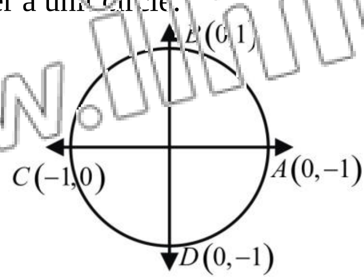
NOTE:

Where a ratio is $+ve$, its reciprocal is also $+ve$. And remaining ratios are $-ve$.

Trigonometric Ratios of Quadrantal Angles (0°, 90°, 180°, 270° & 360°)

Trigonometric Ratios of 0° (K.B + U.B + A.B)

Consider a unit circle.



Then $m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = 1$

$$r = \sqrt{x^2 + y^2} = \sqrt{(1)^2 + (0)^2} = \sqrt{1+0} = \sqrt{1} = 1$$

$$\sin 0^o = \frac{y}{r} = \frac{0}{1} = 0$$

$$\cos 0^o = \frac{x}{r} = \frac{1}{1} = 1$$

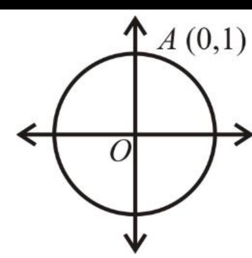
$$\tan 0^o = \frac{y}{x} = \frac{0}{1} = 0$$

$$\text{cosec } 0^o = \frac{1}{0} = \text{Undefined}$$

$$\sec 0^o = \frac{1}{\cos 0^o} = \frac{1}{1} = 1$$

$$\cot 0^o = \frac{1}{0} = \text{Undefined}$$

Trigonometric Ratios of 90° (K.B)



$$\sin 90^\circ = \frac{y}{r} = \frac{1}{1} = 1$$

$$\cos 90^\circ = \frac{x}{r} = \frac{0}{1} = 0$$

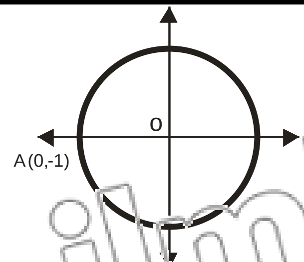
$$\tan 90^\circ = \frac{y}{x} = \frac{1}{0} = \text{Undefined}$$

$$\text{cosec } 90^\circ = \frac{1}{\sin 90^o} = 1$$

$$\sec 90^\circ = \frac{1}{0} = \text{Undefined}$$

$$\cot 90^o = \frac{1}{\tan 90^o} = 0$$

Trigonometric Ratios of 180° (K.B)



$\sin 180° = \frac{y}{r} = \frac{0}{1} = 0$

$\cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$

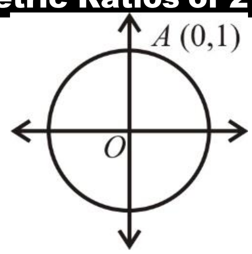
$\tan 180° = \frac{y}{x} = \frac{0}{-1} = 0$

$\text{cosec } 180° = \frac{1}{\sin 180^o} = \frac{1}{0} = \text{Undefined}$

$\sec 180° = \frac{1}{\cos 180^o} = \frac{1}{-1} = -1$

$\cot 180° = \frac{1}{\tan 180^o} = \frac{1}{0} = \text{Undefined}$

Trigonometric Ratios of 270° (K.B)



$\sin 270° = \frac{y}{r} = \frac{-1}{1} = -1$

$\cos 270° = \frac{x}{r} = \frac{0}{1} = 0$

$\tan 270° = \frac{y}{x} = \frac{-1}{0} = \text{Undefined}$

$\text{cosec } 270° = \frac{1}{\sin 270^o} = \frac{1}{-1} = -1$

$\sec 270° = \frac{1}{\cos 270^0} = \frac{1}{0} = \text{Undefined}$

$\cot 270° = \frac{1}{\tan 270^o} = 0$

Note (K.B + U.B)

$\sin(-\theta) = -\sin\theta$

$\text{cosec}(-\theta) = -\text{cosec}\theta$

$\cos(-\theta) = \cos\theta$

$\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

Example 1: (Page # 159) (K.B + A.B)

If $\sin\theta = \frac{-3}{4}$ and $\cos\theta = \frac{\sqrt{7}}{4}$, then find the values of $\tan\theta, \cot\theta, \sec\theta$ and $\text{cosec}\theta$

Solution:

Applying the identities that express the remaining trigonometric functions in terms of sine and cosine, we have,

$\sin\theta = \frac{-3}{4}$

$\Rightarrow \text{cosec}\theta = \frac{1}{\sin\theta} = \frac{-4}{3}$

$\Rightarrow \text{cosec}\theta = \frac{-4}{3}$

$\cos\theta = \frac{\sqrt{7}}{4}$

$\sec\theta = \frac{1}{\cos\theta}$

$\Rightarrow \sec\theta = \frac{4}{\sqrt{7}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-3}{4}}{\frac{\sqrt{7}}{4}}$

$\Rightarrow \tan\theta = \frac{-3}{\sqrt{7}}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{-\sqrt{7}}{3}$

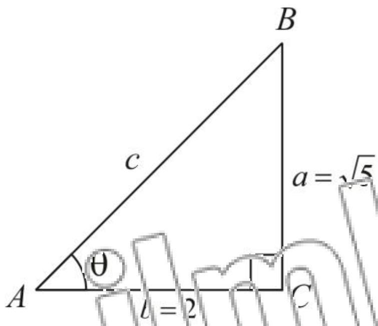
Example 2: (Page # 159) (K.B + A.B)
(GRW 2014)

If $\tan\theta = \frac{\sqrt{5}}{2}$, then find the values of other trigonometric ratios at θ.

Solution:

In any right triangle ABC,

$\tan\theta = \frac{\sqrt{5}}{2} = \frac{a}{b} \Rightarrow a = \sqrt{5}, b = 2$



Now by Pythagorean theorem

$$a^2+b^2=c^2 \Rightarrow \left(\sqrt{5}\right)^2+\left(2\right)^2=c^2$$

$$c^2=5+4=9 \Rightarrow c=\pm 3 \text{ or } c=3$$

$$\because \quad \cot\theta=\frac{1}{\tan\theta}$$

$$\therefore \quad \cot\theta=\frac{1}{\frac{\sqrt{5}}{2}} \Rightarrow \cot\theta=\frac{2}{\sqrt{5}}$$

$$\sin\theta=\frac{a}{c}=\frac{\sqrt{5}}{3}, \quad \operatorname{cosec}\theta=\frac{1}{\sin\theta}$$

$$\Rightarrow \operatorname{cosec}\theta=\frac{1}{\frac{\sqrt{5}}{3}} \quad \therefore \operatorname{cosec}\theta=\frac{3}{\sqrt{5}}$$

$$\cos\theta=\frac{b}{c}=\frac{2}{3}, \quad \sec\theta=\frac{1}{\cos\theta}$$

Also

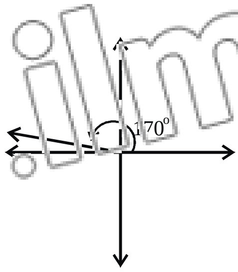
$$\Rightarrow \sec\theta=\frac{1}{\frac{2}{3}} \quad \therefore \sec\theta=\frac{3}{2}$$

Exercise 7.3

Q.1 Locate each of the following angles in standard positions using protractor or fair free hand guess. Also find a positive and negative coterminal with each given angle.

Solution:

(i) 170° **(A.B)**



Positive conterminal angle of 170°

= 360° + 170°

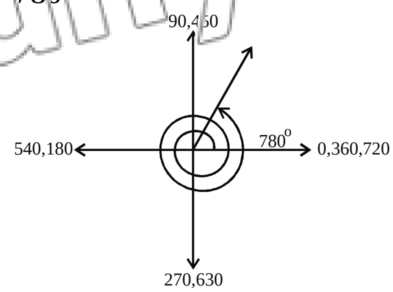
= 530°

Negative coterminal angle

= –360° + 170

= –190°

(ii) 780° **(A.B)**



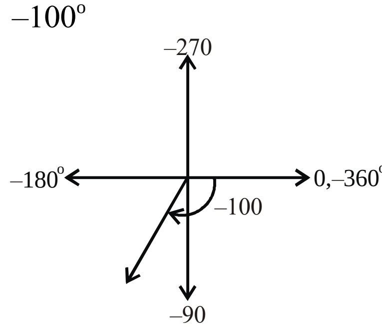
Positive coterminal angle =780°–2(360°)

= 60°

Negative coterminal angle = –360° + 60°

= –300°

(iii) –100° **(A.B)**



Positive coterminal angle

= 360° + (–100°)

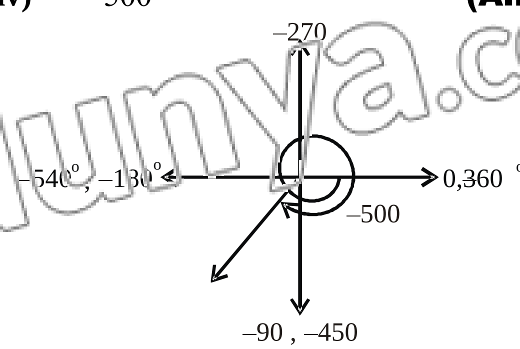
= 260°

Negative coterminal angle

= –360° + (–100°)

= –460°

(iv) –500° **(A.B)**



Negative coterminal angle =–500° + 360°

= –140°

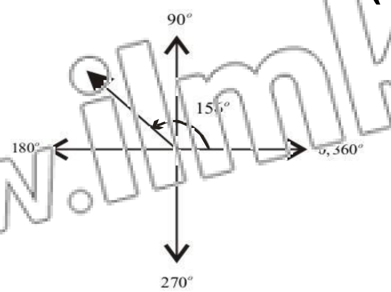
Positive coterminal angle =–140° + 360°

=220°

Q.2 Identify the closest quadrantal angles between which the following angles lie.

