

Circle
(ITHR 2 (14 URVL201, D.G.K 2014, 16)
Tis arset opoints on a plane which are efvilistant from a fixed point from a circle". Or
Locus of a point in a plane equidistant from a fixed point is called a circle.


Centre of the Circle
(K.B)
"The fixed point is called center of the circle". In the given figure, $O$ is centre.

## Radius of the Circle

(K.B)
"The distance between the centre to any point of the circle is called radius of the circle". In the given figure, $m \overline{O A}$ is radius.


## Radial Segment (LHR 2015) (K.B)

"A line segment which joins centre to any point of the circle is called radial segment" In the given figure, $\overline{O A}$ is radial stgnent.

## Circumference <br> I

(LHR 2015, 15, 16, GRW 201S, FSD 201s. SWL2010, P G.K 20 14.)
"Boundary of the circie is called circonferenve".

## OR

" t is the length of the segment joining all the points of circle is called circumference". Circumference is measured by the formula $2 \pi r$, where $r$ is the radius.

Arc of a Circle
(BWP 2014, RWP 2014, SGD 2015, D.G.K 2015) "It is the part of circumference".


## Major Arc

(K.B)
(LHR 2016, FSD 2015, SWL 2016, RWP 2014)
"An arc which is greater than semi-circle is called major arc". In the given figure, $A C B$ is major arc.

## Minor Arc

(K.B)
(LHR 2016, FSD 2015, SWL 2016, RWP 2014)
"An arc which is less than semi-circle is called minor arc". In the given figure, $A D B$ is minor arc.

## Chord of the Circle

(LHR 2015, GRW 2014, MTN 2014)
"A line segment whose two end points are any two points of the circle".

OR
"A line segment which joins two points of the circle". In the given figare, $\overline{A B}$ anc


Diameter
(K.B)
"A chord passing through centre of the circle is called diameter".

OR
"The largest chord is called diameter". In the given figure, $\overline{P Q}$ is diameter.

Segment of a Circle
"A chord divides a circular region in two parts called segment of a circle". In the given figure, $\overline{A B}$ divides the circle into two segments.

## wrixermid


(H) 2016, FSD 2015, SWL 2016, RWP 2014) "Clicular region bounded by a chord and minor arc is called minor segment". In the given figure, shaded part is minor segment.

## Major Segment

(K.B)
"Circular region bounded by a chord and major arc is called major segment". In the given figure, non shaded part is major segment.

## Sector of a Circle

(LHR 2014, GRW 2014, 17, FSD 2014, SWL 2015, 16, D.G.K 2015)
"Circular region bounded by an arc and its two corresponding radial segments is called sector
of a circle". In the given figure, $A O B$ is a sector of the circle.

Central Angle of an Arc
(K.B)
"An angle subtended by an arc at the center of the circle is called central angle".
"An angle formed by two radial segments at the centre of the circle is called central angle".
In the given figure, $\angle A O B$ is central angle of $A B$.


## Theorem 1

(FSD 2015)
Statement:
One and only one circle can pass through three non-collinear points.
Given:
$A, B$ and $C$ are three non collinear points in a plane.
To Prove:
One and only one circle can pass through three non-collinear points $A, B$ and $C$.


Construction:
Join $A$ with $B$ and $B$ with $C$.
Draw $\overline{D F} \perp$ bisector to $\overline{A B}$ and $\overline{F A M} \perp$ bisectrino $\overline{B C}$. S PDA and $\bar{H}$ are iot parallel and they intersect each other at poinsp. Also jp n. A, $B$ and $C$ with poont $O$.
Proof:

| ()Shatita $n$ | Reasons |
| :---: | :---: |
| Every point on $\overline{\bar{L}} \overline{\overline{4}}$ is eculdiktint trom $A$ and B. <br> Similarly every point on $\overline{H K}$ is equidistant from $B$ and $C$. <br> In particular $m \overline{O B}=m \overline{O C} \rightarrow($ ii $)$ | $\overline{D F} \perp$ bisector to $\overline{A B}$ (construction) <br> $\overline{H K}$ is $\perp$ bisector to $\overline{B C}$ (construction) |

Now $O$ is the only point common to $\overline{D F}$ and $\overline{H K}$ which is equidistant from $\mathrm{A}, \mathrm{B}$ and C .
i.e., $m \overline{O A}=m \overline{O B}=m \overline{O C}$

However there is no such other roint excer. $O$.
Hence a circle y th chitre $O$ and radius QA wir pass inrough $A, B$ and $C$. Ultimately there is on yre circle which passes through three given points $\mathrm{A}, \mathrm{B}$ and C .

