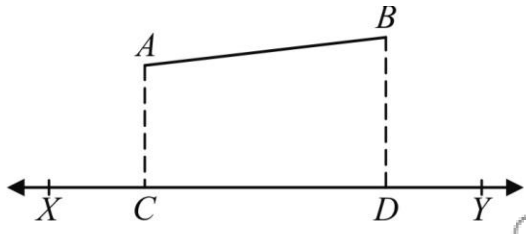


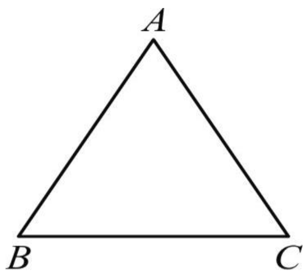
UNIT 8

PROJECTION OF A SIDE OF A TRIANGLE

Acute Angled Triangle **(K.B)**

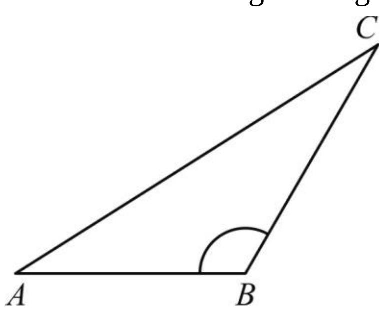
A triangle having all angles less then 90^o is called acute angled triangle.

i.e In the fig, ΔABC is an acute angle triangle.



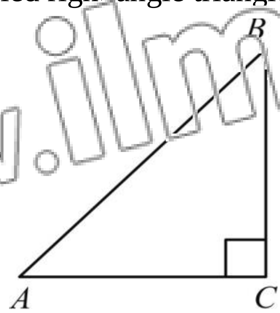
Obtuse Angled Triangle **(K.B)**

A triangle having one of its angles greater than $90°$ is called obtuse angle triangle.



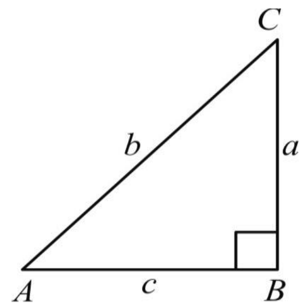
Right Angled Triangle **(K.B)**

A triangle having one of its angles equal to $90°$ is called right angle triangle.



Pythagoras's Theorem **(K.B)**

It states that **"**In a right angled triangle, square of hypotenuse is equal to, sum of squares of other two sides". From the ΔABC, $b^2 = a^2 + c^2$

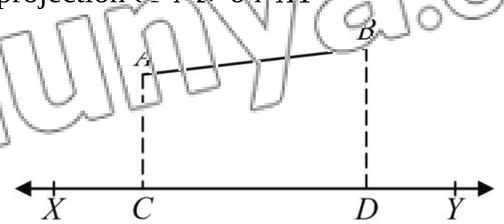


Apollonius Theorem **(K.B)**

It states that **"**In any triangle, the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side".

Projection **(K.B)**

Image on Footing is called projection. $\overline{CD}$ is projection of $\overline{AB}$ on $\overleftrightarrow{XY}$



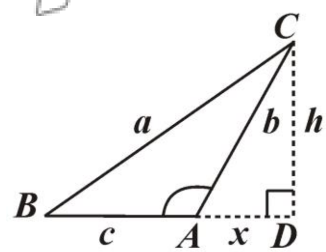
Theorem 1 **(A.B)**

8.1(*i*)

In an obtuse angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the squares on the sides containing the obtuse angle together with twice the rectangle contained by one of the sides, and the projection on it of the other.

Given:

ABC is a triangle having an obtuse angle BAC at A. Draw $\overline{CD}$ perpendicular on $\overline{BA}$ produced. So that $\overline{AD}$ is the projection of $\overline{AC}$ on $\overline{BA}$ produced. Take $m\overline{BC}=a, m\overline{CA}=b$, $m\overline{AB}=c, m\overline{AD}=x$ and $m\overline{CD}=h$.