Solution:

(i) 156^{o} **(A.B)**

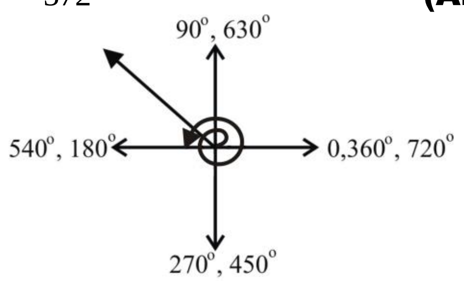


Closest quadrantal angles are 90^{o} and 180°

(ii) 318^{o} **(A.B)**

Closest quadrantal angles are 270^{0} and 360^{o}

(iii) 572^{o} **(A.B)**



Closest quadrantal angles are 540^{o} and 630^{o}

(iv) -330^{o} **(A.B)**

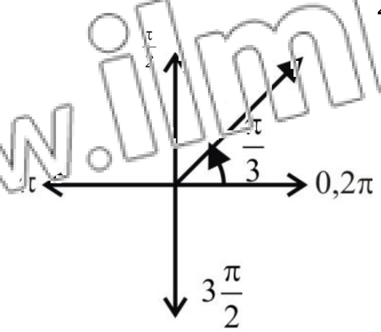
Closest quadrantal angles are -270^{o} and -360^{o}

Q.3 Write the closest quadrantal angles between which the angle lies. Write your answer in radian measure.

Solution:

(i) $\frac{\pi}{3}$ **(A.B)**

Closest quadrantal angles are 0 and $\frac{\pi}{2}$



(ii) $\frac{3\pi}{4}$ **(A.B)**

Closest quadrantal angles are $\frac{\pi}{2}$ and π

(iii) $\frac{-\pi}{4}$ **(A.B)**

Closest quadrantal angles are 0 and $-\frac{\pi}{2}$

(iv) $\frac{-3\pi}{4}$ **(A.B)**

Closest quadrantal angles are $-\frac{\pi}{2}$ and $-\pi$

Q.4 In which quadrant θ lies, when

(i) $\sin\theta > 0, \tan\theta < 0$

(ii) $\cos\theta < 0,\ \sin\theta < 0$

(iii) $\sec\theta > 0,\ \sin\theta < 0$

(iv) $\cos\theta > 0,\ \tan\theta < 0$

(v) $co\sec\theta > 0,\ \cos\theta < 0$

(vi) $\sin\theta > 0,\ \sec\theta < 0$

Solution:

(i) $\sin\theta > 0, \tan\theta < 0$ **(K.B)**

$\sin\theta > 0$, then θ lies I, II quadrant.

$\tan\theta < 0$, then θ lies in II, IV quadrant.

II quadrant is common

$\therefore$ θ lies in second quadrant

(ii) $\cos\theta < 0,\ \sin\theta < 0$ **(K.B)**

$\cos\theta < 0$, then θ lies in II, III quadrant.

$\sin\theta < 0$, then θ lies in III, IV quadrant.

III quadrant is common.

$\therefore$ θ lies in III quadrant.

(iii) $\sec\theta > 0,\ \sin\theta < 0$ **(K.B)**

$\sec\theta > 0$, then θ lies I, IV quadrant.

$\sin\theta < 0$, then θ lies in III, IV quadrant.

IV quadrant is common

$\therefore$ θ lies in fourth quadrant

(iv) $\cos\theta > 0,\ \tan\theta < 0$ **(K.B)**

$\cos\theta > 0$, then θ lies I, IV quadrant.

$\tan\theta < 0$, then θ lies in II, IV quadrant.

IV quadrant is common

$\therefore$ θ lies in fourth quadrant

(v) $\text{cosec}\,\theta > 0,\ \cos\theta < 0$ **(K.B)**

$\text{cosec}\,\theta > 0$, then θ lies I, II quadrant.

$\cos\theta < 0$, then θ lies in II, III quadrant.

II quadrant is common

$\therefore$ θ lies in second quadrant

(vi) $\sin\theta > 0, \sec\theta < 0$ **(K.B)**

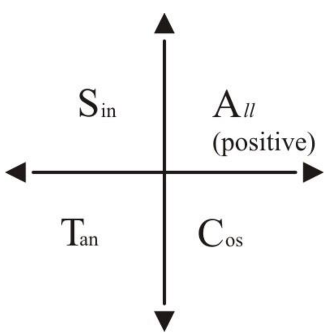
$\sin\theta > 0$, then θ lies I, II quadrant.

$\sec\theta < 0$, then θ lies in II, III quadrant.

II quadrant is common

$\therefore$ θ lies in second quadrant

Note



(After school to college, (where a ratio is positive its reciprocal is also positive))

Q.5 Fill in the blanks.

(i) $\cos(-150^o) = \cos 150°$ **(K.B)**

$\cos(-\theta) = \cos\theta$

(ii) $\sin(-310°) = -\sin 310°$ **(K.B)**

$\because \sin(-\theta) = -\sin\theta$

(iii) $\tan(-210°) = -\tan 210°$ **(K.B)**

$\because \tan(-\theta) = -\tan\theta$

(iv) $\cot(-45°) = -\cot 45°$ **(K.B)**

$\because \cot(-\theta) = -\cot\theta$

(v) $\sec(-60°) = \sec 60°$ **(K.B)**

$\because \sec(-\theta) = \sec\theta$

(vi) $\text{cosec}(-137°) = -\text{cosec}\,137°$ **(K.B)**

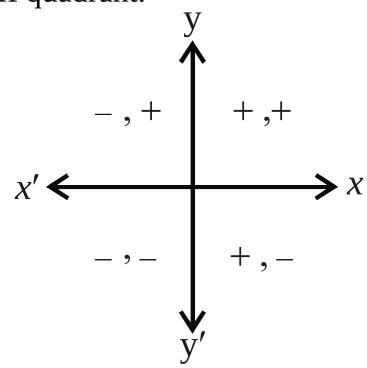
$\because \text{cosec}(-\theta) = -\text{cosec}\,\theta$

Q.6 The given point P lies on the terminal side of θ. Find quadrant of θ and all six trigonometric ratios.

Solution:

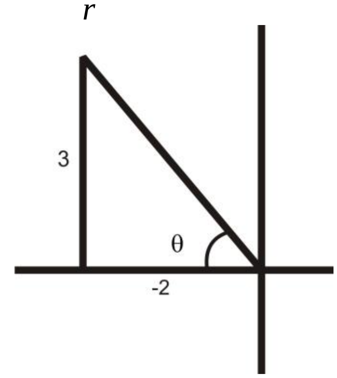
(i) $P(-2,3)$ **(K.B + A.B)**

Since x is $-ve$ and y is $+ve$, θ lies in II quadrant.



Trigonometric Ratios:

$\sin\theta = \dfrac{y}{r}$



$r = \sqrt{x^2 + y^2}$

$= \sqrt{(-2)^2 + (3)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

$\sin\theta = \dfrac{3}{\sqrt{13}}$

$\cos\theta = \dfrac{x}{r}$

$= \dfrac{-2}{\sqrt{13}}$

$\tan\theta = \dfrac{y}{x}$

$= \frac{3}{-2}$

$= -\frac{3}{2}$

cesecθ$= \frac{1}{\sin\theta}$

$= \frac{\sqrt{13}}{3}$

secθ$= \frac{1}{\cos\theta}$

$= -\frac{\sqrt{13}}{3}$

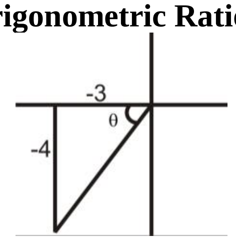
cotθ$= \frac{1}{\tan\theta}$

$= -\frac{2}{3}$

(ii) P$(-3,-4)$ **(K.B + A.B)**

Since both x and y are $-ve$, l lies in III quadrant.

Trigonometric Ratios:



$= \sqrt{(-3)^2 + (-4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

Now

$\sin\theta = \frac{y}{r} = \frac{-4}{5} = -\frac{4}{5}$

$\cos\theta = \frac{x}{r} = \frac{-3}{5} = -\frac{3}{5}$

$\tan\theta = \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3}$

$\cos ec\theta = \frac{1}{\sin\theta} = -\frac{5}{4}$

$\sec\theta = \frac{1}{\cos\theta} = -\frac{5}{3}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$

(iii) $(\sqrt{2},1)$ **(K.B + A.B)**

Since both x and y are $+ve$, θ lies in I quadrant.