## Theorgmit

(RWP 2015)
starinont:
A straight line, draw from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.
Given:
$M$ is the mid point of any chord $\overline{A B}$ of a circle with centre at $O$. Where chord $\overline{A B}$ is not the diameter of the circle.
To Prove:
$\overline{O M} \perp$ the chord $\overline{A B}$.
Construction:
Join $A$ and $B$ with centre $O$. Write $\angle 1$ and $\angle 2$ as shown in the figure.


Proof:

| Statements |
| :--- |
| In $\triangle O A M \leftrightarrow \triangle O B M$ |
| $m \overline{O A}=m \overline{O B}$ |
| $m \overline{A M}=m \overline{B M}$ |
| $m \overline{O M}=m \overline{O M}$ |
| $\therefore \Delta O A M \cong \triangle O B M$ |
| $\Rightarrow m \angle 1=m \angle 2 \rightarrow(i)$ |
| i.e., $m \angle 1+m \angle 2=m \angle A M B=180^{\circ} \rightarrow(i i)$ |
| $\therefore m \angle 1=m \angle 2=90^{\circ}$ |
| i.e., $\overline{O M} \perp \overline{A B}$ |

## Theorem 3

Statement:
Perpendicatier from the eentre of a circle on chord hisects it.
Given:
$\overline{A B}$ is the cho of cracle aitin centre at $O$.
$\sqrt{20}$ that $\overline{C P M}$. 1 ciord $\overline{A B}$.

## IoPProv:

$M$ is the mid point of chord $\overline{A B}$ i.e., $m \overline{A M}=m \overline{B M}$

## Construction:

Join $A$ and $B$ with centre $O$.

Proof:

## Statements

## Reasons

In $\angle r t \Delta^{S} O A M \leftrightarrow O B M$
$m \angle O M A=m \angle O M B=90^{\circ}$
hyp. $\overline{O A}=$ hyp. $\overline{O B}$.
$m \overline{O M}=m \overline{O M}$
$\therefore \triangle O A M \cong \triangle A M$
Hence, $m \overline{A M}=m \overline{\beta M}$


Given
Fosii of the same circile
Comino
In ㄴ. $\leq . t \Delta^{\prime} \quad H . S \cong H . S$
Corresponding sides of congruent triangles
Distco the chord $\overline{A B}$.
$\perp$ Bisector of the chord of a circle passes through the centre of a circle.

## Corollary 2

$\mathbf{( K . B + U . B )}$
The diameter of a circle passes through the mid points of two parallel chords of a circle.

## Exercise 9.1

Q. 1 Prove that, the diameters of a circle bisect each other.

Given
In a circle with centre ' $O$ ', $\overline{A B}$ and $\overline{C D}$ are two diameters.
To prove
$\overline{A B}$ and $\overline{C D}$ bisect each other.
i.e $O$ is midpoint of $\overline{A B}$ and $\overline{C D}$.


Proof

| Statements | Reasons |
| :--- | :--- |
| $\overline{O A} \cong \overline{O B} \rightarrow(\mathrm{i})$ | Radii of same circle |
| $A O B$ is a state line |  |
| $\therefore O$ is midpoint of $\overline{A B} \rightarrow(\mathrm{ii})$ | As in (i) |
| Similarly $\overline{O C} \cong \overline{O D}$ |  |
| Or O is midpoint of $\overline{C D}$ | From (ii) and (iii) $\overline{A B}$ and $\overline{C D}$ bisect each other. |
| Hence $\overline{A B}$ |  |

Q. 2 Two chords of a circle do not pass througi the centre. Pro that theyanmeris each other.
Given
In a circit with cente $O$ chood $\angle B$ and chod $C D$ imersect each other at point $P$.
To prove
$P$ is neithe mid po n of $\overline{A D}$ nor $\overline{C D}$.
Contraction
Draw $O K \perp \overline{A B}$ and $\overline{O H} \perp \overline{C D}$.


