بسم اللہ الرحمٰن الرحیم

شروع اللہ کے نام سے جو بڑا مہربان نہایت رحم کرنے والا ہے

A TEXTBOOK OF

PHYSICS

XI

BALOCHISTAN TEXTBOOK BOARD, QUETTA

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Unit 1

MEASUREMENTS

Major Concepts (19 PERIODS)

- The scope of Physics
- SI base, supplementary and derived units
- Errors and uncertainties
- Use of significant figures
- Precision and accuracy
- Dimensionality

Conceptual Linkage

This chapter is built on Measurement Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- Describe the scope of Physics in science, technology and society.
- State SI base units, derived units, and supplementary units for various measurements.
- Express derived units as products or quotients of the base units.
- State the conventions for indicating units as set out in the SI units.
- Explain why all measurements contain some uncertainty.
- Distinguish between systematic errors (including zero errors) and random errors.
- Identify that least count or resolution of a measuring instrument is the smallest increment measurable by it.
- Differentiate between precision and accuracy.
- Assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties.
- Quote answers with correct scientific notation, number of significant figures and units in all numerical and practical work.
- Check the homogeneity of physical equations by using dimensionality and base units.
- Derive formulae in simple cases using dimensions.

INTRODUCTION

Among the creatures, human being is the only creature who has ability of thinking, reasoning and researching. On account of this ability, he is trying to gain knowledge about the origin, creation and organization of this vast universe and the different related laws governing it. He also endeavours to explain the hidden natural reservoirs and forces (acting and reacting) which cause various events in the universe. In the past, man was reluctant to think about the universe but at present, he wishes to make an abode on moon or on any other planet. Quantization rules of electrons in an atom, a solar system and a massive body like galaxy all these have become part of the study and observation of the mankind. Similarly, a man researches to reason out that how the life of living things (plants and animals) get possibility to exist and evaluate only on earth? How days and nights are formed by the spin motion of the earth? How changing of seasons are timed by orbital motion of the earth? In the same way, how the process of evaporization, condensation and sterilization take place? In short, the knowledge about the nature in terms of research, observations and practical applications are known as science. Gradually, due to rapid research, the volume of knowledge about science increases. Therefore, science is basically classified into two main classes; Biological sciences which deals with study of living things and Physical sciences which deals with study of non-living things. Physical sciences can be further sub divided into five main branches i.e.; Chemistry, Geology, Astronomy, Meteorology and Physics. The word Physics comes from the Greek word 'Physis' Physika or Physikos meaning the knowledge of the nature and natural world.

On the other hand, Physics is the study of properties of matter and energy and the mutual relationship between them. This chapter deals with the scope of physics in science, technology and our society. Therefore, we will explain the international system (SI) for weights and measures. Similarly, we will also study errors, uncertainties, significant figures, precision, accuracy and dimensions of physical quantities and their usage in this chapter.

1.1 THE SCOPE OF PHYSICS

Physics is based on experimental observations, quantitative measurements and concerned with the fundamental laws of the universe. Therefore, physics is the most basic branch of physical sciences.

Like electrons around the nucleus, all the other subjects of physical sciences are revolving around the physics. Now there is no denial of the fact that physics has countless contributions in the field of science and technology and its role in the development of our society is dynamic.

The principles of physics have not only brought tremendous changes in every walk of life but also changed the life style of mankind by the wonderful contexts of infrastructures. On one hand, the appliances of physics have introduced an industrial revolution in the world. On the other hand, these have turned the world into a global village by fast audio and video communication system via radio, television, mobile, computer and internet system. In all these modes of telecommunication systems, the carrier signal is electromagnetic wave whose speed is equal to speed of light (3×10^8 ms^{-1}). In addition, the existed vast libraries and archives which contained millions of books and documents, all these information and knowledge have been confined to tiny chips. These chips have been developed from the basic ideas of physics. Similarly, in medical sciences, diagnose and treatment of incurable diseases now have become possible by introducing considerable advancements and achievement of modern technology such as transplantation, radiotherapy, chemotherapy, x-rays, magnetic resonance imaging (MRI), computer tomography (CT Scan), LASER surgery, operation without surgery by nano robots.

In the field of engineering, all sort of appliances such as microwave ovens, vacuum cleaners, washing machines, air-conditioners, refrigerators, engines, electric motors, generators, submarines, airplanes, excavators, robots and many more which are working under the various laws and principles of physics and they have made our lives easy and comfort.

In the same way, the role of physics in genetic engineering and transgenetic organism in the development of new species cannot be neglected. Summing up, the involvement of physics in each section of life is a universal truth.

Physics has a number of branches which are listed in the box. But all the work and research of physics has been basically classified into two main

Physics and Technology

One of the most exciting technological advances in the world today is the field of nanotechnology. Nanorobots or nanomachines have been used to remove obstructions in the circulatory system and kills cancerous tumors with precision: Researchers from McGill University have achieved a spectacular breakthrough in nanotechnology.

Different Branches of Physics

Classical Physics

Mechanics
Optics (Light)
Sound (Acoustics)
Electromagnetism
Heat & Thermodynamics

Modern Physics

Atomic Physics
Nuclear Physics
Molecular Physics
Plasma Physics
Quantum Physics
Space Physics
Solid State Physics
Nanotechnology
Laser Physics
Fluid Dynamics
Aero Dynamics
Hydro Dynamics

classes, which are named as classical physics and modern physics.

The physics upto the end of 19^{th} century is known as classical physics which consists of Newton's laws of motion, gravitational laws, laws of thermodynamics, kinetic theory, Maxwell's theory of electromagnetic wave and the laws about optical phenomenon. However, the physics after the 19^{th} century is known as modern physics which includes discovery of x-rays and radioactivity, Michelson-Morley experiment, Max Planck's quantum theory, Einstein's special theory of relativity, Bohr's atomic theory, De-Broglie hypothesis, Schrodinger wave equations and Heisenberg uncertainty principle. All these new researches have brought a revolution in the field of physics and other scientific disciplines.

Interdisciplinary Branches of Physics

Astro Physics
Bio Physics
Chemical Physics
Relatavistics Physics
Low Temperature Physics
Condensed Matter Physics
Engineering Physics
Geo Physics
Mathematical Physics
Medical Physics

1.2 PHYSICAL QUANTITIES

Physics is an experimental science where the measurements are made and we usually use some quantities to describe the results of these measurements. Thus the quantities which can be measured are known as physical quantities. For example, mass length, time, distance, velocity, force, weight, momentum, work, power etc. On the other hand, all the laws and equations of Physics can be expressed in terms of these physical quantities. Physical quantities can be classified into two classes, i.e. base quantities and derived quantities.

Base Quantities

The quantities which are independent and cannot be expressed in terms of other physical quantities are known as base quantities. There are seven base quantities such as; mass, length, time, temperature, current, amount of substance and intensity of light.

Derived Quantities

The quantities which can be expressed in terms of fundamental quantities using arithmetical operations of product or quotient rule are known as derived quantities. For example, velocity, acceleration, momentum, force, work, power etc.

Product rule:

According to this rule two or more physical quantities are multiplied such that their product gives a new resultant physical quantity. For example, momentum is derived by the product of mass and velocity. i.e.,

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

$$\vec{p} = m\vec{v}$$

Quotient rule:

According to this rule, one physical quantity is divided by another and their quotient gives a new resultant physical quantity. For example, velocity is the quotient of displacement and time. i.e.,

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\vec{v} = \frac{\vec{d}}{t}$$

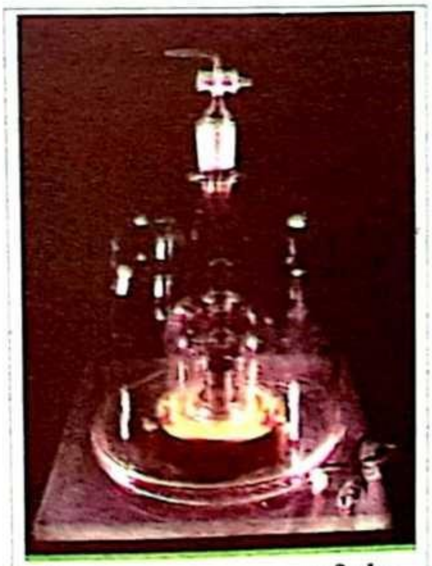
1.3 INTERNATIONAL SYSTEM OF UNITS

The standard and justified measurement of a physical quantity is called unit. e.g. kilogram, meter, second, newton, joule, watt, radian, etc.

In 1960, a general conference on weight and measure was held in Paris. After prolong discussion, the international committee for weights and measures agreed to introduce a common system of units all over the world and they recommended a metric system for measurements which is called International System of Units (SI). The SI unit is an improved form of MKS (Metre, Kilogram and Second) system. SI unit is replaced by CGS (centimeter, gram and second) and FPS (Foot, pound and second).

There are three main routes of the SI units

(i) Base Units
(ii) Derived Units
(iii) Supplementary Units



An accurate copy of the International Standard Kilogram kept at Sevres, France, is housed under a double bell jar in a vault at the National Institute of Standards and Technology.

1.3.1 Base Units

The units of base quantities are known as base units. Base units are isolated and cannot be derived from any other units. There are seven base units which are listed in table 1.1.

Table 1.1: Base S.I Units

Base Quantity	Unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Electric Current	ampere	A
Temperature	kelvin	K
Intensity of Light	candela	cd
Amount of substance	mole	mol

1.3.2 Derived Units

The units of derived quantities are called derived units. These derived units can be obtained under the arithmetical operations of product or quotient rules which are explained as:

Product Rule

According to this rule, when two or more units and multiplied such that their product gives a new resultant unit. e.g. metre cube (m^3) is the unit of volume and it can be obtained by the product of base units as:

$$\text{Volume} = (\text{length}) \times (\text{breadth}) \times (\text{height})$$

The derived unit for volume is $(m)(m)(m) = m^3$

Quotient Rule

According this rule, when a unit is divided by another unit such that their quotient gives a new resultant unit. e.g. Watt (W) which is the unit of power and it can be obtained by the quotient of base units as

$$P = \frac{\text{work}}{\text{time}} = \frac{\text{Joule}}{\text{second}}$$

$$= \frac{kg\,m^2\,s^{-2}}{s} = kg\,m^2\,s^{-3}$$

$$P = \text{watt}$$

Some other derived SI units are mentioned in table 1.2.

Table 1.2: Derived SI Units

Derived Quantity	Unit	Symbol	In terms of Base units
Force	newton	N	$kg\,m\,s^{-2}$
Work, Energy	joule	J	$N\,m = kg\,m^2\,s^{-2}$
Power	watt	W	$Js^{-1} = kg\,m^2\,s^{-3}$
Pressure, stress	pascal	Pa	$\frac{N}{m^2} = kg\,m^{-1}\,s^{-2}$
Co-efficient of Viscosity	poise	H	$\frac{N - s}{m^2} = kg\,m^{-1}s^{-1}$
Electric Charge	coulomb	C	A . s
Frequency	hertz	Hz	s^{-1}
Moment of Inertia	kilogram metre square	$kg\,m^2$	$kg\,m^2$

1.3.3 Supplementary Units

The two units could not find a room in base units nor in derived units in the general conference for weights and measures held in 1960 in Paris. These two units are called supplementary SI units. Supplementary SI units are radian and steradian, both are geometrical units and dimensionless.

(a) Radian

Radian is a two dimensional plane angle and it is defined as, "the angle subtended at the centre of a circle by an arc equal in length to the radius of circle", as shown in Fig.1.1. and it is equal to the ratio between lengths of the arc to the radius.

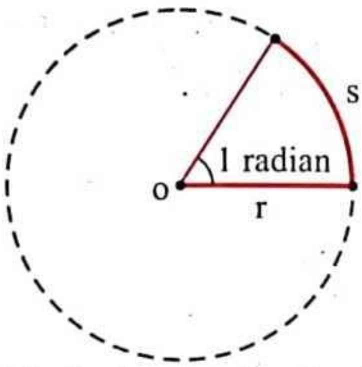


Fig.1.1: A plane angle of radian subtended by an arc whose length is equal to the radius.

Let an angle 'θ', which is subtended by an arc of length 'S' along a circle of radius 'r' then;

$$\theta = \frac{S}{r}(\text{rad}) \quad ......(1.1)$$

For one revolution,

Length of the boundary of the circle (S) = $2\pi r$ (circumference)

Thus, equation 1.1. becomes

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ radians}$$

Now if length of the arc 's' = radius 'r'.

Again equation (1.1) becomes $\theta = \frac{r}{r}(\text{rad})$.

Thus, $\theta = 1\,\text{radian}$, as shown in Fig 1.1.

For Your Information

NIST F-1 Cesium Atomic Clock

NIST-F1 is the world's most accurate time and frequency standard. This atomic clock developed at the NIST (National Institute of Standard and Technology) laboratories in Boulder, Colorado USA. NIST-F1 defines Coordinated Universal Time (UTC). The official world time.

The uncertainty of NIST-F1 is continually improving. In 2000 the uncertainty was about 1×10^{-15}, but as of January 2013, the uncertainty has been reduced to about 3×10^{-16}, which means it would neither gain nor lose a second in more than 100 million years. First atomic clock was developed in 1945.

Importance of Units

In September 1999, after nine months and travelling 650 million kilometers, the Mars climate Orbiter (robotic space probe) suddenly disappeared. The 'root cause' of this loss was the faulty unit conversion.

(b) Steradian

Steradian is a three dimensional solid angle. It is defined as "the angle subtended at the center of a sphere by a surface of sphere and it is equal to the ratio between subtended spherical area to the square of the radius".

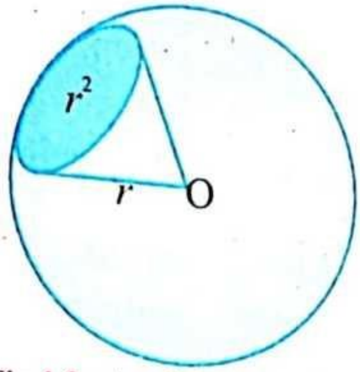


Fig.1.2: A solid angle of steradian subtended by surface area equal to the square of the radius.

$$\text{Angle } \theta = \frac{\text{spherical area}}{r^2} \text{(steradian)} \ldots\ldots (1.2)$$

But, spherical area $= 4\pi r^2$

$$\text{Angle } \theta = \frac{4\pi r^2}{r^2} \text{(steradian)} = 4\pi \text{ steradians}$$

Now if the spherical area = 1 m^2 and radius r = 1 m

Then equation (1.2) becomes

$$\text{Angle } \theta = \frac{1 m^2}{1 m^2} \text{ (steradian)}$$

Angle θ =1 steradian as shown in Fig.1.2.

1.3.4 Conventions for indicating SI units

A unit system has a great importance in physics as well as in any other subject of science, because a value or a result without unit is meaningless. Therefore, a special care is required in the expression of unit and writing of prefixes. In this regard, there are some rules which are related with the using of units and these are summarized as;

I. The unit's name should not be written with a capital initial letter, even if named after a scientist.
For example: newton, pascal, watt, kelvin

II. The symbols of the units named after scientist should be written by an initial capital letter.
For example: N for newton, Pa for pascal, W for watt, K for kelvin

III. The prefix should be written before the unit without any space.
For example: 1×10^{-6} m is written as $1 \mu m$.

IV. One space is always to be left between the numbers and the symbols of the unit and also between the symbols for a compound unit such as force, momentum, etc.

Approximate values of some time intervals in seconds

Event	Time Interval
Life span of most unstable particle	10^{-24}
Time required for light to cross a nuclear distance	10^{-22}
Period of X-rays	10^{-19}
Period of atomic vibration	10^{-15}
Life time of an excited state of an atom	10^{-8}
Period of radio wave	10^{-6}
Period of sound wave	10^{-10}
Wink of eye	10^{-1}
Time between successive heart beats	10^{0}
Time taken by light from Sun to Earth	10^{2}
Time period of satellite	10^{4}
Rotation period of Earth	10^{5}
Rotation & revolution periods of the moon	10^{6}
Revolution period of the Earth	10^{7}
Travel time for light from the nearest star	10^{8}
Age of Egyptian pyramids	10^{11}
Time since dinosaurs became extinct	10^{17}

For example: It is not correct to write 2.3m. But the correct representation is 2.3 m; Similarly, kg m s^{-2} and not as $kgms^{-2}$.

V. Compound prefixes are not allowed. For example: 1μμm may be written as 1pm.

VI. No full stop or other punctuation marks should be used within or at the end of symbols.
For example: 50 m and not as 50 m.

VII. The symbols of the units do not take plural form. For example: 10 kg not as 10 kgs.

VIII. When temperature is expressed in Kelvin, the degree sign is omitted.
For example: 273 K not as 273°K
(If expressed in Celsius or Fahrenheit scale, degree sign should be included. For example, 100°C and not 100 C)

IX. When a multiple of a base unit is raised to a power, the power applies to the whole multiple and not the base unit alone. Thus $1km^2 = 1\ (km)^2 = 1 \times 10^6\ m^2$.

X. Only accepted symbols should be used.
For example: Ampere is represented as A and not as amp. or am; second is represented as s and not as sec.

1.4 SCIENTIFIC NOTATION

Sometimes, we come across a value or a result of a physical quantity which has extremely large or small magnitude. For example, the number of molecules in one mole is 602,300,000,000,000,000,000,000. Similarly, the radius of hydrogen atom is 0.000,000,000,053 m. Such notations are difficult ways of expression. Therefore, these values can be simplified in a decimal form under a process known as scientific notation or standard form. According to this process the given value is expressed in term of some power of ten multiplied by a number which lies between 1 and 10. Thus the number of molecules in one mole is written in terms of scientific notation is 6.023×10^{23} molecules and the radius of the hydrogen atom is 5.3×10^{-11} m. According to the rule only one non-zero digit should be written on the left of the decimal point. For example, the standard form of 120000 is 1.2×10^5 but not 12×10^4.

1.5 PREFIXES

The magnitude of physical quantities vary over a wide range. When the values of physical quantities are very large or very small then it is difficult to express them in terms of the fundamental units. For this, we introduce larger and smaller units by using specific letters before the fundamental units. These letters are known as prefixes and these are represented by fixed values in the form of power of ten. For example, the prefix 'kilo' abbreviated as 'k'. It is always equal to 1000 or 10^3. The prefixes can further be explained by the following examples.

I. Mass of bag is 2000 g
   $2000\ g = 2 \times 10^3 g = 2$ kilogram $= 2\ kg$

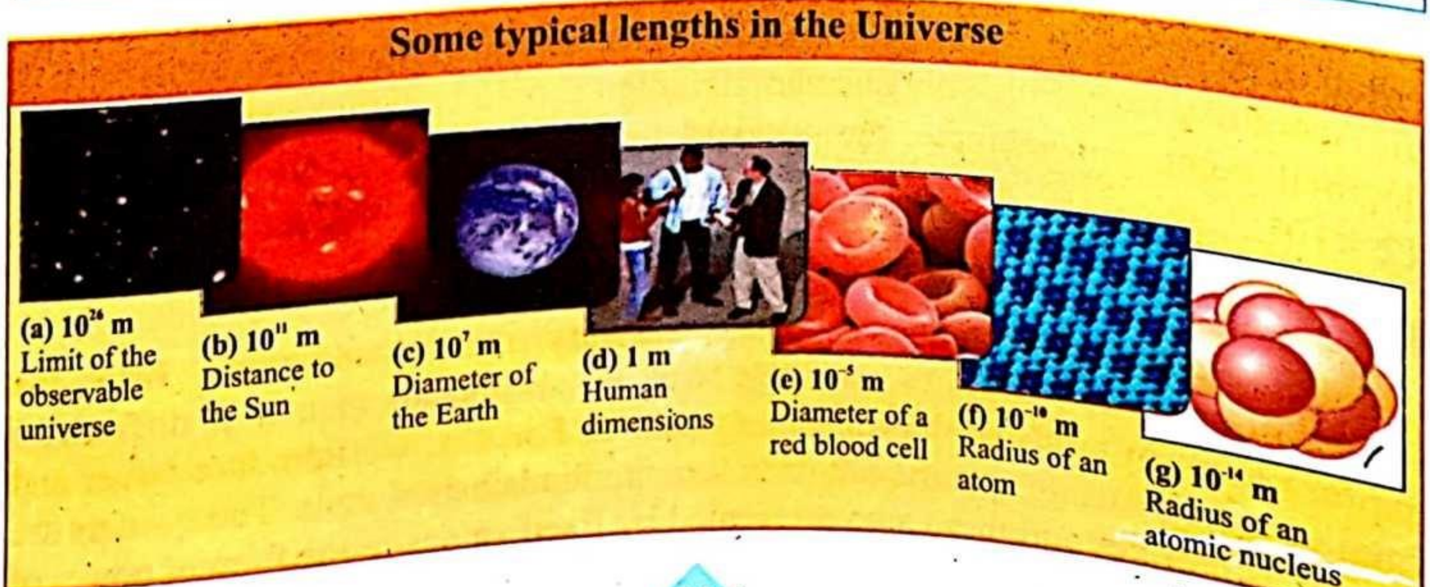
II. Size of living cell is 0.000001 m
   $0.000001\ m = 1 \times 10^{-6}\ m = 1$micro m $= 1\mu m$

III. Time for speed of sound to travel in air through 0.35 m is 0.001 s.
   $0.001 s = 1 \times 10^{-3}\ s = 1$milli second $= 1\ ms$

   Different prefixes and their values are given in table 1.3.

Table 1.3: Prefixes and their values

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	K	10^{-18}	atto	a
10^{2}	hecto	H	10^{-21}	zepto	z
10^{1}	deca	Da	10^{-24}	yocto	Y



1.6 SIGNIFICANT FIGURES

The number of accurately known digits and the first doubtful digit are called significant figures. It is explained as;

The measurement of physical quantities made by related instruments often involves some errors or uncertainties. These uncertainties are due to the following factors:

(i) The least count of measuring instruments
(ii) Quality and condition of the apparatus
(iii) Skill of the observer
(iv) Different recorded observations by the same apparatus.

In the presence of these complications, the reported result contains both certain and uncertain digits and the total number of all these certain and uncertain digits are known as significant figures.

For example, let the reported mass of a sphere is 1.53 kg. In this case 1 and 5 are certain digits while 3 is uncertain and the measured value has three significant figures. Similarly, the reported length of a simple pendulum is 102.5 cm, this value has four significant figures, the digits 1, 0 and 2 are certain while 5 is uncertain.

1.6.1 Rules for determining the number of significant figures

The number of significant figure of a measured value can be determined under the following rules.

I. All the non-zero digits are significant (1,2,3,4,5,6,7,8,9).

II. Zero may or may not be significant and it is explained as;

a) All the zeros between two non-zero digits are significant, whether decimal point exists or does not exist.
e.g. 2003, 2.003, 20.03, in all these cases significant figures are four.

b) Zero to the right of a significant figure may or may not be significant.
e.g. 6000 calories can be written as
6×10^3 calories (1 Significant figure)
6.0×10^3 calories (2 Significant figure)
6.00×10^3 calories (3 Significant figure)
6.000×10^3 calories (4 Significant figure)

c) The terminate zero in a number with decimal point are significant e.g. 0.2300, 0.1540, 3.600
All these three numbers have four significant figures each.

d) If the number is less than one, the zero on the right of decimal point and to the left of the 1st non zero digit are not significant. e.g. 0.00123 in this case zeros are not significant and the number of significant figures is three, i.e., $0.00123 = 1.23\times10^{-3}$.

e) When the measurement is reported in scientific notation, then the figures other than power of ten are significant figures. e.g. 6.40×10^{24}kg has three significant figures.

III. No change occurs in the number of significant figures by changing the unit of the measured value. e.g. 23.15 mm = 2.315 cm = 0.02315 m
All these numbers have four significant figures each.

IV. When measurements are added or subtracted, the answer contains no more decimal places than the least accurate measurement (less decimal number value).
The following examples will clarify these rules.

2355.2342	15600.00	15600	13.7
+ 23.24	+ 172.49	+ 172.49	+ 1.3
2378.47	15772.49	15772	15.0

Keep the same number of decimal places as the factor with the least amount.

V. When measurements are multiplied or divided, the answer contains no more significant figures than the least accurate measurement (least significant figure value). Some examples:

13.1	13.100	15310	1.00
× 2.25	× 2.2500	× 2.3	× 10.04
29.5	29.475	35213	10.04

Keep the same number of significant figures as the factor with the least number of significant figures.

Examples

4767	4 significant figures (all 1-9 digits are significant)
0.0008	1 significant figure (zeros locate only the decimal position)
14.90	4 significant figure (In decimal figure zero is significant)
7000.0	5 significant figure (In decimal figure zero is significant)
8500	4 significant figure
1.121	4 significant figures
1.08701	6 significant figures
0.00254	3 significant figures (2,5 and 4 are significant)
0.2540	4 significant figures (2, 4, 5 and last 0 are significant)
2.15×10^3	3 significant figures (2, 1, and 5)

1.7 ERRORS AND UNCERTAINTIES

When an observer is making a measurement by using some measuring instruments, he makes an effort to determine precise and accurate result. But it is a difficult job for him because the measuring instruments contain some uncertainties.

These uncertainties are called errors and these are due to the following factors.

(a) Zero error and faulty or poor condition of the instrument.
(b) Irrelevant experimental technique or procedure.
(c) Lack of experience in the setting and using of the apparatus.
(d) Taking observation without precautions.

The error in the measurement can be minimized by using the instrument which contains small uncertainty. For example, between metre rod and vernier callipers, the measurement by using vernier callipers is more reliable than metre rod, because it has small uncertainty.

The errors in measurement can be classified into two main classes.

(i) Systematic Error (ii) Random Error

1.7.1 Systematic Errors

The errors that appear in measurement and repeat in same magnitude and sign under the same conditions are called systematic errors. Such errors are due to the following factors i.e., the zero error in instrument, poor calibration of instrument and incorrect marking.

Systematic error can be removed by using some standard instruments.

1.7.2 Random Errors

The errors that appear in measurement and repeat in different magnitude and sign under the same conditions are called random errors. Such errors occur due to the following reasons, i.e., personal error, lack of sensitivity of the instrument, environmental factors (temperature, humidity etc.) and wrong technique of measurement.

Random error can be minimized by applying statistical method i.e. repeating the measurement several times and taking an average of these measurements.

1.8 PRECISION AND ACCURACY

When physical quantities are measured by using the measuring instruments then there are some uncertainties or errors exist in the measurement and it is due to the various factors. These uncertainties or errors can be explained in terms precision and accuracy which are related with the reported result of any measured quantity. Now the terms precision and accuracy can be distinguished as:

1.8.1 Precision

The precision of a measurement is associated with the least count of the measuring instrument. This shows that precise measurement depends upon the resolution or limit of measuring instrument. Precision depends upon absolute uncertainty e.g. absolute uncertainty of a metre rod is 1 mm or 0.1 cm and the absolute uncertainty of a Vernier callipers is 0.1 mm or 0.01 cm. It may be noted that the smaller the least count of the measuring instrument, the more precise will be the measurement. e.g. screw gauge is more precise than that of vernier callipers and vernier callipers is more precise than that of metre rod.

1.8.2 Accuracy

The accuracy of a measurement is associated with the fractional uncertainty or relative uncertainty. This shows that accuracy depends upon the closeness of calculated value with the actual value of the quantity. Lesser is the fractional uncertainty or percentage uncertainty of the result, the more accurate will be the measurement.

> All the measurements are subject to uncertainties.
> The accuracy of a measurement describes how well the result agrees with an accepted value.
> The precision of an instrument is limited by the smallest division on the measurement scale.

$$\text{Fractional Uncertainty} = \frac{\text{Least Count}}{\text{Measured value}}$$

$$\text{Percentage Uncertainty} = \frac{\text{Least Count}}{\text{Measured value}} \times 100$$

Now precision and accuracy can further be explained by the following examples. Let, the length of an object is measured by metre rod is 19.5 cm as shown in Fig.1.3, then;

Absolute Uncertainty (least count) = $\pm 0.1\,\text{cm}$

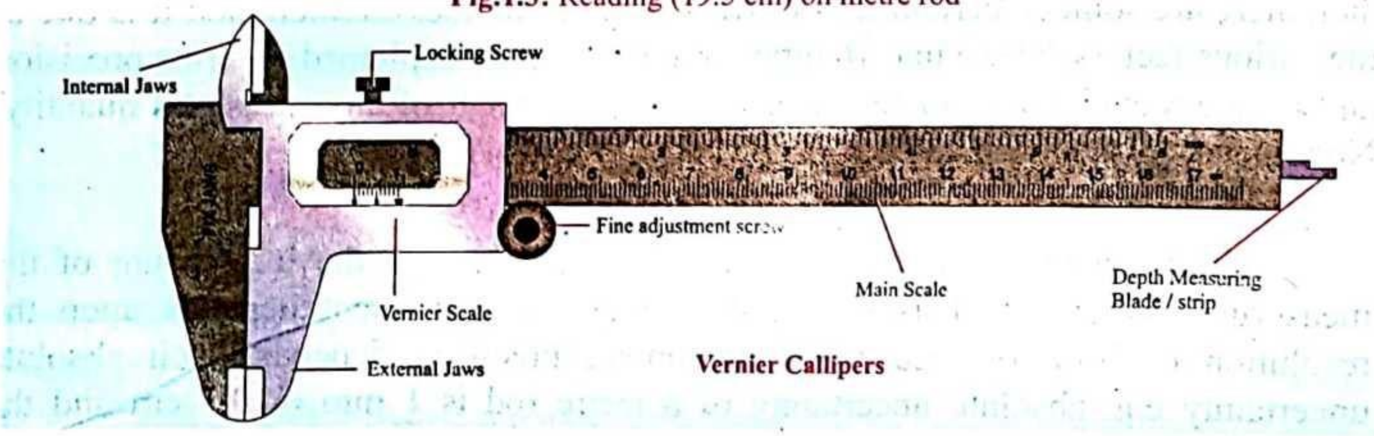
$$\text{Fractional Uncertainty} = \frac{0.1\,\text{cm}}{19.5\,\text{cm}}$$

Fractional Uncertainty = 0.005

$$\text{Percentage Uncertainty} = \frac{0.1}{19.5} \times 100$$

Percentage Uncertainty = 0.5%

Fig.1.3: Reading (19.5 cm) on metre rod



Vernier Callipers

Similarly, the length of another object as measured by a vernier callipers is 0.51 cm as shown in Fig.1.4 then;

Absolute Uncertainty (least count) = ±0.01cm

$$\text{Fractional Uncertainty} = \frac{0.01\,\text{cm}}{0.51\,\text{cm}}$$

$$\text{Fractional Uncertainty} = 0.019 = 0.02$$

$$\text{Percentage Uncertainty} = \frac{0.01\,\text{cm}}{0.51\,\text{cm}} \times 100$$

$$\text{Percentage Uncertainty} = 2\%$$

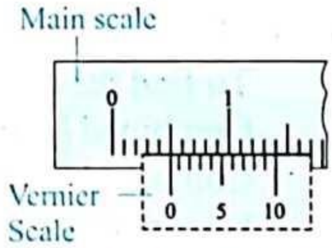


Fig.1.4: Reading (0.5 cm) on vernier Calliper

In the first example the reading 19.5cm is taken by metre rod and it is less precise but is more accurate having more absolute uncertainty and less percentage uncertainty or error. Whereas in the second example, the reading 0.51 cm is taken by vernier callipers which is more precise but less accurate having less absolute uncertainty or least count and more percentage uncertainty.

1.8.3 Assessment of total uncertainty in the final result

The experiments show that the measurement of a physical quantity contains some errors or uncertainties and to calculate the final result, the arithmetic operations may have to be performed. The total uncertainty in the final result does not depend only on the uncertainty of the individual but also on the arithmetic operations as well. The total uncertainty in the final result can be found using the following rules.

(a) If two measured quantities are added or subtracted, then their absolute uncertainties are added.

(b) If two (or more) measured quantities are multiplied or divided, then their relative uncertainties are added.

(c) If a measured quantity is raised to a power, then the relative uncertainty is multiplied by that power.

1.8.4 For addition and subtraction:

Absolute uncertainties are added.

For example, suppose we measure a length of a rod by using metre rod. The positions of two ends 'A' & 'B' of the rod are recorded as:

$A = 11.6 \pm 0.1$ cm and $B = 39.8 \pm 0.1$ cm.

To find the total length of rod we subtract the two points that is

$$\text{Length of rod} = B - A = (39.8 \pm 0.1) - (11.6 \pm 0.1)\ \text{cm}$$

$$\text{Length of rod} = B - A = 28.2 \pm (0.1 + 0.1)\ \text{cm}$$

$$\text{Length of rod} = B - A = 28.2 \pm 0.2\ \text{cm}$$

For example, Akmal and Ajmal are acrobats. Akmal is 165 ±2 cm tall, and Ajmal is 135 ±3 cm tall. If Ajmal stands on top of Akmal's head, how far is his head above the ground?

To find the combined height of Akmal and Ajmal, we add the two heights.
Combined height = Height of Akmal + Height of Ajmal
Combined height = 185 cm + 145 cm = 330 cm
Uncertainty in combined height = 2 cm + 3 cm = 5 cm
So the uncertainty in Combined height = 330 cm ±5 cm

1.8.5 For multiplication and division:

Percentage uncertainties are added.

For example, Find the value of force 'F' and determine the total uncertainty by using F = ma, where m = 60 ± 0.5 kg and a = 5.0 ± 0.2 ms^{-2}.
The maximum possible uncertainty in the value of force is determined as follows:

$$\text{The Percentage uncertainty for m} = \frac{0.5}{60} \times 100 = \text{about } 0.8\%$$

$$\text{The Percentage uncertainty for a} = \frac{0.2}{5} \times 100 = \text{about } 4\%$$

The result is thus given as F = ma = 60 × 5 = 300 N with a percentage uncertainty of 4.8 %.

Thus the total result

$$F = 300\text{ N} \pm 4.8\%$$

$$F = 300\text{ N} \pm \frac{4.8}{100} \times 300$$

$$F = 300 \pm 14.4\text{ N}$$

> Precision is the degree of correctness to which a measurement can be reproduces.

1.8.6 For power factor

If absolute uncertainty of a measurement is known and that measurement occurs interms of power in the given formula, then total percentage uncertainty is calculated by multiplying the power with percentage uncertainty.

For Example

The calculation of the area of cross-section of a cylinder, we use the formula $A = \pi r^2$.

Percentage uncertainty in area of cross-section = 2 × %age uncertainty in radius 'r'.

When uncertainty is multiplied by power factor, then it increases the precision demand of measurement. If the radius of the cylinder is measured as 1.95 cm by vernier callipers with least count 0.01 cm, then the radius 'r' is recorded as:

$$r = 1.95 \pm 0.01\text{ cm}$$

Absolute uncertainty = least count = ± 0.01 cm

Percentage uncertainty in r (radius) = $\frac{0.01}{1.95}\times 100\% = 0.512\% = 0.5\%$

Total percentage uncertainty in area of cross-section = A = 2 × 0.5

Total percentage uncertainty in area of cross-section = 1.0 %

Thus area of cross-section $A = \pi r^2$

$$A = 3.14 \times (1.95)^2 = 11.94\ cm^2$$

So the area of cross section = A = 11.939 cm^2 with 1.0 % uncertainty

The uncertainty is $\frac{1.0}{100} \times 11.939\ cm^2 = 0.12\ cm^2$

Hence the result should be recorded as $A = 11.94 \pm 0.12\ cm^2$.

1.9 DIMENSIONS

The concept of dimension was introduced by Joseph Fourier. It is a method of analysis in which different physical quantities are expressed in terms of their base quantities, such as, mass, length and time. On the other hand, a dimension analysis is a mathematical technique which is being used for the following purposes; to explain the nature of physical quantities, to test the correctness of an equation, to provide a method of changing of units, to assist in recapitulating the formula and to suggest relations between fundamental constants.

For example; torque, work and energy are different physical quantities but dimensionally, they have same nature. Similarly, a distance can be measured in any unit such as feet, metre, kilometre or even in light year but it is always a distance and its dimension is length. In order to learn the expression of physical quantities in terms of their dimension, we use some rules which are related to the process of dimensions and these are summarized as;

I. Dimensions of physical quantities are represented by capital letters in square brackets such as [M L T]

II. The dimension of mass is [M], the dimension of length is [L] and the dimension of time is [T].

III. The dimensions of the majority of physical quantities are expressed in terms of three dimensions [M], [L] and [T].

IV. The quantity which does not exist in the given expression then the power of its dimension is taken as zero such as, the dimension of velocity $\left[M^0L^1T^{-1}\right]$.

V. The quantity which is placed in denominator, the power of its dimensions is taken as a negative integer. e.g. $v = \frac{s}{t} = \frac{[L]}{[T]} = \left[M^0LT^{-1}\right]$

VI. The integers or specific physical quantities which are defined in terms of ratio has no dimension, such as 2, 3, π, angle, strain etc.

Different physical quantities and their dimensions are mentioned in the table 1.4.

Table 1.4: Different physical quantities and their dimensions

Physical Quantity	Formula	SI Unit	Dimensional Formula
Distance	Length	m	[L]
Displacement	Length	m	[L]
Wavelength	Length	m	[L]
Area	Length×Breadth	m^2	$[L] \times [L] = [M^0 L^2 T^0]$
Volume	Length × breadth × height	m^3	$[L] \times [L] \times [L] = [M^0 L^3 T^0]$
Density	Mass / Volume	$kg\ m^{-3}$	$\frac{[M]}{[L^3]} = [M L^{-3} T^0]$
Speed	Distance / time	$m\ s^{-1}$	$\frac{[L]}{[T]} = [M^0 L T^{-1}]$
Velocity	Displacement / time	$m\ s^{-1}$	$\frac{[L]}{[T]} = [M^0 L T^{-1}]$
Acceleration	Velocity / time	$m\ s^{-2}$	$\frac{[L]}{[T^2]} = [M^0 L T^{-2}]$
Momentum	Mass × Velocity	$kg\ m\ s^{-1}$	$[M] \times [LT^{-1}] = [M L T^{-1}]$
Force	Mass × Acceleration	N (Newton)	$[M] \times [L T^{-2}] = [M L T^{-2}]$
Pressure	Force / Area	$N\ m^2$ or Pa	$\frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$
Work	Force × displacement	J (joule)	$[MLT^{-2}] \times [L] = [ML^2 T^{-2}]$
Torque	Force × moment arm	N m	$[MLT^{-2}] \times [L] = [ML^2T^{-2}]$
Power	Work / Time	W (watt)	$\frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$

Physical Quantity	Formula	SI Unit	Dimensional Formula
Impulse	Force × Time	$kg\ ms^{-1} = Ns$	$[MLT^{-2}] \times [T] = [M L T^{-1}]$
K.E	$\frac{1}{2} m v^2$	J (joule)	$[M][LT^{-1}]^2 = [ML^2T^{-2}]$
P.E	Mgh	J (joule)	$[M][LT^{-2}] [L] = [ML^2T^{-2}]$
Stress	Force / Area	Pa (Pascal)	$\frac{[MLT^{-2}]}{[M^0L^2T^0]} = [M^0L^{-1}T^{-2}]$

1.9.1 Principle of homogeneity of dimensions

As a mathematical equation is developed under the various arithmetical operations. If the dimensions of both sides i.e., right hand side and left hand side of the given equation are identical then it is considered as homogenous equation. For example, we test the dimensional homogeneity of the equation $v_f = v_i + at$

Dimensional formula of final velocity $v_f = [LT^{-1}]$

Dimensional formula of initial velocity $v_i = [LT^{-1}]$

Dimensional formula of acceleration and time, $at = [LT^{-2}] \times [T] = [LT^{-1}]$

$L.H.S = v_f = [LT^{-1}]$

$R.H.S = v_i + at = [LT^{-1}] + [LT^{-1}] = 2[LT^{-1}]$

Here 2 is an integer and dimensionless. So $R.H.S = [LT^{-1}]$

This proves that L.H.S = R.H.S

∴ Dimensions on both sides of the equation are the same. Hence, the equation is dimensionally correct.

Example 1.1

The rotational kinetic energy of a body is given by $K.E_{rot} = \frac{1}{2}I\omega^2$, where ω is the angular velocity of the body. Using this equation, find out the dimensional formula for the moment of inertia I.

Solution:

As we know that rotational kinetic energy $= (K.E.)_{rot} = \frac{1}{2}I\omega^2$

or $$I = \frac{2(K.E.)_{rot}}{\omega^2}$$

Using principle of homogeneity of dimensions.

Dimensions of rotational kinetic energy $= (K.E.)_{rot} = [ML^2T^{-2}]$

Dimensions of angular velocity = $\omega = [T^{-1}]$

We get $$I = \frac{2\left[ML^2T^{-2}\right]}{\left[T^{-1}\right]^2}$$

$$I = \frac{2\left[ML^2T^{-2}\right]}{\left[T^{-2}\right]}$$

$$I = 2\left[ML^2T^0\right]$$

As 2 is an integer and dimensionless, so it should be neglected.

Hence the dimensions of moment of inertia are $=\left[ML^2T^0\right]$

Example 1.2

Test the dimensional homogeneity of the following equation

$$E = mgh + \frac{1}{2}mv^2$$

where E is the total energy, m is the mass, g is the acceleration due to gravity, h is the height and v is the velocity.

Solution: LHS = Total Energy = E = $[ML^2T^{-2}]$

$$RHS = mgh + \frac{1}{2}mv^2 = [ML^2T^{-2}] + \frac{1}{2}[ML^2T^{-2}]$$

$$RHS = \frac{3}{2}[ML^2T^{-2}]$$

> **Check Your Concept**
> - What are the dimensions of g/G?
> - What are the dimensions of η/ρ?

Here $\frac{2}{3}$ is a numerical constant and dimensionless, so it should be ignored.

Therefore,

$$RHS = [ML^2T^{-2}]$$

Hence it is proved that dimensions of LHS = dimensions of RHS

1.9.2 Deriving a possible formula

If we have some idea about the physical quantities which depend to one another then we can use the method of dimensional analysis to develop an equation or formula relating these physical quantities.

To derive an equation or formulae we must consider the following rules.

(i) Identify that how many factors depend upon the required quantity.

(ii) All these factors are written in terms of mass, length and time dimensions.

(iii) Equating the powers of M, L and T on both sides of the dimensional equation, three equations are formed by which the value of unknown powers can be calculated.

(iv) By substituting these values in the equation, the real form of relation is achieved.

Example 1.3

Derive an expression for the time period (T) of a simple pendulum by using dimensional analysis.

Solution:

Let 'T' be the time period of a simple pendulum and it may depend upon the following factors,

(i) mass of the bob of the pendulum (m).

(ii) length of the pendulum (ℓ)

(iii) acceleration due to gravity (g)

(iv) angle θ which the thread makes with the vertical.

All these relations can be expressed as;

Let $T \propto m^a$, $T \propto \ell^b$, $T \propto g^c$, $T \propto \theta^d$

By combining all these results;

$$T \propto m^a \ell^b g^c \theta^d$$

$$T = K m^a \ell^b g^c \theta^d \qquad ......(1.1)$$

where K is constant and dimensionless

Equation (1.1) in terms of dimension is expressed as,

- $[T]^1 = [M]^a [L]^b [LT^{-2}]^c [LL^{-1}]^d$
- $M^0L^0T^1 = M^aL^bL^cT^{-2c}L^dL^{-d}$
- $M^0L^0T^1 = M^aL^{b+c+d-d}T^{-2c}$
- $M^0L^0T^1 = M^aL^{b+c}T^{-2c}$

By comparing the powers of respective terms we get,

$$a = 0 \qquad ......(1.2)$$

$$b + c = 0 \qquad ......(1.3)$$

$$-2c = 1$$

and $\quad c = -\dfrac{1}{2} \qquad ......(1.4)$

Putting the value of c in equation (1.3) we get the value of b i.e.

$$b = \frac{1}{2}$$

For Your Information

$\text{Angle} = \dfrac{\text{Arc length}}{\text{Radius}}$

$\text{Angle} = \dfrac{\text{Meter}}{\text{Meter}}$

$\text{Angle} = LL^{-1}$

Angle is dimensionless

Thus, putting the values of a, b and c in eq. (1.1)

$$T = K m^0 \ell^{\frac{1}{2}} g^{\frac{-1}{2}} \theta^0$$

$$T = K m^0 \ell^{\frac{1}{2}} g^{\frac{-1}{2}} \theta^0 \qquad m^0 = 1 \text{ and } \theta^0 = 1$$

$$T = K \ell^{\frac{1}{2}} g^{\frac{-1}{2}}$$

$$T = K \frac{\ell^{\frac{1}{2}}}{g^{\frac{1}{2}}} = K \frac{\sqrt{\ell}}{\sqrt{g}}$$

$$T = K \sqrt{\frac{\ell}{g}}$$

Example 1.4

Derive the relation for speed of sound 'v' through a gas using dimension analysis.

Solution:

The velocity of sound depends upon pressure 'P', and density 'ρ'of the medium i.e.

$$V \propto P^a \rho^b$$

$$V = K P^a \rho^b \quad ......(1.5)$$

where 'K' is a dimensionless constant and hence eq. (1.5) can be expressed in terms of dimensions as;

$$[M^0 L^1 T^{-1}] = [ML^{-1}T^{-2}]^a [M L^{-3}]^b$$

$$[M^0 L^1 T^{-1}] = [M^a L^{-a} T^{-2a}] [M^b L^{-3b}]$$

$$[M^0 L^1 T^{-1}] = [M^{a+b} L^{-a-3b} T^{-2a}] \quad ......(1.6)$$

By comparing the powers of the respective terms in eq. (1.6) we get;

$$a + b = 0 \quad ......(1.7)$$

$$-a - 3b = 1 \quad ......(1.8)$$

$$-2a = -1$$

$$a = \frac{1}{2} \quad ......(1.9)$$

Putting the value of a in eq. (1.7)

$$\frac{1}{2} + b = 0$$

$$b = -\frac{1}{2} \quad ......(1.10)$$

Thus, putting the values of a and b in equation (1.5), we get

Hence,
$$v = K\, P^{\frac{1}{2}} \rho^{\frac{-1}{2}}$$
$$v = K\sqrt{\frac{P}{\rho}}$$

Where K is an arbitrary constant and experimentally its value is 1.
$$v = \sqrt{\frac{P}{\rho}}$$

1.10 LIMITATIONS OF DIMENSIONAL ANALYSIS

Although the technique of dimensional analysis is useful and helpful in many cases but, it has some limitations which are listed below:

I. The method cannot be used to determine the value of dimensionless constants. They have to be determined either by experimental or mathematical analysis

II. This method cannot be used to relations involving trigonometric, logarithmic and exponential functions.

III. Dimensional analysis does not indicate whether a physical quantity is a scalar or vector. For example, speed and velocity both have same dimensions $\left[M^0LT^{-2}\right]$.

IV. Dimensional analysis cannot be used to derive the exact form of a physical relation if the physical quantity depends upon more than three physical quantities (i.e., M, L and T).

V. Dimensional analysis provides the correctness of the given relation only dimensionally but it does not give the physical correctness of the relation e.g.

$T = 2\pi\sqrt{\frac{\ell}{g}}$ It is correct dimensionally.

$T = \frac{1}{2\pi}\sqrt{\frac{\ell}{g}}$ It is correct both dimensionally but not physically.

SUMMARY

- **Physics:** Physics is the branch of science which deals with the study of matter, and energy and their mutual interaction.
- **Physical Quantities:** The quantities which can be measured and have proper units are called physical quantities.
- **Unit:** The quantity used as a standard of measurement is called unit.
- **International system (SI):** The international committee for weights and measures introduced a metric system for measurement which is called

international system (SI) of unit and it consists of seven base units, two supplementary units and a number of derived units.

- **Scientific notation**: A method of expressing of too large or too small value in terms of some power of ten multiply by a number is called scientific notation or standard form.
- **Significant Figures:** A reported result by an observer always contain both certain and uncertain digits. The number of these certain digits and first uncertain digit are known as significant figures.
- **Uncertainty:** Due to the poor condition of the instrument, irrelevant experimental technique and carelessness of the observer the reported result contains some errors which are called uncertainty.
- **Systematic Error:** The errors which appear in measurement due to known causes are known as systematic error. These errors repeat in same magnitudes and signs.
- **Random Error:** The errors which appear in measurement due to unknown causes are known as random errors. These errors repeat in different magnitudes and signs.
- **Precision:** Precision of a measurement depends upon the least count of the instrument. Smaller the least count, more precise is the measurement.
- **Accuracy:** The accuracy of the measurement depends upon fractional uncertainty. Smaller the fractional uncertainty more is the accuracy of the measurement.
- **Dimensional analysis:** A mathematical technique which explains the nature of the physical quantities is known as dimensional analysis. It can be used to analyze the homogeneity of a mathematical equation and deriving a possible formula.

EXERCISE

Choose the best option.

1. The main contribution of modern physics is

(a) Newton's laws of motion (b) Thermodynamics laws

(c) Kinetic theory (d) Special theory of relativity

2. The branch of physics which deals with the properties and interaction of nuclear particles (protons and neutrons) is called

(a) Molecular physics (b) Plasma physics

(c) Nuclear physics (d) Solid State Physics

3. The fundamental physical quantities which form the basis of the SI units are
(a) Force weight and time (b) Mass, length and time
(c) Mass, length and force (d) Mass, energy and time

4. Which one of the following is base physical quantity?
(a) Pressure (b) Temperature (c) Density (d) Energy

5. Which list of units contains three base quantities and two derived quantities?
(a) Kelvin, newton, second, kilogram, ohm
(b) Volt, joule, ampere, coulomb, meter
(c) Kilogram, meter, second, mole, kelvin
(d) Mole, hertz, kelvin, joule, newton

6. Light year is the unit of
(a) Time (b) Distance (c) Speed (d) Velocity

7. μm × mm is equal to
(a) 10^9m (b) 10^{-9}m (c) 10^9m^2 (d) 10^{-9}m^2

8. The derived unit joule in terms of base units is
(a) kg m s^{-2} (b) kg m^2 s^{-2} (c) kg m^{-1} s^{-2} (d) kg m^{-2} s^{-2}

9. The scientific notation of a measured value 0.0092 m
(a) 9.2×10^3m (b) 9.2×10^{-3}m (c) 9.2×10^5 m (d) 9.2×10^{-5}m

10. By using vernier callipers, the length of an object is measured by four students. Which one of the following is correct?
(a) 4.5 cm (b) 4.51 cm (c) 4.510 cm (d) 4.5100 cm

11. What is the absolute precision of the referred result 8.52 cm.?
(a) 1 cm (b) 0.1 cm (c) 0.01 cm (d) 0.001 cm

12. The number of significant figures of the value 0.0202 is
(a) Two (b) Three (c) Four (d) Five

13. Which one of the following measurement is the most significant?
(a) 203000 (b) 203×10^3 (c) 20.3×10^4 (d) 2.03×10^5

14. The length of a body is measured as 3.51 m, if the accuracy is 0.01 m, then the percentage uncertainty in the measurements is
(a) 3.51 % (b) 0.035 % (c) 0.28 % (d) 28.65%

15. The dimensions of moment of inertia is
(a) $[MLT^0]$ (b) $[ML^2T^0]$ (c) $[ML^{-2}T^0]$ (d) $[M^2LT^0]$

16. Which pair of physical quantities has same dimensions?
(a) Velocity and acceleration (b) Mass and weight
(c) Inertia and moment of inertia (d) Work and potential energy

SHORT QUESTIONS

1. Differentiate between base and derived quantities.
2. How can you obtain the derived physical quantities by using the arithmetic operations?
3. Convert one year into months, days, hours, minutes and seconds.
4. When and where the system of international (SI) for weight and measure came into being?
5. Which physical quantity has unit but has no dimension?
6. Write down the following in scientific notation
(a) Angstrom (A^o) (b) Pico metre (c) mega pixels
7. How can you derive the unit of watt in term of base units?
8. Identify three physical quantities which have no units and no dimensions?
9. What is the difference between systematic error and random error?
10. Between precise and accurate measurement, which one is more reliable?
11. What are three causes of errors in measurement by instruments?
12. How can you minimize the error of a reported result?
13. How accuracy is increased by decreasing the limit of precision?
14. Write three examples when zeros are not considered as significant figures.
15. How many expected number of significant figures are in 7000?
16. What do you know about the uncertainty of an instrument?
17. Which physical quantities have the same dimension? Give an example.
18. Find the dimension of gravitational constant G by using $F = G\frac{m_1m_2}{r^2}$.
19. According to Hook's law, the restoring force F due to a body attached to a spring is given by $F = -kx$. Calculate the dimensions of the spring constant k.

COMPREHENSIVE QUESTIONS

1. State and explain the scope, importance and applications of physics in our daily life activities.
2. Describe the physical quantities with all its classes and justify that how can you obtain derived quantities by using product and quotient rules?
3. When and where was established the international system of units? Explain all the branches of SI units.
4. Explain conventions used for indicating SI units.

5. What is role of scientific notation and prefixes in the expression of too large or too small quantities.
6. Explain the significant figures with all its rules.
7. What do you know about;
   a) errors and uncertainties.
   b) precision and accuracy.
8. What are the dimensions of physical quantities? How can you explain the nature of physical quantities by using the dimensions analysis.
9. What is the role of dimension analysis in the derivation of formula and explanation of homogeneity of a mathematical equation.

NUMERICAL PROBLEMS

1. How much distance is covered by light in one year when its speed in space is 3×10^8 m s^{-1}. **(9.5×10^{15} m)**
2. Mass of neutron is 1.67×10^{-27} kg. Calculate the number of neutrons in a piece of metal whose net mass is one gram. **(5.99×10^{23} neutrons)**
3. Prove that nano seconds in one second is more than the number of seconds in one year.
4. Convert the following (a) 20 m s^{-1} into km h^{-1}, (b) 3×10^8 m s^{-1} into km h^{-1}, (c) 220 km h^{-1} into m s^{-1} **(a) 72 km h^{-1}, (b) 1.08×10^{10} km h^{-1} (c) 61 m s^{-1}**
5. (a) Express the following values in terms of prefixes.
   (I) 0.62×10^4 g, (II) 2×10^7 m, (III) 4×10^{-5}s
   (b) Express the following values in terms of scientific notation.
   (I) 0.000036, (II)140000, (III) 107000000
   (a) (I) 6.2 kg, (II) 20 Mm, (III) 40μs
   (b) (I) 3.6×10^{-5}, (II) 1.4×10^5 (III)1.07×10^8
6. The radius of a rod is 0.24 cm. Find its cross sectional area with appropriate significant figures. **(0.18 cm^2)**
7. How many significant figures are there in the given values?
   (a) 32.900, (b) 2003, (c) 2.0, (d) 0.0007, (e) 2.73×10^6
   (a) 5, (b) 4, (c) 2, (d) 1, (e) 3
8. Add the given values 12, 13.5, 15.432. Give the answer to correct significant figures. **(41)**
9. Verify that the given equation $S = v_i t + at^2$ is dimensionally correct.
10. The centripetal force 'F' acting on a particle (moving uniformly in a circle) depends on the mass 'm' of the particle, its velocity 'v' and radius 'r' of the circle. Derive dimensionally the formula for the centripetal force 'F'.

For example, velocity vector is represented by $\vec{v}$, momentum vector by $\vec{p}$, and acceleration vector by $\vec{a}$ etc.

The magnitude of a vector $\vec{A}$ is denoted by the same letter used for the vector without arrow A or $|\vec{A}|$ read as magnitude of $\vec{A}$. For example, the magnitude of displacement vector $\vec{d}$ is denoted as d or $|\vec{d}|$.

It is important to note the following points;

- The magnitude of a vector is always positive and scalar.
  The magnitude of a vector is also called modulus of the vector and is represented by enclosing the vector symbol between two vertical lines, for example, the modulus of displacement vector $\vec{d}$ can be denoted as $|\vec{d}|$.
- Vectors can be added, subtracted and multiplied. However, the division of a vector by another vector is not valid operation in vector algebra. It is because the division of a vector by a direction is not possible.

2.1.2 Graphical representation of vectors

A vector is represented graphically by a straight line with an arrow on its one end as shown in Fig.2.1. The length of the line (OA) represents the magnitude of the given vector $\vec{d}$ (on suitable scale) while arrow shows the direction of the vector. The starting point (O) of the vector $\vec{d}$ is called tail and the end point (A) is called head of the given vector. The graphical representation of vector is further explained by some examples.

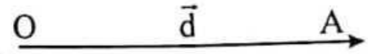
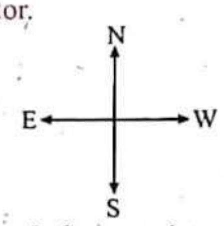


Fig.2.1: A straight line with an arrow on its one end represents a vector.

Suppose a bike travels 10 km from east to west, we say that the bike undergoes a displacement vector of 10 km towards east.



Reference points

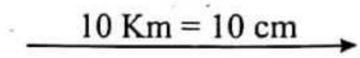


Fig.2.2: A displacement vector of 10 Km towards West

Graphically, it is represented by a straight line with an arrow using scale. Let 10 km = 10 cm is the magnitude of the given displacement vector and arrow toward the west is in its direction as shown in Fig. 2.2.

Similarly, in case of a vector in two dimensional plane, let 100 m length of thread of a flying kite making an angle 'θ' with ground. In this case, the length of thread (100 m = 10 cm) is the magnitude of the given displacement (vector) while angle 'θ' shows its direction as shown in Fig. 2.3.

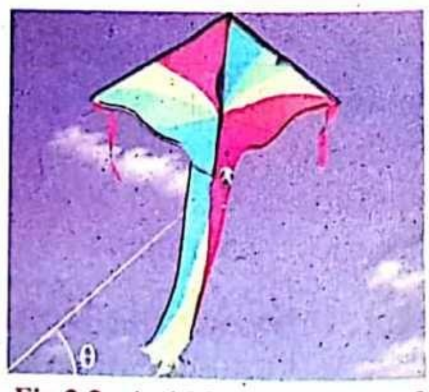


Fig.2.3: A 100 m thread of flying kite in two dimensions plane making angle 'θ' with ground.

2.2 CARTESIAN CO-ORDINATE SYSTEM

In common life we use the reference points East-West and North-South for the determination of position of an object. But for the graphical representation of a vector, we use cartisen co-ordinate system which consists of two straight lines which are perpendicular to each other and it is known as plane rectangular co-ordinate system or Cartesian co-ordinate system. The horizontal line is known as x-axis while the vertical line is known as y-axis.

The point of intersection of these two lines is known as origin 'O' as shown in Fig. 2.4 positive x-axis is taken to the right and negative x-axis is taken to the left from origin 'O'. Similarly, positive y-axis is taken upward and negative y-axis is taken downward from the origin 'O'.

In Cartesian co-ordinate system, there are four quadrants. A vector can be drawn in any one of them according to the angle 'θ' made by the given vector with x-axis as shown in Fig. 2.5.

In the first quadrant both x and y components of a vector are positive, in the second quadrant, x-component is negative and y-component is positive. In the third quadrant both x and y components are negative and in the fourth quadrant x-component is positive while y-component is negative.

In space, a third axis also exists which is called z-axis and it is perpendicular to both x-axis and y-axis. Now when a vector is drawn in this three dimensional space, it makes angles α, β and γ with x, y and z respectively. These angles provide the direction of the given vector as shown in Fig. 2.6.

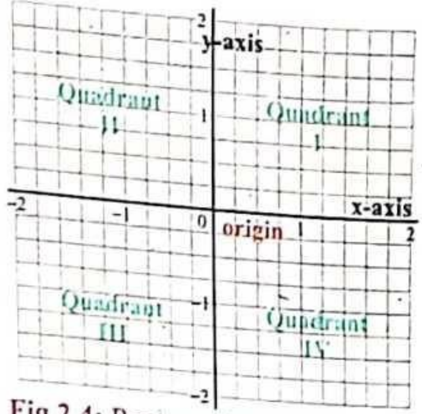


Fig.2.4: Rectangular Co-ordinate System

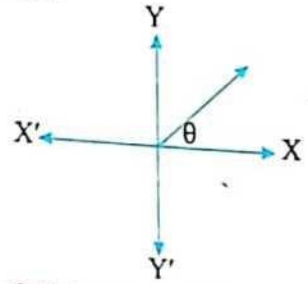


Fig.2.5: A vector in 1st quadrant making angle 'θ' with x-axis.

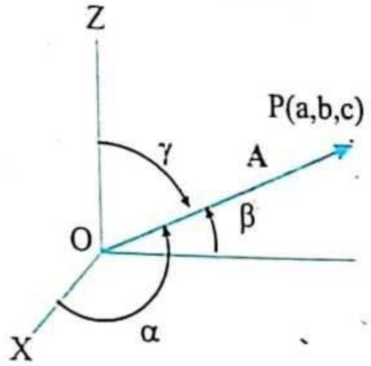
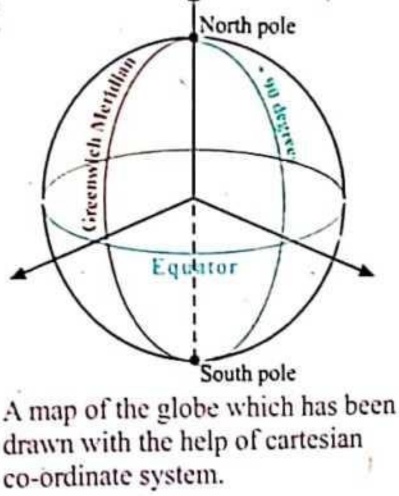


Fig.2.6: A vector in three dimensional space, making angles a, b, and y with x-axis, y-axis and z-axis respectively.



A map of the globe which has been drawn with the help of cartesian co-ordinate system.

2.3 KINDS OF VECTORS

2.3.1 Null or Zero Vectors

A vector of zero magnitude and with arbitrary direction is called null vector. This is also called zero vector and is denoted by ' $\vec{O}$ '. It is used to balance vector equations. For example, if

$$\vec{A} = \vec{B}, \text{ then } \vec{A} - \vec{B} = \vec{O}$$

Thus, we can say that if two vectors $\vec{A}$ and $\vec{B}$ are equal then their difference $\vec{A} - \vec{B}$ is defined as zero vector.

> **Point to ponder**
> How can you draw graphically the direction of a null vector?

On the other hand, the vector whose magnitude is not zero is called proper vector.

2.3.2 Unit Vector

A vector whose magnitude is equal to unity is known as unit vector. It has the same direction as that of the given vector. It is represented by a bold face letter with a cap and is read as 'A cap' or 'A hat'. For example, $\hat{A}, \hat{B}, \hat{C}$ and $\hat{D}$. A unit vector $\hat{A}$ in the direction of the given vector $\vec{A}$ is expressed as;

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} \qquad \text{...... (2.1)}$$

or

$$\vec{A} = \hat{A}|\vec{A}|$$

Thus, any vector in the direction of a unit vector may be written as the product of the unit vector and the magnitude of that vector of the given vector.

The unit vector has no units and dimensionless vector. It represents only direction of the given vector.

The unit vectors along x, y and z-axes of three dimensional Cartesian co-ordinate system are represented by vectors, $\hat{i}, \hat{j}$ and $\hat{k}$ respectively as shown in Fig.2.7.

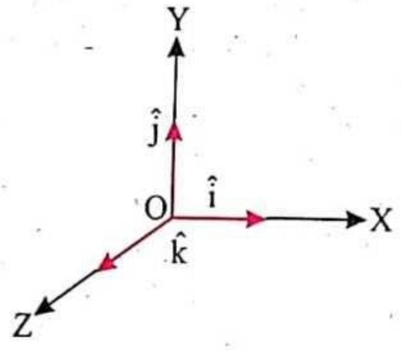


Fig.2.7: Unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ in three dimensional Cartesian Co-ordinate system.

The magnitude of each unit vector is 1. i.e.,

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

Example 2.1

Find the unit vector $\hat{A}$ in the direction of vector $\vec{A}$ where $\vec{A} = 3\hat{i} + 4\hat{j}$.

Solution: $\vec{A} = 3\hat{i} + 4\hat{j}$

We know that $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

Putting the value of $\vec{A}$ in above equation

$$\hat{A} = \frac{3\hat{i}+4\hat{j}}{|3\hat{i}+4\hat{j}|} = \frac{3\hat{i}+4\hat{j}}{\sqrt{3^2+4^2}}$$

$$\hat{A} = \frac{3\hat{i}+4\hat{j}}{\sqrt{9+16}} = \frac{3\hat{i}+4\hat{j}}{\sqrt{25}}$$

$$\hat{A} = \frac{3\hat{i}+4\hat{j}}{5} = \frac{3\hat{i}}{5}+\frac{4\hat{j}}{5}$$

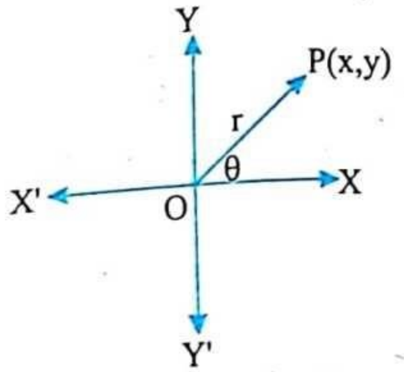


Fig.2.8: A position vector $\vec{r}$ in xy-plane

2.3.3 Position Vector

A vector which is used to specify the position of an object or a point with respect to the origin is known as position vector. It is represented by $\vec{r}$. Graphically, a position vector $\vec{r}$ in two dimensional XY-plane is represented by a straight line with an arrow head from origin to point P(x,y) as shown in Fig. 2.8 and it is expressed as;

$$\vec{r} = x\hat{i} + y\hat{j}$$

and $$|\vec{r}| = \sqrt{x^2+y^2} \quad ......(2.2)$$

Similarly a position vector $\vec{r}$ in three dimensional space from origin 'O' to point P(x, y, z) as shown in Fig. 2.9 and it is expressed as;

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

and $$|\vec{r}| = \sqrt{x^2+y^2+z^2} \quad ......(2.3)$$

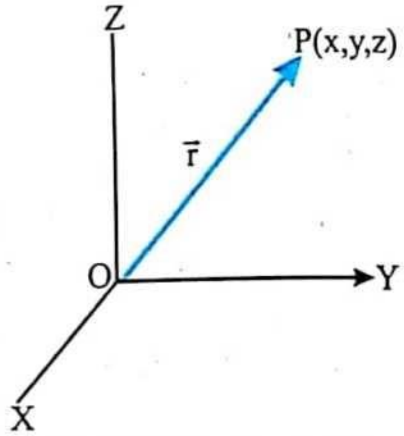


Fig.2.9: A position vector $\vec{r}$ in three dimension space

Example 2.2:

The position vector of the points P and Q in space with respect to the origin are $\vec{r}_1$ and $\vec{r}_2$. If $\vec{r}_1 = 2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{r}_2 = 4\hat{i}-3\hat{j}+2\hat{k}$. Determine $\overrightarrow{PQ}$ and its magnitude.

Solution:

$$PQ = \vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\vec{r} = (4\hat{i}-3\hat{j}+2\hat{k})-(2\hat{i}+3\hat{j}-\hat{k})$$

$$\vec{r} = 4\hat{i}-3\hat{j}+2\hat{k}-2\hat{i}-3\hat{j}+\hat{k}$$

$$\vec{r} = 2\hat{i}-6\hat{j}+3\hat{k}$$

$$|\overrightarrow{PQ}| = |\vec{r}| = \sqrt{2^2+(-6)^2+3^2}$$

$$\left|\overline{PQ}\right| = |\bar{r}| = \sqrt{4+36+9}$$

$$\left|\overline{PQ}\right| = |\bar{r}| = \sqrt{49} = 7$$

$$\left|\overline{PQ}\right| = |\bar{r}| = 7$$

Point to Ponder
Equal vectors are called parallel vectors but you cannot say that parallel vectors are also called equal

2.3.4 Equal Vectors

Two vectors which have same magnitude and same direction are known as equal vectors. Mathematically, two vectors $\bar{A}$ and $\bar{B}$ having same magnitude and direction as shown in Fig. 2.10 are expressed as;

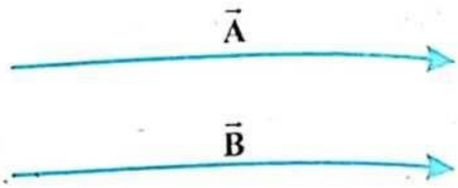


Fig.2.10: Two equal vectors $\bar{A}$ and $\bar{B}$ of same magnitude and direction.

$$\bar{A} = \bar{B}$$

$$|\bar{A}| = |\bar{B}| \quad \ldots\ldots(2.4)$$

Equal vectors are also known as parallel vectors. Angle 'θ' between parallel vectors is 0°.

2.3.5 Negative Vector

A vector which has same magnitude but opposite in direction to the given vector is called the negative vector. The negative vector of the given vector $\bar{A}$ is represented by $-\bar{A}$ as shown in Fig.2.11.

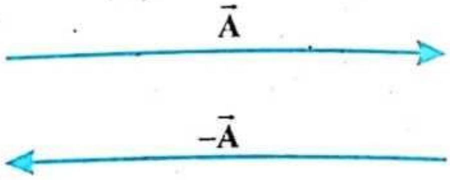


Fig.2.11: Two negative vectors $\bar{A}$ and $-\bar{A}$ of same magnitude but in opposite direction.

Mathematically $\bar{A}$ and $-\bar{A}$ are expressed as;

$$\bar{A} = -\bar{A} \quad \ldots\ldots(2.5)$$

Negative vectors are also known as anti-parallel vectors and angle 'θ' between them is 180°.

2.4 MULTIPLICATION OF A VECTOR BY A SCALAR

When a vector $\bar{A}$ is multiplied by a positive integer 'm' say a scalar then its resultant vector $m\bar{A}$ is another vector whose magnitude is 'm' times that of vector $\bar{A}$ and its direction is the same as that of vector $\bar{A}$. Similarly, when the vector $\bar{A}$ is multiplied by a negative integer '–m', then the magnitude of resultant vector is $m\bar{A}$ but its direction is opposite as that of the $\bar{A}$.

For example, let a vector $\bar{A}$ is multiplied by a number 2, it gives vector $2\bar{A}$ and shows that the magnitude of the resultant is increased 2 times in the direction of

$\vec{A}$ as shown in Fig. 2.12. Similarly, if we multiplied a vector $\vec{A}$ by -2 then we get a vector $-2\vec{A}$.

This shows that the magnitude of the resultant vector is increased by 2 times but in the reverse direction of $\vec{A}$ as shown in Fig. 2.12.

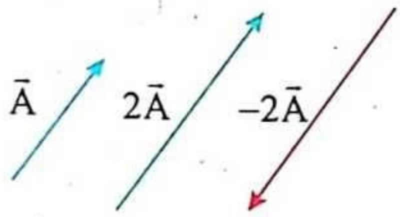


Fig 2.12: Multiplication of vector $\vec{A}$ by a number ± 2.

2.5 ADDITION AND SUBTRACTION OF VECTORS

2.5.1 Addition of Vectors

Vectors can be added under three different methods.

I. Parallelogram Method

Let two vectors $\overline{MN} = \vec{A}$ and $\overline{MP} = \vec{B}$ which are represented the two adjacent sides of a parallelogram MNOP.

According to the law of parallelogram, the diagonal $\overline{MO} = \vec{R}$ is the resultant vector of the given vectors $\vec{A}$ and $\vec{B}$ as shown in Fig. 2.13.

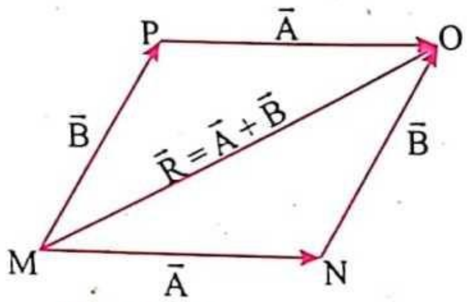


Fig.2.13: Addition of vectors by Parallelogram Method

Thus, $\overline{MO} = \overline{MN} + \overline{NO}$

$$\vec{R} = \vec{A} + \vec{B} \quad ......(2.6)$$

Similarly, $\overline{MO} = \overline{MP} + \overline{PO}$

$$\vec{R} = \vec{B} + \vec{A} \quad ......(2.7)$$

From eq. 2.6 and eq. 2.7

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \quad ......(2.8)$$

This is a commutative law vector addition.

II. Triangle Method

When two vectors $\overline{OP} = \vec{A}$ and $\overline{PQ} = \vec{B}$ which are represented the two adjacent sides of a triangle OPQ, then according to law of triangle, the 3rd side of the triangle $\overline{OQ} = \vec{R}$ is the resultant vector of the given vectors $\vec{A}$ and $\vec{B}$ as shown in Fig. 2.14.

$$\vec{R} = \vec{A} + \vec{B} \quad ...... (2.9)$$

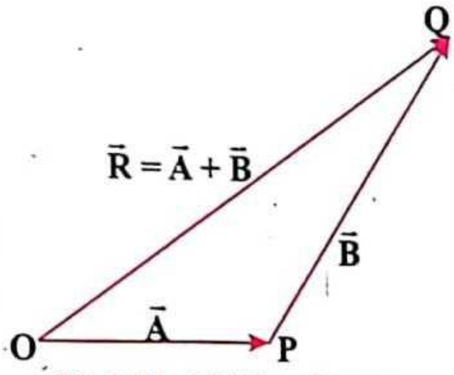


Fig 2.14: Addition of vectors by triangle method

III. Head to Tail Rule

Graphically, two or more than two vectors are added by a rule which is known as "Head to Tail Rule". First, select a suitable scale and draw the representative lines (i.e. in terms of magnitude and direction) of all given vectors. Then apply this rule as; join the head of the first vector with the tail of the second vector according to their respective directions. Similarly, join the head of the second vector with the tail of third vector.

Keep on repeating the same process till the last vector is also drawn.

Now the resultant vector of all these vectors is a straight line from the tail end of the first vector to the head end of the last vector.

For example, we have four vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ which are acting in their given directions i.e., they are represented by arrow lines with suitable scale. Thus, all these vectors can be added according to their directions and scale by using head-to-tail rule. Join the head of vector $\vec{A}$ with the tail of vector $\vec{B}$, similarly join the head of vector $\vec{B}$ with the tail of vector $\vec{C}$ and then join the head of vector $\vec{C}$ with the tail of vector $\vec{D}$.

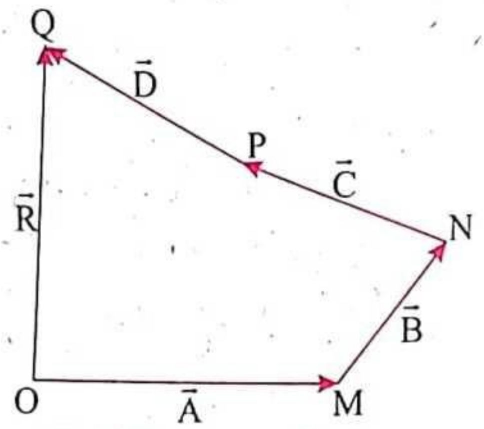


Fig.2.15: Addition of vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ by Head to Tail rule

Thus, the resultant vector $\vec{R}$ of these vectors is a straight line from the tail of the first vector $\vec{A}$ to the head of the last vector $\vec{D}$ as shown in Fig.2.15. Mathematically it can be expressed as;

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} \quad \ldots\ldots (2.10)$$

It is noted that when two vectors are parallel to each other, then the magnitude of their resultant vector will be maximum and it is equal to the sum of their magnitudes. Similarly, when two vectors are anti-parallel then the magnitude of their resultant vector will be minimum and it is equal to the difference of their magnitudes.

2.5.2 Subtraction of Vectors

When we want to subtract a vector $\vec{B}$ from vectors $\vec{A}$ then we draw the representative lines of vector $\vec{A}$ and $-\vec{B}$ i.e. negative of vector $\vec{B}$ and apply head-to-tail rule on vectors $\vec{A}$ and $-\vec{B}$ in order to get the resultant.

Let us have two vectors $\vec{A}$ and $\vec{B}$. The subtraction of vector $\vec{B}$ from vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ (negative of vector $\vec{B}$) to vector $\vec{A}$.

Thus, $$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \quad \ldots\ldots (2.11)$$

Graphically, the subtraction of vector $\vec{B}$ from vector $\vec{A}$ is explained as; first take the negative vector of vector $\vec{B}$ i.e. $-\vec{B}$. Now according to head to tail rule join the head of vector $\vec{A}$ with the tail of vector $-\vec{B}$ as shown in Fig. 2.16. The resultant vector $\bar{C}$ of these two vectors is the line from tail of vector $\vec{A}$ to the head of vector $-\vec{B}$ and it is equal to $\vec{A}-\vec{B}$.

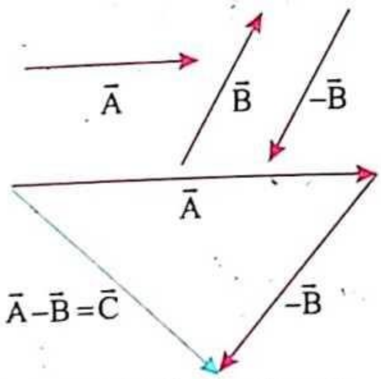


Fig.2.16: Subtraction of vectors $\vec{A}$ and $\vec{B}$

Example 2.3

If $\vec{A}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{B}=2\hat{i}-\hat{j}-4\hat{k}$. Find

(a) $|\vec{A}+\vec{B}|$ (b) $|\vec{A}-\vec{B}|$

Solution: (a)

$$\vec{A}+\vec{B}=(\hat{i}+2\hat{j}-3\hat{k})+(2\hat{i}-\hat{j}-4\hat{k})$$
$$\vec{A}+\vec{B}=\hat{i}+2\hat{j}-3\hat{k}+2\hat{i}-\hat{j}-4\hat{k}$$
$$\vec{A}+\vec{B}=3\hat{i}+\hat{j}-7\hat{k}$$
$$|\vec{A}+\vec{B}|=\sqrt{(3)^2+(1)^2+(-7)^2}$$
$$|\vec{A}+\vec{B}|=\sqrt{9+1+49}=\sqrt{59}$$
$$|\vec{A}+\vec{B}|=7.68$$

(b)

$$\vec{A}-\vec{B}=(\hat{i}+2\hat{j}-3\hat{k})-(2\hat{i}-\hat{j}-4\hat{k})$$
$$\vec{A}-\vec{B}=\hat{i}+2\hat{j}-3\hat{k}-2\hat{i}+\hat{j}+4\hat{k}$$
$$\vec{A}-\vec{B}=-\hat{i}+3\hat{j}+\hat{k}$$
$$|\vec{A}-\vec{B}|=\sqrt{(-1)^2+3^2+1^2}$$
$$|\vec{A}-\vec{B}|=\sqrt{1+9+1}$$
$$|\vec{A}-\vec{B}|=\sqrt{11}=3.3$$

2.6 RESOLUTION OF A VECTOR

The process of splitting or decomposing a single vector into two or more vectors in different direction called components such that their resultant is again equal to the given vector is called resolution of a vector.

If a vector is resolved into two components which are perpendicular to each other, then these are called rectangular components of the given vector. It is explained as;

Consider a vector $\vec{A}$, represented by a line $\overline{OP}$ which is lying in a XY-plane. In order to determine its rectangular components, we draw two perpendiculars $\overline{PN}$

and $\overline{PM}$ on X and Y axes respectively. Then the vectors $\vec{A}_x$ and $\vec{A}_y$ drawn from O to N and O to M are the rectangular components of the vector $\vec{A}$. Indeed, these rectangular components A_x and A_y are the projections of the given vector $\vec{A}$ on x-axis and y-axis respectively.

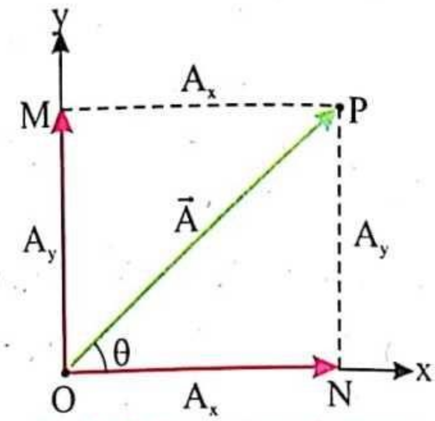


Fig.2.17: Resolution of vector $\vec{A}$ into its rectangular components A_x and A_y.

As shown in Fig.2.17, $\overrightarrow{PM}$ is equal and parallel to $\overline{ON}$, and $\overline{PN}$ is equal and parallel to $\overline{OM}$.

Thus, applying law of vector addition for the right angle triangle ONP and we have

$$\overline{OP} = \overline{ON} + \overline{NP}$$

or $$\vec{A} = \vec{A}_x + \vec{A}_y \quad ......(2.12)$$

As $\vec{A}_x = A_x\hat{i}$ and $\vec{A}_y = A_y\hat{j}$

So, eq. 2.12 can also be expressed in terms of $\hat{i}$ and $\hat{j}$.

$$\vec{A} = A_x\hat{i} + A_y\hat{j} \quad ......(2.13)$$

This is the resultant vector $\vec{A}$ in terms of its rectangular components.

As the rectangular components A_x and A_y are at right angle, therefore, their magnitude can be calculated by using the trigonometric ratios. From ΔONP,

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{ON}{OP} = \frac{A_x}{A}$$

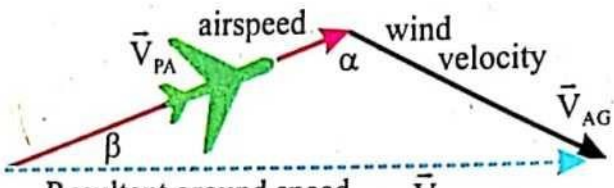
$$A_x = A\cos\theta \;...... (2.14)$$

This is the horizontal or x-component of the vector $\vec{A}$. Similarly,

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{NP}{OP} = \frac{A_y}{A}$$

$$A_y = A\sin\theta \quad ......(2.15)$$

This is the perpendicular or y-component of the vector $\vec{A}$. The magnitude and direction of the given vector $\vec{A}$ can be calculated by using Pythagorean Theorem. Using triangle ONP, we have



Velocity components of an aeroplane

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$$A^2 = A_x^2 + A_y^2$$

$$A = \sqrt{A_x^2 + A_y^2} \quad ......(2.16)$$

This is the magnitude of the given vector $\vec{A}$. Dividing equation (2.15) by equation (2.14), we get

$$\frac{\sin\theta}{\cos\theta} = \frac{A_y}{A_x} \Rightarrow \tan\theta = \frac{A_y}{A_x}$$

or $$\theta = \tan^{-1}\frac{A_y}{A_x} \quad ......(2.17)$$

where 'θ' shows the direction of the given vector $\vec{A}$.

Table 2.1: The values of trigonometric ratios at different angles

θ	Sin θ	Cos θ	Tan θ
0°	0	1	0
30°	$\frac{1}{2} = 0.5$	$\frac{\sqrt{3}}{2} = 0.866$	$\frac{1}{\sqrt{3}} = 0.577$
45°	$\frac{1}{\sqrt{2}} = 0.707$	$\frac{1}{\sqrt{2}} = 0.707$	1
60°	$\frac{\sqrt{3}}{2} = 0.866$	$\frac{1}{2} = 0.5$	$\sqrt{3} = 1.73$
90°	1	0	∞

Example 2.4

A person is climbing up on a ladder of length 10 m which is lying with wall making angle 60° with floor. Calculate the horizontal distance from the floor end of the ladder to the wall and height of the wall from the floor to the upper end of the ladder.

Solution:

Length of ladder $= A = 10\,m$

Angle $= \theta = 60^\circ$

Horizontal distance $= A_x = ?$

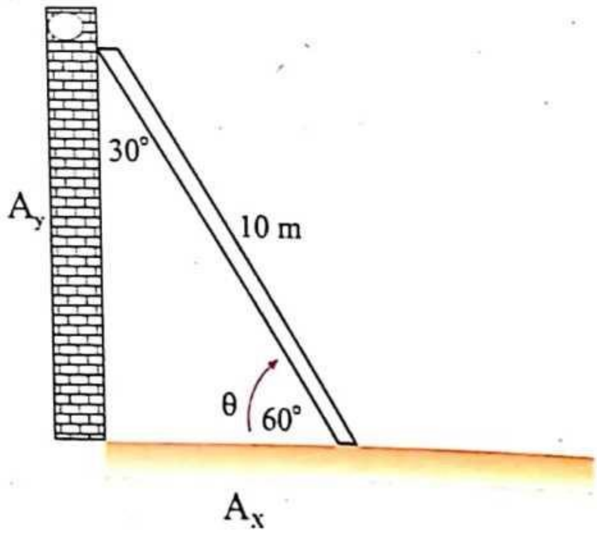
Vertical height $= A_y = ?$

$$A_x = A\cos\theta$$

$$A_x = 10 \times \cos 60^\circ$$

$$A_x = 10 \times (0.5)$$

$$A_x = 5\,m$$



Similarly, $$A_y = A\sin\theta$$

$$A_y = 10\sin 60^\circ$$

$$A_y = 10(0.866)$$

$$A_y = 8.66\,m$$

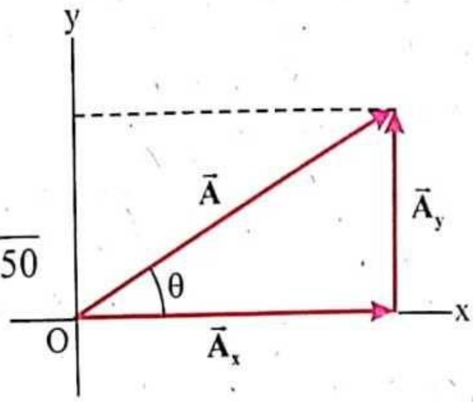
Example 2.5

What is the magnitude and direction of a vector $\vec{A}$ which is lying in the first quadrant and when its both perpendicular components are of 5 units?

Solution:

According to the question $A_x = A_y = 5$ units

$$|\vec{A}| = ? \text{ and } \theta = ?$$



$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$|\vec{A}| = \sqrt{5^2 + 5^2} = \sqrt{25 + 25} = \sqrt{50}$$

$$|\vec{A}| = 7.1 \text{ units}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\frac{5}{5} = \tan^{-1}1$$

$$\theta = 45^\circ$$

2.7 ADDITION OF VECTORS BY RECTANGULAR COMPONENTS

We have learnt addition of two or more than two vectors by the method of head-to-tail rule in the previous section. This method of addition was graphical and there were more chances of error in the determination of the resultant vector.

Now we introduce another analytical method of addition of two or more vectors which is named as addition of vectors by rectangular components. This method of addition of vectors is more reliable and accurate than head-to-tail rule. It is explained as;

Considering two vectors $\overline{OP} = \bar{A}_1$ and $\overline{OQ} = \bar{A}_2$ are lying in XY-plane making angles θ_1 and θ_2 with x-axis respectively. Let from point 'P' we draw another vector $\overline{PR}$ which is parallel to $\bar{A}_2$ as shown in Fig. 2.18. According to head to tail rule, vector $\bar{A}$ be the resultant of vectors $\bar{A}_1$ and $\bar{A}_2$ making angle 'θ' with x-axis, i.e.,

$$\vec{A} = \vec{A}_1 + \vec{A}_2 \quad \ldots\ldots (2.18)$$

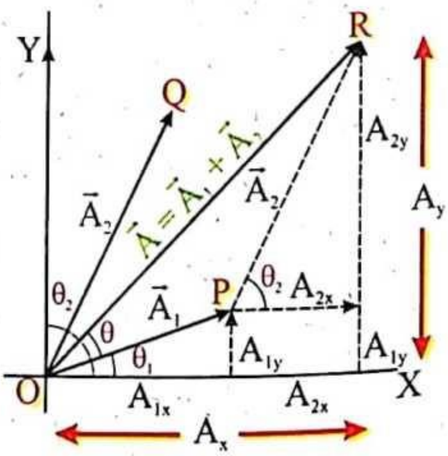


Fig.2.18: Addition of two vectors by their rectangular components

Now by resolving all the three vectors $\bar{A}, \bar{A}_1$ and $\bar{A}_2$ into their rectangular components, then we get;

$$\vec{A} = A_x\hat{i} + A_y\hat{j} \quad \ldots\ldots(2.19)$$

$$\vec{A}_1 = A_{1x}\hat{i} + A_{1y}\hat{j} \ldots\ldots(2.20)$$

$$\vec{A}_2 = A_{2x}\hat{i} + A_{2y}\hat{j} \quad \ldots\ldots(2.21)$$

Putting eq. (2.19), eq. (2.20) and eq. (2.21) in eq. (2.18)

$$\vec{A} = \vec{A}_1 + \vec{A}_2$$

$$A_x\hat{i} + A_y\hat{j} = \left(A_{1x}\hat{i} + A_{1y}\hat{j}\right) + \left(A_{2x}\hat{i} + A_{2y}\hat{j}\right)$$

$$\left(A_x\right)\hat{i} + \left(A_y\right)\hat{j} = \left(A_{1x} + A_{2x}\right)\hat{i} + \left(A_{1y} + A_{2y}\right)\hat{j}$$

Equating the coefficients of $\hat{i}$ and $\hat{j}$ we get

$$A_x = A_{1x} + A_{2x} \quad \ldots\ldots(2.22)$$

$$A_y = A_{1y} + A_{2y} \quad \ldots\ldots(2.23)$$

Magnitude of the resultant vector $\vec{A}$

$$A = \left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}$$

$$A = \left|\vec{A}\right| = \sqrt{\left(A_{1x} + A_{2x}\right)^2 + \left(A_{1y} + A_{2y}\right)^2} \quad \ldots\ldots(2.24)$$

Direction of the resultant vector $\vec{A}$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\frac{A_{1y} + A_{2y}}{A_{1x} + A_{2x}} \quad \ldots\ldots(2.25)$$

Addition of 'n' number of Coplanar vectors

The addition of two vectors by the method of rectangular components can further be extended for '*n*' number of coplanar vectors $A_1, A_2, A_3, A_4, \ldots, A_n$ which are making angles $\theta_1, \theta_2, \theta_3, \theta_4, \ldots, \theta_n$ with x-axis respectively as shown Fig. 2.19.

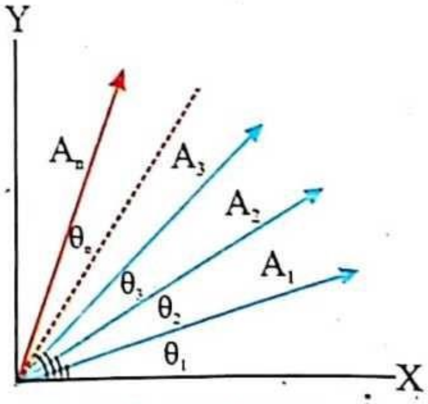


Fig.2.19: *n* number of coplanar vectors in XY-plane

By resolving all the vectors into rectangular components then equation 2.22 and 2.23 become;

$$A_x = A_{1x} + A_{2x} + A_{3x} + \ldots + A_{nx}$$

$$= A_1\cos\theta_1 + A_2\cos\theta_2 + A_3\cos\theta_3 + \ldots + A_n\cos\theta_n$$

$$A_x = \sum_{i=1}^{n} A_i \cos\theta_i$$

$$A_y = A_{1y} + A_{2y} + A_{3y} + \ldots + A_{ny}$$

$$= A_1 \sin\theta_1 + A_2 \sin\theta_2 + A_3 \sin\theta_3 + \ldots + A_n \sin\theta_n$$

$$A_y = \sum_{i=1}^{n} A_i \sin\theta_i$$

Magnitude of the resultant vector

$$A = \sqrt{A_x^2 + A_y^2}$$

$$A = \sqrt{\left(\sum_{i=1}^{n} A_i \cos\theta_i\right)^2 + \left(\sum_{i=1}^{n} A_i \sin\theta_i\right)^2} \quad \ldots\ldots(2.26)$$

Direction of the resultant vector

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\frac{\sum_{i=1}^{n} A_i \sin\theta_i}{\sum_{i=1}^{n} A_i \cos\theta_i} \quad \ldots\ldots\ (2.27)$$

where θ gives the direction of the resultant vector $\vec{A}$ which depends upon the position of A_x and A_y and it can be determined by using the following equation.

$$\phi = \tan^{-1}\left|\frac{A_y}{A_x}\right| \quad \ldots\ldots(2.28)$$

Now the value of θ with the help of φ in the four quadrants can be determined as;

(a) In the first quadrant both the components A_x and A_y of the resultant vector $\vec{A}$ are positive as shown in Fig. 2.20 (a). Thus the direction of the resultant vector is $\theta = \phi$.

(b) In the second quadrant A_x is negative and A_y is positive as shown in Fig. 2.20 (b). Thus the direction of the resultant vector is $\theta = 180^\circ - \phi$.

(c) In the third quadrant both components A_x and A_y of the resultant vector $\vec{A}$ are negative as shown in Fig. 2.20 (c). Thus the direction of the resultant vector is given $\theta = 180^\circ + \phi$.

(d) In the fourth quadrant A_x is positive and A_y is negative as shown in Fig. 2.20 (d) the direction of the resultant vector $\bar{A}$ is given as $\theta = 360^\circ - \phi$.

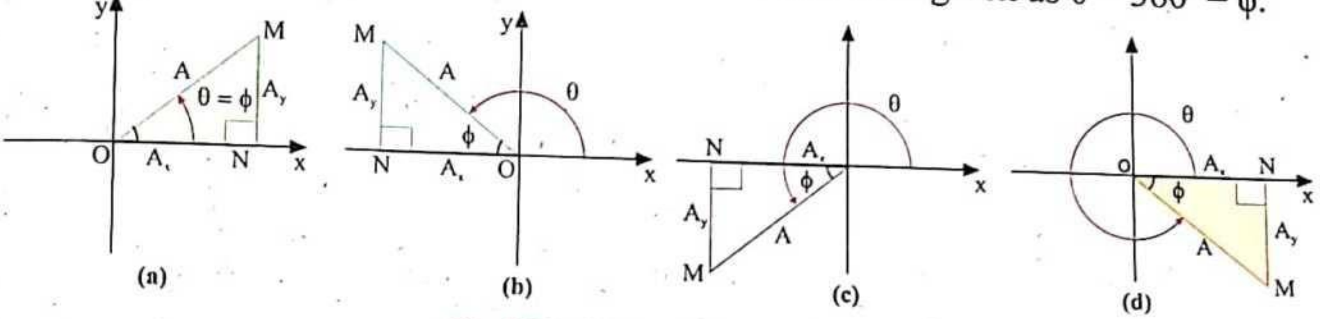
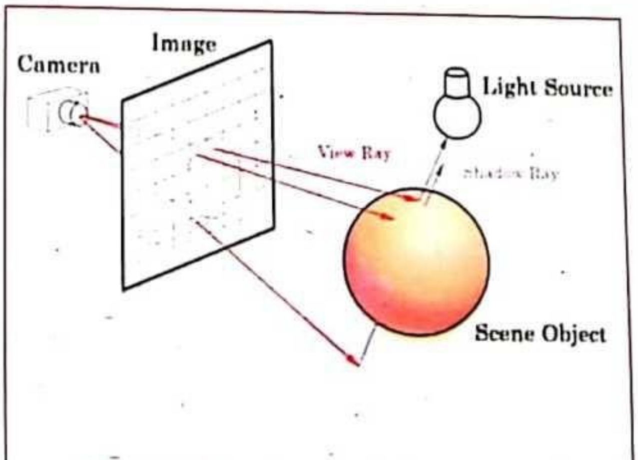


Fig.2.20: Vector in four quadrants

(i) When $A_x > 0$ and $A_y = 0$, $\theta = 0^\circ$
(ii) When $A_x = 0$ and $A_y > 0$, $\theta = 90^\circ$
(iii) When $A_x < 0$ and $A_y = 0$, $\theta = 180^\circ$
(iv) When $A_x = 0$ and $A_y < 0$, $\theta = 270^\circ$



The role of vectors in the formation of an image by using a camera.

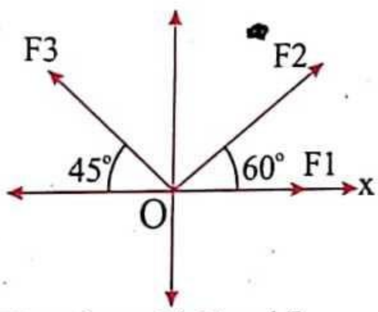
Example 2.6

Three concurrent forces are acting on a body at point 'O' as shown in figure. Calculate the magnitude and direction of their resultant force.

Solution:

We have $F_1 = 19N$ at $\theta = 0^\circ$
$F_2 = 15N$ at $\theta = 60^\circ$
$F_3 = 16N$ at $\theta = 135^\circ$ $(180^\circ - 45^\circ = 135^\circ)$

Resolve all the forces into their rectangular components.



Three forces F_1, F_2 and F_3 are acting in different direction

$$F_{1x} = F_1 \cos\theta$$
$$F_{1x} = 19\cos 0^\circ = 19(1) = 19N$$
$$F_{1y} = F_1 \sin\theta$$
$$F_{1y} = 19\sin 0^\circ = 19(0) = 0$$
$$F_{2x} = F_2 \cos\theta_2$$
$$F_{2x} = 15\cos 60^\circ = 15(0.5) = 7.5N$$
$$F_{2y} = F_2 \sin\theta_2$$
$$F_{2y} = 15\sin 60^\circ = 15(0.866) = 12.99 = 13N$$
$$F_{3x} = F_3 \cos\theta_3$$
$$F_{3x} = 16\cos 135^\circ = 16(-0.707) = -11.312N$$

$$F_{3y} = F_3 \sin\theta_3$$

$$F_{3y} = 16\sin 135^\circ = 16(0.707) = 11.312 \text{ N}$$

The magnitude of x-component F_x of the resultant force $\vec{F}$.

$$F_x = F_{1x} + F_{2x} + F_{3x}$$

$$F_x = 19 + 7.5 + (-11.312)$$

$$F_x = 15.2 \text{ N}$$

The magnitude of y-component F_y of the resultant force $\vec{F}$.

$$F_y = F_{1y} + F_{2y} + F_{3y}$$

$$F_y = 0 + 13 + 11.312 = 24.3 \text{ N}$$

The magnitude of the resultant force $\vec{F}$ is given by,

$$F = \sqrt{F_x^2 + F_y^2}$$

$$F = \sqrt{(15.2)^2 + (24.3)^2} = \sqrt{231.04 + 590.5} = \sqrt{821.54}$$

$$F = 28.66 \text{ N}$$

If the resultant force F makes an angle θ with the x-axis, then

$$\theta = \tan^{-1}\frac{F_y}{F_x}$$

$$\theta = \tan^{-1}\frac{24.3}{15.2} = \tan^{-1} 1.6$$

$$\theta = 58^\circ$$

2.8 PRODUCT OF TWO VECTORS

When two vector quantities are multiplied, then their product may be either scalar or a vector quantity. This shows that two vectors can be multiplied in two different ways, one is called scalar product and other is called vector product.