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{3}}$

$\cos\theta = \frac{x}{r} = \frac{\sqrt{2}}{\sqrt{3}}$

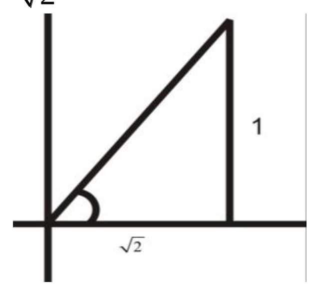
$\tan\theta = \frac{y}{x} = \frac{1}{\sqrt{2}}$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(\sqrt{2})^2 + (1)^2}$

$= \sqrt{2+1}$

$= \sqrt{3}$



$\cos ec\theta = \frac{1}{\sin\theta} = \sqrt{3}$

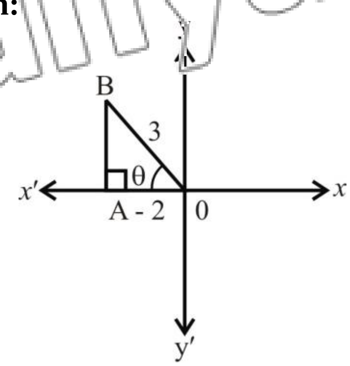
$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{3}}{\sqrt{2}}$

$\cot\theta = \frac{1}{\tan\theta} = \sqrt{2}$

Q.7 If $\cos\theta = -\frac{2}{3}$ and terminal arm of the angle θ is in quad. II, find the values of remaining trigonometric functions. **(K.B + A.B)**

(SGD 2014, D.G.K 2014)

Solution:



Form ΔABC

$$\left(\overline{AB}\right)^2 = \left(\overline{OB}\right)^2 - \left(\overline{OA}\right)^2$$
$= 3^2 - (2)^2$
$= 9 - 4$
$= 5$

$\Rightarrow \overline{AB} = \sqrt{5}\,(\theta \text{ lies in II quad})$

$$\sin\theta = \frac{\text{Perp}}{\text{Hyp}} = \frac{m\overline{AB}}{m\overline{OB}} = \frac{\sqrt{5}}{3}$$

$$\cos\theta = \frac{m\overline{OA}}{m\overline{OB}} = \frac{-2}{3} = -\frac{2}{3}$$

$$\tan\theta = \frac{m\overline{AB}}{m\overline{OB}} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{3}{\sqrt{5}}$$

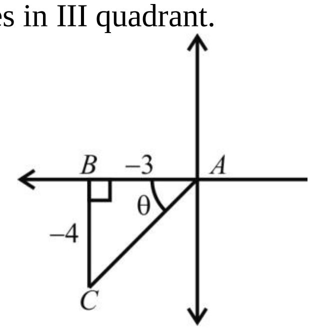
$$\sec\theta = \frac{1}{\cos\theta} = -\frac{3}{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{2}{\sqrt{5}}$$

Q.8 If $\tan\theta = \frac{4}{3}$ and $\sin\theta < 0$, find the values of other trigonometric functions at θ **(K.B + A.B)**

Solution:

Since $\tan\theta$ is $+ve$ and $\sin\theta$ is $-ve$, θ lies in III quadrant.



Form ΔABC

$$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$$
$= (-3)^2 + (-4)^2$
$= 9 + 16$
$= 25$

$m\overline{AC} = 5$

$$\sin\theta = \frac{m\overline{BC}}{m\overline{AC}} = \frac{-4}{5}$$
$$= -\frac{4}{5}$$

$$\cos\theta = \frac{m\overline{AB}}{m\overline{AC}} = \frac{-3}{5}$$
$$= -\frac{3}{5}$$

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}} = \frac{-4}{-3}$$
$$= \frac{4}{3}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = -\frac{5}{4}$$

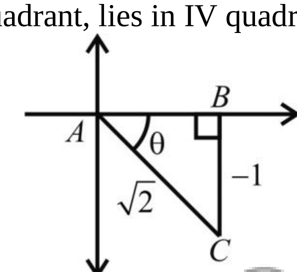
$$\sec\theta = \frac{1}{\cos\theta} = -\frac{5}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$$

Q.9 If $\sin\theta = -\frac{1}{2}$ and terminal side of the angle is not in quadrant III, find the values of $\tan\theta$, $\sec\theta$ and $\text{cosec}\,\theta$. **(K.B + A.B)**

Solution:

Since $\sin\theta$ is $-ve$ and θ is not in III quadrant, lies in IV quadrant.



Form ΔABC

$$\left|\overline{AB}\right|^2 = \left|\overline{AC}\right|^2 - \left|\overline{BC}\right|^2$$
$= \left(\sqrt{2}\right)^2 - (-1)^2$
$= 2 - 1$
$= 1$

$\Rightarrow m\overline{AB} = 1$

$$\sin\theta = -\frac{1}{\sqrt{2}}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = -\sqrt{2}$$

$$\sec\theta = \frac{m\overline{AC}}{m\overline{AB}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}} = \frac{-1}{1} = -1$$

Result:

$\tan\theta = -1$

$\sec\theta = \sqrt{2}$

$\text{cosec}\,\theta = -\sqrt{2}$

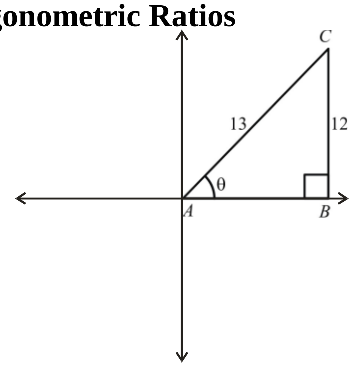
Q.10 If $\text{cosec}\,\theta = \frac{13}{12}$ and $\sec\theta > 0$, find the remaining trigonometric functions.
(K.B + A.B)

Solution:

Since both Cosecθ and Secθ are $+ve$, θ lies in I quadrant.

Now

Trigonometric Ratios



Form ΔABC

$$\left|\overline{AB}\right|^2 = \left|\overline{AC}\right|^2 - \left|\overline{BC}\right|^2$$

$$= (13)^2 - (12)^2$$

$= 169 - 144$

$= 25$

$$\Rightarrow m\overline{AB} = 5 \left(\theta \text{ is in I quad}\right)$$

$$\sin\theta = \frac{1}{\text{cosec}\,\theta} = \frac{12}{13}$$

$$\cos\theta = \frac{m\overline{AB}}{m\overline{AC}} = \frac{5}{13}$$

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}} = \frac{12}{5}$$

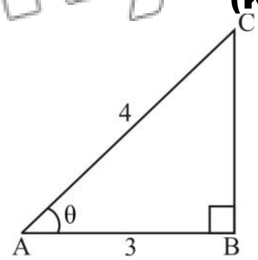
$$\text{cosec}\,\theta = \frac{13}{12}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{13}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{5}{12}$$

Q.11 Find the values of trigonometric functions at the indicated angle θ in the right angle triangle.

(i) **(K.B + A.B)**



Here

$$\left|m\overline{BC}\right|^2 = \left|m\overline{AC}\right|^2 - \left|m\overline{AB}\right|^2$$

$$= (4)^2 - (3)^2$$

$= 16 - 9$

$= 7$

$$\Rightarrow m\overline{BC} = \sqrt{7}$$

Now Trigonometric Ratios are:

$$\sin\theta = \frac{m\overline{BC}}{m\overline{AC}} = \frac{\sqrt{7}}{4}$$

$$\cos\theta = \frac{m\overline{AB}}{m\overline{AC}} = \frac{3}{4}$$

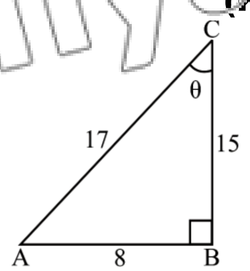
$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}} = \frac{\sqrt{7}}{3}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{4}{\sqrt{7}}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{4}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{\sqrt{7}}$$

(ii) Trigonometric Ratios are:
(K.B + A.B)



$$\sin\theta = \frac{m\overline{AB}}{m\overline{AC}} = \frac{8}{17}$$

$$\cos\theta = \frac{m\overline{BC}}{m\overline{AC}} = \frac{15}{17}$$

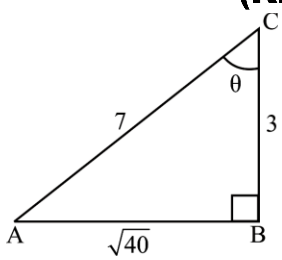
$$\tan\theta = \frac{m\overline{AB}}{m\overline{BC}} = \frac{8}{15}$$

$$\text{cosec} = \frac{1}{\sin\theta} = \frac{17}{8}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{17}{15}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{15}{8}$$

(iii) **(K.B + A.B)**



Here

$$\left|m\overline{AB}\right|^2 = \left|m\overline{AC}\right|^2 - \left|m\overline{BC}\right|^2$$

$$= (7)^2 - (3)^2$$

$$= 49 - 9$$

$$= 40$$

$$\Rightarrow m\overline{AB} = \sqrt{40}$$

Now Trigonometric Ratios are:

$$\sin\theta = \frac{m\overline{AB}}{m\overline{AC}} = \frac{\sqrt{40}}{7}$$

$$\cos\theta = \frac{m\overline{BC}}{m\overline{AC}} = \frac{3}{7}$$

$$\tan\theta = \frac{m\overline{AB}}{m\overline{BC}} = \frac{\sqrt{40}}{3}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{7}{\sqrt{40}}$$

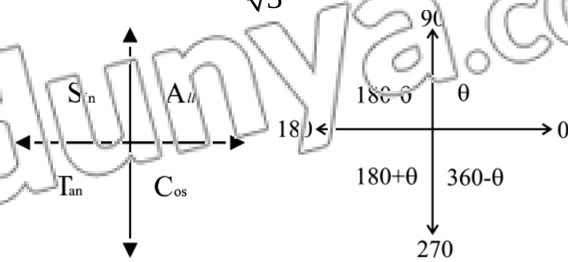
$$\sec\theta = \frac{1}{\cos\theta} = \frac{7}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{\sqrt{40}}$$

Q.12 Find the values of the trigonometric functions. Do not use trigonometric tables or calculator.