Proof

## Statements

$\overline{A K} \cong \overline{B K}$
$m \overline{A K}=m \overline{B K}=m \overline{K P}+m \overline{P B}$
$m A K=m B K=m K P+m P B$
$\therefore \mathrm{P}$ is not midpoint of $\overline{A B} \rightarrow$ (i)
Similarly
$\overline{C H} \cong \overline{H D}$
Or $m \overline{C P}+n \bar{P} \bar{Y}=m \bar{F}$

- Fispladrident or $C D \rightarrow$ (ii)

So $\overline{A B}$ and $\overline{C D}$ cannot bisect each other.

## roof

## Reasonsom


Q. 3 If length of the chord $\overline{A B}=8 \mathrm{~cm}$. Its distance from the centre is 3 cm , then find the diameter of such circle.
(GRW 2014, D.G.K 2014)
(A.B)

## Solution:

From the fig, $m \angle O C A=90^{\circ}$
( $\overline{O C} \perp \overline{A B}, \perp$ is shortest distance)
$m \overline{A C}=m \overline{B C}=4 \mathrm{~cm}$,
From $\triangle$ OCA,

$$
\begin{aligned}
(\overline{O A})^{2} & =(\overline{A C})^{2}+(\overline{O C})^{2} \\
& =(4)^{2}+(3)^{2} \\
& =16+9 \\
& =25
\end{aligned}
$$



Taking square root on both side
$\sqrt{(\overline{O A})^{2}}=\sqrt{25}$
$m \overline{O A}=\sqrt{25}$
$m \overline{O A}=5 \mathrm{~cm}$
Since $\overline{O A}$ is the radius then diameter $=2 \mathrm{~m} \overline{O A}$

$$
=2(5 \mathrm{~cm})
$$

## Result:

Diameter $=10 \mathrm{~cm}$
Q. 4 Calculate the length of a chord which stands at a distance 5 cm firn ora dratee of a circle whose radius is 9 cm . Solution:
Here $\sqrt{204}=9 \mathrm{~cm}, \overline{\mathrm{mOC}}=5 \mathrm{~cm}$
From $\triangle T Q C$
$\left.(m \bar{A} \bar{C})=(m \bar{C} A)^{2}-\frac{1 m O C}{}\right)^{2}$

$$
\begin{aligned}
& =(9)^{2}-(5)^{2}=81-25 \\
& =56
\end{aligned}
$$

Taking square root on both sides


$$
\begin{aligned}
m \overline{A C} & =\sqrt{56} \\
& =2 \sqrt{14} \mathrm{~cm}
\end{aligned}
$$

$m \overline{A B}=2 m \overline{A C} \quad \because \mathrm{C}$ is midpoint of $\overline{A B}$, then by thoorem. 9. 2 $=2(2 \sqrt{14} \mathrm{~cm})$ $=-2 \sqrt{12} \mathrm{~cm}$

## Statement:

If two chords of a circle are congruent then they will be equidistant from the centre.
Given:
$\overline{A B}$ and $\overline{C D}$ are two equal chords of a circle with centre at $O$. So that $\overline{O H} \perp \overline{A B}$ and $\overline{O K} \perp \overline{C D}$.

## To Prove:

$$
m \overline{O H}=m \overline{O K}
$$

## Construction:

Join $O$ with $A$ and $O$ with $C$. So that


We have $\angle r t \Delta^{S} O A H$ and $O C K$.