To Prove:

$(\overline{BC})^2=(\overline{AC})^2+(\overline{AB})^2+2(m\overline{AB})\ (m\overline{AD})$ *i.e.*, $a^2=b^2+c^2+2cx$

Proof:

Statements	Reasons
In $\angle rt\Delta CDA$,	
$m\angle CDA=90°$	Given
$\therefore (\overline{AC})^2=(\overline{AD})^2+(\overline{CD})^2$	Pythagoras Theorem
or $b^2=x^2+h^2$ (i)	
In $\angle rt\Delta CDB$,	
$m\angle CDB=90°$	Given
$\therefore (\overline{BC})^2=(\overline{BD})^2+(\overline{CD})^2$	Pythagoras Theorem
or $a^2=(c+x)^2+h^2$	$\overline{BD}=BA+\overline{AD}$
$=c^2+2cx+x^2+h^2$ (ii)	
Hence $a^2=c^2+2cx+b^2$	Using (i) and (ii)
i,e., $a^2=b^2+c^2+2cx$	
or $(\overline{BC})^2=(\overline{AC})^2+(\overline{AB})^2+2(\text{m}\,\overline{AB})(\text{m}\,\overline{AD})$	

Theorem 2 **(A.B)**

8.1(*ii*)

In any triangle, the square on the side opposite to acute angle is equal to sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projection on it of the other.

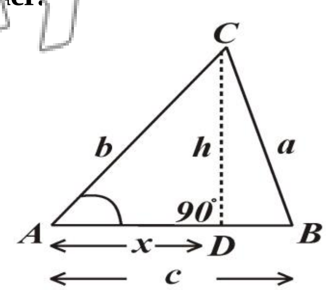
Given:

ΔABC with an acute angle CAB at A.

Take $m\overline{BC}=a$, $m\overline{CA}=b$ and $m\overline{AB}=c$

Draw $\overline{CD}\perp\overline{AB}$ so that $\overline{AD}$ is projection of $\overline{AC}$ on $\overline{AB}$

Also, $m\overline{AD}=x$ and $m\overline{CD}=h$



To Prove:

$\left(\overline{BC}\right)^2=\left(\overline{AC}\right)^2+\left(\overline{AB}\right)^2-2\left(m\overline{AB}\right)\left(m\overline{AD}\right)$

i.e $a^2=b^2+c^2-2cx$

Proof:

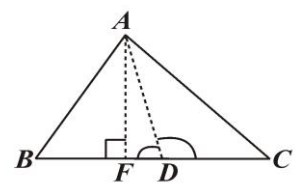
Statements	Reasons
In $\angle rt\Delta CDA$	
$m\angle CDA = 90°$	Given
$\left(\overline{AC}\right)^2 = \left(\overline{AD}\right)^2 + \left(\overline{CD}\right)^2$	Pythagoras Theorem
i.e., $b^2 = x^2 + h^2$ (i)	
In $\angle rt\Delta CDB,$	
$m\angle CDB = 90°$	Given
$\left(\overline{BC}\right)^2 = \left(\overline{BD}\right)^2 + \left(\overline{CD}\right)^2$	Pythagoras Theorem
$a^2 = (c-x)^2 + h^2$	Form the figure
or $a^2 = c^2 - 2cx + x^2 + h^2$ (ii)	
$a^2 = c^2 - 2cx + b^2$	Using(i) and (ii)
Hence, $a^2 = b^2 - c^2 - 2cx$	
i.e., $\left(\overline{BC}\right)^2 = \left(\overline{AC}\right)^2 + \left(\overline{AB}\right)^2 - 2\left(m\overline{AB}\right)\left(m\overline{AD}\right)$	

Theorem 3 **(A.B)**

(Apollonius' Theorem)

8.1(*iii*)

In any triangle, the sum of the squares on any two sides is equal to twice the square on half the third side together with twice the square on the median which bisects the third side.