Solution:

(i) $\tan 30^o = \dfrac{1}{\sqrt{3}}$ **(K.B + A.B)**



(ii) $\tan 330^o = -\tan 30^o$ **(K.B + A.B)**

$$= -\frac{1}{\sqrt{3}}$$

| 360° - θ = 330°
| θ = 360° - 330°
| = 30°

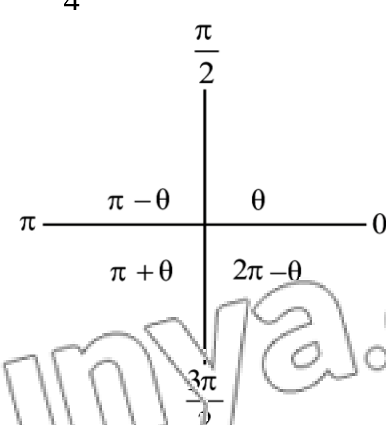
(iii) $\text{Sec}330^o = \sec 30^o$ **(K.B + A.B)**

$$= \frac{1}{\cos 30^o}$$

$$= \frac{1}{\sqrt{3}/2}$$

$$= \frac{2}{\sqrt{3}}$$

(iv) $\cot\dfrac{\pi}{4} = 1$ **(K.B + A.B)**



(v) $\text{Cos}\dfrac{2\pi}{3} = -\text{Cos}\dfrac{\pi}{3}$ **(K.B + A.B)**

$$= -\frac{1}{2}$$

$$\frac{2\pi}{3} = \pi - \theta$$

$$\theta = \pi - \frac{2\pi}{3}$$

$$= \frac{\pi}{3}$$

(vi) $\text{Cosec}\frac{2\pi}{3} = \text{Cosec}\frac{\pi}{3}$ **(K.B + A.B)**

$= \frac{2}{\sqrt{3}}$

(vii) $\text{Cos}(-450^o) = \text{Cos}\,450^o$

$= \text{Cos}(360^o + 90^o)$

$= \text{Cos}\,90°$

$= 0$

(viii) $\tan(-9\pi) = -\tan 9\pi$

$= -\tan(4\times 2\pi + \pi)$

$= -\tan\pi$

$= -0$

$= 0$

(ix) $\text{Cos}\left(\frac{-5\pi}{6}\right)$ $\because \cos(-\theta) = \cos\theta$

$= -\text{Cos}\frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

$\frac{5\pi}{6} = \pi - \theta$

$\theta = \pi - \frac{5\pi}{6}$

$= \frac{\pi}{6}$

(x) $\text{Sin}\frac{7\pi}{6} = -\text{Sin}\frac{\pi}{6}$

$= -\frac{1}{2}$

$\pi + \theta = \frac{7\pi}{6}$

$\theta = \frac{7\pi}{6} - \pi$

$\theta = \frac{\pi}{6}$

(xi) $\text{Cot}\frac{7\pi}{6} = \cot\frac{\pi}{6}$ **(K.B + A.B)**

$= \sqrt{3}$

(xii) $\text{Cos}225°$

$= -\text{Cos}45°$ \`

$= -\frac{1}{\sqrt{2}}$

$225^o = 180^o + \theta$

$\theta = 45^o$

(5) **Trigonometric identities:**

(K.B + U.B)

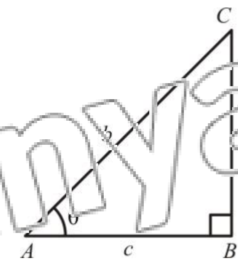
(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \text{cosec}^2\theta$

Proof:

Consider a ΔCAB in which $m\angle CAB = \theta$ radians



(i) By Pythagorean Theorem:

$b^2 = a^2 + c^2$

Div by 'b^2'

$\frac{b^2}{b^2} = \frac{a^2}{b^2} + \frac{c^2}{b^2}$

$1 = \left(\frac{a}{b}\right)^2 + \left(\frac{c}{b}\right)^2$

$1 = \sin^2\theta + \cos^2\theta$

Or $\sin^2\theta + \cos^2\theta = 1$

Proved

(ii) By Pythagorean theorem

$b^2 = a^2 + c^2$

Div by 'c^2'

$\frac{b^2}{c^2} = \frac{a^2}{c^2} + \frac{c^2}{c^2}$

$\left(\frac{b}{c}\right)^2 = \left(\frac{a}{c}\right)^2 + 1$

$\sec^2\theta = \tan^2\theta + 1$

Or $1 + \tan^2\theta = \sec^2\theta$

Proved

(iii) By Pythagorean theorem

$b^2 = a^2 + c^2$

Div by 'a^2'

$\frac{b^2}{a^2} = \frac{a^2}{a^2} + \frac{c^2}{a^2}$

$\left(\frac{b}{a}\right)^2 = 1 + \left(\frac{c}{a}\right)^2$

$\text{cosec}^2\theta = 1 + \cot^2\theta$

Proved

Example 2: (Page # 164) **(A.B)**

Verify that $\tan^4\theta + \tan^2\theta = \tan^2\theta\sec^2\theta$

Solution:

$L.H.S = \tan^4\theta + \tan^2\theta$

$= \tan^2\theta(\tan^2\theta + 1)$

$\because \tan^2\theta+1=\sec^2\theta$

$=\tan^2\theta\sec^2\theta$

= R.H.S

Example 3: (Page # 164) **(A.B)**

Show that $\dfrac{\cot^2\alpha}{\operatorname{cosec}\alpha-1}=\operatorname{cosec}\alpha+1$

Solution:

$$\text{L.H.S}=\frac{\cot^2\alpha}{\operatorname{cosec}\alpha-1}$$

$$\left(\begin{array}{l}\because \operatorname{cosec}^2\theta-\cot^2\theta=1\\ \cot^2\theta=\operatorname{cosec}^2\theta-1\end{array}\right)$$

$$=\frac{\left(\operatorname{cosec}^2\alpha-1\right)}{\operatorname{cosec}\alpha-1}$$

$$=\frac{\left(\operatorname{cosec}\alpha-1\right)\left(\operatorname{cosec}\alpha+1\right)}{\left(\operatorname{cosec}\alpha-1\right)}$$

$$=\operatorname{cosec}\alpha+1=R.H.S$$

Example 4: (Page # 164) **(A.B)**

Express the trigonometric functions in terms of $\tan\theta$.

Solution:

By using reciprocal identity, we can express $\cot\theta$ in terms of $\tan\theta$.

i.e. $\cot\theta=\dfrac{1}{\tan\theta}$

Since $1+\tan^2\theta=\sec^2\theta$

$\Rightarrow \sec\theta=\pm\sqrt{\tan^2\theta+1}$

we have expressed $\sec\theta$ in terms of $\tan\theta$.

$\because \quad \cos\theta=\dfrac{1}{\sec\theta}$

$\Rightarrow \cos\theta=\dfrac{1}{\pm\sqrt{\tan^2\theta+1}}$

Since $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$

$\therefore \sin\theta=\tan\theta\left(\dfrac{1}{\pm\sqrt{\tan^2\theta+1}}\right)$

$$=\frac{\tan\theta}{\pm\sqrt{\tan^2\theta+1}}$$

$\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$

$$=\frac{\pm\sqrt{\tan^2\theta+1}}{\tan\theta}$$

Note **(K.B)**

We can express all the trigonometric functions in terms of one trigonometric function.

Exercise 7.4

In problems 1-6, simplify each expression to a single trigonometric function.

Q.1 $\dfrac{\sin^2 x}{\cos^2 x}$ **(K.B + A.B)**

Solution:

$$\frac{\sin^2 x}{\cos^2 x}=\frac{(\sin x)^2}{(\cos x)^2}$$

$$=\left(\frac{\sin x}{\cos x}\right)^2 \quad \left(\because \tan\theta=\frac{\sin\theta}{\cos\theta}\right)$$

$$=(\tan x)^2$$

$$=\tan^2 x$$

Q.2 $\tan x\sin x\sec x$ **(K.B + A.B)**

Solution:

$\tan x\sin x\sec x$

$$=\frac{\sin x}{\cos x}\times\sin x\times\frac{1}{\cos x}$$

$$=\frac{\sin^2 x}{\cos^2 x} \quad \left(\because \frac{\sin\theta}{\cos\theta}=\tan\theta\right)$$

$$=\left(\frac{\sin x}{\cos x}\right)^2 \quad \left(\because \sec\theta=\frac{1}{\cos\theta}\right)$$

$$=\tan^2 x$$

Q.3 $\dfrac{\tan x}{\sec x}$ **(K.B + A.B)**

Solution:

$$\frac{\tan x}{\sec x}=\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} \quad \because \tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$\because \sec\theta=\frac{1}{\cos\theta}$$

$$\frac{\tan x}{\sec x}=\frac{\sin x}{\cancel{\cos x}}\times\frac{\cancel{\cos x}}{1}$$

$$=\sin x$$

Q.4 $1-\cos^2 x$ **(K.B + A.B)**

Solution:

$1-\cos^2 x$

$=1-\left(1-\sin^2 x\right) \quad \left(\because \sin^2\theta+\cos^2\theta=1\right)$

$=1-1+\sin^2 x$

$=\sin^2 x$

Q.5 $\sec^2 x-1$ **(LHR 2014)** **(K.B + A.B)**

Solution: $\sec^2 x-1$

$=\left(\dfrac{1}{\cos x}\right)^2-1$

$=\dfrac{1}{\cos^2 x}-1$

$=\dfrac{1-\cos^2 x}{\cos^2 x} \quad \left(\because \sin^2\theta+\cos^2\theta=1\right)$

$=\dfrac{\sin^2 x}{\cos^2 x}$

$=\left(\dfrac{\sin x}{\cos x}\right)^2 \quad \left(\because \dfrac{\sin\theta}{\cos\theta}=\tan\theta\right)$

$=\tan^2 x$

Q.6 $\sin^2 x.\cot^2 x$ **(K.B + A.B)**

Solution: $\sin^2 x.\cot^2 x$

$=\sin^2 x.\left(\dfrac{\cos x}{\sin x}\right)^2$

$=\sin^2 x.\dfrac{\cos^2 x}{\sin^2 x}$

$=\cos^2 x$

In problems 7-12, verify the identities.