2.8.1 Scalar product or dot product

When the product of two vectors is a scalar quantity, then such product is called scalar or dot product.

Let two vectors $\vec{A}$ and $\vec{B}$ are making an angle 'θ' with each other as shown in Fig. 2.21. The scalar product of these two vectors $\vec{A}$ and $\vec{B}$ is defined as;

$$\vec{A} \cdot \vec{B} = A\,B \cos\theta \;......(2.29)$$

where A and B are the magnitude of the given vectors $\vec{A}$ and $\vec{B}$, and 'θ' is the angle between them.

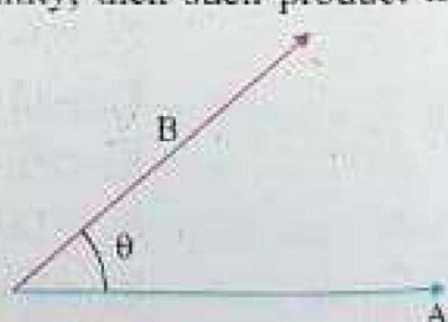


Fig.2.21: Angle θ between two vectors $\vec{A}$ and $\vec{B}$

Explanation:

To explain the scalar product, we draw two vectors $\vec{A}$ and $\vec{B}$ with their tails at the same point such that there is an angle 'θ' between them. The Fig.2.22 shows that $B_x = B\cos\theta$ is the projection of vector $\vec{B}$ along the direction of vector $\vec{A}$.

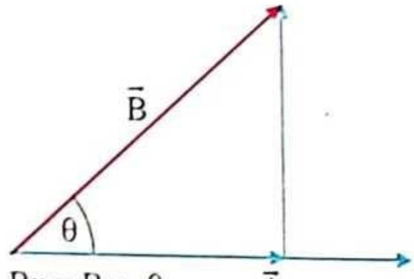


Fig.2.22: Projection of vector $\vec{B}$ in the direction of vector $\vec{A}$

Thus, $\vec{A}\cdot\vec{B}$ is defined as the product of magnitude of $\vec{A}$ and the component of $\vec{B}$ along the direction of $\vec{A}$. i.e.,

$$\vec{A}\cdot\vec{B} = A\ (\text{Projection of vector } \vec{B} \text{ on vector } \vec{A})$$

$$\vec{A}\cdot\vec{B} = AB\cos\theta \quad \text{......(2.30)}$$

Similarly, projection of $\vec{A}$ on $\vec{B}$ as shown in Fig.2.23 is expressed as;

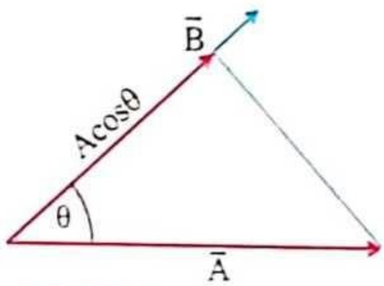


Fig.2.23: Projection of vectors $\vec{A}$ in the direction of vector $\vec{B}$

$$\vec{B}\cdot\vec{A} = B\ (\text{Projection of vector } \vec{A} \text{ on vector } \vec{B}$$

$$\vec{B}\cdot\vec{A} = B(A\cos\theta)$$

$$\vec{B}\cdot\vec{A} = AB\cos\theta \quad \text{......(2.31)}$$

From equation (2.30) and equation (2.31) it is cleared that

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

Examples of scalar product of two vectors

(1) **Work** is equal to the scalar product of force $(\vec{F})$ and displacement $(\vec{d})$. i.e., $W = \vec{F}\cdot\vec{d}$.

(2) **Power** is equal to the scalar product of force $(\vec{F})$ and velocity$(\vec{v})$. i.e., $P = \vec{F}\cdot\vec{v}$

Properties of scalar product of two vectors

I. Scalar or dot product of two vectors is commutative

Since $\vec{A}\cdot\vec{B} = AB\cos\theta$ and $\vec{B}\cdot\vec{A} = AB\cos\theta$

Thus, $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ i.e., the scalar product is commutative.

This simply means that the order of vectors in the dot product does not a matter.

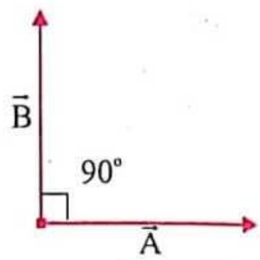
II. The scalar or dot product of two mutually perpendicular vectors is zero.

Let $\vec{A}$ and $\vec{B}$ are two vectors which are mutually perpendicular to each other i.e. angle 'θ' between them is 90°.

Then, $\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = AB\cos 90^\circ$

$\vec{A}\cdot\vec{B} = AB(0) = 0 \quad (\because \cos 90^\circ = 0)$



Two vectors $\vec{A}$ and $\vec{B}$ perpendicular to each other.

In case of unit vectors, $\hat{i},\hat{j}$ and $\hat{k}$ which are perpendicular to each other therefore,

$$\hat{i}\cdot\hat{j} = \hat{i}\,\hat{j}\cos 90^\circ = (1)(1)(0) = 0$$

Similarly $\hat{j}\cdot\hat{k} = |\hat{j}||\hat{k}|\cos 90^\circ = (1)(1)(0) = 0$ and

$$\hat{k}\cdot\hat{i} = \hat{k}\,\hat{i}\cos 90^\circ = (1)(1)(0) = 0 \qquad \therefore \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

III. The scalar or dot product of two parallel vectors is maximum and equal to the product of their magnitudes.

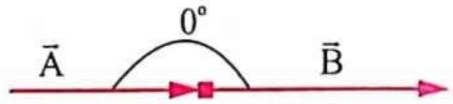
If $\vec{A}$ and $\vec{B}$ are two vectors parallel to each other then angle 'θ' between them is 0°, so

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = AB\cos 0^\circ$

$\vec{A}\cdot\vec{B} = AB(1) \quad \because \cos 0^\circ = 1$

$\vec{A}\cdot\vec{B} = AB$



Vectors $\vec{A}$ and $\vec{B}$ parallel to each other

Similarly, the scalar or dot product of two anti-parallel vectors is equal to the product of their magnitudes with −ve sign.

If $\vec{A}$ and $\vec{B}$ are anti-parallel to each other than angle 'θ' between them is 180°, so;

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = AB\cos 180^\circ$

$\vec{A}\cdot\vec{B} = AB(-1) \quad \because \cos 180^\circ = -1$

$\vec{A}\cdot\vec{B} = -AB$

Vectors $\vec{A}$ and $\vec{B}$ antiparallel to each other

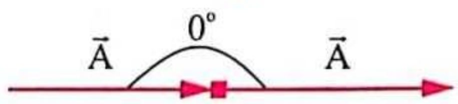
IV. The scalar or dot product of a vector with itself is equal to the square of its magnitude.

$\vec{A}\cdot\vec{A} = AA\cos 0^\circ$

$\vec{A}\cdot\vec{A} = AA(1) \quad \because \cos 0^\circ = 1$

$\vec{A}\cdot\vec{A} = AA$

$\vec{A}\cdot\vec{A} = A^2$



Two identical vectors of same magnitude and direction

In case of unit vectors, $\hat{i}$, $\hat{j}$ and $\hat{k}$. Since $\hat{i}$ is parallel to $\hat{i}$ (i.e., θ = 0°) and each has a unit magnitude, so we have,

$$\hat{i}\cdot\hat{i} = \hat{i}\hat{i}\cos 0^{o} = (1)(1)(1) = 1$$

Similarly $\quad \hat{j}\cdot\hat{j} = |\hat{j}||\hat{j}|\cos 0^{o} = (1)(1)(1) = 1$

and $\quad \hat{k}\cdot\hat{k} = \hat{k}\hat{k}\cos 0^{o} = (1)(1)(1) = 1 \quad \therefore \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

V. The scalar or dot product obeys the distributive law.

Let we have three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ which are directed in their given directions as shown in Fig. 2.24. Then according to distributive law of multiplication

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

where $\quad \vec{A}\cdot(\vec{B}+\vec{C}) = A\left\{\text{Projection of } (\vec{B}+\vec{C}) \text{ on } \vec{A}\right\}$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = A(OQ)$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = A(OP+PQ)$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = A(OP)+A(PQ)$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = A(B_x)+A(C_x)$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

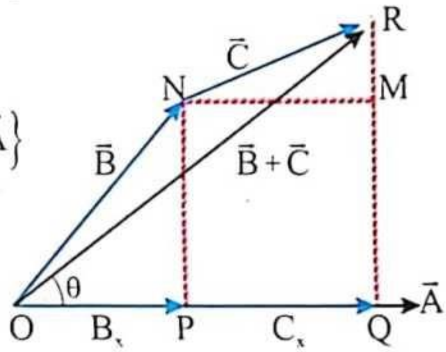


Fig.2.24: Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ explain the distributive law.

This shows that scalar product obeys distributive law of multiplication.

VI. Scalar product of two vectors also obeys Associative law of multiplication

$$m\vec{A}\cdot\vec{B} = \vec{A}\cdot m\vec{B} = m(\vec{A}\cdot\vec{B})$$

VII. Scalar product of two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components.

$$\vec{A} = A_x\hat{i}+A_y\hat{j}+A_z\hat{k};\ \ \vec{B} = B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$

$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right)\cdot\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$

As we know that $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Also $\vec{A}\cdot\vec{B} = AB\cos\theta$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

$$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

Example 2.7

Find the angle between the vector $\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{B} = 5\hat{i} - 2\hat{j} - 3\hat{k}$.

Solution:

We have, $\vec{A} = 3\hat{i} + 2\hat{i} + \hat{k}$ and $\vec{B} = 5\hat{i} - 2\hat{j} - 3\hat{k}$

As, $\vec{A} \cdot \vec{B} = AB\cos\theta$ so, $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{AB}$

where, $\vec{A} \cdot \vec{B} = \left(3\hat{i} + 2\hat{j} + \hat{k}\right) \cdot \left(5\hat{i} - 2\hat{j} - 3\hat{k}\right)$

$$\vec{A} \cdot \vec{B} = (3)(5) + (2)(-2) + (1)(-3) \quad \because \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = 15 - 4 - 3 = 15 - 7 = 8$$

Similarly, $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$A = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14} = 3.74$$

and $B = \sqrt{B_x^2 + B_y^2 + B_z^2}$

$$B = \sqrt{5^2 + (-2)^2 + (-3)^2} = \sqrt{25 + 4 + 9} = \sqrt{38} = 6.16$$

Thus, $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{AB} = \dfrac{8}{3.74 \times 6.16} = 0.347 = 0.35$

$$\theta = \cos^{-1} 0.35 = 69.5^\circ$$

Example 2.8

Prove that vectors $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j}$ are perpendicular to each other.

Solution:

$$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}\ ;\ \vec{B} = 2\hat{i} - \hat{j}$$

The two vectors are perpendicular if $\vec{A} \cdot \vec{B} = 0$

Now $\vec{A} \cdot \vec{B} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) \cdot \left(2\hat{i} - \hat{j}\right)$

$$\vec{A} \cdot \vec{B} = (1)(2) + (2)(-1) + (3)(0) \quad \because \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = 2 - 2 = 2 - 2 = 0$$

Since the scalar or dot product of $\vec{A}$ and $\vec{B}$ is zero, this proves that the two vectors are mutually perpendicular.

2.8.2 Vector product or cross product

When a vector quantity is obtained by the product of two vector quantities then such product of two vectors is called vector product or cross product.

Consider two vectors $\vec{A}$ and $\vec{B}$ making angle 'θ' with each other as shown in Fig. 2.25. The vector product of these vectors is a vector $\vec{C}$ and is written as;

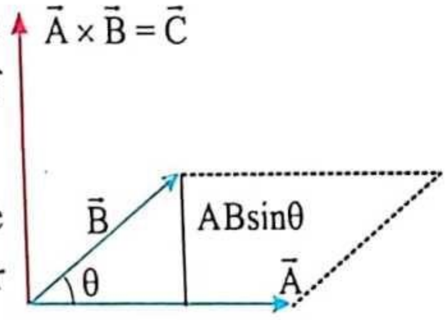


Fig.2.25: Vector product of two vectors $\vec{A}$ and $\vec{B}$

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{C} = AB\sin\theta\hat{n}$$

$$\vec{A} \times \vec{B} = AB\sin\theta\hat{n} \quad ......(2.32)$$

where A and B are the magnitudes of vectors $\vec{A}$, $\vec{B}$ and $\hat{n}$ is called the normal unit vector and it represents the direction of $\vec{C}$. It is always perpendicular to the plane containing vector $\vec{A}$ and $\vec{B}$.

Right hand rule for determination of resultant vector $\vec{C}$

The direction of vector $\vec{C}$ can be determined by right hand rule.

According to this right hand rule, curl the fingers of the right hand in such a way that the first vector $\vec{A}$ would rotate towards the second vector $\vec{B}$ through the smaller angle between them, the stretched perpendicular thumb indicates the direction of $\vec{C} = \vec{A} \times \vec{B}$ as shown in Fig.2.26.

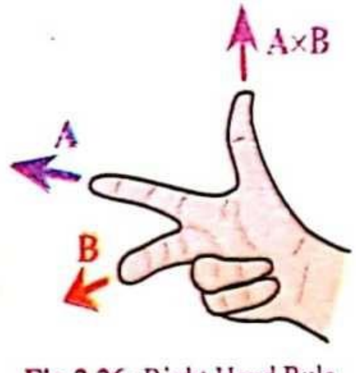


Fig.2.26: Right Hand Rule

Explanation

To explain the vector product of two vectors $\vec{A}$ and $\vec{B}$, we join their tails at the same point, such that there is an angle 'θ' between them and hence we have the plane of vectors $\vec{A}$ and $\vec{B}$.

The direction of $\vec{A} \times \vec{B}$ is determined by rotating vector $\vec{A}$ into the vector $\vec{B}$ through smaller angle 'θ' as shown in Fig. 2.27. According to right hand rule, the direction of $\vec{C}$ is vertically upward.

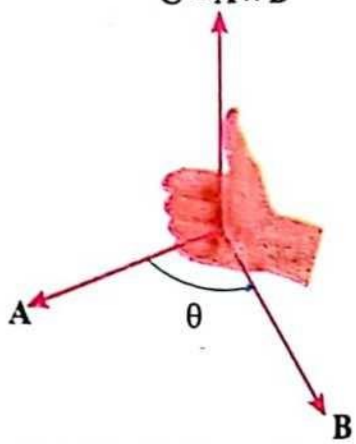


Fig.2.27: Right Hand Rule anticlockwise rotation

Thus, $\vec{A} \times \vec{B} = \vec{C}$ ......(2.33)

Similarly, the direction of $\vec{B} \times \vec{A}$ is determined by rotating vector $\vec{B}$ into the vector $\vec{A}$ as shown in Fig.2.28. According to right hand rule, the direction of $\vec{C}$ is vertically downward.

Thus, $\vec{B} \times \vec{A} = -\vec{C}$

$$-\left(\vec{B} \times \vec{A}\right) = \vec{C} \text{......(2.34)}$$

From eq. (2.33) and eq. (2.34) it is clear that

$$\left(\vec{A} \times \vec{B}\right) = -\left(\vec{B} \times \vec{A}\right) \text{......(2.35)}$$

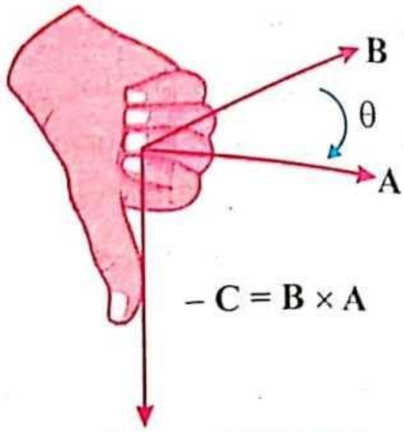


Fig.2.28: Right Hand Rule Clockwise rotation

Examples of vector product of two vectors

(i) **The torque** is equal to cross product of moment arm $(\vec{r})$ and the force applied $(\vec{F})$ i.e., $\vec{\tau} = \vec{r} \times \vec{F}$

(ii) **The angular momentum** $\vec{L}$ is equal to the cross product of position vector $(\vec{r})$ and the linear momentum $(\vec{p})$ i.e., $\vec{L} = \vec{r} \times \vec{p}$

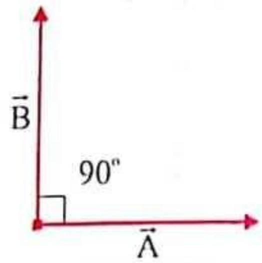
Properties of Vector Product

I. The cross product of the two vectors does not obey commutative law

As discussed earlier $\left(\vec{A} \times \vec{B}\right) = -\left(\vec{B} \times \vec{A}\right)$

i.e. $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

Therefore, vector product of two vectors is not commutative.



Two vectors $\vec{A}$ and $\vec{B}$ which are perpendicular to each other

II. The magnitude vector or cross product of two mutually perpendicular vectors is maximum and is equal to the product of their magnitudes.

$$\left|\vec{A} \times \vec{B}\right| = AB \text{ (Maximum)}$$

Let vector $\vec{A}$ is perpendicular to vector $\vec{B}$ and angle 'θ' between them is 90°

$$\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$$

$$\vec{A} \times \vec{B} = AB\sin 90^\circ\hat{n}$$

$$\vec{A} \times \vec{B} = AB\hat{n} \qquad \because \sin 90^\circ = 1$$

In case of unit vectors, $\hat{i}, \hat{j}$ and $\hat{k}$ along x-axis, y-axis and z-axis as shown in Fig.2.29. According to right hand rule, the direction of $\hat{i}\times\hat{j}$ is perpendicular axis that equal to z-axis with unit vector $\hat{k}$. Thus,

$$\hat{i}\times\hat{j} = \hat{i}\,\hat{j}\sin 90^\circ\,\hat{n}$$

$$= (1)(1)(1)\hat{n} = \hat{n} = \hat{k}$$

Similarly, $\hat{j}\times\hat{k} = \hat{i}$

$$\hat{k}\times\hat{i} = \hat{j}$$

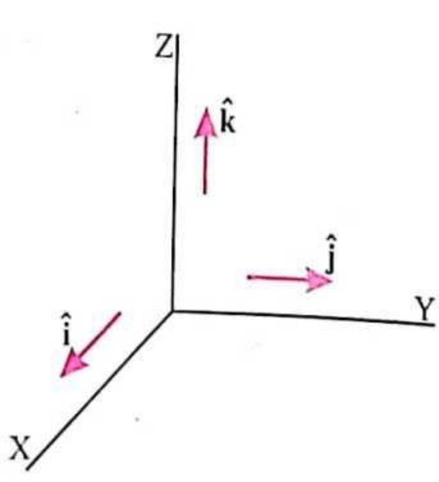


Fig.2.29: Unit vectors along X, Y and Z axes in Cartesian Coordinate system

Using Fig. 2.27, simply reversing the order of the unit vectors gives

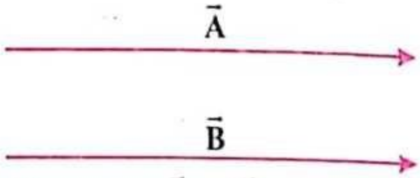
$$\hat{j}\times\hat{i} = -\hat{k} \quad \hat{k}\times\hat{j} = -\hat{i} \qquad \hat{i}\times\hat{k} = -\hat{j}$$

III. The vector or cross product of two parallel vectors is zero.

Let A and B are two non-zero parallel vectors. So in this case, the angle 'θ' between them is 0° then

$$\vec{A}\times\vec{B} = AB\sin 0^\circ\,\hat{n}$$

$$\vec{A}\times\vec{B} = 0 \qquad \because \sin 0^\circ = 0$$



Two vectors $\vec{A}$ and $\vec{B}$ which are parallel to each other.

In case of unit vectors, $\hat{i}$, $\hat{j}$ and $\hat{k}$. We have,

$$\hat{i}\times\hat{i} = \hat{i}\hat{i}\sin 0^\circ\,\hat{n} = (1)(1)(0)\hat{n} = 0$$

and $\hat{j}\times\hat{j} = 0$ and $\hat{k}\times\hat{k} = 0$

Similarly, the vector or cross product of two anti-parallel vectors is equal to zero.

If $\vec{A}$ and $\vec{B}$ are anti-parallel to each other than angle 'θ' between them is 180°, so;

$$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$$

$$\vec{A}\times\vec{B} = AB\sin 180^\circ\,\hat{n}$$

$$\vec{A}\times\vec{B} = AB(0)\hat{n} = 0 \quad \because \sin 180^\circ = 0$$

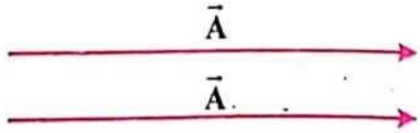
Two vectors $\vec{A}$ and $\vec{B}$ which are anti-parallel to each other.

IV. The vector product of a vector with itself is equal to zero or Null vector.

$$\vec{A}\times\vec{A} = A\vec{A}\sin\theta\,\hat{n}$$

$$\vec{A}\times\vec{A} = AA\sin 0^\circ\,\hat{n} \quad \because \sin 0^\circ = 0$$

$$\vec{A}\times\vec{B} = AB(0)\hat{n} = 0$$



Two identical vectors of same magnitude and direction.

V. The vector product obeys the distributive law.

If we have three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ then,

$$\vec{A}\times(\vec{B}\times\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

VI. Vector product of two vectors obeys Associative law of multiplication

i.e., $m\vec{A}\times\vec{B}=\vec{A}\times m\vec{B}=m(\vec{A}\times\vec{B})$

VII. Vector product of two vectors $\vec{A}$ and $\vec{B}$ in terms of their rectangular components.

$$\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k};\quad \vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$

$$\vec{A}\times\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\times(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$\begin{aligned}\vec{A}\times\vec{B}=\;&A_xB_x(\hat{i}\times\hat{i})+A_xB_y(\hat{i}\times\hat{j})+A_xB_z(\hat{i}\times\hat{k})+\\&A_yB_x(\hat{j}\times\hat{i})+A_yB_y(\hat{j}\times\hat{j})+A_yB_z(\hat{j}\times\hat{k})+\\&A_zB_x(\hat{k}\times\hat{i})+A_zB_y(\hat{k}\times\hat{j})+A_zB_z(\hat{k}\times\hat{k})\end{aligned}$$

As we know that

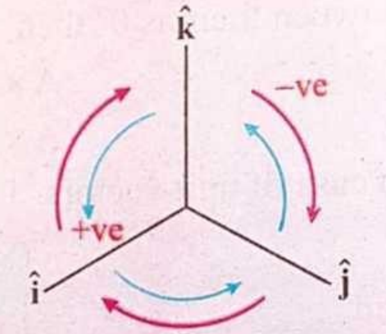
$$\hat{i}\times\hat{j}=\hat{k}\quad \hat{j}\times\hat{k}=\hat{i}\quad \hat{k}\times\hat{i}=\hat{j}$$

$$\hat{j}\times\hat{i}=-\hat{k}\quad \hat{k}\times\hat{j}=-\hat{i}\quad \hat{i}\times\hat{k}=-\hat{j}$$

and $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$

$$\begin{aligned}\vec{A}\times\vec{B}=\;&A_xB_x(0)+A_xB_y(\hat{k})+A_xB_z(-\hat{j})+\\&A_yB_x(-\hat{k})+A_yB_y(0)+A_yB_z(\hat{i})+\\&A_zB_x(\hat{j})+A_zB_y(-\hat{i})+A_zB_z(0)\end{aligned}$$

For Your Information



Anti-clockwise gives +ve unit vectors clockwise gives −ve unit vectors.

$$\vec{A}\times\vec{B}=A_xB_y\hat{k}-A_xB_z\hat{j}-A_yB_x\hat{k}+A_yB_z\hat{i}+A_zB_x\hat{j}-A_zB_y\hat{i}$$

$$\vec{A}\times\vec{B}=A_yB_z\hat{i}-A_zB_y\hat{i}+A_zB_x\hat{j}-A_xB_z\hat{j}+A_xB_y\hat{k}-A_yB_x\hat{k}$$

$$\vec{A}\times\vec{B}=(A_yB_z-A_zB_y)\hat{i}+(A_zB_x-A_xB_z)\hat{j}+(A_xB_y-A_yB_x)\hat{k}$$

In determinant form, this can be written as

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

ve law of multiplication

ms of their rectangular

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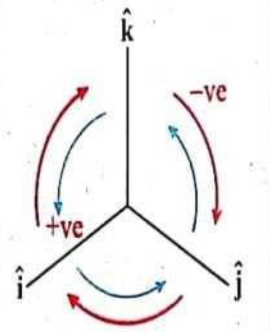


Anti-clockwise gives +ve unit vectors clockwise gives −ve unit vectors.

$-A_zB_y\hat{i}$

$-A_yB_x\hat{k}$

$_y - A_yB_x)\hat{k}$

VIII. The magnitude of vector product of two vectors represents the area of the parallelogram formed by them.

Figure 2.30 shows a parallelogram OPQR whose adjacent sides OP and OR represent vectors $\vec{A}$ and $\vec{B}$ respectively and SR is its height.

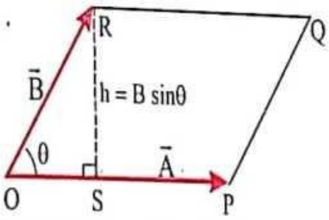


Fig.2.30: Area of parallelogram is equal to the vector product of two vectors $\vec{A}$ and $\vec{B}$.

By definition of the magnitude vector product of two vectors $\vec{A}$ and $\vec{B}$ is given by;

$$\vec{A}\times\vec{B} = AB\sin\theta$$

But in triangle ORS,

$$\frac{h}{B} = \sin\theta$$

$$h = B\sin\theta$$

$$\therefore \quad \left|\vec{A}\times\vec{B}\right| = Ah$$

$$\left|\vec{A}\times\vec{B}\right| = \text{(Base) (Height)}$$

$$\left|\vec{A}\times\vec{B}\right| = \text{Area of parallelogram}$$

Example 2.9

Determine the area of the parallelogram whose adjacent sides are $2\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}-\hat{j}$.

Solution:

Let $\vec{A}$ and $\vec{B}$ be the vectors representing the adjacent sides of the parallelogram.

Here, $\vec{A} = 2\hat{i}+\hat{j}+3\hat{k}$; $\vec{B} = \hat{i}-\hat{j}$

The area of parallelogram is equal to the magnitude of the vector product of $\vec{A}$ and $\vec{B}$.

Now, $\vec{A}\times\vec{B} = \left(2\hat{i}+\hat{j}+3\hat{k}\right)\times\left(\hat{i}-\hat{j}\right)$

$$\vec{A}\times\vec{B} = (2)(1)\left(\hat{i}\times\hat{i}\right)+(2)(-1)\left(\hat{i}\times\hat{j}\right)+(1)(1)\left(\hat{j}\times\hat{i}\right)+(1)(-1)\left(\hat{j}\times\hat{j}\right)$$
$$+(3)(1)\left(\hat{k}\times\hat{i}\right)+(3)(-1)\left(\hat{k}\times\hat{j}\right)$$

$$\vec{A}\times\vec{B} = 2(0)-2\left(\hat{k}\right)+1\left(-\hat{k}\right)-1(0)+3\left(\hat{j}\right)-3\left(-\hat{i}\right)$$

$$\vec{A}\times\vec{B} = 0-2\hat{k}-\hat{k}-0+3\hat{j}+3\hat{i}$$

$$\vec{A}\times\vec{B} = 3\hat{i}+3\hat{j}-3\hat{k}$$

$\therefore$ Magnitude of the area of the parallelogram $= \left|\vec{A}\times\vec{B}\right| = \sqrt{3^2+3^2+3^2}$

Magnitude of the area of the parallelogram $= \sqrt{9+9+9}$

Magnitude of the area of the parallelogram $= \sqrt{27} = 5.19$

Magnitude of the area of the parallelogram $= 5.19$ sq. Units

2.9 TORQUE

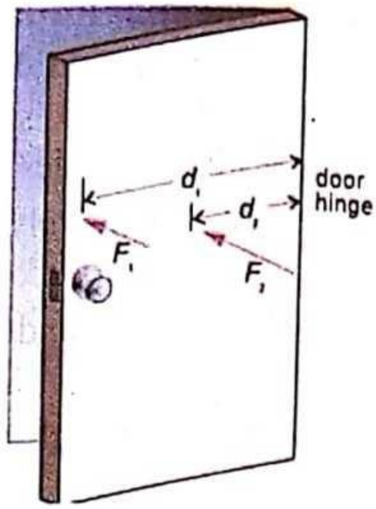


Fig.2.31: Turning effect due to opening or closing the door.

The most common example of turning effect in daily life is the opening or the closing the doors. That is, when we open or close the door, we apply a force perpendicular to the plane of the door. It is our observations that if the force is applied near the hinges, we are likely to face difficulty in opening or closing of door. On the other hand, when the force is applied at the maximum distance from the hinges then it is much easier and we will have to apply less force to open or close it. This example clearly indicates that the turning effect of a force depends upon not only the applied force but also the distance between the line of action of force and the axis of rotation. It is shown in Fig.2.31.

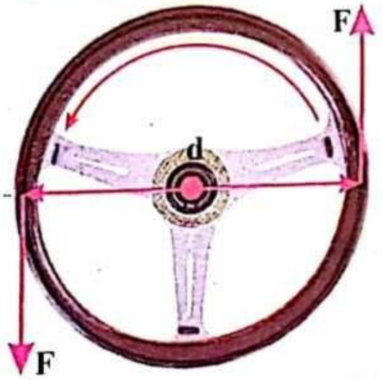


Fig.2.32: Two forces act on the steering to produce a turning effect.

Similarly, the steering wheel of a car is another common example of the turning effect of force. In this case, the turning effect in the car's steering can be observed when two forces of same magnitude but in opposite direction are acting on it. This is also known as a couple. A couple has a turning effect but does not cause an object to accelerate. For a couple, the two forces must be separated at a distance 'd' as shown in Fig.2.32.

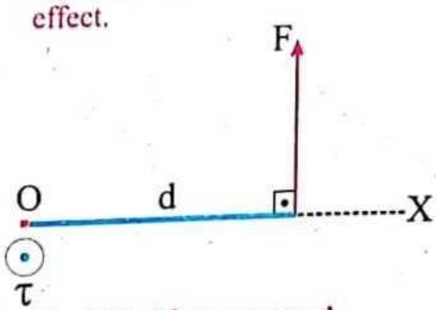


Fig.2.33: Moment arm **d** between the line of action of force and axis of rotation.

A force applied on a body is capable of rotating the body about an axis. This turning effect of a force is called torque or moment of a force.

It is equal to the product of applied force and moment arm. It is represented by 'τ' and is given as;

$$\tau = (d)(F) \qquad ......(2.36)$$

where 'd' is a moment arm and 'F' is the applied force. Momentum arm is the perpendicular distance between line of action of force and axis of rotation as shown in Fig. 2.33.

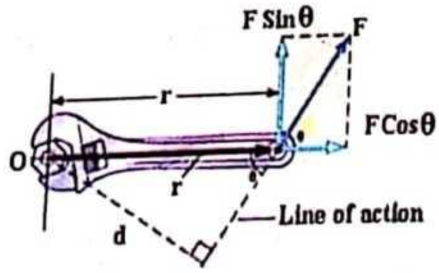


Fig.2.34: A force acts at an angle θ on a wrench

Considering a wrench which is pivoted about an axis through 'O' by applying a force at an angle 'θ' as shown in Fig. 2.34.

If r is the distance between the pivoted point and the point of applied force and d is the perpendicular distance from the pivoted point to the line of action of force then,

$$\frac{d}{r} = \sin\theta \quad \because \sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$d = r\sin\theta = \text{Moment arm}$$

Thus, eq. (2.36) becomes

$$\tau = (r\sin\theta)F$$

$$\tau = rF\sin\theta$$

$$\vec{\tau} = \vec{r}\times\vec{F} \quad .....(2.37)$$

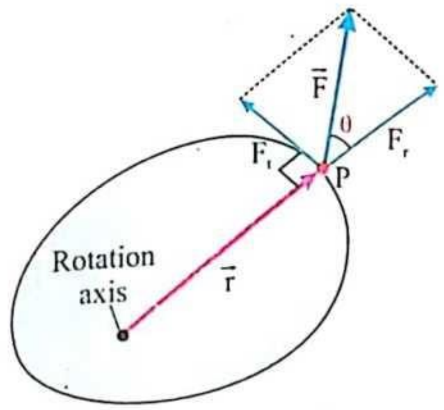


Fig.2.35: Force at an angle with position vector $\vec{r}$

Thus, torque can be defined as vector product of force and moment arm.

This shows that torque is a vector quantity whose direction is along the axis of rotation.

Torque is taken as negative when the body is rotated clockwise and it is taken as positive when the body is rotated anti-clockwise.


Torque due to turning of turn table.

Now considering a torque due to the applied force $\vec{F}$ acting on a rigid body at point 'P' whose position vector with respect to pivot O is $\vec{r}$ as shown in Fig.2.35. As $\vec{F}$ is acting at an angle 'θ' with $\vec{r}$ so we resolve it into its rectangular components.

The component of force along $\vec{r}$ is called radial component $(F\cos\theta)$ and there is no torque due to this component.

The component perpendicular to $\vec{r}$ is called tangential component $(F\sin\theta)$. Actually, the torque is due to this tangential component that is.

$$\tau = rF_t$$

But $\quad F_t = F\sin\theta$

$$\tau = rF\sin\theta$$

$$\vec{\tau} = \vec{r}\times\vec{F}$$

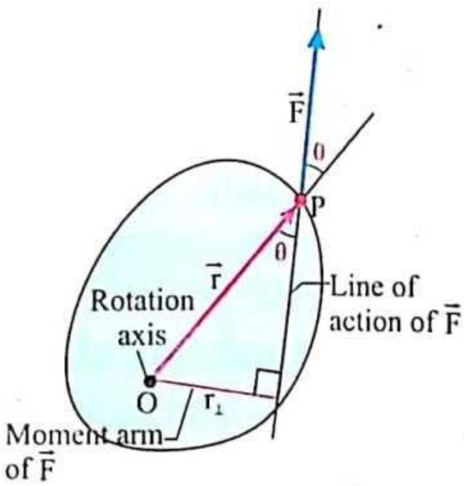


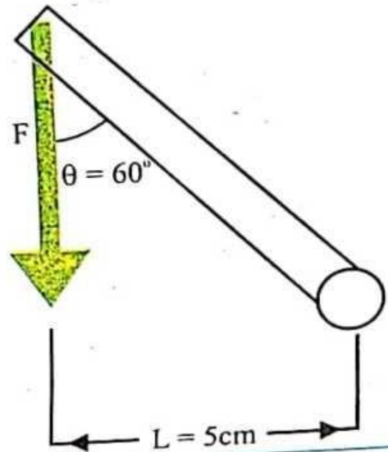
Fig.2.36: Moment arm at an angle with force F.

Similarly, we can also resolve the position vector $\vec{r}$ into its rectangular components as shown in Fig.2.36. In this case the torque is due to F and r sinθ that is;

$$\tau = Fr\sin\theta$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Torque depends upon magnitude of force, position vector and angle 'θ'. The SI unit of torque is $N \cdot m$ and its dimensional formula is $\left[ML^2T^{-2}\right]$.



Example 2.10

Calculate the torque produces by force of 10N which is applied by a man downward at 60° on a crossing level of length 5m.

Solution:

$$\tau = rF\sin\theta$$

$$\tau = (5)(10)\sin 60^{\circ}$$

$$\tau = 50(-0.30)$$

$$\tau = -15\,N \cdot m$$

The negative sign indicates that the torque is in clockwise direction.

2.10 EQUILIBRIUM

When an engineer designs a system, building, bridge etc., his first priority is to maintain its balance. It is possible only when all these are at rest or moving with uniform velocity. This is the basic principle of equilibrium and it is stated as, "a body is in equilibrium when either it is at rest or in uniform motion and its acceleration is zero". There are two forms of equilibrium; static equilibrium (at rest) and dynamic equilibrium (in motion). Equilibrium can be studied under the following two conditions.

A paratrooper moving downward with uniform velocity in state of dynamic equilibrium.

1) First condition of equilibrium or translational equilibrium

A body is said to be in translational equilibrium when the algebraic sum of concurrent forces acting on it is zero and this is the first condition of equilibrium. Consider a number of forces (F_1, F_2, F_3 ......F_n) in different directions are acting on an object as shown in Fig. 2.37. The object will be in state of equilibrium when the result force of all these forces is zero.

Stones in static equilibrium

Mathematically, it is expressed as;

$$F_1 + F_2 + F_3 + F_4 + ... + F_n = 0$$

$$\sum F_n = 0$$

This is the mathematical form of 1st condition of equilibrium. It is further explained by an example.

When an aeroplane is in flight, four forces are acting on it, its weight is acting downward and lift force is upward, while thrust is forward and drag force is backward as shown in Fig. 2.38.

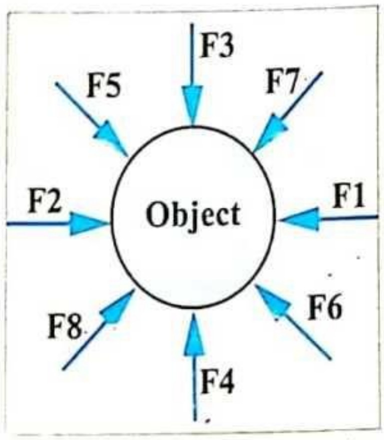


Fig.2.37: Number of forces acting on an object

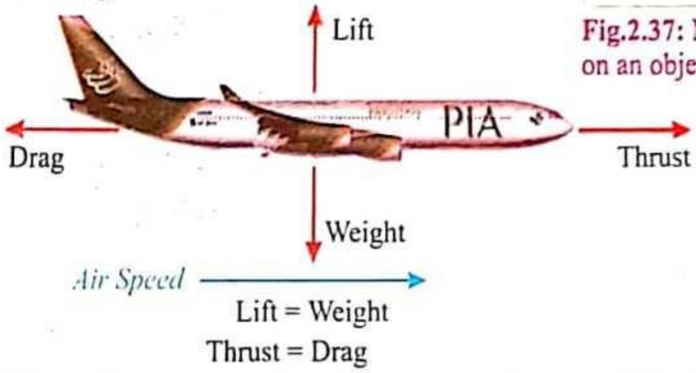


Fig.2.38: Aeroplane in state of equilibrium under the action of four forces

According to 1st condition of equilibrium, the aeroplane will be in equilibrium when;

$$\text{Weight} = \text{Lift}$$

and

$$\text{Thrust} = \text{Drag Force}$$

Now when all the forces are acting on a body along x-axis. Then first condition of equilibrium is written as

$$\sum F_x = 0$$

Similarly, for y-axis $\sum F_y = 0$ and for z-axis $\sum F_z = 0$

2) Second condition of equilibrium or rotational equilibrium

A body is said to be in rotational equilibrium when the algebraic sum of torques acting on it is zero. This is the second condition of equilibrium. It is explained by an example of two boys who are sitting on the opposite ends of a seesaw as shown in Fig.2.39.

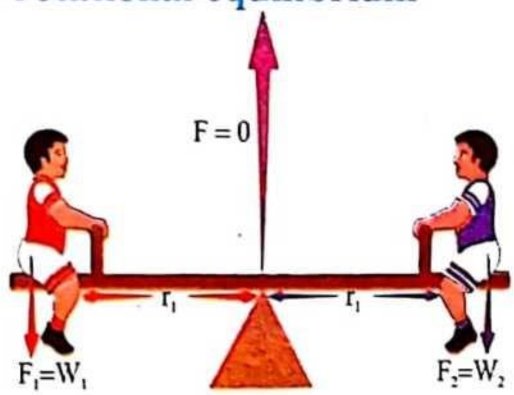


Fig.2.39: A seesaw in state of rotational equilibrium due to clockwise and anti-clockwise torques.

The boy who is sitting at the end of the right hand side of the seesaw produces clock wise torque $(-\tau)$ and the boy who is sitting at the end of left hand side of seesaw produces anti clock wise torque $(+\tau)$. Now if the sum of all torques is zero then seesaw is balanced and the whole system is in rotational equilibrium.

This example can be extended for 'n' number of torques acting on a body and the body is in equilibrium, if the vector sum of all the torques acting on it is zero.

$$\tau_1 + \tau_2 + \tau_3 + \ldots + \tau_n = 0$$

$$\sum \tau_n = 0$$

This is the mathematical form of second condition of equilibrium. It is also called rotational equilibrium.

From the above discussion, it is concluded that if a body satisfies the first condition of equilibrium then its linear acceleration is zero. Similarly, if a body satisfies the second condition of equilibrium its angular acceleration is zero.

It is also worth noting that if a body satisfies both conditions of equilibrium then its linear as well as angular accelerations are zero and such a body is said be a complete or perfect equilibrium.

A crane is working under the principle of equilibrium.

Point to Ponder

The accuracy of a measurement describes how well the result agrees with an accepted value.

Mathematically it is expressed as;

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0 \text{ and } \sum \tau = 0$$

SUMMARY

- **Scalars and Vectors**: The physical quantities which have only magnitude are known as scalars whereas the physical quantities which have both magnitude and direction are known as vectors.
- **Graphical representation of vector:** Graphically, a vector quantity is represented by a straight line with an arrow on its one end. The length of the straight line represents, according to the chosen scale, the magnitude and the arrow indicates the direction of the vector.
- **Cartesian co-ordinates system**: Two lines which are perpendicular to each other is known as Cartesian co-ordinates system. A vector can be drawn in any one of the four quadrant in such Cartesian co-ordinates system.
- **Addition of a vector:** Vectors can be added by the head to tail rule and by rectangular components method.
- **Position vector**: Location of a point in Cartesian co-ordinates system described by a vector known as position vector.

- **Unit vector:** A vector whose magnitude is equal to unity is known as unit vector $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$.
- **Resolution of vectors:** When a vector is split into two or more vectors then it is called resolution of a vector.
- **Rectangular components of a vector:** When a vector $\vec{A}$ is lying in XY-plane then its horizontal and vertical components are given as;

$$\vec{A}_x = \vec{A}\cos\theta\,\hat{i} \qquad \vec{A}_y = \vec{A}\sin\theta\,\hat{j}$$

The magnitude and direction of the resultant vector $\vec{A}$ interms of rectangular components are given as;

$$A = |\vec{A}| = \sqrt{\vec{A}_x^2 + \vec{A}_y^2} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

- **Scalar or dot product:** When the product of two vectors is a scalar quantity then it is called scalar or dot product.
- **Vector or cross product:** When the product of two vectors is a vector quantity then it is called vector or cross product.
- **Torque:** The turning effect of force in a body about its axis is called torque. i.e., $\vec{\tau} = \vec{r} \times \vec{F}$.
- **Equilibrium:** A body is said to be in equilibrium when it is at rest or moving with uniform velocity. There are two conditions of equilibrium.

  First condition of equilibrium: According to first condition of equilibrium, the sum of forces acting on a body is equal to zero. i.e. $\sum F_n = 0$

  Second condition of equilibrium: According to second condition of equilibrium, the sum of torques acting on a body is zero. i.e. $\sum \tau = 0$.

EXERCISE

Multiple choice questions.

1. Which one of the following is a scalar quantity?
(a) Force (b) Torque (c) Momentum (d) Density

2. Which one of the following is a vector quantity?
(a) Work (b) Power (c) Weight (d) Mass

3. Which pair includes a scalar and a vector quantity?
(a) K.E. and momentum (b) P.E. and Work
(c) Weight and force (d) Velocity and acceleration

4. Two vectors $\vec{A}_1$ and $\vec{A}_2$ are making an angle of 90° with each other. What is their resultant magnitude?

(a) $A_1^2 + A_2^2$ (b) $\sqrt{A_1^2 + A_2^2}$

(c) $\sqrt{A_1^2 + A_2^2 + A_1A_2}$ (d) $\sqrt{A_1^2 + A_2^2 + \frac{1}{2}A_1A_2}$

5. The magnitude of two vectors is 3N and 4N respectively. If the angle between them is 90°, then their resultant vector will be:
(a) 5 N (b) 6 N (c) 7 N (d) Zero

6. At what angle the vertical component of a vector is maximum?.
(a) 0° (b) 30° (c) 45° (d) 90°

7. In which quadrant a vector can be drawn when its both x and y components are negative.
(a) First (b) Second (c) Third (d) Fourth

8. What is the possible result of $(-3\hat{i})\cdot(-4\hat{j})$?
(a) Zero (b) One (c) $12\hat{k}$ (d) 12

9. What is the angle of the given vector $2\hat{i} + 2\hat{j}$?
(a) 30° (b) 45° (c) 60° (d) 90°

10. The correct result of the expression $\hat{j}\cdot(\hat{k}\times\hat{i})$ is;
(a) Zero (b) One (c) $\hat{i}$ (d) $\hat{j}$

11. Which law does not obey by the vector product of two vectors?
(a) Associative (b) Commutative (c) Distributive (d) Identitive

12. The scalar product of two non-zero vectors is equal to zero when angle θ between them is;
(a) 0° (b) 30° (c) 60° (d) 90°

13. The vector product of two vectors is maximum when both the vectors are
(a) Parallel (b) Anti-parallel (c) Perpendicular (d) Equal

14. What is the expected result of $(\vec{A}\times\vec{B})^2 + (\vec{A}\cdot\vec{B})^2 = ?$
(a) $A^2B^2\cos\theta$ (b) $A^2B^2\sin\theta$ (c) AB (d) A^2B^2

15. Self cross product of unit vectors is always
(a) One (b) Zero (c) Linear (d) Non-linear

16. The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ and 6 respectively, the angle between the vectors is:
(a) 90° (b) 60° (c) 30° (d) Zero

17. What torque is produced by 30N force which is acting at 60° on a wrench of length 30 cm?
(a) 5.8 Nm (b) 6.8 Nm (c) 7.8 Nm (d) 8.8 Nm

18. A force of 10N at 60° is acting on a block, what force in opposite direction will bring to block at equilibrium.
(a) 5 N (b) 10 N (c) 15 N (d) 100 N

19. If the line of action of the force passes through their axis of rotation or origin, then its torque is:
(a) Zero (b) Maximum (c) One (d) None of these

SHORT QUESTIONS

1. Why a scalar quantity cannot be added or subtracted with a vector quantity?
2. What are the characteristics of vectors addition?
3. When the resultant of two vectors is zero? Explain it with the help of diagram.
4. Under what condition the resultant vector of three vectors acting simultaneously on a particle is zero?
5. What would be the position of a vector when its x-component is positive and y-component is negative? Explain it with the help of a diagram.
6. What change takes place in a vector when it is multiplied by a negative number?
7. Give any three examples, where a vector is divided by a scalar quantity.
8. Under what circumstances the rectangular components of a vector give same magnitude?
9. Can the scalar product of two vector quantities be negative? If your answer is yes, give an example, if no provide a proof?
10. How scalar product of two vectors obeys commutative law?
11. Can the magnitude of any one rectangular components greater than the magnitude of the given vector?
12. When the scalar product of two vectors is maximum?
13. How the direction of the resultant vector for vector product of two vectors can be determined?
14. What are the similarities between torque and work?
15. Why both condition of equilibrium are necessary for the complete equilibrium?
16. Can a body be in equilibrium when three forces are acting on it? Explain with the help of diagram.
17. When a system will be in perfect equilibrium?
18. What do you understand by positive and negative torques?

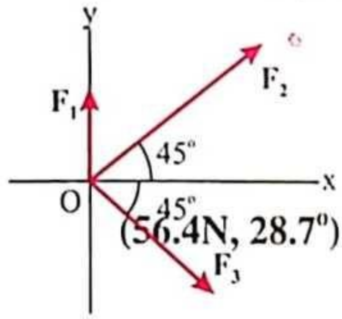
COMPREHENSIVE QUESTIONS

1. What do you know about the scalar and vector physical quantities? Explain the representation of vector quantities.
2. Describe various kinds of vectors.
3. Define Cartesian co-ordinate system and explain that how one can draw a vector quantity in this system.

4. State and explain the addition and subtraction of vector quantities.
5. Explain that how can you multiply a vector quantity by a scalar or a number.
6. Explain the addition of vectors by rectangular components for two and more than two vectors.
7. State and explain scalar product of two vectors with its properties.
8. What do you know about the vector product of two vectors. Discuss all the properties of vector product.
9. Define torque with examples. Also mention the kinds of torque.
10. What is equilibrium? State and explain the first and second conditions of equilibrium.

NUMERICAL PROBLEMS

1. If $\vec{A} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{B} = 3\hat{i} - \hat{j} + 2\hat{k}$ then find out (a) $|2\vec{A} + 3\vec{B}|$ (b) $|3\vec{A} + 2\vec{B}|$ **(11,11)**
2. What is the unit vector $\hat{A}$ in the direction of vector $\vec{A} = 2\hat{i} + 2\hat{j} + \hat{k}$. $\left(\frac{2\hat{i} + 2\hat{j} + \hat{k}}{3}\right)$
3. Find the projection of vector $\vec{A} = 2\hat{i} + 4\hat{j}$ on the vector $\vec{B} = 4\hat{i} + 3\hat{k}$. $\left(\frac{8}{5}\right)$
4. What is the magnitude and direction of a vector when its horizontal component is doubled then its vertical? **(2.236, 26.6°)**
5. Find the X and Y components of a vector $\vec{A} = 4\hat{i} + 7\hat{j}$ making angle 60° with x-axis. **(4,7)**
6. Three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ are acting on a body at point 'O' such that $F_1 = 20\,N$, $F_2 = 40\,N$ and $F_2 = 30\,N$ as shown in the figure. Calculate the magnitude and direction of the resultant force of these three forces. **(56.4N, 28.7°)**

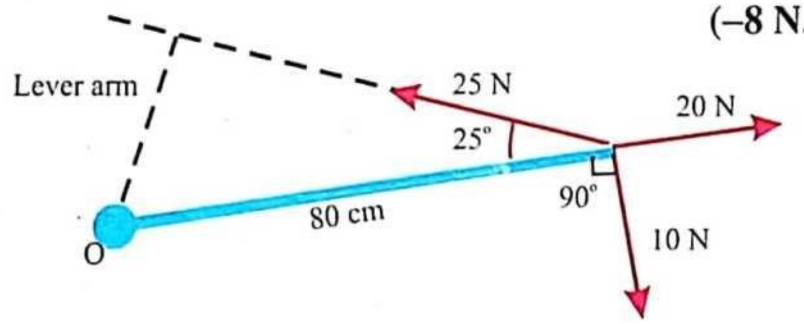


(56.4N, 28.7°)

7. Prove that the three vectors $3\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} - \hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} - 4\hat{k}$ are at right angle to one another.
8. The magnitude of dot and cross products of two vectors are 6 and $6\sqrt{3}$ respectively. Find the angle between them. **(60°)**
9. A force, $F = 4\hat{i} - 3\hat{j} + 2\hat{k}$ acts on an object and the object is displaced along a straight line from point $A(3,2,-1)$ to point $B(2,-1,4)$. If force is measured in newton and displacement in metres then find work done on the object. **(15J)**

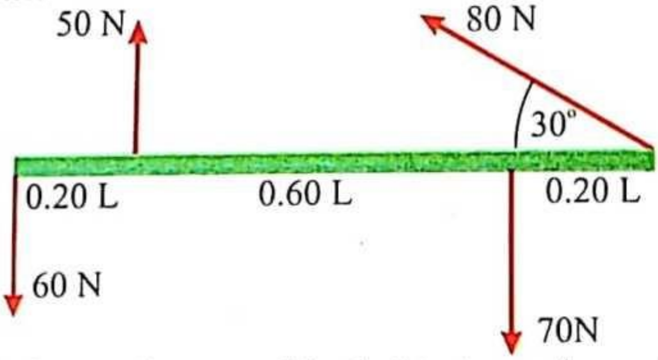
10. Three forces are acting at one end of the rod of length 80cm. What are the expected torques due to each force about an axis of rotation 'O'.

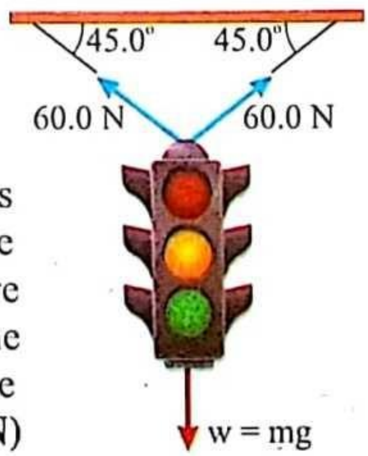
(–8 N.m, +8.5 N.m, 0)



11. A uniform beam of weight 40 N is subjected by forces as shown in figure. What is the magnitude, direction and location of a force that can keep the beam in equilibrium?

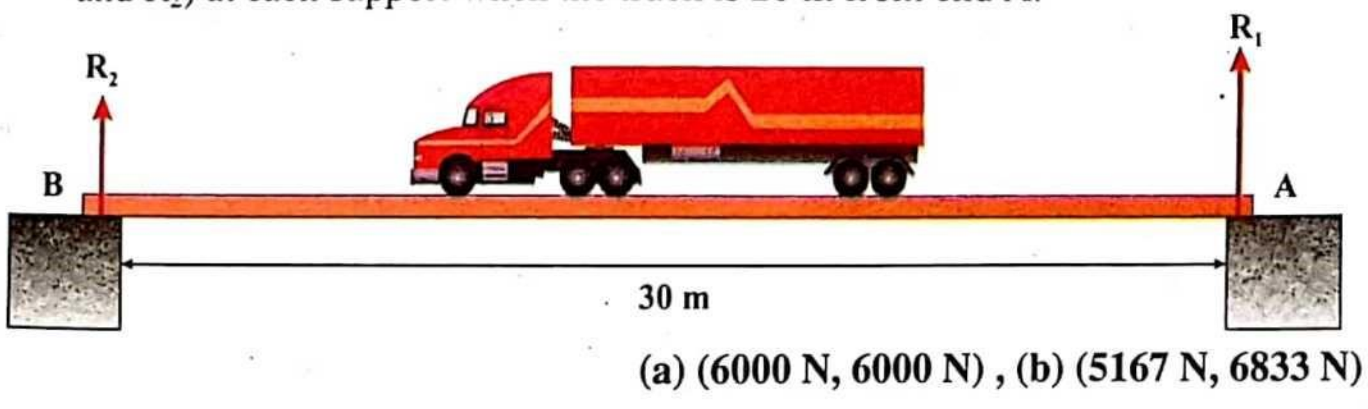
(0.6N, 49°, 0.163L)





12. A traffic light hangs from a cable tied to two other cables fastened to a support as in figure. The upper cables make angles of 45° with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 130 N. Find the weight of the traffic light to keep the system in equilibrium. **(85 N)**

13. A truck of weight 5000 N is driven across a single span bridge of weight 7000 N and of length 30 m as shown in figure. Find (a) the total reaction at the two supports A and B when truck is at the center of the bridge. (b) the reaction (R_1 and R_2) at each support when the truck is 20 m from end A.



(a) (6000 N, 6000 N) , (b) (5167 N, 6833 N)

Unit 3

FORCES AND MOTION

Major Concepts (30 PERIODS)

- Displacement
- Average velocity and instantaneous velocity
- Average acceleration and instantaneous acceleration
- Review of equations of uniformly accelerated motion
- Newton's laws of motion
- Momentum and Impulse
- Law of conservation of momentum
- Elastic collisions in one dimension
- Momentum and explosive forces
- Projectile motion
- Rocket motion

Conceptual Linkage

This chapter is built on Kinematics & Dynamics Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- Describe vector nature of displacement.
- Describe average and instantaneous velocities of objects.
- Compare average and instantaneous speeds with average and instantaneous velocities.
- Interpret displacement-time and velocity-time graphs of objects moving along same straight line.
- Determine the instantaneous velocity of an object moving along the same straight line by measuring the slope of displacement-time graph.
- Define average acceleration (as rate of change of velocity $a_{av} = \Delta v / \Delta t$) and instantaneous acceleration (as the limiting value of average acceleration when time interval Δt approaches zero).
- Distinguish between positive and negative acceleration, uniform and variable acceleration.
- Determine the instantaneous acceleration of an object measuring the slope of velocity-time graph.
- Manipulate equation of uniformly accelerated motion to solve problems.
- Explain that projectile motion is two dimensional motions in a vertical plane.
- Communicate the ideas of a projectile in the absence of air resistance that.

(i) Horizontal component (V_H) of velocity is constant.

(ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.

(iii) The horizontal motion and vertical motion are independent of each other.

- Evaluate using equations of uniformly accelerated motion that for a given initial velocity of frictionless projectile.
  1. How higher does it go?
  2. How far would it go along the level land?
  3. Where would it be after a given time?
  4. How long will it remain in air?
- Determine for a projectile launched from ground height.
  1. Launch angle that results in the maximum range.
  2. Relation between the launch angles that result in the same range.
- Describe how air resistance affects both the horizontal component and vertical component of velocity and hence the range of the projectile.
- Apply Newton's laws to explain the motion of objects in a variety of context.
- Define mass (as the property of a body which resists change in motion).
- Describe and use of the concept of weight as the effect of a gravitational field on a mass.
- Describe the Newton's second law of motion as rate of change of momentum.
- Correlate Newton's third law of motion and conservation of momentum.
- Show awareness that Newton's Laws are not exact but provide a good approximation, unless an object is moving close to the speed of light or is small enough that quantum effects become significant.
- Define Impulse (as a product of impulsive force and time).
- Describe the effect of an impulsive force on the momentum of an object, and the effect of lengthening the time, stopping, or rebounding from the collision.
- Describe that while momentum of a system is always conserved in interaction between bodies some change in K.E. usually takes place.
- Solve different problems of elastic and inelastic collisions between two bodies in one dimension by using law of conservation of momentum.
- Describe that momentum is conserved in all situations.
- Identify that for a perfectly elastic collision, the relative speed of approach is equal to the relative speed of separation.
- Differentiate between explosion and collision (objects move apart instead of coming nearer).

INTRODUCTION

It is our common observation that all bodies are either at rest or in motion. A body is said to be at rest if it does not change its position with respect to its surroundings. For example, a book placed on a table. **"A body is said to be in motion, if it changes its position with respect to its surroundings".** For example, a man, walking a moving car or a train etc. In universe, everything is in perpetual motion, like motion of an electron around the nucleus, the motion of the moon around the earth, earth moves around the sun and so many others. The motion can be categorized into three types i.e., translational, rotational and vibrational. For example, motion of a car along a highway is a translational motion, motion of fan is a rotational motion and to and fro motion of a pendulum is a vibrational motion. All kinds of motion can be explained in terms of displacement, velocity, acceleration and force. These parameters can be studied in the equations of motion and Newton's three laws of motion.

The study of motion of a body under the influence of an applied force is called mechanics and there are two main branches of mechanics such as kinematics and dynamics. In kinematics, we study the motion of bodies without reference of forces or masses while in dynamics we study the forces that change the motion of the bodies. Another important aspect of this chapter is the projectile motion, such motion is a two dimensionally, so the path of the projectile motion is the resultant of simultaneous effect on the horizontal and vertical components of their velocities but these components act independently such as the vertical components determines the time of flight while horizontal component determines the range of flight.

3.1 DISPLACEMENT

Consider two lengths both having same magnitude of 10 m, but one length has no direction while the other length has specific direction from East to West as shown in Fig. 3.1.

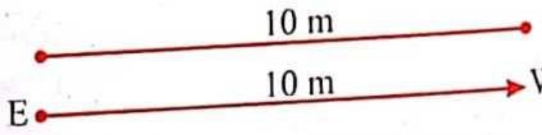


Fig.3.1: Distance vs. Displacement

Thus **the length of actual path between two points in any direction covered by a body during its motion in a given time is called distance. Its unit is metre**. Distance is a scalar quantity. Its value can never be zero or negative, during the motion of an object.

On the other hand, the length between two points in a given direction is called displacement. Displacement is also defined in terms of **the shortest distance between the initial and final points of the body in a particular direction** which is given by the vector drawn from initial to final position.

Displacement is a vector quantity. The SI unit of displacement is also metre and its dimensional formula is $[M^oLT^o]$. The displacement is either less or equal but never greater than the actual distance travelled. It is explained by the following examples.

Let a body moves from point A to point B then from point B to point C in time 't' as shown in Fig.3.2. The shortest distance $\overline{AC}$ from initial point 'A' to final point 'C' is a displacement, while $\overline{AB}+\overline{BC}$ is the actual distance covered.

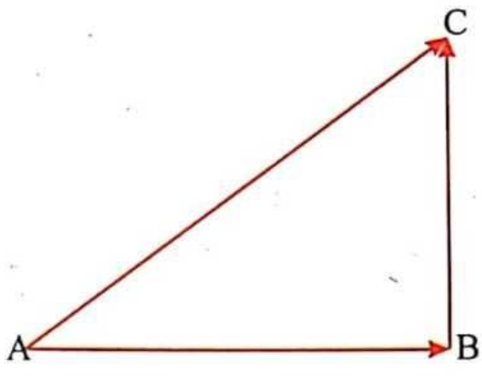


Fig.3.2: Distance vs displacement

The motion of a body along a circular path of circle from point 'A' to point 'B' is shown in Fig. 3.3. In this case arc AB is a distance while chord AB is its displacement.

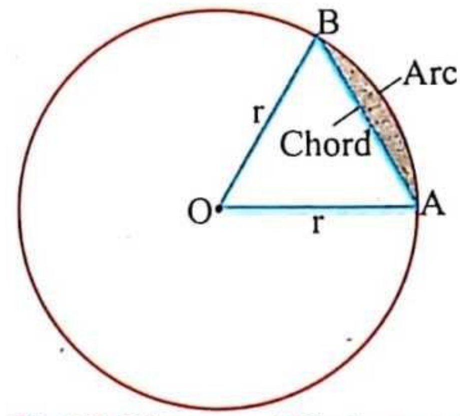


Fig.3.3: Distance and Displacement between two points A and B.

Example 3.1

Compare distance and displacement of a body when its motion is along circular path from point A to B of a hemi sphere of radius 10 cm as shown in Fig 3.4.

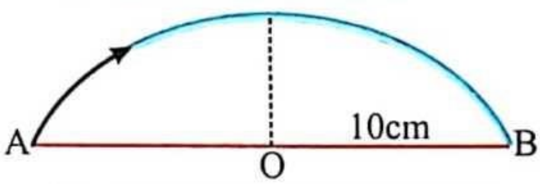


Fig.3.4: Distance along the circular path and displacement along the diameter.

Solution:

Distance = length along curved path of hemi sphere

$$\text{Distance} = \frac{2\pi r}{2} = (3.14)\,(10)$$

Distance = 31.4cm

Displacement = Diameter of a hemi sphere

Displacement = 2r

Displacement = 2(10)

Displacement = 20 cm

This example shows that displacement is the shortest distance.

3.2 SPEED

We can calculate the average speed of a moving object if we know the distance covered by the object and time taken. Thus the average speed is defined as **"The time rate of change of position of the object in any direction"**. It is measured by the distance covered by an object in unit time i.e.,

$$\text{Average speed} = \frac{\text{distance covered}}{\text{time interval}} \quad ......(3.1)$$

Speed is a scalar quantity. Its SI unit is metre/second ($m\ s^{-1}$) and its dimensional formula is $[M^0LT^{-1}]$.

The speed of an object can be zero or positive but never negative.

3.3 VELOCITY

Let there be some displacement between a moving car and a milestone as shown in Fig.3.5.

The displacement between them decreases with time when the car is moving toward the milestone and increases with time when the car is moving away from the milestone. This change in displacement of the car with respect to time is called its velocity and it is defined as;

The rate of change of displacement of body is called its velocity.

Mathematically it is expressed as;

$$\bar{v} = \frac{\Delta \vec{d}}{\Delta t} \quad ......(3.2)$$

where $\Delta \vec{d}$ represents the difference in displacements of the body ($\mathbf{d}_2 - \mathbf{d}_1$) and Δt is the time interval ($t_2 - t_1$).

Velocity is a vector quantity its direction is along the direction of displacement. The SI unit of a velocity is $m\ s^{-1}$ and its dimensional formula is $[M^0LT^{-1}]$.

The magnitude of velocity is equal to speed of the body.

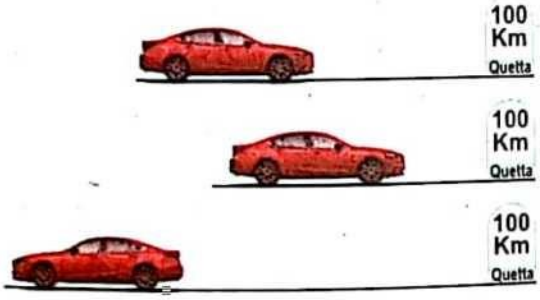


Fig.3.5: Velocity of a car

TYPICAL SPEEDS

Motion	Speed ms^{-1}
Walking Ant	0.01
Human Swimming	2
Human Running	4
Flying Bee	5
Tortoise	9
100 Meters Dash	10
Running Cheetah	29
Falcon in a dive	37
Automobile	62
Jet Airline	267
Sound in Air	333
Moon around the earth	1023
Earth around the Sun	29600
Sun around galaxy	230000
Light (Electromagnetic Wave)	300000000

3.3.1 Uniform Velocity

Velocity of a body is called uniform when it covers equal displacements in equal intervals of time. However, small these intervals may be.

3.3.2 Variable Velocity

The velocity of a moving body is said to be variable (or non-uniform) when it covers unequal displacements in equal intervals of time or vice versa.

Key Points

(i) The magnitude of velocity is called the speed
(ii) Velocity = Speed × direction
(iii) Speed and velocity, both have same unit ms^{-1}
(iv) Speed is a scalar quantity whereas velocity is a vector

If a moving body has constant speed but changes in its direction of motion, then the velocity is variable. In fact, the velocity may be variable due to the two following reasons.

(i) change in magnitude (speed)

(ii) change in direction

3.3.3 Average Velocity

Average velocity is defined as, **"the ratio of total displacement to the total intervals of time during which the displacement is covered".**

Let $\mathbf{d}_1$ be the displacement at time t_1 and $\mathbf{d}_2$ be the displacement at time t_2 as shown in Fig. 3.6. Then

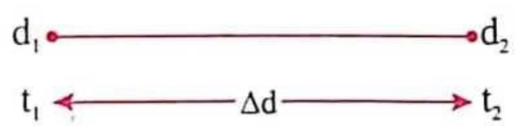


Fig.3.6: Rate of change of displacement

Change in displacement $\Delta\vec{d} = \vec{d}_2 - \vec{d}_1$

Interval of time $\Delta t = t_2 - t_1$

$$\text{Average velocity} = \vec{v}_{avg} = \frac{\Delta\vec{d}}{\Delta t} \quad \ldots\ldots (3.3)$$

3.3.4 Instantaneous Velocity

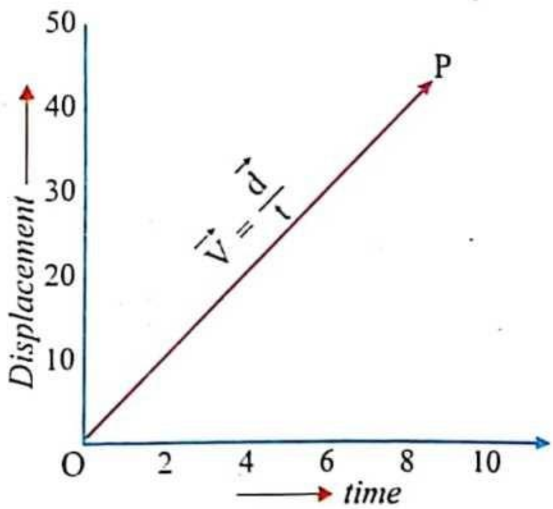
"The velocity of a body at any instant of time where the time approaches to zero then such velocity of body is called its instantaneous velocity".

If $\Delta\vec{d}$ is the change in displacement in time interval Δt which approaches to zero then the corresponding value of the instantaneous velocity is written as;

$$\vec{v}_{ins} = \underset{\Delta t \to 0}{\text{limit}} \frac{\Delta\vec{d}}{\Delta t} \quad \ldots\ldots (3.4)$$

3.3.5 Displacement - time Graph

The velocity of a moving body is defined as the ratio of the change in the displacement to the time taken. Therefore, the graph between displacement and time is more helpful to explain the changing position of the body. The slope of the displacement-time graph is equal to the velocity of the body and it can be studied under different cases.



Fiog.3.7: A straight line in displacement-time graph for uniform velocity of a moving body

When the motion of a body is uniform, a straight line $\overline{OP}$ in displacement-time graph represents the uniform velocity of the body as shown in Fig. 3.7.

When the motion of a body is non-uniform then there is a curved line in displacement-time graph and the chord of this curved is represented the average velocity of the body as shown in Fig. 3.8.

In case of instantaneous velocity, the slope of the tangent at the point P of the curved line in displacement-time graph shows instantaneous velocity of the body as shown in Fig. 3.9.

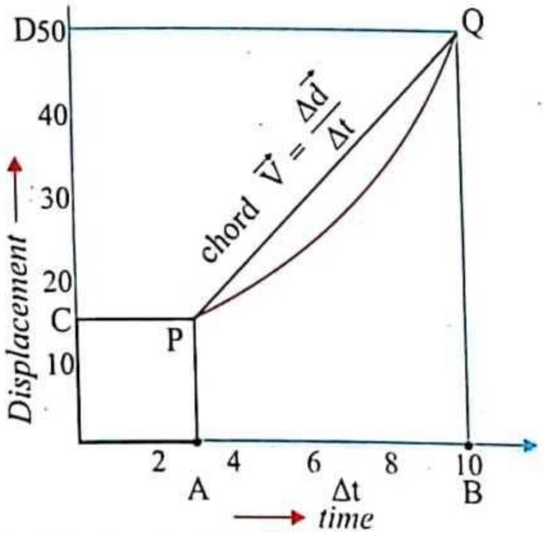


Fig.3.8: The chord in displacement-time graph shows average velocity of a moving body

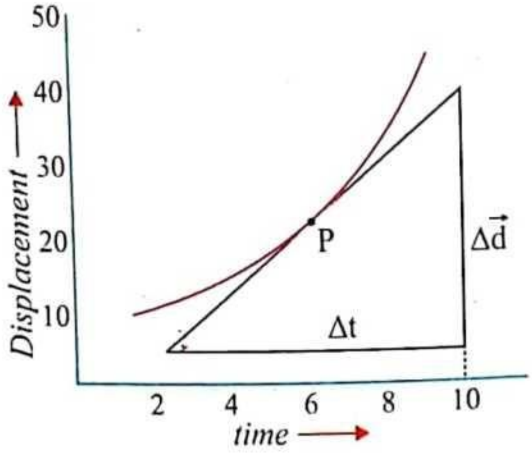


Fig.3.9: A point P in Displacement-time graph shows instantaneous velocity of a moving body

3.4 ACCELERATION

Generally, bodies do not move with constant velocities. When the velocity of a moving body changes with time then it is said to be accelerating. As the velocity is a vector quantity, the change in velocity may be due to the change in magnitude or change in direction or both. The acceleration is a measure of how fast or slow the velocity is changing with time. Therefore, the acceleration is defined as "**the rate of change in velocity of a body with respect to time**". If $\mathbf{v}_i$ be the initial velocity of a body at time t_i and $\mathbf{v}_f$ be its final velocity at time t_f, then the acceleration of the body is given by;

$$\bar{a} = \frac{\bar{v}_f - \bar{v}_i}{t_f - t_i} = \frac{\Delta \bar{v}}{\Delta t} \text{ ...... (3.5)}$$

Acceleration is a vector quantity. Its direction depends upon the nature of change in velocity. The SI unit of acceleration is $ms^{-2}$and its dimensions are $[M^oLT^{-2}]$.

If the rate of change of velocity of a body is increasing, then its acceleration is taken as positive and the direction of acceleration is along the direction of velocity. However, if the rate of change of velocity of a body is decreasing then its acceleration is taken as negative and the direction of acceleration is opposite to the direction of velocity. It is also called **deceleration** or **retardation**.

3.4.1 Average Acceleration

When the acceleration of a body is due to the continuous change in magnitude or direction or both of the velocity then we introduce the average acceleration which is equal to the total change in velocity over the total time interval in which that change takes place in velocity. Mathematically, the average acceleration a_{av} is expressed as;

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} \qquad ......(3.6)$$

3.4.2 Instantaneous Acceleration

The acceleration of the body at particular instant of time if the time interval Δt is infinitesimally small $(\Delta t \rightarrow 0)$, then such acceleration is called instantaneous acceleration and it is given by

$$\vec{a}_{ins} = \lim_{\Delta t \rightarrow 0} it \frac{\Delta\vec{v}}{\Delta t} \qquad ......(3.7)$$

3.4.3 Uniform Acceleration

A body is said to be moving with uniform acceleration (i.e., constant acceleration) if the velocity of the body changes by equal amounts in equal intervals of time. However, small these intervals of time may be.

3.4.4 Variable Acceleration

The acceleration of a body is said to be variable if its velocity changes with time in terms of magnitude or direction or both. The variable acceleration is also called non-uniform acceleration.

3.4.5 Graphical representation of acceleration in velocity-time graph

The graphical representation of acceleration is more easy and helpful to understand its nature. The slope of the velocity-time graph is equal to the acceleration of the body which can be studied under the following various cases.

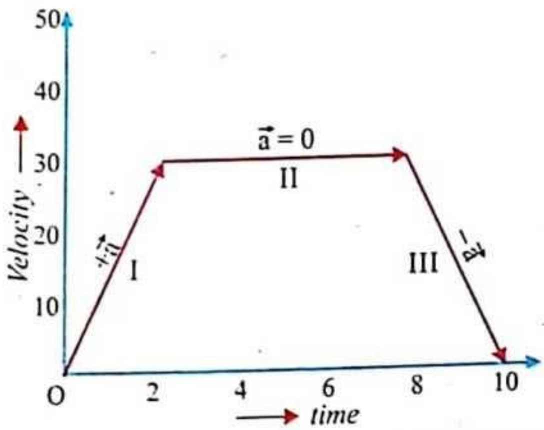


Fig.3.10: Velocity-time graph represents acceleration, uniform acceleration and deceleration

When the velocity of a body is increasing with time then there is a straight line with positive slope in velocity-time graph which shows the positive acceleration of the body as shown in Fig.3.10 (I).

When the velocity of a body is constant than there is a straight horizontal line in velocity-time graph, it represents a uniform velocity of a body as shown in Fig.3.10(II). In this case, $v_i = v_f$ and acceleration of the body is zero.

When the velocity of the body is decreasing with time then there is a straight line with negative slope in velocity-time graph which shows deceleration or retardation or negative acceleration as shown in Fig.3.10(III)

In case of instantaneous acceleration, the slope of the tangent at point P of the curved line in velocity-time graph shows instantaneous acceleration of the body as shown in Fig. 3.11.

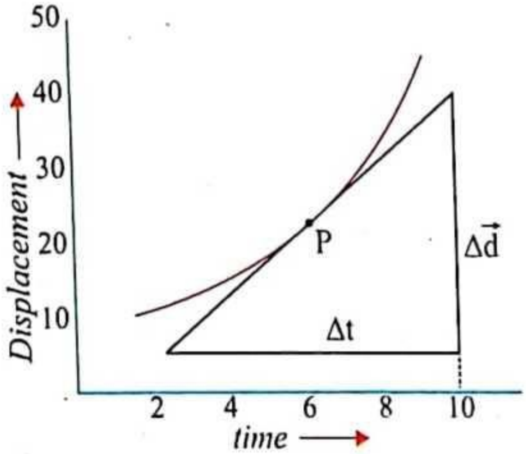


Fig.3.11: The point 'P' in velocity-time showing instantaneous acceleration

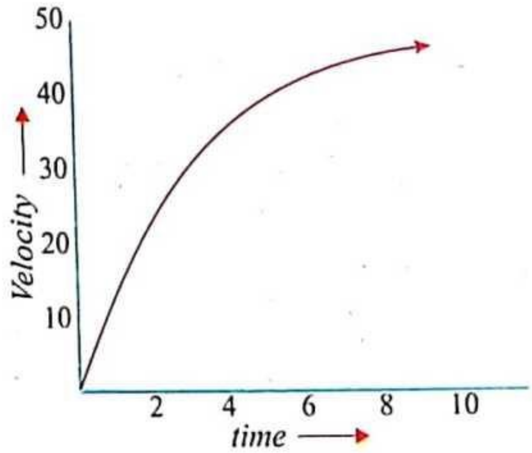


Fig.3.12: A curved line in velocity time graph shows variable acceleration

When there is continues change of velocity of a body with respect to time in magnitude or direction then there is a curved line in velocity-time graph which shows the variable acceleration of the body as shown in Fig. 3.12.

3.5 FREE FALL MOTION

In the absence of resistive forces (air resistance), when a body falls freely under gravity, its rate of change of velocity is termed as gravitational acceleration. It is represented by 'g' and its value at sea level is 9.8m s^{-2}.

The value of 'g' is taken as negative for upward vertical motion of a body and is taken as positive for downward motion. However, in case of motion of a paratrooper, where its weight and normal reaction (air resistance) are equal then the motion of paratrooper becomes uniform and the value of 'a' is zero as shown in Fig. 3.13.

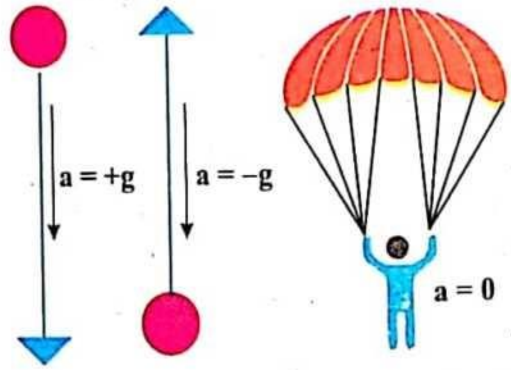


Fig.3.13: Acceleration, deceleration and uniform acceleration of a body under gravity.

Some examples of free falling objects

- A stone dropped from a height.
- A skydiver is in freefall until he pulls his parachute.
- An object, in projectile motion, on its descent.
- A satellite or a spacecraft in continuous orbit.
- The planets are in free fall as they orbit the sun.

FOR YOUR INFORMATION

Planets	g(ms^{-2})
Mercury	3.7
Venus	8.9
Earth	9.8
Mars	3.7
Jupiter	23.1
Saturn	9.0
Uranus	8.7
Neptune	11.0
Sun	274

3.6 REVIEW OF EQUATIONS OF UNIFORMLY ACCELERATED MOTION

There are four parameters time, displacement, velocity and acceleration which are associated with a moving body. To study these parameters, we have three important equations of motion which are expressed as;

Let a body starts its motion with initial velocity '$\mathbf{v_i}$' and after some time 't' its velocity becomes '$\mathbf{v_f}$' after covering a displacement **S** as shown in Fig. 3.14. This change in velocity of body with time is called its acceleration, which can be expressed as;

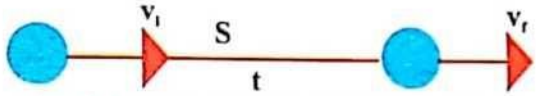


Fig.3.14: Linear motion of a body along a straight path

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t}$$

$$\vec{a}t = \vec{v}_f - \vec{v}_i$$

$$\vec{v}_f = \vec{v}_i + \vec{a}t \text{ ...... (3.8)}$$

This is known as 1[st] the equation of motion. In scalar notation $v_f = v_i + at$.

Now by definition of displacement.

$$S = v_{av}t$$

But, (Average velocity) $v_{av} = \left(\dfrac{v_f + v_i}{2}\right)$

Therefore, $S = \left(\dfrac{v_f + v_i}{2}\right)t$

As $v_f = v_i + at$

$$S = \left(\frac{v_i + at + v_i}{2}\right)t$$

$$S = \frac{2v_i t}{2} + \frac{at^2}{2}$$

$$S = v_i t + \frac{1}{2}at^2 \quad ......(3.9)$$

This is the 2nd equation of motion.

Again $\quad v_f = v_i + at$

$$t = \frac{v_f - v_i}{a}$$

Put it in equation (3.9)

$$S = v_i\left(\frac{v_f - v_i}{a}\right) + \frac{1}{2}a\left(\frac{v_f - v_i}{a}\right)^2$$

$$S = \frac{v_i v_f - v_i^2}{a} + \frac{1}{2}\frac{v_f^2 + v_i^2 - 2v_i v_f}{a}$$

$$S = \frac{2v_i v_f - 2v_i^2 + v_f^2 + v_i^2 - 2v_i v_f}{2a}$$

$$S = \frac{v_f^2 - v_i^2}{2a}$$

$$2aS = v_f^2 - v_i^2 \quad ......(3.10)$$

This is the 3rd equation of motion.

Example 3.2

A vehicle starts from rest and moves with a constant acceleration of 6 m s^{-2}. Find its velocity and the distance traveled after 5 sec.

Solution:

We have,

$$v_i = 0$$
$$a = 6\,ms^{-2}$$
$$v_f = ?$$
$$S = ?$$
$$t = 5s$$

According to 1st equation of motion.

$$v_f = v_i + at$$
$$v_f = 0 + (6)(5)$$
$$v_f = 30\,ms^{-1}$$

Now according to 2nd equation of motion.

$$S = v_i t + \frac{1}{2}at^2$$

$$S = (0)(5) + \frac{1}{2}(6)(5)^2 = 0 + 3(25)$$

$$S = 75\,m$$

Example 3.3

The velocity of a truck is reduced uniformly from 30 m s^{-1} to 8 m s^{-1} while traveling a displacement of 210 m. (a) What is the deceleration of the truck? (b) How much further will the truck move before coming at rest?

Solution:

(a)

$v_i = 30\ m\ s^{-1}$

$v_f = 8\ m\ s^{-1}$

$S = 100\,m$

$a = ?$

Using 3[rd] equation of motion.

$$2aS = v_f^2 - v_i^2$$

$$a = \frac{v_f^2 - v_i^2}{2S}$$

$$a = \frac{(8)^2 - (30)^2}{2(210)}$$

$$a = \frac{64 - 900}{420}$$

$$a = -2\,m^{-2}$$

(b)

$S = ?$

$v_i = 8\,m\,s^{-1}$

$v_f = 0$

$a = -2\,m\,s^{-2}$

Using 3[rd] equation of motion.

$$2aS = v_f^2 + v_i^2$$

$$S = \frac{v_f^2 - v_i^2}{2a}$$

$$S = \frac{0 - (8)^2}{2(-2)}$$

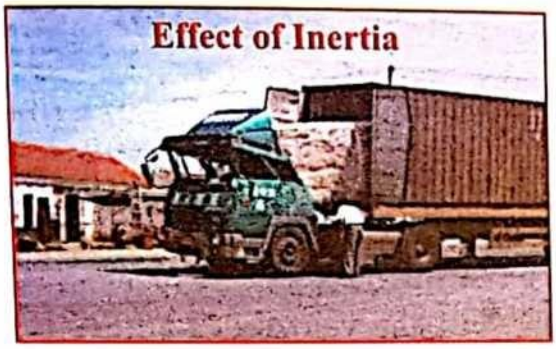
$$S = \frac{-64}{-4}$$

$$S = 16\,m$$

3.7 NEWTON'S LAWS OF MOTION

Newton's laws of motion have great importance in classical physics. A large number of theorems and results may be derived from Newton's laws of motion. To study the basic principles of motion as well as relationship between force and motion, Sir Isaac Newton published "Laws of Motion" in his famous book "Principia" in 1687. These laws can be applied to the motion of massive bodies which have low speed as compared with speed of light. However, for atomic particles which are moving very fast Einstein's, relativistic mechanics can be applied for their motion instead of Newton's laws of motion. The Newton's laws of motion are summarized below;

3.7.1 Newton's first law of motion



This law is based upon law of nature and it states that " In the absence of an external force, if a body is at rest it will remain at rest and if a body is moving with uniform velocity, it will continue its uniform motion". Newton's 1st law is also called law of inertia, that is, the resistive property of a body to resist any change in its state of rest or uniform motion is known as inertia. Inertia is also defined as the inherent property of an object due to which it tends to maintain the state of rest or of uniform motion. The mass of a body is a quantitative measure of its inertia. The bigger is the mass of a body, the higher will be the its inertia. Hence, there is a great resistance to any change in velocity for a big mass.

3.7.2 Newton's second law of motion

Newton's second law of motion is also known as law of acceleration which is stated as; "When a force is applied on a body, an acceleration is produced in a body in the direction of force as shown in Fig.3.15. According to this law, the acceleration is directly proportional to the applied force and inversely proportional to the mass of the body".

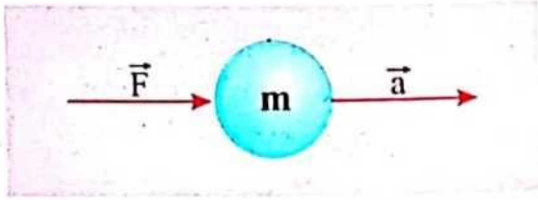


Fig.3.15: Acceleration in the direction of force

Hence, we can develop a relation between mass, acceleration, and force through the following mathematical statement of Newton's second law.

$$\vec{a} \propto \frac{\vec{F}}{m}$$

$$\vec{a} = \text{Constant}\frac{\vec{F}}{m}$$

$$\vec{a} = K\frac{\vec{F}}{m}$$

where 'K' is a constant of proportionality. If its value in S.I units is one than;

$$\vec{F} = m\vec{a} \qquad \text{...... (3.11)}$$

This is a mathematical form of Newton's 2nd law of motion. Force is a vector quantity, its SI unit is newton (N) and its dimensional formula is $[MLT^{-2}]$. One newton force can be defined as; "An applied force is said to be one newton if it produces an acceleration of $1\,m\,s^{-2}$ in a body of mass 1 kg".

Thus, a greater force is required to accelerate a massive body as compared to a light body as illustrated in Fig. 3.16.

Fig 3.16: The same force exerted on system of different masses produces different accelerations.
(a) A basketball player pushes on a basketball to make a pass. (Ignore the gravitational force).
(b) The same player exerts an identical force on a stationary land cruiser and produces less acceleration.

Example 3.4

What is the force which acts on a moving body of mass 10 kg for 10 s and reduces the velocity of the body from $9\,\text{m}\,\text{s}^{-1}$ to $4\,\text{m}\,\text{s}^{-1}$.

Solution:

$F = ?$
$m = 15$ Kg
$t = 5$ sec
$v_i = 9\ \text{m s}^{-1}$
$v_f = 4\ \text{m s}^{-1}$

According to Newton's 2nd law.

$$F = ma$$

$$F = m\left(\frac{v_f - v_i}{t}\right)$$

$$F = 15\left(\frac{4-9}{5}\right)$$

$$F = 3(-5)$$

$$F = -15\,\text{N}$$

> **POINT TO PONDER**
> Why the driver and the passengers wear safety belt during their journey?

The negative sign shows that the applied force is acting in a direction opposite to that of motion of the body. .

3.7.3 Newton's third law of motion

This law is also known as law of forces and it is stated as; **"For every action there must be an equal and opposite reaction"** where action and reaction are forces which have same magnitude, but act in opposite direction.

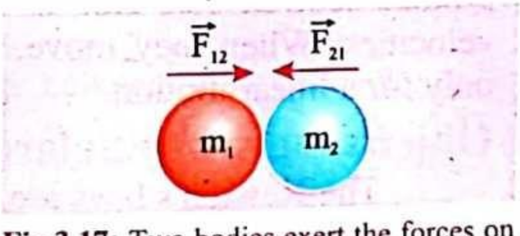


Fig.3.17: Two bodies exert the forces on each other which are same in magnitude but act in opposite directions.

Action and reaction forces always exist in the form of a pair and never act on the same body.

Consider two bodies of masses m_1 and m_2 which exert forces on each other during their collision as shown in Fig.3.17. The force exerted by m_1 on m_2 is F_{12} and the force exerted by m_2 on m_1 is F_{21}. The force F_{12} may be called action force and the force F_{21} may be called reaction or vice versa. Then according to Newton's third law of motion;

> **DO YOU KNOW**
> Which two forces are acting on a flying kite?

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

This is the mathematical form of Newton's 3rd law.

We can observe Newton's third law of motion in our everyday life.

(i) Consider a book lying (motionless) on a table as shown in Fig. 3.18. Its acceleration is zero. Weight of the book acts downward, so another force called normal reaction provided by the table top must act upward on the book.

(ii) A rocket is also moving according to the principle of action and reaction forces. i.e., When its fuel burns, hot gases escape from its tail with a very high speed. The reaction of these gases on the rocket causes it to move in the upward direction i.e. opposite to the direction of gases shown in Fig. 3.19.

(iii) When we walk or run on the ground our feet pushes the ground backward (action) while the ground pushes us forward (reaction).

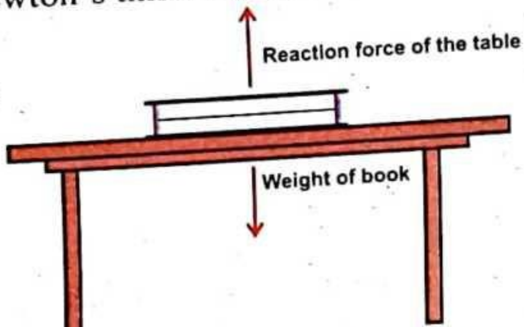


Fig.3.18: Action and reaction forces on book lying on table

Fig.3.19: Action and reaction forces cause of motion of a rocket.

Limitations of laws of motion on microscopic objects

Newton's laws are not valid on the microscopic objects such as electrons, proton, neutron etc. This is because these particles have small masses but large velocities. When they move, they behave as wave. But Newton's laws can apply only for a linear motion.

Objects moving with large velocities

The Newton's laws are not valid for the objects which are moving with large velocities comparable to the speed of light because at large velocities the mass of the objects do not remain constant but increases.

Similarly, we use quantum mechanics instead of Newton's laws for the study of motion of sub-atomic particles.

On macroscopic level, Newton's laws have also limitations because of non-ideal environmental conditions, for example all equations and formulae are derived by assuming frictionless motions but practically we cannot have environment where friction is not present. We can minimize frictional force but cannot eliminate them completely.

3.8 WEIGHT AND MASS

3.8.1 Weight

We know that everybody is attracted to the Earth. The attractive force exerted by the Earth on a body is called the gravitational force. This force is directed towards the centre of the Earth and its magnitude is called the weight of the body. It is represented by 'W' and it is calculated as;

$$W = m\,g \quad ......(3.12)$$

Weight is a variable quantity, because it depends upon 'g'. The value of 'g' decreases with increasing distance from the centre of the Earth and increases with decreasing distance from the centre of the earth. The SI unit of weight is newton and its dimensions are $[MLT^{-2}]$.

3.8.2 Mass

The quantity of matter in a body is called its mass. It is measured in terms of kilogram and it is a constant quantity. In other words, mass is that property of a body which specifies how much resistance a body exhibits to changes in its velocity i.e., greater the mass of a body, the lesser will be the acceleration in the body for a given applied force. The mass of a body can be determined by two different newtons.

I. Gravitational Mass

The gravitational mass of a body is defined in term of the ratio between the weight of the body to the gravitational acceleration i.e.,

$$m = \frac{W}{g} \quad ......(3.13)$$

Gravitational mass is measured at rest on a balance that depends upon gravity.

II. Inertial Mass

The inertial mass of a body is defined as the ratio between the applied force 'F' to the linear acceleration 'a' produced in the body by that applied force i.e.;

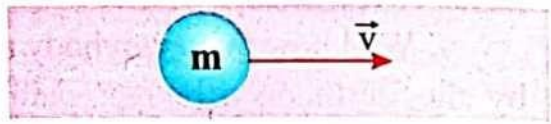
$$m = \frac{F}{a} ......(3.14)$$

Inertial mass is measured dynamically (while moving) and does not depend on gravity.

3.9 LINEAR MOMENTUM

You are quite familiar with the fact that a force is needed to stop a moving object. The force needed to stop the object will depend on at least two factors: the mass and velocity of the moving object.

For example, it would be more difficult to stop a car travelling at 10 m s^{-1} than a bicycle travelling at the same speed. Similarly, it would be more difficult to stop a car moving at 10 m s^{-1} than the same car moving at 5 m s^{-1}.



A body of mass m moving with velocity $\vec{V}$ along a straight path.

This ability to stop a moving object is related to its momentum. The momentum is the property of moving objects. It is represented by $\vec{p}$ and it is defined as "the product of mass (m) and velocity $\vec{v}$".

Mathematically we have

$$\vec{p} = m\vec{v} ...... (3.15)$$

Momentum is a vector quantity and its direction is along the direction of velocity of the body. Its SI unit is kg m s^{-1} or N s and its dimensional formula is $[MLT^{-1}]$.

If there are two bodies of different masses and velocities, but having the same momentum then;

$$p_1 = p_2$$
$$m_1 v_1 = m_2 v_2$$
$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

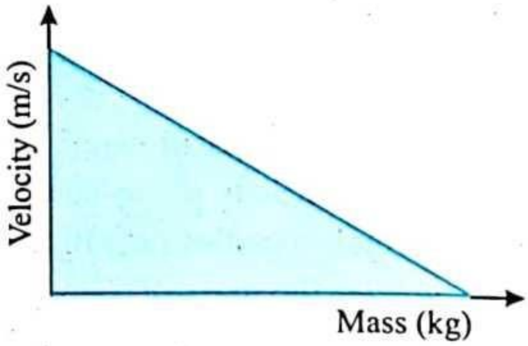


Fig.3.20: Velocity-mass graph showing momentum of a body

This result shows that at constant momentum, velocity of body is inversely proportional to its mass. Graphically, the relationship between mass and velocity is shown in Fig. 3.20.

3.9.1 Momentum and Newton's 2nd law of motion

We have stated that the more is the momentum of an object, the greater will be the force required to stop it. What is exact relationship between the force and momentum? Now we will derive the same.

Consider a force $\vec{F}$ which is applied on a body of mass 'm' which is moving with initial velocity $\vec{v}_i$ along a straight line. After some time Δt its velocity becomes $\vec{v}_f$ due to the applied force as shown in Fig.3.21. This change in velocity of body is called its acceleration and is given by;

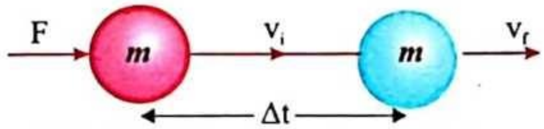


Fig.3.21: Change in velocity of body with time by the applied force

$$\vec{a} = \left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right)$$

But, $$\vec{F} = m\vec{a}$$

$$\vec{F} = m\left(\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right)$$

$$\vec{F} = \frac{m\,\vec{v}_f - m\,\vec{v}_i}{\Delta t}$$

$$\vec{F} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} \quad ......(3.16)$$

This equation gives the relationship between the applied force and the momentum of the body. This is another form of Newton's 2nd law. We can also state Newton's second law of motion as **"The time rate of change of momentum of a body is equal to the force applied on it".**

Example 3.5

What is the momentum of a runner of mass 65 kg who covers a displacement of 100 m in 40 sec?

> **POINT TO PONDER**
>
> When a stone and leaf are dropped from a building simultaneously then why the stone reaches to the ground earlier?

Solution:

m = Mass of man = 65 kg

d = Displacement = 100 m

t = Time = 40 s

The magnitude of momentum is given by

$$p = mv$$

But $$v = \frac{d}{t}$$

therefore, $$p = m\left(\frac{d}{t}\right)$$

$$p = 65\frac{100}{40}$$

$$p = 162.5\ \text{kg m s}^{-1}$$

Note that this momentum is along the direction of velocity of body.

3.9.2 Impulse

It is a daily life experience that in certain cases the forces act on bodies for a short interval of time and these forces are called impulsive forces. For example, when a ball is struck by a tennis racket it exerts a force on the ball for very short interval of time.

The product of impulsive force and short interval of time is called impulse. Mathematically,

$$\text{Impulse} = \vec{F} \times \Delta t \text{ ......(3.17)}$$

Impulse is a vector quantity. Its unit is N s and its dimensional formula is $[MLT^{-1}]$. The observations show that impulsive force does not remain constant but it varies with time and it is shown in Fig.3.22. The area under the curve in force-time graph shows impulse and it is equal to the change in momentum.

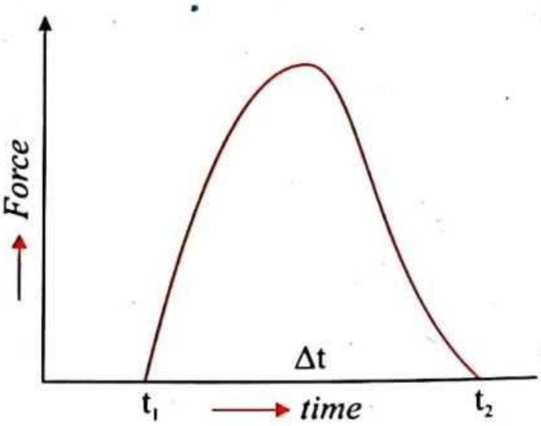


Fig.3.22: Force-time graph show impulse

According to Newton's second law of motion, the time of rate of change of momentum is equal to the applied force i.e.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

Put it in equation (3.17)

$$\text{Impulse} = \frac{\Delta\vec{p}}{\Delta t} \times \Delta t$$

$$\text{Impulse} = \Delta\vec{p}$$

$$\text{Impulse} = p_f - p_i$$

$$\text{Impulse} = m\vec{v}_f - m\vec{v}_i \text{ ......(3.18)}$$

This is termed as impulse-momentum theorem which states that an impulse always changes the momentum of a body. It is based on the fact that if the total change in momentum takes place in a very short time, then the applied force should be very large. If the same change in momentum takes place over a longer interval of time, then the applied force will be small. For example, if two forces F_1 and F_2 act on a body to produce the same impulse, then their respective times of applications t_1 and t_2 should be such that

$$F_1 t_1 = F_2 t_2$$

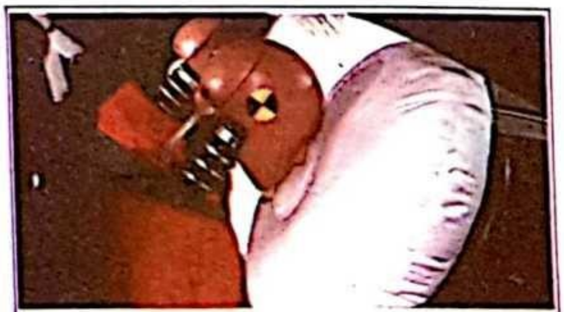
$$\frac{t_1}{t_2} = \frac{F_2}{F_1}$$

Practical applications of impulse

There are some practical applications of impulse, which are listed below:

I. A cricket player draws the hands back while catching a ball

While catching a fast moving cricket ball, a player lowers his hands. In this way the time of catch increases and the force decreases. So the player has to apply a less average force. As a result, the ball will also apply only a small force (reaction) on the hands. In this way the player will not hurt his hands.