Given:

In a ΔABC, the median $\overline{AD}$ bisects $\overline{BC}$.

i.e., $m\overline{BD} = m\overline{CD}$

To Prove:

$$\left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 = 2\left(\overline{BD}\right)^2 + 2\left(\overline{AD}\right)^2$$

Construction:

Draw $\overline{AF} \perp \overline{BC}$

Proof:

Statements	Reasons
In ΔADB Since $\angle ADB$ is acute at D	
$\therefore \left(\overline{AB}\right)^2 = \left(\overline{BD}\right)^2 + \left(\overline{AD}\right)^2 - 2m\overline{BD}.m\overline{FD} \rightarrow$ (i)	Using Theorem 2
Now in ΔADC Since $\angle ADC$ is acute at D	
$\therefore \left(\overline{AC}\right)^2 = \left(\overline{CD}\right)^2 + \left(\overline{AD}\right)^2 + 2m\overline{CD}.m\overline{FD}$	
$= \left(\overline{BD}\right)^2 + \left(\overline{AD}\right)^2 + 2m\overline{BD}.m\overline{FD} \rightarrow$ (ii)	Using Theorem 1
Thus $\left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 = 2\left(\overline{BD}\right)^2 + 2\left(\overline{AD}\right)^2$	Adding (i) and (ii)

Exercise 8.1

Q.1 **(A.B)**

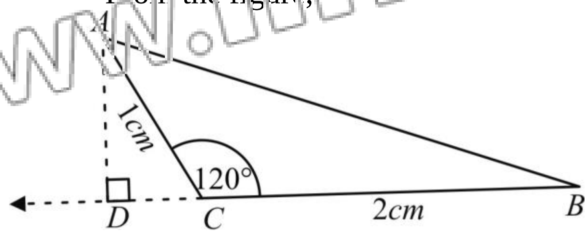
Given:

$m\overline{AC} = 1cm$, $m\overline{BC} = 2cm$, $m\angle C = 120°$ compute the length AB and the area of Δ ABC

Solution:

For area of Δ

From the figure,



$m\angle ACD = 180° - m\angle ACB$

(Supplementary Angles)

= 180° -120°

= 60°

From Δ ACD,

$\text{Sin}60° = \dfrac{m\overline{AD}}{m\overline{AC}}$

$\dfrac{\sqrt{3}}{2} = \dfrac{m\overline{AD}}{1cm}$

$\Rightarrow\ m\overline{AD} = \dfrac{\sqrt{3}}{2} \times 1cm\ \ m\overline{AD} = 0.877cm$

Area of Δ ABC = $cm\dfrac{1}{2}m\overline{BC} \times m\overline{AD}$

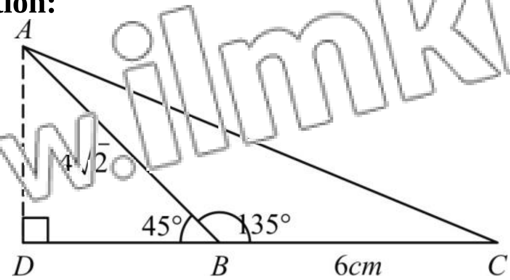
$\therefore$ Area of $\Delta = \dfrac{1}{2}\text{base} \times \text{height}$

$\Rightarrow$ Area of Δ ABC $= 0.877cm^2$

Q.2 **Find** $m\overline{AC}$ **if in** ΔABC $m\overline{BC} = 6cm$, $m\overline{AB} = 4\sqrt{2}$ **and** $m\angle ABC = 135°$.

(A.B)

Solution:



$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 - 2m\overline{BC}m\overline{BD}$$

From ΔABD

$\cos 45° = \dfrac{m\overline{BD}}{m\overline{AB}}$

$\dfrac{1}{\sqrt{2}} = \dfrac{m\overline{BD}}{4\sqrt{2}}$

$4 = m\overline{BD}$

Now putting the values in formula

$\left(m\overline{AC}\right)^2 = \left(4\sqrt{2}\right)^2 + (6)^2 - 2(6)(4)$

$\left(m\overline{AC}\right)^2 = 16(2) + 36 - 48$

$\left(m\overline{AC}\right)^2 = 32 + 36 - 48$

$\left(m\overline{AC}\right)^2 = 20$

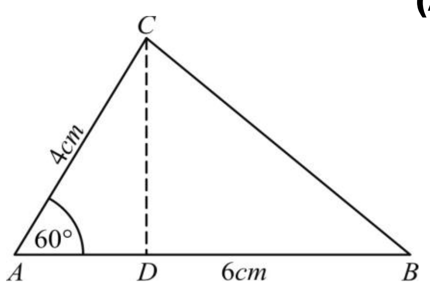
Taking square root $m\overline{AC} = \sqrt{20} = 4.47$