## Proof:



## Theorem 5

## 9.1(v)

Two chords of a circle which are equidistant from the centre, are congruent.
Given:
$\overline{A B}$ and $\overline{C D}$ are two choros of a incly with centre an $\overline{O H} \perp \bar{A} \bar{B}$ and $\overline{O K} \perp \overrightarrow{C D}$, so that $\overline{C D} \bar{H}=\hat{\pi} \bar{O} \bar{K}$
To Prove:

$$
m \overline{A I}=\operatorname{ma} \overline{C L}
$$

Centiaction


Join $A$ and $C$ with $O$. So that we can form
$\angle r t \Delta^{s} O A H$ and $O C K$.
Proof:

| Statements | Reasons |
| :--- | :--- |
| In $\angle r t \Delta^{s} O A H \Leftrightarrow O C K$. | Radii of the same circle. |
| $\because$ hyp $\overline{O A}=$ hyp $\overline{O C}$ | Given |
| $m \overline{O H}=m \overline{O H}$ | H.S Postulate |
| $\therefore \Delta O A H \cong \Delta O C K$ | Corresponding sides of congruent triangles |
| So $m \overline{A H}=m \overline{C K}(\mathrm{i})$ | $\overline{O H} \perp$ chord $\overline{A B}$ (Given) |
| But $m \overline{A H}=\frac{1}{2} m \overline{A B}($ ii $)$ | $\overline{O K} \perp$ chord $\overline{C D}$ (Given) |
| Similarly $m \overline{C K}=\frac{1}{2} m \overline{C D}($ iii $)$ | Already proved in (i) |
| Since $m \overline{A H}=m \overline{C K}$ | Using (ii) \& (iii) |
| $\therefore \frac{1}{2} m \overline{A B}=\frac{1}{2} m \overline{C D}$ | Multiplying both sides by 2 |
| or $m \overline{A B}=m \overline{C D}$ |  |

## Exercise 9.2

Q. 1 Two equal chords of a circle intersect, show that the segments of the ane are and corresponding to the segments of the other.
Given
 that $m \overline{A B}=\overline{m D}$
To prove
$m \overline{A P}=m \bar{D} \cdot \bar{P} \cdot \mathrm{~d} \ln \overline{B P}=m \bar{D} \bar{D}$
Construction

$$
\mathrm{Dr}_{\mathrm{rav}} \overline{\mathrm{OH}} \perp \overline{C D} \text { and } \overline{O K} \perp \overline{A B}
$$

Join O to P .


## Proof

Statements

$$
m \overline{O H}=m \overline{O K}
$$

Also H is midpoint of $\overline{C D}$ and K sidngrio of $\overline{A B}$

In rt $\triangle O P H \leftrightarrow \stackrel{\mathrm{rt}}{\mathrm{rt}} \mathrm{PK}$
$\overline{O H} \cong \overline{O K}$
$\overline{O P} \cong \overline{O^{P}}$
$\therefore \triangle A N=402 \mathrm{CK}$
$A M E=\bar{\Sigma} \bar{P} \rightarrow(\mathrm{i})$
$m \overline{C P}=m \overline{C H}+m \overline{H P} \rightarrow($ ii $)$
$\overline{C H} \cong \overline{B K} \rightarrow(i i)$
$m \overline{C P}=m \overline{B K}+m \overline{K P}$
$=m \overline{B P}$
Similarly
$m \overline{A P}=m \overline{D P}$

## Reasons

Th-9. $(\overline{A B} \cong \overline{\underline{C D}})$

Q. $2 \overline{A B}$ is the chord of a circle and the diameter $\overline{C D}$ is perpendicular bisector of $A B$.
(A.B)

Given
In a circle with centre ' $O$ ', chord $\overline{A B}$ and diameter $\overline{C D}$ intersect each other at point ' $P$ ',
Such that $\overline{C D} \perp \overline{A B}$ and $\overline{A P} \cong \overline{B P}$
To prove

$$
\overline{A C} \cong \overline{B C}
$$



## Construction

Join $C$ to $A$ and $B$
Proof

Q. 3 As shown in the figure, find the distance between two parallel chords $\overline{A B}$ and $\overline{C D}$.

## Solution:

Since $m \overline{O E} \perp \overline{A B}$ and $\overline{O F} \perp \overline{C D}$ then $\overline{A E}=3$ siand $\overline{\bar{F}}=4 \mathrm{cn}$
From $\triangle O C F$,
$(m \overline{O F})^{2}=(5)^{2}-(4)^{2}$
$(m \overline{O F})^{2}=9$
$\sqrt{(m \overline{O F})^{2}}=\sqrt{9}$
$m \overline{O F}=3$