Air bags in automobiles have saved countless lives in accidents. The air bag increases the time interval during which the passenger is brought to rest, thereby decreasing the force on the passenger.

II. Automobiles are provided with spring systems

When the automobile bumps over an uneven road, it receives a jerk. The spring increases the time of the jerk, thereby reducing the force. This minimizes the damage to the automobile.

III. Train bogies are provided with buffers

The buffers increase the time of jerks during shunting and hence reduces force with which the bogies pull each other.

FOR YOUR INFORMATION
Impulsive force is a force which acts on body for a very short time. Examples are; (i) A bat hitting the ball, (ii) The collision between two snooker balls

Example 3.6

A car has a constant force of 1000 N applied for 10 s. What impulse has been applied?

Solution:

$$\text{Impulse} = \text{Force} \times \text{time}$$
$$\text{Impulse} = 1000 \times 10$$
$$\text{Impulse} = 10000 \text{ Ns.}$$

3.9.3 Law of conservation of momentum

The law of conservation of momentum states that in the absence of an external force, the total momentum of an isolated system remains constant.

Consider two spheres of masses m_1 and m_2 which are moving along the same axis and same direction with velocities v_1 and v_2 before collision such that $v_1 > v_2$ and let both bodies collide and their velocities after collision become v'_1 and v'_2. During collision both the bodies exert the forces on each other which are same in magnitude but opposite in direction as shown in Fig.3.23. Let F_{21} be the force exerted on m_1 by m_2 and F_{12} be the force exerted on m_2 by m_1 then according to Newton's third law of motion;

$$F_{21} = -F_{12}$$

As $$F = \frac{\Delta p}{\Delta t}$$

So $$\frac{\Delta p_1}{\Delta t} = -\frac{\Delta p_2}{\Delta t}$$

$$\Delta p_1 = -\Delta p_2$$

$$m_1 v_1' - m_1 v_1 = -(m_2 v_2' - m_2 v_2)$$

$$m_1 v_1' - m_1 v_1 = -m_2 v_2' + m_2 v_2$$

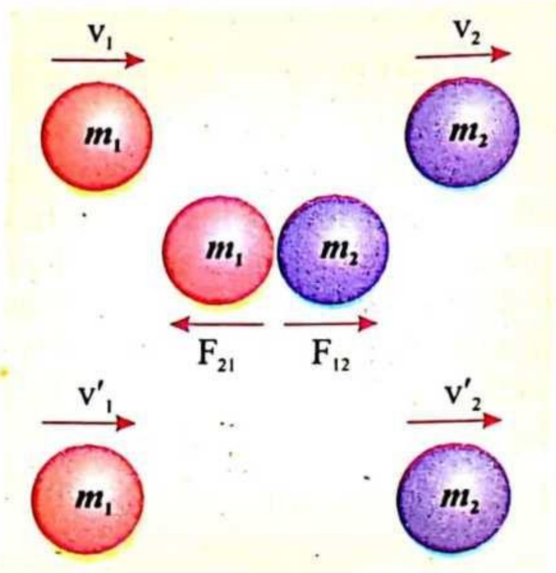


Fig.3.23: Collision of spheres which follows the law of conservation of momentum

Rearranging the above equation

$$-m_1 v_1 - m_2 v_2 = -m_1 v_1' - m_2 v_2'$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \;\ldots\ldots\; (3.19)$$

This is a mathematical form of law of conservation of momentum. According to this law the sum of momentum of the given system before collision is equal to the sum of momentum after collision that is the total momentum of an isolated system remains constant.

3.10 COLLISION

The impact of two bodies due to their interaction with each other is called collision. The magnitude and direction of the velocities of the bodies before and after collision may be same or different. The time in which the bodies remain in contact is known as impulsive or compression time which is very short interval of time and it can be neglected. There are two kinds of collision.

Elastic Collision

A collision in which both kinetic energy and momentum are conserved is called elastic collision. For example, collisions of molecules of a gas is elastic collision. An elastic collision has the following characteristics.

(i) The linear momentum is conserved.

(ii) The kinetic energy is conserved

(iii) The total energy of a system is conserved.

Inelastic Collision

A collision in which the linear momentum of a body is conserved, but total energy is not conserved is called inelastic collision. The experiment shows that there is loss in kinetic energy in inelastic collision. This loss of energy appears in the other forms of energy, such as heat, sound etc. For example, when a bouncing ball is dropped on to a hard floor, the collision between the ball and floor is elastic and the ball would not lose its kinetic energy and so would rebound to its original height. However practically, the actual rebound height is slightly shorter, showing some loss of kinetic energy in collision. Such collision is called inelastic collision.

Similarly, the collision between cars, mud thrown on the wall and sticking to it and the collision between bullet and its target are the examples of inelastic collision.

During inelastic collision kinetic energy is not conserved, it is converted into various forms especially heat and sound. Hence the final kinetic energy is less than initial kinetic energy. An inelastic collision has the following characteristics.

(i) The linear momentum is conserved.

(ii) The total energy of a system is not conserved.

(iii) The whole or a part of kinetic energy energy is converted into any other form of energy (like heat and sound).

3.10.1 Elastic collision in one dimension

The collision between two bodies is said to be in one dimension, if the colliding bodies continue their motion along the same straight line after collision.

It is explained by an example. Let us consider two elastic spheres of masses m_1 and m_2 are moving with velocities v_1 and v_2, where $v_1 > v_2$ before collision. After moving a certain distance, both the bodies collide elastically and their velocities become v'_1 and v'_2 respectively such that they continue their motion along the same straight line in the same direction as shown in Fig. 3.24.

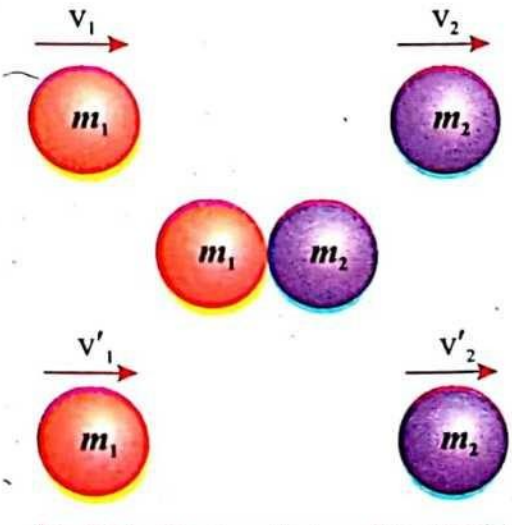


Fig.3.24: Elastic collision of two bodies one dimension

The values of these velocities after collision can be expressed in terms of velocities before collision.

Since in an elastic collision, linear momentum is conserved, therefore according to law of conservation of momentum;

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

As all velocities are in the same direction, we can rearrange the above equation

$$m_1v_1 - m_1v_1' = m_2v_2' - m_2v_2$$

$$m_1(v_1 - v_1') = m_2(v_2' - v_2) \text{......(3.20)}$$

Similarly, according to law of conservation of kinetic energy.

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

Rearrange the above equation

$$\frac{1}{2}m_1v_1^2 - \frac{1}{2}m_1v_1'^2 = \frac{1}{2}m_2v_2'^2 - \frac{1}{2}m_2v_2^2$$

$$m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2 - m_2v_2^2$$

$$m_1\left(v_1^2 - v_1'^2\right) = m_2\left(v_2'^2 - v_2^2\right)$$

$$m_1(v_1 - v_1')(v_1 + v_1') = m_2(v_2' - v_2)(v_2' + v_2) \text{......(3.21)}$$

Dividing eq. (3.21) by eq. (3.22) we get;

$$v_1 + v_1' = v_2' + v_2 \text{......(3.34)}$$

Above equation can be written as;

$$v_1 - v_2 = v_2' - v_1' \text{......(3.23)}$$

$$v_1 - v_2 = -(v_1' - v_2')$$

The crumple zone is designed to absorb energy from a collision and reduce the force of collision. Folding during a crash increases the impact time. Time to come to halt is increased so the force is decreased.

This is an interesting result that the quantity on the left of the equation (3.34) i.e., $(v_1 - v_2)$ is the relative velocity of approach of the two masses while the quantity on the right $(v_1' - v_2')$ is the relative velocity of separation.

Thus for perfectly elastic collision in one dimension, the relative velocity of approach before collision is equal to the relative velocity of separation after collision and they are opposite. If one object is approaching another at a relative velocity of 10 m s^{-1}, then after collision it will be receding at a relative velocity of 10 m s^{-1}.

Now by solving eq. (3.20), eq. (3.22) and eq. (3.23) we get the value of v_1' and v_2' as:

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2 \text{ ......(3.24)}$$

$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 \text{ ......(3.25)}$$

Equation (3.24) and equation (3.25) can be studied for different cases.

3.10.2 Elastic collision in one dimension for different cases

Special cases of elastic collision in one dimension.

Case – I:

When both the colliding bodies have same masses.

i.e., $m_1 = m_2 = m$.

Then eq. (3.24) and eq. (3.25) become.

$$v'_1 = v_2$$

and $$v'_2 = v_1$$

This shows that if $m_1 = m_2$ as shown in Fig.3.31, then after one dimensional elastic collision the velocities of the bodies will be interchanged.

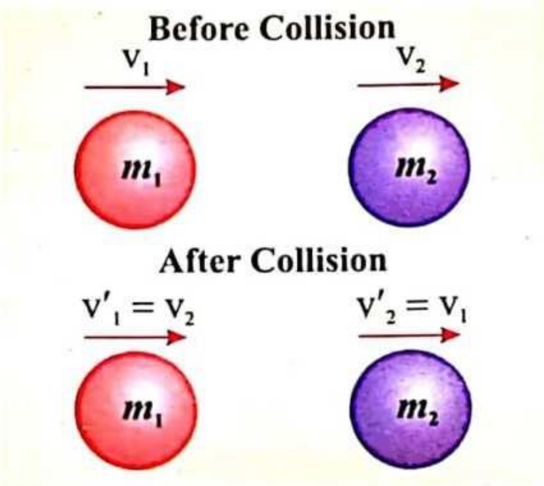


Fig.3.25: One dimension elastic collision of two bodies where $m_1 = m_2$.

Case – II:

When both bodies have same mass i.e. $m_1 = m_2$ and the target body m_2 is at rest ($v_2 = 0$) as shown in Fig. 3.26.

Then eq. (3.24) and eq. (3.25) become;

$$v'_1 = 0$$

and $$v'_2 = v_1$$

This shows that if $m_1 = m_2$ and m_2 at rest, then after collision the m_1 moving with velocity v_1 comes to rest and m_2 which was initially at rest starts moving with velocity v_1. Clearly, both the momentum and kinetic energy of the first body are completely transferred to the second body.

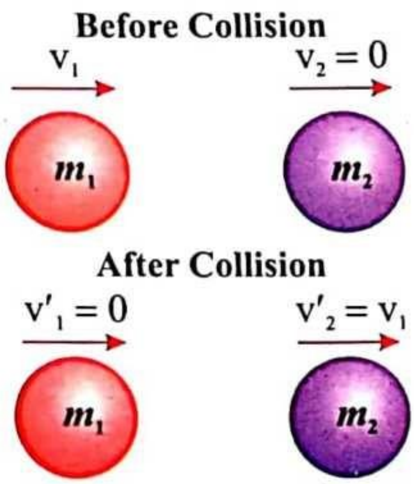


Fig.3.26: One dimension elastic collision of two bodies where $m_1 = m_2$ and $v_2 = 0$

Case – III:

When the body m_1 is much lighter than m_2 ($m_1 << m_2$) and m_2 is at rest ($v_2=0$) as shown in Fig.3.27.

So mass of lighter body m_1 can be neglected ($m_1 \approx 0$) as compared to mass of second heavier body m_2 then eq. (3.24) and eq. (3.25) become

$$v'_1 = -v_1$$

and $$v'_2 = 0$$

It means that when a lighter body m_1 collides against a heavier body m_2 at rest, the lighter body m_1 rebounds with its own velocity or m_1 starts moving with equal velocity in opposite direction after collision while the heavier one will remain at rest.

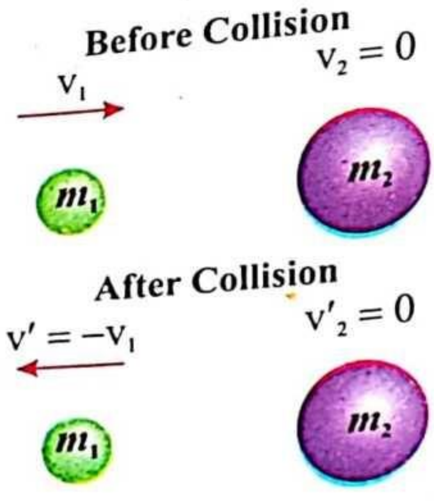


Fig.3.27: One dimension elastic collision of two bodies where $m_1 << m_2$ and $v_2 = 0$.

Case – IV:

When the body m_1 is much heavier than m_2 ($m_1 >> m_2$) and m_2 is at rest ($v_2 = 0$) as shown in Fig.3.28. In this case mass of lighter body m_2 can be neglected ($m_2 \approx 0$) as compared to mass of first heavier body m_1.

So from eq. (3.24) and eq. (3.25) we get

$$v'_1 = v_1$$

and $$v'_2 = 2v_1$$

This shows that the heavier body m_1 collides against a lighter body m_2 at rest, the heavier keeps on moving with the same velocity of its own and the lighter starts moving with a velocity double that of heavier.

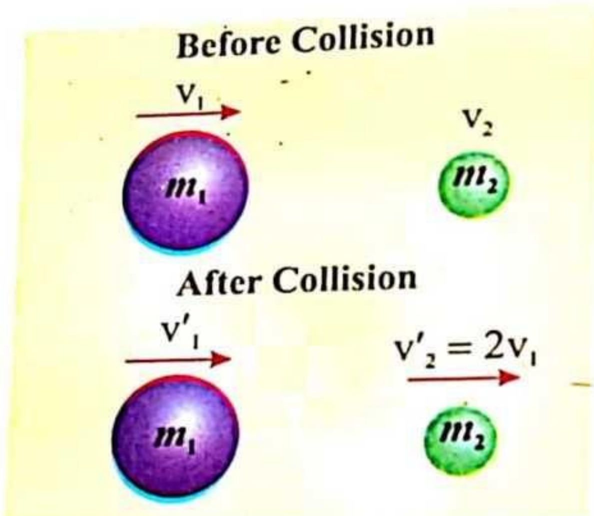


Fig.3.28: One dimension elastic collision of two bodies where $m_1 >> m_2$ and $v_2 = 0$.

Example 3.7

A 10 kg mass traveling with velocity 2 m s^{-1} collides elastically with a 2 kg mass traveling with velocity 4 m s^{-1} in the opposite direction. Find the final velocities of both objects after collision.

Solution:

$m_1 = 10$ kg

$v_1 = 2\ m\ s^{-1}$

$m_2 = 2\ kg$

$v_2 = -4\ m\ s^{-1}$

The negative sign is because of the velocity is in the opposite direction.

$v'_1 = ?$

$v'_2 = ?$

$$v'_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

$$v'_1 = \left(\frac{10-2}{10+2}\right)2 + \left(\frac{2\times 2}{10+2}\right)(-4)$$

$$v'_1 = \left(\frac{8}{12}\right)2 + \left(\frac{4}{12}\right)(-4) = \left(\frac{16}{12}\right) + \left(\frac{-16}{12}\right)$$

$$v'_1 = 1.33 - 1.33 = 0$$

$$v'_2 = \left(\frac{2m_1}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2$$

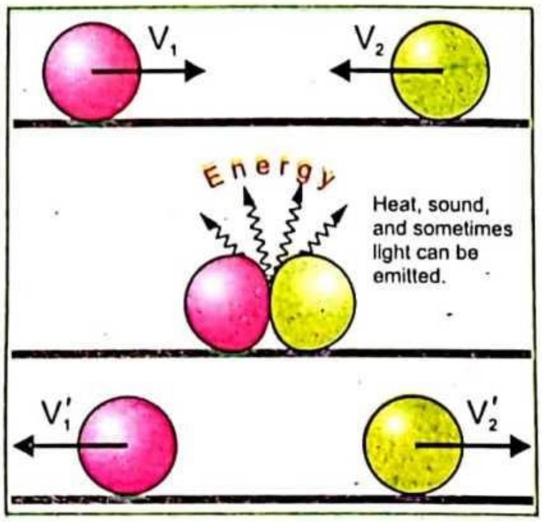
$$v'_2 = \left(\frac{2\times 10}{10+2}\right)2 + \left(\frac{2-10}{10+2}\right)(-4)$$

$$v'_2 = \left(\frac{20}{12}\right)2 + \left(\frac{-8}{12}\right)(-4) = \left(\frac{40}{12}\right) + \left(\frac{32}{12}\right)$$

$$v'_2 = \left(\frac{40}{12}\right) + \left(\frac{32}{12}\right) = \frac{40+32}{12}$$

$$v'_2 = \frac{72}{12} = 6\ ms^{-1}$$

> **ISOLATED SYSTEM**
>
> In the absence of an external and unbalanced force, when two or more than to bodies are exerted the forces to one another during their collision is called isolated system.



3.11 COLLISION AND EXPLOSION

We have studied about the collision i.e. the impact of two bodies to each other. After collision there will be two possibilities i.e. either the bodies stick to each other or bounce from each other. In both cases their total momentum will be conserved but their energy will be either conserved or changed.

An explosion is an event in which a single body breaks apart into a number of fragments. Like inelastic collision, total momentum in an explosion is conserved but total energy of the given system is not conserved, even the potential energy of the bomb is transferred in the form of kinetic energy of its fragments.

Suppose a bomb is at rest, its momentum will be zero because its velocity is zero. Let the bomb explode into seven fragments of masses m_1, m_2, m_3, m_4, m_5, m_6 and m_7 as shown in Fig.3.29. Let their velocities be v_1, v_2, v_3, v_4, v_5, v_6 and v_7. Thus their respective momentum will be given by;

$p_1 = m_1v_1$, $p_2 = m_2v_2$, $p_3 = m_3v_3$, $p_4 = m_4v_4$, $p_5 = m_5v_5$, $p_6 = m_6v_6$ and $p_7 = m_7v_7$.

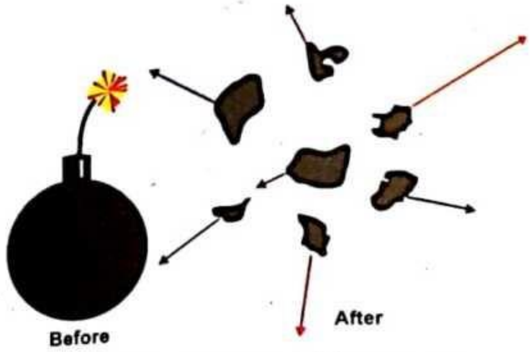


Fig.3.29: Bomb exploded into seven fragments

Now in the absence of an external force, the law of conservation of momentum can be applied.

∴ Momentum after explosion = Momentum before explosion

$\therefore p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 = 0$

The momentum vectors are shown in Fig.3.30. Since the momentum of the bomb was zero before the explosion, it must be zero after explosion as well. Each piece does have momentum, but the total momentum of the exploded bomb must be zero afterwards. This means that it must be possible to place the momentum vectors head to tail and form a closed polygon, which shows the vector sum is zero.

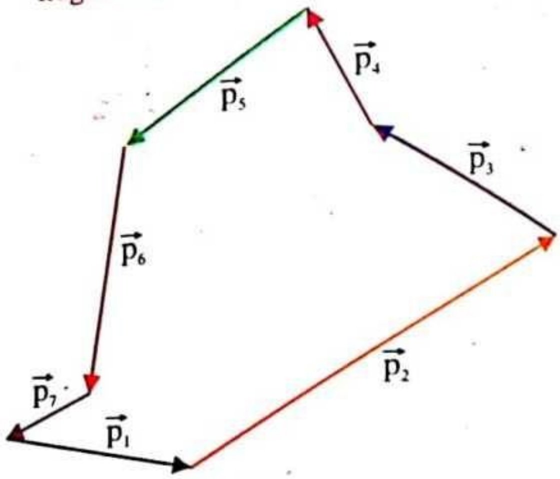


Fig.3.30: Sum of momentum vectors

Similarly, when a bullet of mass 'm' is fired with velocity 'v' from a gun of mass 'M' as shown in Fig.3.31. Initially, the total momentum of bullet and the gun is zero because both are at rest.

Fig.3.31: Conservation of momentum in firing a gun

When the gun is fired, a controlled chemical explosion takes place within the gun. A force $\mathbf{F_{BG}}$ is exerted on the bullet by the gun through the gases caused by the exploding gun powder. But by Newton's third law, an equal but opposite force $\mathbf{F_{GB}}$ is exerted on the gun by the bullet. Since there are no external forces, the net force on the system of bullet and gun is

$$\text{Net Force} = \mathbf{F_{BG}} + \mathbf{F_{GB}}$$

According to Newton's third law of motion

$$\mathbf{F_{BG}} = -\mathbf{F_{GB}}$$

Therefore, in the absence of external forces, the net force on the system of bullet and gun is equal to zero:

$$\text{Net Force} = \mathbf{F_{BG}} - \mathbf{F_{GB}} = 0$$

Hence momentum is conserved, $p_i = p_f$

The initial momentum of system of bullet and gun is zero, $p_i = 0$. Therefore, according to the law of conservation of momentum, in the absence of an external force, when the bullet is fired its final momentum of system of bullet and gun must also be zero.

Since the bullet is moving with a velocity 'v' to the right, and therefore has momentum to the right, the gun must move to the left with the same amount of momentum in order to keep the momentum constant. Thus the total final momentum is;

Momentum of bullet + momentum of gun = 0

$$mv + MV' = 0$$

$$MV' = -mv$$

This shows that the momentum of the gun is equal to momentum of bullet but in opposite direction. Solving for the velocity V′of the gun, which is known as recoil velocity, we get

$$V' = -\,m\,v/M \quad \text{......(3.26)}$$

3.12 PROJECTILE MOTION

The projectile motion of an object is an important form of two dimensional motion.

When an object is thrown in air or space with some initial velocity at an angle θ with the horizontal direction, it moves along a curved path under the effect of gravitational force. Such an object is called projectile and its motion is called projectile motion. The path followed by the projectile is called trajectory and this trajectory is usually a parabola as shown in Fig. 3.15.

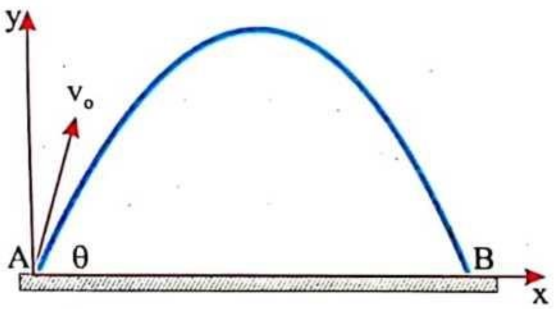


Fig.3.15: A trajectory path of projectile motion with initial velocity v_o making angle θ with the ground.

Some common examples of projectile are given as:

(i) A rocket or missile fired at a target.

(ii) A hammer or javelin thrown by an athlete.

(iii) A body thrown over the edge of a cliff or building with an initial horizontal velocity.

(iv) A long jump attempted by an athlete.

(v) A football kicked by a player.
(vi) A baseball hit by a batter for a home run.
(vii) A cricket ball hit by a batsman for six.

In order to analyze the projectile motion, we make the following three assumptions:

(a) The acceleration due to gravity, g is constant over the range of motion (horizontal motion) and its direction is downward;
(b) The effect of the air resistance is neglected (no horizontal force).
(c) The motion of Earth does not affect the motion of projectile.

Equations of projectile motion

Consider a motion of a projectile in a vertical xy-plane with initial velocity $\mathbf{v}_o$ making angle 'θ' with x-axis (horizontal direction) such that $0 < \theta < 90^{\circ}$ as shown in Fig.3.33.

The most important experimental fact shows that a projectile motion is the combination of horizontal and vertical motion. These two motions are completely independent of each other. Thus we treat its x and y co-ordinates separately. By neglecting air resistance, there is no horizontal force

A motorcyclist launches off a bike from the edge of a cliff at certain angle using principle of projectile motion.

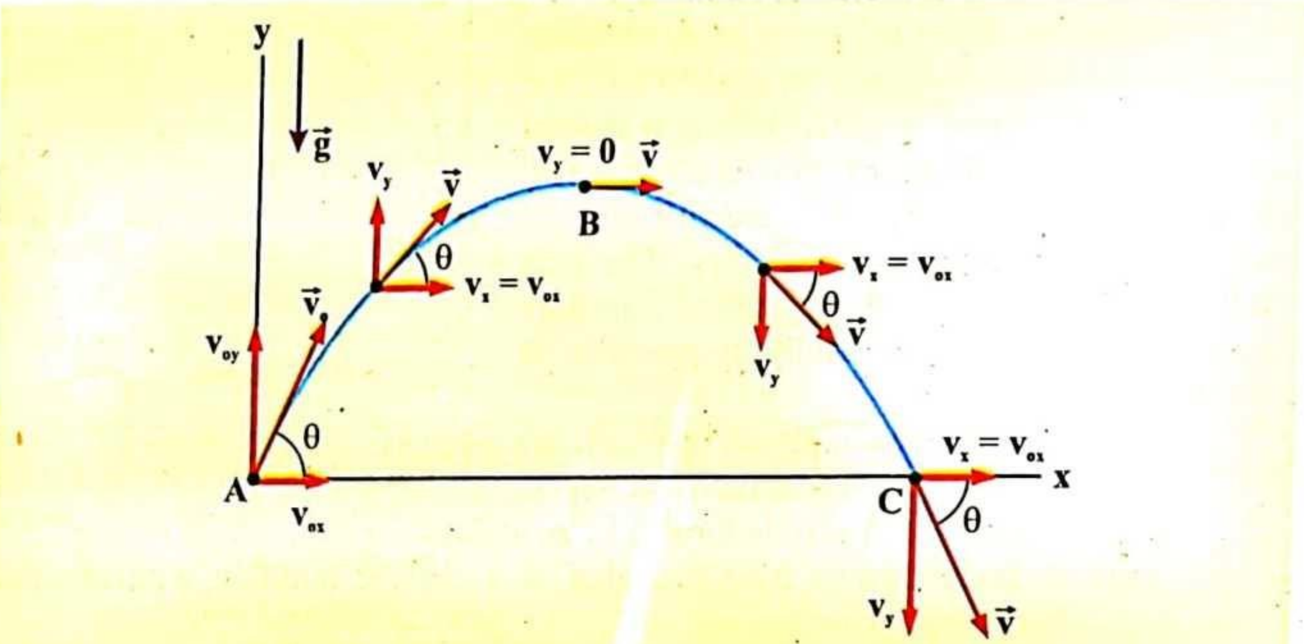


Fig.3.33: Projectile motion of a body with constant horizontal component of velocity v_x. However, its vertical component of velocity v_y varies at each point.

acting on a projectile. So its horizontal velocity v_x is constant and hence, the x-component of acceleration a_x is zero. On the other hand, the y-component of the velocity is variable, its magnitude increases in downward motion and decreases in upward motion. Thus, the y-component of acceleration $a_y = \pm g$ and its direction is downward at each point as shown in Fig.3.33. Now equations for projectile motion can be developed by using equations of motion.

(i) Distances in projectile motion

In projectile motion a body covers distances along both x-axis and y-axis which are calculated as;

Distance along horizontal direction (X-axis)

In projectile motion, the distance covered by a projectile along x-axis remains constant, so we have;

Horizontal distance = S = x =?

Horizontal component of initial velocity = $v_{ox} = v_o \cos\theta$

Horizontal component of acceleration= $a_x = 0$

Thus, by using these data in the 2nd equation of motion.

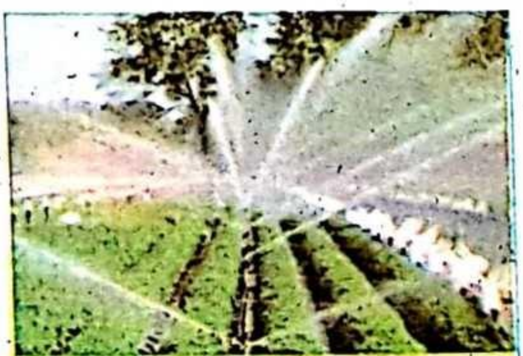
$$S = v_i t + \frac{1}{2} a t^2$$

$$x = v_{ox} t + \frac{1}{2} a_x t^2$$

$$x = v_{ox} t + \frac{1}{2}(0) t^2$$

$$x = v_{ox} t$$

$$x = (v_o \cos\theta) t \quad \text{...... (3.27)}$$



Sprinkle irrigation involves projectile motion.

Distance along vertical direction (Y-axis)

Similarly, for vertical motion of the projectile we have

Vertical distance = S = Y

Vertical component of initial velocity = $v_{oy} = v_o \sin\theta$

Vertical component of acceleration = $a_{oy} = -g$

Again the 2nd equation of motion becomes;

$$S = v_i t + \frac{1}{2} a t^2$$

$$Y = v_{oy} t + \frac{1}{2} a_y t^2$$

$$Y = (v_o \sin\theta)\, t - \frac{1}{2} g t^2 \quad ......(3.28)$$

> Practically it is observed that the shape of the trajectory is greatly affected by the air resistance in the earth's atmosphere.

Equation (3.11) and equation (3.12) represent components of displacement in projectile motion.

(ii) Velocity of projectile

As motion of a projectile is a two dimensional, its velocity also has two components i.e., horizontal (v_x) and vertical (v_y). The values of these two components can be calculated as;

Velocity along the horizontal direction

In projectile motion, the horizontal component of velocity v_x remains constant. Therefore, $a = a_x = 0$.

Thus by using the 1st equation of motion;

$$v_f = v_i + at$$

$$v_x = v_{ox} + a_x t$$

$$v_x = v_o \cos\theta + 0$$

$$v_x = v_o \cos\theta \quad ...... (3.29)$$

Velocity along vertical direction

Similarly, in projectile motion, the velocity along y-axis varies with time.

So, $a_y = -g$

Again the 1st equation of motion becomes

$$v_f = v_i + at$$

$$v_y = v_{oy} + a_y t$$

$$v_y = v_o \sin\theta + (-g)t \qquad (\because a_y = -g)$$
$$v_y = v_o \sin\theta - gt \text{ ...... } (3.30)$$

Equation (3.13) and (3.14) represent components of velocity in projectile motion.

The magnitude of resultant velocity at any instant can be calculated as;

$$v = \sqrt{v_x^2 + v_y^2}$$
$$v = \sqrt{(v_o \cos\theta)^2 + (v_o \sin\theta - gt)^2}$$
$$v = \sqrt{v_o^2 \cos^2\theta + v_o^2 \sin^2\theta + g^2t^2 - 2v_o \sin\theta gt}$$
$$v = \sqrt{v_o^2(\cos^2\theta + \sin^2\theta) + g^2t^2 - 2v_o \sin\theta gt} \qquad \because \cos^2\theta + \sin^2\theta = 1$$
$$v = \sqrt{v_o^2 + g^2t^2 - 2v_o \sin\theta gt} \text{ ......(3.31)}$$

Direction of the resultant velocity

$$\tan\phi = \frac{v_y}{v_x}$$

Projectile motion is a two dimensional motion under the acceleration due to gravity.

$$\tan\phi = \frac{v_o \sin\theta - gt}{v_o \cos\theta} \text{ ......(3.32)}$$

Note that the angle ϕ goes on changing with time.

Characteristics of Projectile Motion

In the study of a projectile motion, there are a number of interesting characteristics like time of the flight (T), maximum vertical height (H) attained by the projectile and the horizontal range (R) of the projectile. The first two characteristics (i.e., T and H) are determined from the vertical motion of the projectile while the third (i.e., R) is calculated from the horizontal motion of the projectile. All these are described below.

Time of Flight

It is the total time taken by a projectile for which it remains in air above the horizontal plane. In other words, it is the time taken by a projectile from the instant it is released till it strikes the target point of projection on the same horizontal plane (i.e., from A to C) as shown in Fig.3.34. It is denoted by T and it consists of two parts:

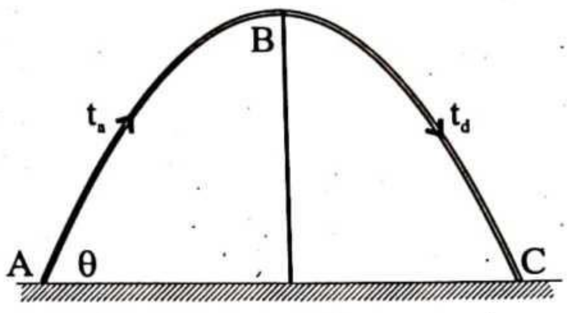


Fig.3.34: Total time of the flight

(a) The time of ascent: It is the time taken by the projectile to reach from its releasing point A to the highest point B.

(b) The time of descent: It is the time taken by the projectile to go from highest point B to the target point C on the ground at the same level.

These two times taken by projectile can be determined as;

In projectile motion at the maximum point the vertical component of velocity of the projectile becomes zero. i.e., $v_y = 0$ and $t = t_{asc}$.

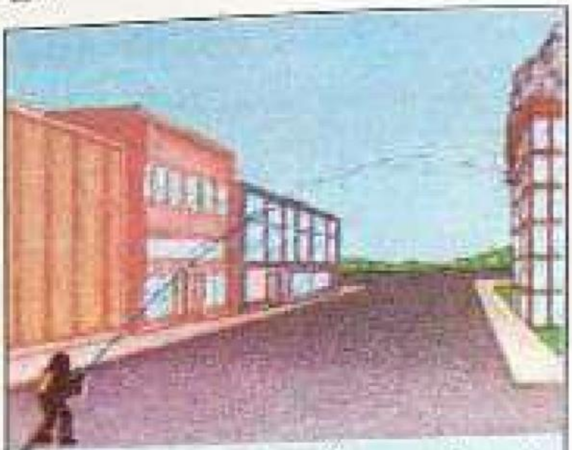
Thus, eq.3.30 becomes,

$$0 = v_o \sin\theta - gt_{asc}$$

$$t_{asc} = \frac{v_o \sin\theta}{g} \quad ......(3.33)$$

This is the time of ascending. The same time will be taken by the projectile for descending i.e.,

$$t_{desc} = \frac{v_o \sin\theta}{g} \quad ......(3.34)$$



A firefighter, at a distance from a burning building, directs a stream of water from a fire hose at angle above the horizontal.

Total time of flight

$$T = t_{asc} + t_{desc}$$

$$T = \frac{2v_o \sin\theta}{g} \quad ......(3.35)$$

Maximum Height

The vertical distance of projectile from the horizontal plane to the peak point is known as maximum height. It is represented by H.

In order to calculate the maximum height of the projectile covered in time t, we use the third equation of motion.

$$2aS = v_f^2 - v_i^2$$

Taking vertical upward motion, we have;

$$S = H,\ v_{oy} = v_o \sin\theta,\ a_y = -g \text{ and } v_f = 0$$

$$2(-g)H = 0 - (v_o \sin\theta)^2$$

$$-2gH = -v_o^2 \sin^2\theta$$

$$H = \frac{v_o^2 \sin^2\theta}{2g} \quad ......(3.36)$$

Horizontal Range

In projectile motion, the distance covered by a projectile along x-axis is known as horizontal range. It is denoted by R.

In this case, x = R and the time is equal to the total time of flight T, and thus eq. 3.11 becomes

$$R = v_o \cos\theta T$$

$$R = v_o \cos\theta \times \left(\frac{2v_o \sin\theta}{g}\right)$$

$$R = \frac{v_o^2}{g} 2\sin\theta\cos\theta \quad [\because 2\sin\theta\cos\theta = \sin 2\theta]$$

$$R = \frac{v_o^2}{g} \sin 2\theta \text{......(3.37)}$$

It is clear that the horizontal range R depends upon angle of projection, for a given speed v_o of the projectile.

Maximum horizontal range

For a given initial velocity v_o, the horizontal range of projectile will be maximum when sin2θ in equation 3.21 is equal to one. i.e.,

$$\sin 2\theta = 1$$

$$2\theta = \sin^{-1}(1)$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

This shows that in order to achieve maximum range, the projectile must be projected at an angle of 45° with horizontal direction as shown in Fig. 3.35. The expression for maximum horizontal range can be obtained by putting θ = 45° in eq. (3.37).

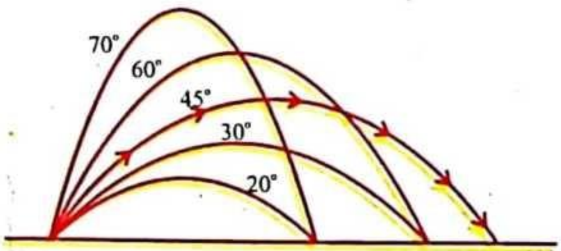


Fig.3.35: The range is maximum at θ = 45°

$$R_{max} = \frac{v_o^2}{g} \sin 2(45^\circ)$$

$$R_{max} = \frac{v_o^2}{g} \sin 90^\circ$$

$$R_{max} = \frac{v_o^2}{g} \quad \therefore \sin 90^\circ = 1 \text{ ......(3.38)}$$

Two angles of projection for same horizontal range

When a projectile is thrown at an angle θ with horizontal direction, having velocity v_o, then its horizontal range is given by;

$$R = \frac{v_o^2}{g} \sin 2\theta$$

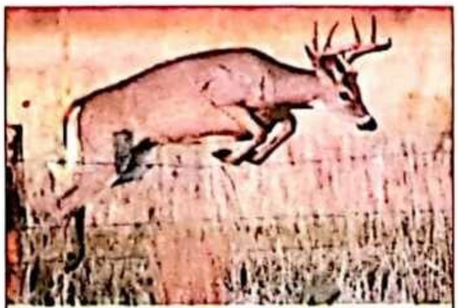
Now, let R′ be the horizontal range of the projectile for angle of projection $(90^\circ - \theta)$, having same velocity v_o, then

$$R' = \frac{v_o^2}{g}\sin 2(90^\circ - \theta)$$

$$R' = \frac{v_o^2}{g}\sin(180^\circ - 2\theta)$$

But $\sin(180^\circ - 2\theta) = \sin 2\theta$, therefore

$$R' = \frac{v_o^2}{g}\sin 2\theta$$



When a doe jumps, she follows a trajectories path.

Thus we see that the horizontal range is same for angle of projection θ and $(90^\circ - \theta)$ i.e.,

$$R = R'$$

It means that for a given velocity there are two angles of projection for which the horizontal range is same.

An angle of projection (90°- θ) with the horizontal is equivalent to an angle θ′ with the horizontal or the sum of two angles is 90° $\left(\theta' = 90^\circ - \theta \text{ or } \theta' + \theta = 90^\circ\right)$

Hence range will remain same for two angles of projection which are complementary of each other.

Effect of air resistance

Up until this point, we have ignored a very important aspect of projectile motion, i.e. air resistance. This force, however, plays a major role in the motion of objects around us. Air resistance is a force, called the drag force that acts in the direction opposite to the object motion.

This air resistance affects the path of a projectile such as a bullet or a ball. When air resistance is taken into account the trajectory of a projectile is changed. The resistance is often taken as being proportional to either the velocity of the object or the square of the velocity of the object.

Both the range of a projectile and the maximum height that it reaches are affected by air resistance. Figure.3.35 and Fig.3.36 show generally how air resistance affects both the trajectory and the velocity of a projectile.

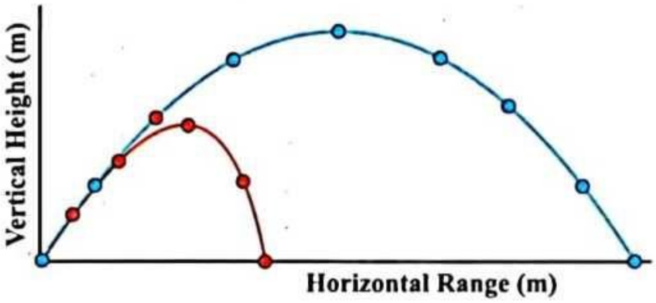


Fig.3.35: Trajectory of Projectile motion under air resistance

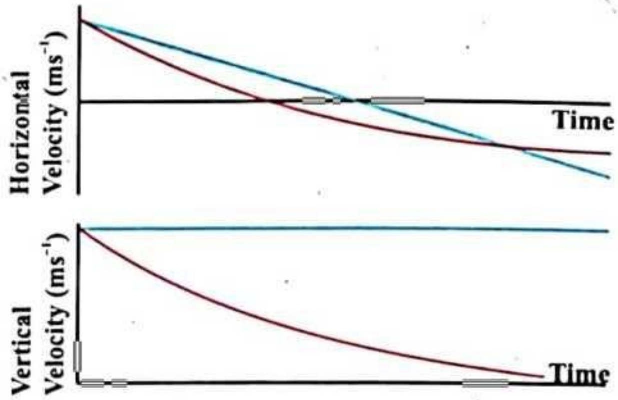


Fig.3.36: Effect of air resistance on vertical and horizontal velocity of projectile trajectory

The blue lines show the projectile with no air resistance and the red lines show what happens when air resistance is taken into account. Thus, in the presence of air resistance the maximum height, the range and the velocity of the projectile are all reduced.

Example 3.8

A body is projected upward from the horizontal plane at an angle 45° with the ground has an initial velocity of $45\ m\ s^{-1}$. (a) How long will it take to hit the ground? (b) How far from the starting point will it strike?

Solution:

Angle of projection $= \theta = 45^\circ$

Initial velocity $= v_o = 45\ ms^{-1}$

(a) Total time of flight = T =?

(b) Horizontal range = R =?

(a) $$T = \frac{2v_o \sin\theta}{g}$$

$$T = \frac{2\times 45\times \sin 45^\circ}{9.8} = \frac{90\times 0.707}{9.8}$$

$$T = 6.5\,s$$

(b) $$R = \frac{v_o^2}{g}\sin 2\theta$$

$$R = \frac{(45)^2}{9.8}\sin 2(45^\circ)$$

$$R = \frac{2025}{9.8}\sin 90^\circ = \frac{2025}{9.8}(1)$$

$$R = 206.6\,m$$

Lava spews from a volcanic eruption. Notice the parabolic paths of embers projected into the air.

Example 3.9

A ball is thrown with a speed of 20 m s^{-1} at an angle 60° above the horizontal plane. Determine (a) The time to reach the ball at maximum height. (b) Maximum height from the ground.

Solution:

Initial velocity = $v_o = 20\ ms^{-1}$

Angle of projection = $\theta = 60°$

$g = 9.8\ ms^{-2}$

(a) Time maximum of height = t =?

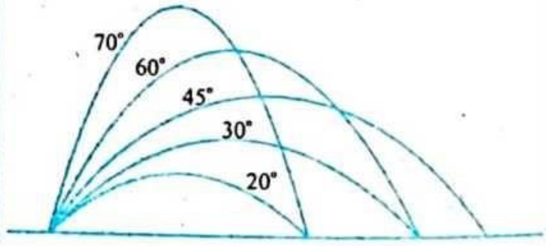
(b) Maximum height = R =?

(a)
$$T = \frac{v_o \sin\theta}{g}$$
$$T = \frac{20 \times \sin 60°}{9.8}$$
$$T = \frac{20 \times 0.866}{9.8}$$
$$T = 1.767s \simeq 1.8s$$

(b)
$$H = \frac{v_o^2 \sin\theta^2}{2g}$$
$$H = \frac{(20)^2(\sin 60°)^2}{2 \times 9.8}$$
$$H = \frac{400 \times (0.866)^2}{19.6}$$
$$H = 15.3m$$

FOR YOUR INFORMATION

Range of projectile is same at different angles when the sum of two angles is 90°.



3.13 ROCKET MOTION

A rocket is a spacecraft vehicle which is capable to carry heavy objects like missiles or satellites at certain height to launch in orbit around the Earth. It is the fastest of all the man made vehicles. The body of a rocket consists of three main sections, such as mass of its structure, mass of fuel and mass of load. It works on the basis of Newton's third law of motion and law of conservation of linear momentum, as shown in Fig.3.37. Before a rocket is fired, the total linear momentum of rocket plus fuel is

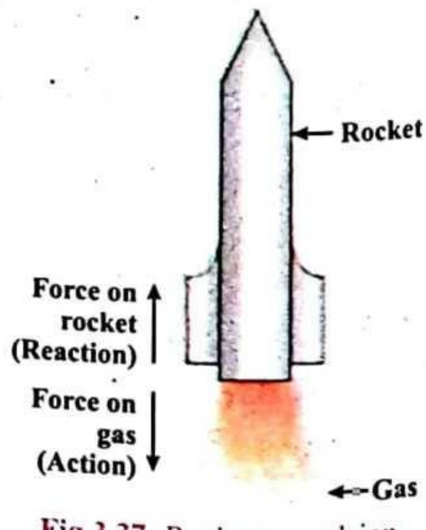


Fig.3.37: Rocket propulsion in space

zero. Since the system is essentially an isolated system, the linear momentum of the system remains the same. When rocket is fired, fuel is burnt and very hot gases are formed. These gases are expelled from the back of the rocket. Since linear momentum acquired by the gases is directed towards the rear, the rocket must acquire an equal linear momentum in the opposite direction (upward) in order to conserve linear momentum.

The exhaust of gases on burning the fuel is action and the up thrust of rocket is its reaction and this causes the acceleration of rocket in upward direction. Thus according to Newton's third law of motion;

$$\text{Up thrust} = -\text{(Force due to exhaust gases)} \quad \text{......(3.39)}$$

Let v_{ex} be the velocity of exhaust gases, and Δm be the mass of fuel which is burnt in time Δt then

$$\text{Rate of burning of fuel} = \frac{\Delta m}{\Delta t} \text{......(3.40)}$$

According to Newton's Second Law of motion

$$F = ma$$

$$F = \Delta m\left(\frac{v_f - v_i}{\Delta t}\right)$$

$$F = \Delta m\left(\frac{0 - v_{exh}}{\Delta t}\right)$$

$$F = \frac{\Delta m}{\Delta t}(-v_{exh})$$

$$F = -\frac{\Delta m}{\Delta t} v_{exh}$$

Interesting Information

Space Shuttle Main Engines (SSME's) each are rated to provide 1.6 million N of thrust. Powered by the combustion of hydrogen and oxygen, the SSME's are throttled anywhere from 65 percent to 99 percent of their rated thrust.

We notice that quantity of fuel ejected ($-\Delta m$) is equal to the loss of mass of the rocket. Negative sign shows decrease in mass.

Thus equation 3.29 becomes

$$\text{Up thrust} = -\left(-\frac{\Delta m}{\Delta t} v_{exh}\right)$$

$$\text{Up thrust} = \frac{\Delta m}{\Delta t} v_{exh} \text{ .....(3.41)}$$

But, the up thrust force = M a

Where 'M' is the total mass of rocket

$$Ma = \frac{\Delta m}{\Delta t} v_{exh}$$

Substituting $\Delta t = 1s$ in above equation we get,

$$Ma = \frac{\Delta m}{1\,sec} v_{exh}$$

$$a = \frac{\Delta m}{M} v_{exh} \quad ......(3.42)$$

This is rocket equation which shows that the acceleration of a rocket depends upon

(i) The rate of mass of burning fuel

(ii) The speed of exhaust gases

(iii) Effective mass of the rocket

It means the speed of a rocket can further be increased.

As the rocket moves under the influence of gravity so its weight can also be included.

Resultant upward force = Up thrust – Weight

SUMMARY

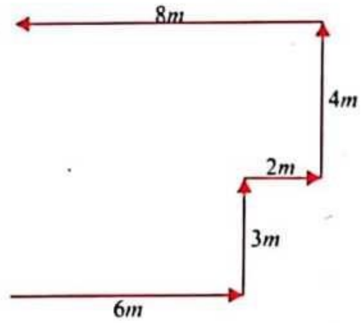
- **Motion:** When a body changes its position with respect to its surroundings then the body is in motion.
- **Displacement:** The shortest distance between two points in given direction is called displacement.
- **Velocity:** The rate of change of displacement is called velocity.
- **Acceleration:** The rate of change of velocity is called acceleration.
- **Newton's laws of motion:** In the absence of an external force, a body at rest will always be at rest and a body in motion will be continue its motion with uniform velocity. This is ***Newton's 1st law*** of motion. The acceleration produces in a body is directly proportional to the force and inversely proportional to mass, this is ***Newton's second law*** and every action has a reaction, this is ***Newton's third law.***
- **Momentum:** The product of mass and velocity is defined as linear momentum. The rate of change of momentum is equal to the applied force while change in momentum is an Impulse.
- **Impulse:** The product of force and short time is called impulse
- **Elastic and Inelastic collisions:** A collision in which energy and momentum both are conserved called elastic collision while the collision in which momentum is conserved but energy is not conserved is called inelastic collision. A collision where the bodies move along the same path and same direction before and after collision is called elastic collision in one dimension.
- **Explosion:** An explosion is an event where a single body breaks apart into a number of fragments.

- **<u>Projectile motion</u>**: Two dimensional motion of a body along a curved path under the action of gravity with initial velocity making angle with horizontal plane is called projectile motion. The projectile motion depends upon initial velocity, angle of projection and gravitational acceleration.
- **<u>Rocket:</u>** A rocket is a vehicle which has more speed than that of any other man-made vehicle. It works on the basis of action and reaction and it is being used to launch a satellite in an orbit or a missile.

EXERCISE

Multiple choice questions.

1. What is the displacement of the moving body, shown in the following figure.



(a) 6 m (b) 7 m (c) 8 m (d) 23 m

2. Acceleration due to uniform velocity of a body is:
(a) positive (b) Negative (c) Maximum (d) Zero

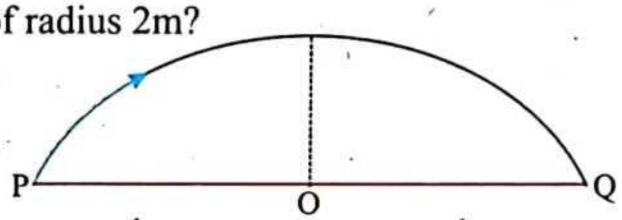
3. Third equation of motion is independent of:
(a) Time (b) Displacement (c) Velocity (d) Acceleration

4. What is the speed of the cyclist between two points S_1 and S_2 as shown in figure?

(a) $10\ km\ h^{-1}$ (b) $20\ km\ h^{-1}$ (c) $30\ km\ h^{-1}$ (d) $50\ km\ h^{-1}$

5. What will be the velocity of a body when it starts its motion from rest and after 5s its acceleration becomes $2\ ms^{-2}$
(a) $5\ m\ s^{-1}$ (b) $10\ m\ s^{-1}$ (c) $25\ m\ s^{-1}$ (d) $50\ m\ s^{-1}$

6. What is the velocity of an object when it reaches from point P to point Q in 2 s along a semicircle of radius 2m?

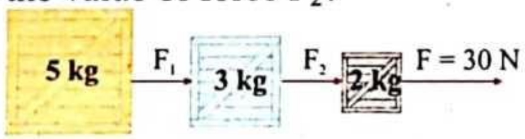


(a) Zero (b) $1\ ms^{-1}$ (c) $2\ ms^{-1}$ (d) $4\ ms^{-1}$

7. The SI unit of weight is;
(a) Gram (b) kilogram (c) Pound (d) Newton

8. What is the acceleration produced by a force of 0.5 N applied on a body of mass 0.1 kg?
(a) 0.1 ms^{-2} (b) 0.5 ms^{-2} (c) 1 ms^{-2} (d) 5 ms^{-2}

9. Three masses which are connected by a massless spring are shown in the figure. What will be the value of force F_2?

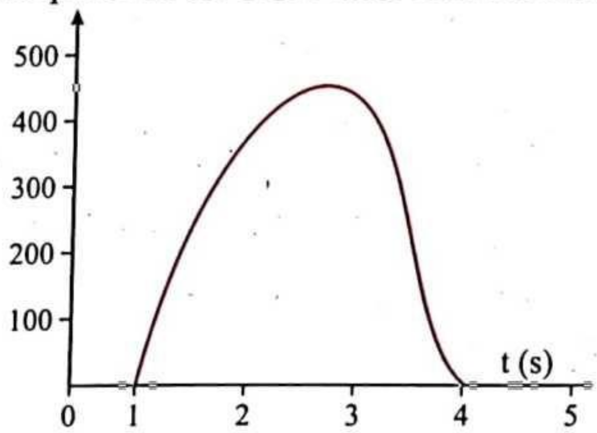


(a) 15 N (b) 18 N (c) 24 N (d) 30 N

10. Rate of change of momentum is
(a) Impulse (b) Force (c) Torque (d) Velocity

11. A body moves from point P to point Q with a speed of 6 m s^{-1} along a straight line then from Q to P with a speed of 4 m s^{-1}. What is its average speed over the entire trip?
(a) 4 m s^{-1} (b) 4.8 m s^{-1} (c) 5 m s^{-1} (d) 5.5 m s^{-1}

12. What is the value of impulse in the force-time curve as shown in figure.



(a) 1200 N s (b) 1350 N s (c) 1500 N s (d) 1650 N s

13. Impulse has always changed;
(a) Energy (b) Momentum (c) Velocity (d) Acceleration

14. In one dimensional elastic collision of two bodies of same masses, what will happen if the moving body collides with the mass which is initially at rest?
(a) Their velocities will be interchanged
(b) Velocities of both bodies will be zero
(c) Moving body will continue its motion
(d) Moving body will come at rest and the mass at rest will start its motion

15. In projectile motion, the horizontal component of acceleration of a body is;

(a) Zero (b) Accelerated (c) Decelerated (d) Maximum

16. At what angle a projectile gains its maximum height.
(a) 0° (b) 45° (c) 60° (d) 90°

17. A rocket works on basis of
(a) Newton's 1st law (b) Newton's 2nd law
(c) Newton's 3rd law (d) Newton's gravitational law

18. The vertical and horizontal distances of the projectile will be equal if angle of projection is:
(a) 45° (b) 56° (c) 66° (d) 76°

19. The quantitative measure of inertia of a body is its:
(a) Mass (b) Weight (c) Velocity (d) Momentum

SHORT QUESTIONS

1. When the magnitude of distance and displacement are equal?
2. Is it necessary, when the acceleration of a body is zero then its velocity is also zero?
3. Under what condition the velocity of a body is zero but its acceleration has some value?
4. Distinguish between mass and weight.
5. Differentiate between inertial mass and gravitational mass.
6. What is the main difference between equations of motion and laws of motion?
7. How can you define Newton's first law of motion in terms of inertia?
8. Why Newton's 2nd law of motion cannot be applied to elementary particles?
9. Why action and reaction are not acting on the same body?
10. State the law of conversation of momentum.
11. What is the meaning of a straight horizontal line in velocity-time graph?
12. Will a body be at rest, when the net force on the body is zero?
13. How is elastic collision in one dimension of two bodies possible?
14. What will happen, if a moving body with large mass collides with very small body at rest?
15. How elastic collision is different from inelastic one?
16. You kick a stone in $\frac{1}{100}$ seconds and $\frac{1}{10}$ seconds time intervals. In which condition you hurt most?
17. At what points the velocity of a body is minimum and maximum on the trajectory of projectile motion?
18. How can a body achieve its maximum range in a projectile motion?

19. At what angle of projection the horizontal range and maximum height are equal?
20. State the values of angle of projection for which the horizontal range of the two trajectory paths is same.
21. How can the speed of a rocket be increased?
22. Explain the circumstances in which the velocity v and acceleration a of a car are (a) Parallel (b) v is zero but a is not (c) a is zero but v is not.

COMPREHENSIVE QUESTIONS

1. Describe the following terms;
   a) Distance and displacement,
   b) speed and velocity
   c) linear acceleration and gravitational acceleration.
2. State and explain the graphical representation of all kinds of velocity and acceleration.
3. State and explain Newton's three laws of motion with examples.
4. What is linear momentum? Describe law of conservation of linear momentum. Also define Newton's second law of motion in terms of rate of change of momentum.
5. What do you know about impulse? Explain impulse in terms of change in momentum.
6. What is elastic collision in one dimension? Calculate the velocities of the bodies after their collision and discuss these final velocities under different cases.
7. What is projectile and projectile motion? Discuss displacement, velocity and acceleration of the projectile along its trajectory path.
8. Explain the various characteristics of projectile motion such as time of flight, maximum height and horizontal range.
9. Define rocket motion and derive an equation for the speed of a rocket.

NUMERICAL PROBLEMS

1. A train moves with a uniform velocity of 24 m s^{-1}. The driver applies the brakes and the train comes to rest with a uniform retardation in 12 s. Find (i) the retardation, (ii) velocity of the train after 4 s and (iii) distance covered by train after the brakes are applied. **(i) –2 m s^{-2} (ii) 16 m s^{-1} (iii) 144 m**

2. A coin is dropped from a tower. If the coin reaches the ground in 5 s then determine (a) height of the tower and (b) Find the speed with which coin hit the ground. **(a) 123 m (b) 49 m s^{-1}**

3. An electron emitted from a source is subjected to a force of 10^{-23}N and the electron is accelerated toward the target. Find (a) the acceleration of electron (b) How long does the electron takes to reach from source to target at 10 cm away. (Take mass of electron as 9.1×10^{-31}kg)
(a) 2.00×10^7m s^{-2} (b) 1.0×10^{-4}s

4. What is the magnitude of the applied force on a body of mass 2 kg which changes its velocity from 2 m s^{-1} to 6 m s^{-1} in 20 s? **(0.4 N)**

5. A force of 12 N acts on a body of mass 6 g for 2 μs. Calculate the impulse and change in velocity of the body. **(2.4×10^{-5} N s, 4×10^{-3} m s^{-1})**

6. What is the recoil velocity of 6 kg gun if its shoots a 9 g bullet with muzzle velocity of 350 m s^{-1}? **(0.6 ms^{-1})**

7. A 4000 kg truck is moving at a speed of 20 m s^{-1} along a straight road, strikes a 800 kg stationary car and couples to it. What will be their combined speed after impact? **(16.7 ms^{-1})**

8. Two spheres of masses 'm' and '2m' both are moving to the right with velocities 4 m s^{-1} and 2 m s^{-1} respectively. If both collides then what will be their final velocities. **(1.33 m s^{-1} to the right, 4 m s^{-1} to the right)**

9. A cricket ball is hitted upward at an angle 45° with velocity of 20 m s^{-1}, find (a) The maximum height (b) time of flight (c) How far away it hits the ground.
(a) 10.2 m (b) 2.9 s (c) 41 m

10. In certain projectile motion, the horizontal range is thrice in the magnitude of the maximum height. Calculate the angle of projection. **(53°)**

11. Prove that the horizontal ranges in projectile motion are same at $(45 + \theta)$ and $(45 - \theta)$.

12. A rocket is fired in space, which has initial mass 8000 kg and ejects gas at the rate of 2500 m s^{-1}. How much gas must it eject in the first second to have acceleration 30 m s^{-2}. **(96 kg)**

13. A projectile rocket emits gases at the rate of 300 m s^{-1}. If it burns 150 kg fuel in each second, then what is the thrust of the rocket. **(4.5×10^5 N)**

Unit 4

WORK AND ENERGY

Major Concepts (17 PERIODS)

- Work done by a constant force
- Work as scalar product of force and displacement
- Work against gravity
- Work done by variable force
- Gravitational Potential at a point
- Escape velocity
- Power as scalar product of force and velocity
- Work energy principle in resistive medium
- Sources and uses of energy
  (i) Conventional sources of energy
  (ii) Non-conventional sources of energy

Conceptual Linkage

This chapter is built on
Work and energy
Physics IX
Gravitation Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- describe the concept of work in terms of the product of force F and displacement d in the direction of force (Work as scalar product of F and d).
- distinguish between positive, negative and zero work with suitable examples.
- describe that work can be calculated from the area under the force-displacement graph.
- explain gravitational field as an example of field of force and define gravitational field strength as force per unit mass at a given point.
- prove that gravitational field is a conservative field.
- compute and show that the work done by gravity as a mass 'm' is moved from one given point to another does not depend on the path followed.
- describe that the gravitational PE is measured from a reference level and can be positive or negative, to denote the orientation from the reference level.
- define potential at a point as work done in bringing unit mass from infinity to that point.
- explain the concept of escape velocity in term of gravitational constant G, mass m and radius of planet r.
- differentiate conservative and non-conservative forces giving examples of each.
- express power as scalar product of force and velocity.
- explain that work done against friction is dissipated as heat in the environment.

- state the implications of energy losses in practical devices and the concept of efficiency.
- utilize work – energy theorem in a resistive medium to solve problems.
- discuss and make a list of limitations of some conventional sources of energy.
- describe the potentials of some nonconventional sources of energy.

INTRODUCTION

Work and energy are not two different things, but they are correlated to each other. When work is done by one system on another, indeed energy is transferred between the two systems. For example, when an engine pulls a train along a horizontal track, the engine does work. The engine transfers energy to the train. If we assume that there is no loss of energy, then the amount of energy of the moving train is equal to the work done. Similarly, when we walk upstairs, we do work; our work is equal to gain in gravitational Potential Energy (P.E).

Energy is present in the universe in various forms. It can be converted from one form into other form but neither be created nor be destroyed. This is the principle of conservation of energy. For example, a heat engine converts heat energy into mechanical energy. A fan converts electrical energy into mechanical energy, a bulb converts electrical energy into light and heat energy etc. All forms of energy can be explained in terms of kinetic energy or potential energy. These two are the most important types of energies. The Kinetic Energy (K.E) is due to motion while P.E is a stored energy.

Here a question arises, how much work is done or how much energy is consumed? It is measured in terms of rate of doing work or rate of consumption of energy which is called power. For example, a boy may carry a box upstairs in 3 minutes while a man may do it in 1 minute. Obviously, the power of the man is more than the power of the boy. Thus, time factor is important for power. A body which has the capacity to do work is said to possess energy. The greater the capacity of the body to do work, the greater is the energy possessed by it. Thus work, energy and power are related to each other. In this unit, we shall deal with these three most important parameters of physics.

4.1 WORK

In our daily life, we use the meaning of work in terms of any physical or mental activity. For example, reading a book, cooking, shopping etc. all are regarded as work but in physics, the word work has a different meaning. The work is only done when the force acting on an object produces a displacement in it in the direction of force. Thus, for work to be done by a force on an object, the two aspects must be considered:

(i) Displacement of the object.
(ii) Component of force in the direction of the displacement.

4.1.1 Work done by a constant force

The work done on an object by a constant force is defined as the product of the magnitude of displacement and the component of force in the direction of the displacement. It is explained as;

Consider an object which is pulled by an applied constant force **F** at an angle θ with a horizontal axis and the body is displaced through a displacement **d** as shown in Fig. 4.1.

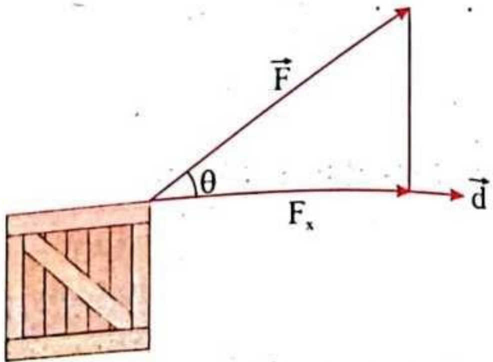


Fig.4.1: The horizontal component of force (F_x) in the direction of displacement (d) does work on the body.

The horizontal component of force **F** in the direction of displacement is F cosθ.

Thus, according to the definition, work is done on the body. This work done by a constant force is expressed as:

Work = (Horizontal component of force) (Displacement)

Work = F_x d

Work = F cosθ d

Work = Fd cosθ ......(4.1)

Work = $\vec{F}\cdot\vec{d}$ ......(4.2)

This is a work done in terms of the scalar product of force and displacement. Therefore, work is a scalar quantity. It has magnitude but no direction. The SI unit of work is joule and its dimensional formula is $[ML^2T^{-2}]$.

Joule

One joule is defined as the amount of work done when a force of one newton displaces a body through one metre in its direction. Hence

1 Joule = 1 newton × 1 metre

1J = 1Nm

When the applied force remains constant during the whole path then graphically, there is a straight horizontal line in force and displacement graph as shown in Fig.4.2. The area under this straight horizontal line is equal to the work done on the body under a constant force.

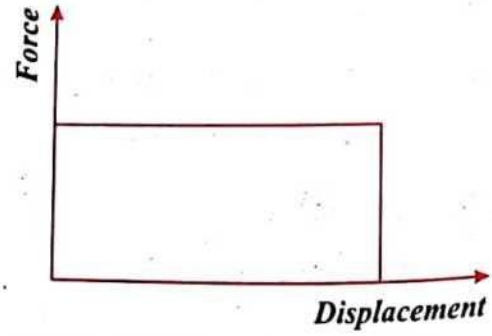


Fig.4.2: The area under a straight Horizontal line in Force-displacement graph showing work done by a constant force

Equation (4.1) shows that work done on the body depends upon force, displacement and angle 'θ' between force and displacement. The term cosθ in this equation indicates that the work done may be positive, negative or zero depending upon the value of θ. Therefore, the work done can be studied under the following three cases;

I. Positive Work (Maximum)

If $\theta < 90^\circ$, $\cos\theta$ is positive so work done is also positive. Under this condition there is a component of force in the direction of the displacement. If the and angle between force and displacement is zero i.e. $\theta = 0^\circ$ then;

$$\text{Work} = Fd\cos 0^\circ = Fd$$

This result shows that work done is maximum when the applied force is parallel to the displacement.

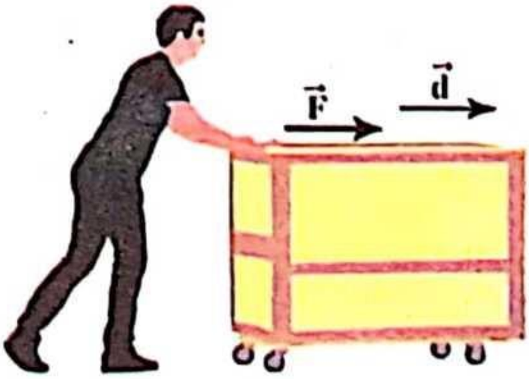


Fig.4.3: Positive work done by a man on cart

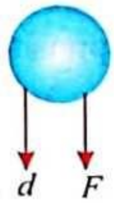


Fig.4.4: Positive work on a rolling ball

Examples

(a) When a man pushes a cart on horizontal smooth surface, as shown in Fig. 4.3, the force and displacement are in the same direction.

(b) When a body falls freely under gravity, then gravitational force and displacement are in same direction as shown in Fig.4.4.

Fig.4.5: Zero work done by a man on wall

II. Zero Work

If $\theta = 90^\circ$, then $\cos\theta = 0$ and no work is done by the force on the body. Note that work done is also zero when either force or displacement
or both are zero.

$$\text{Work} = Fd\cos 90^\circ = Fd(0) = 0 \because \cos 90^\circ = 0$$

Examples

(a) When a man pushing a rigid wall with a force and fails to move it, then work done is zero as shown in Fig. 4.5.

(b) When a man holding a pail in his hand while moving on a horizontal level surface as shown in Fig. 4.6 then angle between force and displacement is 90°. Work done by the man is zero.

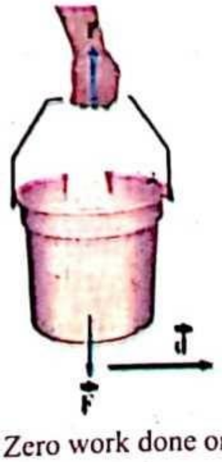


Fig.4.6: Zero work done on a pail.

III. Negative Work (-ve Maximum)

If $\theta > 90^\circ$, then $\cos\theta$ is negative so work done is also negative. If the component of force is opposite to the direction of the displacement then the angle between force and displacement is 180°.

$$\text{Work} = Fd\cos 180^\circ = Fd(-1) = -Fd$$

This result shows that when the applied force is anti parallel to the displacement and angle 'θ' between them is 180° then the work done will be negative maximum.

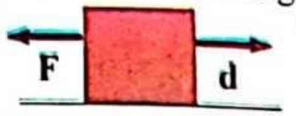


Fig.4.7: Negative work done by frictional force

Examples

(a) When a body makes to slide over the a rough horizontal surface as shown in Fig. 4.7, the frictional force is opposite to the displacement and hence work obtained is –ve (displacement).

(b) When a body is thrown up, the gravitational force is vertically downward while the displacement is vertically upward. The work done by the body is negative and against the gravitational field.

(c) Two masses m_1 and m_2 connected by a string which is suspended from a frictionless pulley, as shown in Fig. 4.8. Gravitational force and displacement are opposite to each other. So the work done in this case will be negative if m_1 is lifted in the upward direction.

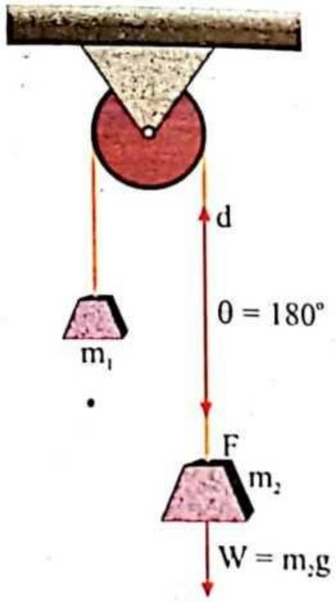


Fig.4.8: Negative work done by gravitational force

4.1.2 Work done by a variable force

When the applied force on a body remains constant in terms of magnitude and direction then its work done can be calculated by the following equation;

$$\text{Work} = Fd\cos\theta \text{ ......(4.3)}$$

So far we have considered the work done by constant forces. But sometimes the applied forces in terms of magnitude or direction, are not constant. For example, when a rocket moves away from the earth, the work is done against the gravitational force which varies as the square of the distance from the centre of the Earth, thus in case of variable force, the work done of an object cannot be determined by using eq.4.3 directly. It requires another relation which is developed as under;

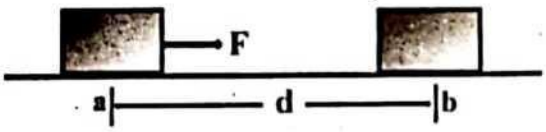


Fig.4.9: A body is displaced by a variable force from point a to point b.

Consider an object being displaced along the x-axis in a straight line under the action of a variable force as shown in Fig. 4:9. To find the total work done, we divide the total displacement (whole path) along x-axis into 'n' number of very short segments; $\Delta d_1, \Delta d_2, \Delta d_3, ..., \Delta d_n$ such that for each segment (displacement) the forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, ..., \vec{F}_n$ respectively may be treated as constant as shown in Fig.4.10.

Thus when the object moves through the small distance $\Delta \vec{d}_1$ under the action of approximately constant force $\vec{F}_1$, the small amount of work done W_1 is given by;

$$W_1 = F_1 \cos\theta_1 \Delta d_1$$

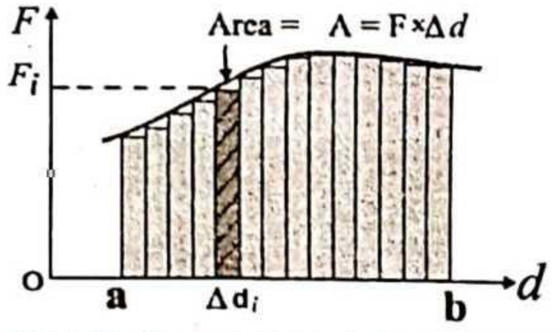


Fig.4.10: The path which is divided into 'n' number of small and equal displacements

Similarly, when the object moves through displacement $\Delta \vec{d}_2$ under the action of force F_2 then the work done W_2 is given as;

$$W_2 = F_2 \cos\theta_2 \Delta d_2$$

Since the motion of the object from point 'a' to point 'b' is divided into 'n' number of small and equal segments therefore the total work done on the body is given as,

$$W_{Total} = F_1 \cos\theta_1 \Delta d_1 + F_2 \cos\theta_2 \Delta d_2 + F_3 \cos\theta_3 \Delta d_3 + ... + F_n \cos\theta_n \Delta d_n \text{ .... (4.4)}$$

$$W_{Total} = \sum_{i=1}^{n} F_i \Delta d_i \cos\theta_i \text{ ......(4.5)}$$

In the $\lim_{\Delta d \to 0}$ then, we have,

$$Work_{Total} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \, \Delta d_i \cos\theta_i \text{ ......(4.6)}$$

This is a resultant work done by a variable force and it shows that the total work done by a variable force is equal to the sum of the areas of all the segments (rectangles) from point 'a' to point 'b'. Graphically, when it is plotted on a variable force-displacement graph then we have a curved path as shown in Fig.4.11 and the area under this curved path is equal to the work done by a variable force on a body.

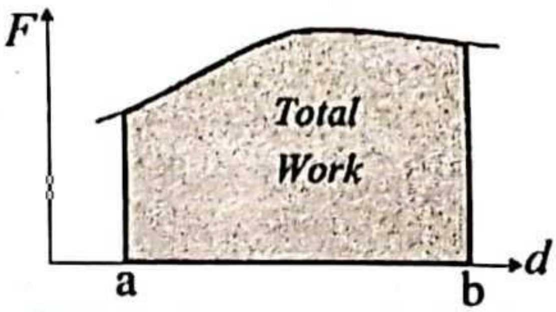


Fig.4.11: Area under curved path in the F-d graph is equal to the total work done by variable force.

Example 4.1

A man pushes a lawn roller through a distance of 40 m under the action of force of 50 N which makes an angle of 60° with the direction of motion. Calculate the work done.

Solution:

$F = 50\ N$
$d = 40\ m$
$\theta = 60^\circ$
Work = ?
Work = $Fd\cos\theta$
Work = $(50)(40)\cos 60^\circ$
Work = $2000\ (0.5)$
Work = $1000\ J$

The work is done 100% when the applied force is acting at angle '0' in the direction of displacement, what would be the angle of applied force when work is done 50%.

4.2 WORK DONE IN A GRAVITATIONAL FIELD

The space around the Earth in which the Earth can attract a body toward its centre is known as a gravitational field. Similarly, the gravitational force per unit mass on a body is called gravitational field strength and its SI unit is $N\ kg^{-1}$.

Consider a force F which is applied on a body of mass 'm' and the body is raised from the surface of the Earth with uniform velocity in a gravitational field through a height 'h' as shown in Fig. 4.12. It means that work is done on the body against the force of gravity and it is given as;

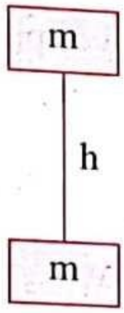


Fig.4.12: Work done on a body in a gravitational field

$$\text{Work} = F\,d\cos\theta$$

As the body moves under the force of gravity;
so $F = W = mg$ and $d = h$, and angle θ between weight (force) and displacement is 180°.

Thus, $\text{Work} = W\,h\cos 180^\circ$

$\text{Work} = mg\,h\,(-1) = -mgh$

The gravitational field is a conservative and it has the following properties;

(i) The work done in a field does not depend upon its path but it depends upon its initial and find points.

(ii) The work done in a field along a closed path is zero.

(iii) The work done on a body against the direction of gravitational field is stored in terms of its P.E.

4.2.1 Work done in a gravitational field is independent of path

Consider an object of mass 'm' which can be displaced in a gravitational field with a constant velocity from point 'A' to 'C' along the following two different paths. The first path is the direct path from 'A' to 'C' and the other path is from 'A' to 'B' and then 'B' to 'C' as shown in Fig. 4.13.

Let us consider the first path which is direct path from 'A' to 'C', the work done along this path is calculated as,

$$\underset{A\to C}{\text{Work}} = Fd\cos\theta$$

$$\underset{A\to C}{\text{Work}} = Wd\cos\theta \quad \because F = W$$

Using triangle ABC

$$\frac{d_1}{d} = \cos\theta$$

$$d_1 = d\cos\theta$$

$$\therefore \underset{A\to C}{\text{Work}} = W\,d_1 \text{......(4.7)}$$

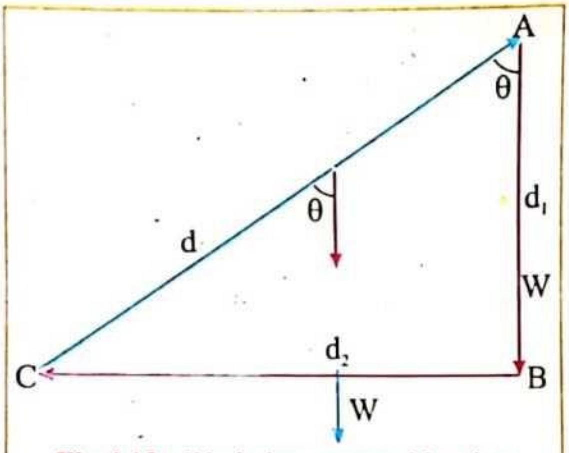


Fig.4.13: Work done on an object in a gravitational field along two different paths

Let us now consider the second path i.e. path AB and path BC. Thus, the work done along AB is give as;

$$\underset{A\to B}{\text{Work}} = Fd_1\cos\theta$$

$$\underset{A\to B}{\text{Work}} = Wd_1\cos\theta \quad \because F = W$$

As the angle 'θ' between W and d_1 is 0°

So, $$\underset{A\to B}{\text{Work}} = Wd_1\cos 0 \quad (\text{because } \cos\theta = 1)$$

$$\underset{A\to B}{\text{Work}} = Wd_1 \text{......(4.8)}$$

Similarly, the work done from B to C is,

$$\underset{B\to C}{\text{Work}} = Fd_2\cos\theta$$

$$\underset{B\to C}{\text{Work}} = Wd_2\cos\theta$$

As the angel 'θ' between W and d_2 is 90°

So, $$\underset{B\to C}{\text{Work}} = Wd_2\cos 90^\circ \quad \because \cos 90^\circ = 0$$

$$\underset{B\to C}{\text{Work}} = Wd_2\,(0)$$

$$\underset{B\to C}{\text{Work}} = 0 \text{......(4.9)}$$

From eq. (4.8) and eq. (4.9), we get

$$\underset{A\to B\to C}{\text{Work}} = Wd_1 + 0$$

$$\underset{A\to B\to C}{\text{Work}} = Wd_1 \text{......(4.10)}$$

> We can calculate the work done by a force on an object, but that force is not necessarily the cause of the displacement. For example, if you lift a body, work is done on the object by the gravitational force, although gravity is not the cause of the object moving upward.

> The change in the gravitational potential energy of an object does not depend on the path it takes.

By comparing eq.4.7 and eq.4.10, we conclude that work done in a gravitational field is independent of path followed.

4.2.2 Work done in a gravitational field along a closed path is zero

Let us find the work done in a gravitational field along a closed path ABCA. We have already calculated work along the path AC, AB and BC in previous section and these are as under.

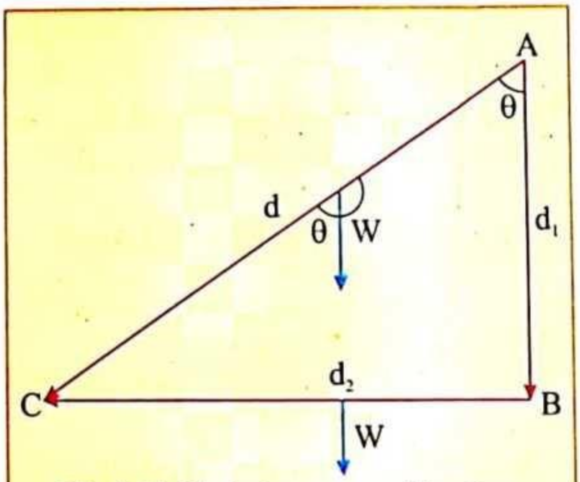


Fig.4.14: Work done on an object in a gravitational field along a closed path

$$\underset{A\to B}{\text{Work}} = Wd_1$$

$$\underset{B\to C}{\text{Work}} = 0$$

$$\underset{A\to C}{\text{Work}} = Wd_1$$

$$\underset{C\to A}{\text{Work}} = -Wd_1$$

It is noted that work from A to C and work from C to A are same in magnitude but in opposite direction.

Total work done in a closed path ABCA = $\underset{A\to B}{\text{Work}} + \underset{B\to C}{\text{Work}} + \underset{C\to A}{\text{Work}}$

Work done in a closed path ABCA = $Wd_1 + 0 + (-Wd_1)$

Work done in a closed path ABCA = 0 ......(4.11)

Equation (4.11) shows that work done along a closed path in a gravitational field is zero.

From the above discussion, it is concluded that the work done on the body in a gravitational field is independent of path followed and the work done on the body in a gravitational field along a closed path is zero. Thus, the gravitational field is a conservative field.

4.3 ENERGY

The word energy is derived from the Greek word "Energeia" which means work. Hence energy is defined as the ability (or capacity) of a body to do work. This implies that energy is associated with the performance of work because the more work is done; the greater the quantity of energy is needed. Work is always done by a force. It means that a body possessing energy can exert force on any other body to do work. In other words, when a work is done on a body, an equal amount of energy is stored in it.

Energy is a scalar quantity. The SI unit of energy is same as that of the work, i.e., joule (J) and its dimensional formula is $[ML^2T^{-2}]$.

There are two basic form of energies.

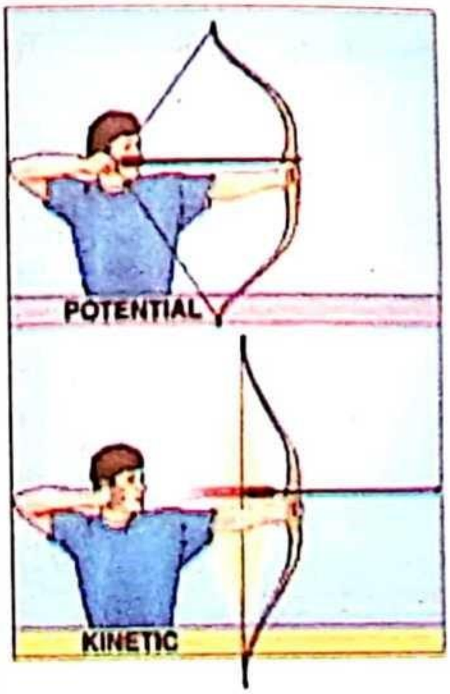
(i) Kinetic Energy (ii) Potential Energy

4.3.1 Kinetic Energy

The energy possesses by a body due to its motion is called kinetic energy. For example, a moving ball can break a window glass and a hammer can drive a nail into wood. These examples show that the K.E of a body depends upon its motion. The faster a body moves, the greater is it's K.E. The mathematical expression of K.E of a body of mass 'm' moving with velocity 'v' is given as;

$$K.E = \frac{1}{2}mv^2 \quad ......(4.12)$$

4.3.2 Potential Energy



The potential energy of a body is defined as the energy possesses by the body by virtue of its position or configuration.

A body can gain potential energy only when work is done on it. For example,

I. When a body is lifted to some height against the gravitational force, then there is increase its gravitational P.E.

II. The water at the top of a water fall, or water stored in a dam possess gravitational P.E

III. If a spring is compressed, an elastic P.E is stored in it because a work is done in compressing the spring against the elastic force.

The mathematical relation for gravitational P.E. can be expressed as;

Consider a force is applied vertically upward on a body of mass 'm'. The body is raised up to a certain height 'h' above the Earth's surface as shown in Fig.4.15. It means there is a work done on the body against the direction of gravitational field and this work is stored in the body in terms of its potential energy. The value of such P.E is calculated as; by definition of work done.

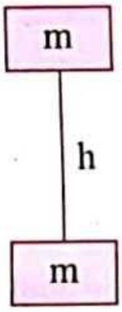


Fig.4.15: P.E due to the work done on the body at height h against the gravitational field.

$$Work = F.d$$

As the body is under gravity, so $F = W = mg$ and $d = h$.

$$Work = mg\,h$$

This work is stored in the body in the form of gravitational potential energy.

Thus, $\quad P.E = mgh \quad ......(4.13)$

The eq. (4.13) shows that the P.E. of a body depends upon height. That is, higher a body is above the Earth, the greater will be its P.E. If the height of object from the surface of Earth is zero then its P.E is also zero. In this expression, the surface of Earth is considered as reference point for zero potential energy.

Example 4.2

A neutron of mass 1.67×10^{-27} kg travels a distance of 12 m in 3.6×10^{-4} s. If its speed remains constant, then what is its K.E?

FOR YOUR INFORMATION

The matter in 0.453 kg of anything, when it is completely converted into energy according to, $E = mc^2$, will produce 11400 million kilowatt-hours of energy.

1 kWh = 3.6×10^6 joules

Solution: Mass = m = 1.67×10^{-27} kg

Distance = S = 12 m

Time = t = 3.6×10^{-4} sec

Kinetic Energy = K.E = ?

$$v = \frac{S}{t}$$

$$v = \frac{12}{3.6\text{x}10^{-4}} = 3.33\times10^4 \text{ m s}^{-1}$$

$$K.E = \frac{1}{2}mv^2$$

$$K.E = \frac{1}{2}\left(1.67\times10^{-27}\right)\left(3.33\times10^4\right)^2$$

$$K.E = 9.26\times10^{-19} \text{ J}$$

4.4 WORK - ENERGY THEOREM

This theorem is stated as; **"The work done by an applied force on a body is equal to the change in its energy either K.E or P.E".** It is explained as;

When a force is applied on a body in the direction of motion of the body, the speed and hence kinetic energy of the body increases. According to work-energy theorem, the increase in kinetic energy of the body is equal to the work done by the force on the body.

Similarly, if a force is applied on a body in the direction opposite to its motion, kinetic energy of the body decreases. This decrease in kinetic energy is equal to the work done by the body against the retarding force. In either case, the change in kinetic energy of the body is equal to the work done (positive or negative work done). A mathematical relation for work-energy theorem is derived as under.

Consider a force 'F' which is applied on a body of mass 'm' moving with initial velocity 'v_i' and after some time it covers a displacement 'd' and its final velocity becomes 'v_f' as shown in Fig. 4.16. Now work done on the body is given as;

$$\text{Work} = W = Fd \text{ ......(4.14)}$$

According to Newton's second law of motion.

$$F = ma \text{ ......(4.15)}$$

According to 3[rd] equation of motion.

$$2ad = v_f^2 - v_i^2$$

or

$$d = \frac{v_f^2 - v_i^2}{2a} \text{ ......(4.16)}$$

Substituting eq. (4.15) and eq. (4.16) in eq. (4.14), we have

$$W = ma \times \left(\frac{v_f^2 - v_i^2}{2a}\right)$$

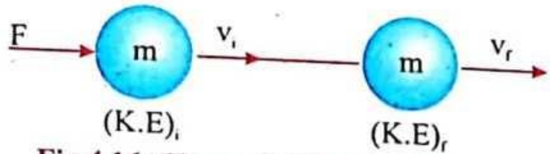


Fig.4.16: Change in K.E of a body due to Work done

$$W = \frac{mv_f^2 - mv_i^2}{2}$$

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \text{ ......(4.17)}$$

$$W = K.E_f - K.E_i$$

$$W = \Delta K.E$$

This is a mathematical form of work-energy theorem. Here work is expressed in terms of change in K.E. Similarly, the same principle can be used for P.E i.e.,

$$\text{Work} = (P.E.)_f - (P.E)_i$$

$$\text{Work} = \Delta P.E.$$

Example 4.3

A force of 1500 *N* is acting horizontally on a vehicle of mass 200 *kg* and the vehicle starts its motion from rest. What will be the speed of the vehicle after covering a distance of 30 m?

Solution: $F = 1500\text{ N}$

$m = 200\text{ kg}$

$\theta = 0^\circ$

$v_i = 0\text{ ms}^{-1}$

$v_f = ?$

$d = 30\text{ m}$

$$W = K.E_f - K.E_i$$

$$Fd\cos\theta = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$Fd\cos\theta = \frac{1}{2}mv_f^2 - \frac{1}{2}m(0)^2$$

$$F d\cos\theta = \frac{1}{2}mv_f^2$$

$$v_f^2 = \frac{2Fd\cos\theta}{m}$$

$$v_f^2 = \frac{2(1500)(30)\cos 0^\circ}{200} = \frac{90000}{200}$$

$$v_f^2 = 450\,m^2s^{-2}$$

$$v_f = 21.2\,ms^{-1}$$

4.5 GRAVITATIONAL POTENTIAL ENERGY

When a body is raised to certain height 'h' from the surface of Earth in a gravitational field, then work is said to be done on the body against the gravitational field. This work is stored in the body in the form of its gravitational potential energy. Its value is given as,

$$\text{Gravitational P.E} = mgh \quad ......(4.18)$$

This shows that the gravitational P.E depends upon height. That is, when the body gains height, its P.E increases while the value of 'g' decreases. Now at very large height where the value of 'g' becomes zero and the P.E due to the work done on a body from the surface of Earth to above stated point is called absolute gravitational potential energy whose value can be calculated as;

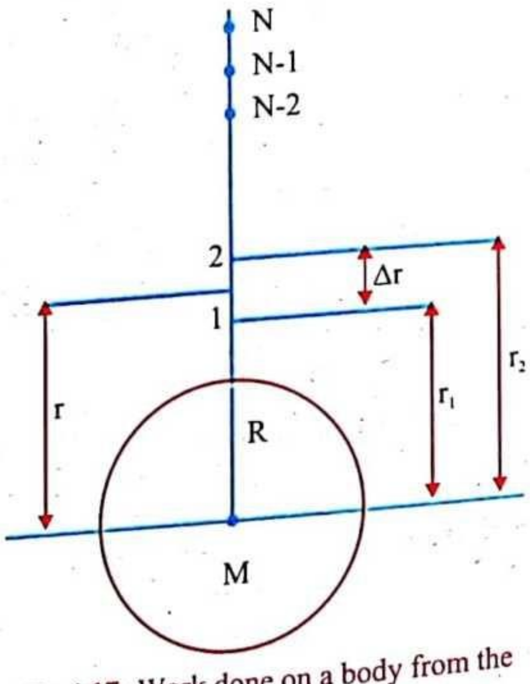


Fig.4.17: Work done on a body from the surface of earth to the point 'N'.

Consider a body of mass 'm' which is raised above the surface of Earth at a distance 'r' from the centre of Earth. Then the gravitational force between the body and the Earth is given by;

$$F = G\frac{mM}{r^2} \quad ......(4.19)$$

where 'G' is a gravitational constant whose value is $6.67\times10^{-11}\,Nm^2kg^{-2}$ and 'M' is mass of the Earth. Equation (4.19) shows that the gravitational force is inversely proportional to the square of the distance. Therefore, this relation cannot be used directly to calculate the total work on the body. For this we divide the whole path into 'N' number of points (1,2,3 ...N) at distances (r_1, r_2, r_3, r_4,..., r_N) respectively from the centre of the Earth such that the force in each step almost remains constant because the distance between every two consecutive points is 'Δr' and it remains the same.

First we consider the points 1 and 2 at distances r_1 and r_2 respectively from the center of the earth, to get an average force, we consider the midpoint between 1 and 2 at a distance 'r' from centre of earth such that;

$$r = \frac{r_1 + r_2}{2}$$

As $$\Delta r = r_2 - r_1$$

and $$r_2 = r_1 + \Delta r$$

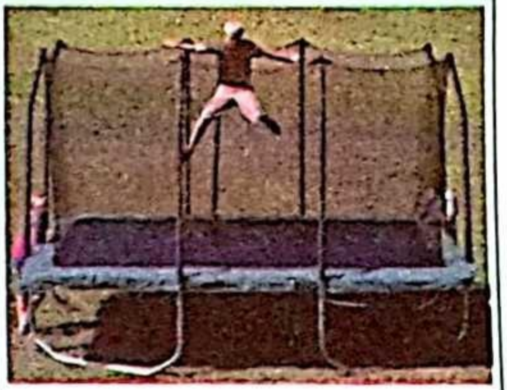
Thus $$r = \frac{r_1 + r_1 + \Delta r}{2}$$

$$r = \frac{2r_1 + \Delta r}{2}$$

Squaring both sides

$$r^2 = \left(\frac{2r_1 + \Delta r}{2}\right)^2$$

$$r^2 = \frac{4r_1^2 + \Delta r^2 + 4r_1\Delta r}{4}$$



A boy bounces on a trampoline. The boy moves upward with an initial speed v and reaches maximum height with a final speed of zero. So energy changes from elastic P.E to K.E and then into gravitational P.E

Δr^2 is very small, so this term can be neglected.

$$r^2 = r_1^2 + r_1\Delta r$$

Substituting the value of Δr in above equation

$$r^2 = r_1^2 + r_1(r_2 - r_1) = r_1^2 + r_1 r_2 - r_1^2$$

$$r^2 = r_1 r_2$$

Hence, Eq. (4.19) becomes

$$F = G\frac{mM}{r_1 r_2}$$

Work from 1 to 2 we have,

$$W_{1\to 2} = Fd\cos\theta$$

$$W_{1\to 2} = F\Delta r\cos 180^\circ$$

$$W_{1\to 2} = F\Delta r(-1)$$

$$W_{1\to 2} = -F\Delta r$$

Negative sign shows that the work is done against the gravitational field.
Substituting the values of F and Δr

$$W_{1\to 2} = -G\frac{mM}{r_1 r_2}(r_2 - r_1)$$

$$W_{1\to 2} = -GmM\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

$$W_{1\to 2} = -GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad ......(4.20)$$

Similarly, the work done from 2 to 3 is

$$W_{2\to 3} = -GmM\left(\frac{1}{r_2} - \frac{1}{r_3}\right) \quad ......(4.21)$$

Finally the work done from r_{N-1} to r_N is

$$W_{N-1\to N} = -GmM\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right) \quad ......(4.22)$$

Hence, the total work done from point 1 to N is

$$W_{Total} = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + ... + W_{N-1\to N}$$

$$W_{Total} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + ... + \frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

$$W_{Total} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_N}\right) \quad ......(4.23)$$

This work cause of P.E of the body from point (1) to point (N). It is represented by Δu. Thus according to work energy theorem.

$$\Delta u = -W_{Total}$$

$$u_N - u_1 = GmM\left(\frac{1}{r_1} - \frac{1}{r_N}\right) \quad ......(4.24)$$

If the r_N is very large distance say at infinity $(r_N = \infty)$, at that point $u_N = 0$ and u_1 at distance r_1 from the centre of the earth is known as absolute potential energy. Thus, eq. 4.24 becomes;

$$0 - u_1 = GmM\left(\frac{1}{r_1} - \frac{1}{\infty}\right) \qquad \because \left(\frac{1}{\infty} = 0\right)$$

$$u_1 = -GMm\left(\frac{1}{r_1}\right)$$

or

$$u_1 = -\frac{GMm}{r_1} \quad ......(4.25)$$

The gravitational potential energy is called absolute gravitational potential energy on the surface of Earth. If $r_1 = R$ and $u_1 = u$. Then, eq.4.25 becomes;

$$U = -\frac{GMm}{R}$$

This is the absolute potential energy on the surface of the Earth.

4.6 ESCAPE VELOCITY

When a body is projected vertically upward from the surface of Earth then due to the gravitational force of attraction, the velocity of the body decreases and finally becomes zero at some height and the body returns to the ground. If we keep on increasing the initial projection velocity of the object, a stage will be reached such that its final velocity becomes zero at the point where the gravitational field becomes zero as shown in Fig.4.18. The body escapes the gravitational field of the Earth or any other planet and it will not return back. This projected velocity is called its escape velocity. Its value can be calculated as;

When a body is projected upward with maximum initial velocity, then it loses its K.E and gains P.E. Thus the escape velocity can be calculated by using the law of exchange of energy.

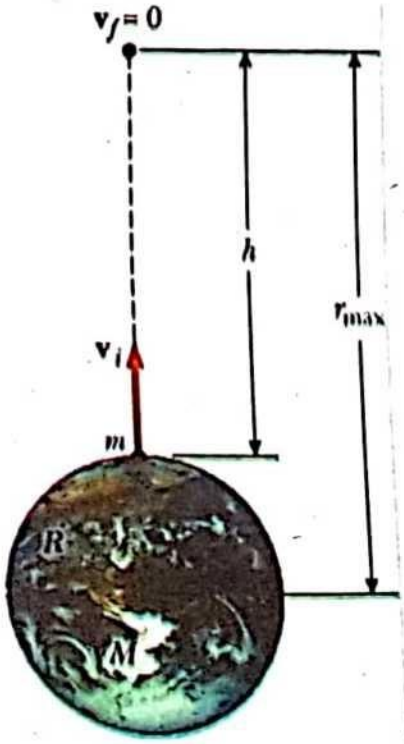


Fig.4.18: Escape velocity of a body from surface of the earth

Loss of initial K.E. = Gain in absolute P.E.

$$\frac{1}{2}mv_{esc}^2 = \frac{GMm}{R}$$

$$v_{esc}^2 = \frac{2GM}{R}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}} \quad ......(4.26)$$

We know that

$$g = \frac{GM}{R^2}$$

or

$$gR = \frac{GM}{R}$$

Hence,

$$v_{esc} = \sqrt{2gR}$$

Escape Velocities for Planets

Planets	Velocity ms^{-1}	Velocity $k\ ms^{-1}$
Sun	618033.60	618.03
Mercury	4247.56	4.25
Venus	10360.79	10.36
Earth	11174.36	11.17
Mars	5021.09	5.02
Jupiter	59542.35	59.54
Saturn	35457.55	35.46
Uranus	21284.62	21.28
Neptune	23439.59	23.44
Moon	2375.18	2.38

$$v_{esc} = \sqrt{2\left(9.8\,\text{m s}^{-2}\right)\left(6.4\times10^{6}\,\text{m}\right)}$$

$$v_{esc} = 11200\,\text{m s}^{-1}$$

$$v_{esc} = 11.2\times10^{3}\,\text{m s}^{-1}$$

$$v_{esc} = 11.2\,\text{km s}^{-1}$$

This result shows that if a body is thrown upward from the surface of Earth with a velocity of 11.2 km s^{-1} or more, then it will never return to the Earth.