$\therefore m\overline{AC} = 4.47cm$

Exercise 8.2

Q.1 **In a** ΔABC **calculate** $m\overline{BC}$ **when** $m\overline{AB} = 6cm$, $m\overline{AC} = 4cm$ **and** $m\angle A = 60°$

(A.B)



Solution:

By Theorem 8.2, we get

$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - 2m\overline{AB}.m\overline{AD}$

For value of $m\overline{AD}$

From Δ ADC,

$\cos 60° = \dfrac{m\overline{AD}}{m\overline{AC}}$

$\dfrac{1}{2} = \dfrac{m\overline{AD}}{m\overline{AC}}$

$\dfrac{1}{2}\ m\overline{AC}\ =\ m\overline{AD}$

Or

$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - 2m\overline{AB}.\dfrac{1}{2}m\overline{AC}$

Putting the values

$$\left(\overline{BC}\right)^2 = (6)^2 + (4)^2 - 2.6.\frac{1}{2}.4$$

$$\left(\overline{BC}\right)^2 = 36 + 16 - 24$$

$$\left(\overline{BC}\right)^2 = 28$$

Taking square root on both sides

$m\overline{BC} = \sqrt{28}$

Result

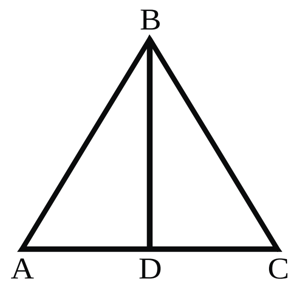
$m\overline{BC} = \sqrt{28}cm$

Q.2 In a $\Delta ABC, m\overline{AB} = 6cm, m\overline{BC} = 8cm$, $m\overline{AC} = 9cm$ and D is the mid point of side $\overline{AC}$. Find length of the median $\overline{BD}$. **(A.B)**

Ans:

By Theorem 8.3, we have

$$(AB)^2 + (BC)^2 = 2\left[(AD)^2 + (BD)^2\right]$$



Putting the values

$$(6)^2 + (8)^2 = 2\left[\left(\frac{9}{2}\right)^2 + (BD)^2\right]$$

$$36 + 64 = 2\left[\frac{49}{4} + (BD)^2\right]$$

$$100 = 2\left[\frac{49}{4} + (BD)^2\right]$$

$$50 = \frac{49}{4} + (BD)^2$$

$$50 - \frac{49}{4} = (BD)^2$$

$$\frac{200-49}{4} = (BD)^2$$

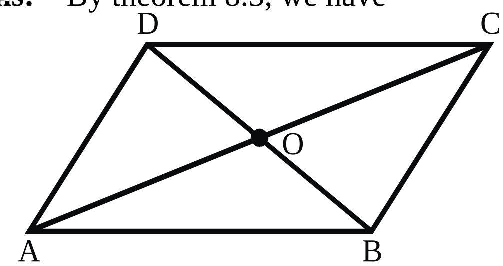
$$\frac{151}{4} = (BD)^2$$

Taking square root on both sides

$$\frac{\sqrt{151}}{2} = \overline{BD}$$

Q.3 In a parallelogram $ABCD$ prove that $\left(\overline{AC}\right)^2 + \left(\overline{BD}\right)^2 = 2\left[\left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2\right]$ **(A.B)**

Ans: By theorem 8.3, we have



$$(AB)^2 + (BC)^2 = 2\left(\frac{1}{2}\overline{AC}\right)^2 + 2\left(\frac{1}{2}\overline{BD}\right)^2$$

$$(AB)^2 + (BC)^2 = \frac{1}{2}(BD)^2 + \frac{1}{2}(AC)^2$$

$$(AB)^2 + (BC)^2 = \frac{1}{2}\left[(BD)^2 + (AC)^2\right]$$

$\because$ O is mid point of $\overline{BD}$ and $\overline{AC}$

Or $2\left[\left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2\right] = \left(\overline{AC}\right)^2 + \left(\overline{BD}\right)^2$