Q.7 $(1-\sin\theta)(1+\sin\theta)=\cos^2\theta$

(K.B + A.B)

(FSD 2015, SWL 2014, SGD 2014)

Proof:

L.H.S $=(1-\sin\theta)(1+\sin\theta)$

$=(1)^2-(\sin\theta)^2 \quad \left[\because (a+b)(a-b)=a^2-b^2\right]$

$=1-\sin^2\theta$

$=\cos^2\theta \quad \left(\because \sin^2\theta+\cos^2\theta=1\right)$

$=$R.H.S

Q.8 $\dfrac{\sin\theta+\cos\theta}{\cos\theta}=1+\tan\theta$ **(K.B + A.B)**

(LHR 2016, GRW 2014)

Proof:

L.H.S $=\dfrac{\sin\theta+\cos\theta}{\cos\theta}$

$=\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\cos\theta}$

$=\tan\theta+1 \quad \left(\because \tan\theta=\dfrac{\sin\theta}{\cos\theta}\right)$

$=1+\tan\theta$

= R.H.S

Hence Proved

Q.9 $(\tan\theta+\cot\theta)\tan\theta=\sec^2\theta$

(K.B + A.B)

Proof:

L.H.S $=(\tan\theta+\cot\theta)\tan\theta$

$=\tan^2\theta+\cot\theta.\tan\theta$

$=\tan^2\theta+\dfrac{1}{\tan\theta}\times\tan\theta \quad \left(\because \cot\theta=\dfrac{1}{\tan\theta}\right)$

$=\tan^2\theta+1$

$=\sec^2\theta$

=R.H.S

Hence Proved

M-II

L.H.S $=(\tan\theta+\cot\theta)\tan\theta$

$=\left(\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}\right)\times\dfrac{\sin\theta}{\cos\theta}$

$=\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta.\cos\theta}\times\dfrac{\sin\theta}{\cos\theta}$

$=\dfrac{1}{\cos\theta}$

$=\sec\theta$

=R.H.S

Proved

Q.10

$(\cot\theta+\operatorname{cosec}\theta)(\tan\theta-\sin\theta)=\sec\theta-\cos\theta$

(FSD 2017) **(K.B + A.B)**

Proof:

L.H.S $=(\cot\theta+\operatorname{cosec}\theta)(\tan\theta-\sin\theta)$

$=\left(\dfrac{\cos\theta}{\sin\theta}+\dfrac{1}{\sin\theta}\right)\left(\dfrac{\sin\theta}{\cos\theta}-\sin\theta\right)$

$\left(\because \operatorname{cosec}\theta=\dfrac{1}{\sin\theta}, \cot\theta=\dfrac{\cos\theta}{\sin\theta}, \tan\theta=\dfrac{\sin\theta}{\cos\theta}\right)$

$=\dfrac{\cos\theta+1}{\sin\theta}.\dfrac{\sin\theta-\sin\theta\cos\theta}{\cos\theta}$

$= \frac{1+\cos\theta}{\sin\theta}\cdot\frac{\sin\theta(1-\cos\theta)}{\cos\theta}$

$= \frac{(1+\cos\theta)(1-\cos\theta)}{\cos\theta}$

$= \frac{1-\cos^2\theta}{\cos\theta}$ $\left(\because (a+b)(a-b)=a^2-b^2\right)$

$= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$

$= \sec\theta - \cos\theta$ $\left(\because \sec\theta = \frac{1}{\cos\theta}\right)$

= R.H.S

Hence Proved

Q.11 $\frac{\sin\theta+\cos\theta}{\tan^2\theta-1} = \frac{\cos^2\theta}{\sin\theta-\cos\theta}$ **(K.B + A.B)**

(FSD 2018, MTN 2017, SGD 2015)

Proof:

$\text{L.H.S} = \frac{\sin\theta+\cos\theta}{\tan^2\theta-1}$

$= \frac{\sin\theta+\cos\theta}{\frac{\sin^2\theta}{\cos^2\theta}-1}$ $\left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$= \frac{\sin\theta+\cos\theta}{\frac{\sin^2\theta-\cos^2\theta}{\cos^2\theta}}$

$= (\sin\theta+\cos\theta)\times\frac{\cos^2\theta}{\sin^2\theta-\cos^2\theta}$

$= (\sin\theta+\cos\theta)\times\frac{\cos^2\theta}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$

$= \frac{\cos^2\theta}{\sin\theta-\cos\theta}$

= R.H.S

Hence Proved

Q.12 $\frac{\cos^2\theta}{\sin\theta}+\sin\theta = \text{cosec}\,\theta$ **(K.B + A.B)**

Proof:

$\text{L.H.S} = \frac{\cos^2\theta}{\sin\theta}+\sin\theta$

$= \frac{\cos^2\theta+\sin^2\theta}{\sin\theta}$ $\left(\because \sin^2\theta+\cos^2\theta=1\right)$

$= \frac{1}{\sin\theta}$ $\left(\because \text{cosec}\,\theta = \frac{1}{\sin\theta}\right)$

$= \text{cosec}\,\theta$

= R.H.S

Hence Proved

Q.13 $(\sec\theta-\cos\theta) = \tan\theta\times\sin\theta$

(BWP 2015, D.G.K 2015, 16, 17) **(K.B + A.B)**

Proof:

L.H.S = $\sec\theta - \cos\theta$

$= \frac{1}{\cos\theta}-\cos\theta$ $\left(\because \sec\theta = \frac{1}{\cos\theta}\right)$

$= \frac{1-\cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$ $\left(\because \sin^2\theta+\cos^2\theta=1\right)$

$= \frac{\sin\theta}{\cos\theta}\times\sin\theta$

$= \tan\theta\times\sin\theta$ $\left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

= R.H.S

Hence Proved

Q.14 $\frac{\sin^2\theta}{\cos\theta}+\cos\theta = \sec\theta$ **(K.B + A.B)**

(SWL 2017, SGD 2014, D.G.K 2016, 17)

Proof:

$\text{L.H.S} = \frac{\sin^2\theta}{\cos\theta}+\cos\theta$

$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}$ $\left(\because \sin^2\theta+\cos^2\theta=1\right)$

$= \sec\theta$ $\left(\because \sec\theta = \frac{1}{\cos\theta}\right)$

= R.H.S

Hence Proved

Q.15 $\tan\theta+\cot\theta = \sec\theta\,\text{cosec}\,\theta$

(K.B + A.B)

Proof:

L.H.S = $\tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$

$\left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}\right)$

$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta \times \sin\theta}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$

$= \sec\theta \times \text{cosec}\,\theta$

$\left(\because \sec\theta = \frac{1}{\cos\theta}, \text{cosec}\,\theta = \frac{1}{\sin\theta}\right)$

= R.H.S

Hence Proved

Q.16 $(\tan\theta + \cot\theta)(\cos\theta + \sin\theta)$
$= \sec\theta + \text{cosec}\,\theta$ **(K.B + A.B)**

Proof:

L.H.S $= (\tan\theta + \cot\theta)(\cos\theta + \sin\theta)$

$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)(\cos\theta + \sin\theta)$

$\left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} \times (\cos\theta + \sin\theta)$

$= \frac{1}{\sin\theta\,\cos\theta}(\cos\theta + \sin\theta)$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}$

$= \frac{\cancel{\cos\theta}}{\sin\theta\cancel{\cos\theta}} + \frac{\cancel{\sin\theta}}{\cancel{\sin\theta}\cos\theta}$

$= \frac{1}{\sin\theta} + \frac{1}{\cos\theta}$

$\left(\because \text{cosec}\,\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}\right)$

$= \text{cosec}\,\theta + \sec\theta$

= R.H.S

Hence Proved

Q.17 $\sin\theta(\tan\theta + \cot\theta) = \sec\theta$ **(K.B + A.B)**

Proof:

L.H.S $= \sin\theta(\tan\theta + \cot\theta)$

$= \sin\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= \cancel{\sin\theta}\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\cancel{\sin\theta}}\right)$

$= \frac{1}{\cos\theta}$

$= \sec\theta$

= R.H.S

Hence Proved

Q.18 $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\text{cosec}\,\theta$

(K.B + A.B)

Proof:

L.H.S $= \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{(1+\cos\theta)^2 + \sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{1 + 2\cos\theta + 1}{\sin\theta(1+\cos\theta)}$

$= \frac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$= \frac{2}{\sin\theta}$

$= 2\text{cosec}\,\theta$

= R.H.S

Hence Proved

Q.19 $\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} = 2\text{cosec}^2\theta$

(K.B + A.B)

(FSD 2015, BWP 2017, RWP 2016, 17, SGD 2016, MTN 2016, D.G.K 2017)

Proof:

L.H.S $= \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$

$= \frac{1 + \cancel{\cos\theta} + 1 - \cancel{\cos\theta}}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{2}{1-\cos^2\theta}$

$= \frac{2}{\sin^2\theta}$

$= 2\text{cosec}^2\theta$

= R.H.S

Hence Proved

Q.20 $\frac{1+\sin\theta}{1-\sin\theta}-\frac{1-\sin\theta}{1+\sin\theta}=4\tan\theta\sec\theta$

(D.G.K 2015) (K.B + A.B)

Proof:

$$\text{L.H.S}=\frac{1+\sin\theta}{1-\sin\theta}-\frac{1-\sin\theta}{1+\sin\theta}$$
$$=\frac{(1+\sin\theta)^2-(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$
$$=\frac{4(1)(\sin\theta)}{1-\sin^2\theta}$$
$$=\frac{4\sin\theta}{\cos^2\theta}$$
$$=\frac{4\sin\theta}{\cos\theta}\times\frac{1}{\cos\theta}$$
$$=4\tan\theta\sec\theta$$
= R.H.S