Similarly from $\triangle O A E$,
$(m \overline{O E})^{2}=(m \overline{O A})^{2}-(m \overline{A E})^{2}$
$(m \overline{O E})^{2}=(5)^{2}-(3)^{2}$
$(m \overline{O E})^{2}=25-9$
$(m \overline{O E})^{2}=16$
Taking sq. root
$\sqrt{(m \overline{O E})^{2}}=\sqrt{16}$
$m \overline{O E}=4$
Distance between parallel chords $\overline{A B}$ and $\overline{C D}$

$$
\begin{aligned}
m \overline{E F} & =m \overline{O E}+m \overline{O F} \\
& =4 \mathrm{~cm}+3 \mathrm{~cm} \\
& =7 \mathrm{~cm}
\end{aligned}
$$

## Result:

Distance between parallel chords $\sqrt[A B]{ }$ and $\frac{\square}{\mathrm{CD}}=7 \mathrm{~m}$

## Miscellaneous Exercise 9

## Q. 1 Multiple choice questions

Four possible answers are given for the foliphing question Tich ( $v$ ) the corcect answer.
(i) In the circular figure, $A L$ is caller
(a) An arc
(b) A secant
(c) A chord
(d) A diameter
(ii) In the circular figure, $A C B$ is called
(K.B)

(a) An arc
(b) A secant
(c) A chord
(d) A diameter
(iii) In the circular figure, $A O B$ is called
(SWL 2014)

(a) An arc
(b) A secant
(c) A chord
(d) A diameter
(iv) In a circular figure, two chords $\overline{A B}$ and $\overline{C D}$ areq iitis antfron the cene. They will be
(a) Parallel
(b) Non congruent
(c) Congruent
(d) Perpendicular
(v) Radii of a circle are (LHR 2014, GRW 2014, SWL 2015, 16)
(a) All equal
(b) Double of diameter
(c) All unequal
(d) Half of any chora
(vi) A chord passing through the centriz of a ir cle is ralled
(a) Radius
(b) Dianete:
(c) Cir urur feence
(d) Secant
(vii) Right tiscetor the hord of a circie always passes through the
(a, Radius
(b) Circumference
(v) Cemre
(d) Diameter

The circular region bounded by two radii and the corresponding arc is called (K.B)
(a) Circumference of a circle
(b) Sector of a circle
(c) Diameter of a circle
(d) Segment of a circle
(ix) The distance of any point of the circle to its centre is called
(K.B)
(a) Radius
(b) Diameter
(c) A chord
(d) An arc
(x) Line segment joining any point of the circle to the centre is called
(K.B)
(SGD 2014, ,MTN 1015, RWP 2015)
(a) Circumference
(b) Diameter
(c) Radial segment
(d) Perimeter
(xi) Locus of a point in a plane equidistant from a fixed point is called
(K.B)
(D.G.K 2014)
(a) Radius
(b) Circle
(c) Circumference
(d) Diameter
(SGD 2014, D.G.K 2014)

The symbol for a triangle is denoted by
(K.B)
(a) $\angle$
(b) $\Delta$
(c) $\perp$
(d) $\square$
(xiii) A complete circle is divided into
(a) 90 degrees
(b) 180 degrees
(c) 270 degrees
(d) 360 degrees
(xiv) Through how many non collinear points, a circle can pass?
(a) One
(c) Three
Q. 2 Differentiate between the following terms and illustrate them by diagrams.
(i) A circle and a circumference.

Ans:
Differentiation

ii) 1 chord and the diameter of a circle.