Hence Proved

Miscellaneous Exercise 8

Q.1 In a $\Delta ABC, m\angle A = 60°$, prove that $\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - m\overline{AB}.m\overline{AC}$. **(A.B)**

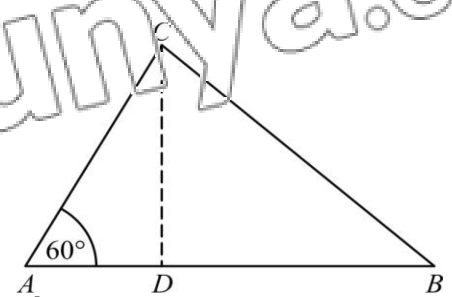
Given:

In a Δ ABC, m$\angle$ A = 60°

To prove:

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - m\overline{AB}.m\overline{AC}$$

Proof:



By Theorem 8.2, we get

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - 2m\overline{AB}.m\overline{AC} \rightarrow \text{(i)}$$

For value of $m\overline{AD}$

From Δ ADC,

$\text{Cos}60^o = \dfrac{m\overline{AD}}{m\overline{AC}}$

$\dfrac{1}{2} = \dfrac{m\overline{AD}}{m\overline{AC}}$

$\dfrac{1}{2}\, m\overline{AC} = m\overline{AD}$

Put in equation (i)

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - 2m\overline{AB}.\frac{1}{2}m\overline{AC}$$

Or

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - m\overline{AB}.m\overline{AC}$$

Hence Proved

Q.2 In a $\Delta ABC, m\angle A = 45^\circ$**, prove that** $\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - \sqrt{2}m\overline{AB}.m\overline{AC}$.

(A.B)

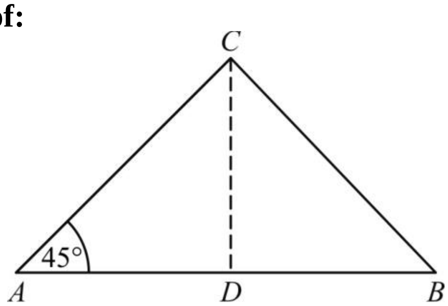
Given:

In a $\Delta ABC, m\angle A = 45^\circ$

To prove

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - \sqrt{2}m\overline{AB}.m\overline{AC}$$

Proof:



By Theorem 8.2, we get

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - 2m\overline{AB}.m\overline{AD} \rightarrow \text{(i)}$$

For value of $m\overline{AD}$

From Δ ADC,

$\cos 45^\circ = \dfrac{m\overline{AD}}{m\overline{AC}}$

$\dfrac{1}{\sqrt{2}} = \dfrac{m\overline{AD}}{m\overline{AC}}$

$\dfrac{1}{\sqrt{2}}\, m\overline{AC} = m\overline{AD}$

Put in equation (i)

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - 2m\overline{AB}.\frac{1}{\sqrt{2}}m\overline{AC}$$

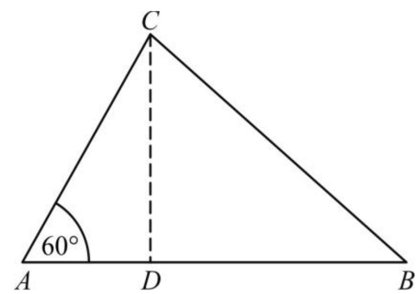
Or $\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - \sqrt{2}m\overline{AB}.m\overline{AC}$

Hence Proved

Q.3 In a ΔABC, **calculate** $m\overline{BC}$ **when** $m\overline{AB} = 5cm, m\overline{AC} = 4cm, m\angle A = 60^\circ$.