It is worth noting that the escape velocity does not depend on the mass of the body. It is the same for all masses for a given planet.

4.7 POWER

The rate at which energy is transferred or work is done by a body is a sign of power of that body. For example, a boy may carry a box upstairs in 3 minutes while a man may do it in 1 min. Obviously, the power of the man is more than the power of the boy. This example also shows that the time factor is important for the power. That is, power is defined interms of the ratio between work and time. Thus, it is stated as **the rate of doing work of a body or rate of transfer of energy of a system** is called its power.

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Energy}}{\text{Time}} \quad ......(4.27)$$

We can also find another expression for power. Suppose a force '**F**' acts on a body so that it moves with velocity '**v**' and it converse a displacement d than by definition of work

$$W = \vec{F}\cdot\vec{d}$$

Eq. 4.27 becomes,

$$P = \frac{\text{Work}}{\text{Time}} = \frac{\vec{F}\cdot\vec{d}}{t} = \vec{F}\cdot\frac{\vec{d}}{t}$$

$$P = \vec{F}\cdot\vec{v} \;......(4.28) \qquad \because\ \vec{v} = \frac{\vec{d}}{t}$$

This is a power may be defined as dot product of the force and the velocity of the body. It is also written as;

$$P = \vec{F}\cdot\vec{v} = Fv\cos\theta$$

Since power is the dot product of F and v, so it is a scalar quantity i.e., power has magnitude but no direction. The dimensional formula of power is $[ML^{2}T^{-3}]$.

Unit of Power

The SI unit of power is watt.

Watt

The power of a body is 1 watt if it is doing 1 J of work in 1s, or One watt is equal to work of one joule per second.

When the rate of doing work is different, then we introduce average power, if ΔW be the total amount of work which is done in time Δt. Then the average power of the body is given as;

$$P_{avg} = \frac{\Delta W}{\Delta t} \qquad ......(4.29)$$

Now when the rate of doing work of a body is for very short interval of time which approaches to zero then the power is called instantaneous power.

$$P_{inst} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} ......(4.30)$$

Watt is SI unit of power and its use is very common in electrical engineering. However, in mechanical engineering, horse power is the practical unit of power. The relation between horse power (hp) and watt (W) is as under;

$$1 \text{ h.p} = 746 \text{W}$$

Example 4.4

What is the power of an electric motor when it performs work of 6.45×10^7 J in 12 hours?

Solution:

Power = P =?

Work = W = 6.45×10^7 J

Time = t = 12 hours

Time = t = 12×3600 s

$t = 4.32 \times 10^4$ s

$$P = \frac{\text{Work}}{\text{Time}}$$

$$P = \frac{6.45 \times 10^7 \text{J}}{4.32 \times 10^4 \text{s}}$$

$$P = 1.49 \times 10^3 \text{W}$$

4.8 WORK DONE AGAINST FRICTION

When a body does work then there is a resistive force against the motion of the body called friction. This friction dissipates the kinetic energy of the body and it causes decreasing the efficiency of the body in performing work. But its role in the working of a body cannot be neglected. Thus in the presence of friction, the work of the body is called frictional work. To calculate the frictional work, we consider a block of mass 'm' lying on a rough horizontal surface such that there is a coefficient of friction between the two surfaces. In order to move the block, we applied a constant force F in the horizontal direction against the kinetic friction force f_k. Thus, the resultant force acting on the block is given by;

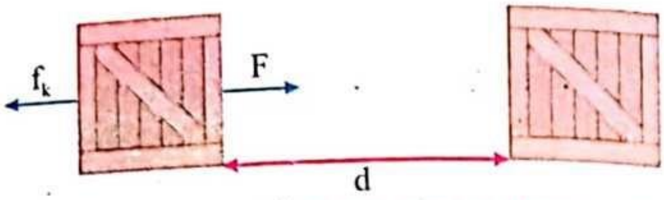


Fig.4.19: A block is sliding over the horizontal surface under the action of applied constant force against the friction force.

$$\text{Resultant force} = \text{applied force} - \text{friction force}$$

$$ma = F - f_k \quad \text{...... (4.31)}$$

As the applied force causes change in velocity of the body from v_i to v_f through a horizontal displacement 'd'. So according to 3rd equation of motion;

$$2ad = v_f^2 - v_i^2$$

$$a = \frac{v_f^2 - v_i^2}{2d}$$

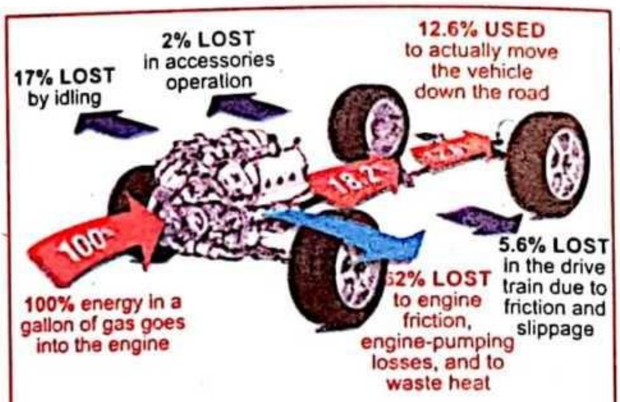
Hence equation (4.31) becomes;

$$m\left(\frac{v_f^2 - v_i^2}{2d}\right) = F - f_k$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = Fd - f_k d$$

$$Fd = \Delta K.E + f_x d$$

$$\text{Work} = \Delta K.E + f_x d \text{ ....(4.32)}$$



A schematic diagram illustrates the energy losses in different parts of a car to overcome the resistive forces by resistive medium against motion of the car.

This is the work done on a block which is sliding on a horizontal surface. This shows that a part of work is used for change in K.E. of the block. The remaining part is used against the friction between the two surfaces.

On the other hand, the work done against the friction is definitely converted into heat or thermal energy. As a result, both the surfaces warm up and their temperature is raised. The gain in thermal energy by the surfaces will then transfer into the environment. The same process of transfer of heat into the environment will be observed, when the work is done on a body in a gravitational field or work is done on a machine. Thus, equation (4.32) can also be expressed as;

$$\text{Work} = \Delta E_{mech} + \Delta E_{therm} \text{ ...... (4.33)}$$

4.9 IMPLICATION OF ENERGY LOSSES IN PRACTICAL DEVICES AND EFFICIENCY:

According to law of conservation of energy, the energy can transform from one form to another form through a system or a device but the total energy remains constant. It is possible only when a system or a device is free from friction. In real life situation, frictional forces are always present and a device cannot do any work without friction. Due to presence of frictional forces, it is not possible to convert the available energy completely into useful work. Only a fraction of energy is converted into useful work and the rest is wasted in form of heat. How much energy is utilized for the useful work by the system or device and how much energy is wasted?

In this regard, we can determine the efficiency of any device or system in terms of ratio using the following relation;

$$\text{Efficiency} = \frac{\text{Utilised output energy}}{\text{Total input energy}} \times 100 \quad ...... (4.33)$$

An ideal machine or engine (Carnot engine) is a theoretical machine or engine whose efficiency is 100%. Because, there is no loss of energy in an ideal machine. i.e., their output is equal to their input. Practically, it is not possible to design a machine which will have 100% efficiency because frictional forces are always present. These frictional forces can be minimized but cannot be eliminated completely.

4.10 CONSERVATIVE AND NON-CONSERVATIVE FORCES

All forces can be classified into two classes on the basis of their different properties i.e., conservative and non-conservative forces.

4.10.1 Conservative Force

A force is said to be a conservative force, if the work done in moving a body between any two points is independent of path followed but it depends on the initial and final positions of the body. In other words, we can also say that a force is conservative if the work done on a body is zero when the body moves around any closed path returning to its initial position.

This definition shows an important feature of conservative force i.e., work done by a conservative force is recoverable.

The work done by a conservative force is always stored in a body in the form of potential energy and in the presence of a conservative force; law of conservation of energy of an isolated system is valid.

Some common examples of conservative forces are:

(i) The gravitational force.

(ii) The force exerted by a spring.

(iii) The electrostatic force between two charges.

4.10.2 Non-Conservative Force

FOR YOUR INFORMATION	
Conservative Forces	Non Conservative Forces
Gravitational	Friction
Elastic	Air Resistance
Electric	Tension in cord
	Motor or rocket Propulsion
	Push or pull by a person

A force is said to be a non-conservative if the work done by that force in moving a body between two points depends on the path followed. Similarly, the work done by a non-conservative force in moving a body along a closed path is not zero. In other words, work done by a non-conservative force cannot be represented by potential energy and the law of conservation of energy is not valid in the presence of non-conservative forces.

Some common examples of non-conservative forces are:

(i) The frictional force.

(ii) The resistance force exerted by resistive mediums force.

(iii) The tension.

Let us explain a non-conservative force with an example. Suppose you have to displace a book between two points A and B on a rough horizontal surface such as; a table as shown in Fig.4.20.

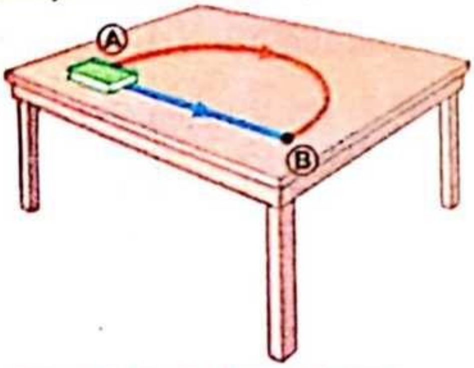


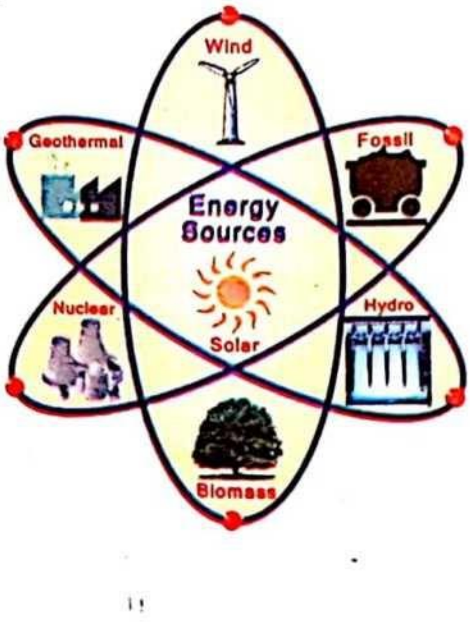
Fig.4.20: Work done by a non-conservative force (Frictional force)

If the book is displaced in a straight line between the two points, then the work done by friction force is given as;

$$W = Fd$$

However, if the book is moved along a semi-circular path between the two points, the work done by the frictional force would be greater than the work in a straight line. Finally, if the book is moved around a closed path, the work done by the frictional force is not zero. Thus the work done by a non-conservative force is not recoverable.

4.11 NATURAL SOURCES OF ENERGY



In this technological age, the demand of energy is increasing rapidly day by day. Energy is one of the most important factor of the economic infrastructure and it is the basic input source to maintain economic growth. Thus, one can say that all the developments of the countries directly or indirectly depend upon energy. This is a reason that why over the last few decades; the scientists are paying attention towards the exploring of new energy resources in order to fulfill the energy demands. The energy sources are classified into two groups.

- Conventional sources or Non-renewable sources of energy
- Non-conventional or Renewable source of energy

4.11.1 Conventional or non-renewable sources of energy

Conventional or non-renewable sources of energy are those which can be used for a long time. These sources are exhaustible and they cannot replenish easily. Coal, oil and natural gas called fossil fuels are the examples of non-renewable sources of energy. All these are remnants of plants and animals and their formation took billion of years. For example, plants and animals store energy under process of photosynthesis. This stored energy remains with them when they die. Therefore, it has been estimated that the fossil fuels were formed by natural process over millions of years ago when decomposed plants and animals matter was buried in earth's crust.

Although the world's major energy sources are fossil fuels but they are hydro-carbons. That is they contain high percentage of carbon. So, when these fossil fuels are burnt they release carbon dioxide, methane and nitrogen into the atmosphere. This causes the pollution, which leads to smog, acid rain and a greenhouse effect.

The greenhouse effect refers to the rising temperature caused by the sun's energy being trapped in our atmosphere by these extra gases. This raises the earth's temperature. It is estimated that 71% of the energy is being used in the world from these conventional sources. The pie chart about the contribution of various energy sources is shown in Fig.4.21. On the other hand, uranium and hydro are also conventional sources of energy but they are neither fossil fuels, nor exhaustible and they also make no pollution.

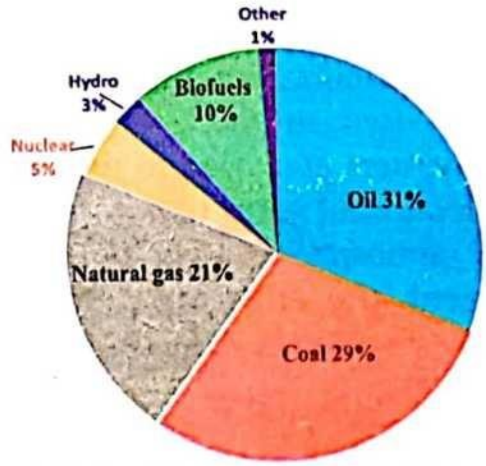


Fig.4.21: Pie chart natural sources of energy

I. Coal:

Coal is also known as black diamond which is the most abundant solid form of fossil fuel on the Earth. The world total coal reserves are estimated around in trillion metric tons, which is sufficient for more than 200 years for the energy generation. Pakistan has the 7th largest coal reserves in the world.

Fig.4.22: Coal thermal power plant

Thar coal reserves in Sindh contain 175-bil
like; Shahrag, Marwaar, Duki, Much, Chamalung etc. also contain coal at large scale. The usage of coal at the large scale played a vital role in the industrial revolution in the 19th and 20th centuries and it still remains essential for the industrial sector in the 21st century.

Coal is used for electric power generation and also used directly in heavy industries like steel making. A typical coal thermal power plant is shown in Fig.4.22.

II. Oil:

Oil is a liquid fossil fuel which is found und
form of crude oil through drilling. This crude oi
products such as, gasoline, diesel fuel, jet fuel,
distillation process.

Oil is not only being used for transportation but by products of crude oil are also used in the production of plastic tyres, polyesters etc. Due to the process of burning of oil, harmful gases like carbon dioxide are emitted in atmosphere and it is a major cause of greenhouse effect and global warming. An oil power station is shown in Fig.4.23.

III. Natural Gas:

Natural gas is a 3rd form of Fossil fuels. It is
oil deposits. It is mainly composed of methane w
ethane. It burns completely and leaves no ashes. T
emits less carbon dioxide than coal and oil an
friendly fuel. Natural gas is used for cooking, heating and transporting as well as in industries.

Natural gas is the 2nd largest energy source in Pakistan. The first gas field in Pakistan was discovered in 1952 at Sui in Balochistan.

Sui gas field is contributing 46 percent of the total installed gas capacity of the country. A Natural gas power plant is shown in Fig.4.24.

illion-ton coal. Some other areas

Interesting Information

Fuel	Approximate Time Scale of Formation (million years)	Approximate Energy per mass of fuel (kWh/Kg)
Residual fuel oil	165	12.5
Coal	325	8
Diesel	165	12.9
Natural gas	165	10.8
Uranium	6000	22500000

derground and it is obtained in the
oil is refined into various energy
heating oil etc. Using fractional

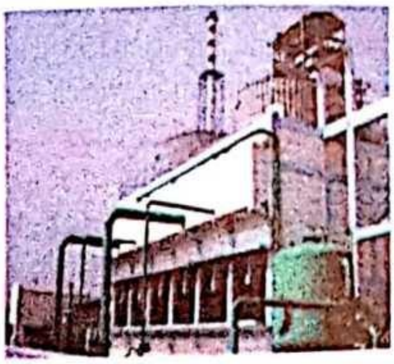


Fig.4.23: Oil Power Plant

s found under the oceans and near
with small amount of propane and
This is the reason that Natural gas
d it is known as environmental

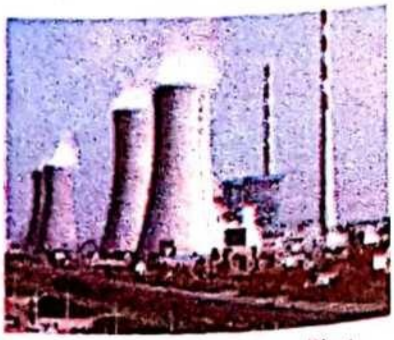


Fig.4.24: Natural Gas Power Plant

IV. Nuclear Power:

Nuclear power is not a fossil fuel. It is a conventional and non-renewable wonderful source of energy. It adds up 12% of the world's total installed electric capacity. In nuclear power plant, fission process is being used to get the nuclear energy.

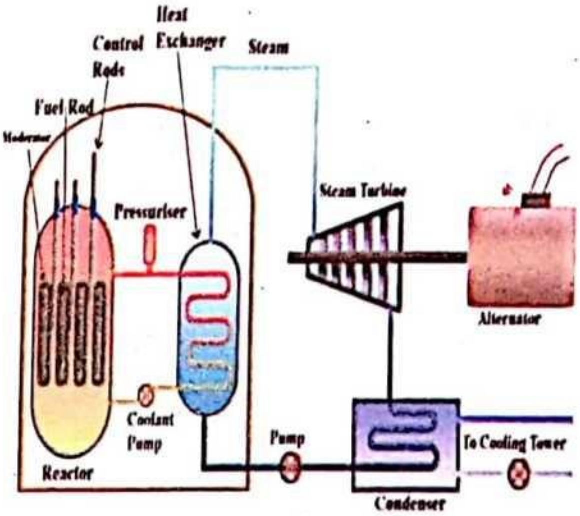


Fig.4.25: Nuclear Power Plant

Nuclear fission is the most common technique to harness nuclear energy in which uranium, sometimes plutonium is used as a fuel like fossil fuels. The fission of 1 kg of uranium has the capacity to produce the same amount of energy as one million ton of coal. Nuclear power plant does not exhaust any greenhouse gases in atmosphere, but it has its own drawbacks along with benefits. The radioactive waste products remain dangerous for thousands of years and must be safely locked away so that they cannot get into the environment. In Pakistan, there are five functional nuclear power plants i.e. KANUPP in Karachi, CHASNUPP-I, CHASHNUPP-II, CHASNUPP-III, CHASNUPP-IV Miawali district. The total installed capacity of these plants is 1430 MW while 2500MW is under construction.

V. Hydro Power:

Hydro power is conventional and renewable form of energy and it is obtained from hydro power generator. Such generator operates on running water or water falling from high potential to low potential. In this connection a large hydroelectric dam reserviours are being constructed.

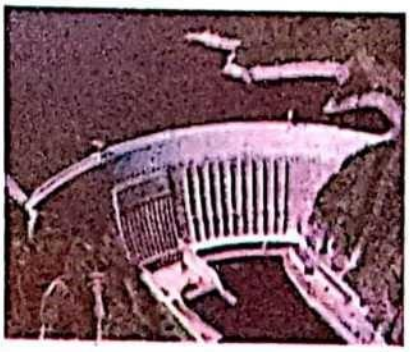


Fig.4.26: Hydro Power Plant

Hydro power is generating 17% of the world's total installed electricity. In the field of renewable energy, it adds up 70% of the total installed capacity. In Pakistan the total hydro power stations are generating about 6700 MW. The main sources of hydro power of the country are Tarbela dam, Ghazi-Brotha hydro plant, Mangla dam, Neelam-Jehlum hydro plant, and Warsak dam.

4.12.2 Non-Conventional or renewable sources of energy

Non-conventional sources of energy are still under development. These sources are inexhaustible and they are replenished quickly. On the other hand, renewable sources of energy are inexpensive and pollution free. The range of non-conventional sources is limited and these can be used at domestic level, some non-conventional energy sources are explained below:

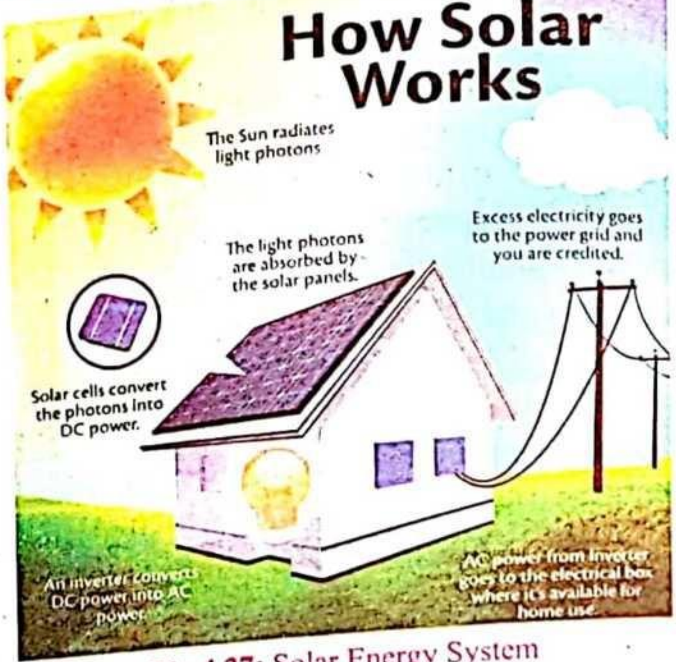


Fig.4.27: Solar Energy System

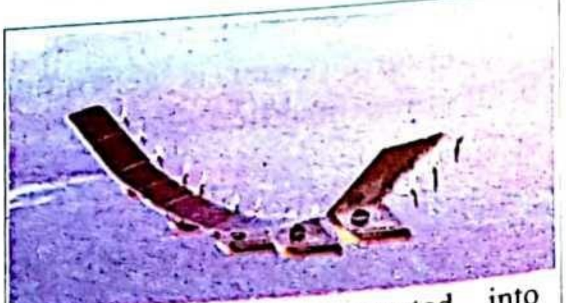
I. Solar Energy:

Sun is a tremendous source of energy. It is 150 million kilometres away from our Earth and we receive solar energy in the form of heat and light from it.

On a clear and cloudless day the incident solar power at the Earth's surface may be up to 10^7 joules per second or 1.4 kW m^{-2}. There are several techniques to harness solar energy. A direct conversion of solar energy into electrical energy can be done by using semiconductor devices called photocells or photovoltaic cells or silicon solar cells. These solar cells can also be connected to rechargeable batteries which store the energy collected, so that it can be used during the darkness or in cloudy weather.


Fig.4.28: Solar Thermal Power Plant

Similarly, solar energy can also be converted into thermal energy by using thermal collectors to heat water in order to produce steam for running turbines to produce electricity.


Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar power aircraft

Solar thermal power systems concentrate solar radiation by mirrors. In this method solar radiation heat a fluid such as molten salt to much higher temperatures, >450°C. Thermal energy from a source at high temperature can be converted into mechanical energy (to drive a turbine). Solar water heaters are a common sight on roofs of the houses and their use is becoming more widespread.

II. Wind Energy:

Wind energy is pollution free and cheap source but it requires a vast area of land and a method of storing electricity for use when the wind drops. Due to the uneven heating and cooling of the atmosphere by the Sun as well as the rotation of earth, wind blows from areas of high pressure to areas of low pressure. Wind mills are being installed in such areas to rotate turbines and produce electricity. Wind in coastal and high altitude areas can be harnessed up to 5MW power from a single turbine. It is estimated that the capacity of wind energy is 2% of the total energy produced in the world.

Wind Energy Farm: The Gansu wind farm in China is the biggest wind farm in the world. It generates up 7900 MW There are 7000 wind turbines in Gobi Desert.

III. Biomass Energy:

Biomass is an organic material which originates from plants, trees, crops, cattle dung, sewages, agricultural and urban wastes and so many other things. Biomass energy is the conversion of biomass into heat, electricity and liquid fuels. Biomass energy can be achieved under various processes.

A biomass fuel can be achieved using a biological method. According to this method, trees and plants store energy from the Sun in the form of carbohydrate through the process of photosynthesis. The carbohydrates are then converted into ethanol or methanol which can be used as a liquid fuel in vehicles.

Similarly, direct burning of biomass such as wood, agriculture residue etc., can be used for heating and cooking purpose.

Energy can also be extracted from the waste biomasses such as animal dung, household waste, urban waste. Ethane and other biogases are produced from the bacterial decomposition of these wastes. As a result, heat energy is produced by burning biogas which can be used to generate steam and operate turbines.

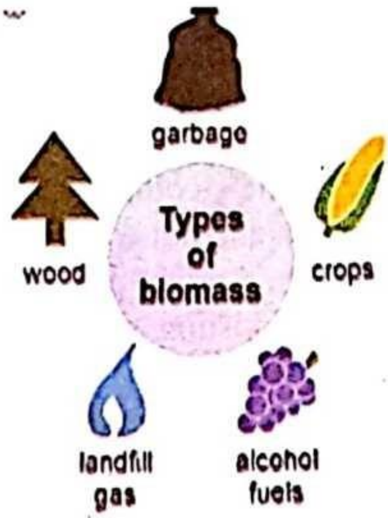


Fig.4.29: Bio Mass Energy

IV. Geothermal Energy

Geothermal energy is the natural heat present inside the Earth, which is available in the depth of 10 km and it is present inside the earth due to the following three reasons,

(a) Ancient heat stored in the core of Earth at temperature 4000°C,

(b) Friction of Earth plates

(c) Decay of radioactive elements which occur naturally.

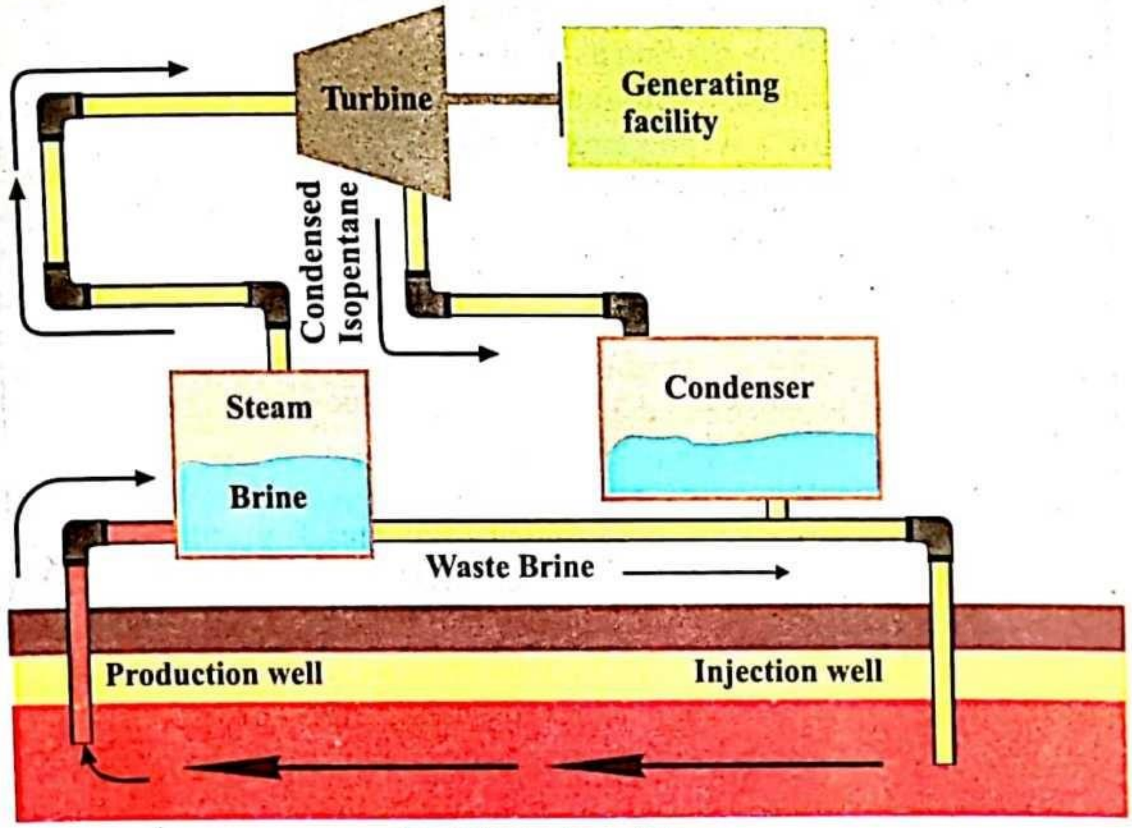


Fig.4.30: Geothermal Energy

The Geothermal energy can be extracted from inside of the Earth in the form of steam and hot water under the following process.

In the first process, underground hot steam and water which are emitted from natural existing springs, can operate turbines to produce electricity as shown in Fig.4.30.

In the second process, holes are drilled into the Earth's crest to put cool water in and pump out the steam. This steam can be used to rotate the electrical turbines for the production of electricity. The amount of geothermal energy is enormous. It has been estimated that only one percent of heat contains in the upper most 10km of Earth's crest is equivalent to 500 times of the present energy obtained from oil and gas sources.

V. Tidal Energy

Tidal energy is also known as gravitational energy which is obtained by the tides of the oceans. Tides in ocean are due to the gravitational force of moon as well as the rotation of the earth. These tides can operate turbines to produce electricity. A tidal barrage system is developed to store water in a tidal basin behind a large dam; by this system the tides are trapped in the basin to operate the turbine as shown in Fig.4.31. Similarly, oceans tidal streams and tidal current below the surface of the sea can also be used for the generation of electricity.

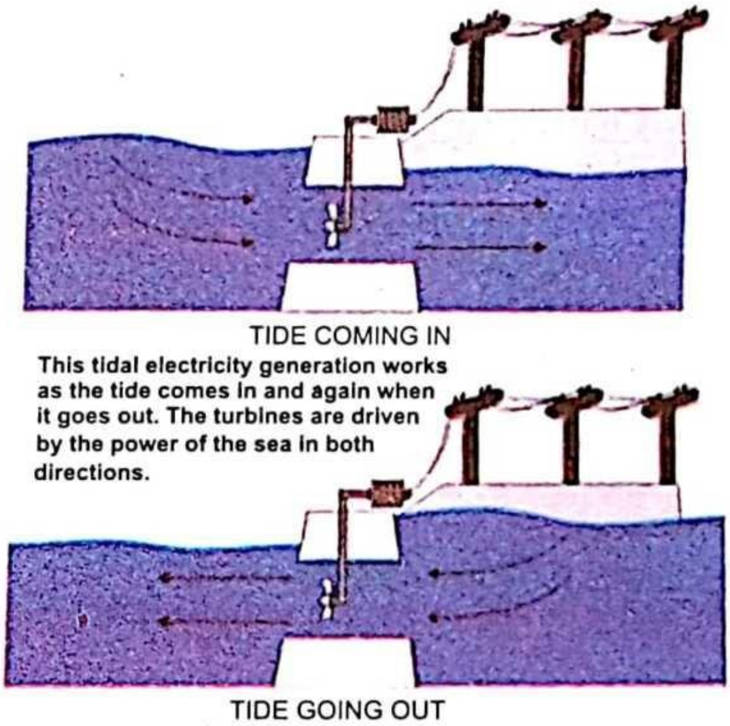


Fig.4.32: Tidal Energy

SUMMARY

- **Work:** The scalar product of force and displacement is called work. The work under a ***constant force*** = Fd $\cos\theta$ and work under ***a variable force*** = $\Sigma F_i \Delta d_i \cos\theta_i$.
- Work done in ***gravitational field*** does not depend upon path. Work done in a ***gravitational field along a closed path*** is zero.
- **Energy:** The ability of a body to do work is called energy.
- **Kinetic and Potential energies:** Kinetic energy is an energy of a body due to its motion while potential energy is the energy of a body due to its position.
- **Work-energy theorem:** Work has always changed the energy (K.E, P.E) of a body. This is work-energy theorem.
- **Absolute P.E.:** Work on a body from the surface of earth to the infinity where $g = 0$ is called absolute P.E.
- $U = \dfrac{-GmM}{R}$
- **Escape velocity:** The projected initial velocity of a body, such that it gets out of the gravitational field is called escape velocity. $V_{esc} = \sqrt{2gR}$
- **Power:** The rate of doing work is called power.

- **Law of conservation of energy:** This law is stated as, "the energy in an isolated system can be transformed from one form to another or transformed from one body to another; but the total amount of energy remains constant".
- **Conservative and non-conservative forces;** If work done by a force does not depend upon path followed, then such force is called conservative force. If the work done by a force depends on path followed then such force is called non-conservative force.
- **Conventional sources of energy:** Sources of energy which are exhaustible and have been in use for long time are called conventional sources of energy such as coal, oil, gas, hydro and uranium.
- **Non-conventional sources of energy**: Sources of energy which are inexhaustible and they exhaust no greenhouse gases in environment are called non-conventional. Solar power, wind powered, Tidal power, geothermal power, biomass power.

EXERCISE

Multiple choice questions.

1. When the applied force and covered displacement are parallel to each other then work done of the body is;
   (a) Zero (b) Negative (c) Maximum (d) Minimum
2. A man pulling a bag with force of 15N at angle 60° with horizontal plane. If bag covers a distance of 10 m, then work done by the man is
   (a) 50 J (b) 75 J (c) 100 J (d) 150 J
3. The area under a curved in a force and displacement graph shows that
   (a) Work under a constant force (b) Work under a variable force
   (c) Work under a maximum force (d) Work under a minimum force
4. If the velocity is doubled then the K.E of the body will be;
   (a) Remain same (b) Double (c) Three times (d) Four times
5. A bullet of mass 20 g is fired with velocity of 2000 ms^{-1}, the K.E of the bullet is;
   (a) 2000 J (b) 4000 J (c) 20000 J (d) 40000 J
6. A body of mass 100 g is raised vertically from surface of Earth in a gravitational field. The P.E of the body at height 100 m is;
   (a) 0.98 J (b) 9.8 J (c) 98 J (d) 980 J
7. What is the power of an electric motor when it consumes energy of 9×10^3J in 3 s?
   (a) 1 hp (b) 2 hp (c) 3 hp (d) 4 hp

8. Absolute potential energy of a body at the surface of the earth is
(a) Gm/R (b) Gm/R^2 (c) GmM/R (d) GmM/R^2

9. What is the value of potential energy of a body at the height where the value of 'g' is zero?
(a) Zero (b) Negative (c) Maximum (d) mgh

10. When velocity of body of mass 10 kg is increased from 2 m s^{-1} to 8 m s^{-1} then the work done of the body is
(a) 100 J (b) 200 J (c) 300 J (d) 400 J

11. Which force is a non-conservative?
(a) Gravitational force (b) Friction force
(c) Electrostatic force (d) Magnetic force

12. Which the following source of energy is not a fossil fuel?
(a) Coal (b) Uranium (c) Oil (d) Gas

13. Conventional method of energy extracted is
(a) Hydro power (b) Wind power (c) Tidal power (d) Biomass power

14. One megawatt hour is equal to:
(a) 3.6×10^7 J (b) 3.6×10^9 J (c) 3.6×10^{12} J (d) 3.6×10^{18} J

15. If the speed of an object is tripled, its kinetic energy is increased by;
(a) $\frac{1}{9}$ times (b) $\frac{1}{3}$ times (c) 6 times (d) 9 times

SHORT QUESTIONS

1. Using work formula, at what angle, the work will be negative? Give example of negative work.
2. Calculate the work of a body when it is moving with uniform velocity.
3. Why the work done of a body in a gravitational field along a closed path is zero?
4. Why the value of K.E is always positive?
5. How can you prove mathematically that power is a scalar quantity?
6. How can absolute potential energy be achieved?
7. What would be the value of P.E of a body when it gets out the gravitational field?
8. Does the escape velocity of a body depend upon its mass?
9. Is there any conversion of energy when law of conversation is not valid?
10. What are the three properties of a conservative force?
11. What are the differences between conventional and non-conventional sources of energy?

12. How many conventional and non-conventional power plants are working in Pakistan?
13. A meteor burns into ashes when it enters into outer atmosphere of Earth. Why?
14. Can a centripetal force do any work? If yes then explain it.
15. What are the essential conditions for conservative field?
16. A bucket is taken to the bottom of a well, does the bucket possess any potential energy.
17. A boy drops a glass from a certain height, which breaks into pieces. What energy changes are involved?
18. Does the kinetic energy of a car changes more when it speeds up from 10 $m\ s^{-1}$ to 15 $m\ s^{-1}$ or from 15 $m\ s^{-1}$ to 20 $m\ s^{-1}$ explain.

COMPREHENSIVE QUESTIONS

1. Define work done by a constant force and explain with examples of positive work, negative work and zero work.
2. State and explain the work done by a variable force with its graphical representation.
3. What is gravitational field? Verify that; (i) the work done in a gravitational does not depend upon its path, (ii) the work done in a gravitational field along a closed path is zero.
4. State and explain energy with its two forms such as; kinetic energy and potential energy. Also describe the work-energy theorem.
5. What is gravitational potential energy? Derive an expression for the absolute gravitational potential energy.
6. Define escape velocity and derive its mathematical relation.
7. What do you know about power? Define power in terms of the scalar product of force and velocity.
8. Study the work done against friction and show that energy is lost due to friction.
9. State and explain conventional and non-conventional sources of energy.

NUMERICAL PROBLEMS

1. A man pulls a bag along the ground with a force of 80 N at an angle of 30° from the ground. How much work done the man do in pulling the bag 10 m?
(693 J)
2. A 250 kg cart is pushed up on an inclined surface. How much work does the pushing force when the cart moves up and at 3 m above the ground, friction is neglected? **(7350 J)**

3. A pump lifts water from a well to a tank 30 m above the well. If there are 100 m^3 water stored in tank, then how much work against the gravity is done by the pump. Density of water is 1000 kg m^{-3}. **(3×10^7J)**

4. A force of 6000 N is applied horizontal on a van of mass 2500 kg. The van starts its motion from rest and if it has traveled a distance of 110 m. What will be its speed and it's K.E.? **(23 ms^{-1}, 660 kJ)**

5. A proton of mass 1.67×10^{-27} kg is being accelerated along a straight line with acceleration of 3.6×10^{15} m s^{-2}. If the proton's initial velocity is 2.4×10^7m s^{-1} and travels a distance of 250 m, what is its final velocity and increase in its K.E.? **(5.56×10^7 m s^{-1}, 2.6×10^{-12} J)**

6. A car of mass 1500 kg is accelerated from rest to a speed of 30 ms^{-1} in a time of 10 sec. What is the power of car in hp when friction is neglected? **(90.5 hp)**

7. What power is required to raise a block of mass 500 kg vertical to height of 15 m in a time of 50 s? Express your answer in hp. **(2 hp)**

8. How much work is required to accelerate a body of mass 200 kg from 5m s^{-1} to 15 m s^{-1}? If its covers a distance of 150 m what is the net force acting on it. **(2×10^4 J, 133 N)**

9. A body of mass 'm' is dropped from a tower 100 m above the ground. What will be the height from ground to the point at which the velocity of body becomes 30 m s^{-1}. Air resistance is neglected. **(54 m)**

10. A block starts from rest at the top of an inclined surface of height 10 m above the ground, what is its speed when it reaches at the ground and friction is neglected. Now by including friction and when it reaches at the ground with a speed of 10 m s^{-1} then what is its energy loses in percent. **(14 m s^{-1}, 49%)**

Unit 5 ROTATIONAL AND CIRCULAR MOTION

Major Concepts **(21 PERIODS)**

- Kinematics of angular motion
- Centripetal force and centripetal acceleration
- Orbital velocity
- Artificial satellites
- Artificial gravity
- Moment of inertia
- Angular momentum

Conceptual Linkage

This chapter is built on
Dynamics Physics IX
Turning Effect of Forces
Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- Define angular displacement, angular velocity and angular acceleration and express angular displacement in radians.
- Solve problems by using $S = r\theta$ and $v = r\omega$.
- State and use of equations of angular motion to solve problems involving rotational motions.
- Describe qualitatively motion in a curved path due to a perpendicular force.
- Derive and use centripetal acceleration $a = r\omega^2$, $a = v^2/r$.
- Solve problems using centripetal force $F = mr\omega^2$, $F = mv^2/r$.
- Describe situations, in which the centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, or a normal force.
- Explain when a vehicle travels round a banked curve at the specified speed for the banking angle, the horizontal component of the normal force on the vehicle causes the centripetal acceleration.
- Describe the equation $\tan\theta = v^2/rg$, relating banking angle θ to the speed v of the vehicle and the radius of curvature r.
- Explain that satellites can be put into orbits round the earth because of the gravitational force between the earth and the satellite.
- Explain that the objects in orbiting satellites appear to be weightless.
- Describe how artificial gravity is created to counter balance weightless.
- Define the term orbital velocity and derive relationship between orbital velocity, the gravitational constant, mass and the radius of the orbit.

- Analyze that satellites can be used to send information between places on the earth which are far apart, to monitor conditions on earth, including the weather, and to observe the universe without the atmosphere getting in the way.
- Describe that communication satellites are usually put into orbit high above the equator and that they orbit the earth once a day so that they appear stationary when viewed from earth.
- Define moment of inertia of a body and angular momentum.
- Derive a relation between torque, moment of inertia and angular acceleration.
- Explain conservation of angular momentum as a universal law and describe examples of conservation of angular momentum.
- Use the formulae of moment of inertia of various bodies for solving problems.

INTRODUCTION

In universe, everything is going on systematically. Summer or winter and spring or autumn no one comes either before or after, similarly, day and night never replace each other. All these are taking place at their proper time, because in the whole universe from electrons in an atom to the galaxies are in uniform rotational motion.

In daily life activities, there are a number of examples of rotational motion such as wheels, propeller, a ceiling fan, a motor pulley, a car shaft, CDs, DVDs, computer hard disk and so many others. All these rotating bodies require some study to analyze their motions. For example, what are the rules of centripetal and centrifugal forces in the rotational motion of a body? How the displacement, velocity, acceleration, momentum and kinetic energy of a rotating body can be determined? All these will not be only a part of our discussion in this chapter but we will also explain rotational inertia, motion of a satellite, orbital velocity, weightlessness and other parameters which are related to a rotational motion.

5.1 ANGULAR DISPLACEMENT

Consider the motion of a particle 'P' in XY-plane along the circumference of a circle of radius OP = r about an axis through centre of circle 'O' and perpendicular to the plane of the circle as shown in Fig. 5.1. This axis of rotation is taken as Z-axis, Let at time t_1 the particle is at point P_1 and the position OP_1 is making angle θ_1 with X-axis as shown in Fig. 5.2. After some time t_2 the particle is at point P_2 and the position OP_2 making angle θ_2 with x-axis, such that $\Delta\theta = \theta_2 - \theta_1$ be the angle between OP_1 and OP_2.

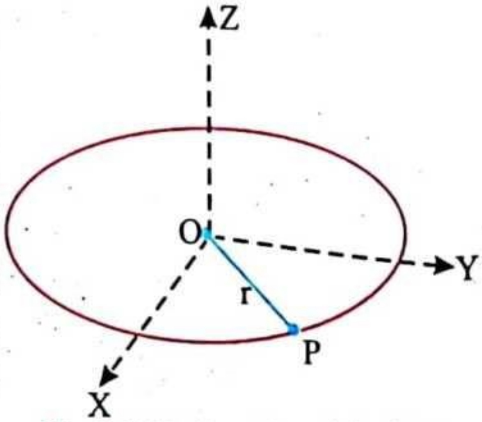


Fig.5.1: Motion of particle along circle about an axis of rotation (Z-axis).

Thus in the interval of time Δt the angle Δθ which is subtended by an Arc P_1P_2 is known as angular displacement and it is defined as;

"The angle subtended at the centre of circle by an arc along which it moves in a given time is known as angular displacement".

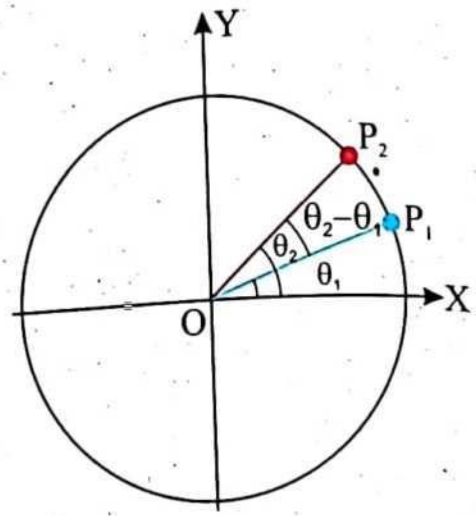


Fig.5.2: Angular displacement (Δθ) of a particle between points P_1 and P_2

The following are the properties of angular displacement.

(i) It depends upon length of arc.
(ii) For very small angle it is a vector quantity.
(iii) For anti-clock wise rotation, angular displacement is positive.
(iv) For clock wise rotation, angular displacement is negative.
(v) Its direction can be determined by right hand rule.

The direction of angular displacement is along the axis of rotation (Z-axis) as shown in Fig. 5.1 and is given by right hand rule.

"Hold the axis of rotation in right hand then curl the fingers in the direction of rotation, the erect thumb will indicate the direction of angular displacement".

The angular displacement can be measured in terms of revolution, degree or radian. All these are explained as;

Revolution:

When a particle completes one round trip along a circular path of a circle, then it is called one revolution.

Degree:

When a circle is divided into 360° equal parts then each part is known as one degree.

Radian:

Radian is the SI unit of angular displacement and it is defined as; **"One radian is the angle subtended at the centre of a circle by an arc whose length is equal to radius of the circle"** as shown in Fig. 5.3.

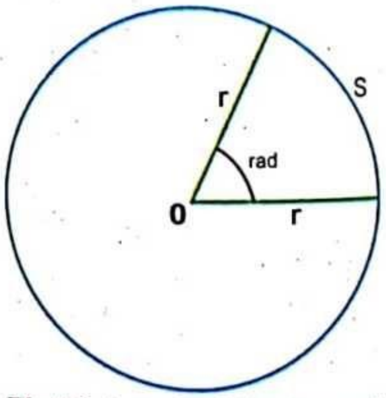


Fig.5.3: Length of the arc equal to radius and the corresponding angle is one radian.

Relation between radian and degree

A mathematical relation among the three units of angular displacement can be expressed as;

Consider an arc of length 'S' along a circular path of a circle of radius 'r' which subtends an angle 'θ' at the centre of the circle, as shown in Fig.5.4. It has been observed that at constant radius, the length of the arc is directly proportional to the subtended angle.

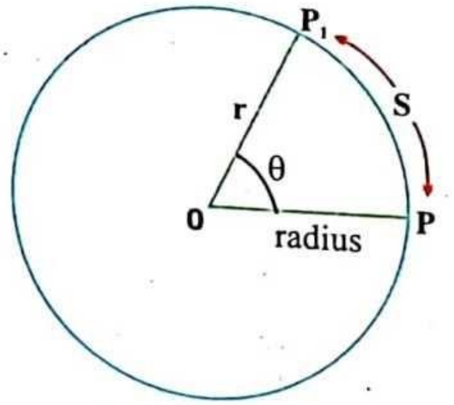


Fig.5.4: Arc vs. angle

$$S \propto \theta$$
$$S = r\theta$$
$$\theta = \frac{S}{r}\text{(rad)}......(5.1)$$

Now for one complete revolution.
$S = 2\pi r$ (Circumference) and $\theta = 360°$
Thus eq. (5.1) becomes

$$360° = \frac{2\pi r}{r}\text{(rad)} = 2\pi\text{(rad)}$$

Hence, 1 revolution $= 2\pi\text{(rad)} = 360°$

$$1\,\text{rad} = \frac{360°}{2\pi}$$
$$1\,\text{rad} = \frac{180°}{\pi}$$
$$1\,\text{rad} = 57.3°$$
or $$1° = 0.01745\,\text{rad}$$

Now for one complete revolution.
$S = 2\pi r$ (Circumference)
and $\theta = 360°$
$$360° = \frac{2\pi r}{r}\text{(rad)}$$
$$1\,\text{rad} = \frac{360°}{2\pi}$$
$1\,\text{rad.} = 0.159$ rev.

Example 5.1

Khawar goes around a circular track that has a diameter of 20 m. If he runs around the entire track for a distance of 160 m, what is his angular displacement?

Solution:

According to situation, Khawar's linear displacement is
$$S = 160\text{ m}$$
Also, the diameter of the circular path, $d = 20$ m
As we know that $d = 2r$

So $$r = \frac{d}{2} = \frac{20}{2} = 10\text{ m}$$

According to formula for angular displacement, $\theta = \frac{S}{r}$ (rad)

$$\theta = \frac{160}{10}\text{radians}$$
$$\theta = 16\text{ radians}$$

Example 5.2

a) Convert the following angles from degrees to radians: 30 degrees, 45 degrees, 90 degrees, 180 degrees, 360 degrees

b) Convert the following angles from radians to degrees: $\frac{2\pi}{3}, 1, 2, \frac{2\pi}{5}, \frac{3\pi}{4}$

Solution:

a) As we know that $1\,\text{rad} = \frac{180^\circ}{\pi}$ or $1^\circ = \frac{\pi}{180}\,\text{rad}$

$$30^\circ = 30^\circ \times \frac{\pi}{180}\,\text{rad} = \frac{\pi}{6}\,\text{rad}$$

$$45^\circ = 45^\circ \times \frac{\pi}{180}\,\text{rad} = \frac{\pi}{4}\,\text{rad}$$

$$90^\circ = 90^\circ \times \frac{\pi}{180}\,\text{rad} = \frac{\pi}{2}\,\text{rad}$$

$$180^\circ = 180^\circ \times \frac{\pi}{180}\,\text{rad} = \pi\,\text{rad}$$

$$360^\circ = 360^\circ \times \frac{\pi}{180}\,\text{rad} = 2\pi\,\text{rad}$$

- Angle swept by minute hand in one complete rotation is 360°.
- Angle swept by minute hand in one minute is 6°.
- Angle swept by minute hand in 5 minutes is $5 \times 6^\circ = 30^\circ$.

b) As we know that $1\,\text{rad} = \frac{180^\circ}{\pi}$

$$\frac{2\pi}{3} = \frac{2\pi}{3} \times \frac{180^\circ}{\pi} = 120^\circ$$

$$1 = 1 \times \frac{180^\circ}{\pi} = 57.3^\circ$$

$$2 = 2 \times \frac{180^\circ}{\pi} = 114.6^\circ$$

$$\frac{2\pi}{5} = \frac{2\pi}{5} \times \frac{180^\circ}{\pi} = 72^\circ$$

$$\frac{3\pi}{4} = \frac{3\pi}{4} \times \frac{180^\circ}{\pi} = 135^\circ$$

5.1.1 Angular Velocity

When a body is moving along the circumference of a circle, then it is often of our interest to know that how its rotation gets fast or slow. By its fast or slow rotation means how much angular displacement is covered by a body over a period of time. This is the angular velocity of body which is defined as; **"The rate of change of angular displacement is called angular velocity".**

It is generally denoted by ω(omega), a Greek letter and is measured in radians per second (rad. s^{-1}) or revolution per second (rev. s^{-1}). Its dimensional formula is $[M^0L^0T^{-1}]$.

Let a rotating particle makes an angle 'θ_1' with x-axis at time 't_1'. After some time t_2, the angle has changed to θ_2 as shown in Fig. 5.5. Then the average angular velocity $\vec{\omega}_{av}$ of the body is the ratio between the change in angular displacement $\Delta\vec{\theta} = \theta_2 - \theta_1$ to the time interval $\Delta t = t_2 - t_1$ and given as;

$$\vec{\omega}_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\vec{\theta}}{\Delta t} \ldots\ldots(5.2)$$

When the change in angular displacement takes for very short interval of time which approaches to zero then its corresponding velocity is called instantaneous velocity.

$$\vec{\omega}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} \ldots\ldots(5.3)$$

It is a vector quantity and its direction is along the axis of rotation and its sense is given by right hand rule. According to the right hand rule, if we hold the axis of rotation with the right hand so that the fingers are curled in the sense of the rotation, the erected thumb then points in the direction of ω.

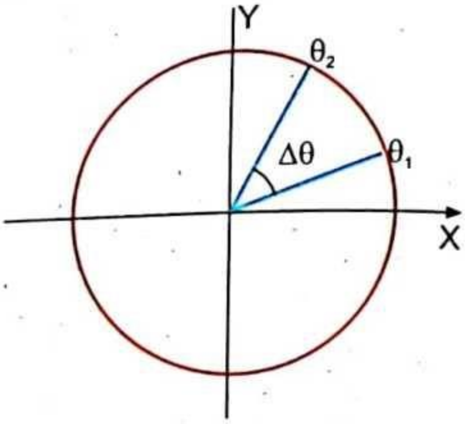
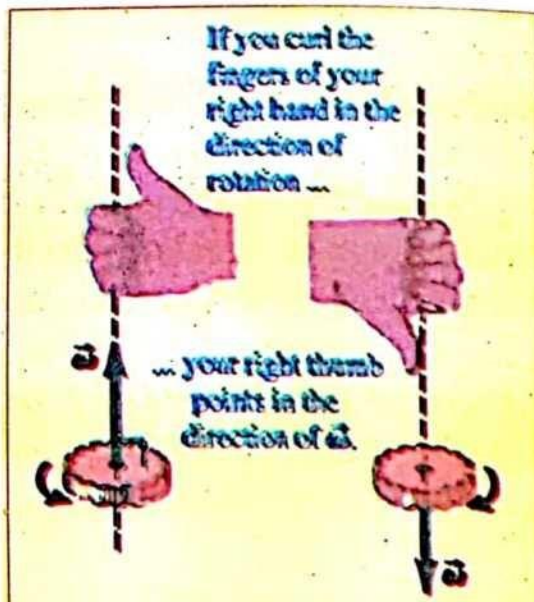


Fig.5.5: Rate of change of angular displacement Δθ in time Δt.



Right Hand Rule

Hold the axis of rotation in your right hand then curl your fingers along the direction of rotation the erect thumb will point in the direction of angular velocity 'ω'.

Example 5.3

Find the angular velocity of Earth about its own axis.

Solution:

We know that Earth completes one revolution about its own axis in one day.

Angular velocity = ω = ?

Angular displacement = $\vec{\theta} = 2\pi$ radians

Time for one revolution =1 day = 24 hours' × 3600 seconds

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{1 \text{ day}}$$

$$\omega = \frac{2(3.14)\text{ rad}}{24 \times 3600\text{ s}}$$

$$\omega = \frac{6.28\text{ rad}}{86400\text{ s}}$$

$$\omega = 7.27 \times 10^{-5}\text{ rad s}^{-1}$$

KEY POINT

The direction of $\vec{\omega}$ simply represents that the rotational motion is taking place perpendicular to it.

5.1.2 Angular Acceleration

It is our daily life observation that the angular velocity of a ceiling fan can be increased or decreased by its regulator. Indeed, this is an angular acceleration or deceleration of the fan.

Angular acceleration of a body is defined as; **"the change of angular velocity of a body with time is called its angular acceleration"**. It is generally denoted by 'α'. If 'ω_i' be initial angular velocity at time 't_i' and 'ω_f' be the final angular velocity at time 't_f' as shown in Fig. 5.6, then the average angular acceleration $\vec{\alpha}_{av}$ of the body is given as;

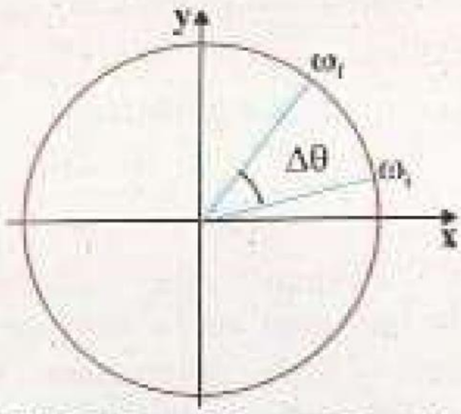


Fig.5.6: Rate of change of angular velocity Δω in time Δt.

$$\vec{\alpha}_{av} = \frac{\vec{\omega}_f - \vec{\omega}_i}{t_f - t_i}$$

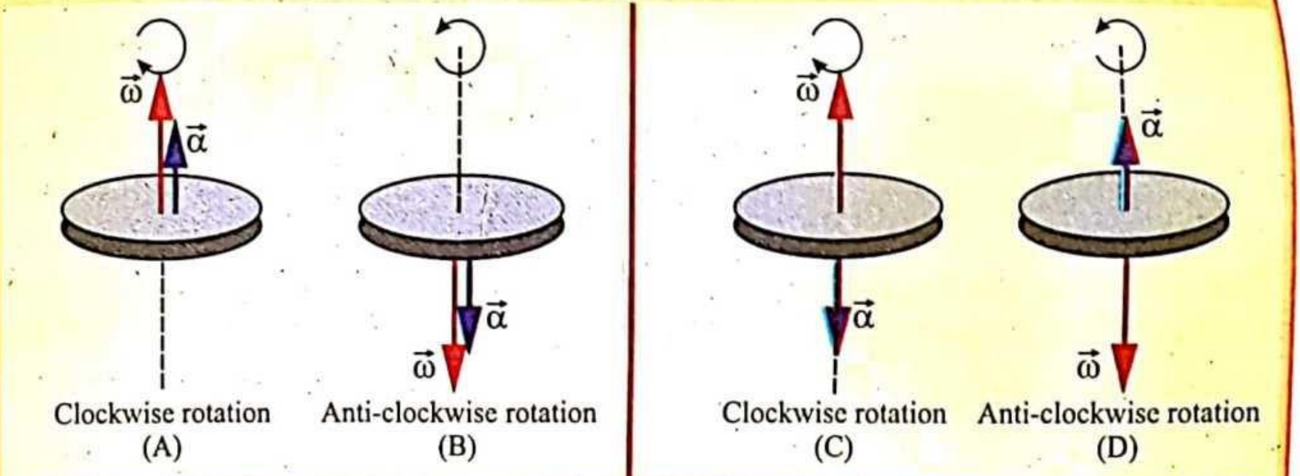
$$\vec{\alpha}_{av} = \frac{\Delta\vec{\omega}}{\Delta t} \quad ......(5.4)$$

If the time interval Δt is infinitesimally small (i.e., $\Delta t \to 0$) then instantaneous angular acceleration is given by

$$\vec{\alpha}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} \quad ......(5.5)$$

Angular acceleration is a vector quantity. Its direction is along the axis of rotation according to the right hand rule. The SI unit of angular acceleration is rad s^{-2} and the dimensional formula is $[M^0L^0T^{-2}]$.

Direction of Angular Acceleration



When the angular velocity is increasing, the angular acceleration vector points in the same direction as the angular velocity, as shown in Fig. (A) and (B).

When the angular velocity is decreasing, the angular acceleration vector points in the direction opposite to the angular velocity, as shown in Fig. (C) and (D).

Example 5.5

The angular velocity of a body is increased from 6 rad s^{-1} to 18 rad s^{-1} in 16 s. Calculate the angular acceleration and the number of revolution in this time.

Solution:

Initial angular velocity = $\omega_i = 6$ rad s^{-1}

Final angular velocity = $\omega_f = 18$ rads^{-1}

Time = t = 16 s.

Angular acceleration = α =?

Numbers of revolution = n =?

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$\alpha = \frac{18-6}{16}$$

$$\alpha = 0.75 \text{ rad s}^{-2}$$

Now $\Delta\omega = \omega_f - \omega_i = 18 - 6 = 12 \text{ rad s}^{-1}$

But $1\text{rad} = \frac{1}{6.2827}\text{rev} = 0.159 \text{ rev.}$

Therefore, $\omega_f - \omega_i = 12(0.159 \text{ rev.})\ s^{-1}$

$$\Delta\omega = \omega_f - \omega_i = 1.91 \text{ rev s}^{-1}$$

Thus the number of revolutions in given time = $\left((\omega_f - \omega_i)\text{rev.s}^{-1}\right) \times \text{time}$

Numbers of revolutions = $(1.91 \text{ rev. s}^{-1}) \times 16\text{s}$

Numbers of revolutions = 30.56 rev.

5.2 RELATION BETWEEN LINEAR AND ANGULAR VARIABLES

Consider a rigid body which is rotating with angular velocity 'ω' about an axis perpendicular to the plane of the circle of radius 'r'. Then the body does not only sweep out an angle 'θ' but it also covers a linear distance in the form of an arc of length 'S'.

Thus one can say that the motion of each particle of a rigid rotating body has both linear and angular motions. Hence the important relations among the linear variables 'S' 'v' and 'a' and angular variables 'θ' 'ω' and 'α' can be established as;

5.2.1 Relation between linear and angular displacements

Consider the motion a particle along the circumference of a circle of radius 'r'. At time't_1', the particle moves from point A to point B and its angular position is θ_1 which is subtended by arc AB of length 'S' as shown in Fig. 5.7. At time 't_2' the particle is at point B′, and its angular position is 'θ_2' which is subtended by arc AB′ has the same length as that of the radius of the circle. Thus angle $\theta_2 = 1$ rad. We know that angle 'θ' is always proportional to the arc length, therefore ratio of two arcs on the circumference of a circle will be equal to the ratio of the corresponding angles at the centre of the circle i.e.

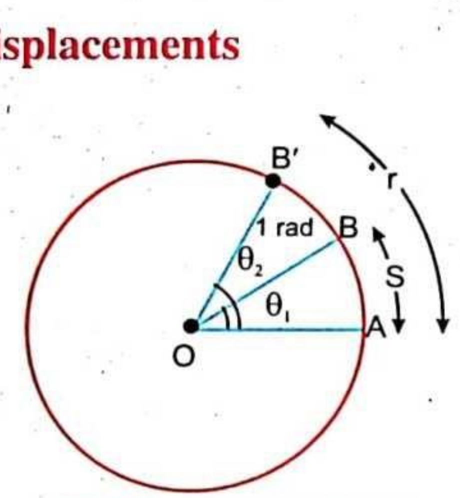


Fig.5.7: Linear variables vs. angular variables

$$\frac{\text{Arc AB}}{\text{Arc AB}'} = \frac{\angle AOB}{\angle AOB'}$$

$$\frac{S}{r} = \frac{\theta}{1\,\text{rad}}$$

$$S = r\theta \quad \ldots\ldots(5.6)$$

This is the relationship between linear and angular displacements.

5.2.2 Relation between linear and angular velocities

Let S_1 and θ_1 be the initial linear and angular displacements respectively at time t_1 and S_2 and θ_2 be the final linear and angular displacements respectively at time t_2, then

$$\Delta S = S_2 - S_1$$

$$\Delta\theta = \theta_2 - \theta_1$$

$$\Delta t = t_2 - t_1$$

Again eq. (5.6) can be written as;

$$\Delta S = r\,\Delta\theta$$

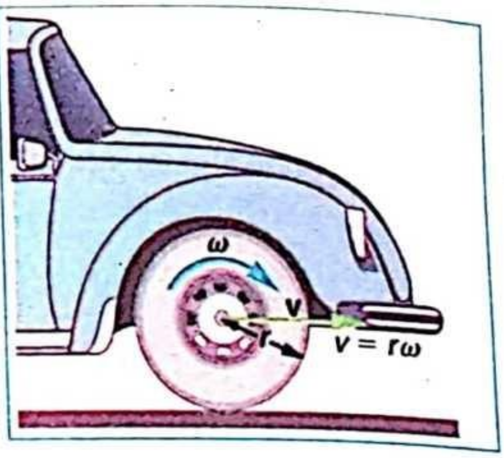
Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

$$v = r\omega \text{ ......(5.7)}$$

In vector form $\vec{v} = \vec{r} \times \vec{\omega} = r\omega \sin\theta$

This is the relationship between linear and angular velocities, where θ is the angle between $\vec{r}$ and $\vec{\omega}$. The direction of $\vec{v}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{\omega}$. Right-hand rule is used to determine the sense of direction of $\vec{v}$ i.e. same as for the cross product of two vectors.



Example 5.4

A particle moves in a circle of radius 200 cm with a linear speed of 20 m s^{-1}. Find the angular velocity.

Solution:

Angular velocity = ω = ?

Radius of the circle = r = 200 cm = 2 m

Linear velocity = v = 20 ms^{-1}

$$\omega = \frac{v}{r} = \frac{20\ ms^{-1}}{2\ m} = 10\ rad\ s^{-1}$$

5.2.3 Relation between linear and angular accelerations

If v_i and ω_i are the initial linear and angular velocities at time t_1 of a particle which is moving in a circular path, while v_f and ω_f are the final linear and angular velocities at time t_2, then,

$$\Delta v = v_f - v_i$$

$$\Delta\omega = \omega_f - \omega_i$$

$$\Delta t = t_2 - t_1$$

Eq. (5.7) can be written as;

$$\Delta v = r\Delta\omega$$

Dividing by Δt

$$\frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t}$$

$$a = r\alpha \text{ ......(5.8)}$$

POINT TO PONDER

If the diameter of a truck tyre is doubled than the diameter of car tyre and both are moving with the same speed. The truck covers a distance 'd' in time 't' then how much distance covers by the car in the same time 't'?

In vector form $\vec{a} = \vec{r} \times \vec{\alpha} = r\,\alpha \sin\theta\,\hat{n}$

This is the relationship between linear and angular accelerations, where θ is the angle between $\vec{r}$ and $\vec{\alpha}$. The direction of $\vec{v}$ is perpendicular to both $\vec{r}$ and $\vec{\alpha}$.

The equations of motion in terms of linear and angular variables

The three equations of motion for linear variables (S, v and a) and angular variables (θ, ω and α) are given as;

$$v_f = v_i + at \quad , \quad \omega_f = \omega_i + \alpha t \quad ......(5.9)$$

$$S = v_i t + \frac{1}{2}at^2 \quad , \quad \theta = \omega_i t + \frac{1}{2}\alpha t \quad ......(5.10)$$

$$2aS = v_f^2 - v_i^2 \quad , \quad 2\alpha\theta = \omega_f^2 - \omega_i^2 \quad ......(5.11)$$

5.3 CENTRIPETAL FORCE

As we know that when the velocity of a body is changing, it has some acceleration. In the case of uniform circular motion, the acceleration is quite different because we have seen, the speed of the body does not change but its velocity does. According to Newton's first law of motion, the acceleration of a moving body with uniform velocity along a straight path is zero in the absence of an external force. In case of uniform circular motion, the acceleration is due to the continuous change of direction of velocity of the body.

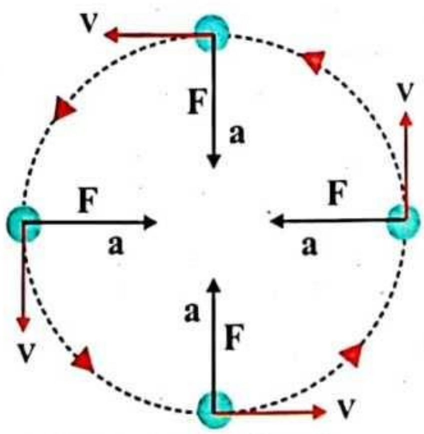


Fig.5.8: The direction of velocity particle is tangent and the acceleration towards the centre

The direction of such acceleration is perpendicular to the tangent of the circular path and is always directed towards the centre of the circle as shown in Fig.5.8. This acceleration is called centripetal acceleration. Thus it can be defined as; **"the acceleration of a body moving with uniform speed in a circle is directed towards the centre of the circle"**.

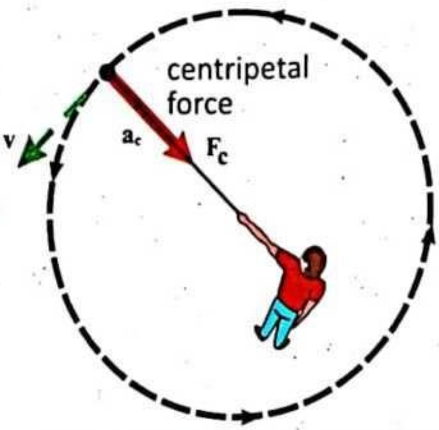


Fig.5.9: The string is pulling in ward

The direction of velocity of a body, moving in a circle with constant speed, at each point of the circular path is tangent as shown in Fig. 5.8.

For example, a ball connected with a string is whirled in a horizontal circle by a boy as shown in Fig. 5.9. Unless the boy is pulling the string inward the circle, the ball continues its motion along the same circular path. Now if the string breaks then the ball follows a straight line path which is along the tangent of the circle as shown in Fig. 5.10.

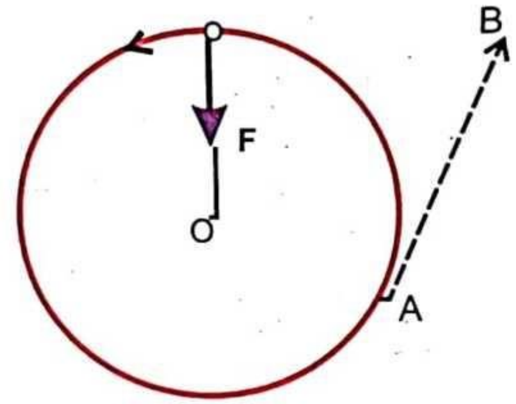


Fig.5.10: The ball moving outward

This example shows that the applied force by the string changes the direction of velocity of a rotating body at each point. Such force is called centripetal force and thus it can be defined as; **"the centripetal force is a force that makes a body to follow a circular path"**. The centripetal force is always directed towards the centre of the circle. i.e., the direction of the centripetal force is the same as that of centripetal acceleration.

In order to calculate the magnitude of centripetal acceleration and centripetal force, we consider a uniform circular motion of a particle of mass 'm' along the circumference of a circle of radius 'r'.

A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

Consider the motion of a particle in a circle. The arc AB of length 'S' subtends an angular displacement 'θ' in time 'Δt' as shown in Figure 5.11.

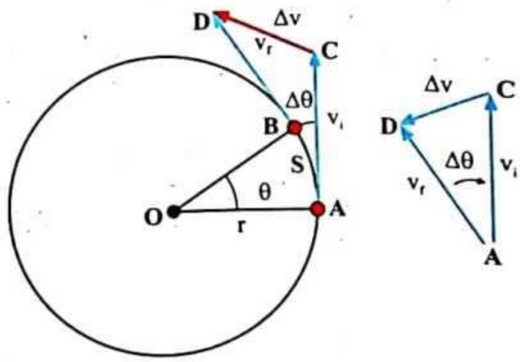


Fig.5.11: Uniform motion of a particle along a circular path of circle

At point 'A, the initial velocity of the particle is v_i which is represented by the line $\overline{AC}$. Similarly at point 'B' the final velocity v_f which is represented by the line $\overline{BD}$. The figure shows that there are two isosceles triangle OAB and ACD. Now by comparing these two triangles we have,

$$\overline{AB} = \overline{CD}$$

$$\Rightarrow S = \Delta v \quad ......(5.12)$$

$$OA = AC$$

$$r = v_i = v \quad ......(5.13)$$

$$\angle A\hat{O}B = \angle C\hat{A}D$$

$$\theta = \Delta\theta ......(5.14)$$

According to the relation between linear and angular velocities, we have;

$$S = r\,\theta$$

Putting the values of S, r and θ from eq. (5.12), eq. (5.13) and eq. (5.14)

$$\Delta v = v\,\Delta\theta$$

Dividing by Δt, $\frac{\Delta v}{\Delta t} = v\frac{\Delta \theta}{\Delta t}$

$$a_c = v\,\omega$$

But $v = r\,\omega$

or $\omega = \frac{v}{r}$

Therefore $a_c = v\frac{v}{r}$

$$a_c = \frac{v^2}{r} \quad ......(5.15)$$

POINT TO PONDER

How a motorcyclist maintains his position in a death well?

KEY POINT

If speed of particle is not uniform but changes during circular motion, then particle possess tangential acceleration (a_T) in addition to centripetal acceleration (a_c). Both these accelerations are perpendicular to each other.

This is the value of centripetal acceleration and its direction is toward the centre of circle.

According to Newton's 2nd law of motion

$$F = ma$$

$$F_c = ma_c$$

$$F_c = \frac{mv^2}{r} \quad ......(5.16)$$

This is a centripetal force which keeps the body in a uniform circular motion.

5.3.1 Forces causing centripetal acceleration

We have discussed that a body moves in a circular path due to a centripetal acceleration. This acceleration is being caused by some external forces in various forms which are explained as;

(i) Consider a conical pendulum which is swinging in a circle. The weight of the pendulum is acting downward. while the tension 'T' acts along the string. The tension in the string can be resolved into vertical and horizontal components as Tcosθ and Tsinθ respectively. As there is no motion in the vertical direction therefore, vertical component of tension Tcosθ is exactly balanced by weight (w = mg) of conical pendulum i.e. Tcosθ = mg. The horizontal component of the tension, Tsinθ, is equal to the centripetal force and it causes the centripetal acceleration as shown in Fig.5.12.

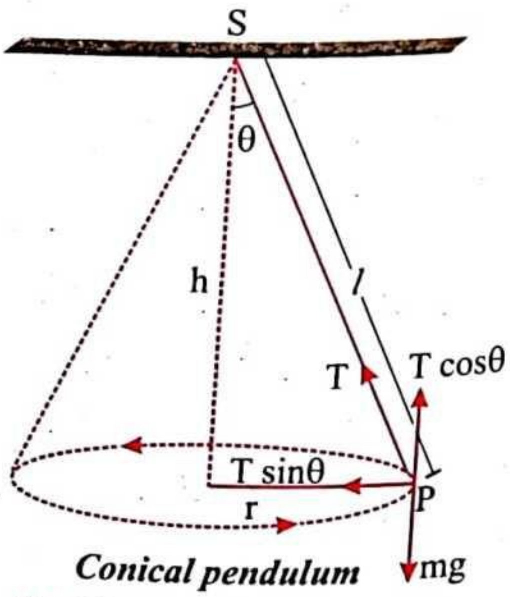


Fig.5.12: Vertical component of T equal to the centripetal acceleration

(ii) In case of a vehicle moving on a flat circular track. The friction force between the tyres of vehicle and the circular track produces a centripetal acceleration inward as shown in Fig. 5.13.

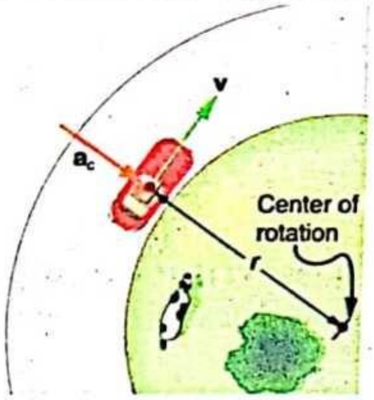


Fig.5.13: The friction force between road and tyres causes centripetal acceleration

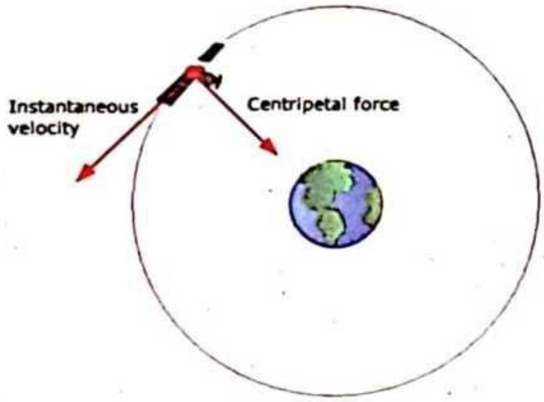


Fig.5.14: The gravitational force of attraction between the earth and satellite provides centripetal acceleration

(iii) When a satellite is revolving in its orbit around the Earth then there exists a gravitational force of attraction between satellite and Earth which is responsible for centripetal acceleration. It is shown in Fig. 5.14.

Example 5.6

A body of mass 0.5 kg moves along a circular path of radius 30 cm at a constant speed of 1.5 revs^{-1}. Calculate (a) tangential speed (b) the centripetal acceleration (c) the required centripetal force.

Solution:

Mass = m = 0.5 kg

Radius = r = 30 cm = 0.3 m

Angular velocity = ω = 1.5 rev s^{-1}

Angular velocity = 1.5×6.28 rad s^{-1}

Angular velocity = 9.42 rad s^{-1}

(a) $v = ?$, (b) $a_c = ?$, (c) $F_c = ?$

(a) $$v = r\omega$$

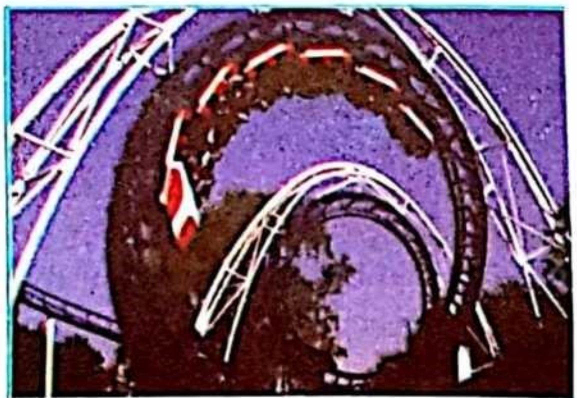
$$v = 0.3\,\text{m} \times 9.42\,\text{rad s}^{-1}$$

$$v = 2.83\,\text{m s}^{-1}$$

(b) $$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(2.83\,\text{m s}^{-1})^2}{0.3\,\text{m}}$$

$$a_c = 26.70\,\text{m s}^{-2}$$



Roller coaster is the application of centripetal force.

POINT TO PONDER

A pail of water can be whirled in a vertical path such that none is spilled. Why does the water stay in, even when the pail is above your head?

(c) $F_c = ma_c$

$F_c = (0.5\text{ kg})(26.70\text{ m s}^{-2})$

$F_c = 13.35\text{ N}$

Example 5.7

What is the centripetal force of a car of mass 750 kg driving at 47 Km h^{-1} on a circular track of radius 24 m?

Solution:

$m = 750\text{ kg}$

$v = 47\text{ km h}^{-1}$

$= \frac{(47)(1000)}{3600}\text{ms}^{-1} = 13\text{ ms}^{-1}$

$r = 24\text{ m}$

$F_c = ?$

$F_c = m\,a_c = \frac{mv^2}{r}$

$$F_c = \frac{750\text{ kg}\left(13\text{ m s}^{-1}\right)^2}{24\text{ m}}$$

$F_c = 5281\text{ N}$

> **FOR YOUR INFORMATION**
> A stone that is stuck in a tyre of an automobile moving at highway speeds experiences a centripetal acceleration of about 2500 m/s^2 or 250 g.

5.4 BANKING OF ROADS AT THE TURN

We often face a portion of curved path (circular arc) on a road when we drive in our car. At this stage, a centripetal force must act on the car to maintain its uniform speed. This centripetal force is provided by the friction between the tyres and the road but this frictional force is inadequate and the car has tendency to skid. As a result the car may leave the curved path.

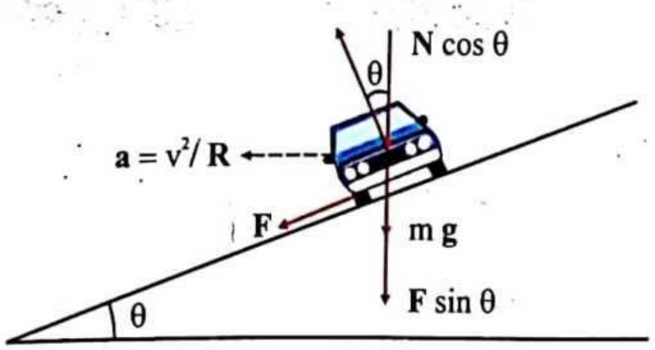


Fig.5.15:
A car at the portion of banked road

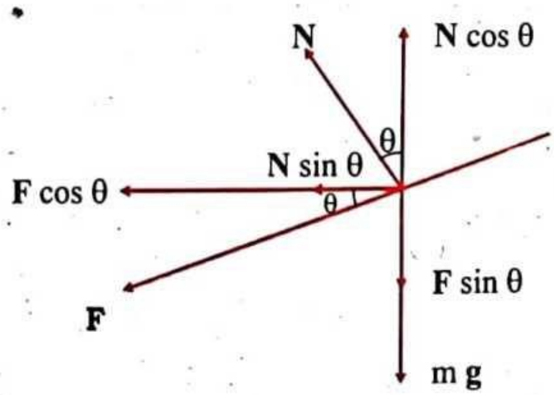


Fig.5.16:
Various forces acting on an object at Banked road

To overcome this problem, the road is constructed such that its outer edge is slightly raised by an angle 'θ' above the level of the inner edge as shown in Fig.5.15. Such construction of road is known as "banking of road" and it provides the necessary centripetal force to a vehicle.

Consider a car which is moving with speed 'v' around the curved road of radius 'R', is banked at angle θ, the forces acting on a car at this curved road can be explained as under. The weight of the car 'mg' is acting vertically downward while its normal reaction 'N' is at angle 'θ' with y-axis.

The horizontal component of normal reaction $N\cos\theta$ is acting vertically upward and it is equal to the weight of the car.

$$N\cos\theta = mg \quad ......(.5.17)$$

The vertical component of the normal reaction N Sin θ is acting horizontally toward the inner edge of the road and it is equal to centripetal force;

$$N\sin\theta = \frac{mv^2}{R} \quad ......(5.18)$$

Dividing eq. 5.18 by eq. 5.17

$$\frac{N\sin\theta}{N\cos\theta} = \frac{\frac{mv^2}{R}}{mg}$$

$$\tan\theta = \frac{v^2}{Rg} \quad ......(5.19)$$

This shows that the banking angle θ depends upon the speed of the car and radius of the turn.

In the eq. (5.19), we have neglected the friction between the tyres and the road. But if the friction between the tyres and the road is also considered then the forces acting on a car at the curved road as shown in Fig. 5.16 which are explained as; The friction 'F' is at an angle 'θ' with N sinθ and it resists the tendency of the car to skid towards the outside of curved path. The horizontal component Fcosθ provides the necessary centripetal force to the car. Thus the resultant centripetal force is given as;

$$N\sin\theta + F\cos\theta = \frac{mv'^2}{R} \quad ......(5.20)$$

where F= μN and μ is the co-efficient of friction between the tyres and road.
If the friction is neglected i.e.$\mu = 0$, then

$$v' = \sqrt{gR\tan\theta} \quad ......(5.21)$$

Example 5.8

What is the speed of train when it passes through a curved track of radius 150 m which has been banked at 5°.

Solution:

Velocity = v = ?

Radius = r = 150 m

Banked Angle = $\theta = 5^\circ$

$$\tan\theta = \frac{v^2}{rg}$$

$$v^2 = rg\tan\theta$$

$$v^2 = (150\,m)(9.8\,m\,s^{-2})\tan 5^\circ$$

$$v^2 = 128.6\ m^2 s^{-2}$$

$$v = 11.34\ m\,s^{-1}$$

$$v = 41\ km\,h^{-1}$$

CHECK YOUR CONCEPT

Why does a pilot tend to blackout when pulling of a steep dive?

5.5 MOMENT OF INERTIA

It is a natural phenomenon that a body always resists to any kind of motion (translational, vibrational and rotational) to be produced in it. This resistive property of a body is called inertia.

In case of a rotational motion, a tendency of a body to resist any change in its state of rest or rotational motion is called moment of inertia or rotational inertia. It depends upon mass and the distance between axis of rotation and centre of mass of the body.

This shows that the greater the moment of inertia, the greater is the torque required to rotate or stop the body about an axis of rotation. Thus by definition of torque.

$$\vec{\tau} = \vec{r}\times\vec{F}$$

As the 'θ' between $\vec{r}$ and $\vec{F}$ is 90°

$$\tau = rF\sin 90^\circ$$

$$\tau = rF \quad ......(5.22)$$

According to Newton's 2nd law

$$F = ma \quad ......(5.23)$$

Putting eq. (5.23) in eq. (5.22)

$$\tau = r\,ma$$

As $\quad a = r\alpha$

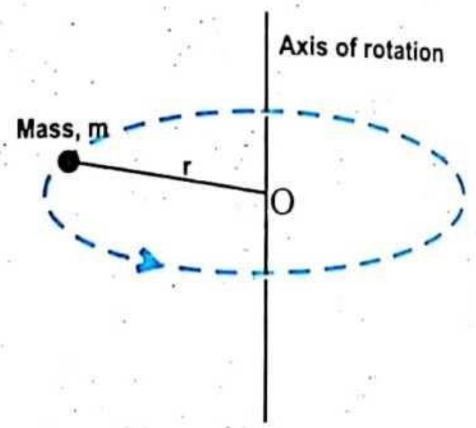


Fig.5.17: A rotating mass in a circle of radius r about an axis perpendicular to the plane

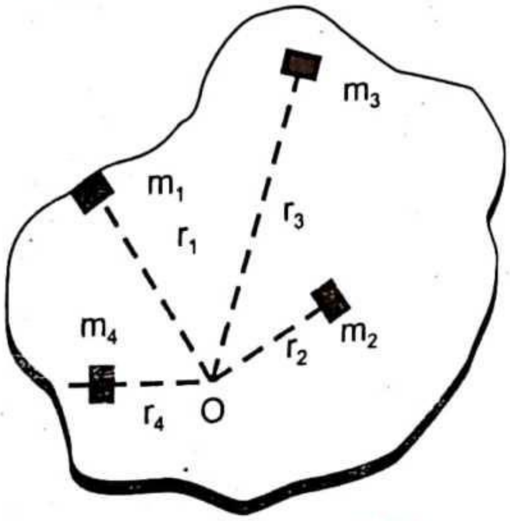


Fig.5.18: : A rotating body has irregular shape

$$\tau = rm(r\alpha)$$

$$\tau = mr^2\alpha \quad ......(5.24)$$

Torque is proportional to the angular acceleration while mr^2 is constant and it is known as moment of inertia of the body which is represented by I, and it is expresses as;

$$I = mr^2 \quad ......(5.25)$$

Equation 5.25 can be used to calculate the moment of inertia of a rigid body when it has a regular geometrical shape as shown in Fig.5.17.

Now consider a rigid body of an irregular shape which is rotating about its axis of rotation as shown in Fig.5.18. In order to calculate its moment of inertia about a vertical axis passing through O we divide it into 'n' number of point masses $(m_1, m_2, m_3, ... m_n)$ at distances $(r_1, r_2, r_3, ... r_n)$ respectively perpendicular to the axis of rotation 'O'. Now when the body is rotating with angular acceleration 'α' then its total torque is given as;

$$\tau = \tau_1 + \tau_2 + \tau_3 ... + \tau_n$$

$$\tau = m_1 r_1^2 \alpha_1 + m_2 r_2^2 \alpha_2 + m_3 r_3^2 \alpha_3 + ... + m_n r_n^2 \alpha_n$$

As the body is rigid so the angular acceleration 'α'of its all point masses remains constant. So,

$$\tau_{Total} = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + ... + m_n r_n^2 \alpha$$

$$\tau_{Total} = (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ... + m_n r_n^2)\alpha$$

$$\tau_{Total} = \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

or $\tau = I\alpha$

where I is the moment of inertia of the rigid body and is expressed as

$$I = \sum_{i=1}^{n} m_i r_i^2 \quad ......(5.26)$$

where m_i is the mass and r_i is the distance of the *i*th point mass from the axis of rotation. Thus, the moment of inertia of a rigid body about a given axis is the sum of the masses of its constituent particles and the square of their respective distances from the axis of rotation.

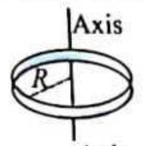
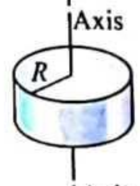
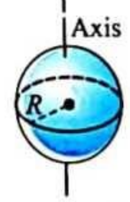
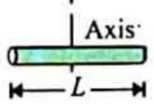
Object	Location of axis		Moment of inertia
Thin hoop, radius ***R***,	Through centre		MR^2
Solid cylinder, radius ***R***,	Through centre		$\frac{1}{2}MR^2$
Uniform sphere, radius ***R***,	Through centre		$\frac{2}{5}MR^2$
Long uniform rod, length ***L***,	Through centre		$\frac{1}{12}ML^2$

Fig.5.19: Moment of inertia of different bodies having different geometrical shape

The SI unit of moment of inertia is kg m^2. Its dimensional formula is $[ML^2T^0]$.

The moments of inertia of various rigid bodies having different regular geometrical shapes are given as in Fig. 5.19.

Example 5.9

What is the moment of inertia of a uniform solid sphere of mass 5 kg and diameter of 100 cm.

Solution:

$$M = 5 \text{ kg}$$
$$D = 100 \text{ cm}$$
$$R = \frac{100}{2} = 50 \text{ cm} = 0.5 \text{ m}$$

Moment of inertia of the sphere about its diameter is given by

$$I = \frac{2}{5}MR^2$$
$$I = \frac{2}{5}(5)(0.5)^2$$
$$I = 2(0.25)\,\text{kg}\,\text{m}^2$$
$$I = 0.5\,\text{kgm}^2$$

5.6 ANGULAR MOMENTUM

We have already discussed that when a body of mass 'm' is in translational motion with velocity 'v' then the product of its mass and velocity is its linear momentum. Similarly, the momentum of a rotating body about an axis is called its angular momentum. It is represented by 'L' and it is equal to the vector product of linear momentum and position vector as shown in Fig.5.20.

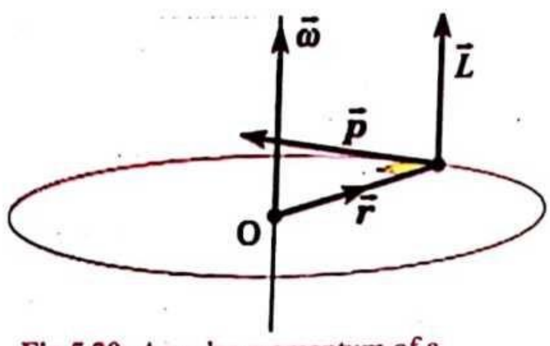


Fig.5.20: Angular momentum of a rotating body

$$\vec{L} = \vec{r} \times \vec{p} \quad ......(5.27)$$

Angular momentum is a vector quantity. Its direction is along the axis of rotation and its unit is kg m^2s^{-1} or J s.

Angular momentum is the product of an object's mass, velocity, and distance from centre of rotation.

If angle θ between $\vec{r}$ and $\vec{p}$ is 90° then eq. 5.27 becomes.

$$L = rp\sin 90^\circ \quad \therefore \sin 90^\circ = 1$$
$$L = rmv(1)$$
As $$v = r\omega$$
$$L = (rm)(r\omega)$$
$$L = mr^2\omega$$
As $$I = mr^2$$
$$L = I\omega \quad ......(5.28)$$

This is angular momentum in terms of moment of inertia. When a rigid body has no regular geometrical shape as shown in Fig.5.21, then we can divide it into 'n' number of point masses $(m_1, m_2, m_3, ...m_n)$ at the distances $(r_1, r_2, r_3, ...r_n)$ from the axis of rotation. If the body is rotating with angular velocity then its total angular momentum is given as;

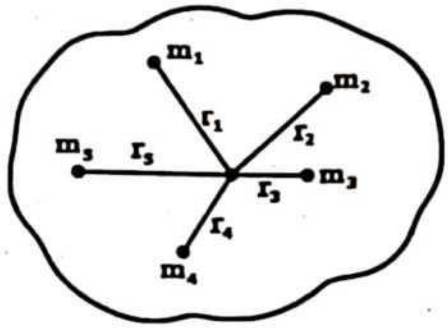


Fig.5.21: A rotating body has irregular shape

$$L = L_1 + L_2 + L_3 + ... + L_n$$
$$L = m_1r_1^2\omega + m_2r_2^2\omega + m_3r_3^2\omega + ...m_nr_n^2\omega$$
$$L = \left(\sum_{i=1}^{n} m_ir_i^2\right)\omega$$
$$L = I\omega \quad ......(5.29)$$

5.6.1 Law of conservation of angular momentum

Just like the law of conservation of linear moment, angular momentum of a rotating system is also conserved in the absence of external torque. Mathematically, it can be explained as;

According to the definition of torque
$$\vec{\tau} = \vec{r} \times \vec{F}$$
But $$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$
$$\vec{\tau} = \vec{r} \times \frac{\Delta\vec{p}}{\Delta t}$$
$$\vec{\tau} = \frac{\Delta}{\Delta t}(\vec{r} \times \vec{p})$$
$$\vec{\tau} = \frac{\Delta\vec{L}}{\Delta t} \quad ......(5.30)$$

In the absence of an external torque, $\tau = 0$ and eq. (5.30) becomes

FOR YOUR INFORMATION

This hurricane was photographed from space. The huge rotating mass of air pressure possesses a large momentum.

$$\frac{\Delta\vec{L}}{\Delta t} = 0$$

$$\Delta\vec{L} = 0$$

$$\vec{L}_1 - \vec{L}_2 = 0$$

$$\vec{L}_1 = \vec{L}_2$$

$$\vec{I}_1\omega_1 = \vec{I}_2\omega_2$$

where $\vec{L}_1$ and $\vec{L}_2$ are the angular momenta of the rigid body before and after the change in angular velocity.

In scalar notation, the above equation becomes;

$$I_1\omega_1 = I_2\omega_2$$

This is a mathematical form of law of conservation of angular momentum and it shows that in the absence of an external torque, initial angular momentum of a rigid body is equal to its final angular momentum.

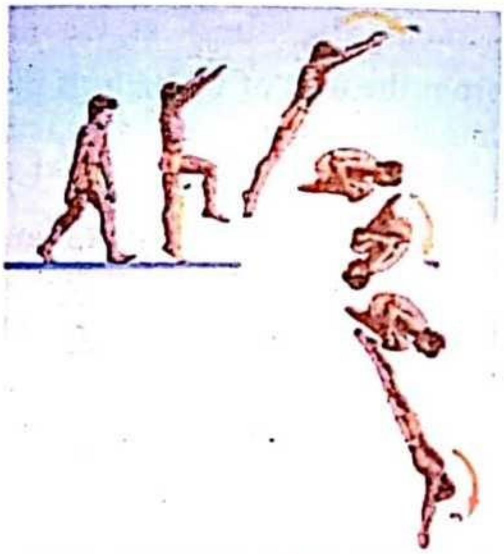


Fig.5.22: A diver using angular momentum

Examples of conservation of angular momentum

A Man Diving from a Diving Board

A diver jumping from spring board has to take a few somersaults in air before touching the water surface, as in Fig. 5.22. After leaving the spring board he curls his body by rolling arms and legs in.

Due to this his moment of inertia decreases and he spins in mid air with large angular velocity. When he is about to touch the water surface, he stretches out his arms and legs. He enters into water at gentle speed and gets a dive. This is an example of law of conservation of angular momentum.

Fig.5.23: An ice skater using angular momentum

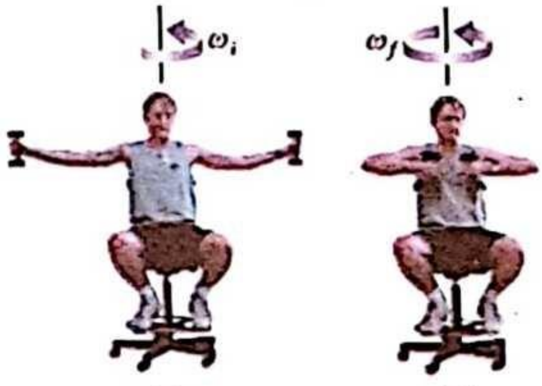
The Spinning Ice Skater

The familiar picture of the spinning ice skater, as shown in Fig.5.23, gives another example of the conservation of angular momentum. An ice skater can increase his angular velocity by folding arms and bringing the stretched leg close to the other leg. By doing so he decreases his moment of inertia. As a result angular speed increases. When he stretches his hands and a leg outward, the moment of inertia increases and hence angular velocity decreases.

A person holding some weight in his hands sitting on a rotating stool.

A person is sitting on a rotating stool with heavy weight in his hands stretched out on both sides as shown in Fig.5.24 As he draws his hands towards the chest, his angular speed at once increases.

This is because the moment of inertia decreases on drawing the hands towards the chest, which increases the angular speed.



(a) (b)

Fig.5.24: A person holding weights on a rotating stool

Angular momentum is conserved during the performance of figure skater. To rotate fast, he has closed his arms and legs, his moment of inertia is small and his angular speed is large. To slow down for the finish of his spin, he moves his arms and legs outward, which increases his moment of inertia and lowers the angular speed.

Example 5.10

The Earth rotates around its axis once in 24 hours (1 day). If it were to expand to twice its present diameter, what would be its new period of revolution?

Solution:

Given data

Radius $= R_1 = R$

Time period $T_1 = 24$ hours $= 1$ day

$R_2 = 2R_1$

$T_2 = ?$

Applying the law of conservation of angular momentum, we have,

$$I_1\omega_1 = I_2\omega_2$$

But $\quad I = \frac{2}{5}MR^2 \quad \because$ Earth is a sphere

and $\quad \omega = \frac{2\pi}{T}$

$$\therefore \left(\frac{2}{5}MR_1^2\right)\frac{2\pi}{T_1} = \left(\frac{2}{5}MR_2^2\right)\frac{2\pi}{T_2}$$

or $\quad \frac{R_1^2}{T_1} = \frac{R_2^2}{T_2}$

$$T_2 = \frac{R_2^2}{R_1^2} \times T_1$$

Putting the given values, we get,

$$T_2 = \frac{(2R_1)^2}{R_1^2} \times T_1$$

$$T_2 = \frac{4R_1^2}{R_1^2} \times 24 \text{ hours}$$

$$T_2 = 4 \times 24 \text{ hours} = 96 \text{ hours} = 4 \text{ days}$$

5.7 ROTATIONAL KINETIC ENERGY

We are familiar with translational K.E. of a body i.e. the energy possesses by a body due to its translational motion. Similarly, when a body is rotating with angular velocity as shown in Fig.5.25, then it also possesses K.E. which is known as rotational K.E.

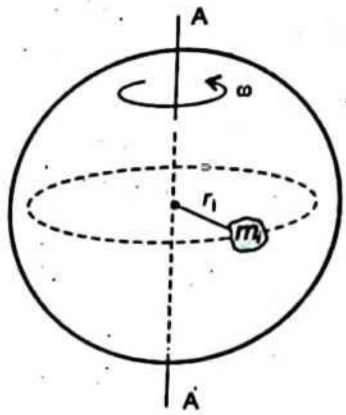


Fig.5.25: Rotational K.E of a body

By definition of translation K.E.

$$K.E = \frac{1}{2}mv^2$$

But $\quad v = r\omega$

$$(K.E) = \frac{1}{2}mr^2\omega^2$$

$$K.E = \frac{1}{2}I\omega^2$$

This is the rotational K.E of the body.