Hence Proved

Q.21 $\sin^3\theta=\sin\theta-\sin\theta\cos^2\theta$

(LHR 2014, SWL 2015) (K.B + A.B)

Proof:

$$\text{R.H.S}=\sin\theta-\sin\theta\cos^2\theta$$
$$=\sin\theta(1-\cos^2\theta)$$
$$=\sin\theta\times\sin^2\theta$$
$$=\sin^3\theta$$
= L.H.S

Hence Proved

Q.22 $\cos^4\theta-\sin^4\theta=\cos^2\theta-\sin^2\theta$

(K.B + A.B)

Proof:

$$\text{L.H.S}=\cos^4\theta-\sin^4\theta$$
$$=(\cos^2\theta)^2-(\sin^2\theta)^2$$
$$=(\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta)$$
$$=1(\cos^2\theta-\sin^2\theta)$$
$$=\cos^2\theta-\sin^2\theta$$
= R.H.S

Hence Proved

Q.23 $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}=\frac{\sin\theta}{1-\cos\theta}$

(LHR 2016, FSD 2015, 16, SWL 2016)

(K.B + A.B)

Proof:

$$\text{L.H.S}=\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

Multiply and divide by $1-\cos\theta$

$$=\sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}$$
$$=\sqrt{\frac{1-\cos^2\theta}{(1-\cos\theta)^2}}$$
$$=\sqrt{\frac{\sin^2\theta}{(1-\cos\theta)^2}}$$
$$=\sqrt{\left(\frac{\sin\theta}{1-\cos\theta}\right)^2}$$
$$=\frac{\sin\theta}{1-\cos\theta}$$
= R.H.S

Hence Proved

Q.24 $\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=\frac{\sec\theta+1}{\tan\theta}$ **(K.B + A.B)**

Proof:

$$\text{L.H.S}=\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$$

Multiply & divide by $\sec\theta+1$

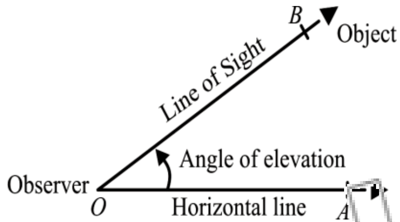
$$=\sqrt{\frac{\sec\theta+1}{\sec\theta-1}\times\frac{\sec\theta+1}{\sec\theta+1}}$$
$$=\sqrt{\frac{(\sec\theta+1)^2}{\sec^2\theta-1}}$$
$$=\sqrt{\frac{(\sec\theta+1)^2}{\tan^2\theta}}$$
$$=\sqrt{\left(\frac{\sec\theta+1}{\tan\theta}\right)^2}$$
$$=\frac{\sec\theta+1}{\tan\theta}$$
= R.H.S

Hence Proved

Angle of Elevation **(K.B + A.B)**

(FSD 2015, MTN 2015)

Angle of elevation is the angle between the horizontal line and the line of sight to an object above the horizontal line.

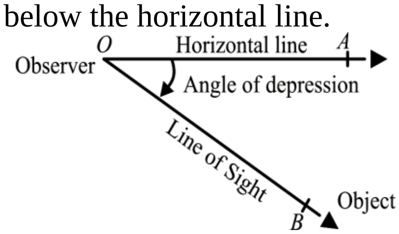


Angle between horizontal line OA and a line segment $\overline{OB}$ from eye O to object B above point A is called angle of elevation.

Angle of Depression **(K.B + A.B)**

(LHR 2015, BWP 2014, RWP 2015)

Angle of depression is the angle between the horizontal line and the line of sight to an object below the horizontal line.

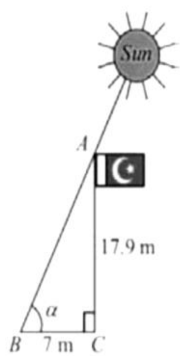


Angle between horizontal line $\overline{OA}$ and a line segment $\overline{OC}$ from eye O to object C below point A is called angle of depression.

Example 1: (Page # 166) **(K.B + A.B)**

A flagpole 17.9 meter high casts a 7 meter shadow. Find the angle of elevation of the sun.

Solution:



From the figure, we observe that a is the angle of elevation.

Using the fact that

$$\tan\alpha = \frac{AC}{BC} = \frac{17.9}{7}$$

Solving for α gives us

$$\alpha = \tan^{-1}\left(\frac{17.9}{7}\right)$$

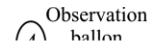
$$= (68.6666)^\circ$$

$\Rightarrow \quad \alpha = 68°40'$

Example 2: (Page # 167) **(K.B + A.B)**

An observation balloon is 4280 meter above the ground and 9613 meter away from a farmhouse. Find the angle of depression of the farmhouse as observed from the observation balloon.

Solution:



For problems of this type the angle of elevation of ***A*** from ***8*** is considered equal to the angle of depression of ***B*** from ***A*,** as shown in the diagram.

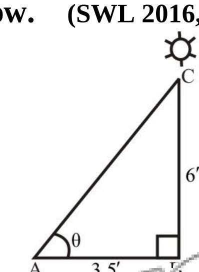
$$\tan\alpha = \frac{AC}{BC} = \frac{4280}{9613} = 0.44523$$

$$\alpha = \tan^{-1}(0.44523) = 24°$$

So, angle of depression 24°

Exercise 7.5

Q.1 Find the angle of elevation of the sun if a 6 feet man casts a 3.5 feet shadow. **(SWL 2016, 17)** **(A.B)**



Given:

Height of man $= m\overline{BC} = 6'$

Length of shadow $= m\overline{AB} = 3.5'$

Required:

Angle of elevation $= \theta = ?$

Solution:

From Δ ABC

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$= \frac{6}{3.5}$$

$\tan\theta = 1.714$

$\Rightarrow \theta = \tan^{-1} 1.714$

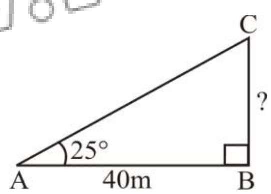
$\theta = 59.74^{o}$

or $\theta = 59^{o}\,44'\,37''$

Result:

$\therefore$ Angle of elevation of the sun is $59^{o}\,44'\,37''$

Q.2 A tree casts a 40 meter shadow when the angle of elevation of the sun is 25°. Find the height of the tree. **(A.B)**



Given:

Length of shadow $= m\overline{AB} = 40\text{m}$

Angle of elevation $= \theta = 25°$

Required:

Height of tree $= m\overline{BC} = ?$

Solution:

From ΔABC,

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan 25^{o} = \frac{m\overline{BC}}{40\text{m}}$$

$$m\overline{BC} = \tan 25^{o} \times 40\text{m}$$

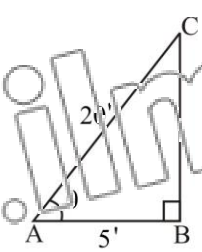
$= 18.65\text{m}$

Result

$\therefore$ Height of tree 18.65m

Q.3 A 20 feet long ladder is leaning against a wall. The bottom of the ladder is 5 feet from the base of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground. **(A.B)**

Given:



Length of ladder

$= m\overline{AC} = 20'$

Distance between ladder and wall $= m\overline{AB} = 5'$

Required:

Angle of elevation $= \theta = ?$

From Δ ABC,

$$\cos\theta = \frac{m\overline{AB}}{m\overline{AC}}$$

$$\cos\theta = \frac{5}{20}$$

$$\theta = \cos^{-1}\frac{1}{4}$$

$= 75.52$

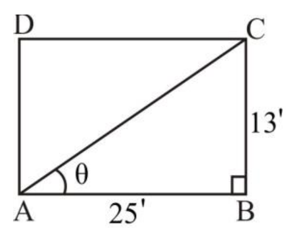
$= 75^{o}\,31'\,21''$

Result:

$\therefore$ Angle of elevation $= 75^{o}\,31'\,21''$

Q.4 The base of a rectangle is 25 feet and the height of the rectangle is 13 feet. Find the angle that the diagonal of the rectangle makes with base. **(MTN 2015)** **(A.B)**

Given:



Length of rectangle $= m\overline{AB} = 25'$

Width of rectangle $= m\overline{BC} = 13'$

Required:

Angle of elevation $= \theta = ?$

Solution:

From Δ ABC,

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan\theta = \frac{13}{25}$$

$$\theta = \tan^{-1}\frac{13}{25}$$

$= 27.47°$

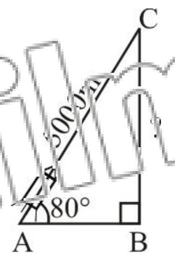
$\theta = 27^{o}\,28'\,28''$

Result:

$\therefore$ Angle of elevation $= 27^{o}\,28'\,28''$

Q.5 A rocket is launched and climbs at a constant angle of 80°. Find the altitude of the rocket after it travels 5000 meters. (SGD 2015) **(A.B)**

Given:



Distance traveled by rocket

$= m\overline{AC} = 5000\text{m}$

Angle of elevation = 80°

Required:

Height (altitude) of rocket $= m\overline{BC} = ?$

Solution:

From Δ ABC,

$$\sin\theta = \frac{m\overline{BC}}{m\overline{AC}}$$

$$\sin 80^{o} = \frac{m\overline{BC}}{5000\text{m}}$$

$$m\overline{BC} = \sin 80^{o} \times 5000\text{m}$$

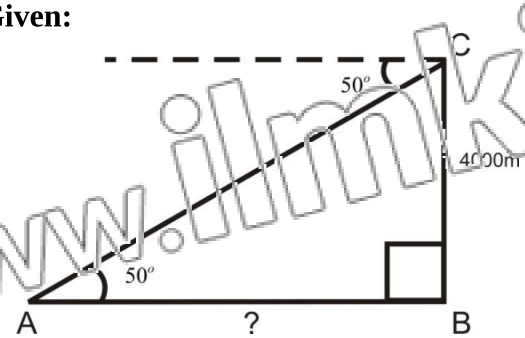
= 4924.04m

Result:

∴ Height of rocket = 4924.04m

Q.6 An aero plane pilot flying at an altitude of 4000m wishes to make an approach to an airport at an angle of 50° with the horizontal. How far from the airport will the plane be when the pilot begins to descend? (A.B)

Given:



Height of plane $= m\overline{BC} = 4000\text{m}$

Angle of depression = 50°

Required:

Distance between plane and airport $= m\overline{AB} = ?$

Solution:

From Δ ABC,

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan 50^{o} = \frac{4000\text{m}}{m\overline{AB}}$$

$$m\overline{AB} = \frac{4000\text{m}}{\tan 50^{o}}$$

= 3356.40m

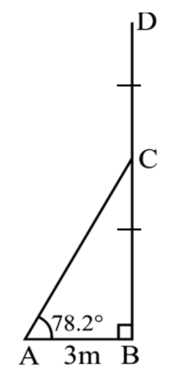
Result:

∴ Plane is 3356.40m away from airport

Q.7 Required: **(A.B)**

Height of pole $= m\overline{BD} = ?$

Given:



Distance between wire and pole $= m\overline{AB} = 3\text{m}$

Angle of elevation = 78.2°

Solution:

From Δ ABC,

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan 78.2^{o} = \frac{m\overline{BC}}{3\text{m}}$$

$$m\overline{BC} = \tan 78.2^{o} \times 3\text{m}$$

$$m\overline{BC} = 14.36\text{m}$$

$$m\overline{BD} = m\overline{BC} + m\overline{CD}$$

$= 2m\overline{BC}$

=2(14.36m)

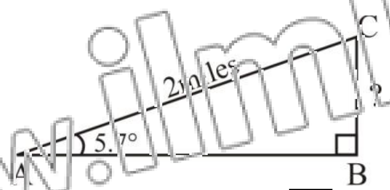
= 28.72 m

Result:

∴ Height of pole = 28.78m

Q.8 A road is inclined at an angle 5.7°. Suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we? **(A.B)**

Given:



Distance travelled by car $= m\overline{AC} = 2\text{miles}$

Angle of elevation = 5.7°

Height of car from sea level $= m\overline{BC} = ?$

From ΔABC,

$$\sin\theta = \frac{m\overline{BC}}{m\overline{AC}}$$

$$\sin 5.7° = \frac{m\overline{BC}}{2\text{miles}}$$

$$m\overline{BC} = \text{Sin}5.7° \times 2\text{miles}$$

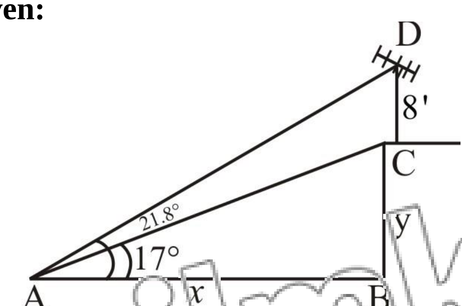
= 0.1986 miles

Result

$\therefore$ Car is 0.1986 miles up from sea level.

Q.9 A television antenna of 8 feet height is located on the top of the house. From a point on the ground the angle of elevation to the top of the house is 17° and the angle of elevation to the top of the antenna is 21.8°. Find the height of the house. **(A.B)**

Given:



Height of antenna $= m\overline{CD} = 8$ feet

$m\angle BAC = 17°$

$m\angle BAD = 21.8°$

Required:

Height of house $= m\overline{BC} = y = ?$

Solution:

Let $m\overline{AB} = x$

From ΔBAC,

$$\tan 17° = \frac{m\overline{BC}}{m\overline{AB}}$$

$$0.3057 = \frac{y}{x}$$

$$0.3057x = y \rightarrow (\text{i})$$

From ΔBAD,

$$\tan 21.8° = \frac{m\overline{BD}}{m\overline{AB}}$$

$$0.4 = \frac{y+8}{x}$$

$$m\overline{BD} = m\overline{BC} + m\overline{CD}$$

$$0.4x = \text{y} + 8 \rightarrow (\text{ii})$$

Putting the value of 'y'

$0.4x = 0.3057x + 8$

$0.4x - 0.3057x = 8$

$0.0943x = 8$

$$x = \frac{8}{0.0943}$$

$\Rightarrow x = 84.86$

Put in equation (i)

$y = 84.86\ (0.3057)$

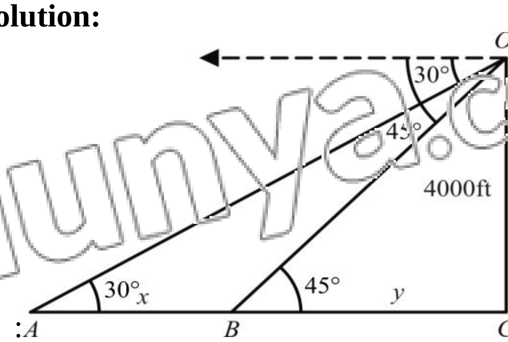
$y = 25.94$

Result:

$\therefore$ Therefore, height of house = 25.94 feet

Q.10 From an observation point, the angles of depression of two boats in line with this point are found to be $30°$ **and** $45°$**. Find the distance between the two boats if the point of observation is 4000 feet high.** **(A.B)**

Solution:



Let A and B are two Boat and O is the point of observation.

$m\angle OAC = 30°$

$m\angle OBC = 45°$

$\overline{\text{OC}} = 4000\text{ft}$

Let distance between two boat = $AB = x$

And $\overline{BC} = y$

In right angled ΔOBC

$$\tan m\angle OBC = \frac{\overline{OC}}{\overline{BC}}$$

$$\tan 45^{o} = \frac{4000}{y}$$

$$1 = \frac{4000}{y}$$

$$y = 4000$$

In ΔOAC

$$\tan m\angle OAC = \frac{\overline{OC}}{\overline{AC}}$$

$$\tan 30^{o} = \frac{4000}{x+y}$$

$$x + y = \frac{4000}{\text{Tan}30^{o}} = \frac{4000}{\frac{1}{\sqrt{3}}}$$

$$= 4000 \times \sqrt{3}$$

$$x + 4000 = 4000\sqrt{3}$$

$$x = 4000\sqrt{3} - 4000$$

$$= \left(\sqrt{3} - 1\right)4000$$

$$= (1.732 - 1)4000$$

$$= 0.732 \times 4000$$

$$x = 2928\text{ft}$$

Result:

$\therefore$ Distance between the two boats

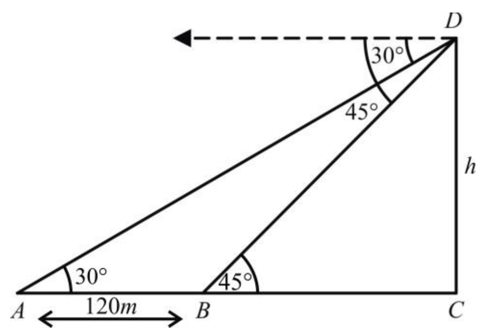
$$= m\overline{AB} = 2928\,ft$$

Q.11 Two ships, which are in line with the base of a vertical cliff, are 120 meters apart. The angles of depression from the top of the cliff to the ships are $30°$ and $45°$, as show in the diagram. **(A.B)**

(a) Calculate the distance *BC*

(b) Calculate the height *CD*, of the cliff.

Solution:



In the figure A and B are two boats and $\overline{CD}$ is height of cliff. height of cliff $= \overline{CD} = h$

Distance between two boats

$$= \overline{AB} = 120m$$

Let $\overline{BC} = x$

$m\angle DAC = 30°$ and $m\angle DBC = 45°$

In ΔDBC

$$\tan m\angle DBC = \frac{\overline{DC}}{\overline{BC}}$$

$$\tan 45° = \frac{h}{x}$$

$$1 = \frac{h}{x}$$

$x = h \rightarrow (i)$

In ΔDAC

$$\tan m\angle DAC = \frac{\overline{DC}}{\overline{AC}}$$

$$\tan 30^\circ = \frac{h}{120+x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{120+x}$$

$$120 + x = \sqrt{3}h$$

$$120 + h = \sqrt{3}h$$

$$120 = \sqrt{3}h - h$$

$$= \left(\sqrt{3}-1\right)h$$

$$= (1.732-1)h$$

$$120 = 0.732h$$

$$h = \frac{120}{0.732}$$

$$h = 163.93$$

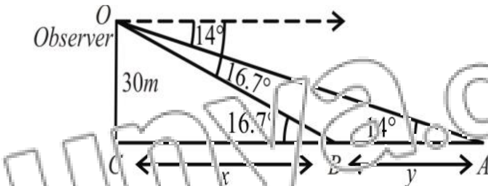
$$\therefore \overline{CD} = h = 163.93m$$

Result:

Hence height of cliff is 163.93 m, and $\overline{BC}$ is also 163.93m.