## Differentiation

| Chord | Diameter |
| :--- | :--- | :--- |
| $\bullet$ <br> A line segment whose two end points <br> are any two points of the circle | $\bullet$A chord passing through centre of the circle is <br> called diameter |
| - It may/may not pass through centre of <br> the circle | $\bullet \quad$ It always passes through centre of circle |
| - It may/may not the longest line segment | $\bullet \quad$ It is longest line segment inside the circle. |

(iii) A chord and an arc of a circle.

## Differentiation

| Chord | Arc |  |
| :--- | :--- | :--- |
| - | A line segment whose two end points <br> are any two points of the circle | $\bullet$Any portion of circumference is called <br> an arc |
| $\bullet$ | It lies inside the circle | $\bullet$ |
| $\bullet$ | It is part of circumference |  |

(iv) Minor arc and major arc of a circle.

Ans: Differentiation

| Major Arc | $\rightarrow$ Mingry $\rightarrow^{4}$ |
| :---: | :---: |
| - It is bigger than minor arc | - It is sudles the najor arc |
| - Major arc is always gi ea er 41 an semicirc | - Mojor are i. al vays le ss than semi circle |
| $A C B$ is a major arc | $A D B$ is a minor arc |

(v) Interior and exterior of a circle.

Ans:
Differentiation

## Interior of the Circle

## er

- All the points which lie insice of he - A l the point whicin lie outside of the circle circle ferm, interior of the crele arm exterior of the circle
- It includes the centie of hecitice
of ir ne pait of circle
- It does not include centre of the circle
- It is outer part of circle

Ans:

## Differentiation

## Sector of Circle

## Segment of Circle

- "A chord divides a circular region in two parts called segment of a circle". In the given figure, $\overline{A B}$ divides the circle into two segments.
- It includes one chord
- It may/may not include centre of the circle



## SELF TEST

Time: 40 min
Mares 25
Q. 1 Four possible answers $(A),(B),(C) \&(L)$ to garl quastion are gien. Nark the correct answer.
1 In the circular figure, $A O B$ is called.
(A) Arrarc
(B) A secant
(C) A cror 1
(D) A diameter


2 Right bisector of the chord of a circle always passes through the:
(A) Radius
(B) Circumference
(C) Centre
(D) Diameter

3 In a circular figure, two chords AB and CD are equidistant from the centre. They will be:

(A) Parallel
(B) Non congruent
(C) Congruent
(D) Perpendicular

4 The union of a circle and its interior is called:
(A) Diameter
(B) Chord
(C) Circular region
(D) Circumference

5 In the adjacent circular figure with centre $O$ and radius 5 cm . The length of the chord intercepted at 4 cm away from the centre of this circle is:

(A) 4 cm
(B) 6crí
(C) 7 cm
(i) 9 cm

6 In figure the projection of a lineger $\bar{C} L$ on line segment $\bar{A} \bar{B}$ is the portion.
(A) $\frac{1}{E F}$
(B) $\overline{D F}$
(C) $\overline{E F}$
(D) $\overline{A F}$

7 The triangle with sides $8 \mathrm{~cm}, 15 \mathrm{~cm}, 17 \mathrm{~cm}$ is:
(A) Acute
(B) Obtuse
(C) Right
(D) Can't determine
(ii) What is the difference between mi segment maj segment?
(iii) Differential betweenimurior en ex te cor a circle and illustrate them by diagrams.
(iv) In a triangle $A B C$ pa'cl late $m \overline{D D} \bar{C}$ when $m \overline{A B}=6 \mathrm{~cm}, m \overline{A C}=4 \mathrm{~cm}, m \angle A=60^{\circ}$.

1) a triangle $\mathrm{ABC}, m \overline{B C}=21 \mathrm{~cm}, m \overline{A C}=17 \mathrm{~cm}, m \overline{A B}=10 \mathrm{~cm}$. Measure the length of projection of $\overline{A C}$ upon $\overline{B C}$.

## Q. 3 Prove that,

A straight line, drawn from the centre of a circle to bisect a chord (Which is not a diameter) is perpendicular to the chord.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.