Now consider a rigid body of mass 'm' with irregular shape which is rotating with uniform angular velocity 'ω' about its axis. To calculate its rotational kinetic energy, we can divide its mass into '*n*' number of small point masses $(m_1, m_2, m_3, \ldots m_n)$ at distances $(r_1, r_2, r_3, \ldots r_n)$ respectively from the axis of rotation as shown in Fig. 5.26. Now total K.E. of all the particles is given as;

$$K.E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \ldots + \frac{1}{2}m_nv_n^2$$

As $\quad v = r\,\omega$

$$K.E_{rot} = \frac{1}{2}\left(m_1v_1^2\omega^2 + m_2v_2^2\omega^2 + m_3v_3^2\omega^2 + \ldots + m_nv_n^2\omega^2\right)$$

$$K.E_{rot} = \frac{1}{2}\left(\sum_{i=1}^{n} m_i r_i^2\right)\omega^2$$

$$K.E_{rot} = \frac{1}{2} I\omega^2 ......(5.31)$$

This is the rotational K.E. of a body in terms of moment of inertia.

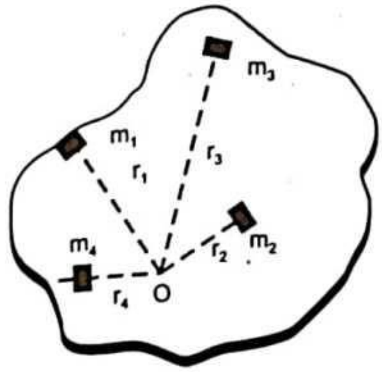


Fig.5.26: A rotating rigid body has irregular shape.

Example 5.11

Calculate the rotational kinetic energy of a 15 kg wheel rotating at a 9 rev s^{-1} and the radius of the wheel is 20 cm.

Solution:

Mass = m = 15 kg

Angular velocity = ω = 9 rev s^{-1}

Angular velocity = $\omega = 9 \times (6.28)$rad s^{-1}

Angular velocity = ω = 56.5 rad s^{-1}

Radius = r = 20 cm = 0.20 m

Rotational Kinetic Energy =?

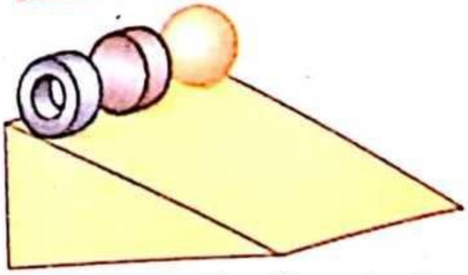
$$\text{Rotational K.E.} = \frac{1}{2} I\omega^2$$

$$\text{Rotational K.E.} = \frac{1}{2} m r^2 \omega^2 \quad (\because I = mr^2)$$

$$\text{Rotational K.E.} = \frac{1}{2}(15)(0.20)^2(56.5)^2$$

$$\text{Rotational K.E.} = 958 \text{ J}$$

CHECK YOUR CONCEPT



Three objects of uniform density a hollow cylinder, a solid cylinder and a solid sphere, are placed at the top of an incline surface. They are all released from the rest at the same height and roll without slipping. Which object reaches the bottom first? Which reaches at last?

5.8 ROLLING OF DISC AND HOOP AT AN INCLINED PLANE

Consider a hoop (Hollow cylinder) and a disc (Solid cylinder) each of mass 'm' which are rolling down on an inclined plane at an angle 'θ' with horizontal plane and at height 'h' from the ground as shown in Fig. 5.27. When they start rolling then they gain both translational and rotational kinetic energies due to increase in their velocities but they lose their potential energies with decreasing height. Mathematically it is explained as;

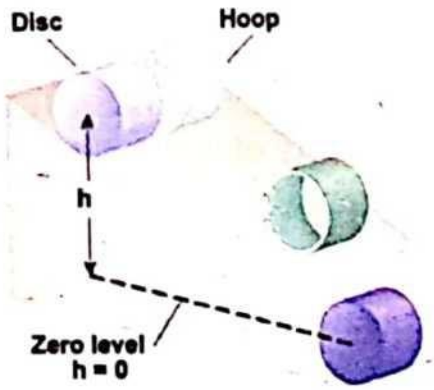


Fig5.27: Motion of hoop and disc on an inclined surface

Rolling of Hoop (Hollow cylinder/Ring)

Loss of P. E= Gain in $(K.E)_{Trans}$ + Gain in $(K.E)_{Rot}$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

But moment of inertia of Hoop (I) = mr^2

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\omega^2$$

$$2gh = v^2 + v^2 \qquad (\because v = r\omega)$$

$$2gh = 2v^2$$

$$v = \sqrt{gh} \text{ ......(5.32)}$$

Rolling of Disc (Solid cylinder)

Loss of P. E= Gain in $(K.E)_{Trans}$ + Gain in $(K.E)_{Rot}$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

But moment of inertia of Disc (I) = $\frac{1}{2}mr^2$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\times\frac{1}{2}mr^2\omega^2$$

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 \qquad (\because v = r\omega)$$

$$gh = \frac{3v^2}{4}$$

$$v = \sqrt{\frac{4}{3}gh} \text{ ......(5.33)}$$

Eqs. (5.32) and (5.33) clearly indicate that the velocities of hoop and disc are independent of their masses. It is worth noting that the velocity of disc is greater than the velocity of hoop due to its relatively large value of the moment of inertia.

5.9 ARTIFICIAL SATELLITE

In space, every smaller celestial body is revolving around every bigger celestial body. For example, moon revolves around the Earth, the Earth revolves around the Sun and so on. Like a natural planet such as moon, earth and so many others, an artificial satellite is a man-made planet. A rocket is used to launch it in an orbit around the Earth at certain height and speed.

The orbital motion of a satellite is due to the gravitational force of attraction between the Earth and satellite. A satellite is being used for the purpose of communication system (T.V, Telephone, Mobile, and Radio transmission), weather prediction, spying, guiding missile system, exploration of mineral resources and others scientific researches.

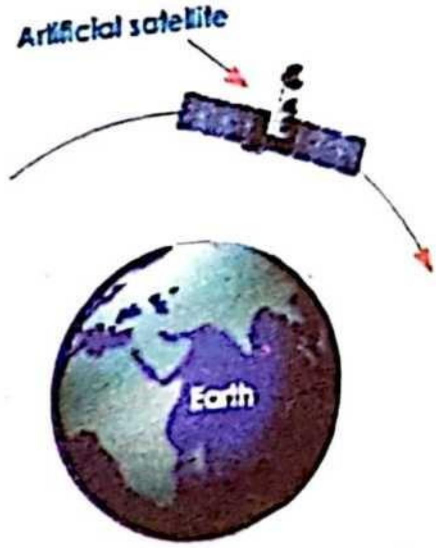


Fig.5.28: Artificial satellite revolving in its orbit around the earth

Consider a satellite of mass 'm' which is launched in an orbit around the Earth at certain height 'h' from the surface of the Earth as shown in Fig.5.28. The centripetal acceleration is provided to satellite by acceleration due to gravity of the Earth. Thus by definition of centripetal acceleration;

$$a_c = g = \frac{v^2}{R}$$

$$v = \sqrt{Rg} \quad ......(5.34)$$

$$v = \sqrt{(6.4\times10^6\,\text{m})(9.8\,\text{m}\,\text{s}^{-2})}$$

$$v = \sqrt{62.72\times10^6\,\text{m}^2\,\text{s}^{-2}}$$

$$v = 7.9\times10^3\,\text{m}\,\text{s}^{-1}$$

$$v = 7.9\,\text{km}\,\text{s}^{-1}$$

This is the minimum velocity required by a satellite to move in an orbit around the earth. This is also called critical velocity of satellite.

Now the time period of the satellite when its velocity is 7.9Kms^{-1} can be calculated as;

$$v = \frac{S}{t} \text{ or } t = \frac{S}{v}$$

For one revolution

$$S = 2\pi R$$

$$t = T$$

$$T = \frac{2\pi R}{v} \quad ......(5.35)$$

$$T = \frac{2(3.14)(6.4\times10^6\,\text{m})}{7.9\times10^3\,\text{m}\,\text{s}^{-1}}$$

FOR YOUR INFORMATION

According to the data of objects launched into outer space, maintained by the United Nations office for Outer Space Affairs (UNOOSA), there are 4635 satellites currently orbiting the Earth. Only 1738 satellites are active and 2897 are the piece of junk metal revolving around the Earth at a speed of 7.5 km/s.

$$T = 5087.59 \text{ s}$$
$$T = 84.79 \text{ min} = 85 \text{ min}$$

By increasing the height of the orbit from the surface of the earth, the velocity of the satellite decreases and its time period increases.

If a satellite completes its one revolution in 85 min. at the speed of 7.9 km s^{-1} then such orbit is at height 400 km from the surface of the Earth and this is the nearest orbit.

5.10 GEOSTATIONARY SATELLITE

An orbit around the Earth that lies in the plane of the equator and has the time period equal to the period of the Earth's rotation on its own axis (23 hours 55 minutes and 5 seconds) is known as geosynchronous orbit.

A satellite that revolves in the geosynchronous orbit is called geostationary or geosynchronous satellite. It appears from Earth to be stationary and it always remains over the same point on the equator as shown in Fig.5.29.

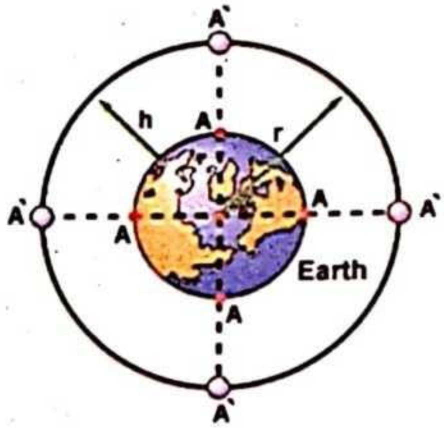


Fig.5.29: Geo-stationary Satellite revolving in its orbit around the earth.

Due to this advantage, the geostationary satellites are more useful for communication system, weather forecasting, navigation etc.

A geostationary satellite revolves round the Earth at a suitable height and velocity. Its velocity is same in magnitude and direction as the Earth does about its own axis and its relative velocity with respect to the earth is zero.

Consider a geostationary satellite of mass 'm' that is revolving in its orbit of radius 'r' from centre of the Earth as shown in Fig. 5.29. The gravitational force of attraction between the Earth and the satellite provides the necessary centripetal force. i.e.,

$$F_c = F_g$$

$$\frac{mv^2}{r} = G\frac{mM}{r^2}$$

$$v = \sqrt{\frac{GM}{r}} \quad ......(5.36)$$

Radius of geostationary satellite

The radius of the geostationary orbit can be calculated by using the relation for the speed of a satellite. i.e.,

$$v = \frac{2\pi r}{T} \quad ......(5.37)$$

Comparing Eq. (5.36) and Eq. (5.37) we get,

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}} \quad ......(5.38)$$

Orbit name	Orbit Altitude (km)	Comments
Low Earth Orbit (LEO)	200 - 1200	-
Medium Earth Orbit (MEO)	1200 - 35790	-
Geosynchronous Orbit (GSO)	35790	Orbits once a day, but not necessarily in the same direction as the rotation of the earth
Geostationary Orbit (GEO) or Geosynchronous equatoriat orbit	35786	Orbits once a day, and moves in the same direction as the earth, Can only be above the equator.
High Earth Orbit (HEO)	Above 35790	-

$$r = \left(\frac{6.673\times10^{-11}\,Nm^2kg^{-2}\times6\times10^{24}\,m\times(24\times3600\ s)^2}{4(3.14)^2}\right)^{\frac{1}{3}}$$

$$r = 4.23\times10^7\,m = 4.23\times10^4\,km$$

Height of the geostationary satellite above earth's surface

The height of satellite from surface of the earth is given as;

$$h = r - R$$

$$\therefore h = (42.4\times10^7\,m) - (6.4\times10^6\,m)$$

$$\therefore h = 3.59\times10^7\,m \simeq 3.6\times10^7\,m = 3.6\times10^4\,km$$

This shows that the orbit of the geostationary satellite is independent of the mass of the satellite.

Orbital speed of geostationary satellite

The speed of a geostationary satellite can be calculated as;

$$v = \frac{2\pi r}{T}$$

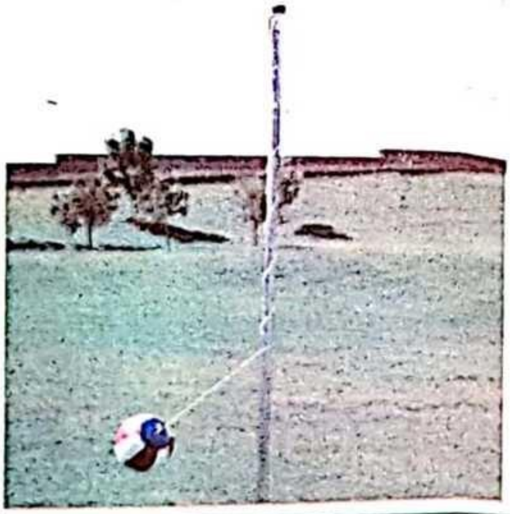
$$\therefore v = \frac{2(3.14)(4.23\times10^7\,m)}{(24\times3600s)} = 3.1\times10^3\,ms^{-1}$$

$$\therefore v = 3.1\ km\ s^{-1}$$

This shows that the geostationary satellite revolves around the earth at a height of 36000 km above Earth's surface with an orbital speed of 3.1 km s^{-1}.

POINT TO PONDER

Why a tetherball seems to speed up as it wraps around the pole?



Example 5.11

A satellite is revolving in an orbit around the earth at height 600 km from the surface of Earth. Calculate the speed and time period of satellite. Given that the radius of the earth is 6.4×10^6 m.

Solution:

$h = 600 \text{ km} = 0.6 \times 10^6 \text{m}$

$v = ?$

$R = 6.4 \times 10^6 \text{m}$

Radius of the orbit of satellite (r) = R + h

$$r = 6.4\times10^6\text{m} + 0.6\times10^6\text{m}$$

$$r = 7\times10^6\text{m}$$

By definition of orbital velocity;

$$v = \sqrt{\frac{GM}{r}} \text{ ...... (1)}$$

At the surface of Earth

$$F = \frac{GmM}{R^2}$$

But, $F = W = mg$

So, $$mg = \frac{GmM}{R^2}$$

$$GM = gR^2$$

Eq. (1) becomes

$$v = \left(\sqrt{\frac{g}{r}}\right)R = \left(\sqrt{\frac{9.8}{7\times10^6}}\right)6.4\times10^6 = 7.57\times10^3\text{ms}^{-1}$$

$$v = 7.57 \text{ kms}^{-1}$$

$$T = \frac{2\pi r}{v} = \frac{2(3.14)(7\times10^6)}{7.57\times10^3} = 5807 \text{ s}$$

$$T = 97 \text{ min.}$$

5.11 COMMUNICATION SATELLITES

Like a football, our Earth is of a spherical shape, so there are some technical complications to set up the communication system among the whole countries of the world by using towers. These problems are overcome by introducing a satellite communication system. It consists of several geostationary satellites which are orbiting at different points above the surface of the Earth. This satellite communication system has converted the world in a global village.

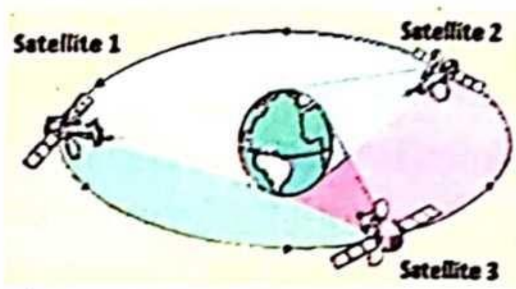


Fig.5.30: Three communications Geostationary Satellites which covers the whole earth.

One such a geostationary satellite has a capacity to cover 120° of longitude. So three satellites are sufficient to cover the whole Earth as shown in Fig. 5.30.

Since these geostationary satellites are appear to be stationary over one place on the Earth, thus continuous communication with any place of the Earth can be made. Microwaves signals are used for communication. The energy needed to amplify and retransmit the signals is provided by a large solar cell panels installed on the satellites. There are over 200 Earth stations which transmit signals to satellites and receive signals via satellites from other countries.

You can also pick up the signals from the satellites using a dish antenna placed on the roof of your house. The largest satellite system is managed by 126 countries, international telecommunication satellite organization (INTELSAT). The INTELSAT VI satellite operates at microwave frequencies 4,6,11 and 14 GHz and has a capacity of 30,000 two-way telephone circuits including three TV channels.

5.12 REAL AND APPARENT WEIGHTS

Weight is a force which is produced in a body by gravity of the Earth. It depends upon 'g' and is always directed towards the centre of the earth.

According to Newton's third law of motion, the weight of a body has normal reaction (N) which is also known as supporting force of the body and it is acting normally upward. Now when the supporting force is equal to the weight of the body then the weight is called real weight.

Fig.5.31: Two forces are acting on a suspanded block, weight of the block downwards and tension of the string upwards.

If the supporting force is greater or less than the weight of the body, then the weight is called apparent weight. When the supporting force is zero then the weight of the body is weightless and this condition is called weightlessness. The weightlessness can be observed when the body falls freely under the action of gravity and the body in a satellite orbiting around the Earth.

All the conditions of weights that is real weight, apparent weight and weightlessness can be studied with the help of spring balance, connects with a block of mass 'm' and is suspended by a string of Tension 'T' from the ceiling of the elevator, as shown in Fig.5.31. It may be noted that weight of the block is acting downward while tension of the string is acting upward.

Case I: When the elevator is at rest or moving with uniform motion

When the elevator is at rest or moving with uniform motion then its acceleration is zero($a = 0$) as shown in Fig.5.32. The net force acting on the body will be;

$$F = W - T$$

$$ma = W - T$$

As $a = 0$

Then $T = W$ ......(5.39)

This is the real weight which can be measured using a spring balance.

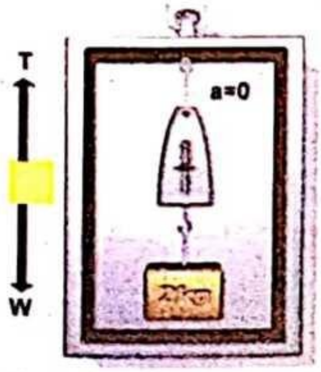


Fig.5.32: Elevator is at rest or moving with uniform velocity.

Objects in freefall experience weightlessness.

Case II: When the elevator is moving upward

When the elevator is moving upward with acceleration 'a' as shown in Fig.5.33, then $T > W$ and the net force will be;

$$F = T - W$$

$$T = W + ma \text{ ......(5.40)}$$

This shows that the apparent weight is increased by an amount 'ma'.

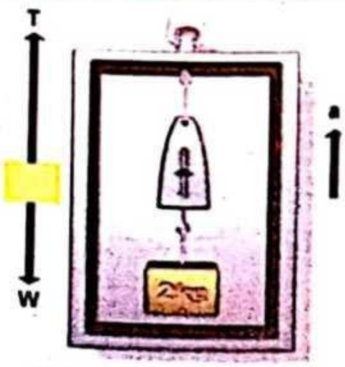


Fig.5.33: Elevator is moving upward

Case III: When the elevator is moving downward

When the elevator is moving downward with acceleration 'a' as shown in Fig.5.34, then $T < W$ and the net force acting on the body will be;

$$F = W - T$$

$$T = W - ma \text{ ......(5.41)}$$

This shows that the apparent weight is decreased by an amount 'ma'.

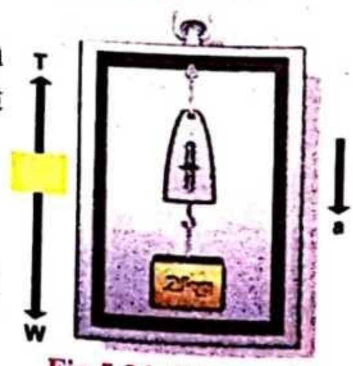


Fig.5.34: Elevator is moving downward.

Case IV: When the elevator is falling freely

Suppose the string is broken and the elevator falls freely under the force of gravity, then $a = g$;

$$F = W - T$$
$$T = W - ma$$
$$T = mg - mg$$
$$T = 0$$

The spring balance will show zero reading and this condition is called weightlessness of a body.

5.13 WEIGHTLESSNESS IN SATELLITES

When a satellite is launched by a rocket in an orbit around the Earth then it has been observed experimentally that everything inside the satellite experiences weightlessness because the satellite is a freely falling body.

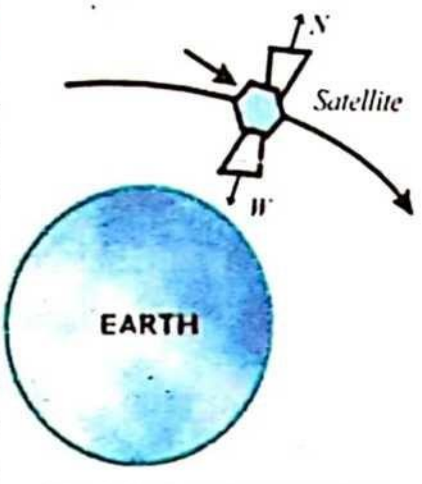


Fig.5.35: Two forces acting on a satellite which is revolving in its orbit

Consider a satellite of mass 'm' that is revolving in its orbit of radius 'r around the Earth of mass 'M'. Two forces are acting on it, that is, weight 'mg' of the satellite is acting downward while the supporting force N is acting upward as shown in Fig.5.35. The normal force is less than the weight and the difference between them provides the centripetal force. According to Newton's 2nd law the net force on the satellite is given as;

$$F = mg - N$$

But

$$F_c = \frac{mv^2}{r}$$

$$\frac{mv^2}{r} = mg - N \text{ ......(5.42)}$$

It may be noted that the centripetal force acting on satellite is provided by gravitational force of attraction between Earth and satellite. i.e.,

$$F_g = F_c$$

$$Mg = \frac{Mv^2}{r}$$

$$g = \frac{v^2}{r}$$

Eq. 5.42 becomes

$$mg = mg - N$$

$$N = mg - mg = 0$$

$$N = 0 \;......(5.43)$$

Since **N** is zero, the force exerted by the support force on the body revolving in a satellite is zero. Hence, the force that the body exerts on the support is also zero. The body and the astronaut in a satellite therefore, find themselves in a state of apparent weightlessness.

5.14 ARTIFICIAL GRAVITY

It has been observed that when a spacecraft is revolving in its orbit around the Earth, then it is in state of weightlessness. The astronauts inside the satellite face difficulties to perform their routine work. To overcome these problems, an artificial gravity is developed by rotating the spacecraft with certain frequency about its axis.

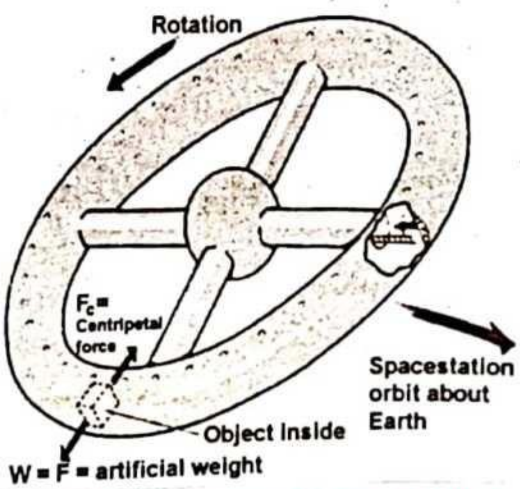


Fig.5.36: Two forces acting on a satellite which is revolving in its orbit

Considering a spacecraft of outer radius *R* and it is rotating about its axis with angular velocity 'ω' as shown in Fig.5.36. So its centripetal acceleration is given as;

$$a_c = R\omega^2 \;......(5.44)$$

Since $\omega = \frac{2\pi}{T}$, therefore eq. (5.44) becomes

$$a_c = R\left(\frac{4\pi^2}{T^2}\right)$$

$$a_c = 4\pi^2 R\left(\frac{1}{T^2}\right)$$

$$a_c = 4\pi^2 R f^2 \qquad \therefore f = \frac{1}{T}$$

$$f^2 = \frac{a_c}{4\pi^2 R}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

Gravitational force diminishes as you go away from earth, but it is never zero.

The environment inside the satellite will be same as that on the surface of Earth, if $a_c = g$ and above equation reduces to;

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}} \quad ......(5.45)$$

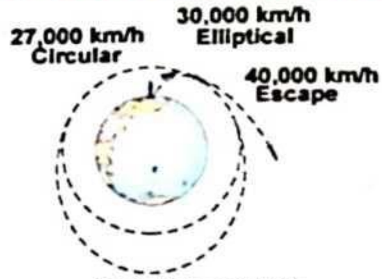
Hence, the spacecraft can produce the required necessary artificial gravity if it is rotated at the frequency given by Eq. (5.45).

Planets' orbital velocities and distance from sun			
	Mean Distance from Sun (million Km)	Velocity m/s	Velocit y Km/s
Mercury	57.9	47685.39	47.69
Venus	108.2	35095.49	35
Earth	149.6	29779.38	29.78
Mars	227.9	24154.38	24.15
Jupiter	778.6	13059.18	13.06
Saturn	1433.5	9641.47	9.64
Uranus	2872.5	6799.73	6.8
Neptune	4495	5431.54	5.43

Example 5.12

A spacecraft consists of two chambers connected by a tunnel of length 20 m. How many revolutions per second must be made by the space craft to provide the required artificial gravity for the astronauts?

FOR YOUR INFORMATION



Satellite Orbits

Solution: $\ell = 20\ m$

$$R = \frac{\ell}{2} = 10\ m$$

$$g = 9.8\ ms^{-2}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

$$f = \frac{1}{2(3.14)}\sqrt{\frac{9.8\ ms^{-2}}{10\ m}} \quad \text{or} \quad f = 0.158\ rev\,s^{-1} \text{ or } 0.158\ Hz$$

5.15 ORBITAL VELOCITY

A small heavenly body revolves around a massive body due to the gravitational pull of massive body. The mass of the bigger body controls the orbit of small body and also speed with which it revolves around it. The more massive the bigger body, the greater is the gravitational pull and faster the smaller body must revolve.

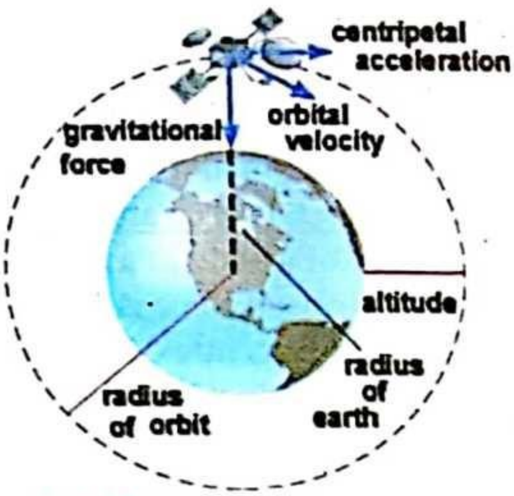


Fig.5.37: A satellite is revolving in its orbit around the earth with an orbital velocity.

It has been observed that all the planets, stars, satellites and space crafts are revolving in nearly circular path. These circular paths are known as orbits. The motion of all these bodies in their orbits is called orbital motion and their velocities are called orbital velocity.

In order to obtain a relation for the orbital velocity, we consider a massive body of mass M around which a smaller body of mass m is revolving. Let the speed of revolution be 'v' and the radius of the orbit be 'r' from the centre of the Earth as shown in the Fig.5.37.

It is a well-known fact that the centripetal force is provided by the gravitational force between satellite and Earth that is:

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}} \quad ......(5.46)$$

The velocity of the satellite is independent of its mass.

This is the required orbital velocity of the satellite.

SUMMARY

- **Circular motion**: The motion of a body along a circular path of constant radius is called circular motion.
- **Angular displacement:** The angle subtended at the centre of circle by an arc along which it moves in a given time is known as angular displacement. Its direction can be found by right hand rule.
- **Angular velocity ω:** The rate of change of angular displacement is called angular velocity.
- **Angular Acceleration α:** The rate of change of angular velocity is called angular acceleration.
- **Relationship between linear and angular variables:** Relationship between linear and angular variables are ; $S = r\theta$, $v = r\omega$, $a = r\alpha$
- **Centripetal force and Centripetal acceleration**: The force which keeps the motion of a body in a circle is called centripetal force. The acceleration produced by centripetal force is called centripetal acceleration. These are always directed towards the centre of the circle.
- **Moment of inertia**: The resistive property of a body to oppose any change in its state of rest or rotational motion is called moment of inertia.
- **Angular momentum:** The vector product of radius and linear momentum is called angular momentum and it is conserved in the absence of an external torque.

- **Rotational kinetic energy:** The kinetic energy of body due to its rotational motion is called rotational kinetic energy
- **Geostationary satellite:** A man made artificial planet revolving around the Earth at certain speed and height is known as satellite and satellite whose time period is 24 hours is called geostationary satellite
- **Apparent weight**: The weight of the object in equilibrium state is called real weight, while variable weight is apparent weight. When the supporting force is equal to zero then the object is in the state of weightlessness.

EXERCISE

Select the best option of the following question.

1. 85.95° degree in terms of radian is

(a) $\frac{1}{2}$ radian (b) 1 radian (c) $1\frac{1}{2}$ radian (d) 2 radian

2. What is the circumference of a circle, having radius of 50cm?

(a) 3.12 m (b) 3.14 m (c) 3.16 m (d) 3.18 m

3. What is the angular velocity of a particle when its frequency is 50 Hz?

(a) 312 rad s^{-1} (b) 313 rad s^{-1} (c) 314 rad s^{-1} (d) 315 rad s^{-1}

4. The direction of angular velocity is along;

(a) Tangent the circular path (b) Axis of rotation
(c) Inward the radius (d) Outward the radius

5. Angular speed for annual rotation of Earth in radian per day.

(a) $\frac{\pi}{2}$ radian/day (b) π radian/day
(c) 2π radian/day (d) 365 radian/day

6. A body moves with constant angular velocity in a circle. Magnitude of angular acceleration is

(a) $r\omega^2$ (b) Constant (c) zero (d) rω

7. Banking angle does not depend upon

(a) Mass (b) Speed (c) Radius
(d) Gravitational acceleration

8. What will happen if the height of an orbit of a satellite from surface of earth is increased,

(a) Speed increases (b) Angular velocity increases
(c) Time period increases (d) Gravitational acceleration increases

9. What is the ratio of translational and rotational kinetic energies of a solid sphere.

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$

10. Angular momentum in terms of moment of inertia is

(a) $I\omega$ (b) $I\omega^2$ (c) $I^2\omega$ (d) $\frac{I}{\omega^2}$

11. Which one of the following rolling body has 50% translational K.E and 50% rotational K.E.

(a) Disc (b) Ring (c) Rod (d) Sphere

12. A solid cylinder of mass 20 kg rotates about its axis with angular velocity of $10 m\,s^{-1}$ radius 0.2m. The moment of inertia of the cylinder is

(a) $0.2\ kg\ m^2$ (b) $0.4\ kg\ m^2$ (c) $0.6\ kg\ m^2$ (d) $0.8\ kg\ m^2$

13. The condition of weightlessness is (N = Normal Reaction and W is weight)

(a) $N > W$ (b) $N < W$ (c) $N = W$ (d) $N = 0$

14. A sphere is rolling without slipping on a horizontal plane. The ratio of its rotational kinetic energy and translational kinetic energy is

(a) 2:3 (b) 2:5 (c) 2:7 (d) 2:9

15. The frequency of artificial gravity is

(a) $2\pi\sqrt{\frac{g}{R}}$ (b) $2\pi\sqrt{\frac{R}{g}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{R}{g}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

16. A particle of mass 100g starts its motion from rest along a circular path of radius 10 cm. If its velocity becomes $10\,m\,s^{-1}$ then its centripetal force is;

(a) 0.1 N (b) 1 N (c) 10 N (d) 100 N

17. Which one of the following is conserved when the torque acting on a system is zero?

(a) K. E (b) Angular momentum
(c) Rotational K. E (d) Linear momentum

18. Orbital velocity of earth's satellite near the surface is 7 km/s. If the radius of the orbit is 4 times than that of Earth's radius, what will be the orbital velocity in that orbit?

(a) $3.5\ kms^{-1}$ (b) $7\ kms^{-1}$ (c) $7\sqrt{2}\ kms^{-1}$ (d) $14\ kms^{-1}$

19. When a parachutist is moving downward with uniform motion then its weight is;

(a) Decreasing (b) Increasing (c) Remain same (d) Zero

20. An object of mass 1.5 kg is suspended by a string having tension T = 5 N in a lift. When the lift is moving upward with acceleration $a = 2\ ms^{-2}$ then its apparent weight is:
(a) 8 N (b) 5 N (c) 2 N (d) 1.5 N

COMPREHENSIVE QUESTIONS

1. Define the following terms;
   (i) Angular displacement **(ii)** Angular velocity **(iii)** Angular acceleration.
2. Derive the relationship between linear and angular variables.
3. State and explain centripetal acceleration and centripetal force. Also derive their mathematical relations.
4. Define banking of road and justify that how does it provide a necessary centripetal force to a vehicle.
5. What is moment of inertia? Show that moment of inertia depends upon mass and radius of the circle in which the body is moving.
6. State and explain angular momentum and law of conservation of momentum.
7. What do you know about the artificial satellite. Discuss the speed, time period and height of an artificial satellite.
8. Define geostationary satellite and it role in the communication system.
9. Explain the terms real weight, apparent weight and weightlessness of a body.
10. Describe weightlessness in satellite and the artificial gravity.

SHORT QUESTIONS

1. How many different units are used for measurement of angular displacement? Explain.
2. How can you define one radian?
3. What is the relationship between arc length and angular displacement?
4. How can a body move along a circle? What is the direction of its velocity?.
5. How can you calculate the angular velocity of Earth about its axis in $rad.s^{-1}$?
6. Why the speed of a rolling disc is greater than the speed of a rolling hoop while both have same masses?
7. How does banking road provide a centripetal force to a moving car?
8. Why torque and work done are not possible by centripetal force?
9. Under what condition the angular momentum of a body is conserved?

10. There are two spheres of copper and lead of same mass. It is found that the lead sphere can be rotated more easily. Explain why?

11. What are the values of speed, height and time period of a geostationary satellite?

12. Is it possible to launch an artificial satellite in an orbit such that it always remains visible directly over Quetta? Explain.

13. What is the minimum number of geostationary satellites for world T.V communication system.

14. Distinguish between real and apparent weights.

15. Is there any difference between orbital motion and rotational motion?

16. How can an artificial gravity be produced?

17. What is the frequency of oscillations of a simple pendulum inside an artificial satellite?

18. The cylinders A and B are of the same mass but the radius of A is greater than that of B. Which one will require more force to come into rotation? Why?

19. What is the angular velocity of the Earth spinning about its axis?

20. How the rotation of Earth will be affected if its density becomes uniform?

NUMERICAL PROBLEMS

1. The Earth completes one rotation about its axis in 24 hours. Calculate (a) the angular speed of the Earth (b) tangential speed of body at the equator. (Radius of the Earth is 6.4×10^{6} m) **7.3×10^{-5} rad/s, 467 m s^{-1}.**

2. The diameter of the wheels of a car is 70 cm. It starts from rest and accelerates uniformly to a speed of 12 m s^{-1}, in time 6 s. Calculate the angular acceleration of the wheels and the number of revolutions made in this time.

(5.7 rad s^{-2}, 16 rev.)

3. A geostationary satellite is revolving around the Earth in the orbit of radius 42.4×10^{6} m in time period of 24 hours. Calculate (a) tangential speed (b) centripetal acceleration. **(3×10^{3} m s^{-1}, 0.23 m s^{-2})**

4. A body of mass 'm' connected with a string of length 'ℓ', is whirled in a horizontal circle. Find the centripetal force (a) when the length of the string is doubled (b) when the tangential velocity of the body is doubled.

(a) $\frac{1}{2}F_c$, (b) $4F_c$

5. What is the banking angle of a curved road of radius 25 m if a car may make the turn at a speed of 11 m s^{-1}? **(26^o)**

6. A 3 kg pulley of radius 30 cm is rotating at the rate of 400 rev/m. Calculate its moment of inertia and its rotational K.E. **(0.27 kg m^2, 0.24 kJ)**

7. The gravitational force on a satellite exerted by Earth on its surface is 'F'. What will be the gravitational force on the satellite, when it is at a height of R/50 where 'R' is the radius of the Earth. (0.96 F)

8. An electron of mass 9.1×10^{-31} kg is revolving in its allowed orbit around the nucleus of radius 5.3×10^{-11}m with velocity 2.2×10^{6} m s^{-1}. Calculate angular momentum and rotational K.E. of electron about the nucleus.

(1.06×10^{-34} Js, 2.2×10^{-18} J)

9. What is the resultant force acting on 70 kg man in a lift which is accelerating upward with 9.8 m s^{-2}? Also calculate the resultant force when the lift falls freely under gravity. **(1372N, 0)**

10. What is the orbital velocity and time period of moon when it is revolving in its orbit around the Earth at height 384000 km, from surface of the earth? Mass of Earth is 6×10^{24} kg and its radius is 6400 km. **(1.01 km s^{-1}, 27.5 days)**

11. At what speed of the outer rim of space craft is rotated in order to produce an artificial gravity equal to 9.8m s^{-2}. The radius of space craft is 60 m, also calculate its time period of rotation. **(42.2 m s^{-1}, 15.6 s)**

12. With what speed a space station should rotate in order to produce at its outer rim an artificial gravity equal to 9.8m s^{-2}? The radius of space station is 85 m? Also calculate its period of rotation? **(29 m s^{-1}, 18.5 s)**

Unit 6

FLUID DYNAMICS

Major Concepts (18 PERIODS)

- Streamline and Turbulent flow
- Equation of continuity
- Bernoulli's equation
- Applications of Bernoulli's equation
- Viscous fluids
- Fluid Friction
- Terminal velocity

Conceptual Linkage

This chapter is built on
Work & Energy Physics IX
Dynamics Physics IX
Properties of Matter Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- Define the terms: steady (streamline or laminar) flow, incompressible flow and non viscous flow as applied to the motion of an ideal fluid.
- Explain that at a sufficiently high velocity, the flow of viscous fluid undergoes a transition from laminar to turbulence conditions.
- Describe that the majority of practical examples of fluid flow and resistance to motion in fluids involve turbulent rather than laminar conditions.
- Describe equation of continuity Av = Constant, for the flow of an ideal and incompressible fluid and solve problems using it.
- Identify that the equation of continuity is a form of the principle of conservation of mass.
- Describe that the pressure difference can arise from different rates of flow of a fluid (Bernoulli Effect).
- Derive Bernoulli equation in the form $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$ for the case of horizontal tube of flow.
- Interpret and apply Bernoulli Effect in the: filter pump, Venturi meter, in, atomizers, flow of air over an aerofoil and in blood physics.
- Describe that real fluids are viscous fluids.
- Describe that viscous forces in a fluid cause a retarding force on an object moving through it.
- Explain how the magnitude of the viscous force in fluid flow depends on the shape and velocity of the object.
- Apply dimensional analysis to confirm the form of the equation $F = A\,\eta r v$ where 'A' is a dimensionless constant (Stokes' Law) for the drag force under laminar conditions in a viscous fluid.
- Apply Stokes' law to derive an expression for terminal velocity of spherical body falling through a viscous fluid.

INTRODUCTION

Basically, there are three states of matter namely solid, liquid and gas. Each state has different nature on the basis of its different properties. For example, the atoms in liquids and gases are not closely bounded but they are at some distance. Typically, the distance between two molecules of liquid is 10^{-7} m and the distance between molecules of the gases is 10^{-1} m. Due to this large space between molecules of liquid and gases, they have the ability to flow under the influence of some applied forces and hence they are called fluid. A liquid flows and acquires the shape of the container. A gas also flows into a container and spreads out until it occupies the entire volume of the container.

Moreover, the distance between the molecules of gas is more than the liquids, so a gas can be compressed while the compression of liquid is almost negligible. Fluid plays a vital role in many aspect of our everyday life. For example, we drink them, breath them, swim in them, they circulate through our bodies, airplanes fly through them, ships float in them. The study of fluids at rest in equilibrium situations is called fluid statics and the study of fluids in motion is called fluid dynamics and it is a most complex branch of mechanics. In this chapter, all the parameters which are related to fluids such as; viscosity, density, pressure, equation of continuity, Bernoulli's equation, Torricelli's theorem and Venturi relation will be studied. All these are related to incompressible and steady flow and these have been derived on the basis of law of conservation of mass and law of conservation of energy.

6.1 VISCOSITY

The property of a liquid by virtue of which it opposes relative motion between its two adjacent layers is called viscosity.

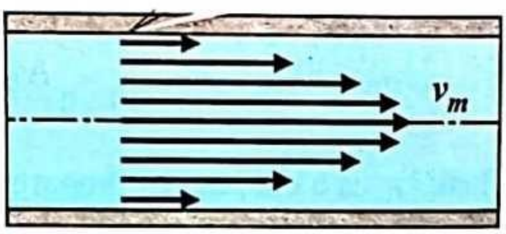


Fig.6.1: Different layers of fluid having different velocities.

Some liquids flow more easily than others. For example, honey is very "thick" and flows very slowly while, water is very "thin" as compared to honey and flows very quickly. In other words, honey offers more resistance than water. This resistive property of a liquid is called its viscosity and it is due to the friction between the two relative layers of a fluid. It explained by, considering a flow of liquid between two solid surfaces which consists of 'n' number of layers. The layer in contact with solid surface i.e. top and bottom solid surfaces is almost stationary, because its velocity is zero. Consider the layers of liquid above the bottom fixed solid surface velocities of upper layers are increasing step by step with distance i.e., the greater the distance of a layer from the surface, the greater is its velocity. A similar phenomenon can be observed for successive layers of liquid below the top fixed solid surface. Hence, the velocity of central layer is maximum as shown in Fig. 6.1.

Now consider the two parallel relative layers AB and A′B′ separated by distance 'y' from each other as shown in Fig. 6.2. The upper layer A′B′ has greater velocity than the velocity of the lower layer AB. Therefore, the layer A′B′ is sliding over the layer AB with velocity 'v', so there is a frictional force between the layers AB and A′B′. This force is called viscous drag force which exists between every two parallel and relative layers of the fluid.

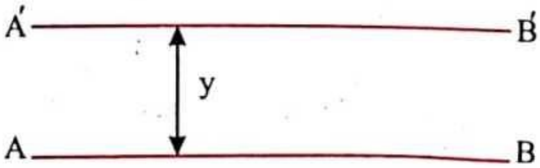


Fig.6.2: Two layers of fluid at a distance Y

Due to this viscous dragging force, the slower layer exerts a tangential retarding force F on the faster upper layer and experiences itself an equal and opposite tangential force due to the upper layer. To overcome the drag force, an external force must be applied. The applied force depends upon the factors like area of layer of fluid (A), velocity of drag of layer (v) and separation between two layers y. The dependence of applied force F is as under:

Liquid	Temperature (°C)	Viscosity (centipoise) (cP)
Water	20	1
Nitric Acid	20	1.7
Milk	10	2
Sulfuric Acid	20	18
Olive Oil	20	84
Glycerin	20	648
Shampoo	36	3000
Castor Oil	20	1000
Honey	36	2000-10000

$$F \propto A$$

$$F \propto v$$

$$F \propto \frac{1}{y}$$

Combine all the above results.

$$F \propto \frac{Av}{y}$$

$$F = \eta \frac{Av}{y} \quad ......(6.1)$$

where 'η' is a constant of proportionality and is known as co-efficient of viscosity. It depends upon temperature and nature of the fluid. Rearranging Eq. (6.1), we get;

$$\eta = \frac{Fy}{Av}$$

If $A = 1m^2$, $v = 1m\ s^{-1}$ and $y = 1m$, then, $F = \eta$, thus the co-efficient of viscosity may be defined as;

The coefficient of viscosity of a fluid is the force required per unit area to maintain the unit relative velocity between the two relative layers of liquid separated by distance of 1m to each other.

The SI unit of 'η' is Ns m^{-2} and its dimensional formula is $[ML^{-1}T^{-1}]$.

Example 6.1

A plate of area $0.1m^2$ is separated from another plate by a layer of glycerin of thickness 2 mm. If the co-efficient of viscosity of glycerin is 0.950 Ns m^{-2}, calculate the horizontal force required to the plate moving with velocity 0.1 ms^{-1}.

Solution:

$A = 0.1\ m^2$

$y = 2\ mm = 2 \times 10^{-3}\ m$

$\eta = 0.950\ N\ s\ m^{-2}$

$v = 0.1\ ms^{-1}$

$F = ?$

$$F = \eta \frac{Av}{y}$$

$$F = \frac{0.950\ Ns\ m^{-2} \times 0.1\ m^2 \times 0.1\ ms^{-1}}{2 \times 10^{-3}\ m}$$

$F = 4.75\ N$

6.2 STOKE'S LAW AND TERMINAL VELOCITY

When a solid body falls free through a viscous medium, its motion is opposed by a force called viscous drag force which is due to the relative motion between the layers of the viscous medium. The magnitude of this viscous drag force increases with the velocity of the body. It was studied by an English physicist Sir George Gabriel Stoke and the corresponding law is named after him.

Consider a solid sphere of mass 'm' radius 'r' which is moving with velocity 'v' in a viscous medium whose coefficient of viscosity is 'η' as shown in Fig.6.3.

According to Stoke's the drag force is directly proportional to the velocity of the sphere, radius of the sphere and coefficient of viscosity of the medium. Hence,

$$F \propto v$$

$$F \propto r$$

$$F \propto \eta$$

Combine all these results.

$$F \propto \eta v r$$

$$F = k \eta v r$$

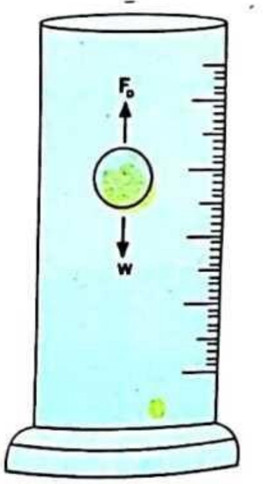


Fig.6.3: Two opposite forces acting on a body moving through a viscous medium

where 'k' is a constant of proportionality. For a small perfectly rigid sphere, the value of k is found to be 6π.

$$F = 6\pi\eta vr \;......(6.2)$$

The above relation is called Stoke's law.

6.2.1 Dimensional analysis of Stokes law

The Stokes' law can further be analyzed by the method of dimensions. Stokes observed that the drag force on slow moving body through viscous fluid depends upon the following factors

1. Velocity of the body ($F \propto v$)
2. Radius of the body ($F \propto r$)
3. Co-efficient of viscosity ($F \propto \eta$)

For dimensional analysis, we can combine these relations as;

$$F \propto v^a r^b \eta^c$$

$$F = \text{constant } v^a r^b \eta^c$$

$$F = k v^a r^b \eta^c \;......(6.3)$$

where 'k' is dimensionless constant and a, b and c are the dimensional coefficient of v, r and η respectively.

Now putting the dimensions of the given terms in equation (6.3) we get,

$$[M\,L\,T^{-2}] = [L\,T^{-1}]^a\,[L]^b\,[M\,L^{-1}\,T^{-1}]^c$$

$$[M\,L\,T^{-2}] = [L]^a\,[T]^{-a}\,[L]^b\,[M]^c\,[L]^{-c}\,[T]^{-c}$$

$$[M\,L\,T^{-2}] = [L]^{a+b-c}\,[T]^{-a-c}\,[M]^c$$

Comparing the respective terms

$$[M]^1 = [M]^c$$

$$[L]^1 = [L]^{a+b-c}$$

$$[T]^{-2} = [T]^{-a-c}$$

$$c = 1,\ -a - c = -2 \text{ and } a + b - c = 1$$

By solving these relations, we get

$$c = 1,\ a = 1 \text{ and } b = 1$$

Putting the values of a, b and c in equation (6.3), we get

$$F = kvr\eta$$

As the value of 'k' was calculated by Stokes for a small sphere as 6π, therefore;

$$F = 6\pi\eta rv$$

6.2.2 Terminal Velocity

Consider a solid sphere of mass 'm' and radius 'r' that falls vertically downward under gravity in a long column of a viscous liquid. When a body is dropped in a viscous medium (fluid), two forces act on it; the weight of the body 'W' acting downward and the viscous drag force 'F' acting upward as shown in Fig.6.4.

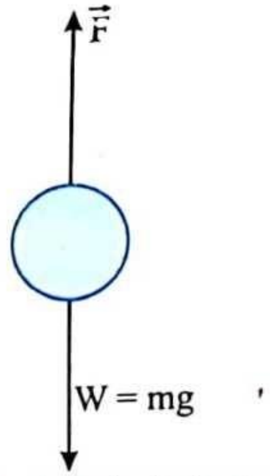


Fig.6.4: A body (sphere) is moving with terminal velocity in viscous medium under the action of two opposite forces.

The analysis shows that the drag force is proportional to the velocity 'v'. Initially the weight of the body is greater and the viscous drag force is zero. So the sphere is accelerating downward as shown graphically in Fig.6.5 (at point A).

Now when the velocity of the sphere increases, the viscous force also increases as shown in Fig.6.5 (at point B). At a certain instant, the viscous drag force becomes equal to weight 'W' of the sphere. The net force then becomes zero and now the sphere falls with constant velocity. This constant velocity is known as the terminal velocity as shown in Fig.6.5 (at points C & D). The value of this terminal velocity can be calculated by using Stokes law.

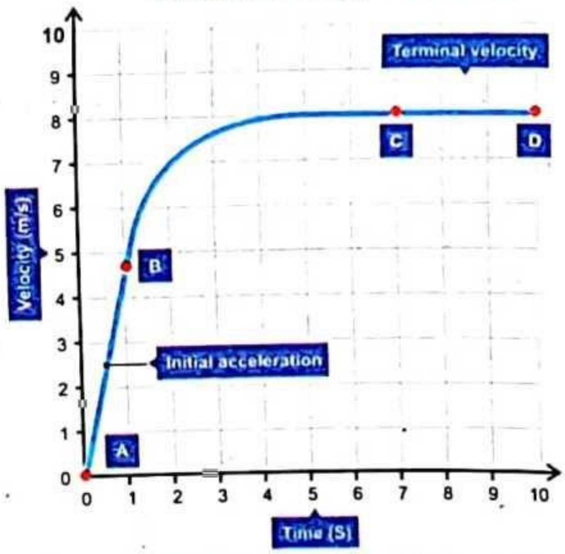


Fig.6.5: Graph between velocity and time shows various stages of the velocity of a body in a viscous medium

$$F = 6\pi\eta r v_t$$

As body is falling under gravity therefore

$$F = mg$$

$$mg = 6\pi\eta r v_t$$

$$v_t = \frac{mg}{6\pi\eta r} \quad ......(6.4)$$

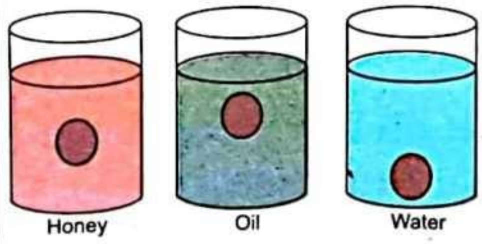
From the definition of density

$$\rho = \frac{\text{mass}}{\text{volume}}$$

As the volume of sphere is $\frac{4}{3}\pi r^3$

$$\rho = \frac{m}{\frac{4}{3}\pi r^3}$$

INFORMATION



Falling balls of same size and density through different liquids with the greatest viscosity hinders the falling balls speed the greatest.

So, $$m = \frac{4}{3}\pi\rho r^3$$

Substitute this value of m in eq. (6.4)

$$v_t = \left(\frac{4}{3}\pi\rho r^3\right)\left(\frac{g}{6\pi\eta r}\right)$$

$$v_t = \frac{2gr^2\rho}{9\eta} \quad ......(6.5)$$

POINT TO PONDER

Why rain drops do not produce any unpleasant effect on us?

This result shows that at constant density and viscosity, the terminal velocity of a spherical body falling freely through a viscous fluid is directly proportional to the square of its radius. It means that for a given medium, the terminal velocity of a large sphere is greater than that of a small sphere of the same material.

Example 6.2

What is the terminal velocity of a ball of diameter 4 cm and of average density of 90 kg m^{-3} which is allowed to fall in oil of viscosity 0.03 Ns m^{-2}?

Solution:

$v_t = ?$

D = Diameter = 4 cm = 0.04 m

$R = \text{Radius} = \frac{D}{2} = \frac{0.04}{2}\,m = 0.02\,m$

$\rho = 90\,kg\,m^{-3}$

$\eta = 0.03\,Ns\,m^{-2}$

$g = 9.8\,ms^{-2}$

$$v_t = \frac{2gr^2\rho}{9\eta}$$

$$v_t = \frac{2(9.8\,ms^{-2})(0.02\,m)^2(90\,kg\,m^{-3})}{9(0.03\,Ns\,m^{-2})}$$

$$v_t = \frac{0.7056}{0.27}$$

$$v_t = 2.6\,ms^{-1}$$

FOR YOUR INFORMATION

Type of particle	Diameter (μm)	Terminal Velocity (m/s)
Condensation nuclei	0.2	0.0000001
Typical cloud droplet	20	0.01
Large cloud droplet	100	0.25
Large droplet or drizzle	200	0.7
Small raindrop	1000	4
Typical raindrop	2000	6.5
Large raindrop	5000	9

6.3 FLUID FLOW

The study of motion of fluids is an important practical subject and it plays a vital role in various fields like automobile engineering, aeronautics, civil engineering, mechanical engineering, marine engineering, sports engineering and meteorology. It is an established fact that there is a large distance between the atoms or molecules of a fluid as compared with a solid. Due to this property, a fluid has ability to flow when an external force is applied on it. This is called fluid flow. There are two kinds of the flow of fluid i.e. steady flow and turbulent flow.

(i) Steady or laminar Flow

The flow of fluid is said to be steady or laminar if its each particle passing through a certain point follows exactly the same velocity as its preceding particles. The path taken by the particle of fluid is known as stream line. Stream lines in steady flow do not cross each other as shown in Fig.6.6. In steady flow the velocity of the liquid may be different at different points, but the velocity of its each particle at a particular point and at given instant remains constant. The stream line may be straight line or curved. The condition of stream line motion depends on the velocity of the flow of a fluid. The motion of a fluid remains streamlined when the average velocity of the fluid remains smaller than a certain value called critical velocity.

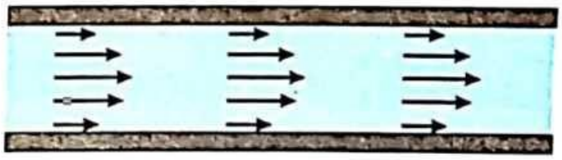


Fig.6.6: Steady or Laminar flow

(ii) Turbulent Flow

The irregular or non-steady flow of fluid is called turbulent flow. In turbulent flow there are continuous fluctuation in velocity and pressure at each point as shown in Fig. 6.7.

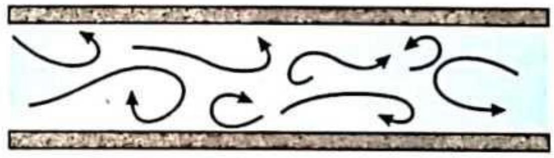


Fig.6.7: Turbulent or Irregular flow

If the velocity of fluid is greater than critical velocity, the motion loses all its orderliness and becomes zigzag. The velocity of fluid molecules at any point is different in magnitude as well as in direction in a random manner and eddies and whirlpools are formed in the fluids. Such a flow of fluid is called turbulent flow.

6.3.1 Ideal flow

The experiments show that the study of fluids flow is extremely complex, but it can be simplified by making a few assumptions. These assumptions are summarized as:

(i) The fluid is non-viscous

A non-viscous fluid is one in which there is no friction between the two adjacent layers i.e., its viscosity is zero.

(ii) The flow is steady:

In steady flow, the velocity of each particle of the fluid at each point remains same. (Fluid experiences no viscous force).

(iii) The fluid is incompressible:

The fluid is incompressible i.e. its density remains constant. The flow which possesses such properties of non-viscous, steady and incompressible is known as an ideal flow because no flow exists in practice which have all these properties.

> **CHECK YOUR CONCEPT**
> Why the shapes of objects are streamlined?

6.4 EQUATION OF CONTINUITY

In fluid dynamics, equation of continuity is based upon law of conservation of mass and it is stated as; **"When fluid is flowing through a pipe then its total mass at any instant and at any cross-section area of the pipe remains same".** It is possible only when the fluid is incompressible and flow is steady.

To derive a mathematical relation for equation of continuity, we consider a steady flow of fluid of density ρ along the streamlines through a pipe of non-uniform size as shown in Fig. 6.8. At point 'P' the cross sectional area of pipe is 'A_1' and the velocity of fluid is v_1. If Δx_1 is the displacement of the fluid in time Δt then the mass of the fluid in volume element ΔV will be;

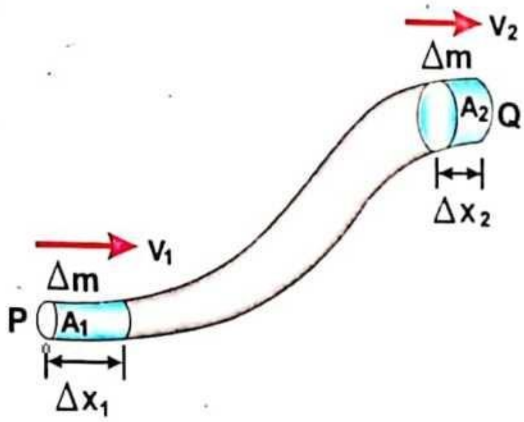


Fig.6.8: Ideal flow of fluid through a pipe of non uniform cross section area.

$$\Delta m_1 = \rho \times \text{volume}$$

$$\Delta m_1 = \rho \times \Delta V \text{.....(6.6)}$$

As the pipe is cylindrical, so the small element of volume of fluid is given by the product of the cross-sectional area A_1 and the length of the pipe Δx_1 containing the mass Δm_1, that is,

$$\Delta V = A_1 \Delta x_1$$

But, $$v_1 = \frac{\Delta x_1}{\Delta t}$$

and $$\Delta x_1 = v_1 \Delta t$$

So $$\Delta V = A_1 v_1 \Delta t \text{.......(6.7)}$$

Substitute equation (6.7) into equation (6.6) then the mass Δm_1 of the fluid becomes;

$$\Delta m_1 = \rho A_1 v_1 \Delta t \text{......(6.8)}$$

Similarly, the fluid moves with velocity v_2 through the upper end 'Q' of the pipe of cross-section area A_2. In the same time interval Δt, the mass m_2 of the fluid flowing at the point Q at distance Δx_2 is given as

$$\Delta m_2 = \rho A_2 v_2 \Delta t \text{......(6.9)}$$

Assume that the fluid is incompressible, so mass is conserved this according to the ***law of conservation of mass.***

Mass of fluid flowing into the pipe = mass of fluid flowing out of the pipe

$$\Delta m_1 = \Delta m_2$$

$$\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2 \text{ ......(6.10)}$$

This is a mathematical form of equation of continuity and is in fact the indirect statement of the law of conservation of mass. The eq.6.10 can be extended to 'n' number of sections. i.e.,

$$A_1 v_1 = A_2 v_2 = A_3 v_3 = ... = A_n \ v_n$$

or $$Av = \text{Constant ......(6.11)}$$

This relation shows that the speed of fluid is increased by decreasing the cross-section area through which the fluid flows. On the other hand, the product of cross-sectional area and speed of fluid is equal to the rate of volume flow and has same values at all points along the pipe.

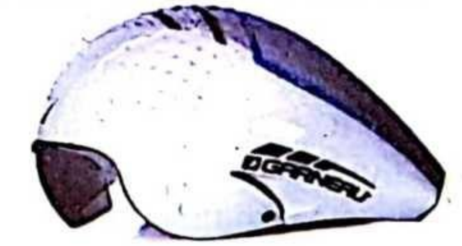
POINT TO PONDER

Can you apply the equation of continuity for the flow of current through a conductor?

Example 6.3

Water flows through a fire hose of inner diameter 6 cm at the rate of $10 ms^{-1}$. The fire hose ends on a nozzle with an inner diameter of 2 cm. What is the speed of water at the nozzle?

FOR YOUR INFORMATION



Aerodynamic designed helmets such as tear drop shaped helmet is helpful for cyclist to improve the speed.

Solution:

$d_1 = 6 cm = 0.06 m$

$r_1 = 0.03$ m

$v_1 = 10$ m/s

$d_2 = 2 cm = 0.02 m$

$r_2 = 0.01$ m

$v_2 = ?$

Equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$\therefore A = \pi r^2$$

$$\pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$v_2 = v\frac{r_1^2}{r_2^2} = 10\frac{(0.03)^2}{(0.01)^2}$$

$$v_2 = 90 \ ms^{-1}$$

6.5 BERNOULLI'S EQUATION

Bernoulli's equation is based upon law of conservation of energy and it is stated that "for steady flow of an ideal fluid, the total energy of the fluid remains constant throughout the flow. According to equation of continuity the speed of fluid flow varies along the path of the fluid. Similarly, the pressure also varies and it depends upon height as well as on the speed of the flow. Daniel Bernoulli studied the variation of speed and pressure of an ideal fluid flow at different heights and derived an equation which is known as Bernoulli's equation and it is derived as under.

Consider a steady flow of incompressible and non-viscous fluid through a pipe which has a non-uniform cross-sectional area at different heights as shown in Fig. 6.9.

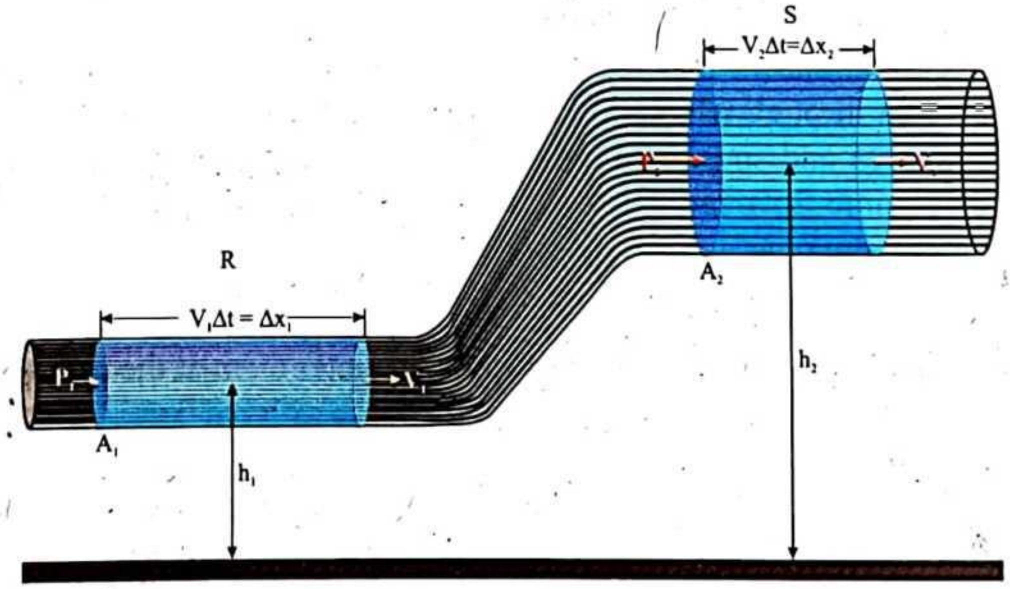


Fig.6.9:.An ideal flow of fluid through a non-uniform cross-section pipe at different heights

At point 'R' and at height h_1, the cross sectional area of the pipe is A_1. The velocity of the fluid at this point is v_1 and its pressure is P_1. Thus the work done on the fluid at distance Δx_1 in time Δt by the applied force F_1 is given as;

$$W = Fd\cos\theta$$

$$W_1 = F_1\Delta x_1 \cos 0^\circ \quad \therefore \cos 0^\circ = 1$$

$$W_1 = F_1\Delta x_1$$

Since, $$P_1 = \frac{F_1}{A_1}$$

> **CRITICAL THINKING**
> Under what process the nozzle of fire brigade vehicle is working?

$$F_1 = P_1 A_1$$

So, $$W_1 = P_1 v_1 \Delta x_1 \text{ ......(6.12)}$$

Now at point S at height h_2 and the cross sectional area of the pipe is A_2 while the velocity of the fluid is v_2 and pressure is P_2. Thus the work done on the fluid at distance Δx_2 in the same time Δt by the applied forces F_2 is given by;

$$W = F_2 d\cos\theta$$
$$W_2 = F_2\Delta x_2\cos 180^\circ \qquad \therefore \cos 180^\circ = -1$$
$$W_2 = -F_2\Delta x_2$$

Since, $P_2 = \dfrac{F_2}{A_2}$

$$F_2 = P_2 A_2$$
$$W_2 = -P_2A_2\Delta x_2 \quad ......(6.13)$$

The total work done on the system is equal to the sum of the work done on the lower point 'R' and the work done on the upper point 'S'.

POINT TO PONDER

Why we construct our water tank at the top roof of the building?

Hence, net work done on the system $= W_1 + W_2$

$$\text{Work} = P_1A_1\Delta x_1 - P_2A_2\Delta x_2 \quad ......(6.14)$$

By definition of velocity

$$v = \frac{\Delta x}{\Delta t}$$

or $\Delta x = v\Delta t$

Equation 6.14 becomes

$$\text{Work} = P_1A_1v_1\Delta t - P_2A_2v_2\Delta t \quad ......(6.15)$$

From equation of continuity,

$$A_1v_1 = A_2v_2 = Av$$

So, eq. 6.15 becomes

$$\text{Work} = P_1 Av\Delta t - P_2 Av\Delta t$$
$$\text{Work} = (P_1 - P_2)Av\Delta t$$
$$\text{Work} = (P_1 - P_2)\ \text{Volume}$$
$$\text{Work} = (P_1 - P_2)V \quad ......(6.16)$$

We know that for an incompressible and non-viscous fluid both mass and density remain constant.

$$\text{Density} = \rho = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Volume} = \frac{\text{Mass}}{\text{density}} = \frac{m}{\rho}$$

Thus, $\text{Work} = (P_1 - P_2)\dfrac{m}{\rho} \quad ......(6.17)$

According to work and energy theorem, work done causes both change in K.E and change in P.E. i.e.;

Work done = Change in K.E + Change in P.E

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gh_2 - \rho gh_1$$

If we place all the terms related with the fluid at position 'R' on the left-hand side of the equation and all the terms related with the fluid at position 'S' on the right-hand side, then we obtain

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 \quad ......(6.18)$$

This is the mathematical form of Bernoulli's equation and it can be extended to 'n' number of sections.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 = P_3 + \frac{1}{2}\rho v_3^2 + \rho gh_3$$

$$= .... = P_n + \frac{1}{2}\rho v_n^2 + \rho gh_n \quad ......(6.19)$$

In general

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{Constant} \quad ......(6.20)$$

Equation 6.20 is the mathematical statement of **Bernoulli's equation.**

It states that **"the sum of pressure, K.E per unit volume and P.E per unit volume of an ideal fluid throughout its steady flow remains constant".**

Example 6.4

Water flows through a horizontal pipe of non-uniform cross sectional area. At one point, the pressure in 4.5×10^4 Pa where as the speed of water is $2\ ms^{-1}$. How much pressure falls down at another point where the speed of water is $8\ ms^{-1}$?

Solution:

As pipe is horizontal so, $h_1 = h_2 = h$

$P_1 = 4.5 \times 10^4$ Pa

$v_1 = 2\ ms^{-1}$

$P_2 = ?$

$v_2 = 8\ ms^{-1}$

ρ = Density of water = $1000\ kg\ m^{-3}$

Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

CRITICAL THINKING

To save fuel in the airplane, does it need to fly at low altitude or high altitude. Why?

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh \qquad \because h_1 = h_2 = h$$

$$P_2 = P_1 - \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$P_2 = 4.5\times10^4 - \frac{1}{2}(1000)\left((8)^2 - (2)^2\right)$$

$$P_2 = 4.5\times10^4 - 500\,(64-4)$$

$$P_2 = 4.5\times10^4 - 500\,(60) = 4.5\times10^4 - 30000$$

$$P_2 = 4.5\times10^4 - 3\times10^4$$

$$P_2 = (4.5-3)\times10^4$$

$$P_2 = 1.5\times10^4 \text{ Pa}$$

Fall in pressure $= P_1 - P_2$

$$P_1 - P_2 = 4.5\times10^4 - 1.5\times10^4$$

$$P_1 - P_2 = 4.5\times10^4 - 1.5\times10^4 P_1 - P_2 = 3\times10^4 \text{ Pa}$$

6.6 APPLICATIONS OF BERNOULLI'S EQUATION

6.6.1 Torricelli`s Theorem

Consider a large tank that contains a fluid of density ρ at height h_1 from bottom to the upper surface of fluid as shown in Fig. 6.10. This tank has also an orifice at height h_2 from its bottom. Thus, $h_1 - h_2 = h$ be the height of fluid from orifice to the upper surface of fluid. We assume that the fluid level falls so slowly that the liquid velocity at the upper level may be assumed to be zero. Let v_1 be the velocity of the fluid at the upper surface and v_2 be the velocity of fluid at orifice which is called velocity of efflux. As both the ends are open to the atmosphere, so the pressure on the upper and the bottom surfaces is equal to the atmospheric pressure 'P' that is,

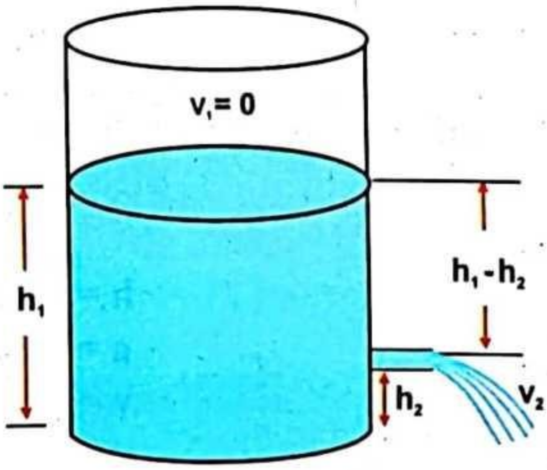


Fig.6.10: A tank contains fluid with a orifice, where fluid flows through orifice with velocity v_2.

$$P_1 = P_2 = P$$

According to Bernoulli's equation.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

$$P+\frac{1}{2}\rho(0)+\rho g h_1 = P+\frac{1}{2}\rho v_2^2+\rho g h_2 \qquad \therefore v_1 = 0 \text{ and } P_1 = P_2 = P$$

$$\rho g h_1 = \frac{1}{2}\rho v_2^2+\rho g h_2$$

$$\frac{1}{2}\rho v_2^2 = \rho g h_1 - \rho g h_2$$

$$\frac{1}{2}\rho v_2^2 = \rho g(h_1 - h_2)$$

$$v_2^2 = 2g(h_1 - h_2)$$

$$v_2 = \sqrt{2g(h_1 - h_2)}$$

$$v_2 = \sqrt{2gh}$$

In general, $$v = \sqrt{2gh} \quad \text{......(6.21)}$$

This is Torricelli's theorem which states that "The velocity of efflux of the fluid through an orifice is directly proportional to the square root of the height of liquid from orifice to the upper surface of fluid". The eq.(6.21) also shows that the velocity of efflux is independent of the nature of liquid, quantity of liquid in the tank, and the area of orifice.

Example 6.5

A tank containing water has an orifice on one vertical side. If the centre of the orifice is 10 m below the surface level of water in the tank, calculate the velocity of efflux.

Solution:

$h = 10\text{ m}$

$g = 9.8\text{ ms}^{-2}$

$v = ?$

$$v = \sqrt{2gh} = \sqrt{2(9.8)(10)}$$

$$v = 14\text{ ms}^{-1}$$

6.6.2 Venturi Relation and Venturimeter

A relation in which we study the variation of pressure as a function of density and speed of fluid flow along a pipe is known as venture relation. This can be derived as under;

Consider a steady flow of incompressible liquid through horizontal pipe of non uniform cross-section area as shown in Fig 6.11.

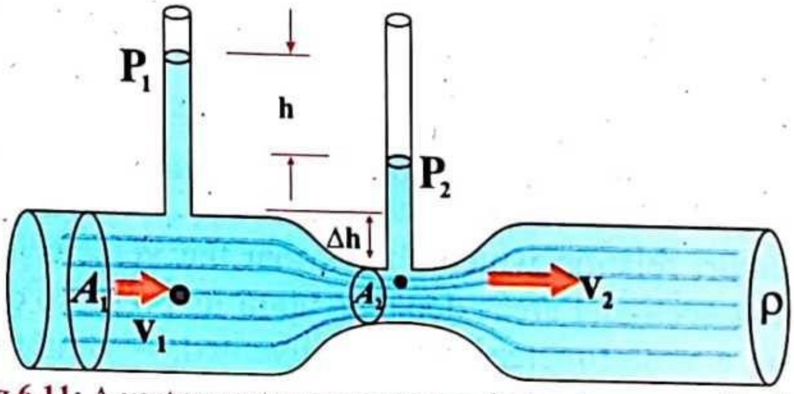


Fig.6.11: A venture meter measures speed of an incompressible fluid. Pressure P_1 is greater than pressure P_2 while the velocity v_1 is less than v_2.

According to equation of continuity at a point of large cross-sectional area A_1, velocity v_1 is low and pressure P_1 is high.

However, at small cross-sectional area A_2, velocity v_2 is high and pressure P_2 is low. As the pipe is horizontal so the height remains same i.e. ($h_1 = h_2 = h$).

Applying Bernoulli's equation, we have

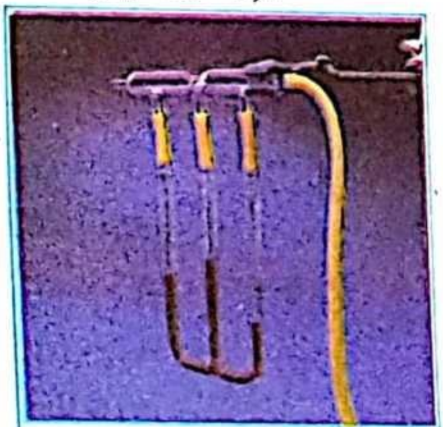
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2\left(1 - \frac{v_1^2}{v_2^2}\right) \quad \ldots\ldots(6.22)$$



A Venturi meter or tube

Using equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \frac{A_2}{A_1} v_2$$

$\frac{A_2}{A_1}$ is very small and it can be neglected.

$$v_1 = 0$$

Equation 6.22 becomes

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 (1 - 0)$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2$$

Sky divers in a horizontal "spread eagle" formation maximize the (air resistance) drag force.

$$v_2^2 = \frac{2(P_1 - P_2)}{\rho}$$

$$v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}} \quad \text{......(6.23)}$$

This is a Venturi relation which shows that the velocity of fluid through a pipe of different cross-sectional areas depends upon its pressure difference. On the basis of this relation, a venturimeter has been invented. A venturimeter is a device which is being used for the measurement of velocity of an incompressible fluid through a horizontal pipe.

Example 6.6

A venturimeter is connected to two points along the main pipe, where its radius at A_1 is 30 cm and at A_2 is 14 cm while the velocity at A_1 is 0.3 m s^{-1} and at A_2 is 2 m s^{-1}. The level of water column in the Venturi tubes differ by 10 cm. If the pressure at A_1 is 3×10^3 Pa, what is the pressure P_2 in the constricted pipe?

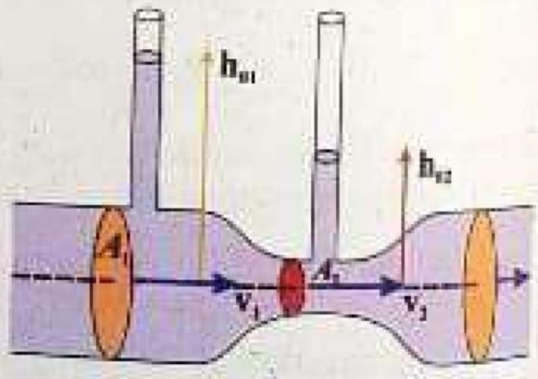
Solution:

$$A_1 = \pi r_1^2 = 3.14 \times (30)^2 \text{cm}^2$$

$$A_1 = 2826 \text{ cm}^2 = 0.28 \text{ m}^2$$

$$A_2 = \pi r_2^2 = 3.14 \times (14)^2 \text{cm}^2$$

$$A_2 = 615 \text{ cm}^2 = 0.0615 \text{ m}^2$$



$v_1 = 0.3$ m s^{1}

$v_2 = 2$ m s^{1}

$P_1 = 3 \times 10^3$ Pa

$P_2 = ?$

$g = 9.8$ ms^{-2}

$h = 10$ cm $= 0.1$ m

Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh$$

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2).$$

$$P_2 = (3\times10^3) + \frac{1}{2}(1000)\left((0.3)^2 - (2)^2\right)$$

$$P_2 = (3\times10^3) + 500(0.09 - 4)$$

$$P_2 = (3\text{x}10^3) + 500(-3.91)$$

$$P_2 = 3000 - 1955$$
$$P_2 = 1045\,\text{Pa}$$

Thus, the pressure of the water in the constricted portion of the tube has decreased to 1045 Pa.

6.6.3 Filter Pump

A filter pump works on the basis of reducing pressure in a vessel. It consists of a tube which contains three pipes A, B and C. The pipe 'A' is used for flow of water from reservoir and the cross sectional area of its outer end is made narrow orifice in the form of a jet.

The pipe 'B' is used to supply air from vessel to the tube and the pipe 'C' is used to sink the water from the tube as shown in Fig. 6.12.

When the water is allowed to pass through the narrow orifice, the velocity of water decreases causing a gradual fall in its pressure and its value soon becomes comparable to the atmospheric pressure. On the other hand, the air from vessel rushes to towards water at low pressure and it carries the water to be filtered through a sink as shown in Fig 6.12.

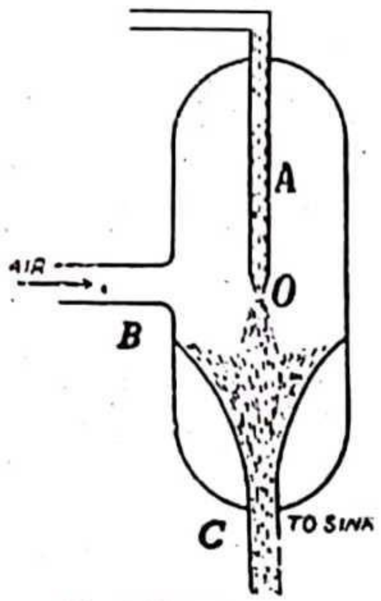


Fig.6.12: Filter Pump

6.6.4 Atomizer

Atomizer or sprayer is an instrument used for spraying scents, paints or other fluids. Its working principle is also based on Bernoulli's equation. When the rubber ball of atomizer is squeezed, then the air is blown through tube and it rushes out through the narrow aperture with high velocity and it causes fall of pressure. So the atmospheric pressure pushes the perfume up the tube leading to the narrow aperture. The perfume spreads out in form of fine spray as shown in Fig.6.13.

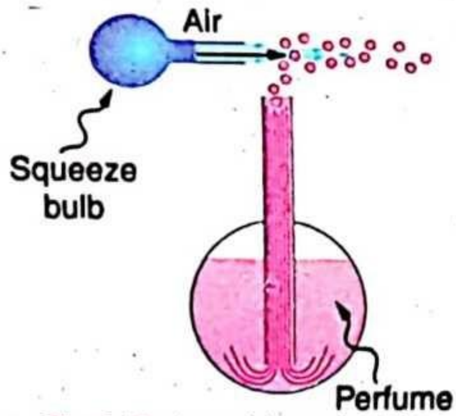


Fig.6.13: A working principle of an atomizer

6.6.5 Aerofoil Lift

The flow of streamlines of air around an aeroplane wing is shown in Fig. 6.14. The wings of an aeroplane are designed such that the air speed above the wing is greater than the speed below the wing. According to Bernoulli's effect the air pressure above the wing is lower due to the higher speed of the air and the air pressure below the wing is greater due to the lower speed of the air.

This pressure difference between the upper surface region and the lower surface region causes a net upward force and it is called aerofoil lift or aerodynamic lift as shown in Fig 6.14.

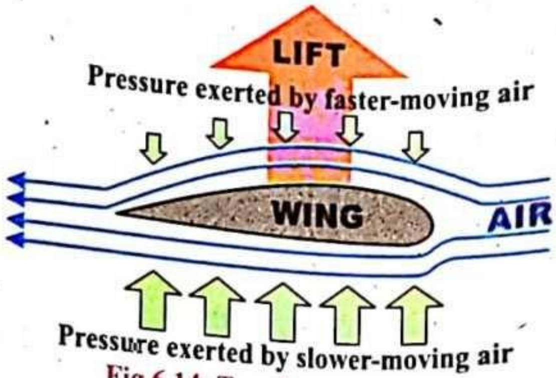


Fig.6.14: Two forces on wing

6.6.6 Sphygmomanometer and measurement of blood Pressure

Blood is an incompressible fluid that has a density slightly more than that of the water. Blood circulates in all parts of the body through arteries, veins and capillaries due to its pumping by the heart. The rate of blood circulation is very fast. For instant, in twenty-eight seconds blood is taken from the left foot back to the heart and lungs and then to the right foot. The blood vessels (arteries and capillaries) are not rigid but they can stretch like a rubber pipe.

Under normal conditions, the volume of the blood is sufficient to keep the vessels inflated all the times. It means there is a tension in walls of the blood vessels and consequently, the pressure of the blood inside is greater than the atmospheric pressure. Due to the tension in the walls of the blood vessels, considerable pressure is needed to force blood through them. This pressure is supplied by the heart during a compression stroke called systole. The peak pressure in the vessels when the heart completes a contraction is called the systolic pressure. When the heart has finished contracting, the pressure falls gradually, and heart is being refilled with blood from the veins.

During the falling stroke called the diastole, the pressure falls to its minimum value which is called diastolic pressure. Typically, the peak (systole) value of pressure is 120 torr and the resting (diastole) is 80 torr. Graphically, these two values are shown in Fig.6.15. The unit of pressure can be taken in torr. It is employed for medical instrument called sphygmomanometer, where $1 \text{torr} = 133.3 \text{ Nm}^{-2}$.

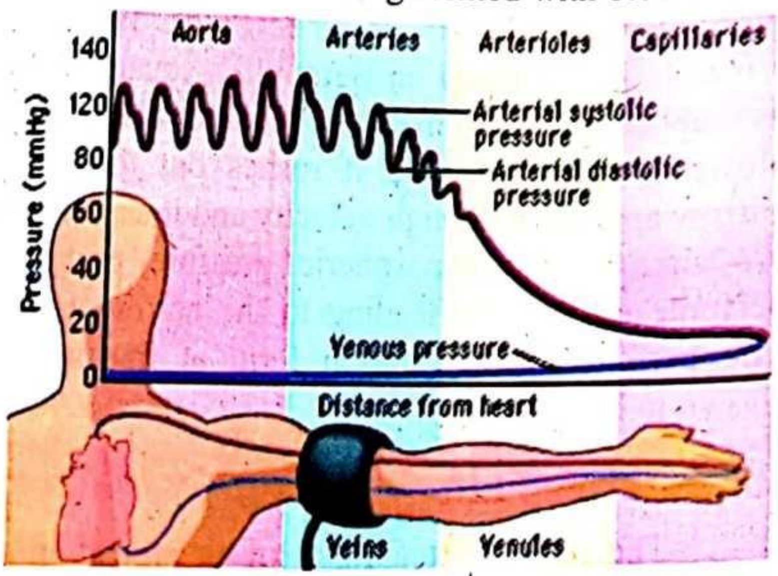


Fig.6.15: A graphical representation of a blood pressure of a man.

Sphygmomanometer

It is a device used to measure the blood pressure. The schematic diagram of Sphygmomanometer is shown in Fig. 6.16. It consists of an inflatable rubber cuff, which is wrapped around the arm of the person and a manometer is also attached with it. When the external pressure is increased beyond the systolic pressure by pumping the rubber ball, then a force is exerted on the cuff such that the arm is squeezed and flow of blood through arteries is stopped.

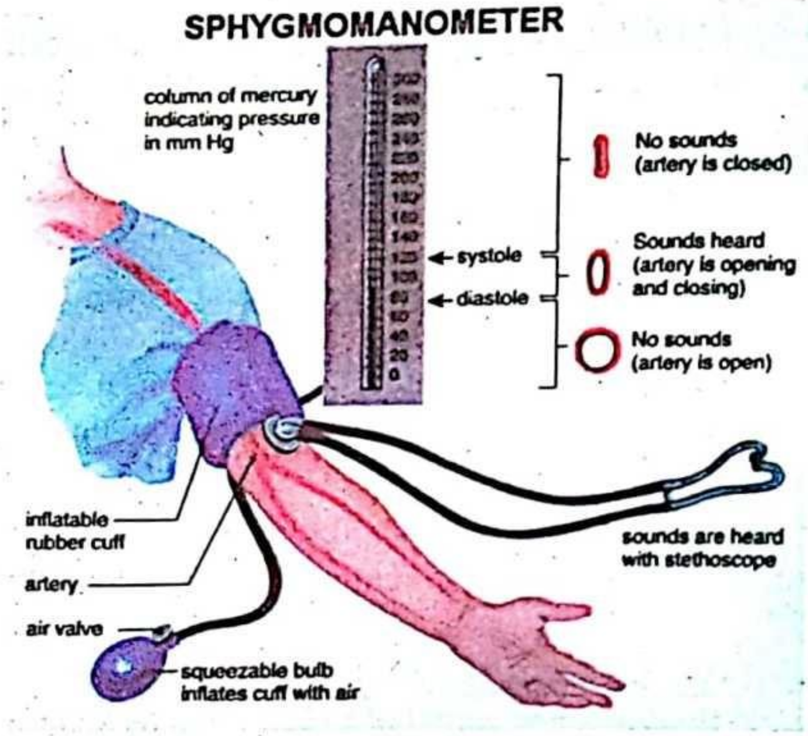


Fig.6.16: A method of usage of sphygmomanometer

When the valve on the rubber ball is opened and the external pressure starts decreasing then the observer listens a sound with a stethoscope. When the external pressure becomes equal to the systolic pressure, then the velocity of the blood becomes high and turbulent.

The external pressure is further decreased such that its value becomes equal to the diastolic pressure. At this point, the arteries are relaxed. A continuous sound is heard by the observer and the flow switches from turbulent to laminar. In this way, the blood pressure can be measured by using the sphygmomanometer.

SUMMARY

- **Fluid:** Anything which can flow is called a fluid. The examples of fluids are liquids and gases.
- **Viscosity**: The resistive property of a fluid due to the friction between its two consecutive relative layers during its alterative motion is called viscosity.
- **Drag Force:** A resistive force experienced by a body moving through a viscous medium is called drag force and according to Stokes law this force depends upon, viscosity, velocity and radius. $F = 6\pi\eta v r$.
- **Terminal velocity:** When the weight of the body moving through a fluid becomes equal to the drag force then it moves with uniform velocity. This velocity is called terminal velocity.
- **Ideal flow:** An ideal flow is a steady, non-viscous and incompressible.

- **Laminar flow:** A steady flow of fluid is called laminar or streamline flow.
- **Turbulent Flow:** Irregular flow of fluid is called turbulent.
- **Equation of continuity:** Equation of continuity is based upon conservation of mass and shows that the product of cross sectional area and velocity of fluid remains constant.
- **Bernoulli equation**: Bernoulli equation is based upon the law of conservation of energy and is applicable to steady flow of ideal fluid i.e. non-viscous and incompressible fluid. It states that the sum of the pressure, K.E. per unit volume and P.E. per unit volume remains constant.
- **Torricelli's theorem:** Torricelli's theorem states that the velocity of efflux depends upon the height of the fluid.
- **Venturimeter:** A venturimeter is a device that is used to measure the velocity of fluid.
- **Venturi effect:** The decrease in pressure with the increase in velocity of the fluid in a horizontal pipe is known as Venturi effect.
- **Sphygmomanometer:** A sphygmomanometer is a device used to measure blood pressure of a person.

EXERCISE

Multiple choice questions.

1. Viscosity of a fluid depends upon;

(a) Mass (b) Density (c) Volume (d) Temperature

2. Drag force exerted by the fluid on a body does not depend upon:

(a) Viscosity of fluid (b) Terminal velocity

(c) Shape of a body (d) Volume of fluid

3. A steel ball of radius 'r' is moving with uniform velocity 'v' in the mustard oil, the drag force acting on the ball is 'F'. What would be the drag force on the steel ball of radius 2r moving with uniform velocity 2v in the mustard oil;

(a) F (b) 2F (c) 4F (d) 8F

4. When two spheres of same volume but different mass fall through a fluid then;

(a) Both gain their terminal velocity simultaneously

(b) Neither lighter nor heavier body gains its terminal velocity

(c) Lighter body gains its terminal velocity earlier

(d) Heavier body gains its terminal velocity earlier

5. Laminar flow usually occurs at speeds.
(a) Low (b) High
(c) Very High (d) Some time high & some time low

6. In incompressible fluid, which parameter remains constant?
(a) Pressure (b) Volume (c) Temperature (d) Density

7. Equation of continuity has been derived on the basis of law of conservation of
(a) Energy (b) Momentum (c) Mass (d) Force

8. According to equation of continuity $A_1v_1 = A_2v_2 =$ constant. The constant is equal to
(a) Flow rate (b) Volume of fluid (c) Mass of fluid (d) Density of fluid

9. When cross-sectional area of a tube is decreased then the speed of fluid through it is;
(a) Increased (b) Decreased (c) Same (d) Zero

10. As the water falls its speed increases, so its cross-sectional area:
(a) Increases (b) Decreases (c) Remains constant (d) Zero

11. Bernoulli's equation has been derived on the basis of conservation of;
(a) Mass (b) Energy (c) Momentum (d) Force

12. What is the speed of water flow through a tap which is connected with tank that contains water at height 2.5 m.
(a) 5 ms^{-1} (b) 6 ms^{-1} (c) 7 ms^{-1} (d) 8 ms^{-1}

13. A two-metre high tank is full of water. If a hole appears at its middle, then the speed of efflux is:
(a) 4.42 ms^{-1} (b) 5.42 ms^{-1} (c) 6.42 ms^{-1} (d) 7.42 ms^{-1}

14. Venturi-meter is an instrument which is being used for the measurement of;
(a) Density of fluid (b) Velocity of fluid
(c) Pressure of fluid (d) Viscosity of fluid

15. When velocity of fluid is increased then its pressure is;
(a) Increased (b) Decreased
(c) Same (d) Not velocity dependent

16. Systolic pressure is called;
(a) Low blood pressure (b) High blood pressure
(c) Normal pressure (d) Irregular blood pressure

SHORT QUESTIONS

1. What are the causes of viscosity of a fluid?
2. How does a body gain at terminal velocity when it falls through a fluid?
3. Does a non viscous fluid exist? If yes, how a fluid can be made non-viscous.
4. Why rain drops do not hurt us?
5. How a steady flow differs from turbulent flow?
6. Why liquid is incompressible while gas is compressible?
7. How can the laminar flow be changed into the turbulent flow?
8. How variation in pressure is affected by speed of fluid?
9. What are the three factors associated with an ideal flow?
10. How can a Venturi meter be constructed?
11. How does filter pump work?
12. How can aerofoil lift be produced?
13. What is the process of measurement of blood pressure of a person?
14. What is the difference between systolic and diastolic blood pressure?
15. If a high wind blows near a window, the window may break outwards. Give reason.
16. When water falls from a tap, its cross-sectional area decreases as it comes down. Why?
17. If you blow between two limp pieces of paper held hanging down a few inches apart, will the pieces of paper come closer together or farther apart? Explain.
18. Why a fog droplet appears to be suspended in air?

COMPREHENSIVE QUESTIONS

1. Define fluid and describe the viscosity of a fluid. Also express the relation for viscosity of fluid.
2. State and explain the terminal velocity with the help of Stokes' law.
3. Discuss flow of fluid and compare steady flow and turbulent flow.
4. State equation of continuity and derive equation of continuity on the basis of conservational of mass.
5. State and explain Bernoulli's equation and derive it on the basis of law of conservation of energy.
6. Discuss the various application of Bernoulli's equation such as; (1) Torricelli's Theorem, (2) Venturi relation, (3) Filler Pump, (4) Atomizer, (5) Aerofoil lift.
7. Discuss briefly the measurement of blood by using sphygmomanometer.

NUMERICAL PROBLEMS

1. A metal sheet of area $0.4\ m^2$ is attracted by a force of 0.98 N placed over a liquid thin film which lies between sheet and surface of table. The thickness of liquid film is 0.2 mm. If the sheet starts its motion with uniform velocity $0.2\ ms^{-1}$, calculate the co-efficient of viscosity of the liquid.

   ($2.45\times10^{-3}\ N\ m^{-2}\ s$ or Pa s)

2. What is the terminal velocity of a rain drop of diameter 0.5 mm? The co-efficient of viscosity of air is taken as 1.83×10^{-5} poise, density of air is 1.3 $kg\ m^{-3}$, density of water is 1000 $kg\ m^{-3}$ and value of 'g' is 9.8 ms^{-2}. **($7.4\ m\ s^{-1}$)**

3. Calculate the average speed of water flowing through a pipe of diameter 10 cm and delivered 5 m^3 of water per hour. **($0.18\ ms^{-1}$)**

4. The speed of water is 0.5 $m\ s^{-1}$ flowing in a pipe of diameter 5 cm. What will be it speed in a pipe of 2.5 cm diameter that is connected with it. **($2\ m\ s^{-1}$)**

5. Water flows through a horizontal pipe of non-uniform cross section area. The pressure at a point is 130k Pa where the velocity is 0.4 ms^{-1}. Calculate the pressure at the point where the velocity is 4 ms^{-1}. **(128.08k Pa)**

6. What will be the gauge pressure in a large fine hose if the nozzle is to shoot water straight upward to a height of 25 m? **(2.45×10^5 Pa)**

7. What is the height of water inside the tank above the orifice if the velocity of efflux of water through orifice is 9.9 $m\ s^{-1}$. **(5m)**

8. A liquid of density $8\times10^3\ kgm^{-3}$ is flowing through a horizontal pipe of different cross section. If the pressure difference between two points is $4\times10^4\ N\ m^{-2}$ then what is the speed of liquid in the tube. **($3.16\ m\ s^{-1}$)**

9. An airplane wing is designed such that speed of the air across the top of the wing in 450 $m\ s^{-1}$ and the speed of the air below the wing is 410 ms^{-1}. What is the pressure difference between the top and the bottom of the wings? (Density of air is 1.29 $kg\ m^{-3}$) **(22k Pa)**

Unit 7

OSCILLATION

Major Concepts **(23 PERIODS)**

- Simple Harmonic Motion (SHM)
- Circular motion and SHM
- Practical SHM system (mass spring and simple pendulum)
- Energy conservation in SHM
- Free and forced oscillations
- Resonance
- Damped oscillations

Conceptual Linkage

This chapter is built on Circular Motion Physics XI Oscillation & Waves Physics XI

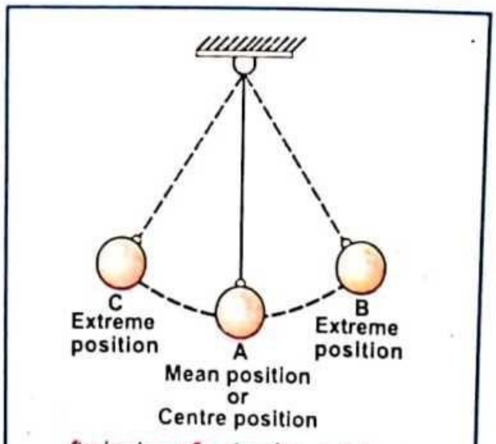
Students Learning Outcomes

After studying this unit, the students will be able to:

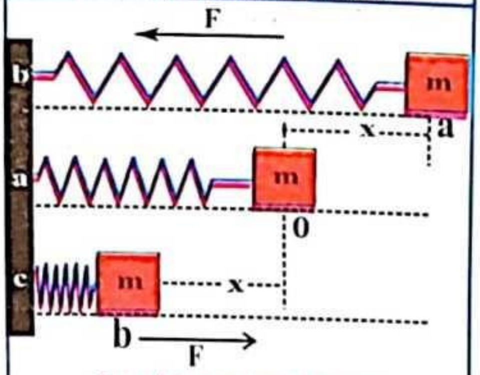
- Describe simple examples of free oscillations.
- Describe necessary conditions for execution of simple harmonic motions.
- Describe that when an object moves in a circle, the motion of its projection on the diameter of the circle is SHM.
- Define the terms amplitude, period, frequency, angular frequency and phase difference and express the period in terms of both frequency and angular frequency.
- Identify and use the equation; $a = -\omega^2 x$ as the defining equation of SHM.
- Prove that the motion of mass attached to a spring is SHM.
- Describe the interchanging between kinetic energy and potential energy during SHM.
- Analyze the motion of a simple pendulum is SHM and calculate its time period.
- Describe practical examples of free and forced oscillations (resonance).
- Describe graphically how the amplitude of a forced oscillation changes with frequency near to the natural frequency of the system.
- Describe practical examples of damped oscillations with particular reference to the efforts of the degree of damping and the importance of critical damping in cases such as a car suspension system.
- Describe qualitatively the factors which determine the frequency response and sharpness of the resonance.

INTRODUCTION

Besides translational and rotational motion, there is another important kind of motion that is vibrational motion which has too many applications in physics as well as in our daily life. This kind of motion of a body is a to and fro motion about its mean position and its nature is a periodic motion because the oscillating body repeats itself after a regular intervals of time. Some examples of oscillations are given below.



Swinging of a simple pendulum



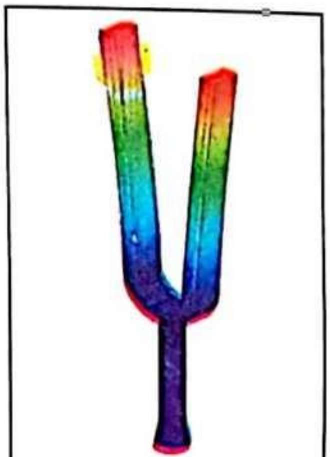
To and fro motion of a body attached to a spring

(i) Swinging of a simple pendulum when it is displaced from its mean position and is made to free.

(ii) Motion of a body attached to a spring when it is pulled and then released.

(iii) Vibration of prongs of the tuning fork when it is struck on a rubber paid.