Q.12 Suppose that we are standing on a bridge 30 **feet above a river watching a log (piece of wood) floating toward we. If the angle with the horizontal to the front of the log is** 16.7° **and angle with the horizontal to the back of the log is** 14° **how long is the log? (A.B)**

Solution:



Here O is the observer

Length of log $= m\overline{AB} = y$

$m\overline{CB} = x$

$m\overline{OC} = 30m$

$m\angle OBC = 16.7^\circ$

and $m\angle OAC = 14^\circ$

In ΔOBC

$$\tan m\angle OBC = \frac{m\overline{OC}}{m\overline{BC}}$$

$$\tan 16.7^\circ = \frac{30}{x}$$

$$x \times \tan 16.7^\circ = 30$$

$$x = \frac{30}{\tan 16.7} = \frac{30}{0.30001}$$

$$x = 99.996$$

In ΔOAC

$$\tan m\angle OAC = \frac{m\overline{OC}}{m\overline{CA}}$$

$$\tan 14^\circ = \frac{30}{x+y}$$

$$x + y = \frac{30}{\tan 14^\circ}$$

$$x + y = \frac{30}{0.24933} = 120.3224$$

$$x + y = 120.3224$$

$$99.996 + y = 120.3224$$

$$y = 120.3224 - 99.996$$

$$y = 20.33m$$

Result:

$\therefore$ Lenght of log is $20.33m$

Miscellaneous Exercise 7

Q.1 Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

(1) The union of two non-collinear rays, which have common end point is called **(K.B)**

(a) An angle (b) A degree
(c) A minute (d) A radian

(2) The system of measurement in which the angle is measured in radians is called **(K.B)**

(a) CGS system (b) Sexagesimal
(c) MKS system (d) Circular system

(3) $20° =$ **(K.B)**

(a) $360'$ (b) $630'$
(c) $1200'$ (d) $3600'$

(4) $\frac{3\pi}{4}$ **radians =** **(GRW 2014, RWP 2015, MTN 2015, FSD 2018) (K.B)**

(a) $115°$ (b) $135°$
(c) $150°$ (d) $30°$

(5) If $\tan\theta = \sqrt{3}$**, then** θ **is equal to** **(LHR 2014, D.G.K 2015) (K.B)**

(a) $90°$ (b) $45°$
(c) $60°$ (d) $30°$

(6) $\sec^2\theta =$ **(K.B)**

(a) $1-\sin^2\theta$ (b) $1+\tan^2\theta$
(c) $1+\cos^2\theta$ (d) $1-\tan^2\theta$

(7) $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$ **(SGD 2014) (K.B)**

(a) $2\sec^2\theta$ (b) $2\cos^2\theta$
(c) $\sec^2\theta$ (d) $\cos\theta$

(8) $\frac{1}{2}\text{cosec}45°$ **(GRW 2014, FSD 2015, D.G.K 2014) (K.B)**

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$
(c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{2}$

(9) $\sec\theta\cot\theta =$ **(K.B)**

(a) $\sin\theta$ (b) $\frac{1}{\cos\theta}$
(c) $\frac{1}{\sin\theta}$ (d) $\frac{\sin\theta}{\cos\theta}$

(10) $\text{cosec}^2\theta - \cot^2\theta =$ **(LHR 2014, FSD 2014, SWL 2014, RWP 2014) (K.B)**

(a) -1 (b) 1
(c) 0 (d) $\tan\theta$

ANSWER KEY

1	2	3	4	5	6	7	8	9	10
a	d	c	b	c	b	a	b	c	b

Q.2 Write short answers. of the following questions.

(i) Define an angle. **(K.B)**

Ans. See definition Page # 216

(ii) What is the sexagesimal system of measurement of angles? **(K.B)**

Ans. See definition Page # 216

(iii) How many minutes are in two right angles? **(K.B)**

Ans: $\text{Angle} = \theta = 2\times 90^\circ$

$= 180^\circ$

$= (180\times 60)'$

$= 10800'$

(iv) Define radian measure of an angle. **(K.B)**

(v) Convert $\frac{\pi}{4}$ radian to degree measure. **(K.B)**

Ans: Convert $\frac{\pi}{4}$ radian to degree measure.

$\frac{\pi}{4}$ radian $= \frac{\pi}{4}\times 1$ radian

$= \frac{\pi}{4}\times\frac{180}{\pi}$

$= 45^\circ$

(vi) Convert 15° to radians. **(K.B)**

Ans: **(D.G.K 2015)**

Convert 15° to radian

$15^\circ = 15\times 1^\circ = 15\times\frac{180}{\pi} = \frac{\pi}{12}$ radian

(vii) What is radian measure of the central angle of an arc 50m long on the circle of radius 25m? **(K.B)**

Ans: Arc length = l = 50m

Radian of circle = r = 25m

Radian measure = θ = ?

$l = r\theta$

$\theta = \frac{l}{r} = \frac{50}{25}$

$\theta = 2$ radian

(viii) Find r when $l = 56cm$ and $\theta = 45^\circ$ **(K.B)**

Ans: $r = ?$

$l = 56cm$

$\theta = 45^\circ$

$= 45\times 1^\circ$

$= 45\times\frac{\pi}{180}$ radian

$= \frac{\pi}{4}$

Putting the values

$l = r\theta$

$r = \frac{l}{\theta} = \frac{56}{\pi/4}$

$= 56\times\frac{4}{\pi} = \frac{56\times 4}{22/7}$

$= 56\times 4\times\frac{7}{22}$

$= \frac{784}{11}$

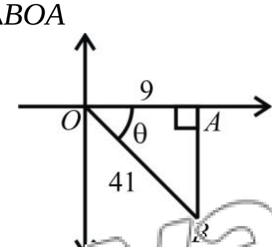
$r = 71.27cm$

(ix) Find $\tan\theta$ when $\cos\theta = \frac{9}{41}$ and terminal side of the angle θ is in fourth quadrant. **(K.B + A.B)**

Ans: Solution:

$\cos\theta = \frac{9}{41}$ and terminal side of the angle θ is in fourth quadrant.

In ΔBOA



Base $= m\overline{OA} = 9$

Hyp $= m\overline{OB} = 41$

Perp $= m\overline{AB} = ?$

$(\text{Perp})^2 = (\text{Hyp})^2 - (\text{Base})^2$

$(m\overline{AB})^2 = (m\overline{OB})^2 - (m\overline{OA})^2$

$= (41)^2 - (9)^2$

$= 1681 - 81$

$(m\overline{AB})^2 = 1600$

$m\overline{AB} = \sqrt{1600}$

$m\overline{AB} = 40$

$\text{Tan}\theta = \dfrac{m\overline{AB}}{m\overline{OA}}$

$\text{Tan}\theta = \dfrac{40}{9}$

(x) **Prove that** $(1-\sin^2\theta)(1+\tan^2\theta)=1$

(K.B + A.B)

Ans: L.H.S=$(1-\sin^2\theta)(1+\tan^2\theta)$

$=\cos^2\theta\times\sec^2\theta$

$=\cos^2\theta\times\dfrac{1}{\cos^2\theta}$

$=1$

=R.H.S

Q.3 Fill in the blanks

(i) π radians = _________degree. **(K.B)**

(ii) The terminal side of angle $235°$ lies in _________quadrant. **(K.B)**

(iii) Terminal side of the angle $-30°$ lies in _________quadrant. **(K.B)**

(iv) Area of a circular sector is _________. **(K.B)**

(v) If $r = 2cm$ and $\theta = 3$ radian, then area of the circular sector is_________. **(K.B)**

(vi) The general form of the angle $480°$ is _________. **(K.B)**

(vii) If $\sin\theta = \dfrac{1}{2}$, then $\theta =$ _________. **(K.B)**

(viii) If $\theta = 300°$, then $\sec(-300)° =$_________. **(K.B)**

(ix) $1+\cot^2\theta =$ _________. **(K.B)**

(x) $\sec\theta - \tan\theta =$_________. **(K.B)**

ANSWER KEY

(i) $180°$

(ii) III

(iii) IV

(iv) $\dfrac{1}{2}r^2\theta$

(v) $6cm^2$

(vi) $2k\pi + 120°$ where $k=1$

(vii) $\theta = 30°$ or $\dfrac{\pi}{6}$ and

(viii) 2

(ix) $\text{cosec}^2\theta$

(x) $\dfrac{1-\sin\theta}{\cos\theta}$

CUT HERE

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 The system of measurement in which the angle is measured in radians is called:
(A) CGS system **(B)** Sexagesimal system
(C) MKS system **(D)** Circular system

2 $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$
(A) $2\sec^2\theta$ **(B)** $2\cos^2\theta$
(C) $\sec^2\theta$ **(D)** $\cos\theta$

3 Two or more than two angles with the same initial and terminal sides are called _________ angles.
(A) Adjacent **(B)** Acute
(C) Conterminal **(D)** All

4 $\operatorname{cosec}30^o =$ ___________
(A) $\sqrt{3}$ **(B)** 2
(C) $\frac{2}{\sqrt{3}}$ **(D)** $\frac{1}{\sqrt{2}}$

5 $2\cos^2\theta-1=$ ___________
(A) $1-\sin^2\theta$ **(B)** $1-2\sin^2\theta$
(C) $1+\sin^2\theta$ **(D)** $1+2\sin^2\theta$

6 A part of the circle bounded by an arc and a chord is called ________ of a circle.
(A) Arc **(B)** Segment
(C) Sector **(D)** Radius

7 $\frac{3\pi}{4}$ **radians =**
(A) 115^o **(B)** 135^o
(C) 150^o **(D)** 30^o

Q.2 Give Short Answers to following Questions. **(5×2=10)**
(i) How many minutes are in two right angles?
(ii) Find area of the sector inside a central angle of 20^o in a circle of radius 7m.
(iii) Find $\sec(-300^o)$ without using table or calculator.
(iv) Verify the identity $\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=\frac{\sec\theta+1}{\tan\theta}$
(v) A tree casts a 40 meter shadow when the angle of elevation of the sun is 25^o. Find the height of the tree.

Q.3 Answer the following Questions. **(4+4=8)**
(a) If $\tan\theta=\frac{4}{3}$ and $\sin\theta<0$, find the values of other trigonometric functions at θ.
(b) From an observation point, the angles of depression of two boats in line with this point are found to 30^o and 45^o. Find the distance between the two boats if the point of observation is 4000 feet high.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.