(A.B)

Solution:



By Theorem 8.2, we get

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - 2m\overline{AB}.m\overline{AD}$$

For value of $m\overline{AD}$

From Δ ADC,

$\text{Cos}60^o = \dfrac{m\overline{AD}}{m\overline{AC}}$

$\dfrac{1}{2} = \dfrac{m\overline{AD}}{m\overline{AC}}$

$\dfrac{1}{2}\, m\overline{AC} = m\overline{AD}$

Or

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - 2m\overline{AB}.\frac{1}{2}m\overline{AC}$$

Putting the values

$$\left(\overline{BC}\right)^2 = (5)^2 + (4)^2 - 2.5.\frac{1}{2}.4$$

$$\left(\overline{BC}\right)^2 = 25 + 16 - 20$$

$$\left(\overline{BC}\right)^2 = 21$$

Taking square root on both sides

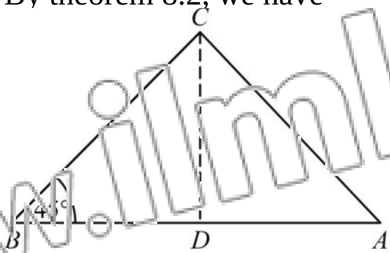
$m\overline{BC} = \sqrt{21}$

Result:

$m\overline{BC} = \sqrt{21}$ cm

Q.4 In a ΔABC, calculate $m\overline{AC}$ when $m\overline{AB} = 5cm$, $m\overline{BC} = 4cm$, m∠B = 45°. **(A.B)**

By theorem 8.2, we have



$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right) + \left(m\overline{BC}\right)^2 - 2m\overline{AB} \times m\overline{BD}$$

For value of $\overline{BD}$

$$\cos 45° = \frac{m\overline{BD}}{m\overline{BC}}$$

$$\frac{1}{\sqrt{2}} = \frac{m\overline{BD}}{4cm}$$

$$\frac{4cm}{\sqrt{2}} = m\overline{BD}$$

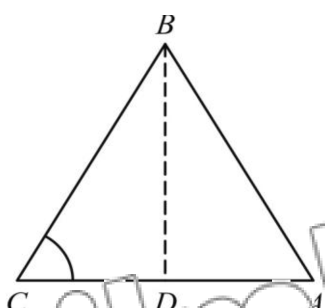
Now putting the values in equation (i)

$$\left(m\overline{AC}\right)^2 = (5)^2 + (4)^2 - 2(5) \times \frac{4}{\sqrt{2}}$$

$$\Rightarrow m\overline{AC} = 12.71cm$$

Q.5 In Δ ABC, $m\overline{BC} = 21cm$, $m\overline{AC} = 17cm$, $m\overline{AB} = 10cm$. Measure the length of projection of $\overline{AC}$ upon $\overline{BC}$ **(A.B)**

Solution:



From the figure, we get

$$\left(\overline{AB}\right)^2 = \left(\overline{BC}\right)^2 + \left(\overline{AC}\right)^2 - 2m\overline{BC}.m\overline{CD}$$

Putting the values

$$(10)^2 = (21)^2 + (17)^2 - 2(21).m\overline{CD}$$

$$100 = 441 + 289 - 42m\overline{CD}$$

$$42m\overline{CD} = 730 - 100$$

$$42m\overline{CD} = 630$$

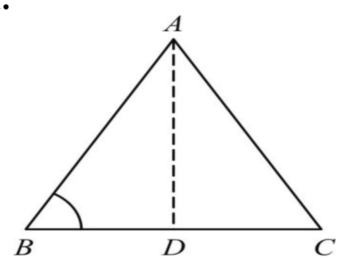
$$m\overline{CD} = \frac{630}{42}$$

$$m\overline{CD} = 15$$

∴ Length of projection of $\overline{AC}$ upon $\overline{BC} = 15cm$

Q.6 In a triangle ABC, $m\overline{BC} = 21cm$, $m\overline{AC} = 17cm$ and $m\overline{AB} = 10cm$. Calculate the projection of $\overline{AB}$ upon $\overline{BC}$. **(A.B)**

Solution:



From the figure, we get

$$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2 - 2\left(m\overline{BD}\right)\left(m\overline{BC}\right)$$

$$(17)^2 = (10)^2 + (21)^2 - 2m\overline{BD}(21)$$

$$289 = 100 + 144 - 42m\overline{BD}$$

$$42m\overline{BD} = 541 - 289$$

$$42m\overline{BD} = 252$$

$$m\overline{BD} = \frac{252}{42}$$

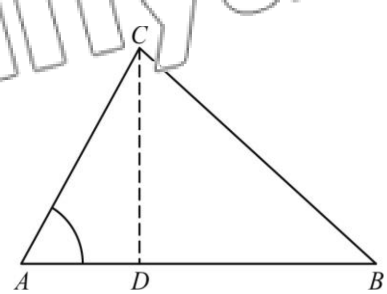
$$m\overline{BD} = 6$$

Result

∴ Project of $\overline{AB}$ upon $\overline{BC} = 6cm$

Q.7 In a ΔABC, $a = 17cm$, $b = 15cm$, $c = 8cm$. Find $m\angle A$ **(A.B)**

Solution:



From fig, we get

$$\left(\overline{BC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{AC}\right)^2 - 2\,m\overline{AB} \times m\overline{AD}$$

Putting the values

$(17)^2 = (15)^2 + (8)^2 - 2\times 8\times m\overline{AD}$

$289 = 225 + 64 - 16\times m\overline{AD}$

$16\times m\overline{AD} = 289 - 289$

$16\times m\overline{AD} = 0$

$m\overline{AD} = 0$

From ΔCAD, $\cos m\angle A = \dfrac{m\overline{AD}}{m\overline{AC}}$

$\cos m\angle A = \dfrac{0}{8}$

$\cos m\angle A = 0$

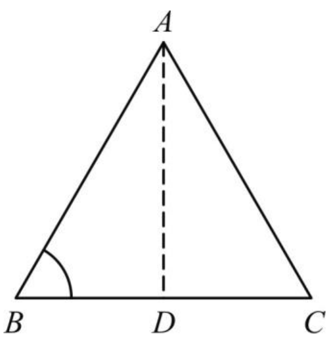
$m\angle A = \cos^{-1} 0$

$m\angle A = 90^o$

Q.8 In a $\Delta ABC, a = 17cm, b = 15cm$ and $c = 8cm$ find $m\angle B$. **(A.B)**

In ΔABC

Solution:



$a = 17cm, b = 15cm$ and $c = 8cm$

By theorem 8.2, we have

$(m\overline{AC})^2 = (m\overline{AB})^2 + (m\overline{BC})^2 - 2m\overline{BC}\times m\overline{BD}$

Putting the values

$(15)^2 = (8)^2 + (17)^2 - 2(17)m\overline{BD}$

$225 = 64 + 289 - 34m\overline{BD}$

$34m\overline{BD} = 353 - 225$

$= 128$

$\therefore m\overline{BD} = 3.76cm$

Now from ΔABD

$\cos m\angle B = \dfrac{m\overline{BD}}{m\overline{AB}}$

$= \dfrac{3.76cm}{8cm}$

$\cos m\angle B = 0.48cm$

$m\angle B = \cos^{-1} 0.48$

$m\angle B = 61.9^o$

Q.9 Whether the triangle with sides 5cm, 7cm, 8cm is acute, obtuse or right. **(A.B)**

Solution:

Let $a = 5cm, b = 7cm$ and $c = 8cm$

$a^2 + b^2 = (5)^2 + (7)^2 \quad c^2 = (8)^2$

$= 25 + 49 \quad = 64$

$= 74$

Since $a^2 + b^2 > c^2 \Delta$ is acute angled.

Or

$a^2 = 5^2 = 25$

$b^2 = 7^2 = 49$

$c^2 = 8^2 = 64$

Since $a^2 + b^2 > c^2$ Δ is acute angled.

Q.10 Whether the triangle with sides $8cm, 15cm, 17cm$ is acute, obtuse or right angled. **(A.B)**

Solution:

Let $a = 8cm, b = 15cm$ and $c = 17cm$

$a^2 = (8cm)^2 = 64cm^2$

$b^2 = (15cm)^2 = 225cm^2$

$c^2 = (17cm)^2 = 289cm^2$

Since $a^2 + b^2 = c^2$

$\therefore \Delta$ with given sides form a right angled triangle.

Note **(A.B + U.B + K.B)**

If $a^2 + b^2 = c^2$, Δ is a right angled Δ

If $a^2 + b^2 < c^2$, Δ is a obtuse angled Δ

If $a^2 + b^2 > c^2$, Δ is a acute angled Δ

Where c is longest side