All the bodies that undergo vibrational or oscillational motion have an equilibrium position or mean position. When the body is displaced from this mean position then there is a restoring force which brings it back to its equilibrium position and it causes of vibration or oscillation motion of the body.


Vibrating tuning fork

The detailed study of vibrational motion helps us in the understanding of waves, sounds, light and alternating current because it has been observed that vibrating bodies produce waves. For example, a violin string produces sound waves in air.

Resonance is a striking phenomenon which is related with vibrational motion and it plays a dynamic role in communication system because maximum communication energy transfer is processed by transmitter and receiver due to the resonance phenomenon.

Though many systems cannot operate without resonance but it should be avoided in some cases such as aeroplane wings or helicopter rotor and suspension bridges etc.

In this chapter we will study not only various parameters related to an oscillating body but will also prove that the motions of a particle along a circle, a body attached to a string and a simple pendulum are simple harmonic motion.

7.1 SIMPLE HARMONIC MOTION (SHM)

The back and forth motion of a body that it repeats in equal interval of time along the same line is called periodic motion. On the other hand, simple harmonic motion is the most important type of the periodic motion and it occurs when the restoring force is directly proportional to the displacement from an equilibrium position. It can be explained with the example of a body of mass 'm' attached to a spring which oscillates about equilibrium position 'O' on a horizontal frictionless surface as shown in Fig.7.1. Consider a force 'F' that is applied to displace the body from its equilibrium position 'O' to an extreme position through a distance 'x'.

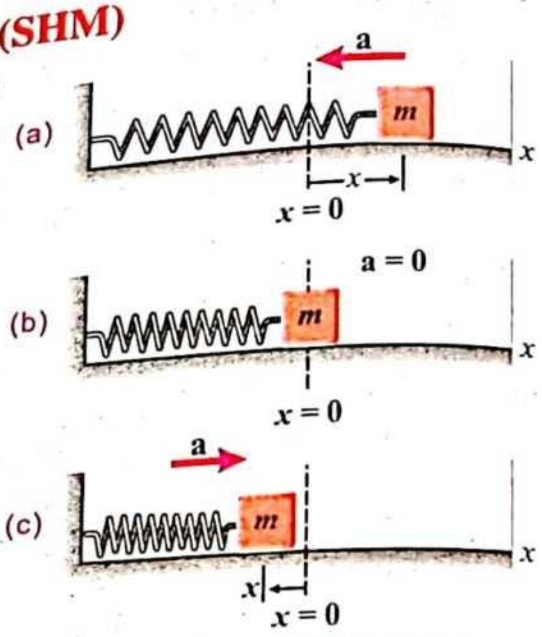


Fig.7.1: Mass attached with spring during its SHM
(a) At the right extreme position
(b) At mean position
(c) At the left extreme position

According to Hook's law, the applied force is equal to kx, Where 'k' is a constant and is called spring constant, and it has the dimensions of force per unit length (Nm^{-1}). Due to the elasticity of the spring, an elastic restoring force $(-kx)$ acts on the body whose magnitude is equal to applied force and its direction is towards the mean position and there is also an acceleration which is produced by such restoring force. This acceleration causes simple harmonic motion in the body and is directly proportional to the displacement and is always directed towards the mean position. These two conditions are known as the conditions that must be obeyed by a body in order to execute simple harmonic motion.

According to Hook's law the elastic restoring force is given by:

$$F = -kx \;\ldots\ldots(7.1)$$

According to Newton's 2^{nd} law of motion

$$F = ma \;\ldots\ldots(7.2)$$

Comparing equation (7.1) and equation (7.2)

$$ma = -kx$$

$$a = -\left(\frac{k}{m}\right)x \;\ldots\ldots(7.3)$$

As the ratio (k/m) is a constant therefore,

$$a = -(\text{Constant})x$$

$$a \propto -x$$

This is the mathematical form of simple harmonic motion. It states that the acceleration of the body executing simple harmonic motion (SHM) is directly proportional to the displacement and negative sign shows that it is directed toward its mean position.

Example 7.1

A body of mass 0.25 kg is connected to a spring and it is oscillating on a horizontal frictionless surface. If the maximum displacement of body is 20cm and the spring constant is 10 N m^{-1} then what is the acceleration of the body?

Solution: $m = 0.25$ kg

$k = 10\ N\ m^{-1}$

$x = 20\ cm = 0.2\ m$

$a = ?$

$$a = -\left(\frac{k}{m}\right)x$$

$$a = -\left(\frac{10}{0.25}\right)0.2 = -8\ m\,s^{-2}$$

Negative sign shows that the motion of body is directed towards its mean position.

POINT TO PONDER

Can a linear motion of a body be SHM?

7.1.1 Characteristic of simple harmonic motion

Simple harmonic motion is a special kind of periodic motion. It can be represented graphically by demonstrating an experiment of mass spring system. The experimental set up consists of a block of mass 'm' attached with a spring which is hanging vertically and remains at its equilibrium position 'O' as shown in Fig. 7.2.

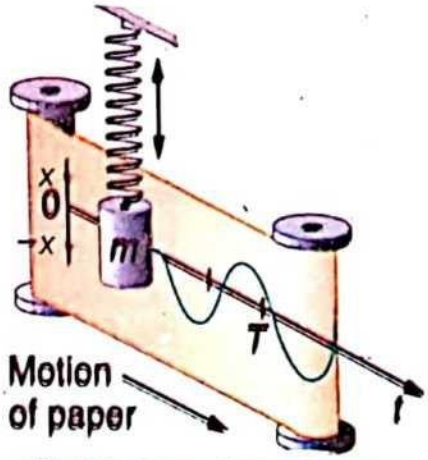


Fig.7.2: Pen and paper arrangement to draw a graph of an oscillating body

A sheet of paper with a suitable time scale is placed behind the block which is moving at a constant speed from right to left. There is also a pen which is attached with the vibrating mass which lightly touches the paper in order to record the variations in displacement with time during the oscillation of mass.

When the block is displaced downward from its mean position to its extreme position at a distance 'x' and is made to free then it starts oscillation. As a result, displacement against time appears on the paper in the form of sinusoidal-wave which is known as wave form of simple harmonic motion as shown in Fig. 7.3.

The various parameters related with simple harmonic motion are summarized as:

I) Instantaneous displacement

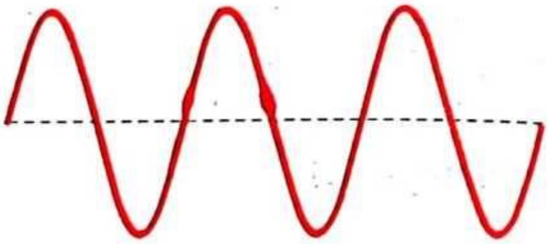


Fig.7.3: A Sine wave shape of an oscillating body

In vibrational motion, the distance from the mean position at any instant is known as instantaneous displacement. It is zero at the instant when the body is at mean position and it is maximum at the extreme position.

II) Amplitude

In Vibrational motion, the maximum distance from the mean position to either extreme position is known as amplitude. The SI unit of amplitude is metre.

III) Vibration

One complete round trip of a body during its vibrational motion is called vibration. For example, when the body starts its motion from its first extreme position ($-x$) to the second extreme position (x) and then from the second extreme position (x) to the first extreme position ($-x$) crossing the mean position (o) is called one vibration as shown in Fig. 7.2.

IV) Time Period

Time period is defined as the time taken to complete one vibration or one cycle. It is represented by 'T' and its SI unit is second 's'.

V) Frequency

Frequency is defined as the number of vibrations completed by the vibrating body in one second. It is expressed in terms of the reciprocal of time period that is;

$$f = \frac{1}{T} \quad \ldots\ldots(7.4)$$

The unit of frequency is hertz (Hz) and it is equal to per second. The dimensional formula of frequency is $[M^0L^0T^{-1}]$.

VI) Angular Frequency

Angular frequency is defined as the number of revolutions per unit time. It is represented by 'ω' and it can be expressed as;

$$\omega = \frac{\theta}{t}$$

Now for one revolution $\theta = 2\pi$ radians and $t = T$ (time period)

$$\omega = \frac{2\pi}{T}$$

As $T = \frac{1}{f}$

Therefore, $\omega = 2\pi f$ ......(7.5)

The SI unit of angular frequency is $rad.s^{-1}$ and its dimensional formula is $[M^{o}L^{o}T^{-1}]$

Example 7.2

A mass connected to a spring makes 15 vibrations in 45 second. Calculate its period and frequency.

POINT TO PONDER

Every vibrating body produces a sound. Does a simple pendulum also produce a sound?

Solution:

Numbers of vibration = 15

Time for 15 vibrations = 45 s

$T = ?$

$f = ?$

$$\text{Time period (T)} = \frac{\text{given time}}{\text{No. of vibs.}}$$

$$\text{Time period (T)} = \frac{45}{15} = 3\ s$$

$$\text{Frequency} = f = \frac{1}{T}$$

$$\text{Frequency} = f = \frac{1}{3} = 0.333\ Hz$$

7.2 CIRCULAR MOTION AND SIMPLE HARMONIC MOTION

To study the simple harmonic motion, consider a turntable of radius 'r' with a ball attached to its rim. A beam of light casts a shadow of the ball on the screen as shown in Fig.7.4.

When the turntable rotates with constant angular speed 'ω' then the ball also moves along it with uniform circular motion. Its shadow on the screen oscillates executing to and fro motion across the screen in the form of simple harmonic motion like a body attached to a spring.

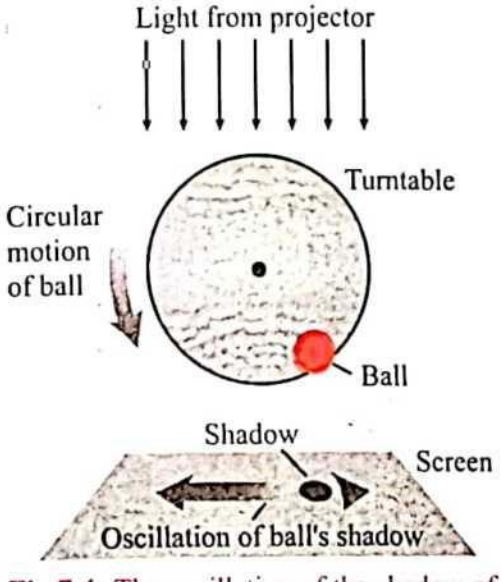


Fig.7.4: The oscillation of the shadow of the ball on screen. The ball is attached with uniformly rotating turn table.

Now we can study the motion of the ball along the circumference of the turn table and its resulting shadow on the screen along the diameter for one complete cycle.

Let the projection of the ball be on the mean position 'O' at $t = 0$ then after some instant $t = \frac{T}{4}$, the projection will be on the left extreme position 'A'.

Similarly, after instant $t = \frac{T}{2}$ the projection is again at the mean position 'O', at $t = \frac{3T}{4}$, the projection is on the right extreme position 'B'. Finally at $t = T$, the projection reaches at its starting point i.e. the mean position O. Hence, one cycle is completed. In the same way, the next cycles will also give the same result. When the graph between displacement and time is plotted then we have a sinusoidal wave as shown in Fig.7.5. This example clearly indicates that when an object moves along the circumference of a circle, its projection on the diameter of the circle executes S.H.M. The parameters such as displacement, velocity, acceleration, time period and phase of the S.H.M by the projection of the particle are explained below.

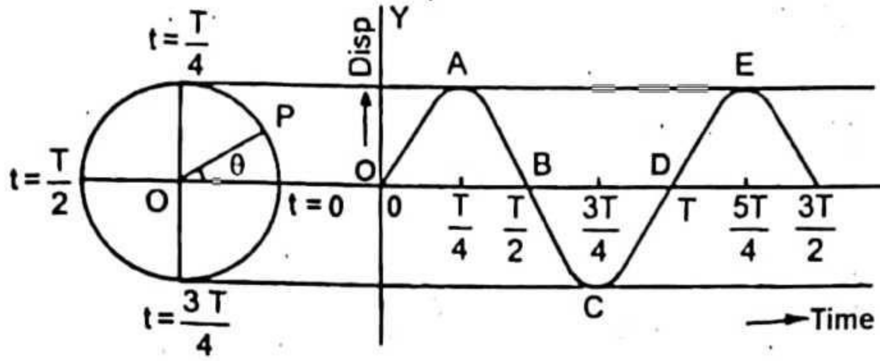


Fig.7.5: The wave shape of the projection of the ball executing S.H.M

7.2.1 Quantitative Analysis

Consider a motion of particle 'P' along the circumference of circle of radius 'r' with uniform angular velocity 'ω'. Its linear velocity at point 'P' is along the tangent ($v_p = r\omega$) and its acceleration a_P is directed towards the centre of circle as shown in Fig. 7.6. The value of acceleration is given as;

$$a_p = \frac{v_P^2}{r} \quad \because v = r\omega$$

$$a_p = \frac{r^2\omega^2}{r}$$

$$a_p = r\omega^2 \quad ......(7.6)$$

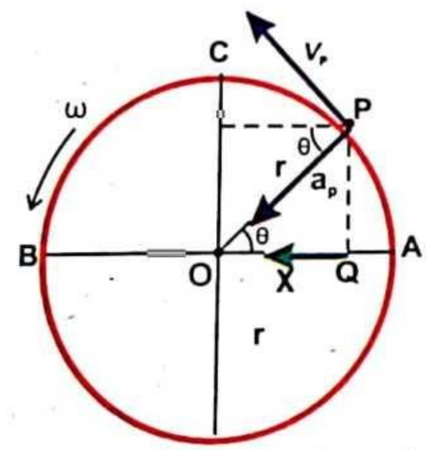


Fig.7.6: A particle which is moving along a circular path of circle with uniform angular velocity and its linear velocity v_p is tangent.

The particle 'P' is making an angle 'θ' and its projection point on a diameter is '*Q*' which are shown in Fig. 7.7.

When the particle moves along the circumference, its projection '*Q*' also starts its motion along the diameter from point *A* to point *B* then point '*B*' to point '*A*' about the mean position '*O*' performing simple harmonic motion.

Displacement

At time t = 0 the particle 'P' subtends an angle $\angle P\hat{O}Q = \theta = \omega t$ with OQ and the displacement of Q is 'x' which is equal to OQ as shown in Fig. 7.7.

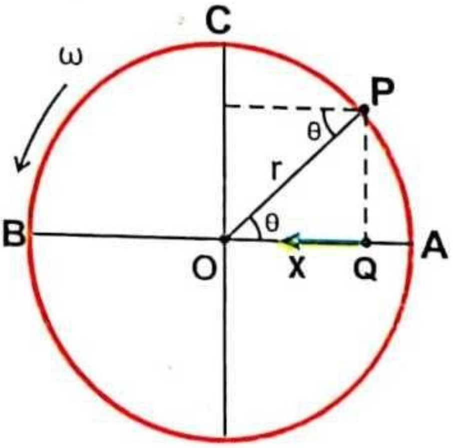


Fig.7.7: Displacement of the projection Q of particle P along the diameter AB.

Considering triangle POQ

$$\frac{OQ}{OP} = \cos\theta$$

$$\frac{x}{r} = \cos\omega t$$

$$x = r\cos\omega t \quad ......(7.7)$$

Eq. (7.7) gives the instantaneous displacement of point Q which is executing simple harmonic motion (SHM).

Velocity

In Fig. 7.8, the line $\overline{PR}$ is the horizontal component of velocity v_P of the particle and it is parallel to the diameter 'AB' of the circle. Therefore,

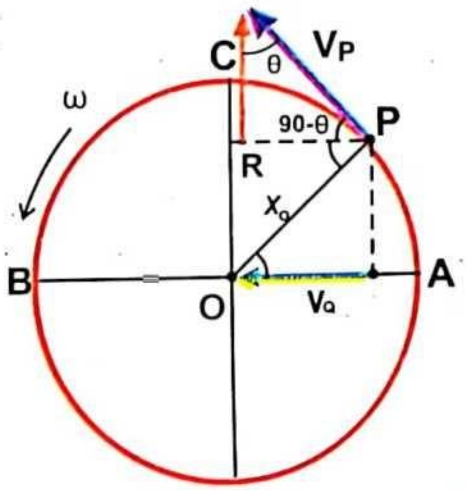


Fig.7.8: Velocity of the projection Q of particle P along the diameter AB.

$$v_Q = (v_P)_x$$

$$v_Q = v_P\cos(90^\circ - \theta)$$

Since $v_P = r\omega$ and $\cos(90^\circ - \theta) = \sin\theta$

$$v_Q = r\omega\sin\theta \, ...(7.8)$$

But $\sin^2\theta + \cos^2\theta = 1$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

Equation 7.8 becomes

$$v_Q = r\omega\sqrt{1-\cos^2\theta}$$

$$v_Q = r\omega\sqrt{1-\frac{x^2}{r^2}} \quad \left[\because \cos\theta = \frac{x}{r}\right]$$

$$v_Q = \omega\sqrt{r^2 - x^2} \quad ......(7.9)$$

It may be noted that at mean position $x = 0$ and velocity is maximum.

$$v_m = v_o = r\omega \quad ......(7.10)$$

Acceleration

Acceleration 'a_P' of the particle at point 'P' is directed towards the centre of the circle as shown in Fig.7.9. The horizontal component of a_P is along the diameter. Thus the acceleration of projection 'Q' is equal to the horizontal component of a_P.

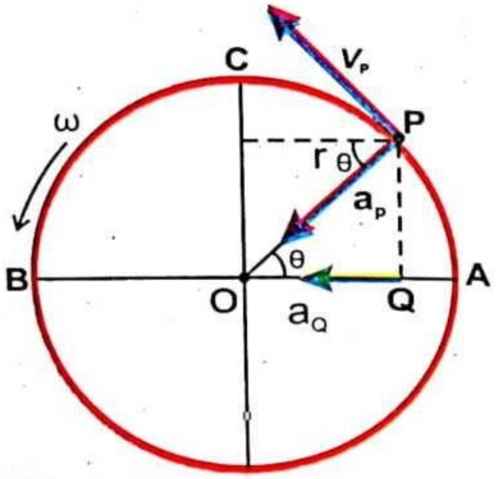


Fig.7.9: A particle which is moving along a circular path of circle its acceleration is directed toward the centre.

$$a_Q = -(a_P)_x$$

$$a_Q = -a_P \cos\theta$$

The negative sign indicates that the direction of acceleration is always directed towards the mean position.

$$a_Q = -r\omega^2\left(\frac{x}{r}\right) \qquad \left[\because a_P = r\omega^2, \cos\theta = \frac{x}{r}\right]$$

$$a_Q = -\omega^2 x \quad ......(7.11)$$

As particle is moving in the circle with uniform angular frequency (ω) therefore, Eq. (7.11) can be rewritten as;

$$a_Q \propto -x$$

This expression is the mathematical condition of S.H.M i.e. acceleration is directly proportional to the displacement and negative sign shows that its direction is towards the mean position. Therefore, it is concluded that when a particle is moving along a circumference of a circle then its projection executes S.H.M.

Time Period

It is defined as the time is required to complete one vibration by 'Q' from point A to B and then B to A. This is the same time in which the particle completes one revolution. It is denoted by 'T'.

Using the relationship $\omega = \dfrac{\theta}{t}$

For one complete cycle, $\theta = 2\pi$ radians and $t = T$ (time period).

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

Phase

The phase of an oscillating body determines its positions and direction of motion at a particular instant.

Consider the particle that moves along the circular path of a circle. Let at time $t = 0$ the particle is at point 'P' and its position vector OP makes an angle 'φ' with OA. After some time 't' the particle is at point P′ as OP makes angle $\theta = \omega t$ with OP. This angle determines both position and direction of the body at any instant and it is called phase angle which varies with time.

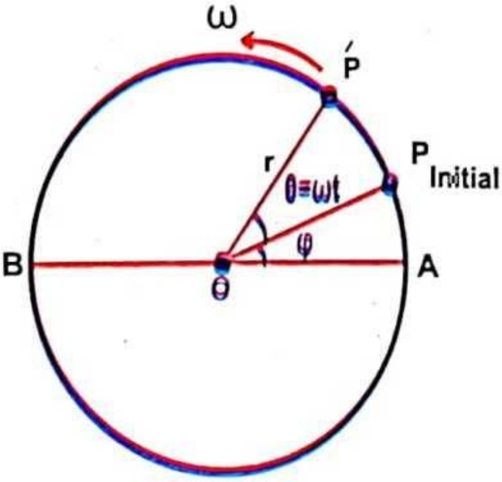


Fig.7.10: Phase angle and phase constant of a particle which is moving along a circle.

Now the total angle at point P is $\theta + \varphi$ is shown in Fig. 7.10. At time $t = 0$, phase $= +\varphi$. Sometimes at $t = 0$ the phase $= -\varphi$. In general the phase can be expressed as $\theta \pm \varphi$ or $\omega t \pm \varphi$, where φ is a phase constant which represents the initial position of a particles and it remains constant.

Example 7.3

A particle vibrates according to the equation $x = 0.3\cos 16t$. Find amplitude, frequency and its position at $t = 0$.

Solution:

As given $x = 0.3\cos 16t$ ......(7.12)

The general equation for displacement of vibrating body is.

$$x = x_o \cos \omega t \quad ......(7.13)$$

Comparing equation (7.12) and equation (7.13)

$x_o = 0.3\text{m}$ and $\omega = 16$ vib/s

But $\omega = 2\pi f$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{16}{2(3.14)}$$

$$f = 2.55 \text{ Hz}$$

FOR YOUR INFORMATION

In SHM, the acceleration a is proportional to the displacement x but opposite in direction, and the two quantities are related by the square of the angular frequency ω.

Position at $t = 0$

$$x = 0.3\cos 0^\circ = 0.3(1) \qquad \therefore \cos 0^\circ = 1$$

$$x = 0.3 \text{ m}$$

7.3 MASS-SPRING SYSTEM AND S.H.M

Consider a block of mass 'm' which is attached to one end of a horizontal spring. The other end of the spring is connected to a rigid support as shown in Fig.7.11. Initially the block is at the mean position on a frictionless horizontal surface i.e. at rest and $x = 0$. When the block is displaced through a small distance 'x' to the right then according to Hook's law there is a restoring force which causes the oscillation of the mass spring system. The acceleration produced by restoring force is directed towards its mean position and is given as;

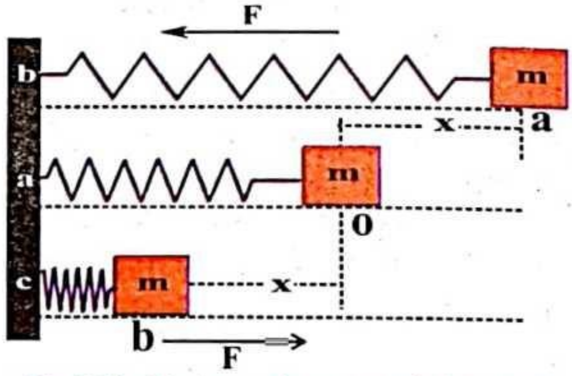


Fig.7.11: Horizontal mass spring system

$$a = -\left(\frac{k}{m}\right)x \;......(7.14)$$

Similarly, the acceleration of the particle moving in a circle executing simple harmonic motion is given as;

$$a = -\omega^2 x \;......(7.15)$$

Comparing equation (7.14) and equation (7.15)

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}} \;......(7.16)$$

Time Period

The time period of a mass attached to a spring, placed on a horizontal frictionless surface and executes S.H.M., is defined as time taken to complete its one round trip. Now,

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad ......(7.17)$$

Displacement

The displacement 'x' of the mass attached to the spring at time 't' is given by;

$$x = r\cos\theta \qquad [\because \theta = \omega t]$$

$$x = r\cos\omega t$$

But in case of mass attached to a spring $r = x_o$ where x_o is its amplitude from mean position to extreme position as shown in Fig. 7.11.

Substitute the values of r and ω in $x = r\cos\omega t$

$$x = x_o \cos\sqrt{\frac{k}{m}}t \qquad \ldots\ldots(7.18)$$

Instantaneous Velocity

We have studied that the velocity of the projection of the particle moving in a circle is along the horizontal direction and at any instant of time t is given by:

$$v = \omega\sqrt{r^2 - x^2}$$

But in mass spring system, we take $r = x_o$ and $\omega = \sqrt{\frac{k}{m}}$

$$v = \sqrt{\frac{k}{m}}\sqrt{x_o^2 - x^2}$$

CONCEPT CHECK
The period is the time required to complete one cycle.

$$v = \sqrt{\frac{k}{m}}\sqrt{x_o^2\left(1 - \frac{x^2}{x_o^2}\right)}$$

$$v = x_o\sqrt{\frac{k}{m}}\sqrt{\left(1 - \frac{x^2}{x_o^2}\right)} \qquad \ldots\ldots(7.19)$$

At mean position $x = 0$ and $v = v_o$ (maximum) equation 7.19 becomes.

$$v = x_o\sqrt{\frac{k}{m}}\sqrt{\left(1 - \frac{0}{x_o^2}\right)}$$

$$v = x_o\sqrt{\frac{k}{m}}\sqrt{(1-0)}$$

$$v = x_o\sqrt{\frac{k}{m}} \qquad \ldots\ldots(7.20)$$

Substitute equation (7.20) in equation (7.19)

$$v = v_o\sqrt{\left(1 - \frac{x^2}{x_o^2}\right)} \qquad \ldots\ldots(7.21)$$

7.4 SIMPLE PENDULUM

A simple pendulum is an ideal pendulum which consists of a solid bob of mass 'm' suspended from a rigid support through a light inextensible string of length 'ℓ'. The pendulum stays at a fixed point if the string is in vertical position. This point is called mean or equilibrium position. The forces acting on the solid bob are,

(a) the weight of the pendulum '*mg*' acting downward and
(b) the tension 'T' of the string acting in the upward direction along the direction of string.

When the pendulum is displaced from its mean position O through an angle 'θ' to the extreme position 'P', then a restoring force acts on the pendulum towards the mean position. Due to this restoring force, the pendulum starts oscillation to and fro under the action of gravity along a curved path about the mean position 'O' as shown in Fig. 7.12. At extreme position 'P', the weight of the body makes an angle 'θ' with the direction of the string. We can resolve it into its rectangular components. As the pendulum has no motion along the direction of the string therefore, the component $mg\cos\theta$ and tension 'T' are along the same line but in opposite direction so they cancel the effects of each other The component mg $\sin\theta$ provides the necessary restoring force and is responsible for the motion of simple pendulum. Thus;

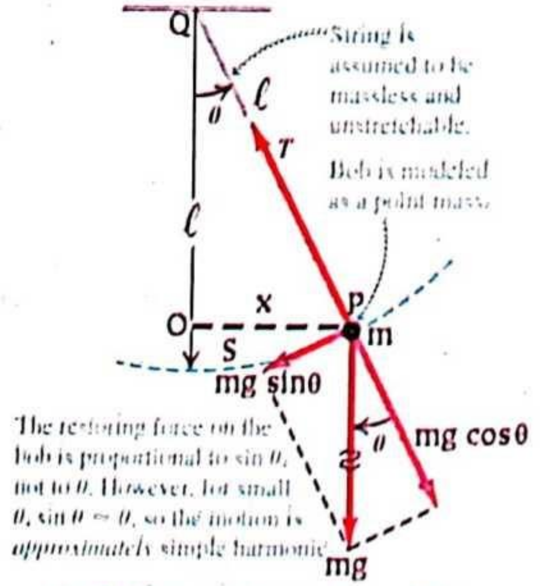


Fig.7.12: An ideal simple pendulum

$$F = -mg\sin\theta$$

Negative sign shows that the acceleration of the pendulum is always directed towards its mean position. According Newton's 2nd law

$$F = ma$$

Comparing above equations

$$ma = -mg\sin\theta$$

$$a = -g\sin\theta \quad ......(7.22)$$

If the angle 'θ' of the simple pendulum is small (i.e., $\theta < 10^\circ$), then the $\sin\theta$ can be replaced by the angle θ itself, expressed in radians. That is, for small angles

$$\sin\theta \approx \theta \cdot$$

So equation (7.22) is written as;

$$a = -g\theta \quad ......(7.23)$$

By the definition of an angular displacement,

$$\theta = \frac{S}{\ell}$$

where 'S' is the actual path length followed by the pendulum. Thus

$$a = -\left(\frac{g}{\ell}\right)S$$

In Figure 7.12, 'x' is very nearly equal to the arc of length 'S' of the circular path when the angle θ is small (about 10° or less). Hence,

$$a = -\left(\frac{g}{\ell}\right)x \quad ......(7.24)$$

If length 'ℓ' of the pendulum is fixed and 'g' remains constant for a given place and (g/ℓ) is constant. Eq. (7.24) can be rewritten as;

$$a = -(\text{constant})\,x$$

$$a \propto -x$$

This is the mathematical form of S.H.M and it is concluded that the motion of a simple pendulum is S.H.M.

As $$a = -\omega^2 x \quad ......(7.25)$$

Comparing equation (7.24) and equation (7.25)

$$\omega^2 = \frac{g}{\ell}$$

$$\omega = \sqrt{\frac{g}{\ell}} \quad ......(7.26)$$

POINT TO PONDER

Does a vibrating simple pendulum produce any sound?

Time period of a simple pendulum is given as

$$T = \frac{2\pi}{\omega} \qquad \left(\because \omega = \frac{\theta}{t} = \frac{2\pi}{T}\right)$$

$$T = \frac{2\pi}{\sqrt{\frac{g}{\ell}}}$$

$$T = 2\pi\sqrt{\frac{\ell}{g}} \quad ......(7.27)$$

The above expression shows that the time period of simple pendulum is directly proportional to the square root of the length of the string and inversely proportional to the square root of acceleration due to gravity. The time period of motion of the pendulum is independent of the mass *m* of the bob and amplitude.

A pendulum that completes one vibration in two seconds, i.e., its time period is two seconds is known a second pendulum.

The simple pendulum can be used to determine the gravitational acceleration at a particular location. We measure the length l of the pendulum and then set the pendulum into motion. The time period T of the simple pendulum is measured using a stopwatch and the acceleration of gravity is calculated by using equation (7.27) in the following form;

$$g = 4\pi^2 \frac{\ell}{T^2} \quad ......(7.28)$$

DO YOU KNOW

If a pendulum is shifted from Karachi to Quetta than its time period will be increased.

Example 7.4

What is the length of a second pendulum?

Solution:

$$\ell = ?$$

$$T = 2\ s$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T^2 = \frac{4\pi^2\ell}{g}$$

$$\ell = \frac{gT^2}{4\pi^2} = \frac{(9.8)(2)^2}{4(3.14)^2} = 0.994\ m$$

$$\ell = 99.4\ cm$$

CONCEPT CHECK

A pendulum making small swings undergoes simple harmonic motion.

7.5 CONSERVATION OF ENERGY IN S.H.M

When a body is executing simple harmonic motion it possesses both potential energy as well as kinetic energy. Its potential energy is on account of its displacement from mean position and the kinetic energy is due to its velocity. These energies vary during the oscillation, but the total energy at any instant remains constant in the absence of unbalanced resistive forces. In case of mass-spring system, when the mass is displaced from the mean position 'O' then there is a restoring force (F) whose value is zero at mean position when $x = 0$ and its value is maximum at either extreme positions where $x = x_o$. Thus average value of force from the mean position to the extreme position is

$$F_{avg} = \frac{F_{mean} + F_{ext}}{2} = \frac{0 + F}{2} = \frac{F}{2}$$

When displacement = 0   Force = 0

When displacement = x_o Force = kx_o

$$F = \frac{1}{2}kx_o$$

When the spring is stretched to its maximum displacement x_o, work is done on the spring which is given as under;

$$W = \vec{F}\cdot\vec{d} = \frac{1}{2}k\,x_o \cdot x_o = \frac{1}{2}k\,x_o^2$$

This work done on the mass attached to a spring stores in terms of potential energy, called elastic potential energy. So we have

$$P.E = \frac{1}{2}k\,x^2 \quad \ldots\ldots(7.29)$$

It is clear from Eq. (7.29) that potential energy of simple pendulum is zero at $x = 0$ and maximum at $x = \pm x_o$ i.e. the extreme position on either side.

After the removal of force, the mass attached to a spring starts its motion with velocity v then the kinetic energy of the mass attached to spring is given as:

$$K.E = \frac{1}{2}m\,v^2$$

From eq. 7.19 $v = x_o\sqrt{\frac{k}{m}}\sqrt{\left(1-\frac{x^2}{x_o^2}\right)}$

$$K.E = \frac{1}{2}m\left(x_o\sqrt{\frac{k}{m}}\sqrt{1-\frac{x^2}{x_o^2}}\right)^2$$

$$K.E = \frac{1}{2}kx_o^2\left(1-\frac{x^2}{x_o^2}\right) \quad \ldots\ldots(7.30)$$

We can study the values of P.E. and K.E. at different positions. Using Eqs.(7.29) and (7.30) respectively.

At mean position

At mean position where $x = 0$

Equation 7.29 and equation 7.30 becomes.

$$P.E. = \frac{1}{2}k\,(0)^2 = 0$$

$$K.E. = \frac{1}{2}kx_o^2\left(1-\frac{0}{x_o^2}\right)$$

$$K.E. = \frac{1}{2}kx_o^2$$

CONCEPT CHECK

The amplitude of vibrating body can be increased by the application of small forces at specific intervals.

$$T.E. = P.E + K.E$$

$$T.E. = 0 + \frac{1}{2}kx_o^2$$

$$T.E. = \frac{1}{2}kx_o^2 \ldots\ldots(7.31)$$

We conclude that the potential energy of a simple pendulum, executing S.H.M., at mean position is zero and its kinetic energy is maximum.

At extreme position

At extreme position we have $x = \pm x_0$ and Eq.(7.30) becomes;

$$K.E = \frac{1}{2}kx_o^2\left(1 - \frac{x_o^2}{x_o^2}\right) = \frac{1}{2}kx_o^2(1-1) = \frac{1}{2}kx_o^2(0)$$

$$K.E = 0$$

$$P.E = \frac{1}{2}kx_o^2$$

$$T.E = P.E + K.E$$

$$T.E = \frac{1}{2}kx_o^2 \ldots\ldots(7.32)$$

We conclude that the kinetic energy of simple pendulum, executing S.H.M., at extreme positions on either side is zero and its potential energy is maximum.

At any position

At any position x, where $-x_o < x < x_o$ then we have,

$$K.E. = \frac{1}{2}kx_o^2\left(1 - \frac{x^2}{x_o^2}\right)$$

$$P.E. = \frac{1}{2}k\,x^2$$

The total energy of simple pendulum, executing S.H.M., can be obtained by adding above two equations i.e.,

$$T.E. = P.E + K.E$$

$$T.E. = \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2\left(1 - \frac{x^2}{x_o^2}\right)$$

$$T.E. = \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2 - \frac{1}{2}kx^2$$

$$T.E. = \frac{1}{2}kx_o^2 \ldots\ldots(7.33)$$

Equations (7.31), (7.32) and (7.33) show that when a body executing SHM, the total energy of the vibrating system remains constant i.e., when the K.E. of the mass is maximum, mass passes through the centre of oscillation, the P.E of the mass spring is zero (x = 0). Conversely when the P.E. of the spring is maximum, mass is at its extreme position on either side, the K.E. of the mass is zero (v = 0). Fig 7.13 shows the variation of P.E. and K.E. with displacement 'x'. But the total energy (T.E.) of the vibrating system remains constant and this is represented by the horizontal line (brown line).

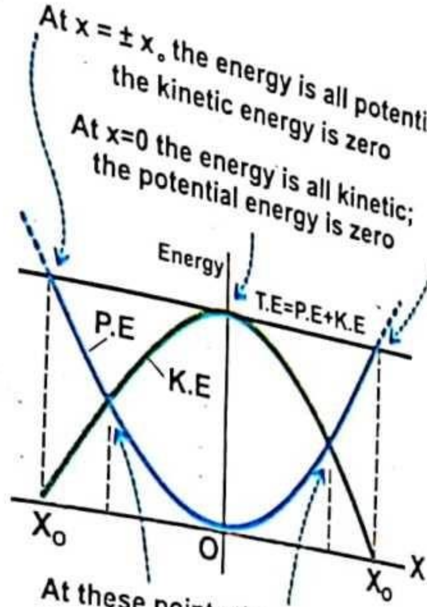


Fig.7.13: Potential energy, Kinetic energy and Total energy for linear harmonic oscillator.

7.6 FREE OSCILLATION

Consider a body or a system capable of oscillating, which is displaced from its mean position to its extreme position and then left free. Due to the restoring force, it starts oscillation with certain frequency which is called its natural frequency and the corresponding period is called its natural time period. If a body is oscillating with its own natural frequency and it is free from all the external resistive forces then such oscillations of the body are called free oscillations.

For example, oscillations of a simple pendulum, vibrations of prongs of a tuning fork, vibrations of string of musical instrument etc.

In free oscillations, the total energy of the body remains constant i.e. it is conserved. As we are assuming the absence of resistance force therefore the amplitude of the oscillation remains constant. Graphically, the free oscillations of a body with constant amplitude are shown in Fig.7.14.

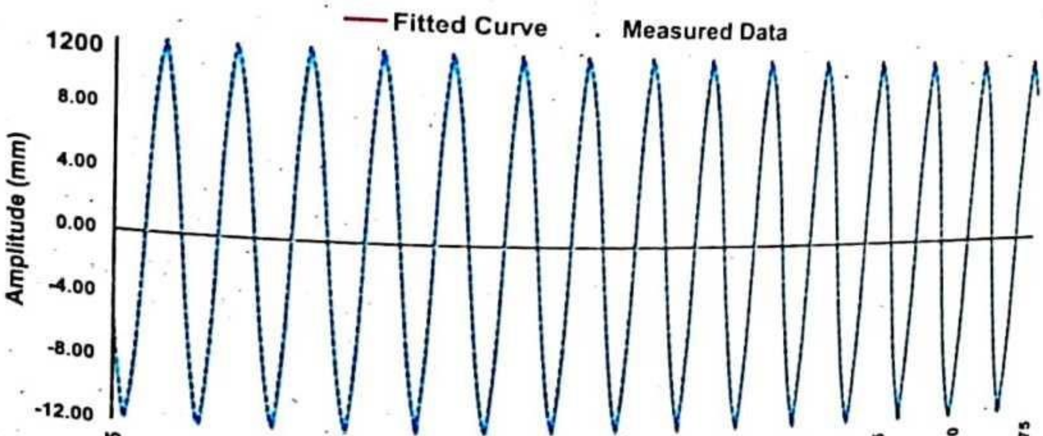


Fig.7.14: A graph of free oscillation of a body with constant amplitude

7.7 DAMPED OSCILLATION

In free oscillation, we have studied that the total mechanical energy of the oscillating body remains constant. But in practices, when a body is oscillating with its natural frequency, the amplitude of the oscillation gradually decreases with time and finally it comes to rest. This is due to the presence of resistive forces such as; air resistance, friction etc. The oscillation with decreasing amplitude in the presence of various resistive forces is called damped oscillation and the resistive forces are called damping forces. Energy dissipates due to negative work done by these damping forces and the body comes to rest in due course of time.

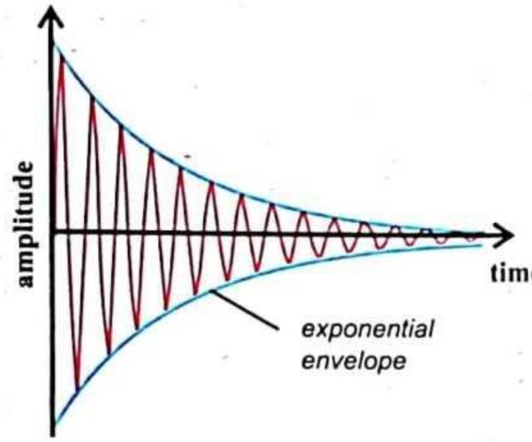


Fig.7.15: Damped oscillation of a body with decreasing amplitude

The damping force depends upon the speed of the oscillating body and is directed opposite to the velocity. Graphically the damped oscillation of the oscillating body is shown in Fig. 7.15.

Now the damped oscillation can be studied under the following three different cases.

(i) When the damping force is greater than the oscillating force, the body does not oscillate, i.e., without performing any oscillation, the body quickly comes at rest position. Such motion is called over-damping; graphically the over damping of a body is shown in Fig. 7.16.

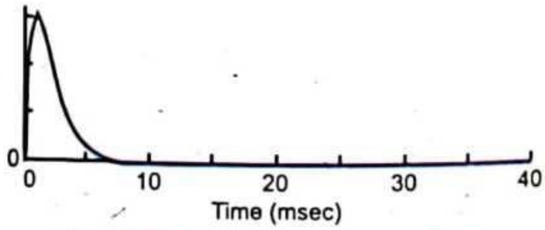


Fig.7.16: Over damping by a body

(ii) When the damping force is equal to the oscillating force, then the motion of body is called critical damping. In this case, the body returns to the equilibrium (mean) position with uniform speed along a curved path without performing oscillation as shown in Fig. 7.17.

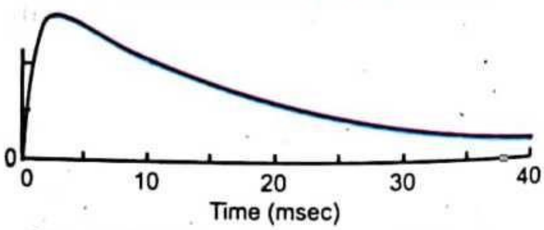


Fig.7.17: Critical damping of a body

(iii) When the damping force is less than the oscillating force then the body is set into oscillation and is called under-damping. Graphically, the under-damping of a body is shown in Fig. 7.18.

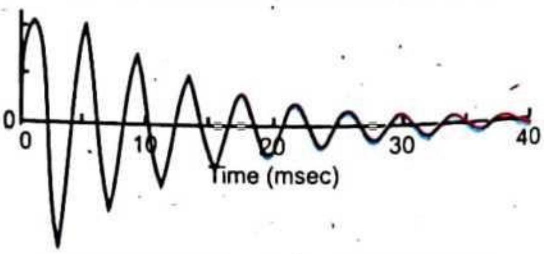


Fig.7.18: Under damping of a body

Examples of damping devices

Shock absorber

In damped oscillation, a small fraction of the energy of the oscillating system is dissipated against the friction but damping in some cases is very useful. One widely used application of damped oscillation is in the suspension system of an automobile. A shock absorber is attached to the frame of the vehicle.

A shock absorber is designed to use damping forces, which reduce the vibrations related with a bumpy ride.

As Fig.7.19 shows, a shock absorber consists of a piston in a reservoir of viscous fluid such as oil. When the piston moves up and down in response to a bump on the road, the oil inside the pressure tube is forced to go through piston valve and the base valve to move into the adjacent chamber. The holes in the valve control the rate of oil flow. Viscous forces that arise during this movement cause the damping effect.

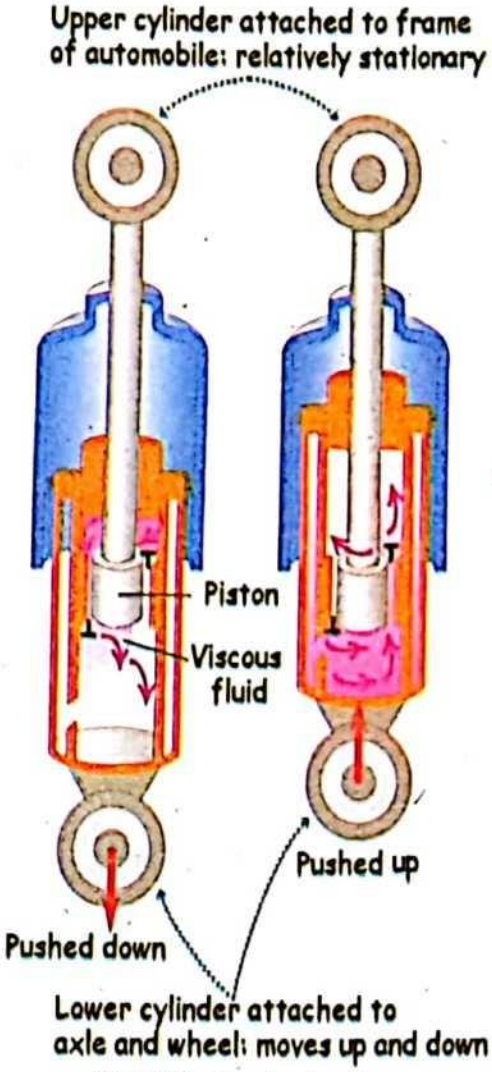


Fig.7.19: Shock absorber

The idea behind a shock absorber is to ease the natural bouncing motion of a spring. The degree of damping of shock absorber is shown in Fig.7.20. If the shocks are worn, and the system becomes under damped motion, then that wheel is going to be bouncing down the road (red-line). If the shocks are too aggressive, then it can create a situation where it delays the time it takes for the tyre to rebound to its position before the bump (green line).

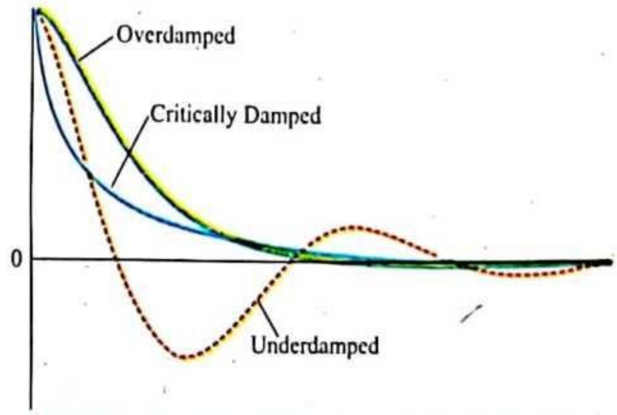


Fig.7.20: Degree of damping of Shock absorber

At critical damping, the tyre will rebound as quickly as it can to the road, without overshoot (blue line). In reality, critical dampening does not occur rather it slightly turns under-damped for a more comfortable ride. Typical automobile shock absorbers are designed to produce underdamped motion somewhat like that red line.

7.8 FORCED OSCILLATION AND RESONANCE

In damped oscillation, the oscillator cannot maintain its natural frequency for long duration due to the resistive forces and the amplitude of the oscillation decreases gradually with time. But we can maintain constant amplitude by applying a periodic external force which is called a driving force. Thus when the oscillating body is subjected to a periodic driving force then such oscillation is called forced oscillation and its frequency is called driving frequency. The vibration of a vehicle caused by the running of engine is an example of forced vibration. In forced oscillation, the amplitude of the oscillation depends upon the relation between the driving frequency and the natural frequency of the body.

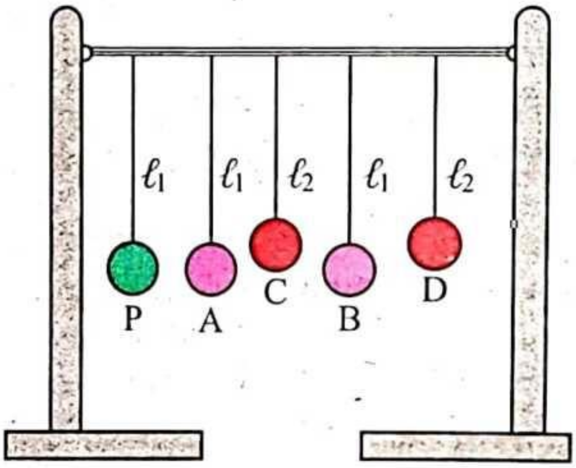


Fig.7.21: A set of five simple pendulums of different lengths suspended from a common rod.

If the frequency of the driving force is same as the natural frequency of the oscillating body, the amplitude of vibration is very much increased. This phenomenon is known as resonance and the oscillations of large amplitude are called resonant oscillations.

To demonstrate the resonance phenomenon, we perform a simple experiment. The experimental set up consists of two pair of pendulums A & B and C & D such that the length of A and B is ℓ_1 and length of C and D is ℓ_2. All the pendulums are suspended by a horizontal rod as shown in Fig. 7.21.

Now we introduce another pendulum 'P' whose length can be varied i.e. either ℓ_1 or ℓ_2. Consider the case when the length of pendulum 'P' is equal to ℓ_1. If the pendulum 'P' is set into vibration, this vibration reaches the other pendulums through the rod. Then the pendulums A and B receive a driving force through the rod and they also start vibration and its amplitude increases due to the resonance phenomenon because their lengths, natural frequency and natural periods are same. At the same time the pendulums C and D whose natural frequencies are different from natural frequency of 'P' do not oscillate i.e. they continue to remain at rest. If the length of the pendulum 'P' is made equal to ℓ_2 and allowed to vibrate, then the pendulums C and D start vibration due to resonance while pendulum 'A and B' remain at rest.

The resonance phenomenon can further be explained by some examples;

(i) The soldiers are advised to break their steps while crossing a bridge. If the soldiers march in steps then it is possible that the frequency of their footsteps become equal to the natural frequency of the bridge and the bridge may be set into vibrations with large amplitude due to the resonance.

(ii) During earthquake, when the frequency of earthquake is equal to the natural frequency of a building then the building will be set into vibrations with large amplitude due to the resonance and the building may collapse.

(iii) In communication system, all the transmitting signals can be received by receivers due to the resonance phenomenon when the frequency of the receiver is made equal to the frequency of incoming signal.

(iv) Microwave ovens generate super high frequency electromagnetic waves (3GHz-30GHz and wavelength of about 12 cm) and scatter them throughout the oven. The frequency of microwave excites water molecules into resonance and causes them to collide with one another. Friction generated by the collisions changes the kinetic energy of the water into heat that warms the food. Food containing water molecules can only be heated by the microwave oven.

(v) The amplitude of a swing can be increased by applying a suitable periodic force on it.

(vi) The tuning of a radio set for a certain station is also based on resonance in its LC-circuit.

7.9 SHARPNESS OF RESONANCE

We have studied in the resonance phenomenon that the amplitude of the oscillation is maximum when the frequency of the driving force is nearly equal to the natural frequency of the oscillating body. The amplitude can be decreased by changing the frequency of driving force.

If the amplitude of oscillation increases rapidly at a frequency 'f' slightly different that from the resonant frequency 'f_o', then the resonance is said to be sharp. Amplitude of the resonance oscillation and its sharpness depend upon damping that is, smaller the damping, greater will be the amplitude and more sharp will be the resonance. Similarly, for greater damping, the amplitude of the resonant oscillation will be small and such resonance is called flat resonance. Fig.7.22 shows the

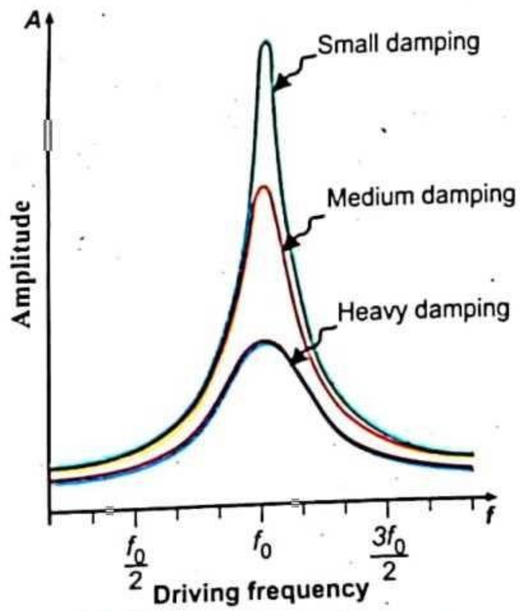


Fig.7.22: Sharpness of resonance

amplitude as a function of the applied frequency of the driving force. We see that the amplitude is large if the damping is small. Also the resonance is sharp in this case, that is the amplitude rapidly falls if 'f' is different from 'f_o'.

In the absence of damping forces, the amplitude of the oscillation (forced vibration) will be infinity but practically it is impossible. In all real cases some damping is always present in mechanical systems and the amplitude remains finite.

However, the amplitude may become very large if the damping is small and the applied frequency is close to the natural frequency.

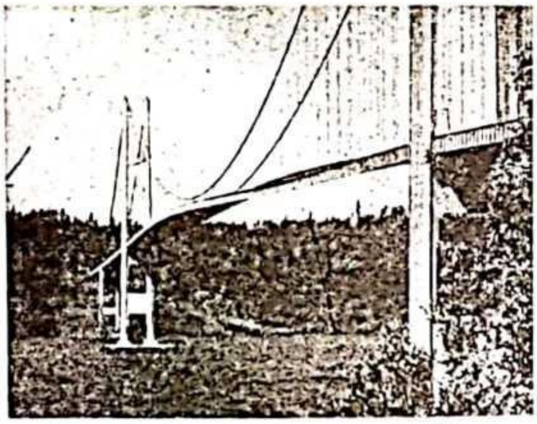


Fig.7.23: Before resonance condition Tacoma narrow bridge

The resonance effect is very important in the design of bridges and other civil engineering projects. On July, 1940 the newly constructed Tacoma Narrow Bridge (Washington) was opened for traffic as shown in Fig.7.23. Only four months after this, a mild wind set up the bridge in resonant vibrations. In a few hours the amplitude became so large that the bridge could not stand the stress and a part broke off and went into the water below (Fig.7.24). After this incident the engineers considered the resonance phenomenon in the design and construction of long span bridges.

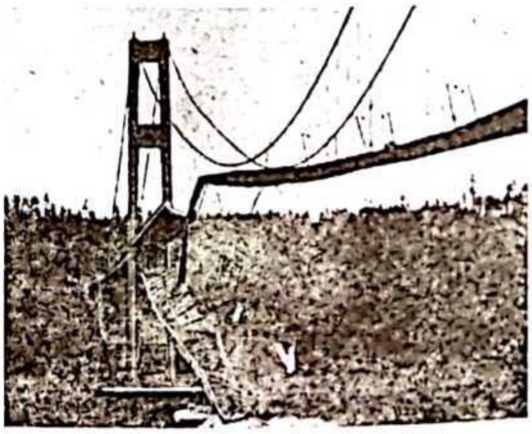


Fig.7.24: After resonance condition Tacoma narrow bridge

SUMMARY

- **Oscillatory motion:** To and fro motion of a body about its mean position is called oscillatory motion.
- **Periodic motion:** Motion that repeats itself in equal intervals of time.
- **Displacement:** The distance of a vibrating body from its equilibrium position to its present position.
- **Simple Harmonic Motion:** The motion of a body is said to be S.H.M. if its acceleration is directly proportional to the displacement and is always directed towards the mean position.
- **Vibration/Cycle:** The complete round trip of an oscillating body is called vibration.

- **Amplitude:** The maximum displacement from mean position of an oscillating object is called its amplitude.
- **Time Period:** The time taken by the vibrating body to complete one vibration / cycle.
- **Frequency:** Number of vibrations in one second is called frequency.
- **Circular Motion related with SHM:** The movement of projection of particle moving in a circle is S.H.M.
- **Angular Frequency:** The number of vibrations per unit time is called angular frequency.
- **Simple Pendulum:** A simple pendulum consists of a solid bob suspended by a string from a rigid support. Its to and fro motion about its mean position is S.H.M and its time period depends upon its length i.e. $T = 2\pi\sqrt{\frac{\ell}{g}}$.
- **Inter conversion of energy in SHM:** When a body is executing S.H.M. then it possesses both K.E. and P.E. which are inter-convertible such that the total energy remains constant.
- **Free Oscillations:** The oscillation of a body in the absence of resistive force is called free oscillation.
- **Forced oscillations:** The oscillation which is driven by frequency of a periodic force is known as forced oscillation.
- **Damped Oscillations:** The oscillation of a body in a resistive medium with decreasing amplitude is known as damped oscillation.
- **Over damped:** When the damping force is greater than the oscillating force then it is called over damping.
- **Critical damping:** When the damping force is equal to the oscillating force then the motion of the body is called critical damping.
- **Under damping:** When damping force is smaller than the oscillating force then the motion of the body is called under-damping.
- **Natural time period and natural frequency:** In the absence of resistive forces, the time period and frequency of the oscillating body is called its natural period and natural frequency.
- **Resonance:** When a force is applied, whose frequency is equal to the natural frequency of the system, the system vibrates at maximum amplitude and the phenomenon is called resonance.

- **Sharpness of resonance:** When the frequency of driving force is slightly different from the resonance frequency then the amplitude will be increased and resonance will be sharp. Sharpness of resonance depends upon damping.

EXERCISE

Multiple choice questions.

1. The acceleration of a body executing simple harmonic motion is;
   (a) Zero at each point (b) Remain same at each point
   (c) Maximum at mean position (d) Minimum at extreme position
2. What is the value of a spring constant when a 100g mass is attached to a spring and it is accelerated $0.5 m s^{-2}$ through a displacement of 5cm?
   (a) $0.1 N m^{-1}$ (b) $0.5 N m^{-1}$ (c) $1 N m^{-1}$ (d) $5 N m^{-1}$
3. If a spring of forced constant K is cut into two equal parts, then the spring constant of each half is
   (a) $\frac{K}{2}$ (b) 2K (c) K (d) $\frac{K}{\sqrt{2}}$
4. When a body is performing S.H.M then at its extreme position.
   (a) Displacement is zero (b) Amplitude is zero
   (c) Velocity is a zero (d) P.E is zero
5. A particle is executing S.H.M along a straight line with amplitude A, its kinetic energy is maximum when its displacement is
   (a) $\pm A$ (b) $\pm\frac{A}{2}$ (c) zero (d) $\pm\frac{A}{\sqrt{2}}$
6. The time period of a body attached to a spring depends upon.
   (a) Amplitude (b) Mass
   (c) Length (d) Displacement
7. When the length of the pendulum is increased four times then its time period is increased.
   (a) One time (b) Two time (c) Three time (d) Four time
8. What is the frequency of the body when its time period is 2 seconds?
   (a) 1 Hz (b) 2 Hz (c) 0.2 Hz (d) 0.5 Hz
9. A second's pendulum is one who has a time period of
   (a) 1 s (b) 2 s (c) $\frac{1}{2}$s (d) 0.2 s
10. In S.H.M., at what distance from mean position in terms of amplitude x_o, K.E. and P.E. both will have equal value?

(a) $0.51x_o$ (b) $0.61x_o$ (c) $0.71x_o$ (d) $0.81x_o$

11. The instantaneous K.E of a mass attached to the end of an elastic spring is:

(a) $\frac{1}{2}k(x_o^2 - x^2)$ (b) $\frac{1}{2}k(x_o^2 + x^2)$ (c) $\frac{1}{2}k(x^2 - x_o^2)$ (d) $\frac{1}{2}k(x - x_o)$

12. In S.H.M., we have the conservation of

(a) Kinetic energy (b) Potential energy
(c) Total energy (d) Mechanical energy

13. A free oscillation has constant

(a) Energy (b) Amplitude (c) Frequency (d) All of these

14. When damping force is equal to the oscillating force then the damping is called.

(a) Under damping (b) Critical damping
(c) Over damping (d) No damping

15. A resonance occurs when driving frequency is

(a) Less than the natural frequency (b) Greater than natural frequency
(c) Equal to the natural frequency (d) Equal to zero

16. In the absence of damping force, when driving frequency is equal to the oscillating frequency then the amplitude of the oscillation will become

(a) Zero (b) Minimum (c) Maximum (d) Infinity

17. Food is cooked in a microwave oven by the effect of:

(a) Interference (b) Mechanical resonance
(c) Magnetic resonance (d) Electric resonance

SHORT QUESTIONS

1. What is the role of the restoring force in the simple harmonic motion?
2. What are the two main conditions that must be met to produce simple harmonic motion?
3. What is the relation of circular motion of body with simple harmonic motion?
4. What is the difference between the time periods of simple pendulum and a body attached to a spring?
5. What is the difference between phase angle and phase constant?
6. When a body is performing S.H.M then at what condition, its total mechanical energy is conserved and at what condition, its energy does not conserve?
7. Show that in S.H.M. the acceleration is zero when the velocity is maximum and the velocity is zero when the acceleration is maximum.
8. In the simple harmonic motion of a mass attached to a spring, the velocity of the mass is equal to zero when the acceleration has its maximum value. How is

this possible? Can you think of other examples in which a body has zero velocity with a nonzero acceleration?

9. What is the difference between free and forced oscillation?
10. Give one practical example each of free and forced oscillations.
11. How natural time period of an oscillating body remains constant.
12. Describe the three kinds of damping?
13. How does sharpness of resonance occur?
14. How the amplitude of resonant oscillation affected by damping?
15. What happens to the time period of a simple pendulum if its length is quadrupled?

COMPREHENSIVE QUESTIONS

1. Define simple harmonic motion with all its characteristics such as; Vibration, Instantaneous displacement, Amplitude, Time period, Frequency and Angular.
2. Show that if a particle is moving along a circle, then its projection on the diameter of the circle executes S.H.M.
3. Prove that the motion of a mass attached to a spring is executing S.H.M.
4. Describe simple pendulum and prove that its time period depends upon its length.
5. Prove that when a body is performing S.H.M, its total energy remains constant.
6. Compare free and damped oscillations. Also discuss the three types of damped oscillations.
7. State and explain with examples the forced oscillation and resonance.

NUMERICAL PROBLEMS

1. When a 600 g mass is suspended at the end of a vertical spring then the spring stretches by 0.45 m. What is the spring constant of the spring and how much further will it be stretched if an additional mass of 600 g is hung from it?

($13\ Nm^{-1}$, 0.45 m)

2. A 2 kg mass attached to a spring is executing S.H.M. and makes 4 vibrations per second. Calculate the acceleration and the restoring force acting on the body when its displacement from mean position is 7 cm. **($44.2\ m\ s^{-2}$, 88.4 N)**

3. A particle performing S.H.M of amplitude 8 cm. If its velocity while crossing the mean position is $4\ m\ s^{-1}$, what is its frequency and time period?

(8 Hz, 0.125 sec)

4. What is the amplitude, frequency, period and position at t = 2s of a vibrating body whose motion is represented by the equation $x = 0.2\cos 0.125\pi t$?

(0.2 m, 0.0625 Hz, 16 s, 0.20 m)

5. Calculate the frequency of simple pendulum of length 0.8 m which is vibrating on Mars, where weight of object is 0.40 times its weight on earth. **(0.35 Hz)**

6. How much time period of a simple pendulum is increased by increasing its length from 0.8 m to 0.993 m? **(0.2 s)**

7. A block of mass 5 kg is dropped from a height of 0.8 m on to a spring of spring constant 1960 N m^{-1}. Find the displacement through which the spring will be compressed. **(0.2 m)**

8. A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant 20000 N m^{-1}. If two people riding in the car have a combined mass of 160 kg, find the frequency of vibration of the car when it is driven over a pot-hole in the road. Assume the weight is evenly disturbed. **(1.18 Hz)**

Unit 8

WAVES

Major Concepts (27 PERIODS)

- Periodic waves
- Progressive waves
- Transverse and longitudinal waves
- Speed of sound in air
- Newton's formula and Laplace correction
- Superposition of waves
- Stationary waves
- Modes of vibration of strings
- Fundamental mode and harmonics
- Vibrating air columns and organ pipes
- Doppler effect and its applications
- Generation, detection and use of ultrasonic

Conceptual Linkage

This chapter is built on
Sound Science VII & VIII
Oscillation & Waves Physics
IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- Describe what is meant by wave motion as illustrated by vibrations in ropes, springs and ripple tank.
- Demonstrate that mechanical waves require a medium for their propagation while electromagnetic waves do not.
- Define and apply the following terms to the wave model; medium, displacement, amplitude, period, compression, rarefaction, crest, trough, wavelength, velocity.
- Solve problems using the equation: $v = f\lambda$.
- Describe that energy is transferred due to a progressive wave.
- Identify that sound waves are vibrations of particles in a medium.
- Compare transverse and longitudinal waves.
- Explain that speed of sound depends on the properties of medium in which it propagates and describe Newton's formula of speed of waves.
- Describe the Laplace correction in Newton's formula for speed of sound in air.
- Identify the factors on which speed of sound in air depends.
- Describe the principle of superposition of two waves from coherent sources.
- Describe the phenomenon of interference of sound waves.
- Describe the phenomenon of formation of beats due to interference of non coherent sources.
- Explain the formation of stationary waves using graphical method

- Define the terms, node and antinodes.
- Describe modes of vibration of strings.
- Describe formation of stationary waves in vibrating air columns.
- Explain the observed change in frequency of a mechanical wave coming from a moving object as it approaches and moves away (i.e. Doppler Effect).
- Explain that Doppler Effect is also applicable to electromagnetic waves.
- Explain the principle of the generation and detection of ultrasonic waves using piezoelectric transducers.
- Explain the main principles behind the use of ultrasound to obtain diagnostic information about internal structures.

INTRODUCTION

The phenomenon of a wave motion is a vast field in the study of physics. because we observe daily various kinds of waves and their propagation such as; sound waves, light waves, waves on the surface of water, waves in a string, seismic (earth quake) waves, radio waves, x-rays and so on. All these waves are disturbance produced by vibrating bodies.

The waves can travel from one place to another place through a medium or without medium. One of the most important properties of waves is that they transfer energy. This transfer of energy is initiated by a vibrational motion. It is the physical manifestation of the form of energy transfer from one place to another. On the other hand, a wave does not transmit matter it transfers only energy. For example, electromagnetic waves from sun carry energy in the form of light and heat, sound energy from musical instruments causes our ear drums to vibrate. The energy carried by seismic waves (earthquakes) can devastate vast areas causing land to move and building to collapse and also produce tsunami.

Certain types of waves can travel only through some material called medium. Those waves which require a medium for their propagation are known as mechanical waves. For example, water waves, sound waves, waves on a string and so on.

On the other hand, the waves which do not require any medium for their propagation are called electromagnetic waves. These waves are capable of traveling even through an empty space without the help of a medium. Such as radio waves, light waves, x-rays, γ-rays, infrared and ultraviolet radiations.

Now according to De-Broglie's hypothesis, the subatomic particles of matter such as electrons, protons, neutrons and other fundamental particles are moving in the form of wave. This kind of wave is termed as matter or De-Broglie wave which is usually studied in modern physics.

In this chapter we discuss not only the general properties of waves such as wavelength, frequency and speed of wave, but also study the behavior of transverse and longitudinal waves. In the later part of this chapter our main focus will be on the sound waves and its characteristics. We will also define the principle of superposition and explain the phenomenons of interference, beats and stationary waves. In the last we will study the apparent change in frequency due to relative motion of source and observer which is called Doppler Effect.

8.1 PROGRESSIVE WAVES

It is common observation that a disturbance is produced on the surface of still water in a pond, when a stone is dropped into it. This disturbance causes the waves which spread out across the surface of water as shown figure 8.1. If we place a leaf on these ripples, we can observe its up and down motion but the leaf does not move along the ripples. This example has confirmed that water waves do carry energy only but there is no movement of matter across the surface of water. Thus a wave which transfers energy from one point to another point by a periodic disturbance is called progressive wave or travelling wave. There are some characteristics of progressive waves which are summarized as;

Fig.8.1: Progressive waves spread out across the surface of water.

Crest

The peak of the portion of the wave above its equilibrium or mean level in a transverse wave is called crest as shown in Fig. 8.1.

Trough

The peak of the portion of the wave below its equilibrium position in a transverse wave is called a trough as shown in Fig. 8.1. The direction of a trough is opposite to that of a crest.

Amplitude

In progressive wave, the maximum displacement of a vibrating particle from the mean level to the peak point of crest or trough is known as amplitude as shown in Fig 8.1. The unit of amplitude is meter and its dimensional formula is $[M^oLT^o]$.

Wave Length

The distance between any two consecutive crests or two consecutive troughs is called wavelength. Wavelength is represented by 'λ' (lambda) and it is measured in metres.

Time period and frequency

The time in which one wave cycle of a wave is passed through a certain point is called time period. It is represented by T. The unit of time period is second.

The numbers of waves passing through a certain point in one second is called frequency. It is measured in Hertz (Hz) and its dimensional formula is $\left[M^{0}L^{0}T^{-1}\right]$. Frequency and time period are reciprocal to each other that is, $f = \frac{1}{T}$. A graph of progressive waves is shown in figure 8.2.

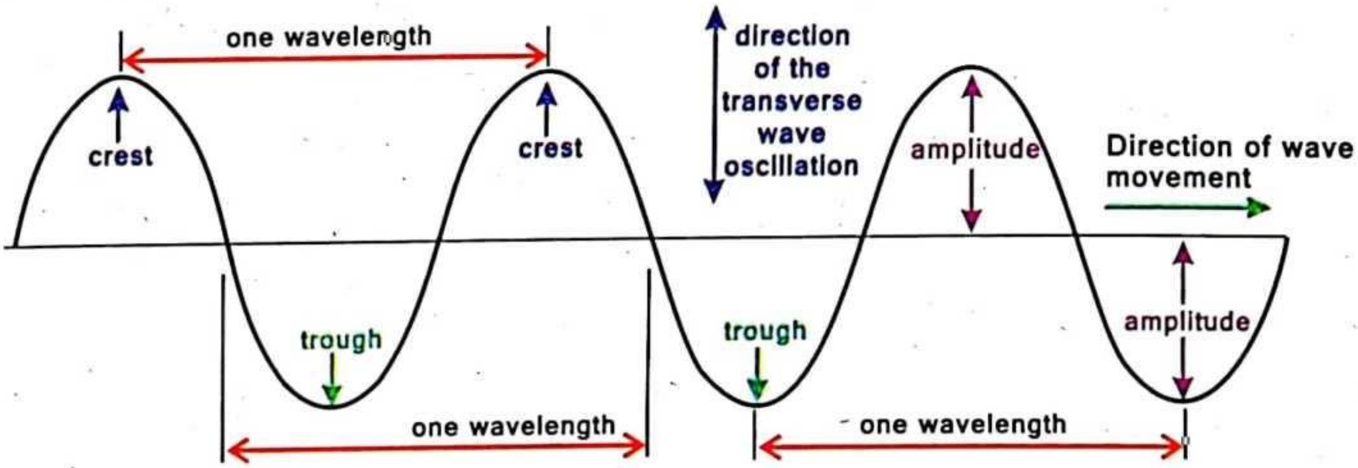


Fig.8.2: Propagation of progressive wave consists of crests and troughs with certain amplitude and wavelength.

8.1.1 Types of progressive waves

Every day, we come across a number of progressive waves. All these waves can be classified into two classes on the basis of their propagations (i) Transverse Waves (ii) Longitudinal Waves.

(i) Transverse Waves

The waves in which the particles of the medium vibrate perpendicular to the direction of propagation of waves are called transverse waves. The wave travelling along a stretched string is an example of a transverse wave which is explained as under;

Let one end of a string of length 'l' be connected to a rigid support and the other free end is moved up and down in a direction perpendicular to its length. A wave consisting of crests and troughs is set up in the string.

This wave is travelling along the length of the string with speed called wave speed. But the particles of the string are vibrating up and down perpendicular to the length of string as shown in figure 8.3.

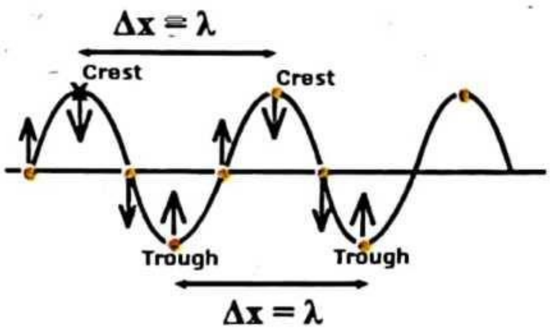


Fig.8.3: A transverse pulse traveling on a stretched string. The direction of disturbance is perpendicular to the direction of propagation.

Another result is also obtained from this example that the wave transfers only its waveform in the forward direction but the particles of the string remain at their own places which are vibrating up and down and they do not move forward.

The speed of transverse wave can be calculated by using the general relation of speed;

$$v = \frac{\Delta x}{\Delta t}$$

If $\Delta x = \lambda$ (Wavelength of the wave)

$\Delta t = T$ (Time period)

Then above equation becomes

$$\because v = \frac{\lambda}{T}$$

But $$T = \frac{1}{f}$$

$$v = f\lambda \quad ......(8.1)$$

This is the fundamental equation for speed of a wave and it is equally applicable to all kinds of wave.

Example 8.1

The radar waves with 3.4 cm wavelength are sent out from a transmitter. If these waves travel with speed is $3\times10^8 ms^{-1}$, what is their frequency?

Solution:

Wavelength (λ) = 3.4 cm. = 0.034 m

Speed of waves (v) = 3×10^8 m s^{-1}

Frequency (f) = ?

According to equation 8.1

$$v = f\lambda$$

$$f = \frac{v}{\lambda}$$

$$f = \frac{3\times10^8 ms^{-1}}{0.034 m}$$

$$f = 8.8\times10^9 Hz$$

$$f = 8.8 GHz$$

(ii) Longitudinal Waves

A wave in which the particles of the medium are vibrating along the direction of propagation of wave is called longitudinal wave. It is explained as under.

Consider two springs of equal lengths which are connected to a body such that the body remains between them. Now let a force is applied on either side to displace the body and then it is made to free. The body sets into oscillation. As a result, a longitudinal wave is produced which consists of compression and rarefaction travelling along the spring. On the other hand, each particle of the spring is also vibrating along the direction of wave in the spring, as shown in Fig.8.4. In longitudinal wave, the distance between the centres of two consecutive compressions or two rarefactions is called its wavelength.

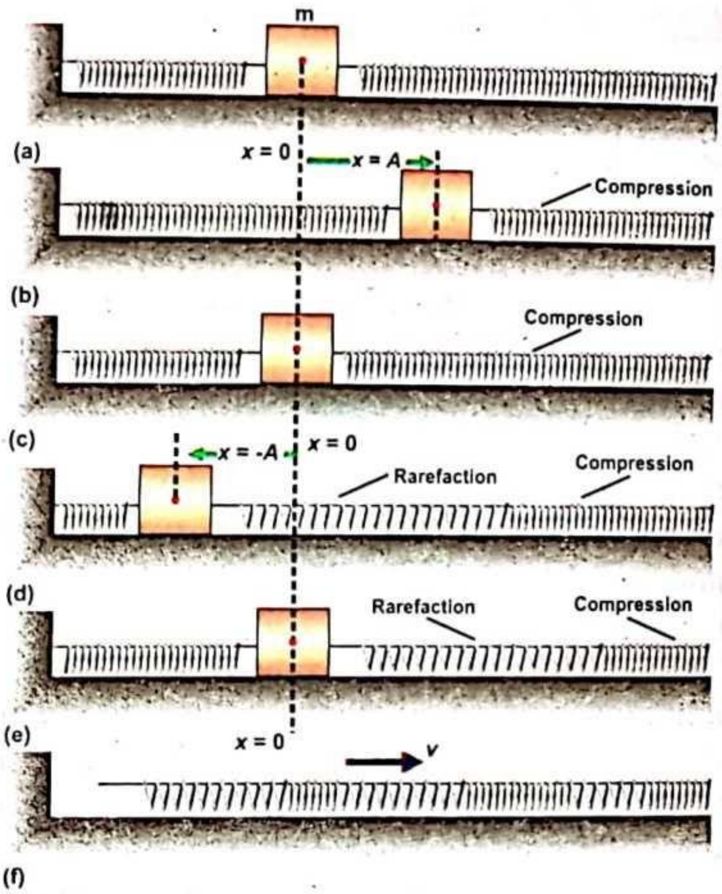


Fig.8.4: A longitudinal pulse along a stretched spring. The displacement of the coils is parallel to the direction of the propagation

8.2 PERIODIC WAVES

We have defined wave in terms of disturbance which is produced by a source and travels in a medium. If a steady vibrating source produces continuous, regular and rhythmic disturbance in a medium, then it is called periodic wave. A vibrating mass spring system as shown in Fig.8.5 is a good example of a periodic vibrator that produces a periodic wave.

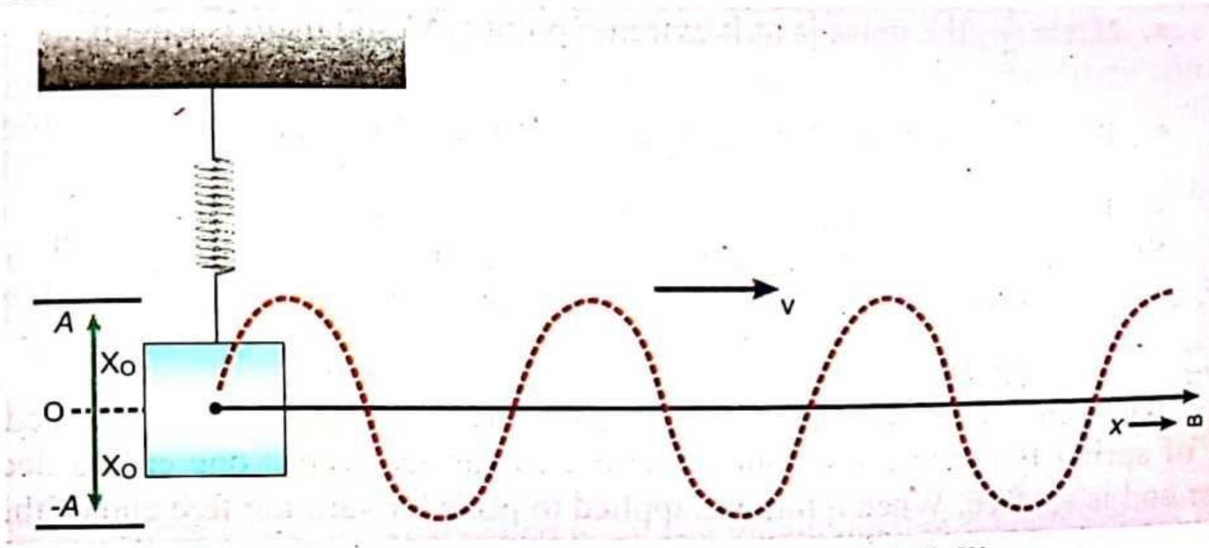


Fig.8.5: The experimental arrangement of Transverse Periodic Wave

8.2.1 Transverse periodic waves

A transverse periodic wave can be demonstrated by the mass-spring experimental setup which consists of a block of mass 'm' connected with a vertical hanging spring, as shown in Fig.8.5. A length of string in the horizontal direction is also connected with the mass. Now when a force is applied to displace the block upward from its mean position 'O' to its extreme position 'A' at a distance 'x_o' and it is made to free, then the block starts vibrating up and down. At the same time, a transverse periodic wave is produced in the string which travels along the length of string. Such waves consist of crests and troughs as shown in Fig.8.5.

The experiment shows that the mass-spring vibrator is executing simple harmonic motion. Its amplitude and time period are equal to the amplitude and time period of a transverse periodic wave. This transverse wave is travelling along the length of the string in the form of a sinusoidal wave.

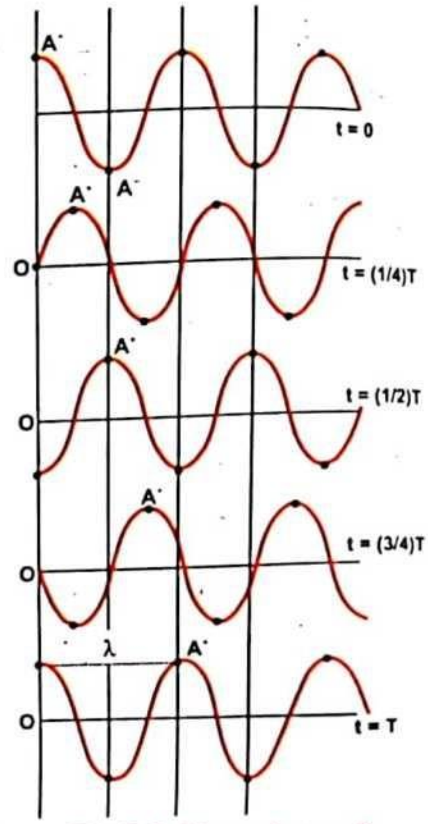


Fig.8.6: Wave shape of Transverse Periodic Wave

Fig.8.6 illustrates the wave shape of such periodic wave. This graph is between the time and displacement and it is explained as under.

- If $t = 0$, the block is at extreme point 'A' and there is crest of transverse periodic wave.
- If $t = \frac{T}{4}$, the mass is at it mean position 'O'.
- If $t = \frac{T}{2}$, the mass is at it extreme point '-A' and there is trough.
- If $t = \frac{3T}{2}$, then mass is again at its mean position 'O'.
- If $t = T$ the mass is again at extreme position and there is again the crest.

Similarly, for the next cycles of wave, the same process is repeated. In this way, a periodic transverse wave moves to the right as shown in Fig.8.6.

8.2.2 Longitudinal periodic waves

The generation of longitudinal periodic waves can also be demonstrated by a coil of spring lying on a horizontal frictionless surface whose one end is tied and other end is set free. When a force is applied to push forward the free end of the coil then a few turns of the spring are compressed. As a result, a compression portion is formed at the free end. This compression portion exerts a force on the next few turns of the spring so that another compression is formed which is transferred in the next section and so on. In this way a compression portion travels along the spring in forward direction.

Similarly, when a force is applied to pull backward the free end then a rarefaction portion is formed in the loops near the free end of the spring. This rarefaction is transferred to the next section and starts travelling along the spring.

Thus, when a regular and steady periodic force is applied to push forward and pull backward the spring at constant rate then a periodic longitudinal wave is set up in the spring, which travels along the spring. It consists of a series of compressions and rarefaction. Graphically, when this longitudinal wave is drawn on a graph then a sinusoidal wave (sine or cosine) is obtained as shown in Fig.8.7.

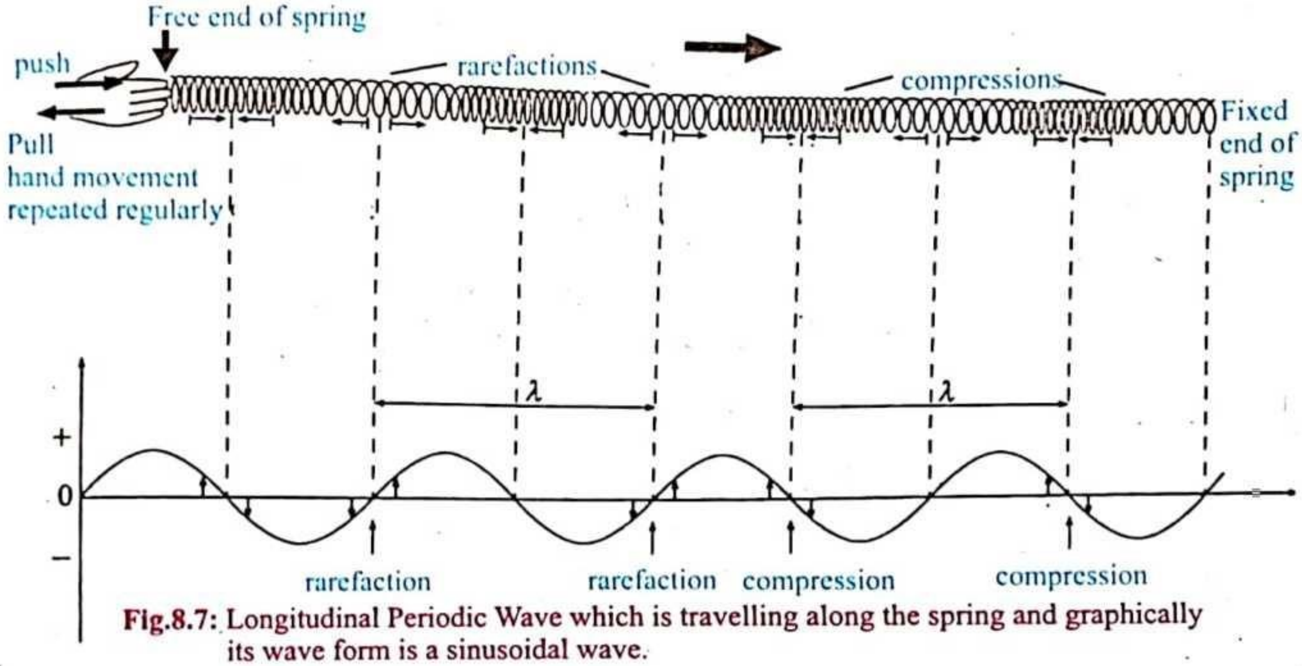


Fig.8.7: Longitudinal Periodic Wave which is travelling along the spring and graphically its wave form is a sinusoidal wave.

Sound wave is also a good example of longitudinal waves and it can be explained with the help of a tuning fork. When the tuning fork is struck on a rubber pad, its prongs start vibration. When the prongs move outward then they compress a small column of air and forms compressions. When the prongs move inward then rarefactions is formed in the air column. In this way, a sound wave, which consists of a series of compressions and rarefactions, travels in air as shown in Fig.8.8.

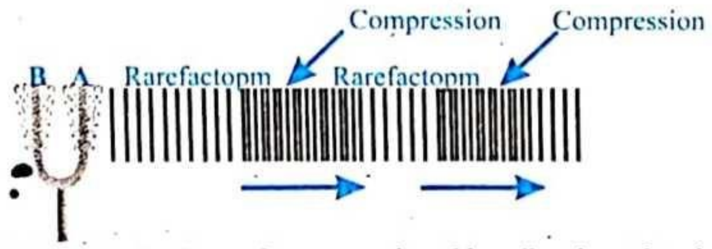


Fig.8.8: Longitudinal sound wave produced by vibration of tuning fork.

Thus, one can say that sound wave is a longitudinal or compressional wave.

8.3 SPEED OF SOUND IN AIR

Sound wave is a longitudinal or compressional wave, and it requires a medium for its propagation. This medium may be solid, liquid or gas. Experiments show that the speed of sound depends upon elasticity 'E' and density 'ρ' of the medium. Mathematically it can be expressed as;

$$v = \sqrt{\frac{E}{\rho}} \text{ ......(8.2)}$$

Stress = F/A
Strain = ΔV/V

But according to Hook's law

$$\text{Elasticity (E)} = \frac{\text{stress}}{\text{strain}}$$

$$E = \frac{\Delta P}{\frac{\Delta V}{V}} \text{ ......(8.3)}$$

Besides, elasticity and density of the medium, the speed of sound also depends upon the nature of the medium, for example the molecules of solids are much closer to one another than in liquids or gases. So a quick disturbance takes place in solid. Therefore the speed of sound is much higher in solids than in liquids or gases. The value of speed of sound in various solids, liquids and gases is given in the table 8.1.

Table 8.1: Speed of Sound in different substances in ms^{-1}

Gases (20°C)		Liquids (25°C)		Solids (20°C)	
Hydrogen	1284	Glycerin	1904	Iron	5960
Carbon Dioxide	259	Sea Water	1535	Pyrex Glass	5640
Oxygen	316	Water	1493	Aluminum	5100
Nitrogen	334	Mercury	1450	lead	2160
Air	344	Methyl Alcohol	1103	Rubber	1550

8.3.1 Newton's formula for the speed of sound in air

Sound is a longitudinal wave and it consists of a series of compressions and rarefactions. We know that the temperature of a gas increases on its compression and fall when allowed to expand. Newton assumed that when a sound travels through air or a gaseous medium, then the process of formation of compressions and rarefactions is very slow. So, the temperature in the regions of compression and rarefaction remains constant i.e. the temperature changes are extremely small and can be neglected. Thus, according to the assumption of Newton the propagation of sound through air or a gaseous medium is under isothermal process in which the temperature of the medium remains constant.

Consider a certain mass of air having volume 'V' and pressure 'P'. When sound is travelling through it, then the pressure of air during compression is increased from 'P' to $P + \Delta P$ and its volume is decreased from V to $V - \Delta V$ as shown in Fig.8.9. The propagation of sound waves through air under isothermal process follows Boyle's law i.e.;

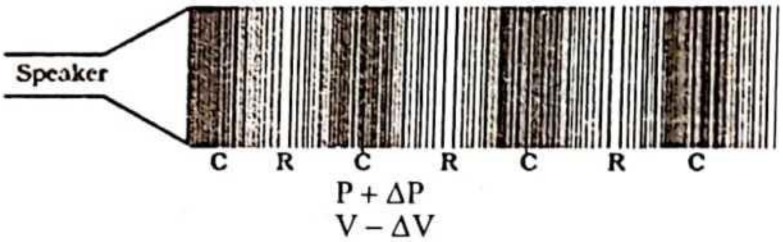


Fig.8.9: When sound wave propagates, a change in pressure and volume of the medium occurs due to the compression and refraction.

$$PV = (P + \Delta P)(V - \Delta V)$$

$$PV = PV - P\Delta V + V\Delta P - \Delta P \Delta V$$

The product of ΔP and ΔV is very small and it can be neglected.

$$0 = -P\Delta V + V\Delta P$$

$$P\Delta V = V\Delta P$$

$$P = \frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$$

CRITICAL THINKING

When explosions due to the fusion reactions occur on the surface of sun then why we cannot hear their sound?

$$P = \frac{\text{Stress}}{\text{Strain}} = E \quad ......(8.4)$$

Substitute the values P for E in equation (8.2), we get;

$$v = \sqrt{\frac{P}{\rho}} \qquad ......(8.5)$$

Eq.(8.5) is referred as Newton's formula for speed of sound.

At S.T.P

Pressure $= 1.01 \times 10^5 \text{ N m}^{-2}$

And the density of air is $\rho = 1.293 \text{ kg m}^{-3}$

By putting these values in equation (8.5), we have

$$v = \sqrt{\frac{1.01 \times 10^5 \text{ N m}^{-2}}{1.29 \text{ kg m}^{-3}}}$$

$$v = 279 \text{ m s}^{-1}$$

This result differs from the experimental result of speed of sound i.e. $333\,ms^{-1}$. The theoretical value of speed of sound is about 16 percent less than the experimental value.

8.3.2 Laplace's correction

Laplace has pointed out that during compression, volume of air is decreased from V to V − ΔV and pressure is increased from P to P + ΔP but its temperature does not remain constant. Because compression and rarefaction are occurred very quickly such that air neither loses heat during compression nor gains during rarefaction. Thus the propagation of sound through air follows adiabatic process and we should apply adiabatic equation rather than of Boyle's law.

$$PV^{\gamma} = (P+\Delta P)(V-\Delta V)^{\gamma} \quad ......(8.6)$$

Where γ is adiabatic constant and its value can be calculated in terms of ratio.

$$\gamma = \frac{\text{Molar specific heat of gas at constant pressure}}{\text{Molar specific heat of gas at constant volume}}$$

Solving equation (8.6)

$$PV^{\gamma} = (P+\Delta P)V^{\gamma}\left(1-\frac{\Delta V}{V}\right)^{\gamma}$$

$$P = (P+\Delta P)\left(1-\frac{\Delta V}{V}\right)^{\gamma} \quad ...(8.7)$$

FOR YOUR INFORMATION

Molar Specific Heat of Various Gases

Monoatomic	1.67
Diatomic	1.41
Polyatomic	1.31

Solving $\left(1-\frac{\Delta V}{V}\right)^{\gamma}$ by binomial theorem and neglecting the higher terms in $\left(\frac{\Delta V}{V}\right)$, we get

$$\left(1-\frac{\Delta V}{V}\right)^{\gamma} = 1-\gamma\frac{\Delta V}{V} + \text{neglected terms}$$

Eq. (8.4) becomes

$$P = (P+\Delta P)\left(1-\gamma\frac{\Delta V}{V}\right)$$

$$P = P-\gamma\frac{P\Delta V}{V}+\Delta P-\gamma\frac{\Delta P\Delta V}{V}$$

The product of ΔP and ΔV is again very small and it can be neglected.

$$-\gamma P\frac{\Delta V}{V}+\Delta P = 0$$

$$\Delta P = \gamma P\frac{\Delta V}{V}$$

FOR YOUR INFORMATION

Binomial series expansion

$$(a+x)^n = 1+nx+\frac{n(n-1)}{2i}x^2+...$$

FOR YOUR INFORMATION

Because radio waves travel at speed $3 \times 10^8\ ms^{-1}$ and sound waves are slower, $3.4 \times 10^2\ ms^{-1}$, a broadcast voice can be heard sooner 1300 miles away then it can be heard at the back of the room in which it originated.

$$\gamma P = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\gamma P = E$$

If we substitute this result in eq.(8.2), we get

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad \text{......(8.8)}$$

This is the required Laplace formula for the speed of sound in gaseous medium.

$$\gamma = 1.4 \text{ for diatomic gas at STP}$$

$$v = \sqrt{\frac{1.4 \times 1.01 \times 10^5 \text{ Nm}^{-2}}{1.29 \text{ kg m}^{-3}}}$$

$$v = 333 \text{ ms}^{-1}$$

The above value of speed of sound in gaseous medium is in close agreement with the experimental value. Hence, Laplace formula for the velocity of sound in gases is correct and widely used.

Example 8.2

Helium is a mono-atomic gas that has a density of 0.179 kg m^{-3} at STP and a temperature of 0°C. Find the speed of sound in helium at this temperature and pressure.

FOR YOUR INFORMATION

Vibrating vocal cords produce the human voice.

The ear can detect very tiny pressure variations.

Solution:

$\rho = 0.179 \text{ kg m}^{-3}$, $P = 1.01 \times 10^5 \text{ N m}^{-2}$

$T = 0°C = 273 \text{ K}$

As helium is monatomic so $\gamma = 1.67$.

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$v = \sqrt{\frac{1.67 \times 1.01 \times 10^5 \text{ Nm}^{-2}}{0.179 \text{ kg m}^{-3}}}$$

$$v = \sqrt{9.42 \times 10^5} \text{ ms}^{-1}$$

$$v = 970.7 \text{ ms}^{-1}$$

8.3.3 Effect of various factors on speed of sound in air

Sound waves cannot propagate without any medium. Therefore the speed of sound is affected by a number of parameters which are related to a medium and these are summarized as;

(1) Effect of pressure

According to Laplace, the speed of sound through air is given as;

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

But pressure is directly proportional to the density that is;

$P \propto \rho$ or $\frac{P}{\rho}$ = constant and 'γ' is also constant

Therefore, v = constant

This shows that the speed of sound is not affected by the variation in pressure of the gas (air).

(2) Effect of density

Again the velocity of sound in a gas is given by

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

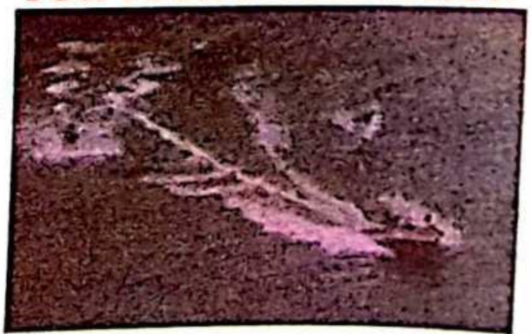
The Laplace relation shows that at the same temperature and pressure, the speed of sound in a gas is inversely proportional to the square root of its density. Now, let us consider two gases which are at the same pressure and the same value of γ. If ρ_1 and ρ_2 be their densities, then velocity of sound in the two gases are

$$v_1 = \sqrt{\frac{\gamma P}{\rho_1}} \text{ and } v_2 = \sqrt{\frac{\gamma P}{\rho_2}}$$

$$\therefore \frac{v_1}{v_2} = \frac{\sqrt{\frac{\gamma P}{\rho_1}}}{\sqrt{\frac{\gamma P}{\rho_2}}}$$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

FOR YOUR INFORMATION



The V-shaped bow wave of a boat is formed because the boat speed is greater than the speed of the waves it generates. A bow wave is analogous to a shock wave formed by an airplane.

If the speed of a body in air exceeds the speed of sound, then it is called supersonic. Such a body leaves behind it a conical region of disturbance which spread continuously. Such a disturbance is called 'shock waves'. These waves may make cracks in window panels.

We know that the density of oxygen is 16 times that of Hydrogen therefore, from equation (8.7), we have

$$\frac{v_H}{v_O} = \sqrt{\frac{\rho_O}{\rho_H}} = \sqrt{\frac{16\rho_H}{\rho_H}} = \sqrt{16} = 4$$

or $$v_H = 4v_O$$

Thus, the speed of sound in hydrogen is greater than (about four times) that in oxygen.

(3) Effect of temperature

The experiments show that at constant pressure, the volume of gas is increased by increasing temperature, hence its density is decreased and speed of sound will be affected. Mathematically it is explained as;

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

As $$\rho = \frac{m}{V}$$

$$v = \sqrt{\frac{\gamma P}{\frac{m}{V}}}$$

$$v = \sqrt{\frac{\gamma PV}{m}}$$

FOR YOUR INFORMATION

- Sound is produced by vibrating objects.
- Sound waves are longitudinal waves.
- Sound has properties of all other waves: reflection, refraction, interference, diffraction.

According to ideal gas equation PV = nRT and for one mole of gas PV = RT

$$v = \sqrt{\frac{\gamma RT}{m}}$$

FOR YOUR INFORMATION

- The speed of sound is higher in liquids and solids than it is in gas.
- The speed of sound in air increases 0.6 ms^{-1} for each °C increase.

The factor $\sqrt{\frac{\gamma R}{m}}$ is constant

So $$v = \text{constant}\sqrt{T}$$

$$v \propto \sqrt{T}$$

Now the speed of sound at 0°C (273K) is v_o and at $t°C = (t + 273)$ K is v_t then

$$\frac{v_t}{v_o} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{v_t}{v_o} = \sqrt{\frac{t+273}{273}}$$

$$\frac{v_t}{v_o} = \left(1 + \frac{t}{273}\right)^{\frac{1}{2}}$$

Using Binomial theorem and expanding $\left(1+\frac{t}{273}\right)^{\frac{1}{2}}$ and neglecting higher power terms, we have $1+\frac{1}{2}\left(\frac{t}{273}\right)+.... = 1+\frac{1}{546}t$

$$\frac{v_t}{v_o} = 1+\frac{1}{546}t$$

$$v_t = v_o\left(1+\frac{t}{546}\right)$$

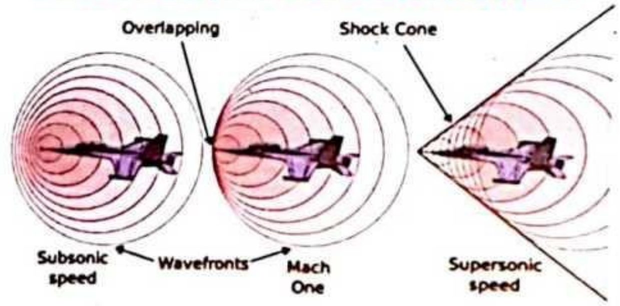
$$v_t = v_o + \frac{v_o t}{546}$$

As for air $\quad v_o = 332 m/s$

$$v_t = v_o + \frac{332\,t}{546}$$

$$v_t = v_o + 0.61\,t \quad ......(8.9)$$

INTERESTING INFORMATION



It has been proved experimentally that the speed of sound is increased by $0.61\ ms^{-1}$ for each 1^oC temperature rise of the air.

Example 8.3

Find the temperature at which the velocity of sound in air will become double of its value at 27^oC.

Solution:

Let v_t be the velocity of sound in air at t^oC and v_0 be the velocity at 27^oC.

$T_1 = 27^oC + 273 = 300\ K$

$T_2 = ?$

$$v_t = 2v$$

or $$\frac{v_t}{v} = 2$$

We know that, $$\frac{v_t}{v_o} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{v_t}{v} = \sqrt{\frac{T_2}{300}} = 2$$

or $$\sqrt{\frac{T_2}{300}} = 2$$

Squaring both sides, we get,

CHECK YOUR CONCEPT

The speed of sound in air is a function of
(a) wavelength
(b) frequency
(c) temperature
(d) amplitude.

$$\frac{T_2}{300} = 4$$

$$T_2 = 4 \times 300 = 1200\ K$$

$$T_2 = 1200\ K - 273 = 927^\circ C$$

Example 8.4

A normal person can hear sound waves ranging in frequency from 20 Hz to 20 kHz. Determine the wavelengths of sounds at these limits. Take the speed of sound in air as $340\ m\ s^{-1}$.

Solution:

Frequency = f_1 = 20 Hz

Frequency = f_2 = 20 kHz = 20×10^3 Hz

$\lambda_1 = ?$

$\lambda_2 = ?$

Speed of sound = v = 340 ms^{-1}

As $v = f\lambda$.

$$\lambda_1 = \frac{v}{f_1} = \frac{340}{20}$$

$$\lambda_1 = 17\ m$$

$$\lambda_2 = \frac{v}{f_2} = \frac{340}{20\times10^3}$$

$$\lambda_2 = 17\times10^{-3}\ m$$

$$\lambda_2 = 17\ mm \text{ or } 1.7\ cm$$

> **CHECK YOUR CONCEPT**
> The sounds are carried by electro-magnetic waves in space. Why our ears cannot hear it?

8.4 PRINCIPLE OF SUPERPOSITION

The principle of superposition was first observed experimentally by Thomas Young in 1801. It is related to the study of combined effect of two or more waves and it is stated as, **"When two or more waves meet at a point in the same medium, the resultant amplitude at that point is the algebric sum of the amplitudes of the individual waves"**.

For example, when a man talks to us while we are listening music, we receive a complex sound but we can still distinguish the sound of speech and the sound of the music from each other. It happens like that because the total sound waves reaching our ears is the algebric sum of the waves produced by a man voice and the waves produced by music. Superposition principle is applicable to all types of waves including the electro-magnetic wave such as light.

Let we consider two waves of amplitudes Y_1 and Y_2 which are in phase. When they are superimposed at the point in the same medium as shown in Fig.8.10(a) then their resultant amplitude 'Y' at that point is given as;

$$Y = Y_1 + Y_2$$

Similarly, the two waves which are in opposite phase or out of phase and they are superimposed at point in the same medium then we get the result which is shown in Fig.8.10(b). The amplitude of their resultant wave is given as;

$$Y = Y_1 - Y_2$$

If $Y_1 = Y_2$

then $Y = 0$

In general, if there are 'n' number of waves passing through the same medium then the amplitude of their resultant wave is given as;

$$Y = Y_1 + Y_2 + Y_3 + \ldots + Y_n \quad \ldots\ldots(8.10)$$

This is the mathematical form of principle of superposition.

The superposition of two waves give rise to the following three important phenomena:

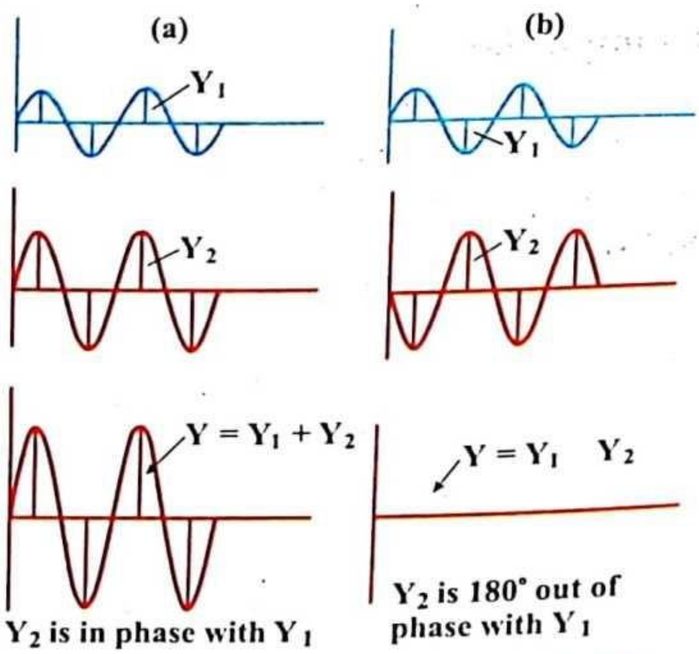


Fig.8.10: Superposition of two waves of same frequency (a) The two waves which are at the same phase and their resultant is increased (b) The two waves which are at the out of phase and their result is decreased (zero).

1. When two waves of same frequency (or wavelength) moving with same speeds in the same direction in a medium superpose on each other, they give rise to an effect called **interference of waves**.
2. When two waves of slightly different frequency (or wavelength) moving with same speeds in the same direction in a medium superpose on each other, they give rise to **beats**.
3. When two waves of same frequency or wavelength moving with same speeds in opposite direction in a medium superpose on each other, they give rise to **stationary waves**.

8.5 INTERFERENCE

When two or more waves having the same frequency travel through the same medium and in the same direction are combined, then this results in a phenomenon called interference. The amplitude of the resultant wave is greater or smaller than the amplitude of combining individual waves and depends upon the relative phase of individual waves.

Now when two coherent waves (the wave having same frequency), which are exactly in phase, are allowed to superimpose such that the crests of one wave coincide with crests of the other wave and troughs with trough then the amplitude of the resultant wave will be increased as shown in Fig.8.11. This is called constructive interference.

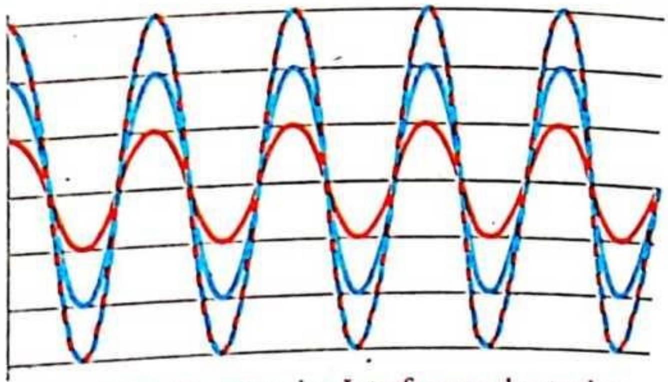


Fig.8.11: Constructive Interference due to the superposition of coherent waves in the same phase

Similarly, when two coherent waves, which are exactly in opposite phase, are allowed to superimpose such that crest of one wave coincide with the trough of second wave then the amplitude of the resultant wave is decreased, as shown in Fig.8.12 and it is called destructive interference.

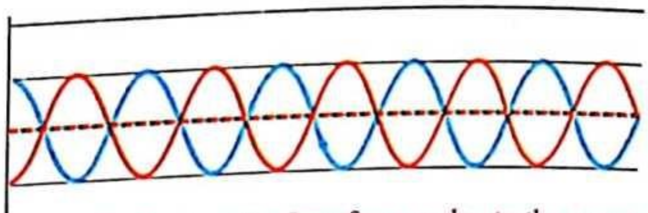


Fig.8.12: Destructive Interference due to the super position of coherent waves, in the opposite phase.

Conditions of Interference

To demonstrate interference phenomenon, considering two identical sources of sound S_1 and S_2 (loud speakers) are placed at some distance. These sources generate continuously spherical waves of same frequency and of same phase which are called coherence waves. These coherent waves are propagated in the outward direction such that they are superimposed at different points as shown in Fig.8.13. Thus, we have both constructive and destructive interferences at different points. In figure, the thick lines represent crests while the thin lines represent troughs.

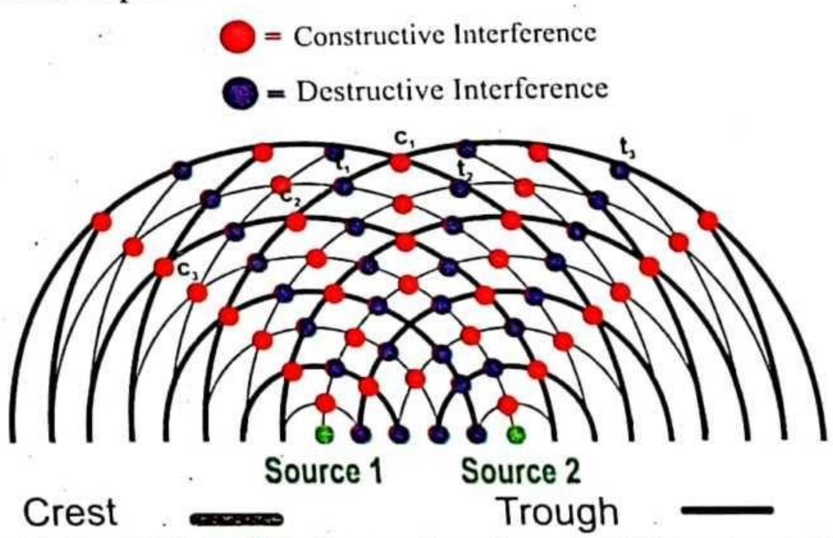


Fig.8.13: Experimental demonstration of interference of sound waves which are generated by two coherent sources

The points where crests coincide with crests and troughs with troughs, a constructive interference is obtained. These are represented by red dots as C_1, C_2, C_3 ...C_n. On the other hand, the points where crests coincide with troughs, a destructive interference is obtained. These are represented by blue dots as $t_1, t_2, t_3, t_4, t_5, \ldots t_n$ crest.

Now mathematical condition of constructive interference can be developed as;

Path difference between two waves at point $C_1 = \Delta S = S_2C_1 - S_1C_1$

$$\Delta S = 4\lambda - 4\,\lambda = 0\,\lambda$$

Path difference between two waves at point $C_2 = \Delta S = S_2C_2 - S_1C_2$

$$\Delta S = 4\lambda - 3\,\lambda = 1\lambda$$

Path difference between two waves at point $C_3 = \Delta S = S_2C_3 - S_1C_3$

$$\Delta S = 4\frac{1}{2}\lambda - 2\frac{1}{2}\lambda = 2\lambda$$

In general, the path difference between two waves for constructive interference is given as:

$$\text{Path difference} = 0, \pm\lambda, \pm 2\lambda, \pm 3\lambda, \pm 4\lambda \ldots. m\lambda$$

$$\text{Path difference} = \Delta S = m\lambda \qquad \ldots\ldots(8.11)$$

where $m = 0, \pm 1, \pm 2, \pm 3, \pm 4 \ldots$

This is a condition for constructive interference and it shows that for constructive interference, the path difference is a whole number of wavelength or the path difference is integral multiple of wavelength.

Similarly, mathematical condition of destructive interference can be developed as;

Path difference between two waves at point $t_1 = \Delta S = S_2t_1 - S_1t_1$

$$\Delta S = 4\lambda - 3\frac{1}{2}\lambda = \frac{1}{2}\lambda$$

Path difference between two waves at point $t_2 = \Delta S = S_2t_2 - S_1t_2$

$$\Delta S = 3\frac{1}{2}\lambda - 4\lambda = -\frac{1}{2}\lambda$$

Path difference between two waves at point $t_3 = \Delta S = S_2t_3 - S_1t_3$

$$\Delta S = 4\lambda - 5\frac{1}{2}\lambda = \frac{3\lambda}{2}$$

In general, the path difference between two waves for destructive interference is given as;

$$\text{Path difference} = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \pm\frac{7\lambda}{2} \ldots$$

$$\text{Path difference} = \left(m + \frac{1}{2}\right)\lambda \quad \ldots\ldots(8.12)$$

or Path difference = $(2m+1)\frac{\lambda}{2}$

Where $m = 0, \pm 1, \pm 2, \pm 3, \pm 4, ...$

This is the condition for destructive interference and it shows that the path difference between two waves for destructive interference is an odd integral multiple of half-wavelengths.

8.6 BEATS

In interference, we have studied the superposition of two waves having the same frequency. But what will be the effect of the superposition of two waves when they have a slight difference in their frequencies? It can be studied in beats phenomenon. When two sound waves of same amplitude but slightly differing in frequencies are travelling through a medium in the same direction and they are allowed to superimpose at a point then a periodic variation in the intensity of resultant wave is observed at that point. This variation in the intensity of resulting wave is in the form of series of loud sound followed by faint sound. This phenomenon is called beats. It is further explained by an example.

Consider two tuning forks A and B of frequencies 100 Hz and 102 Hz. It means that tuning fork 'A' will produce 100 complete waves in one second and tuning 'B' will produce 102 complete waves in one second when each of them is struck against rubber pad. When both tuning forks are sounded together then phenomenon of beats takes place. Let us study the variation in the intensity of resulting sound over a span of one second.

After $\frac{1}{4}$ second, the number of waves produced by A are 25 and distance covered is 25λ and the number of waves produced by B are $25\frac{1}{2}$ and distance $25\frac{1}{2}\lambda$ respectively. The path difference between them is $\frac{\lambda}{2}$. This is the condition of destructive interference. At this point two waves cancel to each other and no sound or faint sound is heard.

Similarly, after $\frac{1}{2}$ second, the number of waves produced by A and B are 50 and 51 and corresponding distances covered are 50λ and 51λ respectively. The path difference between them is '1λ'. This is the condition of constructive interference. At this point two waves reinforce each other and loudness is increased.

After $\frac{3}{4}$ second the number of waves produced by A and B are 75 and $76\frac{1}{2}$ and corresponding distances covered are 75λ and $76\frac{1}{2}\lambda$. The path difference between them is $\frac{3\lambda}{2}$ which is again the condition of destructive interference. Hence, no sound or a faint sound is observed because two waves cancel each other.

After one second, the number of waves produced by A and B are 100 and 102 and corresponding distances covered are 100λ and 102λ. The path difference between them is '2λ' which is the condition of constructive interference. Hence we observe a loud sound because two waves reinforce each other.

This example clearly shows that when the two waves of nearly same frequency are superimposed then there is increase and decrease in loudness at regular interval of time. As a result we have a beats phenomenon. Graphically, it is shown in Fig.8.14.

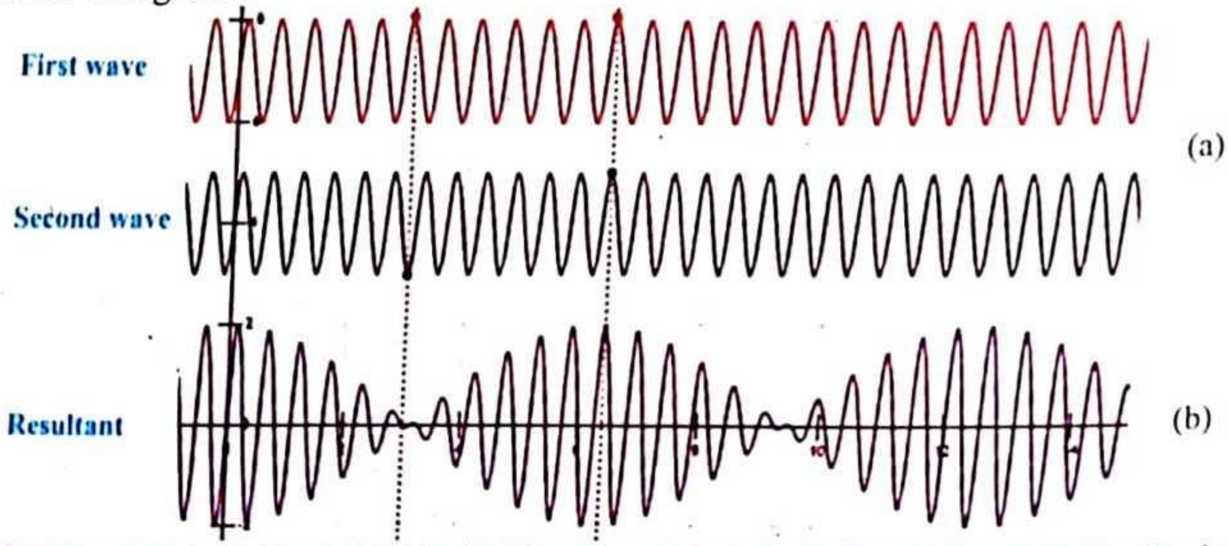


Fig.8.14: Beats are formed by the combination of two waves of slightly different frequencies travelling in the same direction; (a) The two individual waves (b) The combined resultant wave has amplitude that oscillates with time

It may be noted from figure 8.14(b) that rise and fall in the intensity of resulting sound (increase and decrease in loudness) takes place twice in one second and difference in frequency of the two sources is also two. In other words two beats are produced in one second.

INTERESTING INFORMATION

Musicians use beats phenomenon to tune their string instruments like guitar, violin and piano, by beating a note against a note of known frequency. The strings are then adjusted to the desired frequency by tightening or loosening it until no beats are heard.

Thus it is concluded that, "**the difference in frequency of the two sources is equal to the number of beats produced per unit time is called beat frequency**". If f_A and f_B be the frequencies of sound waves of two sources then;

$$f_B - f_A = \frac{\text{number of beats}}{\text{time}} \quad .... (8.13)$$

It is important to note that beats cannot be observed if the difference in frequency is more than 10 Hz.

Uses of beats

The phenomenon of beats can be used in the following cases.

1. To determine the frequency of a note
2. Beats are used to tune musical instruments
3. To detect the hidden metals by using metal detectors.
4. To detect the harmful gases in mines.
5. Beats are used for radio wave reception.

DO YOU KNOW

Metal detector is working under the principle of beats phenomenon.

8.7 REFLECTION OF WAVES

A mechanical wave requires a medium for its propagation and its velocity depends upon the nature of the medium. When the wave comes across the boundary of two media then all or a part of this travelling wave is reflected back. This reflected wave has the same wavelength and frequency but its phase may change depending upon the nature of the boundary.

The reflection of a wave at the boundary of the media can be studied by an example of a stretched string under two different cases.

When one end of the stretched string is connected with a rigid support and the other free end in the hand is shaken up and down then a pulse of transverse wave is produced which travels along the string towards the rigid support with velocity v_i.

When the pulse arrives at the end, it exerts a force on a rigid support. In reaction, the rigid support which acts as a

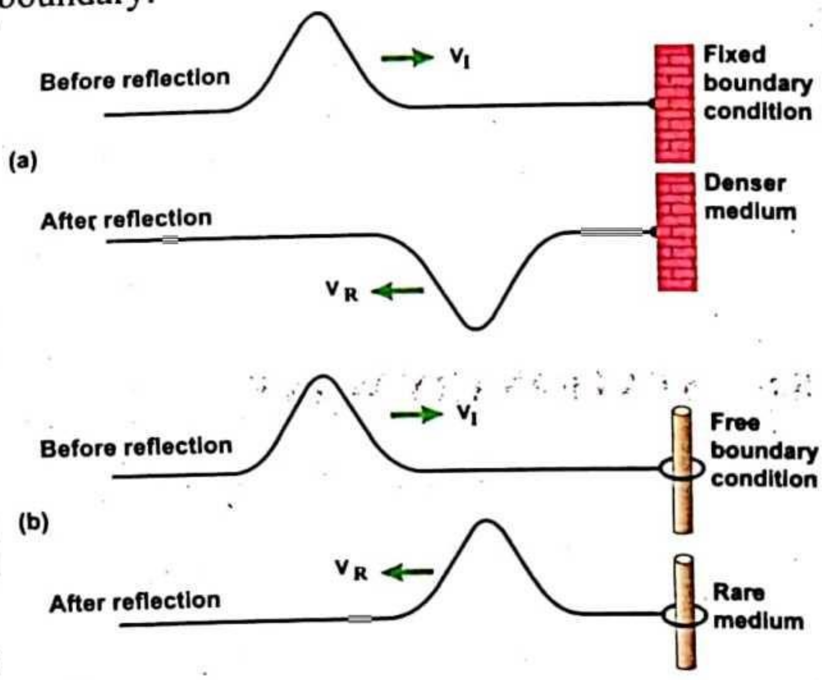


Fig.8.15: A pulse of a wave along a string reflected from
(a) Denser medium (rigid support)
(b) Rare medium (ring & rod)

dense medium also exerts an opposite force on the string. As a result, a reflected inverted pulse starts travelling along the string in the reverse direction with velocity v_R as shown in Fig.8.15 (a).

The incident wave and reflected wave are out of phase a change of 180° in phase and the change in path difference between them is $\frac{\lambda}{2}$.

If one end of the string is connected with a light ring which can move freely up and down. This light ring act as a rare medium and it exerts no force on the string.

FOR YOUR INFORMATION

Type of wave	Boundary of	After reflection	Change of phase
Longitudinal	Denser	Compression as rarefaction Rarefaction as compression	$\pi = 180°$
	Rare	Compression as compression Rarefaction as rarefaction	0
Transverse	Denser	Crest as trough Trough as crest	$\pi = 180°$
	Rare	Crest as crest Trough as trough	0

When the pulse of transverse wave arrives at light ring then it reflects in the reverse direction without any phase change in the reflected wave as shown in Fig.8.15(b).

From the above discussion, it is concluded that a transverse wave which is reflected from a denser medium with phase change of 180° and path difference of $\frac{\lambda}{2}$. But when the transverse wave is reflected from a rare medium, no phase change takes place in the reflected wave.

8.8 STATIONARY WAVE

In interference and beats phenomenons, we have studied the superposition of two waves which are travelling along the same direction. If two waves of same frequency and amplitude move along a straight line in opposite directions and are allowed to superimpose then the new resultant wave is called stationary or standing wave. The formation of standing waves is shown in Fig.8.16.

It can be explained by an example of a string whose one end is connected with a support and the free end in hand is oscillated up and down continuously. Then a transverse waves is produced which travel along the string and reflected from the support. Due to the superposition of the incident waves with the reflected waves, a pattern of the stationary waves is obtained in the string as shown in Fig.8.16. The points in stationary waves which are permanently at rest with zero amplitudes are called nodes (N).

The point between two successive nodes where the particles oscillate with maximum displacement is called as anti-node (A).

It is clear that the distance between two consecutive nodes or consecutive antinodes is $\frac{\lambda}{2}$ and the distance between a node and an adjacent antinode is $\frac{\lambda}{4}$.

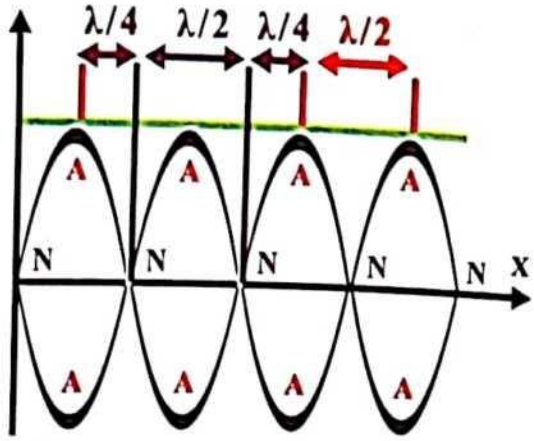


Fig.8.16: The formation of stationary wave due to the superposition of two waves moving in opposite direction. The displacement is marked as node 'N' and anti node 'A'.

Graphically, standing waves can be illustrated by considering two waves 'a' and 'b' having same frequency and amplitude which are travelling along the string in opposite directions as shown in Fig.8.17. The resultant standing wave "c" at instants $0, \frac{T}{4}, \frac{T}{2}, \frac{3T}{4}$ and T is obtained using the principle of superposition (Fig.8.17)

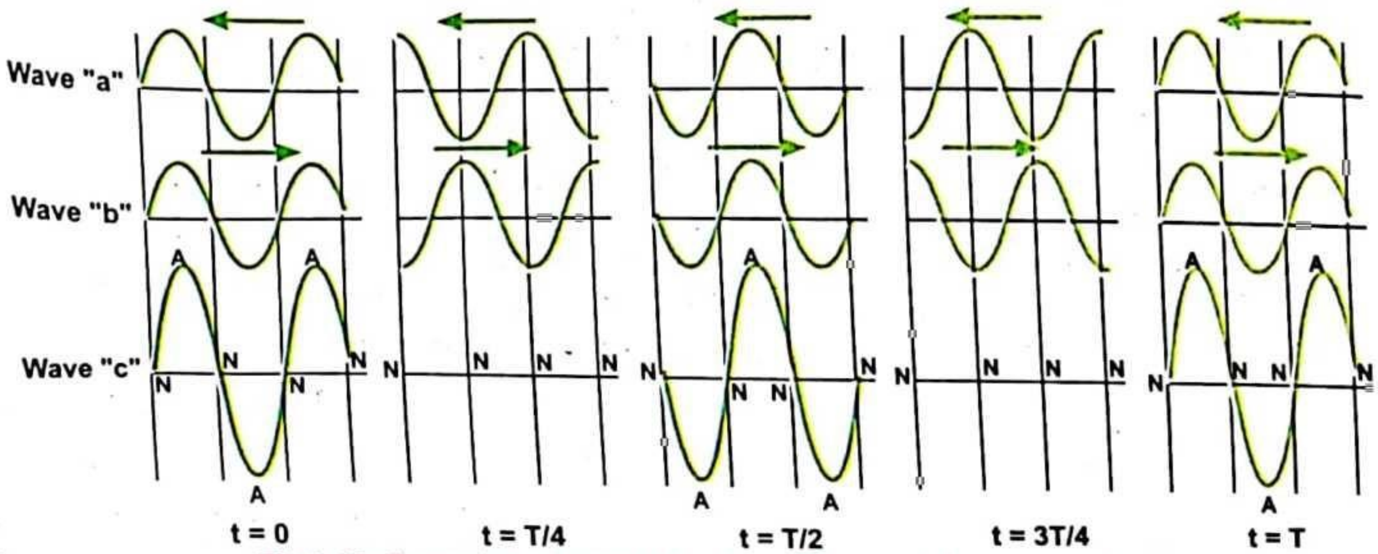


Fig.8.17: Formation of stationary waves. The set of figures show the state of resultant displacement at four different times.

(i) At $t = 0$, both waves "a" and "b" are in phase. After superposition, they produce resultant wave "c" where the amplitude of its nodes is zero and the amplitude of its antinodes is maximum, i.e. equal to the sum of amplitudes of individual waves.

(ii) At $t=\frac{T}{4}$, then wave "a" has travelled a distance $\frac{\lambda}{4}$ to the right and wave "b" has travelled distance $\frac{\lambda}{4}$ to the left. Therefore, both waves are out of phase. This is the condition of destructive interference and the amplitude of the resultant wave "c" is zero at each point.

(iii) At $t=\frac{T}{2}$, wave "a" has travelled a distance $\frac{\lambda}{2}$ to the right and wave "b" has travelled a distance $\frac{\lambda}{2}$ to the left. Both waves are in phase where the amplitude of the nodes of the resultant wave "c" are zero and the amplitudes of anti nodes are maximum.

Similarly, after $t=\frac{3T}{4}$ and $t=T$ the results are same as for $t=\frac{T}{4}$ and $t=0$ or $\frac{T}{2}$ respectively i.e. out of phase and in phase and are shown in Fig.8.17.

The pattern of the resultant wave "c" is known as stationary waves, because neither the patterns move nor the location of nodes and anti nodes changes.

Notice that stationary waves do not travel to left or right. Therefore, they cannot transfer energy because the energy is confined in antinodes. That's why stationary waves are also called standing wave.

POINT TO PONDER
What will happen when a longitudinal wave is reflected from a denser medium and from a rare medium?

Some features of stationary wave are given below:

1. The disturbance produced is confined to the region where it is produced i.e. it does not move forward or backward.
2. Different particles move with different amplitudes.
3. The particles at nodes always remain at rest.
4. All the particles cross their mean positions together.
5. All the particles between two successive nodes are in phase.
6. The energy of one region is always confined in that region.

8.8.1 Stationary waves in a stretched string

Consider a string of length 'L' which is stretched between two rigid supports such that a tension 'T' is developed in the string. When the string is plucked from its centre, two waves, in the form of transverse waves, originate from this point. One of them moves toward the right end and the other toward the left end of the string. When these two waves reach to the two rigid supports, they are reflected back and superimpose to each other. The resulting stationary wave is in the form of a single loop. It has two nodes and one anti node as shown in Fig.8.18.

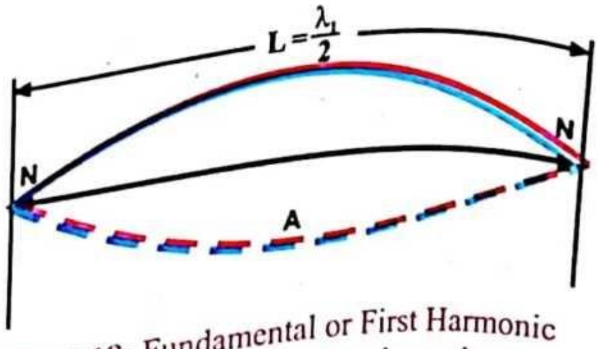


Fig.8.18: Fundamental or First Harmonic with single loop has two nodes and one anti-node.

This single loop stationary wave is called fundamental mode of vibration. Let f_1 be its frequency and 'λ_1' be its wavelength, then these parameters are related to the length of the string 'L' as,

$$L = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2L \quad \ldots\ldots(8.14)$$

The wave travels through the string its speed (v) depends upon tension 'T' and mass per unit length or linear mass density $\left(\mu = \frac{m}{L}\right)$ of the string.

The dependence of speed of wave on tension (T) and mass per unit length (μ) of the string is given by,

$$v = \sqrt{\frac{T}{\mu}} \quad \ldots\ldots(8.15)$$

As

$$v = f_1 \lambda_1$$

$$f_1 = \frac{v}{\lambda_1}$$

$$f_1 = \frac{v}{2L} \quad \ldots\ldots(8.16)$$

$$\therefore f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \quad \ldots\ldots(8.17)$$

This is called the *fundamental frequency or first harmonic* of the string. It is the minimum frequency that can be produced as standing waves in a string.

Second mode of vibration

When the same string is plucked from one-fourth, three-fourth of its length, a stationary wave with two loops is set up in the string. It has three nodes and two anti nodes as shown in Fig.8.19.

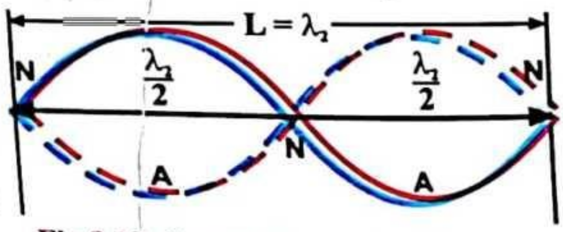


Fig.8.19: Second Harmonic with two loops have three nodes and two anti-nodes.

These two loops of stationary waves are called second mode of vibration. Let f_2 be its frequency and λ_2 be its wavelength which is related with the length of the string.

$$L = \lambda_2$$

As $$v = f_2 \lambda_2$$

$$v = f_2 L$$

$$f_2 = \frac{v}{L} \quad ......(8.18)$$

$$f_2 = 2\left(\frac{v}{2L}\right)$$

But $\frac{v}{2L} = f_1$ (from eq. 8.16)

$$f_2 = 2f_1$$

DO YOU KNOW

The speed of a wave along a stretched ideal string depends only on the tension and linear density of the string and not on the frequency of the wave.

This frequency is called second harmonic or first overtone. It is clear from above equation that if the string vibrates in two loops then the frequency of second harmonic is twice the frequency of first harmonic.

Third mode of vibration

Similarly, if the same string is plucked from one sixth $\left(\frac{5}{6}\right)$th of its length, then the string vibrates in three loops, having four nodes and three antinodes as shown in Fig.8.20. It is called third mode of vibration. Let f_3 be its frequency and λ_3 be its wavelength then;

$$L = \frac{3\lambda_3}{2}$$

$$\lambda_3 = \frac{2}{3}L$$

As $$v = f_3 \lambda_3$$

$$f_3 = \frac{v}{\lambda_3}$$

$$f_3 = \frac{v}{\left(\frac{2}{3}L\right)}$$

$$f_3 = 3\left(\frac{v}{2L}\right)$$

$$f_3 = 3f_1$$

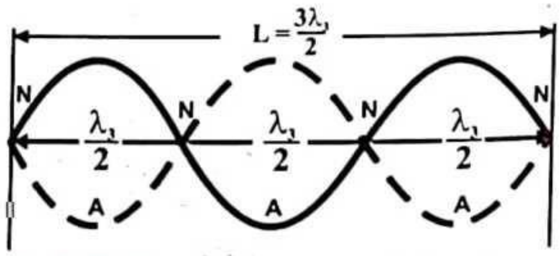


Fig.8.19: Third Harmonic with three loops have four nodes and three anti-nodes.

In general, if the string vibrates in 'n' number of loops then it has 'n + 1' number of nodes and 'n' antinodes. The frequency 'f_n' of such stationary wave setup in the string is expressed as;

$$f_n = nf_1 \text{ where } n = 1,2,3...$$

This is known as quantization of frequencies i.e. the frequencies of the various (or overtones) are whole number (positive integar > 0) multiple of first harmonic (the fundamental frequency).

Example 8.4

A steel wire hangs vertically from a fixed point, supporting a weight of 80 N at its lower end. The diameter of the wire is 0.50 mm and its length from the fixed point to the weight is 1.5 m. Calculate the fundamental frequency emitted by the wire it is plucked? (Density of the steel wire = 7.8×10^3 kg m^{-3})

Solution:

Weight = Tension = T = 80 N

Diameter of wire = d = 0.5 mm = 0.5×10^{-3}m

Length = ℓ = 1.5 m

Density of steel wire = 7.8×10^3 kg m^{-3}

By definition of density = $\rho = \dfrac{\text{mass}}{\text{volume}}$

Mass $= \rho \times$ volume

Mass $= \rho \times$ Area $\times$ length

Mass $= \rho\times\pi r^2\times\ell$

$$\text{Mass} = \rho\times\pi\left(\frac{d}{2}\right)^2\times\ell$$

$$\text{Mass} = \rho\times\pi\frac{d^2}{4}\times\ell$$

$$\text{Mass} = (7.8\times10^3\ \text{kg m}^{-3})(3.14)\frac{(0.5\times10^{-3}\ \text{m})^2}{4}\times1.5\ \text{m}$$

Mass= 2.30×10^{-3} kg

Now linear mass density or mass per unit length is given as;

$$\mu = \frac{\text{mass of wire}}{\text{length of wire}} = \frac{2.30\times10^{-3}\ \text{kg}}{1.5\text{m}}$$

$$= 1.53\times10^{-3}\ \text{kg m}^{-1}$$

FOR YOUR INFORMATION

The energy emitted from sound produced by a crowd of 60,000 at a football match is enough to warm a cup of tea.

GEOPHYSICS

Waves through a solid can be either transverse or longitudinal. An earthquake produces both transverse and longitudinal waves that travel through earth. Geologist studying the waves with seismograph found that longitudinal wave could pass through Earth's core, transverse waves could not. From this evidence, they concluded that Earth's core is liquid. From its density, it is most likely molten iron.

The fundamental frequency

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$f_1 = \frac{1}{2\times1.5\text{m}}\sqrt{\frac{80\text{ N}}{1.53\times10^{-3}\text{kg m}^{-1}}}$$

$$f_1 = 76\text{ Hz}$$

8.8.2 Stationary waves in air column

We have observed stationary waves along a vibrating stretched string, but these waves can also be set up in other media. For example, the vibrating air column of a closed or open organ pipes. Similarly, when air is blown at the mouth of a bottle, sound is produced due to vibrations of air column inside the bottle. Now consider a sound wave from a tuning fork which is allowed to vibrate the air at the one end of the pipe. This wave will travel along the pipe and will be reflected from the far end of the pipe. Thus there are two waves in the pipe i.e. incident wave and reflected wave. The superposition of incident and reflected waves produces a stationary wave in the vibrating air column of organ pipe. The relation between incident wave and reflected wave depends upon the closed and open ends of organ pipe. The open end of the pipe behaves as anti node due to free motion of molecules of air whereas the node is formed at the closed end because the movement of molecules is restricted. Stationary waves in the air column can be studied under the following two cases.

I. When one end of the Pipe is closed

Let us consider a pipe of length 'L' such that its one end is closed and its other end is open. In this case all the sound energy is reflected from the closed end and it causes a stationary wave in the pipe. Node is formed at the closed end and antinode at the open end.

First Harmonic

When the stationary wave is formed in pipe vibrating with a half loop which consists of one node and an anti-node as shown in Fig.8.21 then it is called 1st harmonic. Let f_1 be its frequency and λ_1 be its wavelength which is related with the length of the pipe that is;

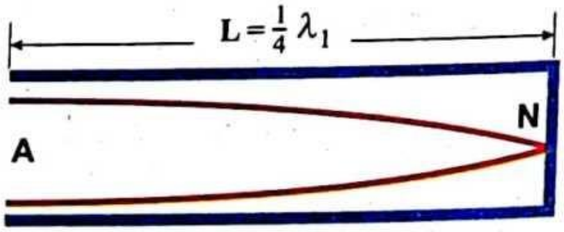


Fig.8.21: Fundamental or First Harmonic stationary wave with a half loop has one node and one anti-node.

$$L = \frac{\lambda_1}{4}$$

$$\lambda_1 = 4L$$

As $$v = f_1 \lambda_1$$

$$f_1 = \frac{v}{\lambda_1}$$

$$f_1 = \frac{v}{4L} \quad \ldots\ldots(8.19)$$

Second Harmonic

The stationary wave which is vibrating, in one and half loops and contains two nodes and two anti-nodes is called 2^{nd} harmonic as shown in Fig.8.22. Let f_2 be its frequency and λ_2 be its wavelength which is related with the length of the pipe as;

$$L = \frac{3\lambda_2}{4}$$

$$\lambda = \frac{4}{3}L$$

As $$v = f_2 \lambda_2$$

$$f_2 = \frac{v}{\lambda_2}$$

$$f_2 = \frac{v}{\frac{4}{3}L} = 3\left(\frac{v}{4L}\right)$$

$$f_2 = 3f_1$$

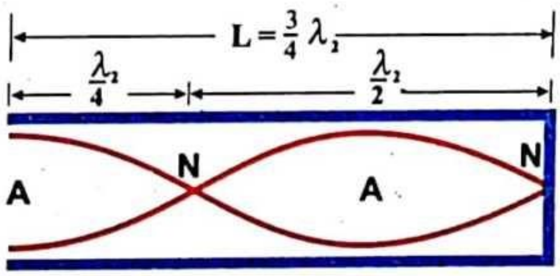


Fig.8.22: Second Harmonic stationary wave with one and half loops have two nodes and two anti-nodes.

Third Harmonic

In third harmonic, the stationary waves are vibrating with two and half loops and contain three nodes and three anti-nodes as shown in Fig.8.23. Let f_3 be its frequency and λ_3 be it wave length which is related with the length of the pipe as;

$$L = \frac{5\lambda_3}{4}$$

$$\lambda_3 = \frac{4}{3}L$$

As $$v = f_3 \lambda_3$$

$$f_3 = \frac{v}{\lambda_3}$$

$$f_3 = \frac{v}{\frac{4}{5}L}$$

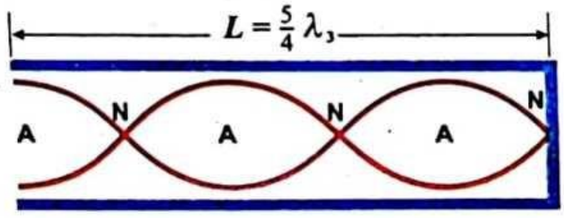


Fig.8.23: Third Harmonic stationary wave with two and half loops have three nodes and three anti-node.

CRITICAL THINKING

Under what principle a sound is produced in a flute?

$$f_3 = 5\left(\frac{v}{4L}\right)$$

$$f_3 = 5f_1$$

A closed pipe resonates when its length is an odd number of quarter wavelengths.

In general, if 'n' represents the numbers of half loops in the above stated pipe then its quantization frequency 'f_n' of stationary wave as;

$$f_n = nf_1$$

where n = 1,3,5,7,...

II. Open Organ Pipe

Consider a pipe of length 'L' whose both ends are open. The most of sound energy is passed outside but some of them is reflected and it causes stationary waves in the open ended pipe.

First Harmonic

The first or fundamental harmonic stationary wave consists of two antinodes and one node as shown in Fig.8.24. Let f_1 be its frequency and λ_1 be its wavelength. The wavelength is related with the length of pipe as;

Then $$L = \frac{\lambda_1}{4} + \frac{\lambda_1}{4} = \frac{\lambda_1}{2}$$

$$\lambda_1 = 2L$$

As $$v = f_1\lambda_1$$

$$f_1 = \frac{v}{\lambda_1}$$

$$f_1 = \frac{v}{2L} \quad \text{......(8.20)}$$

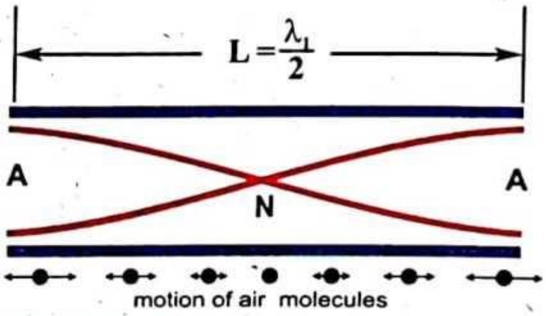


Fig.8.24: Fundamental or first Harmonic stationary wave have one node and two anti-nodes.

Second Harmonic

The second harmonic of stationary wave in open ended pipe consists of three antinodes and two nodes as shown in Fig.8.25. Let f_2 and λ_2 be its frequency and wavelength respectively of the second harmonic stationary wave. The wavelength is related with the length of pipe as;

$$L = \frac{\lambda}{4} + \frac{\lambda}{4} + \frac{\lambda}{4} + \frac{\lambda}{4}$$

i.e. $$L = \frac{4\lambda_2}{4}$$

$$\lambda_2 = L$$

$$v = f_2\lambda_2$$

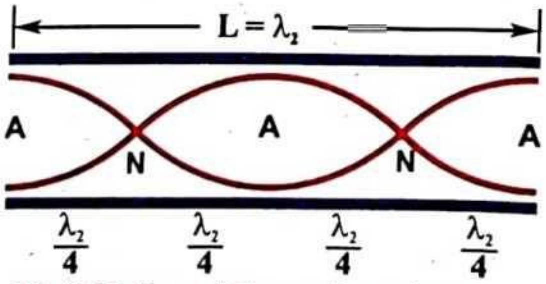


Fig.8.25: Second Harmonic stationary wave have two nodes and three anti-nodes.

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

$$f_2 = 2\left(\frac{v}{2L}\right)$$

$$f_2 = 2f_1$$

Third Harmonic

The third harmonic stationary wave in open ended pipe consists of three antinodes and two nodes as shown in Fig.8.26. Let f_3 be the frequency and λ_3 be the wave of third harmonic stationary wave in the pipe. The wavelength is related with the length of the pipe as;

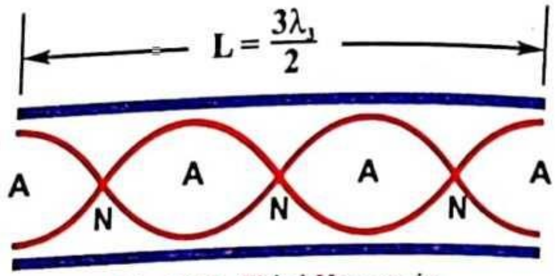


Fig.8.26: Third Harmonic

$$L = 6\frac{\lambda_3}{4} = \frac{3\lambda_3}{2}$$

$$\lambda_3 = \frac{2}{3}L$$

$$v = f_3\lambda_3$$

$$f_3 = \frac{v}{\lambda_3} = \frac{v}{\frac{2}{3}L}$$

$$f_3 = 3\left(\frac{v}{2L}\right)$$

$$f_3 = 3f_1$$

In general, for nth harmonic the quantization frequency of the stationary wave in open ended pipe is given as;

$$f_n = n\left(\frac{v}{2L}\right)$$

$$f_n = nf_1$$

Where $n = 1, 2, 3, 4, \ldots$

> An open pipe resonates when its length is an even number of quarter wavelengths

Example 8.5

What will be the frequencies of fundamental and first three overtones for a 75 cm long organ pipe? (a) If it's both ends are open (b) If its one end is closed. The speed of sound in air is 340 ms^{-1}.

Solution:

Length of organ pipe = L = 75 cm = 0.75 m

Speed of sound = v = 340 ms^{-1}

(a) For an open pipe, the quantized frequency is

$$f_n = n\left(\frac{340\,ms^{-1}}{2\times 0.75\,m}\right) = n(226.7)\,Hz$$

$$n = 1,\ f_1 = (1)(226.7)\,Hz$$

This the fundamental frequency of the given open pipe.

$$n = 2,\ f_2 = 2(226.7)\,Hz = 453.4\,Hz = 453\,Hz$$

$$n = 3,\ f_3 = 3(226.7)\,Hz = 680.1\,Hz = 680\,Hz$$

$$n = 4,\ f_4 = 4(226.7)\,Hz = 906.8\,Hz = 907\,Hz$$

The frequencies of first three overtones are 453 Hz, 680 Hz and 907 Hz respectively.

(b) For a closed pipe, the quantized frequency is given by

$$f_n = n\left(\frac{v}{2L}\right) \text{ where } n = 1,3,5,7,......$$

$$f_n = n\,(113.3s^{-1}) = n\,(113.3)\,Hz$$

$$f_1 = 1\times(113.3)\,Hz = 113.3\,Hz$$

$$f_3 = 3\times(113.3)\,Hz = 339.9\,Hz = 340\,Hz$$

$$f_5 = 5\times(113.3)\,Hz = 566.5\,Hz = 566\,Hz$$

$$f_7 = 7\times(113.3)\,Hz = 793.1\,Hz = 793\,Hz$$

The fundamental frequency of given closed pipe is 113.3 Hz. The frequencies of the first three overtones are 340 Hz, 566 Hz and 793 Hz respectively.

8.9 DOPPLER EFFECT

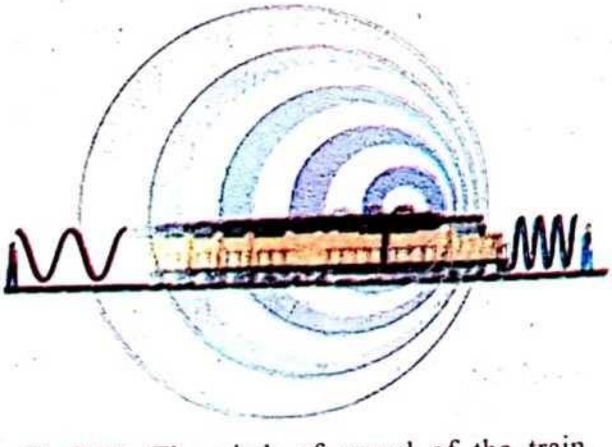


Fig.8.27: The pitch of sound of the train increases when it is moving toward the observer and decreases when it is moving away from the observer

It is a common observation that when the source of sound and the observer both are at rest; the observer receives the frequency of sound in its actual form as the frequency originated from the source. However, the frequency of the sound appears to change if there is relative motion between the source and observer (listener). The frequency appears to increase as the moving source approaches the stationary observer and appears to decrease as the source moves away from the stationary observer. For example, let an observer who is standing at a railway platform. The pitch of the whistle by train increases when the train is approaching toward the observer and the pitch of the sound decreases when the train is moving away from the observers as shown in Fig.8.27. This apparent change in frequency is called Doppler effect and it is stated as; **"there is an apparent change in the frequency of sound due to relative motion between the source of sound wave and the listener"**.

The Doppler effect is named after the Austrian physicist and mathematician Christian Johann Doppler (1803–1853), who did experiments with both moving sources and moving observers. The Doppler effect holds not only for sound waves but also for electromagnetic waves, including microwaves, radio waves, and visible light. However, here we will discuss only the Doppler effect for sound waves. The Doppler effect can be studied under the following four cases.

Consider a source of sound which generates a sound waves of frequency 'f' and wavelength 'λ' in all directions. Let v be the velocity of sound, v_o be the velocity of the observer and v_s is the velocity of source. We assume that the medium (air) between source and observer is stationary.

Case I: Observer is moving towards a stationary source.

In this case the observer is moving towards the stationary source with a velocity v_o as shown in Fig.8.28. If the relative velocity of sound is 'v' then net velocity between observer and source is $v + v_o$. Thus, the number of waves received in one second f_1 is given as;

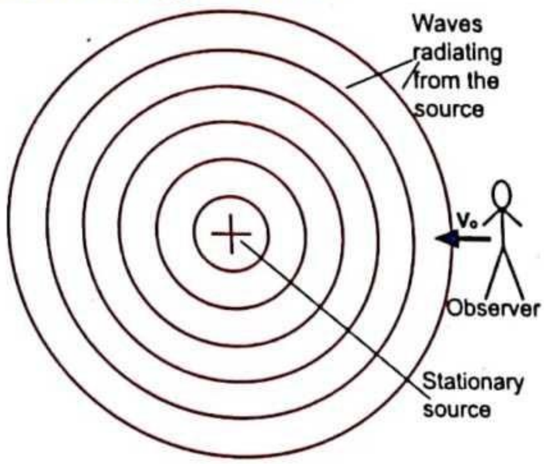


Fig.8.28: Observer is moving towards a stationary source.

$$f_1 = \frac{v + v_o}{\lambda}$$

As $$\lambda = \frac{v}{f}$$

$$f_1 = \frac{v + v_o}{\left(\frac{v}{f}\right)}$$

$$f_1 = \left(\frac{v + v_o}{v}\right) f \quad ......(8.21)$$

This shows that $f_1 > f$, the apparent frequency of sound increases.

Case II: Observer is moving away from a stationary source

Now the observer is moving away from the stationary source with velocity v_o as shown in Fig.8.29. The relative velocity between observer and source is $v - v_o$ and the numbers of waves received in one second f_2 is given as;

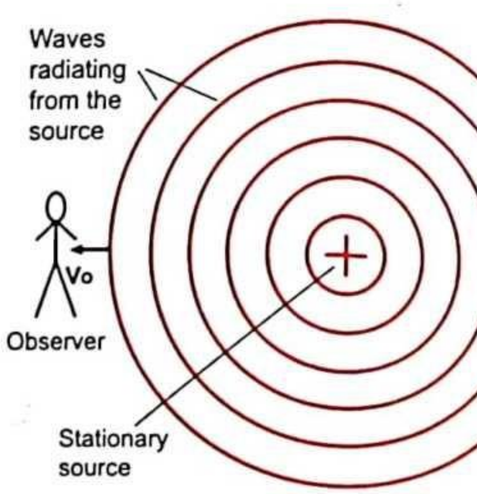


Fig.8.29: Observer is moving away from a stationary source.

$$f_2 = \frac{v - v_o}{\lambda}$$

$$f_2 = \frac{v - v_o}{\frac{v}{f}}$$

$$f_2 = \left(\frac{v - v_o}{v}\right) f \quad ......(8.22)$$

This shows that $f_2 < f$ therefore apparent frequency of sound decreases when the observer is at rest and source is in motion.

Case III: When the source is moving towards the stationary observer

When the source is moving with velocity v_s towards a stationary observer as shown in Fig.8.30. The net velocity is $v - v_s$. In this case, the wavelength λ_3 measured by observer at rest is shorter than the wavelength λ of the source. The waves are compressed and its frequency 'f' remains same, this compression in wavelength is called Doppler shift. It is represented by $\Delta\lambda$.

$$\Delta\lambda = \frac{v_s}{f}$$

Now decrease in wavelength during compression of waves is given as;

$$\Delta\lambda = \lambda - \lambda_3$$

$$\lambda_3 = \lambda - \Delta\lambda$$

$$\lambda_3 = \frac{v}{f} - \frac{v_s}{f}$$

$$\lambda_3 = \frac{v - v_s}{f}$$

As $$v = f_3\lambda_3$$

$$f_3 = \frac{v}{\lambda_3}$$

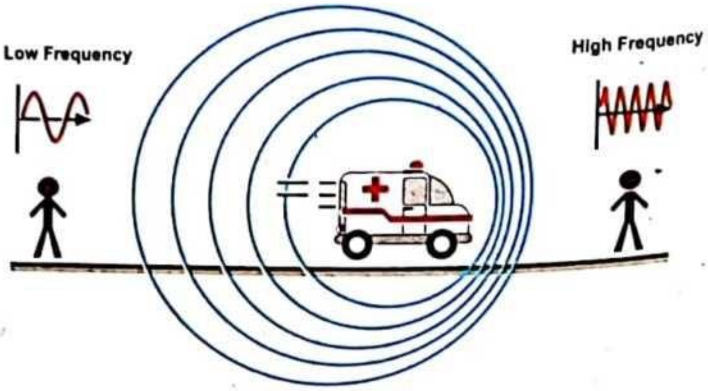


Fig.8.30: Source is in moving towards and moving away from a stationary observer.

$$f_3 = \frac{v}{\frac{v - v_s}{f}}$$

$$f_3 = \left(\frac{v}{v - v_s}\right) f \quad ......(8.23)$$

This shows that $f_3 > f$ thus apparent frequency of sound increases.

Case IV: When the source is moving away from the stationary observer

Similarly, when the sources is moving away from the stationary observer with velocity v_s then the wavelength λ_4 of sound waves increases but its number of waves in one second remains same. In this case the observer measures a wavelength λ_4 that is greater than λ and hears a decreased frequency.

Thus the increase in wavelength is given as;

$$\Delta\lambda = \lambda_4 - \lambda$$

$$\lambda_4 = \Delta\lambda + \lambda$$

$$\lambda_4 = \frac{v_s}{f} + \frac{v}{f} = \frac{v_s + v}{f}$$

$$v = f_4 \lambda_4$$

$$f_4 = \frac{v}{\lambda_4}$$

$$f_4 = \frac{v}{\frac{v_s + v}{f}}$$

$$f_4 = \left(\frac{v}{v_s + v}\right) f \quad ......(8.24)$$

This shows that $f_4 < f$ therefore, the apparent frequency of the observer increases.

BIOPHYSICS

Physicians can detect the speed of the moving heart wall in a fetus by means of Doppler Effect in ultrasound.

POINT TO PONDER

Can you apply Doppler Effect for light wave and source of light?

Example 8.6

A train is approaching a station at 108 km h^{-1} sounding a whistle of frequency 1100 Hz. What will be the apparent frequency of the whistle as heard by an observer standing on the platform? What will be the apparent frequency heard by the same observer if the train moves away from the station with the same speed? Speed of sound is taken as 340 ms^{-1}.

Solution:

Speed of sound = v = 340 m s^{-1}

Speed of the train = V_s = 108 km h^{-1} = 30 m s^{-1}

Frequency of the source = f = 1100 Hz

Apparent frequency of the whistle when the train is approaching towards observer = f =?

$$f = \left(\frac{v}{v - v_s}\right) f$$

$$f = \left(\frac{340\,m s^{-1}}{340\,m s^{-1} - 30\,m s^{-1}}\right) 1100\,Hz$$

$$f = \left(\frac{340\,m s^{-1}}{310\,m s^{-1}}\right) 1100\,Hz$$

$$f = 1206\,Hz$$

Apparent frequency of the whistle when the train is moving away from the observer = f =?

$$f = \left(\frac{v}{v + v_s}\right) f$$

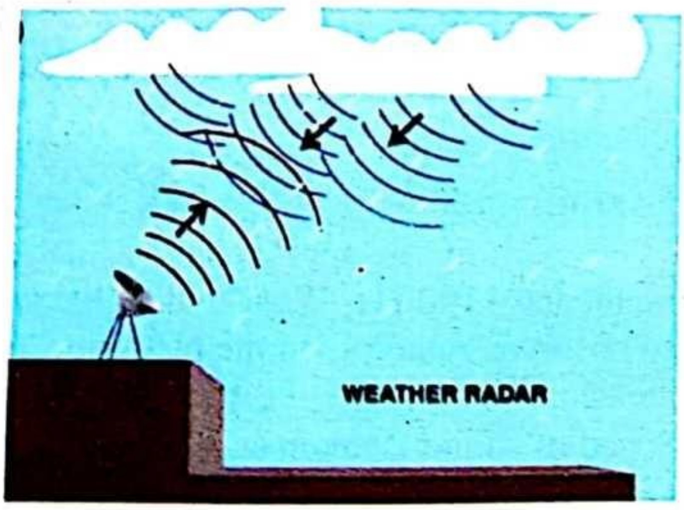
$$f = \left(\frac{340\,ms^{-1}}{340\,ms^{-1} + 30\,ms^{-1}}\right) 1100\,Hz$$

$$f = \left(\frac{340\,ms^{-1}}{370\,ms^{-1}}\right) 1100\,Hz$$

$$f = 1011\,Hz$$

Applications of the Doppler effect

In addition to sound waves, Doppler effect is also applicable to electromagnetic waves and its some application are summarized as:



(i) The Doppler effect provides a method for tracking a satellite.

Suppose the satellite is emitting a radio signal (i.e., an electromagnetic wave) of constant frequency f_s. The frequency f_L of the signal received on the Earth decreases as the satellite is passing.

The received signal is combined with a constant signal generated in the receiver to produce beat. The beat frequency produces an audible note whose pitch changes as the satellite passes overhead.

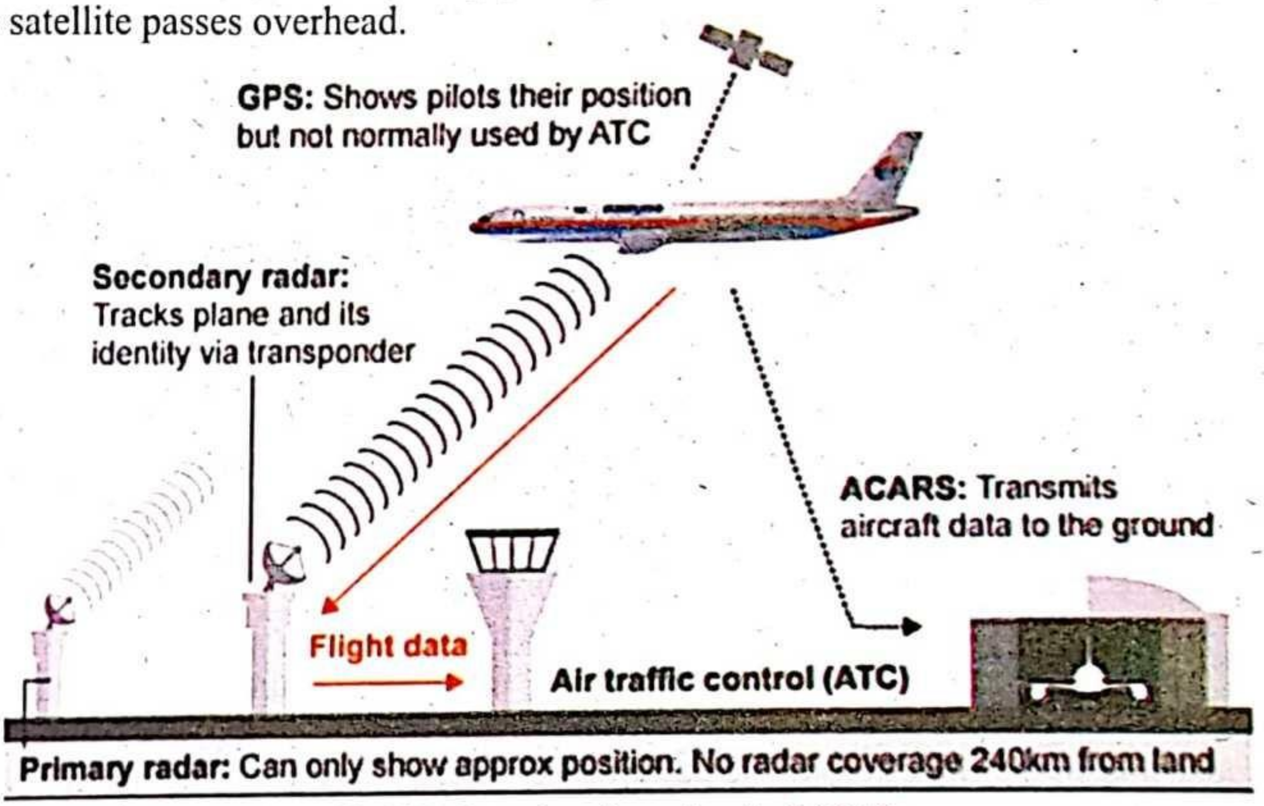


Fig.8.31: Detection of aeroplane by RADAR

Similarly, the radar system uses radio waves to determine the elevation and speed of an aeroplane. Radar is a device, which transmits and receives radio waves. If an aeroplane approaches towards the radar, then the wavelength reflected from aeroplane would be shorter and if it moves away, then the wavelength would be larger as shown in Fig.8.31.

(ii) Sonar is an acronym derived from "sound navigation and ranging". Sonar is the name of the technique for detecting the presence of objects under water by acoustical echo.

In Sonar, "Doppler detection" relies upon the relative speed of the target and the detector to provide an indication of the target speed. It employs the Doppler effect in which an apparent change in frequency occurs when the source and the observer are in relative motion to one another. It is known military applications include the detection and location of submarines, control of antisubmarine weapons, mine hunting and depth measurement of sea.

(iii) Astronomers use the Doppler effect to calculate the speeds of distant stars and galaxies. By comparing the line spectrum of light from the stars with light from a laboratory source, the Doppler shift of the star's light can be measured. Then the speed of the star can be calculated.

(iv) Stars moving towards the Earth show a blue shift. This is because the wavelength of light emitted by the star is shorter than if the star had been at rest. So, the spectrum is shifted towards shorter wavelength, i.e., the blue end of the spectrum as shown in Fig.8.32.

Stars moving away from earth show a red shift. The emitted waves have a longer wavelength than if the star had been at rest. So, the spectrum is shifted towards longer wavelength, i.e., towards the red end of the spectrum as shown in Fig.8.32. Astronomers have also discovered that all the distant galaxies are moving away from us and by measuring their red shifts, they have estimated their speeds.

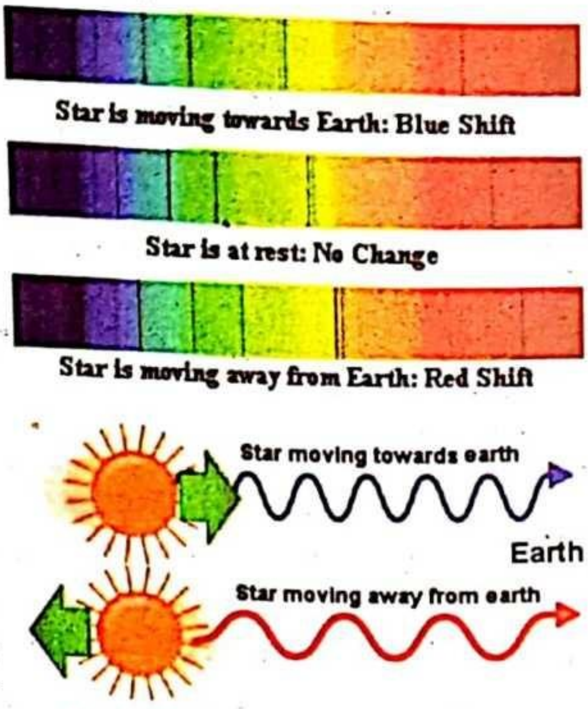


Fig.8.32: Doppler Blue and Red Shift

(v) The Doppler effect is used in measuring the speed of automobile by traffic police. A radar gun is fixed on police car. An electromagnetic signal is emitted by the radar gun in the direction of the automobile whose speed is to be checked. The wave is reflected from the moving automobile and received back.

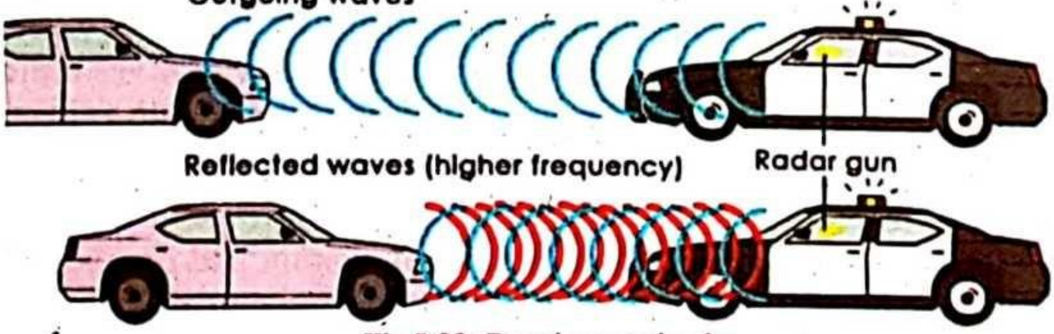


Fig.8.33: Doppler speed radar

The reflected wave is then mixed with the locally generated original signal and beats are produced. The frequency shift is measured using beats and hence the speed of the automobile is determined.

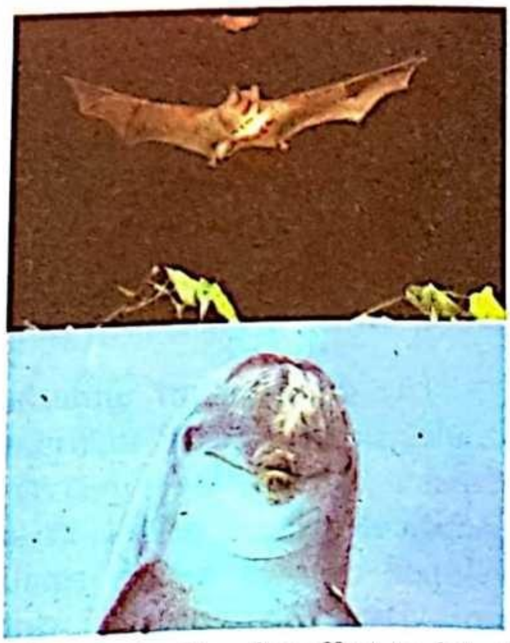
8.10 ULTRASONIC WAVES

Sound waves can be classified into three classes on the basis of their frequencies. That is, the sound which frequency less then 20Hz are called infrasonic waves and it cannot be heard by human ears. Similarly, the waves whose frequency range lie between 20Hz and 20 kHz are known as sonic or audible waves. These waves stimulate the human ear. The sound waves of frequency greater than the upper limit (20 kHz) are called ultrasonic or supersonic. These waves have high frequencies, shortest wavelength and carry much energy. Ultrasonic waves cannot stimulate our ear, but some animals like bats and dogs show response to them. Ultrasonic waves deserve special attention because of its multifarious application in metallurgy, medicines, biology and so many other fields.

There are several methods of generation of ultrasonic vibrations such as, mechanical and thermal but we discuss the electrical method which is named as piezoelectric generator. It was introduced by J and P. Curie in 1880; it is defined as electricity produced by pressure. Now the Piezoelectric method can be explained as; A slice of quartz crystal having regular faces is mounted between the two polished metal plates serving as electrodes. When two opposite faces of a crystal are subjected to pressure (compression or expansion) by the applied forces as shown in Fig.8.34, then there will be equal and opposite charges developed on the two opposite faces of the crystal. The amount of the developed charges is proportional to the subjected pressure. In this way, a potential difference will be developed across these faces. This process is called piezoelectric effect as shown in Fig.8.34.

Conversely, when the two faces of the crystal are subjected to an alternating potential difference as shown in the schematic diagram 8.35, then the crystal set into vibration due to its periodic contraction and expression. The frequency of the vibration is within the ultrasonic range (250Hz – 100000 kHz). This process is called inverse piezoelectric effect.

BIOPHYISCS



Bats use the Doppler effect to detect and catch flying insects. When an insect is flying faster than a bat, the reflected frequency is lower, but when the bat is catching up to the insect, as shown in figure, the reflected frequency is higher this is known as echolocation. This phenomenon is also using by the dolphins and whales to communicate each other and to locate prey. Scientists continue to studying the amazing behavior of dolphins and bats and to use sound waves.

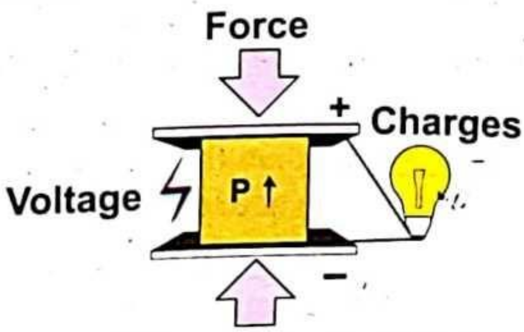


Fig.8.34: A schematic diagram of Piezoelectric effect

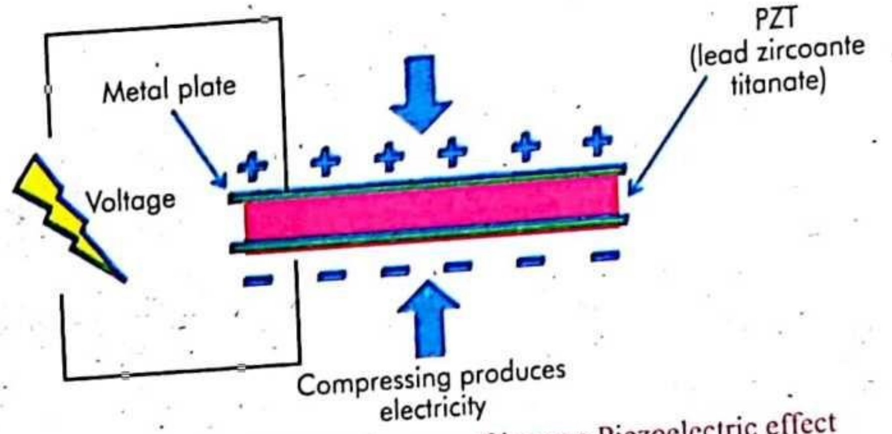


Fig.8.35: A schematic diagram of inverse Piezoelectric effect

The detection of ultrasonic waves can be detected by using the method piezoelectric transducer, that is when the ultrasonic waves fall on the two faces of the quartz crystal, then the varying electric charges are produced on the other perpendicular faces of the crystal as shown in Fig.8.36. The amount of these developed charges is very small but it can be amplified with the help of some means. Thus, this is the way which is being used to detect the ultrasonic waves.

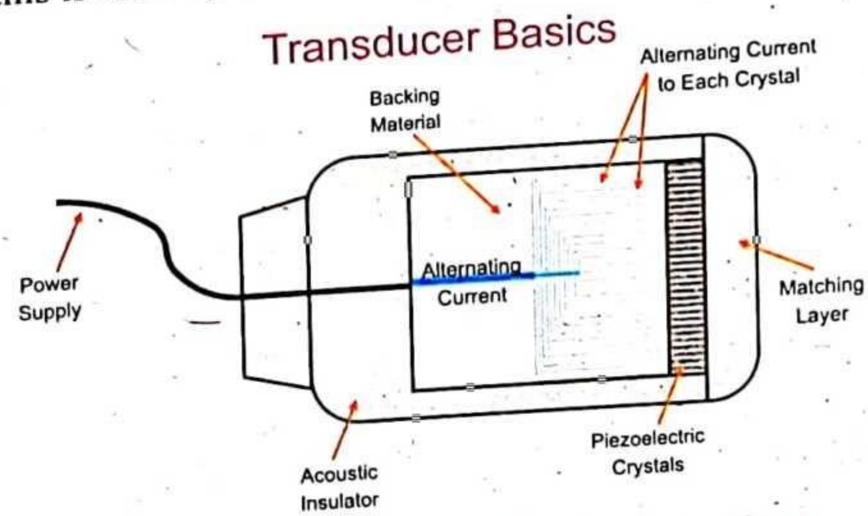


Fig.8.36: A schematic diagram of Piezoelectric transducer

SUMMARY

- **Wave:** A disturbance of medium by a vibrating body which transfers energy from one place to another place is known as wave.
- **Kinds of waves:** There are three kinds of waves, such as mechanical wave, electromagnetic wave and matter wave.
- **Transverse wave**: A wave in which the particles of the medium are vibrating perpendicular to the direction of propagation of wave is called transverse wave such waves consist of crest and trough.

- **longitudinal wave**: A wave in which the particle s of the medium are vibrating parallel to the direction of propagation of wave is called longitudinal wave - such waves consist of compression and rarefaction.
- **Speed of sound**: Speed of sound in air at 0°C is 332 ms^{-1} and it depends upon elasticity, density and temperature of the medium.
- **Principle of superposition:** When two or more waves are travelling in the same medium, their resultant amplitude is equal to the vector sum of all the individual amplitudes. This is called principle of superposition.
- **Interference:** If two or more waves of same frequency travelling in the same direction are superimposed then the amplitude of their resultant wave increases or decreases. This phenomenon is known as interference.
- **Beats:** When two or more waves differing slightly in their frequencies, travelling in the same direction, are superimposed then at regular interval of time the loudness of resulting wave increases or decreases. This phenomenon is known as beats.
- **Stationary waves:** Superposition of two waves of same amplitudes and same frequencies but travelling in the opposite direction are said to form a stationary wave.
- **Doppler's Effect:** The change in the pitch of sound due to the relative motion of the source of sound or the listener is called Doppler's effect.
- **Ultraviolet waves:** The waves with frequency greater than 20kHz are known as ultrasonic waves. These waves cannot be detected by human ears and these can be detected by piezoelectric method.
- **Piezoelectric generator:** A method in which electricity is produced by applying pressure is called piezoelectric and the process of piezoelectric transducer is being used to detect the ultrasonic waves.

EXERCISE

Multiple choice questions.

1. Wave is a mechanism which transmits;
   (a) Wavelength (b) Amplitude
   (c) Mass (d) Energy
2. The wave which requires a medium for its propagation is known as;
   (a) Mechanical waves (b) Electromagnetic waves
   (c) Radio waves (d) Light waves
3. Longitudinal wave consists of;
   (a) Crests and troughs (b) Compression and rarefactions
   (c) Crests and compressions (d) Troughs and rarefactions

4. Transverse wave is different from longitudinal wave, because it possesses a property of
(a) Reflection (b) Interference
(c) Diffraction (d) Polarization

5. Due to high elasticity, the speed of sound is maximum in
(a) Solids (b) Liquids (c) Gases (d) Plazma

6. The speed of sound does not depend upon;
(a) Density (b) Elasticity (c) Temperature (d) Pressure

7. Which of the following phenomenon is based on superposition principle
(a) Interference (b) Standing waves (c) Beats (d) All of these

8. When two waves of same frequency and travelling in the same direction are superimposed than we have
(a) Interference (b) Beats
(c) Standing wave (d) Stationary wave

9. Which one of the following change can be observed in the resultant interference wave?
(a) Amplitude (b) Time period (c) Wavelength (d) Frequency

10. How many beats can be observed when the difference in frequencies of two waves is two;
(a) 1 (b) 2 (c) 3 (d) 4

11. The length between node and antinodes is;
(a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$ (c) λ (d) 2λ

12. Which one of the following wave does not transmit energy
(a) Mechanical wave (b) Standing wave
(c) Matter wave (d) Electromagnetic wave

13. The ratio of the fundamental frequency of an open ended pipe to a pipe whose one end is closed is;
(a) 1:1 (b) 1:2 (c) 2:1 (d) 1:4

14. The number of quantization frequency of stationary wave in pipe when its one end is closed is;
(a) The whole number (b) Natural number
(c) Even number (d) Odd number

15. When wave is reflected from denser medium to rare medium then there is phase change of;

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 2π

16. The sound which stimulates our ear is known as;
(a) Sonic (b) Infrasonic (c) Ultrasonic (d) Tidal

17. Which one of the following parameter of a wave does not change when it transmits through two different media
(a) Amplitude (b) Velocity (c) Frequency (d) Wavelength

18. Piezoelectric effect means to produce the electricity by;
(a) Thermal (b) Mechanical (c) Pressure (d) Tidal

SHORT QUESTIONS

1. Distinguish between transverse and longitudinal waves.
2. How can the wavelength of compression wave be measured?
3. Why transverse wave can travel in a liquid?
4. Why does sound travel faster in solids than the gases?
5. How did Laplace correct the formula for the speed of sound in air?
6. What are the mathematical conditions of constructive and destructive interferences?
7. By what factor would you have to multiply the tension in the string to double the wave velocity?
8. Why standing wave cannot transfer energy?
9. What happens to the wavelength of a wave that passes from a spring into another material with (a) Higher linear density (b) Lower linear density?
10. Does interference of two waves involve a loss of energy? Explain.
11. How many numbers of nodes and antinodes are there in a stationary wave vibrating with 'n' number of loops?
12. What do you know about the Doppler shift in wavelength?
13. Why the ear does not stimulate by sound which is produced by a vibrating simple pendulum?

COMPREHENSIVE QUESTIONS

1. Define wave with all its characteristics such as; crest trough, amplitude, wavelength, time period and frequency.
2. Compare transverse and longitudinal periodic waves.
3. What do you know about the speed of sound? Calculate the Newton's formula for the speed of sound.

4. Discuss that how the Newton's formula for speed of sound was corrected by Laplace.
5. Explain the effect of various parameters, pressure, density and temperature on the speed of sound.
6. What is principle of superposition of waves.
7. State and explain interference of sound waves with its two forms such as; constructive interference and destructive interference.
8. What are beats and how they can be produced? Write done the uses of beats.
9. State and explain the reflection of waves from rare and dense media.
10. Explain stationary waves and their formation.
11. Discuss the stationary waves in a stretched string and in a air column.
12. State and explain Doppler effect under various cases and discuss the applications of Doppler effect.

NUMERICAL PROBLEMS

1. A pulse of a transverse wave on a string moves a distance of 15 m in 0.075 s. If the wavelength of transverse wave is 0.8m then; (a) what is the velocity of the pulse? (b) What is the frequency of a periodic wave on the same frequency?

   (200 ms^{-1}, 250 Hz)

2. What is the wavelength of electromagnetic wave when its frequency is 600 kHz and its speed is 3×10^8 ms^{-1}? **(500 m)**

3. A steel wire 80 cm long has mass of 8 g. If the wire is under a tension of 110 N, what is the speed of transverse wave in the string? **(105 ms^{-1})**

4. What is the speed of sound in a diatomic ideal gas that has density of 3.50 kg m^{-3} and pressure of 215 K Pa.? The value of 'γ' for diatomic gas is 1.40.

   (293 ms^{-1})

5. An 80 m long stretched string has a mass per unit length of 9×10^{-3} kg/m with tension of 20 N. When the string is plucked, a stationary wave is set up in the string. Calculate the fundamental frequency and the next three frequencies?

   (0.295 Hz, 0.59 Hz, 0.885 Hz, 1.18 Hz)

6. The fundamental frequency of an open organ pipe 100 cm long is 180 Hz. What is the speed of sound in the pipe? What is the frequency of the second possible overtone of that open pipe? **(360 ms^{-1}, 360 Hz)**

7. Calculate the length of a pipe that will resonate in air to a sound source of a fundamental frequency 240 Hz, if the pipe is (a) closed at one end and (b) open at both ends. Take the speed of sound in air to be 340 ms^{-1}.

   (35.7 cm, 70.8 cm)

8. Two tuning forks A and B produce 14 beats in 2 seconds. The frequency of the fork 'A' is 512 Hz. When a little wax is attached to the prongs of the fork 'B' the beats disappear. Determine the frequency of fork 'B'. **(519 Hz)**

9. A car travelling at 90 km h^{-1} sounds its horn which has a frequency 800 Hz. What frequency is heard by a stationary distant listener as the car approaches? What frequency is heard after the car has passed? Speed of sound in air is taken as 340 ms^{-1}. **(863.5 Hz, 745 Hz)**

10. Two cars P and Q are travelling along a straight road in the same direction, the leading car P travels at a steady speed of 12 ms^{-1}. The other car Q travelling at steady speed of 20 ms^{-1} sounds its horn to emit a steady note which is estimated by P's driver as a frequency of 830 Hz. What frequency does Q's own driver hear? Speed of sound in air is taken as 340 ms^{-1}. **(810 Hz)**

Unit 9

PHYSICAL OPTICS

Major Concepts (25 PERIODS)

- Nature of light
- Wave front
- Huygen's principle
- Interference
  - o Young's double slit experiment
  - o Michleson's Interferometer
- Diffraction
- Polarization

Conceptual Linkage

This chapter is built on Properties of Waves Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- Describe light waves as a part of electromagnetic waves spectrum.
- Describe the concept of wave front.
- State Huygen's principle and use it to construct wave front after a time interval.
- State the necessary conditions to observe interference of light.
- Describe Young's double slit experiment and the evidence it provides to support the wave theory of light.
- Explain colour pattern due to interference in thin films.
- Describe the parts and working of Michleson Interferometer and its uses.
- Explain diffraction and identify that interference occurs between waves that have been diffracted.
- Describe that diffraction of light is evidence that light behaves like waves.
- Describe and explain diffraction at a narrow slit.
- Describe the use of a diffraction grating to determine the wavelength of light and carry out calculations using $d\sin\theta = n\lambda$.
- Describe the phenomena of diffraction of X-rays through crystals.
- Explain polarization as a phenomenon associated with transverse waves.
- Identify and express that polarization is produced by a Polaroid.
- Explain the effect of rotation of Polaroid on Polarization.
- Explain how plane polarized light is produced and detected.

INTRODUCTION

The properties and the nature of light was studied by many scientists based on different theories but two of them were commendable, that is Newton's corpuscular theory and Christian Huygen's wave theory. Newton believed that light consists of small particles called corpuscles and he was successful in reflection and refraction phenomenons. There are two experiments photoelectric effect and Compton's effect which have been verified the Newton's corpuscular theory and these will be studied in Modern physics in the next class.

In 1676, Huygens explained the light in terms of wave. According to this wave theory, light is travelling in the form of a wave. The wave theory can explain reflection, refraction and the phenomenon of double refraction. The Huygen's wave theory of light was not acceptable by Newton and others. Because the knowledge of waves was confined to mechanical waves only and it requires some medium for its propagation and there was no idea about electromagnetic waves. Therefore, Huygen proposed hypothetical medium Ether. One important difference between the two theories was that the corpuscular theory predicted that light would travel faster in a material medium than air, whereas the wave theory predicted a slower velocity in a dense medium. Later on, it was proved experimentally that the velocity of light is faster in rare medium.

Similarly, James Clark Maxwell presented the idea of electromagnetic waves. Electromagnetic waves can propagate through vacuum. It has same properties as that of the light and the speed of this wave is equal to speed of light i.e. 3×10^{8} m s^{-1}. Thus, Maxwell concluded that light waves are electromagnetic waves and require no medium such as ether for their propagation.

In 1801, Young provided an experimental proof of wave theory of light by performing interference of light. Similarly, the result of diffraction is also a strong evidence of the wave theory of light. The polarization phenomenon has confirmed that transverse nature of light wave. The discussion about the nature of light shows that light possess dual nature. Sometimes it behaves like particles but sometimes it behaves like waves. But it may be noted that these both behaviours cannot be considered simultaneously. The particle nature of light will be studied in modern physics meanwhile we will have studied wave nature of light in the present chapter.

POINT TO PONDER
Can you tell whether the unit of intensity of light is based or derived?

9.1 WAVE FRONT

When a stone is dropped into a pond of still water then there is expanding series of circles formed by crests and troughs. Like water waves, concentric circles of light waves can be drawn that propagate from a source of light in all directions with speed 'c', as shown in Fig.9.1. The radius of each circle is "ct" and each circle has the same displacement from the centre of source 'S' moreover, all the particles on each circle have same phase. **"The surface on which all the points vibrate in the same phase in a homogenous medium is known as a wave front".**

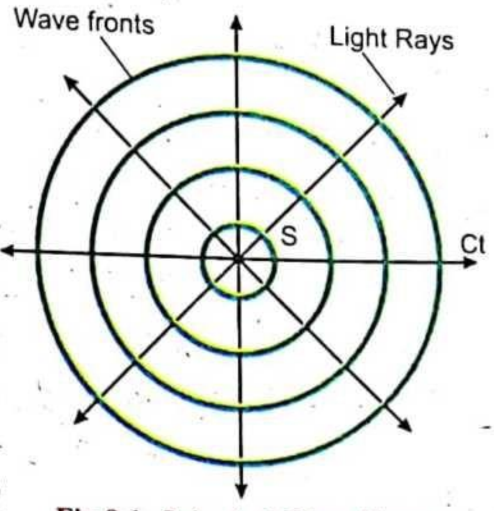


Fig.9.1: Spherical Wave Fronts

A line perpendicular to the wave fronts indicating the direction of motion of the waves is called a ray.

When there is a point source and medium is homogenous and isotropic then we have spherical wave fronts. In this case, the direction of propagation of the wave is always normal to the wave front.

If the disturbance is propagated in a single direction, the waves are then represented as plane waves and its corresponding wave fronts are called plane wave fronts as shown in Fig.9.2.

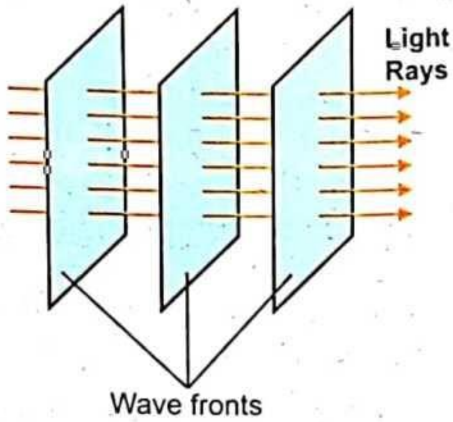


Fig.9.2: Plane wave fronts

9.2 HUYGENS'S PRINCIPLE

Huygens's principle is a geometrical method used to develop a new wave front from the information of shape and position of the primary wave front. According to Huygen wave theory, light travels in the form of waves and all the points of primary wave front behave as secondary sources emitting wavelet in phase with one another which spread out in forward direction with a speed equal to the speed of propagation of the wave.

Let a source S produced a primary wave front AB at instant 't' as shown in Fig.9.3. The dots on the primary wave front AB behave as secondary sources which produce hemisphere each of radius '$c\Delta t$', known as wavelets.

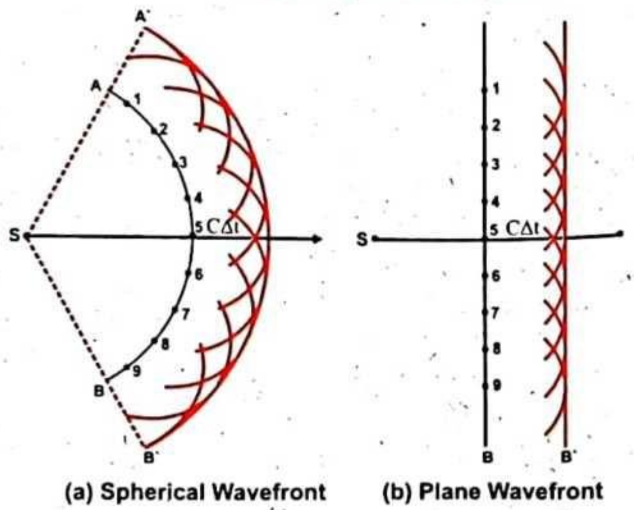


Fig.9.3: The wave fronts which are obtained under the Huygen's principle
(a) Primary spherical wavelets at 't' and secondary spherical wavelets at $t + \Delta t$.
(b) Primary plane wavelets at 't' and secondary plane wavelets at $t + \Delta t$.

The surface which touches all the wavelets from the secondary sources is the new wave front A`B` at instant t + Δt for next wave front, the same process is repeated.

In this way, an infinite number of spherical wave fronts are formed. If the medium is homogenous then equal amount of energy is transmitted in all direction by these waves. Similarly, if the medium is non-homogenous then we have a plane primary wave front AB and also Huygen's principle can be applied for the secondary wave front A`B` as shown in Fig.9.3.

9.3 INTERFERENCE OF LIGHT WAVES

We have studied interference of sound waves in the previous chapter. Now we discuss the interference of light waves. As interference of light is difficult to observe due to the random emission of light from the source. The following conditions should be fulfilled in order to observe the interference phenomenon of light waves.

1. The sources should be monochromatic i.e. these should emit waves of single wavelength.
2. The sources should be coherent which produce waves of same frequency with zero or constant phase difference.
3. The two sources should be closed to each other.

Consider two coherent waves in the same medium which are superimposed with each other. At some points there is enhancement in amplitude and at other points there is cancellation in amplitude. As a result, we have constructive and destructive interference and therefore bright and dark fringes are obtained on a screen as shown in Fig. 9.4. This phenomenon is known as interference of light waves.

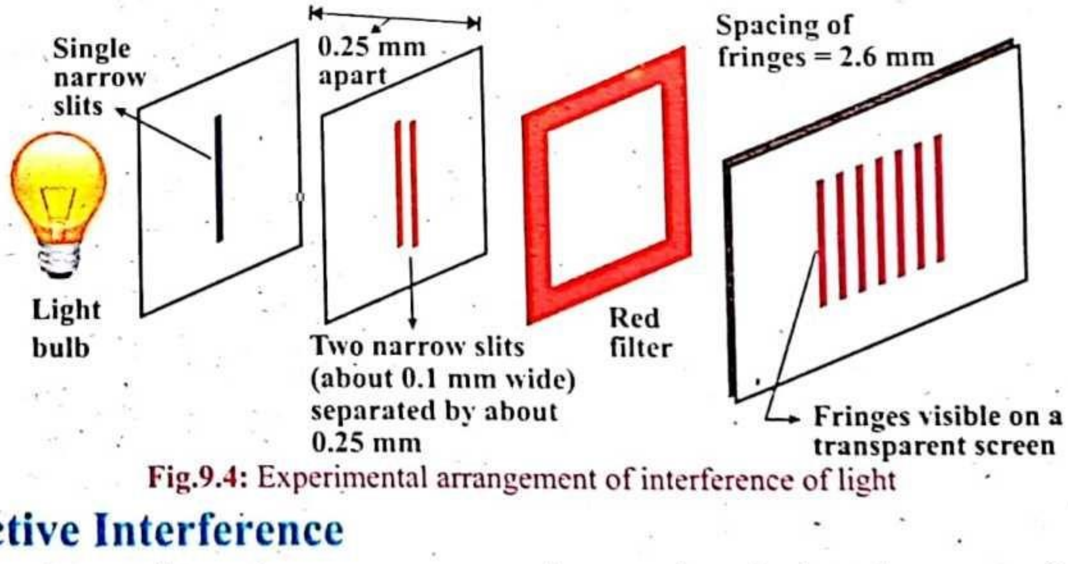


Fig.9.4: Experimental arrangement of interference of light

Constructive Interference

When two coherent waves are superimposed such that the crest of one wave coincide with crest of the other wave and trough with trough then the amplitude of its resultant is greater than that of the amplitude of individual wave as shown in Fig.9.5. This type of interference is called constructive interference.

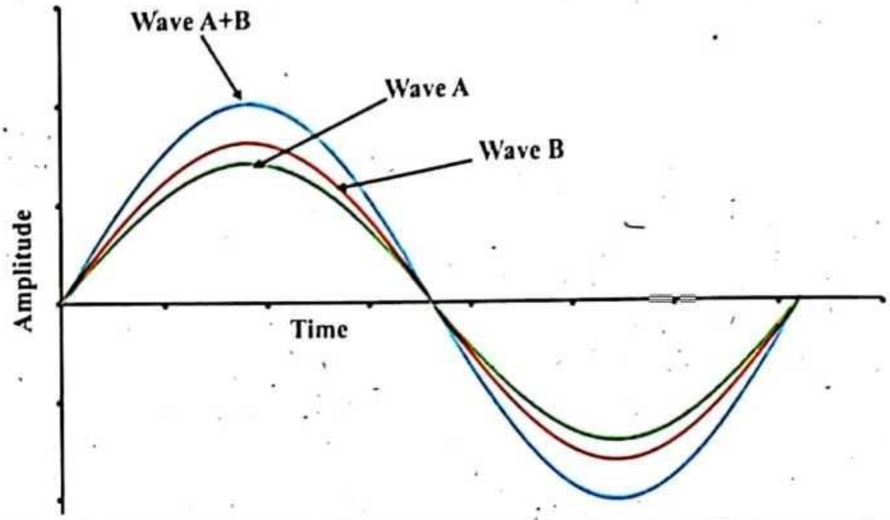


Fig.9.5: Constructive Interference due to the combination of coherent light waves in same phase.

Destructive Interference

In destructive interference, the superposition of two coherent waves, takes place in such a way that the crest coincide with trough and trough with crest and the amplitude of the resultant wave is less than the amplitude of individual wave as shown in Fig. 9.6.

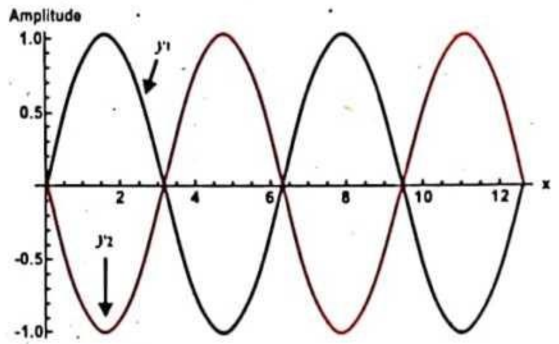


Fig.9.5: Destructive Interference due to the combination of coherent light waves in different phase.

9.4 YOUNG'S DOUBLE SLIT EXPERIMENT

This is the very first experiment on interference of light, which was demonstrated by Young in 1801. The result of this experiment provides a strong evidence for Huygens's wave theory.

The experimental setup consists of a source of monochromatic light which is placed in front of a narrow slit 'S'. Two slits S_1 and S_2 of the same size and separated by small distance are placed in front of narrow slit 'S'. These two slits act as two coherent sources as shown in Fig.9.7. Now the light waves from these two slits are superimposed at different points then interference occurs.

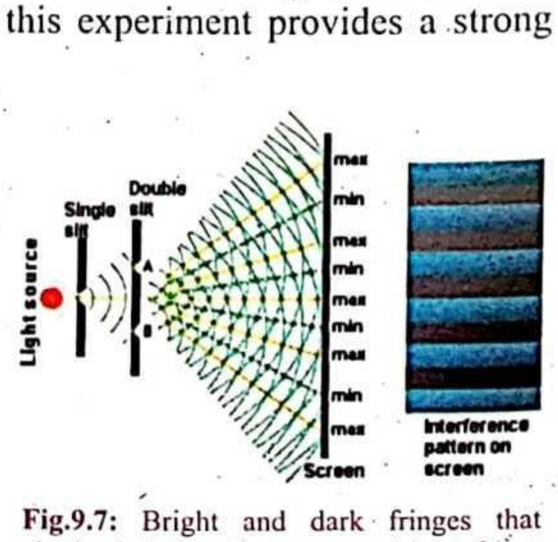


Fig.9.7: Bright and dark fringes that obtained due to the super position of two coherent waves in the Young double slit experimental arrangement.

The points where crests fall on crests or troughs fall on troughs produces constructive interference and we have bright fringes.

On the other hand, those points where

crests fall on troughs produce destructive interference and we have dark fringes. In this way, a series of bright and dark fringes is obtained on the screen which is placed at some distance from the slits. The result of Young's double slit experiment can also be studied analytically as well.

Let 'd' be the distance between two slits S_1 and S_2, 'L' be the distance between centre of slits and centre of the screen and 'y' be the distance of any fringe from the centre of the screen.

In order to derive equations for bright and dark fringes, we consider a point 'P' on the screen at distance QP from the centre of screen. S_1P and S_2P are the two rays from S_1 and S_2 respectively reaching at P. The path difference between S_1P and S_2P can be determined by drawing a perpendicular from S_1 on S_2P.

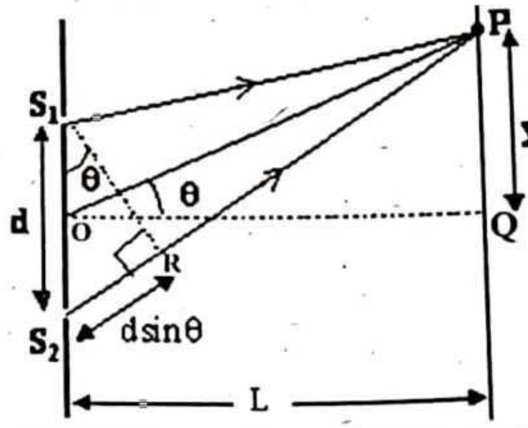


Fig.9.8: Two rays from two slits S_1 and S_2 which are incident on a screen at point P such that S_2R is a path difference between them.

As $S_1P = RP$ as shown in Fig. 9.8, so S_2R is a path difference between the two rays.

In triangle S_1S_2R

$$\frac{S_2R}{S_1S_2} = \sin\theta$$

$$S_2R = S_1S_2 \sin\theta$$

$[\because S_1S_2 = d]$

$$S_2R = d\sin\theta$$

Path difference $(S_2R) = d\sin\theta$ ......(9.1)

In interference pattern, bright fringes will be observed on the screen when path difference between two rays is given by

Path difference $(S_2R) = m\lambda$ ......(9.2)

where $m = 0, \pm1, \pm2, \pm3, ...$

Comparing equations (9.1) and (9.2)

$$d\sin\theta = m\lambda \quad ......(9.3)$$

Similarly, for destructive interferences (dark fringes)

Path difference $(S_2R) = \left(m + \frac{1}{2}\right)\lambda$ ......(9.4)

Comparing eq. 9.1 and eq. 9.4

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad ......(9.5)$$

where $m = 0, \pm1, \pm2, \pm3, ...$

Now if angle 'θ' is very small then

$$\sin\theta \approx \tan\theta$$

From triangle OPQ

$$\tan\theta = \frac{PQ}{OQ}$$

$$\tan\theta = \frac{y}{L}$$

FOR YOUR INFORMATION

θ°	sin θ	tan θ
2	0.035	0.035
4	0.070	0.070
6	0.104	0.105
8	0.139	0.140
10	0.174	0.176

Equation 9.3 becomes.

$$d\tan\theta = m\lambda$$

$$d\frac{y}{L} = m\lambda$$

$$y = \frac{m\lambda L}{d} \quad ......(9.6)$$

Equation (9.6) gives the position of bright fringe

Similarly,

$$d\tan\theta = \left(m + \frac{1}{2}\right)\lambda$$

$$d\frac{y}{L} = \left(m + \frac{1}{2}\right)\lambda$$

$$y = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \quad ......(9.7)$$

Equation 9.7 gives the position of dark fringe

POINT TO PONDER

Can you perform the interference phenomenon by using the sun light?

Fringe Spacing

Fringe spacing is defined as the distance between two consecutive bright fringes or two dark fringes.

Width of bright fringe $\Delta y = y_{m+1} - y_m$

$$\Delta y = \frac{L}{d}(m+1)\lambda - \frac{L}{d}m\lambda$$

$$\Delta y = \frac{L}{d}m\lambda + \frac{\lambda L}{d} - \frac{L}{d}m\lambda$$

$$\Delta y = \frac{\lambda L}{d} \quad ......(9.8)$$

Width of dark fringes $\Delta y = y_{m+1} - y_m$

$$\Delta y = \frac{L}{d}\left(m+1+\frac{1}{2}\right)\lambda - \frac{L}{d}\left(m+\frac{1}{2}\lambda\right)$$

$$\Delta y = \frac{L}{d}m\lambda + \frac{L}{d}\lambda + \frac{L\lambda}{2d} - \frac{L}{d}m\lambda - \frac{L\lambda}{2d}$$

$$\Delta y = \frac{L\lambda}{d} \quad \ldots\ldots(9.9)$$

Eq. (9.8) and eq. (9.9) show that bright and dark fringes are equally spaced.

Example 9.1

In Young double slit experiment, the distance between two slits is 0.25 cm. Interference fringes are formed on the screen placed at a distance of 1 m from the slits. The distance of the third dark fringes from the centre of screen is 0.059 cm. Find the wavelength of the incident light?

Solution:

We have $d = 0.25 \text{ cm} = 2.5\times10^{-3}\text{ m}$

$L = 1 \text{ m}$

$Y = 0.059 \text{ cm} = 5.9\times10^{-4}\text{ m}$

For 3rd dark fringes, order $(m) = 2$

Wavelength (λ) = ?

$$y = \left(m+\frac{1}{2}\right)\frac{\lambda L}{d}$$

$$\lambda = \frac{d\,y}{L\left(m+\frac{1}{2}\right)} = \frac{\left(2.5\times10^{-3}\right)\times\left(5.9\times10^{-4}\right)}{1\times\left(2+\frac{1}{2}\right)} = 5.9\times10^{-7}\text{ m}$$

$\lambda = 590\text{ nm} \quad \because 1\text{nm} = 1\times10^{-9}\text{ m}$

Example 9.2

Yellow sodium light of wavelength 589 nm is emitted by a single source and passes through two narrow slits 1 mm apart. The interference pattern is observed on a screen 225 cm away. How far apart are two adjacent bright fringes?

Solution:

We have

$\lambda = 589\text{ nm} = 589\times10^{-9}\text{ m} = 5.89\times10^{-7}\text{ m}$

$d = 1\text{ mm} = 1\times10^{-3}\text{ m}$

$L = 225\text{ cm} = 2.25\text{ m}$

Width of fringes (Δy) = ?

$$\Delta Y = \frac{\lambda L}{d} = \frac{5.89\times10^{-7}\times2.25}{1\times10^{-3}}$$

$$\Delta Y = 1.33\times10^{-3}\text{m} = 1.33\text{ mm}$$

The adjacent fringes will be 1.33 mm apart.

9.5 INTERFERENCE IN THIN FILMS

A thin film is a transparent medium whose thickness is very small. For example a thin layer of oil floating on water surface or a thin surface of soap bubble. It is a common observation that when light falls on these thin films of oil surface or soap bubble then we observe coloured patterns. This is due to the interference of reflected light from the two surfaces of thin film and it is explained under.

Consider a ray of light AB from a monochromatic source of wavelength 'λ' that is allowed to fall on a transparent thin film of thickness 'd'. This incident ray is partially reflected from the upper surface of the film along BC and partially refracted into the transparent medium of film along BD. At point D, it is again reflected inside the medium along DE and then at point E, the ray refracted along EF as shown in Fig.9.9. Now these two rays BC and EF superimpose with each other in order to produce interference which is detected by our eyes.

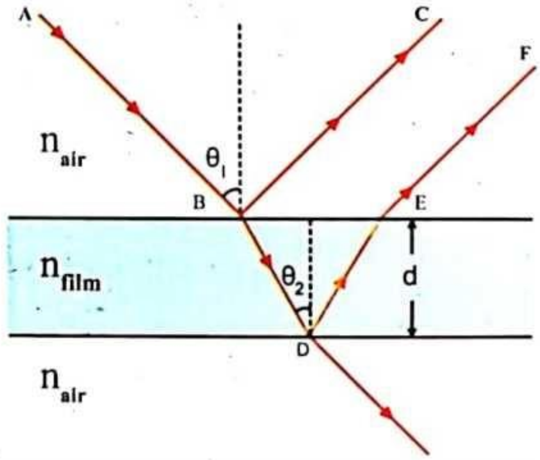


Fig.9.9: The incident ray is reflected from both upper surface and lower surface of thin film and interference is due to the superposition of these two reflected rays.

The Fig.9.9 shows that, the incident ray splits into two parallel reflected rays *BC* and *EF*. The distance covered by these two reflected rays are not same. The path difference between them depends upon angle of incident ray and thickness of the film. It is interesting to note that the point at which the path difference between two reflected rays is zero, a bright fringe should be formed but there is a dark fringe.

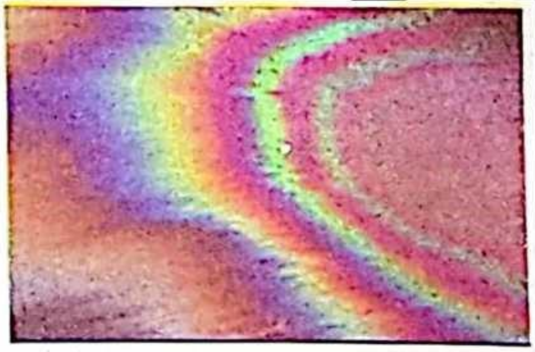
It is due to the fact that when the ray 'EF' is reflected from dense medium (film) to rare medium than there is an extra path difference of $\frac{\lambda}{2}$ added to it and due to this extra path difference the position of bright and dark fringes will be interchanged. Mathematically, it is explained as,

For bright fringes

$$\text{Path difference} = m\lambda + \frac{\lambda}{2}$$

Path difference $= \left(m + \frac{1}{2}\right)\lambda$ .....(9.10)

where m = 0,1,2,3,.......



Interference patterns on soap bubble and a thin layer of oil.

For dark fringes

Path difference $= \left(m + \frac{1}{2}\right)\lambda - \frac{\lambda}{2}$

Path difference $= m\lambda$ ......(9.11)

where m = 0, 1, 2, 3, ...

9.6 MICHELSON INTERFEROMETER

An interferometer is an optical instrument which is widely used to measure lengths or change in length with a great accuracy by means of interference fringes. It was introduced by American Physicists A.A. Michaelson in 1881. A schematic diagram of interferometer is shown in Fig. 9.10. It consists of a source of monochromatic light which is placed in front of a partially silver polished glass plate 'P', inclined at an angle of 45° to the horizontal. This plate is called beam splitter. There are also two highly polished mirrors M_1 and M_2. The mirror M_1 is vertical and fixed. While the mirror M_2 is horizontal and adjustable. Another optical compensator glass plate P′ same as 'P' is fixed between P and M_2 in order to eliminate the path difference between the two rays. To observe the interference pattern, a telescope is placed at the bottom in front of 'P' as shown in Fig. 9.10.

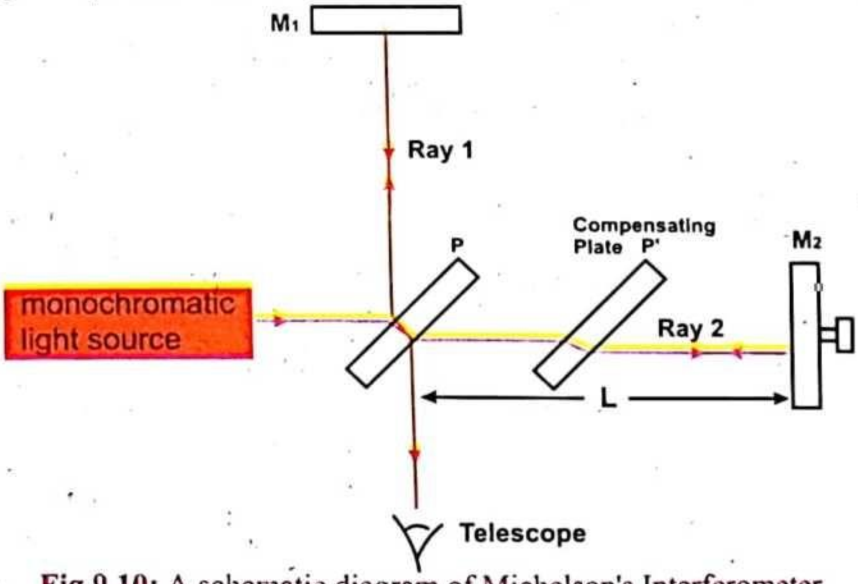


Fig.9.10: A schematic diagram of Michelson's Interferometer

The whole apparatus is mounted on rigid frame. When a ray of monochromatic light from a source is incident on the beam splitter 'P', it is partially reflected and partially refracted. The reflected ray (1) falls on a fixed mirror M_1 and the refracted ray (2) pass through the compensator 'P'' falls on moveable mirror M_2. These two incident rays now reflected from mirrors M_1 and M_2. The rays reflected from mirrors are again refracted and reflected from the plate 'P' and eventually recombine to produce an interference pattern which can be observed by the telescope.

The distance travelled by the two rays (1) and (2) is not same. There is a path difference between them. Hence we have interference pattern that consists of a series of bright and dark fringes. The numbers of these fringes that pass through a given point in certain time can be varied by adjusting the mirror M_2.

Optically, there is also an extra path difference between the two rays due to the reflection and refraction of the rays from one medium to other and to eliminate this path difference the compensator has been used. Hence, the path difference between the two rays is made zero and we get first bright fringe. Similarly, to get the next bright fringe, M_2 is to be shifted at a distance $\frac{\lambda}{2}$. On this basis, a general relation can be obtained. If M_2 is shifted by a distance 'x', and 'm' numbers of bright fringes are obtained due to this shift then,

$$x = m\frac{\lambda}{2} \quad ......(9.12)$$

This is the fundamental relation of Michalson's interferometer where $m = 0, 1, 2, 3, ...$

Example 9.3

When the moveable mirror in the Michelson Interferometer is moved in one direction there are 400 fringes appear to pass through the field of view. If the light of wavelength 500 nm is used, then what is the distance through which the mirror has been moved?

Solution:

We have

Number of fringes = 400

Wavelength = λ= 500 nm = $500 \times 10^{-9} = 5 \times 10^{-7}$ m

Distance through which movable mirror is moved = x =?

Equation of Michelson Interferometer $x = m\frac{\lambda}{2}$

$$x = m\frac{\lambda}{2}$$

$$x = \frac{400 \times 5 \times 10^{-7}}{2} = 1 \times 10^{-4} \text{m}$$

$$x = 0.1 \text{mm}$$

9.7 DIFFRACTION OF LIGHT

Similar to interference phenomenon, the diffraction phenomenon also supports the Huygens's wave theory of light. This phenomenon can be explained by an example of a small opaque ball which is placed between a source of light and a screen. Now when the balls is illuminated by light from a source, then we observed that its shadow is casted on a screen as shown in Fig. 9.11(a). The shadow is completely dark but it has bright spot at its centre and it gives the following two interesting results.

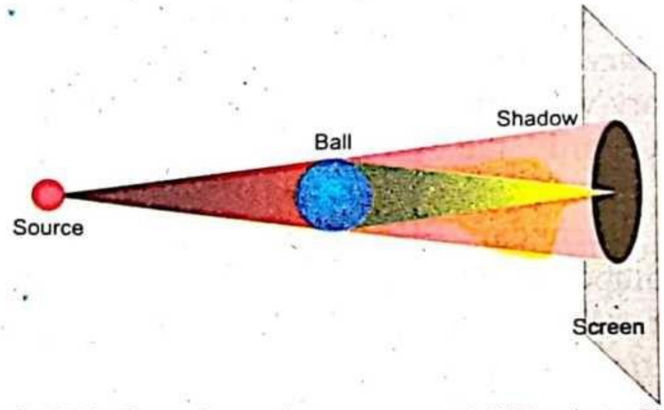


Fig.9.11(a): Experimental arrangemtn of diffraction of light

1. When light travels through an obstacle, it does not proceed exactly along a straight path but bends around the obstacle.
2. When the bending rays of light from the opposite sides of the obstacle are superimposed then there is bright spot at the centre due to the constructive interference.

This phenomenon of bending of light around the corners of an obstacle is called diffraction.

Similarly, when light passes through a narrow slit, the light also bends around the edges of the slit as shown in Fig. 911(b). When this bending of light is allowed to fall on the screen, a diffraction pattern which consists of bright and dark fringes is obtained on it. These results show that diffraction pattern depends upon the size of slit or obstacle. It may be pointed out that diffraction phenomenon will be observed only if the wavelength of the incident light is greater than that of the size of the slit. It means size of obstacle or width of slit should be comparable with the wavelength of light.

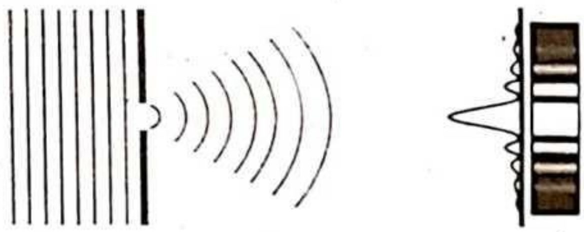


Fig.9.11(b): A diffraction pattern due to bending of light through edges of a narrow slit.

9.8 DIFFRACTION OF LIGHT DUE TO A SINGLE SLIT

To study the diffraction of light, a simple experiment can be demonstrated. The experimental setup consists of slit 'AB' of width 'd' which is illuminated by beam i.e. parallel rays of monochromatic light from a source of wavelength 'λ'. A screen is placed parallel to the slit in order to observe the diffraction pattern on it, as shown in Fig. 9.12.

When beam of light is incident on slit AB in form of a primary wave front then according to Huygen, each point on the wave front at position of slit acts as a secondary source and produces secondary wavelets which propagate toward the screen.

When these secondary wavelets are superimposed at different points then it causes the formation of a diffraction pattern. Such pattern can be studied on the screen as shown in Fig. 9.12.

In order to determine the position of maxima and minima at the screen, we consider the points A,X,Q,Y and B along the width of slit such that the width of AQ, QB and XY is equal to $\frac{d}{2}$.

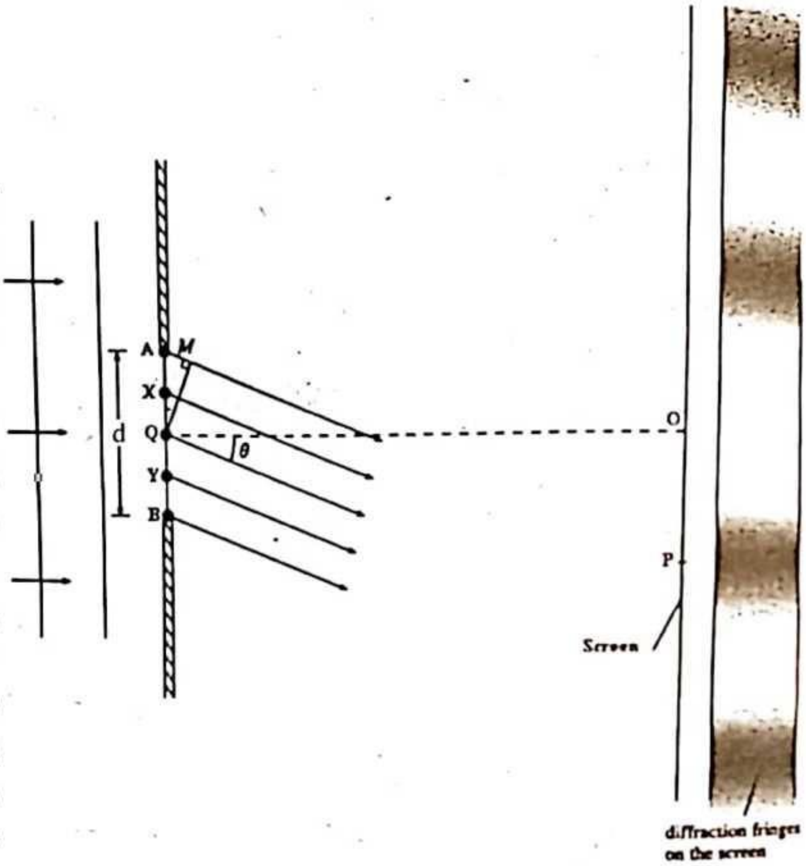


Fig.9.12: Diffraction of light through a single slit

Thus, there is no path difference between the light waves from the points A and B or from X and Y.

When these rays are allowed to meet at the centre of the screen 'O', a bright fringe is formed at that point due to the constructive interference.

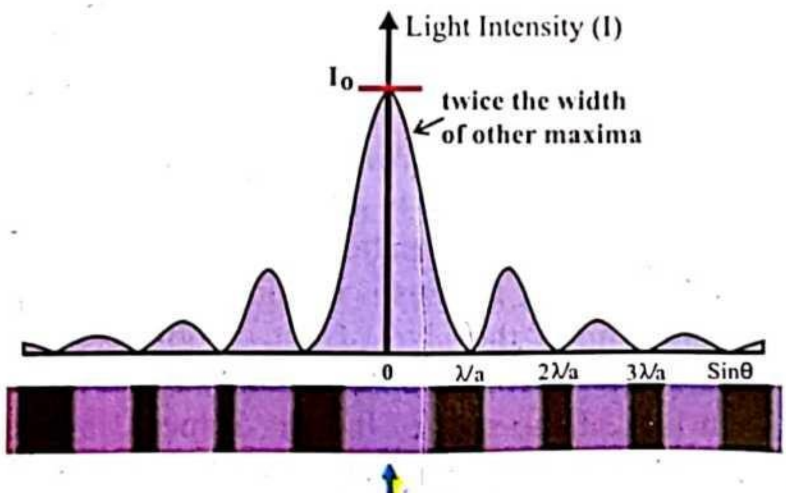


Fig.9.13: Diffraction pattern which consists of bright and dark fringes and it is obtained due to diffraction through a single slit

Similarly, we select another point 'P' at the screen below the point O such that the path difference between the waves from the points A & B at the point P is λ but the path difference between the waves from the pairs of the points A & Q, X & Y and Q & B at the point P is $\frac{\lambda}{2}$.

Hence dark fringe is formed at the point P due to the destructive interference as shown in Fig. 9.13.

A general mathematical relation can be obtained if we take the a half section of slit AQ as $\frac{d}{2}$ and the path difference between the waves from the points A & Q as $\frac{\lambda}{2}$ which is equal to the points from A to M as shown in Fig. 9.12. Thus in triangle AQM

$$\frac{AM}{AQ} = \sin\theta$$

$$\frac{\left(\frac{\lambda}{2}\right)}{\left(\frac{d}{2}\right)} = \sin\theta$$

$$\frac{\lambda}{2} = \frac{d}{2}\sin\theta$$

$$\lambda = d\sin\theta$$

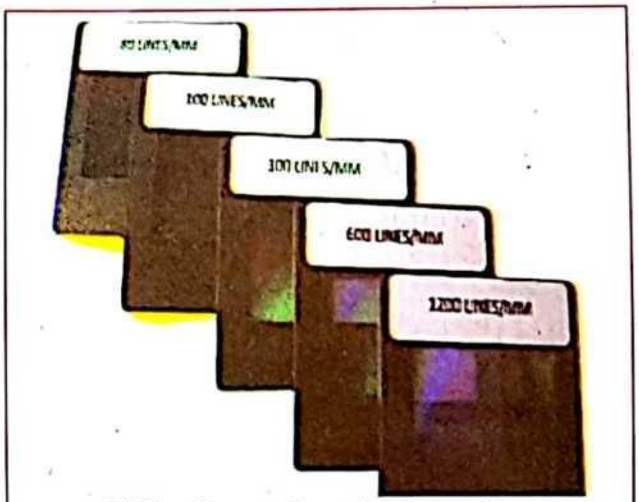
In general, the condition of 'm' orders of maxima on either side from the centre of the screen is given as;

$$d\sin\theta = m\lambda \quad \ldots\ldots(9.1)$$

$$m = \pm 1, \pm 2, \pm 3, \ldots$$

POINT TO PONDER

How the fringes of interference are different than that of the diffraction?



Diffraction grating of various sizes.

9.9 DIFFRACTION GRATING

A diffraction grating is a specially designed transparent glass plate which is used to study the diffraction phenomenon. It consists of a transparent glass slab which contains a large number of parallel and equidistance slits of same width separated by an opaque portion. The number of slits on the diffraction grating depends upon the wavelength of the incident light. For example, the wavelength of visible light is

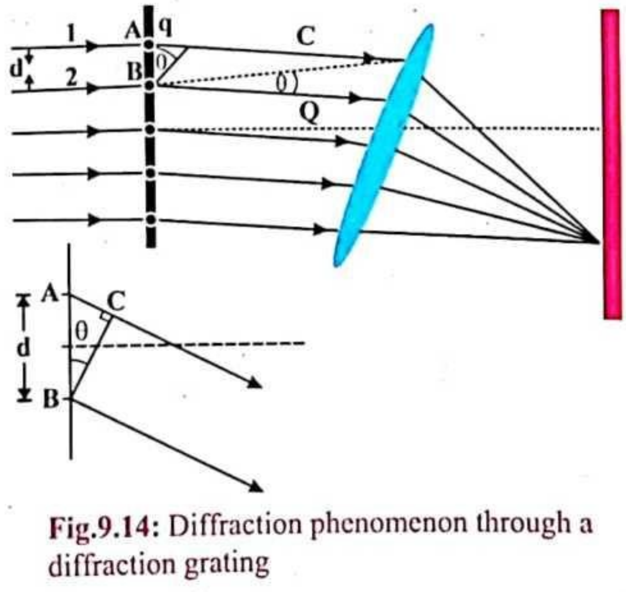


Fig.9.14: Diffraction phenomenon through a diffraction grating

400nm -700nm, it requires 4000 – 6000 lines per centimetre in order to observe diffraction effects by visible light. A typical slit width is given as;

$$d = \frac{1}{5000}\,cm = 2\times10^{-6}\,m = 2000\,nm \quad \because 1\,nm = 1\times10^{-9}\,m$$

The parameter 'd' is called grating element. This example shows that if the size of slit is less than the wavelength of the incident light then the diffraction of the light could be observed.

Practically, the grating which contains too large number of slits can be made by using a diamond point (cutter) to scratch equally spaced grooves (slits) on a glass or metal surface. The opaque paste is used to dip the slab into it. Then it is etched from the surface and the paste remains inside the grooves, thus the lines are opaque lines and the transparent portions between them act as number of slits.

Consider parallel rays of light from a monochromatic source which are incident on a grating. These rays, in form of wave fronts, are passed through slits, such that each slit causes diffraction and the diffracted rays in turn interfere with one another to produce a pattern. This diffraction pattern is focused on the screen with the help of a convex lens as shown in Fig. 9.14. Let we take the two rays r_1 and r_2, such that AC is a path difference between them as shown in Fig. 9.14.

In triangle ABC

$$\frac{AC}{AB} = \sin\theta$$

AC (path difference) = $d\sin\theta$ ......(9.14)

For constructive interference (bright fringes)

Path difference = $m\lambda$ ......(9.15)

Comparing eq. 9.14 and eq. 9.15

$$d\sin\theta = m\lambda \;......(9.16)$$

$$m = 0, \pm1, \pm2, \pm3, ...$$

FOR YOUR INFORMATION

The first man-made diffraction grating was made around 1785 by American inventor **David Rittenhouse**, who stung hairs between two finely threaded screws. This was similar to notable German Physicist **Joseph von Fraunhofer**'s wire diffraction grating.

This is the basic relation of diffraction grating. Where 'm' is the order of fringe or image. For example, if all the diffracted rays are focused at $\theta = 0$ then, m = 0. This is called the zero order maximum. Similarly, if m = 1, m = 2 and so on then there will be bright images.

Example 9.4

The deviation of second order diffracted image formed by an optical grating having 5000 lines per centimetre is 32°. Calculate the wavelength of the light used.

Solution:

We have

Order of diffraction = m = 2

FOR YOUR INFORMATION

Diffraction is maximum when the width of the opening is less than the wavelength of light.

Number of lines per centimetre = 5000 lines

Grating element $= d = \frac{1}{N} = \frac{1}{5000} = 2\times10^{-4}\text{ cm} = 2\times10^{-6}\text{m}$

Angle $= \theta = 32^\circ$

Wavelength $= \lambda = ?$

Equation of diffraction grating $d\sin\theta = m\lambda$

$$\lambda = \frac{2\times10^{-6}\text{m}\times\sin 32^\circ}{2} = 5.30\times10^{-7}\text{m}$$

$$\lambda = 530\text{nm} \quad \because 1\text{nm} = 1\times10^{-9}\text{m}$$

9.10 DIFFRACTION OF X-RAYS

X-rays are a type of electromagnetic waves with extremely short wavelength. Typical wavelength of x-rays is of the order of $1\text{A}^\circ\left(10^{-10}\text{m}\right)$. Therefore a grating which contains 5×10^{7} numbers of slits per centimetre i.e. grating element of size $2\text{A}^\circ\left(2\times10^{-10}\text{m}\right)$ is required for diffraction of x-rays.

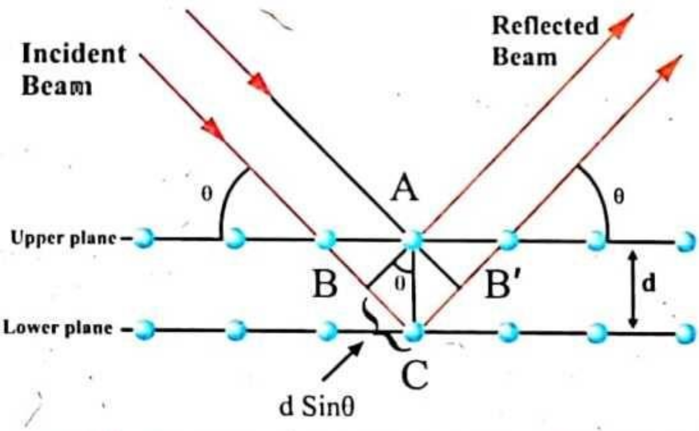


Fig.9.15: A schematic arrangement for diffraction of x-rays by a crystal.

Practically, it is not possible to construct an optical grating of such too small size and large number of slits. To overcome this problem, W.H. Bragg and W.L. Bragg suggested the diffraction of x-rays can be observed by crystals. A crystal is an element whose atoms are arranged in a regular array and they are separated uniformly by a distance of the order of 2.15 A° or 2.15×10^{-10} m. Therefore, the distance between two atoms can act as a slit. The diffraction of X-rays takes place when these are allowed to fall on the crystal. Consider two monochromatic rays which are incident on a crystal at angle 'θ' with the surface of crystal which is called glacing angle. Let these two rays be reflected from the 1st two planes of atoms separated by a distance 'd'. A schematic diagram in Fig. 9.15 shows that the ray reflected from the lower plane travels farther than the ray reflected from the upper plane. The path difference between these two reflected rays is given as,

FOR YOUR INFORMATION

You can produce single slit diffraction by holding the index and middle fingers of one hand together and looking at a bright light through the space between them. Then press the fingers together to change the opening size and observe how the diffraction pattern changes.

Path difference $= BC + CB'$ ......(9.17)

In triangle BAC

$$\frac{BC}{AC} = \sin\theta$$

$$BC = AC\sin\theta \qquad \because AC = d$$

$$BC = d\sin\theta \qquad ......(9.18)$$

Similarly from triangle $B'AC$,

$$CB' = d\sin\theta \qquad ......(9.19)$$

Putting the values of eq. (9.18) and eq. (9.19) in eq. (9.17)

Path difference $= d\sin\theta + d\sin\theta$

Path difference $= 2d\sin\theta$ ......(9.20)

For bright images (Constructive interference)

Path difference $= m\lambda$ ......(9.21)

Comparing eq. (9.20) and eq. (9.21)

$$2d\sin\theta = m\lambda$$

This is a Bragg's law, where m =1,2,3,4...

Example 9.5

How far apart are the diffracting planes in a NaCl crystal for which x-rays of wavelength 1.54 A° makes a glacing angle 16° in the first order?

Solution:

We have

Distance between two planes = d =?

λ = Wavelength of incident x-rays =1.54 A

$\lambda = 1.54\times10^{-10}$ m

Glacing angle = 16°

Order of image = m = 1

According to Bragg's Law

$$2d\sin\theta = n\lambda$$

$$d = (1)(1.54\times10^{-10})/2\sin16°$$

$$d = 2.79\times10^{-10}\text{m} = 2.79\,\text{Å}$$

9.11 POLARIZATION

Wave nature of light has been verified by the interference and diffraction. However, these phenomena cannot explain the transverse or longitudinal behaviour of light wave. For this purpose, the polarization phenomenon is used. Polarization is a property that exhibits only for a transverse wave. It is explained by an example.

Consider a string which is passed through two parallel rectangular slits. When the string is vibrated up and down, a transverse wave which consists of crests and troughs propagate along the string.

If the two slits are also parallel to the vibration of string, then wave is passed through both slits and each part of the string vibrates freely in the slits. The amplitude is not affected as shown in Fig. 9.16. This is a mechanical transverse polarized wave. However, if one slit is rotated by 90^{o} in its plane, the slit will point along the horizontal plane. As the wave arrives at second the slit, the part of string tries to move vertically but the contact force by the horizental slit does not allow it. Then wave does not pass through it. This shows that the incident wave is transverse wave. In case of longitudinal periodic wave along a stretched string, the wave will not be affected by the rotation of slit. This idea helps us in the explanation of transverse wave nature of light.

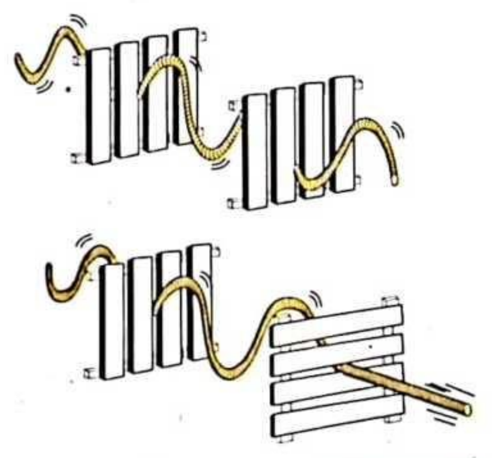


Fig.9.16: Polarization phenomenon due to string and slits arrangement.

Light waves are electromagnetic in nature. An electromagnetic wave consists of electric and magnetic field vectors which are vibrating at right angle to each other. The ordinary light is three dimensional and its components are vibrating in all direction. Therefore, it is called unpolarized light. However, when the vibration of light is restricted only in one plane then such light is called polarized light or plane polarized light.

9.11.1 Production and detection of plane polarized light

The ordinary light by lamp, bulb or the sun is unpolarized because its components are vibrating in all directions. When all the vibrating components are removed except those having vibration along the unidirection plane then it is called plane polarization. It can be achieved by various methods such as selective absorption, reflection, refraction from different surfaces and scattering by small particles.

The selective absorption is being used at large scale to obtain plane polarized light by using a device known as "Polaroid". It was introduced by two American Scientists E.H. Land and Boston. It consists of a transparent sheet of nitrocellulose which has embedded special needle-like crystal of herapathite and it has transmission axis. Thus, the Polaroid transmits those light waves whose electric field vector vibrates parallel to the polarizing direction.

Consider unpolarized light of intensity 'I' from a source which is incident on the Polaroid sheet (P_1), those electric field vectors which are parallel to the axis of transmission are passed through Polaroid (P_1) while the remaining electric field vectors are absorbed by the Polaroid sheet. In output, we have the plane polarized light whose intensity is less than the original unpolarized light waves.

In order to confirm whether the light has been polarized, we introduce another Polaroid 'P_2' same as that of 'P_1' but perpendicular in direction to that of P_1 as shown in Fig.9.17.

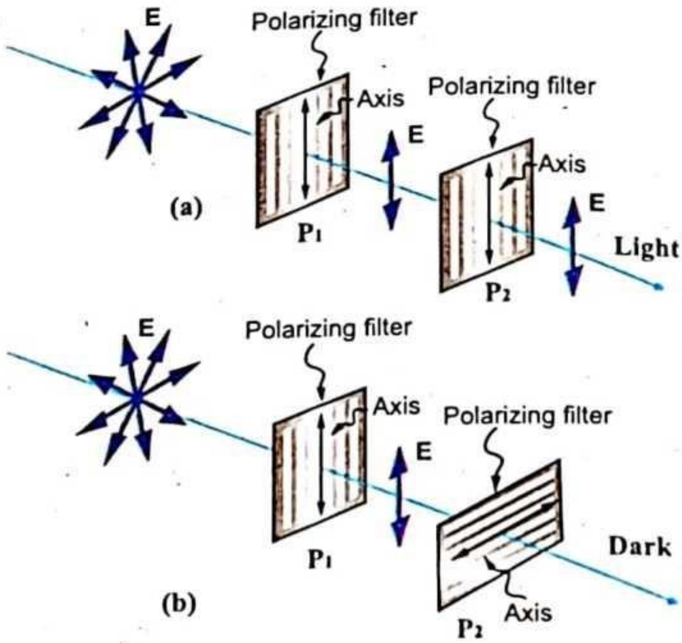


Fig.9.17: Polarization of light through polarizers.
(a) Unidirectional light pass through both polarizers
(b) Unidirectional light pass through only one polarizers

It is being used as analyzer now when the transmission axes of 'P_1' and 'P_2' are parallel, then the intensity of polarized light from both 'P_1' and 'P_2' is same.

POINT TO PONDER

Why the reflected light from surface of water or a mirror cost unpleasing effect in our eyes?

When the analyzer P_2 is rotated such that the transmission axes of P_1 and 'P_2' are perpendicular then no light will be transmitted through the analyzer 'P_2'. Thus, this result has confirmed that light waves are transverse. If it were longitudinal, then light would not disappear by the rotation of the analyzer 'P_2'.

SUMMARY

- **<u>Nature of light:</u>** The light has a dual nature that is Newton defined it in terms of corpuscle while Huygen defined it in terms of wave.
- **<u>Wave front:</u>** The surface on which all the particles vibrate in the same phase and have same displacement from the source is called wave front. When the medium is homogenous then spherical wave fronts are propagated outward from the source.
- **<u>Huygen's Principle:</u>** According to Huygen's principle, each point on the wave front is being considered as the secondary source that produces secondary wavelets.

- **Coherent and monochromatic waves:** The waves which have same frequency with zero phase or constant phase difference are known as coherent waves and the waves which have single wavelength are known as monochromatic waves.
- **Interference:** When two or more coherent waves are superimposed then the amplitude of the resultant wave is increased or decreased and as a result bright and dark fringe are obtained. This phenomenon is known as interference of light.
- **Young's double slits experiment:** The result of Young's double slit experiment is a proof of Huygen's wave theory of light. According to this experiment the position of bright and dark fringe at the screen are given as;

  For bright fringes, $d\sin\theta = m\lambda$

  For dark fringes, $d\sin\theta = \left(m+\frac{1}{2}\right)\lambda$

  Fringe spacing between two successive bright and dark fringes is given as $\Delta y = \frac{\lambda L}{d}$.
- **Thin film:** A thin film is a transparent medium with thickness comparable with the wavelength of light falling on it. Due to an extra path difference, the position of bright and dark fringes is interchanged in thin film.
- **Interferometry:** It is the technique of diagnosing the properties of two or more waves by studying the pattern of interference created by their superposition.
- **Interferometer:** The instrument used to interfere the waves together is called an interferometer.
- **Michelson interferometer:** It is an optical instrument which is used to measure extremely small distance with high precision using interference phenomenon.
- **Diffraction:** Bending of light through edges of slit is known as diffraction.
- **Diffraction grating:** A diffraction grating is a specially designed transparent glass plate which is used to study the diffraction phenomenon. A mathematical relation for diffraction grating is given as $d\sin\theta = m\lambda$
- **X-ray Diffraction:** X-rays have shorter wavelength and these can be diffracted by a crystal, where the inter atomic distance between two atoms acts as slit. Bragg's law for x-ray diffraction is given as $2d\sin\theta = m\lambda$

- **Polarization:** The confinement of beam of light in a given direction or plane is called polarization of light. This phenomenon proves that the nature of light is transverse.
- **Polarized light:** A light wave in which its electric field vectors are vibrating in unidirectional is known as polarized light. Polarization phenomenon proves that the nature of light is transverse.

EXERCISE

Multiple choice questions.

1. Who presented the corpuscular theory of light?
(a) Huygen (b) Newton (c) Young (d) Maxwell

2. Which phenomenon does not explain the of wave theory of light?
(a) Polarization (b) Interference (c) Diffraction (d) Compton effect

3. Interference of light occurs when the source of light are;
(a) Monochromatic (b) Coherent
(c) Closed to each other (d) All of these

4. In Young's double slit experiment, if the distance between two slits is halved and distance between slits and screen is doubled then what will be the fringe width?
(a) Remain same (b) Halved (c) Doubled (d) Quadrupled

5. In Young's double slit experiment, the ratio of fringe width of bright to dark fringe is;
(a) 1:1 (b) 1:2 (c) 2:1 (d) 2:3

6. In the Young's double slit experiment if white light is used then;
(a) Bright fringes will be seen
(b) Dark fringes will be seen
(c) Alternate dark and bright fringes will be seen
(d) No interference fringes will be seen

7. When light is reflected from dense medium to rare medium then its path difference.
(a) Remains same (b) Changes by $\left(\frac{\lambda}{2}\right)$
(c) Changes by λ (d) Changes by $\frac{3\lambda}{2}$

8. Michelson interferometer is an optical instrument which is being used for the measurement of;
(a) Velocity (b) Frequency (c) Amplitude (d) Wavelength

9. Mathematical condition of destructive interference in thin film is;

(a) $m\lambda$ (b) $\frac{m\lambda}{2}$ (c) $\left(m+\frac{1}{2}\right)\lambda$ (d) $(m+\lambda)$

10. Which parameter of light does not change when light is reflected from dense medium to rare medium?

(a) Frequency (b) Wavelength (c) Velocity (d) Amplitude

11. In Michelson interferometer, a fringe is changed by changing the position of movable mirror at a distance

(a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$ (c) λ (d) 2λ

12. Number of slits in a diffraction grating depends upon.

(a) Speed of light (b) Frequency of light
(c) Wavelength of light (d) Amplitude of light

13. If the wavelength of the incident x-rays is 2×10^{-10} m then the required number of slits per centimetre for its diffraction should be

(a) 5×10^{6} (b) 5×10^{7} (c) 5×10^{8} (d) 5×10^{10}

14. Which phenomenon has confirmed that light is transverse wave?

(a) Interference (b) Diffraction (c) Reflection (d) Polarization

15. If the unpolarized incident light with intensity '*I*' is polarized by a Polaroid sheet, then the intensity of plane polarized light will be;

(a) $\frac{I}{2}$ (b) $\sqrt{I}$ (c) I (d) 2I

16. Which one of the following cannot be polarized.

(a) X-rays (b) Radio waves
(c) Ultraviolet waves (d) Sound waves

SHORT QUESTIONS

1. How can you define light?
2. Does the ether exist as proposed by Huygen for wave theory?
3. Under what condition, the spherical wave fronts are formed?
4. What is Huygen's principle?
5. What are the conditions for interference of light?
6. How coloured fringes are obtained on soap bubble?
7. How many phenomenons are there in the favour of wave theory of light?

8. What is the cause of changing the position of bright and dark fringes in interference by thin film?
9. What is the difference between interference and diffraction?
10. How diffraction pattern is obtained on the screen by using principle of superposition?
11. Why diffraction of x-rays is possible only by a crystal?
12. What are Polaroid and polarizer?
13. How can polarized light be detected?
14. Can visible light produce interference fringes?
15. In the Young's double slit experiment, one of the slits is covered with blue filter and other with red filter. What would be the pattern of light intensity on the screen?
16. State whether the fringe width for bright and dark fringes in Young's interference is always constant?
17. Find the grating element of the diffraction grating containing 2000 lines/cm?

COMPREHENSIVE QUESTIONS

1. What is meant by wave front? Under what conditions the spherical and plane wave fronts are formed.
2. What is the Huygens's principle? Explain that how can you obtain the secondary wave front by the primary wave front.
3. State and explain the Young's double experiment for the interference of light. Also discuss the position and width of bright and dark fringes.
4. Explain the interference phenomenon in thin film and derive its mathematical formula for a constructive and destructive interference.
5. State and explain Michelson interferometer and it working principle in the determination of wavelength of bright fringes.
6. What is diffraction of light? Explain the diffraction of light due to a single slit.
7. State and explain diffraction grating and its working principle.
8. Discuss the diffraction of x-rays by a crystal and explain Bragg's law.
9. What do you know about the polarization? State and explain the production and detection of polarization of light.

NUMERICAL PROBLEMS

1. Light of wavelength 400 nm is allowed to illuminate the slits of Young's experiment. The separation between the slits is 0.10 mm and the distance of the

screen from the slits where interference effects are observed is 20 cm. At what angle the first minimum will fall? What will be the linear distance on the screen between adjacent maxima? **(0.11°, 0.8mm)**

2. In a double slit interference experiment, the distance between the slits is 2 mm and the fringe spacing is 0.45 mm on a screen which is 200 cm away from the slits. Find the wavelength of the light. **(450 nm)**

3. Interference fringes were produced by two slits, spaced 0.2 mm apart, on a screen at a distance of 150 cm from the slits. The third bright fringe is found to be displaced 7.5 mm from the central fringes. What is the wavelength of light producing the fringes? **(333 nm)**

4. Green light of wavelength 540 nm is diffracted by grating having 2000 lines/cm. (a) Compute the angular deviation of the third order image. (b) Is 12^{th} order image possible? **(18.9°, impossible)**

5. Sodium light of wavelength 590 nm is incident normally on a grating having 600 lines per millimetre. What is the highest order of the spectrum obtained with the grating? **(2)**

6. How many fringes will pass a reference point if the moveable mirror of Michelson's interferometer is moved through a distance 0.07 mm using light of wavelength 580 nm? **(241)**

7. In a Michelson interferometer, 100 fringes cross the field of view when the movable mirror is displaced by 0.02948 mm, calculate the wavelength of the monochromatic source. **(5896 Å)**

8. Calculate the distance through which the mirror of the Michelson interferometer has to be displaced between two consecutive positions of maximum distinctness of D_1 and D_2 lines of sodium. Wavelength of D_1 line is 5890Å and of D_2 line is 5896Å. **(0.2894 mm)**

9. X-rays of wavelength 1.50 nm are observed to undergo a second order reflection at a Bragg's angle of 15° from a quartz (SiO_2) crystal. What is the interplanar spacing of the reflecting planes in the crystal? **(5.79 nm)**

Unit 10

THERMODYNAMICS

Major Concepts (22 PERIODS)

- Thermal equilibrium
- Heat and work
- Internal energy
- First law of thermodynamics
- Molar specific heats of a gas
- Heat engine
- Second law of thermodynamics
- Carnot's cycle
- Refrigerator
- Entropy

Conceptual Linkage

This chapter is built on
Heat Physics IX
Thermo Chemistry XI

Students Learning Outcomes

After studying this unit, the students will be able to:

- Describe that thermal energy is transferred from a region of higher temperature to region of lower temperature.
- Describe that regions of equal temperatures are in thermal equilibrium.
- Describe that heat flow and work are two forms of energy transfer between systems and calculate heat being transferred.
- Define thermodynamics and various terms associated with it.
- Relate a rise in temperature of a body to an increase in its internal energy.
- Describe the mechanical equivalent of heat concept, as it was historically developed, and solve problems involving work being done and temperature change.
- Explain that internal energy is determined by the state of the system and that it can be expressed as the sum of the random distribution of kinetic and potential energies associated with the molecules of the system.
- Calculate work done by a thermodynamic system during a volume change.
- Describe the first law of thermodynamics expressed in terms of the change in internal energy, the heating of the system and work done on the system.
- Explain that first law of thermodynamics expresses the conservation of energy.
- Define the terms, specific heat and molar specific heats of a gas.
- Apply first law of thermodynamics to derive $C_p - C_v = R$.
- State the working principle of heat engine.
- Describe the concept of reversible and irreversible processes.
- State and explain second law of thermodynamics.

- Explain the working principle of Carnot's engine
- Explain that the efficiency of a Carnot engine is independent of the nature of the working substance and depends on the temperatures of hot and cold reservoirs.
- Describe that refrigerator is a heat engine operating in reverse as that of an ideal heat engine.
- Derive an expression for the coefficient of performance of a refrigerator.
- Describe that change in entropy is positive when heat is added and negative when heat is removed from the system.
- Explain that increase in temperature increases the disorder of the system.
- Explain that increase in entropy means degradation of energy.
- Explain that energy is degraded during all natural processes.
- Identify that system tend to become less orderly over time.

INTRODUCTION

In the past, the scientists believed in the caloric theory of heat. According to this theory, heat is a fluid called caloric which flows from the hot body to the cold body. After the development of kinetic theory, it has become a well known fact that heat is a form of energy called thermal energy and it flows from the hot body to the cold body till the two bodies attain the same temperature. This state of the same temperature of the bodies is called thermal equilibrium.

The kinetic theory also explains the random motion of atoms and molecules of matter. Such motion of atoms and molecules depends upon the temperature. The sum of kinetic energies and potential energies of the moving atoms and molecules of a substance is called its internal energy.

Thermodynamics is the study of heat energy and its transformation into other forms of energy and vice versa. It is an experimental science based on the study of the behaviour of solids, liquids and gases using the concepts of heat and temperature. In this chapter, thermodynamics can be explained under the following two laws. The first law of thermodynamics is based upon law of conservation of energy and it deals with the relationship between work and heat energy, that is, how the heat energy is converted into the other forms of energy and vice versa. The second law of thermodynamics explains not only the proper method of conversion of heat energy into mechanical work but also a specific direction of flow of heat.

We will study the efficiency of a heat engine, theory of Carnot engine, Carnot theorem, working of a refrigerator and the concept of entropy in this chapter.

10.1 THERMAL EQUILIBRIUM

We have studied mechanical equilibrium in unit 2 titled as 'Vectors and Equilibrium'. But in this chapter we deal with thermal equilibrium. The thermal equilibrium can be explained by an example of two bodies at different temperatures which are made in thermal contact. If both the bodies are good conductor of heat, then there is a transfer of heat energy from the hotter body to the cooler body as shown in Fig. 10.1.

In other words, the hotter body loses thermal energy whereas cooler body gains thermal energy. This process of transfer of heat energy continues till both the bodies attain the same temperature. This state of the same temperature of the bodies is termed as thermal equilibrium which is stated as,

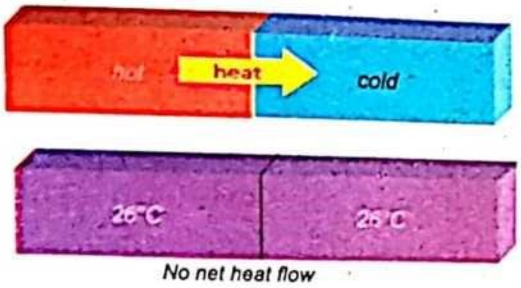


Fig.10.1: A state of Thermal Equilibrium of the two bodies having same temperature and there is no transfer of heat between them.

The bodies are said to be in thermal equilibrium when they have same temperature and there is no transfer of heat energy between them

For example, if we feel that we have fever, we might place a thermometer in our mouth and wait for a few minutes. There is transfer of heat energy between the thermometer and our body. Because our body is hot as compared to the thermometer, therefore, the reading of the thermometer increases. After some time, the rate of transfer of energy between the thermometer and our body becomes equal and our body and the thermometer are then at the same temperature. At this point our body and the thermometer are said to be in the thermal equilibrium.

10.2 INTERNAL ENERGY

All matter is made up of atoms and molecules. According to the kinetic theory, these atoms and molecules are always in motion. For example, atoms in solids vibrate back and forth about their equilibrium positions. In liquids, the molecules wander around the other molecules. In gases, the molecules are in random motion with high speeds and have frequent elastic collisions with one another. The motion may be translational, vibrational and rotational.

POINT TO PONDER

Normal temperature of a man is 37°C. What will be the temperature of a dead body?

The atoms and molecules of a gas possess both kinetic energy due to their translational, rotational and vibrational motion and potential energy associated with the forces between molecules.

Thus, the internal energy of the gas is defined in terms of the sum of the random distribution of kinetic and potential energies of its atoms or molecules.

It is stated as the sum of all kinds of kinetic and potential energies of the system, comprising of gas molecules, is called its internal energy

In the study of thermodynamics, usually ideal gas (mono-atomic gas) is considered as a working substance. The molecules of an ideal gas do not exert inter molecular forces, therefore, its molecules do not possess potential energy. So, the internal energy of an ideal gas is only due to the translational kinetic energy of the molecules. In case of diatomic gas, the molecules possess transitional, vibrational and rotational kinetic energy as well as potential energy. Therefore, the internal energy of dia-atomic gas is a sum of all kinds of kinetic and potential energies of their molecules.

10.2.1 Thermodynamics Systems

Thermodynamics is a branch of physics in which we study about the heat energy and its conversion into other forms of energy and vice versa. For example, the conversion of heat energy into mechanical. This conversion of energy from one form to another can be studied by making a separate environment within a boundary has its own specific values, as shown in Fig.10.2. The boundary may enclose a solid, liquid or gas. This is called system and defined as; **The collection of matter within a distinct boundary is called system.**

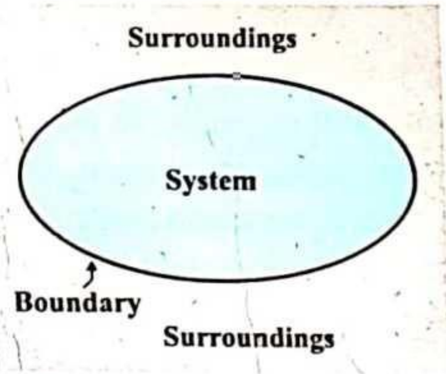


Fig.10.2: A thermodynamics system which is surrounded by a distinct boundary.

Everything outside the boundaries of the system which has a direct bearing on its behaviour is known as surrounding. A system may have the potential to exchange energy with its surrounding. Because the values of thermodynamics variables pressure, volume and temperature of a system are different from the values of the surrounding.

There are three types of systems as shown in Fig.10.3. These are explained as;

(i) Open System

A system which can interact with its surroundings both in terms of heat energy and matter is called open system e.g. plants and animals.

(ii) Closed System

A system which can interact with its surroundings only in terms of heat energy is called closed system e.g. balloon and cylinder.

(iii) Isolated System

A system which has no interaction with its surroundings in terms of heat energy and matter is called isolated system e.g. thermos flask.

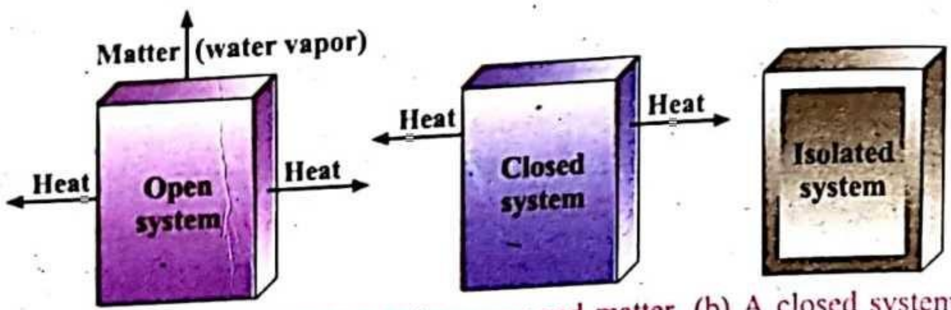


Fig.10.3: (a) An open system which transfer both energy and matter, (b) A closed system which transfers only energy, (c) An isolated system which transfers neither energy nor matter.

10.3 WORK AND HEAT

Work and heat are two different quantities but they are related to each other. A heat engine is a good example to explain such relationship between work and heat. For example, in heat engine, heat energy serves as input and its output is work done. Similarly, when an amount of heat energy in a heat engine enters into a thermodynamics system, it increases the internal energy of this system and work is done by the system. The work done by the system is considered as positive, while work done on the system is considered as negative.

A British engineer Count Rumford observed that a large amount of heat was liberated during boring the barrel of cannon. He concluded from this experiment that heat can also be produced by friction and its amount depends upon the mechanical work against the friction. Later on Joules did a series of experiments and established a relationship between mechanical work and heat energy. According to his results, work done is directly proportional to the amount of heat generated.

$$W \propto Q$$

$$W = J\,Q$$

where J is a mechanical equivalent of heat and its value is 4.186 joule per caloric.

Now we study further the relationship between work and heat by an example of a system (Cylinder) which contains gas at pressure P. The cylinder has a moveable piston of cross-sectional area 'A' and at equilibrium state. The volume occupied by gas is 'V'.

When the pressure is reduced, the volume of the system is increased from V_1 to V_2 at a distance 'Δx' as shown in Fig.10.4. It means that work is done by the system which is given as;

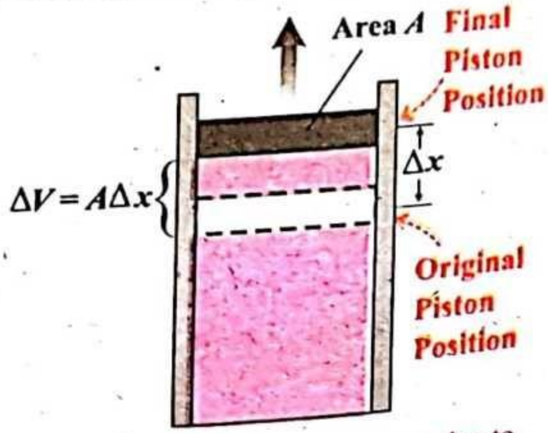


Fig.10.4: Work done by a system due to the expansion of volume by reducing the pressure.

$$W = F\Delta x$$

But $F = PA$

$$W = PA\Delta x$$

$$W = P\Delta V \quad \because \Delta V = A\,\Delta x = \text{(Change in volume)}$$

$$W = P(V_2 - V_1) \quad \text{......(10.1)}$$

This is a work done by the system. The work done at constant and at variable pressure can be represented graphically on a PV-diagram in the form of a straight horizontal line and a curved line respectively as shown in Fig.10.5. The area under these lines is equal to work done on the system.

Thus, these graphs show that the work done on or by the system depends upon the limiting values and the path followed.

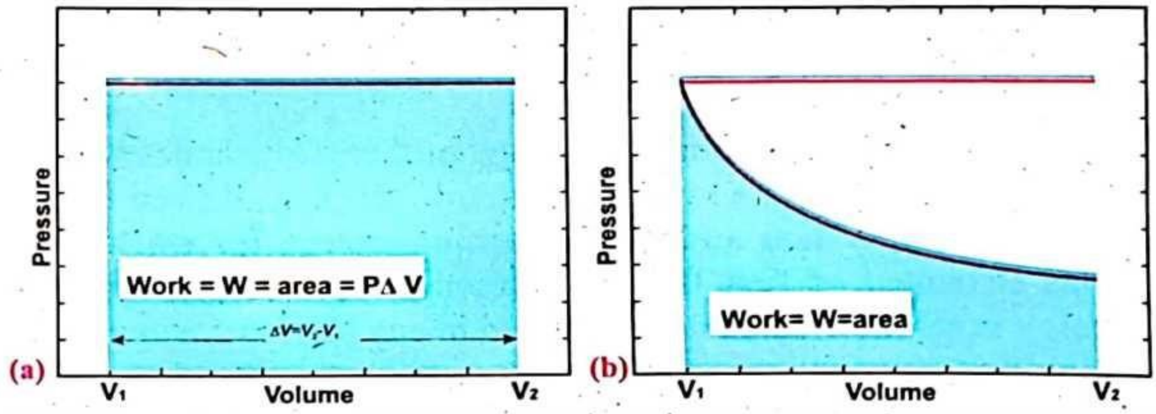


Fig.10.5: Thermodynamics work (a) The area under the straight line in PV-graph shows the work done at constant pressure (b) The area under a curved line shows the work done at variable pressure

Example 10.1

How much work is done by an ideal gas during expansion from its initial volume of 4 litres to a final volume of 24 litres at constant pressure of 8.08×10^5 Nm^{-2}?

Solution:

$$\text{Work} = W = ?$$

$$\text{Initial Volume} = V_1 = 4 \text{ litres} = 4\times10^{-3}\,m^3$$

$$\text{Final Volume} = V_2 = 24 \text{ litres} = 24\times10^{-3}\,m^3$$

$$\text{Change in volume} = \Delta V = V_2 - V_1$$

$$\text{Change in volume} = \Delta V = 20\times10^{-3}m^3$$

$$\text{Pressure} = P = 8.08\times10^5 Nm^{-2}$$

$$\text{Work} = P\Delta V$$

$$\text{Work} = 8.08\times10^5\times20\times10^{-3}$$

$$= 16.2\times10^3\,J$$

$$= 16.2\,kJ$$

10.4 FIRST LAW OF THERMODYNAMICS

First law of thermodynamics is based upon the law of conversion of energy i.e. when heat energy is transformed into other forms of energy or when the other forms of energy is transformed into heat energy, then the total energy of the closed system remains constant. Let heat energy 'Q' is added into a thermodynamic system and the system does not work during the process of transfer of heat but the internal energy of the system increases from its initial state $\mathbf{U_i}$ to its final state $\mathbf{U_f}$. This change in internal energy ΔU is equal to **Q**, that is, $\Delta U = Q$.

When a system does work 'W' due to its expansion but no heat is added during the process then internal energy decreases that is, when 'W' is positive, ΔU is negative and vice versa. Now when heat transfers and both change in internal energy and work done by the system occur then the heat Q is given as;

$$Q = \Delta U + W \quad ......(10.2)$$

This is the mathematical form of first law of thermodynamics which is stated as; **"When heat Q is added to a system, a part of this heat is used to change the internal energy of the system and the remaining energy for work done by the system"**. This statement of first law of thermodynamics also provides universal truth that energy is neither created nor destroyed in any thermodynamic system.

In using first law of thermodynamics, a proper sign should be used. That is, Q is taken as positive when heat energy is supplied to the system and negative when heat energy is taken from the system. Similarly, W is taken as positive when the work is done by the system and negative when it is done on the system.

Example 10.2

When 400 J heat is transferred to the system during expansion then 350 joules of work is done by the system. What is the change in its internal energy?

Solution:

Heat energy added to system $= \Delta Q = 400$ J

Work done by the system $= \Delta W = 350$ J

Internal energy $= \Delta U = ?$

From first law of thermodynamics $\Delta Q = \Delta V + \Delta W$

$$\Delta U = \Delta Q - \Delta W$$

$$\Delta U = 400 - 350$$

$$\Delta U = 50\,J$$

10.4.1 Applications of first law of thermodynamics

The first law of thermodynamics can be studied under the following four processes; each has its different conditions and properties.

(a) Isochoric Process

It is a process in which the volume of gas of the given system remains constant.

Consider a finite volume of gas enclosed in cylinder which has non-conducting walls and piston but a conducting base. Let an amount of heat 'Q' is supplied to the system. When gas is heated at constant volume (fixed piston) its pressure increases from P_1 to P_2 but the work is neither done by the system nor on the system because there is no expansion or compression of the system that is $\Delta V = 0$ as shown in Fig.10.6.

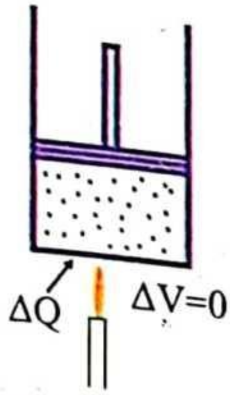


Fig.10.6: Isochoric Process, when volume remains constant and no work is done on or by the system.

Thus the first law of thermodynamics becomes,

$$\Delta U = \Delta Q + \Delta W$$
$$\Delta Q = \Delta U + P\Delta V$$

As $\Delta V = 0$ therefore

$$\Delta Q = \Delta U \quad ......(10.3)$$

This result shows that under isochoric process all the supplied energy is used to increase the internal energy of the system.

Graphically, a straight vertical line in P–V graph represents first law of thermodynamics under isochoric process as given in Fig. 10.7.

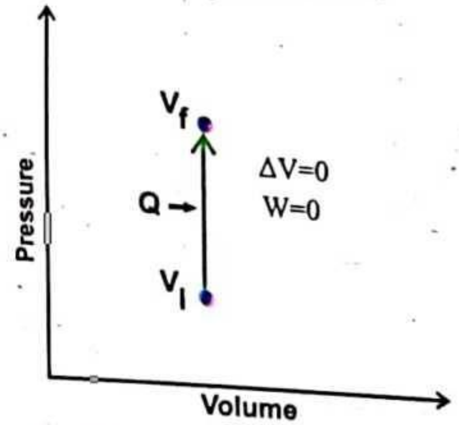


Fig.10.7: A straight vertical line in P-V graph shows isochoric process

(b) Isobaric Process

A process in which the pressure of the gas of the given system remains constant is known as isobaric process.

Consider a finite volume of gas is enclosed in cylinder with moveable piston. The walls and piston of the cylinder are insulator whereas its base is conductor as shown in Fig.10.8. Let an amount of heat 'Q' is added to the system at constant pressure then its temperature increases from T_1 to T_2. As a result, gas is expanded from V_1 to V_2 due the increase in its internal energy. If the piston moves slowly and the displacement of the piston is kept very small, the pressure of the gas will not change much and can be considered constant, and some work is done by the system. Thus first law of thermodynamics under these conditions becomes,

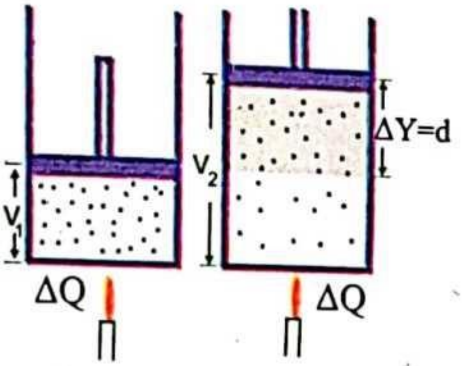


Fig.10.8: Isobaric Process, where pressure remains constant and work is done on the system.

$$\Delta Q = \Delta U + \Delta W$$

As
$$W = Fd$$
$$W = F\Delta y$$
$$W = PA\Delta y = P\Delta V \qquad \because \Delta V = A\,\Delta y = \text{change in volume}$$
$$\Delta Q = \Delta U + P\Delta V \;......(10.4)$$

Under isobaric process, all the supplied heat energy of the system is converted into work done and increase in internal energy.

Graphically, a straight horizontal line in P–V graph represents first law of thermodynamics under isobaric process as shown in Fig.10.9. The P–V graph of isobaric process is called Isobar.

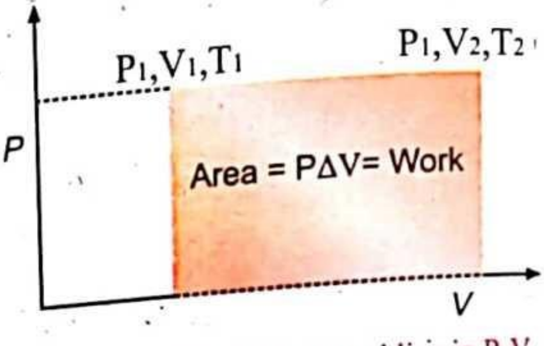


Fig.10.9: A straight horizontal line in P-V graph shows isobaric process

(c) Isothermal Process

A process in which the temperature of gas of the given system remains constant is known as isothermal process.

Consider a gas which is enclosed in a cylinder that has non-conducting walls and piston but its base is conducting as shown in Fig. 10.10. When heat energy ΔQ is added to the system then the temperature of the gas increases. To keep temperature constant, the system is allowed to expand slowly from V_1 to V_2.

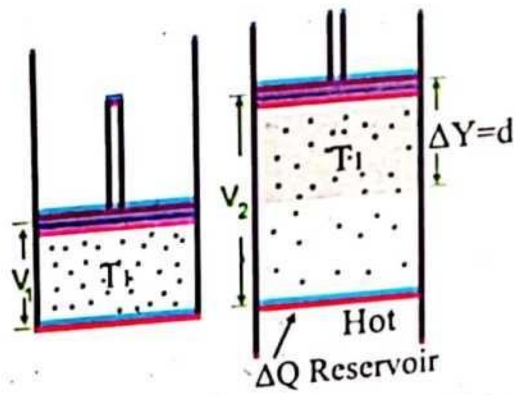


Fig.10.10: Isothermal Process in which temperature remains constant

The internal energy of the gas does not change during this isothermal expansion, as the temperature of the gas remains constant. So, $\Delta U = 0$ and hence at constant temperature, the first law of thermodynamics is written as;

$$\Delta Q = \Delta U + \Delta W$$
$$\Delta Q = P\Delta V \;......(10.5)$$

This equation shows that the heat energy supplied appears in the form of work. Since the work is done by the system, so ΔW is positive.

Graphically, a curved line called isotherm represents the first law of thermodynamics under isothermal process as shown in Fig. 10.11.

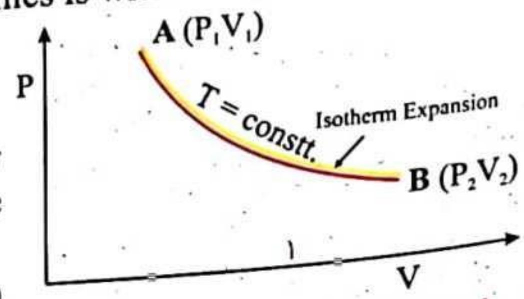


Fig.10.11: A curved line in P-V graph represents the isothermal process.

Conversely, if the gas is compressed, the work is being done on the system and an amount of heat ΔQ has to be allowed to leave the system.

(d) Adiabatic Process

A process in which heat energy neither enters nor leaves the system is called adiabatic process.

Considering a gas which is enclosed in a cylinder that has non-conducting walls and piston as shown in Fig. 10.12. When the system is placed on an insulator stand then there is no transfer of heat into or out of the system, that is $\Delta Q = 0$.

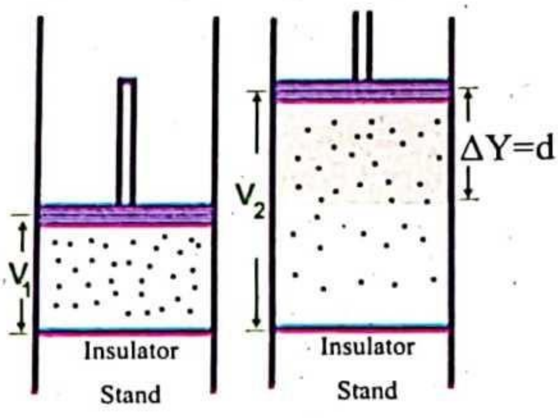


Fig.10.12: Adiabatic Process in which heat energy is not transferred.

Now when the system is allowed to expand by reducing the pressure the internal energy of the system decreases due to decrease in temperature. Similarly, when the system is compressed by applying pressure the internal energy of the system increases by increasing the temperature. Now first law of thermodynamics under the adiabatic process becomes

$$0 = \Delta U + \Delta W$$

$$\Delta U = -\Delta W$$

$$\Delta U = -P\Delta V \quad ......(10.6)$$

In adiabatic process, the work is done at the cost of internal energy of the system. In other words, if the gas expand it will be cooled and it will be heated on compression. Graphically, a curved line in P–V graph shows adiabatic process, but the curve of adiabatic is steeper than that of the curve of isothermal as shown of Fig.10.13, because of the rapid variations in temperature in adiabatic process take place during its expansion or compression.

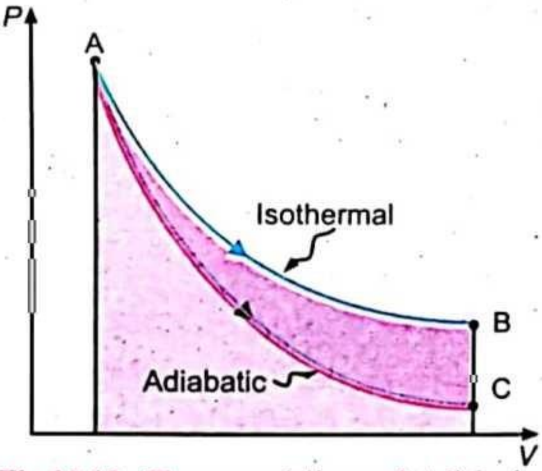


Fig.10.13: The curved lines of Adiabatic and Isothermal P-V graph, where the cure of Adiabatic is faster.

Example 10.3

A gas is enclosed in a cylinder with a moveable piston of cross-section area $0.1\ m^2$. If the piston of the cylinder is allowed to expand through a distance of 5 cm by adding heat energy of 45 J to the gas, then what is the change in internal energy inside the cylinder at constant pressure of $8000\ Nm^{-2}$.

Solution:

Cross-sectional area of the piston = $A = 0.1\ m^2$

Distance covered by the piston = $\Delta x = 5\ cm = 5\times10^{-2}\ m$

Heat energy added to system = $\Delta Q = 45\ J$

Internal energy = $\Delta U = ?$

Pressure = $P = 8000\ Nm^{-2}$

Work done by the system

$$Work = P\Delta V$$

$$Work = PA\Delta x$$

$$Work = 8000 \times 0.1 \times 5 \times 10^{-2}$$

$$Work = 40\ J$$

According to first law of thermodynamics

$$\Delta U = \Delta Q - \Delta W$$

$$\Delta U = 45 - 40$$

$$\Delta U = 5\ J$$

10.5 SPECIFIC HEAT AND MOLAR SPECIFIC HEAT

If a substance is heated then its temperature raises. This raise in temperature of a substance depends upon its mass i.e., more is the mass of the substance, more heat is required to raise its temperature. Thus, **the amount of heat energy required to raise the temperature of any substance through a unit degree is called heat capacity**. The experiment shows that the heat energy is directly proportional to the temperature, that is

$$\Delta Q \propto \Delta T$$

$$\Delta Q = C\Delta T$$

$$\text{Heat capacity (C)} = \frac{\Delta Q}{\Delta T} \quad ......(10.7)$$

Molar Specific Heats of Various Gases

Gas	C_P	C_V	C_P-C_V	$\gamma = C_P/C_V$
	Molar Specific Heats (J/mol. K)			
Monoatomic gases				
He	21	13	8.33	1.67
Ar	21	13	8.33	1.67
Ne	21	13	8.12	1.64
Kr	21	12	8.49	1.69
Diatomic gases				
H_2	29	20	8.33	1.41
N_2	29	21	8.33	1.4
O_2	29	21	8.33	1.4
CO_2	29	21	8.33	1.4
Cl_2	35	26	8.96	1.4
Polyatomic gases				
CO_2	37	29	8.5	1.3
SO_2	40	31	9	1.29
H_2O	35	27	8.37	1.3
CH_2	36	27	8.41	1.31

Some materials are easier to heat than the others. For example, it takes more energy to raise the temperature of 1kg of aluminum through 1°C, than to raise the temperature of 1kg iron by the same temperature. Therefore, **the amount of heat required to raise the temperature of unit mass of a substance through a unit degree is called specific heat** and its value can be calculated as;

$$\Delta Q \propto \Delta T$$

$$\Delta Q \propto m$$

$$\Delta Q \propto m\Delta T$$

$$\Delta Q = cm\Delta T$$

$$c = \frac{\Delta Q}{m\Delta T} \quad ......(10.8)$$

The unit of specific heat 'c' is $J kg^{-1} K^{-1}$. As one kilogram mass of different substances contains different number of molecules and has different specific heats so mass of substance is replaced by mole because one mole of any substance contains the same number of molecules. Thus the specific heat capacity in term of moles is known as molar specific heat. It is stated as; **"the amount of heat energy required to raise the temperature of one mole of a substance through one kelvin is called molar specific heat"**. The equation for molar specific heat is given by,

$$C = \frac{\Delta Q}{n\Delta T} \quad ......(10.9)$$

where 'n' is the number of moles of the given sample.

In case of solids and liquids, the change in their volume and pressure due to increase in temperature are very small and can be neglected. In case of gases the situation is different, because there is variation in pressure as well as in volume of gas with the raise in temperature. To study the heating effect of the gas, either volume or pressure of the gas should be constant. Thus, molar specific heat of a gas is defined under the following two ways.

1) Molar specific heat of gas at constant volume

The amount of heat required to raise the temperature of one mole of gas through 1 K at constant volume is called its molar specific heat at constant volume. It is denoted by C_V and it is expressed as;

$$C_V = \frac{\Delta Q}{n\Delta T} \quad ......(10.10)$$

2) Molar specific heat of gas at constant pressure

The amount of heat required to raise the temperature of one mole of gas through 1 K at constant pressure is called its molar specific heat at constant pressure. It is denoted by C_P and it is expressed as;

$$C_P = \frac{\Delta Q}{n\Delta T} \quad ......(10.11)$$

10.5.1 Prove that $C_P - C_V = R$

Consider two systems that contain equal amount of gas but heat is added to first at constant volume and heat is added to the second at constant pressure as shown in Fig. 10.14. When both systems are heated to raise their temperatures then the heat energy increases their internal energy. But the experiment shows that the system at constant pressure requires more heat than the system at constant volume for the same temperature increase. That is,

$$\Delta Q_P > \Delta Q_V$$

$$\Delta Q_P = \Delta Q_V + \text{Work done due to expansion of the system}$$

$$nC_P\Delta T = nC_V\Delta T + P\Delta V$$

As $$PV = nRT$$

$$P\Delta V = nR\Delta T$$

$$nC_P\Delta T = nC_V\Delta T + nR\Delta T$$

$$nC_P\Delta T = n\Delta T(C_V + R)$$

$$C_P = C_V + R$$

$$C_P - C_V = R \text{......(10.12)}$$

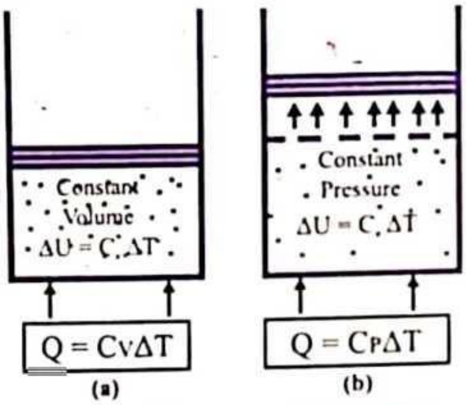


Fig.10.14: Heat transfer into the system (a) at constant volume and (b) at constant pressure

Equation (10.12) shows that $C_P > C_V$ by an amount equal to universal gas constant 'R'.

Example 10.4

The temperature of a silver bar rises by 10°C when it absorbs 1.23 kJ of heat energy. The mass of the bar is 525 g. Determine the specific heat of the silver.

Solution:

$$\Delta T = 10^\circ C - 0^\circ C = 10^\circ C$$

$$\Delta Q = 1.23 \text{ kJ} = 1.23 \times 10^3 \text{ J}$$

$$m = 525 \text{ g} = 0.525 \text{ kg}$$

$$c_{silver} = ?$$

$$c_{silver} = \frac{\Delta Q}{m\Delta T}$$

$$c_{silver} = \frac{1.23\times10^3}{0.525\times10} = 234 \text{ Jkg}^{-1}\text{C}^{o-1}$$

10.5.2 Adiabatic Equation

Consider a system (container) which contains a gas. The walls and moveable piston of the container are perfectly insulated from the surroundings. So heat energy neither enters nor leaves the system and $\Delta Q = 0$. It means the work done under adiabatic process is possible only at the cost of its internal energy during the compression and expansion of the system. Thus, 1st law of thermodynamics under adiabatic process is given by;

$$0 = \Delta U + P\Delta V$$

$$\Delta U = -P\Delta V \quad ......(10.13)$$

According to molar specific heat at constant volume

$$\Delta Q = nC_v\Delta T$$

But 1st law of thermodynamics under isochoric process

$$\Delta Q = \Delta U$$

$$\Delta U = nC_v\Delta T$$

Putting the value of ΔU in equation (10.13), we get

$$nC_v\Delta T = -P\Delta V \quad ......(10.14)$$

As $$PV = nRT$$

$$\Rightarrow V\Delta P = nR\Delta T \text{ (at constant volume)}$$

$$\Rightarrow \Delta T = \frac{V\Delta P}{nR}$$

Substitute this value of ΔT in equation (10.14)

$$nC_v \cdot \frac{V\Delta P}{nR} = -P\Delta V$$

$$\frac{\Delta P}{P} = -\frac{R}{C_v} \times \frac{\Delta V}{V}$$

POINT TO PONDER

Why there is space between the two walls of a thermo flask?

But, $$C_P - C_v = R$$

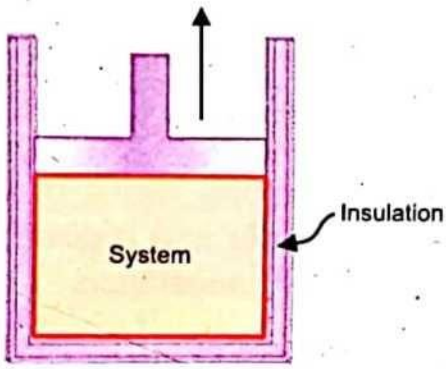
$$\frac{\Delta P}{P} = -\left(\frac{C_P - C_v}{C_v}\right)\frac{\Delta V}{V}$$

$$\frac{\Delta P}{P} = -\left(\frac{C_P}{C_v} - 1\right)$$

But, $$\because \frac{C_P}{C_v} = \gamma = \text{Ratio of specific heats}$$

$$\frac{\Delta P}{P} = -(\gamma - 1)\frac{\Delta V}{V}$$

$$\frac{\Delta P}{P} = -\gamma\frac{\Delta V}{V} + \frac{\Delta V}{V}$$



An diabatic process is that process in which the heat energy neither enters not leaves the system..

As $\frac{\Delta V}{V}$ is very small so it can be neglected.

$$\frac{\Delta P}{P} = -\gamma\frac{\Delta V}{V}$$

$$\frac{\Delta P}{P}+\gamma\frac{\Delta V}{V}=0$$

By the operation of integration, the following result is obtained as;

$$\therefore \quad PV^{\gamma}=\text{Constant} \quad \text{......(10.15)}$$

This is known as adiabatic equation and 'γ' is adiabatic constant.

10.6 REVERSIBLE AND IRREVERSIBLE

When a thermodynamic process is operated by changing the values of the given system then there are two possibilities, the state of the system remains same due to the reverse direction of the process after the succession of an event or the state of the system changes due to proceeding the process in one direction. Thus on the basis of these two reasons, a process of thermodynamics is defined under the following two ways such as reversible and irreversible process.

1) Reversible Process

When a thermodynamics system operates such that a change takes place and it returns to its initial state after a certain fixed interval of time then the process is called a reversible process.

For example, motion of a piston in a heat engine when it completes one cycle under four steps is a reversible process as shown in Fig.10.15. Similarly, the conversion of ice into water then water into ice is also an example of the reversible process. In other word, a reversible process is one which can be retraced in exactly reverse order and it does not produce any change in the surrounding if the work is done by the substance.

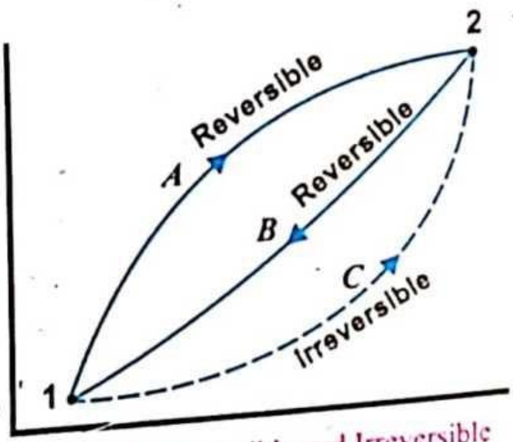


Fig.10.15: Reversible and Irreversible process in one cycle

2) Irreversible Process:

When a thermodynamic system operates such that its function changes from its initial value to its final value but does not returns to its initial value then the process is called irreversible process. For example, the flow of heat from a hot body to a cold body is an irreversible process because heat never flow from a cold body to a hot body. Similarly, burning of fuel and burst tyre are also irreversible process.

10.7 HEAT ENGINE

A heat engine is an important device which converts heat energy into mechanical energy or work. In the beginning a steam heat engine was introduced. In this engine water was boiled in a vessel called steam boiler and engine took heat from this steam boiler and converted a part of it into work. Now in the present age, petrol engines and diesel engines are being used at large scale. These engines consist of a cylinder which contains a gas such as air with a moveable piston, hot reservoir (source) and cold reservoir (sink).

Working of heat engine

A schematic diagram of a heat engine is shown in Fig. 10.16. It shows that a heat engine that works between hot and cold reservoirs. That is, the engine gains heat energy from a source at a high temperature and converts a part of this heat energy into a mechanical work and the remaining part of energy is rejected through a sink as shown in Fig.10.16.

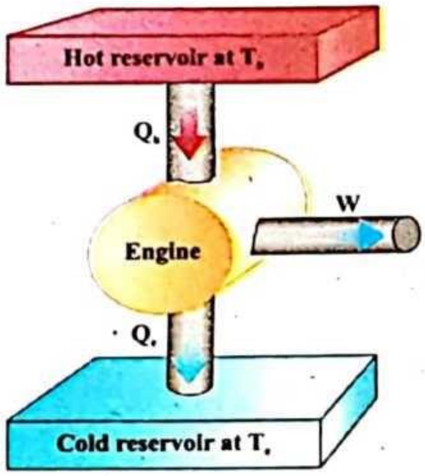


Fig.10.16: Schematic diagram of heat engine, where engine is working between hot and cold reservoirs

In order to get a continuous steady mechanical energy, the heat engine is made to operate in a cyclic process which absorbs heat Q_1 and rejects heat Q_2. The initial and final internal energies of the system under this cyclic process remain same.

Thus first law of thermodynamics becomes

$$\Delta Q = \Delta U - \text{Work}$$

But $$\Delta U = U_2 - U_1 = 0$$

$$Q_1 - Q_2 = \text{Work} \quad ......(10.16)$$

The efficiency of a heat engine is defined as the ratio between the work done by the engine to the supplied heat energy.

POINT TO PONDER

What is the main difference between steam and diesel engine?

$$\text{Efficiency}\ (\eta) = \frac{\text{Output}}{\text{Input}}$$

$$\text{Efficiency}\ (\eta) = \frac{\text{Work done}}{\text{Supplied energy}}$$

$$\text{Efficiency}\ (\eta) = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} \quad ...... (10.17)$$

This result shows that the efficiency of a heat engine depends upon the absorbed heat Q_1 and discarded heat Q_2. As heat energy is proportional to the temperature, so the efficiency of heat engine can also be expressed as;

$$\eta = 1 - \frac{T_2}{T_1} \quad ......(10.18)$$

If $T_2 = 0K$ (−273°C) called absolute temperature then the efficiency of heat engine would be 100% but this is an ideal or a theoretical case.

Example 10.4

Compute the maximum possible efficiency of a heat engine operating between the temperature limits of 150°C and 450°C.

Solution:

Efficiency (η) = ?

Initial temperature $= T_1 = 450°C = 450 + 273 = 723$ K

Final temperature $= T_2 = 150°C = 150 + 273 = 423$ K

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\eta = 1 - \frac{423}{723}$$

$$\eta = 1 - 0.585$$

$$\eta = 0.415$$

Percentage Efficiency $= \eta = 0.415 \times 100 = 41.5\%$

10.8 SECOND LAW OF THERMODYNAMICS

In first law of thermodynamics, we have discussed the conversation of heat energy into a useful work. Now second law of thermodynamics not only verifies the first law of thermodynamics, but it also explains the proper method of conversion of heat into mechanical work and the specific direction of flow of heat.

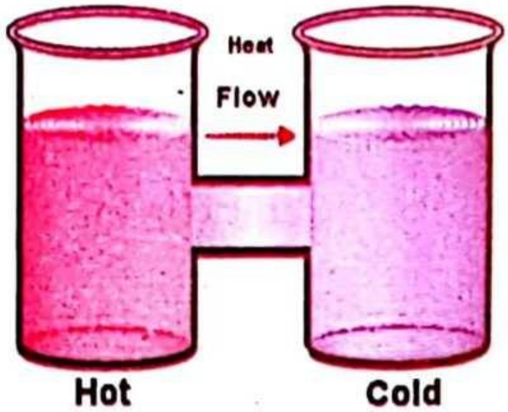


Fig.10.17: Two reservoirs at different temperature where, heat flows from hot body to cold body.

This law is based upon law of nature that is the experimental evidences about the nature show that water flow from higher level to lower level. Similarly, heat energy flows from hot body to cold body and work can be performed as shown in Fig. 10.17. It may be noted that it is impossible to have a cold reservoir at 0K. Based upon these notions, the second law of thermodynamics can be defined under the following two statements.

Lord Kelvin Statement

According to this statement, "it is impossible to construct a heat engine operating in a cycle which absorbs heat energy from a hot reservoir and converts it completely into the mechanical".

Rudolf Clausius Statement

This is another statement of the second law of thermodynamics. According to this statement "Heat energy cannot flow from cold body to hot body without expenditure of energy ". It can be studied in the working principle of refrigerator.

10.9 CARNOT ENGINE AND CARNOT CYCLE

In order to improve the efficiency of a heat engine, French military engineer Sadi Carnot in 1824 introduced a theoretical engine which is known as Carnot engine. It consists of a cylinder that contains gas with a moveable piston. The walls and piston of the cylinder are insulators but its base is conductor. It is assumed that there is no friction between walls and piston. The Carnot engine operates in a cycle known as Carnot cycle which is completed in four steps; two under isothermal and two under adiabatic conditions as shown in Fig.10.18. Here three parameters of gas i.e. P,V and T are considered which control the behaviour of gas contained in the cylinder.

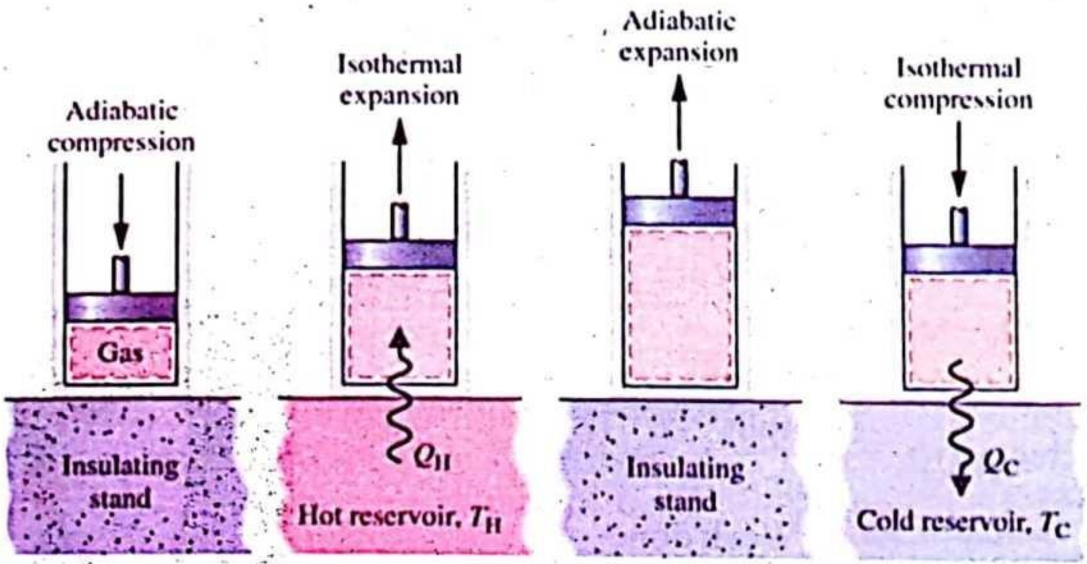


Fig.10.18: Schematic diagrams which explain the operation of a Carnot engine and it completes one cycle in four steps.

(i) Isothermal Expansion

When the system is allowed to expand by reducing the pressure on the piston then the temperature decreases but temperature should remain constant in isothermal process. Therefore, heat energy Q_1 is supplied from the hot reservoir.

Thus, the values of parameters change from $P_1V_1T_1$ to $P_2V_2T_1$. Graphically it is represented by curve AB in P-V graph as shown in Fig.10.19.

(ii) Adiabatic Expansion

When the system is further allowed to expand this time it follows adiabatic process. Therefore, the temperature decreases and thus the values of parameters change from $P_2V_2T_1$ to $P_3V_3T_2$. It is represented by curve BC in Fig.10.19.

(iii) Isothermal Compression

After the expansion, when the system is compressed by applying the pressure on the piston then the temperature should increase but to keep temperature constant heat energy Q_2 is released to the cold reservoir and the values of parameters change from $P_3V_3T_2$ to $P_4V_4T_2$.

Graphically it is represented by curve CD in P–V graph as shown in Fig.10.19.

(iv) Adiabatic Compression

Finally, when the system is further compressed, increased and the reversible cycle is completed.

Thus, the system returns back to its initial stage $P_4V_4T_2$ to $P_1V_1T_1$. Graphically it is represented by curve

Carnot engine operates under reversible p equilibrium is maintained in the whole process by ab rejecting heat energy Q_2.

Thus the internal energy of the system remains c first law of thermodynamics becomes

$$\Delta Q = \Delta U + \text{Work}$$

$$Q_1 - Q_2 = \text{Work}$$

The efficiency of a Carnot engine is defined as:

$$\text{Efficiency}\ (\eta) = \frac{\text{Output}}{\text{Input}}$$

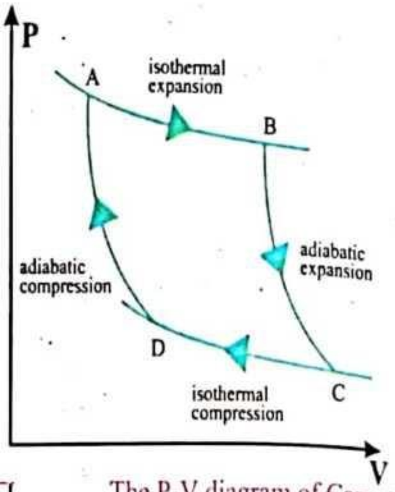
$$\text{Efficiency}\ (\eta) = \frac{\text{Work done}}{\text{Supplied energy}}$$

$$\text{Efficiency}\ (\eta) = \frac{Q_1 - Q_2}{Q_1}$$

$$\eta = 1 - \frac{Q_2}{Q_1} \quad \text{......(10.19)}$$

As heat energy is proportional to the temperature so efficiency in terms of temperature is given as;

$$\eta = 1 - \frac{T_2}{T_1}$$



T[The P-V diagram of Carnot engine executing a Carnot cycle

d, this time the temperature is

ge and the values change from ve DA in Fig.10.19.

process, therefore, thermal absorbing heat energy Q_1 and

s constant that is $\Delta U = 0$. Now

Nicolas Leonard Sadi Carnot (1796 – 1832)

French military engineer and hysicist known as "Father of hermodynamics" His excellent orks are Carnot cycle, Carnot eorem and Carnot heat engine.

The efficiency in percentage is given as

$$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100\% \quad ......(10.20)$$

This shows that the efficiency of a Carnot engine depends on temperature of hot and cold reservoirs.

It is clear from Eq.(10.20) that efficiency of Carnot engine will be 100 percent if either the temperature of hot reservoir is at infinity or the temperature of cold reservoir is 0K. These two conditions cannot be met experimentally. Hence, the efficiency of Carnot engine is always less than 100 percent and depends on the temperature of hot and cold reservoirs.

Carnot Theorem

After the drawing of the nomenclature of a heat engine, Carnot derived the following two results called Carnot's theorems. That is;

1) No heat engine can be more efficient than a reversible engine working between the same two temperatures.
2) All the reversible engines have the same efficiency when they are working between the same two temperatures.

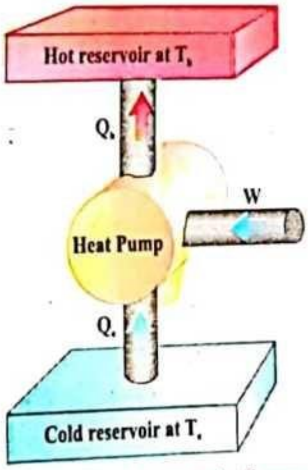


Fig.10.20: A Schematic diagram of a refrigerator which shows that the refrigerator is working between the cold and hot reservoirs.

10.10 REFRIGERATOR

It is a device which maintains the temperature of a body below that of its surrounding. It operates in a cyclic process but in reverse as that of the heat engine as shown in Fig.10.20.

A refrigerator absorbs heat from a cold reservoir and gives it off to a hot reservoir. This shows that in a refrigerator, the work is done on the system while in a heat engine work is done by the system.

A refrigerator consists of a compressor, condenser, expansion valve, exporter and a gas as the working substance which is called refrigerant as shown in Fig.10.21.

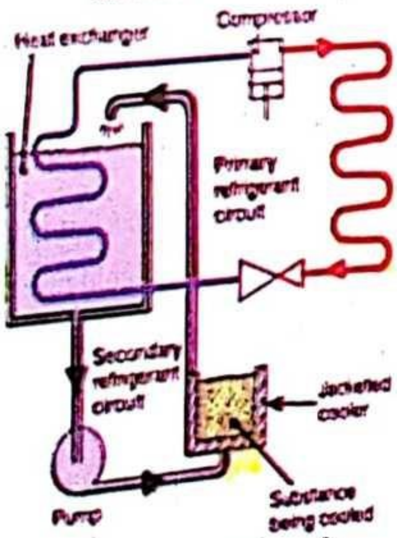


Fig.10.21: A cyclic process of a refrigerator under various steps.

The refrigerant at low pressure and at low temperature from the cold reservoir is compressed by the compressor and the compression is adiabatic. So both pressure and temperature are increased as compared to its surrounding. Now the refrigerant at high pressure and at high temperature passes through the condenser where it loses some of its heat to the surrounding and partly condenses to liquid. The refrigerant now expands adiabatically into the evaporator at a rate controlled by the expansion valve. This adiabatic expansion causes cooling of refrigerant in the evaporator coil, which is cooler than its surrounding. Finally, the refrigerant again enters the compressor to start the next cycle.

Refrigerator operates in a cyclic process; it takes heat energy Q_1 from cold reservoir and leaves heat energy Q_2 to hot reservoir due to work done on the system with constant internal energy $\Delta U = 0$. Now according to first law of thermodynamics

$$\Delta Q = \Delta U + \text{Work}$$

$$Q_2 - Q_1 = 0 + W \qquad \text{...... (10.21)}$$

Co-efficient of performance of a refrigerator is defined as the ratio of heat extracted from reservoir at low temperature to the work done on the system. That is

$$\text{Co-efficient of performance} = \frac{Q_1}{W}$$

$$\text{Co-efficient of performance} = \frac{Q_1}{Q_2 - Q_1} \qquad \text{...... (10.22)}$$

Co-efficient in terms of temperature, where $Q \propto T$

$$\text{Co-efficient of performance} = \frac{T_1}{T_2 - T_1} \qquad \text{......(10.23)}$$

Example 10.5

A refrigerator has a Co-efficient of performance 8. If temperature in the freezer is $-23°C$ then what is the temperature at which it rejects the heat?

Solution:

Co-efficient of performance = 8

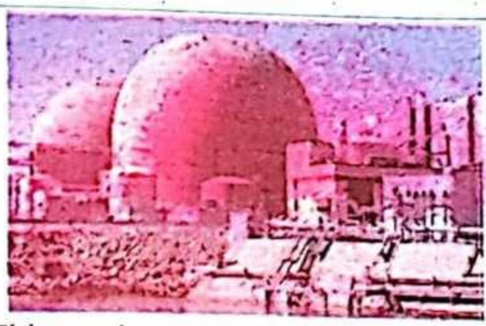
Temperature of cold reservoir (freezer) = $T_1 = -23°C = -23 + 273 = 250$ K

Temperature of hot reservoir (room) = T_2 = ?

$$\text{Co-efficient of performance} = \frac{T_1}{T_2 - T_1}$$

$$8 = \frac{250}{T_2 - 250}$$

$$8(T_2 - 250) = 250$$
$$8T_2 - 8 \times 250 = 250$$
$$8T_2 = 250 + 8 \times 250$$
$$= 250 + 2000$$
$$= 2250$$
$$T_2 = \frac{2250}{8}$$
$$T_2 = 281.25\,K$$
$$T_2 = 8.1^{\circ}C$$



This nuclear power plant generates electric energy at the rate of 1000 MW. At the same time, by design, it discards energy into the nearby river at the rate of 2000 MW. This plant and all others like it throw away more energy than they deliver in useful form. They are real counterparts of the ideal engine.

10.11 ENTROPY

In the laws of thermodynamics, a state of a function has been explained by variables such as pressure, volume, temperature and internal energy. Rudolf Clausius in 1856 introduced another variable named as entropy which describes the state of the system as well as providing a quantitative relationship to the second law of thermodynamics and it is always being used as the measurement of disorder of the system. For example, if a system undergoes a reversible process by taking heat energy ΔQ from a hot reservoir at the thermodynamic temperature 'T' then the increase in the state variable called entropy. It is represented by 'S' and its value is given by;

$$\Delta S = \frac{\Delta Q}{T} \quad ......(10.26)$$

The unit of entropy is J K^{-1}. Change in entropy of the system is positive when heat is added and is negative when heat is removed. It is explained by an example of two reservoirs at different temperatures T_1 and T_2 such that $T_1 > T_2$ as shown in Fig.10.22(a). When both reservoirs are made in thermal contact with each other then there is flow of heat 'Q' from hot reservoir to cold reservoirs, i.e., Heat is lost in hot reservoir with negative entropy $\left(-\frac{Q}{T_1}\right)$ while heat is added in cold reservoir with positive entropy $\left(\frac{Q}{T_2}\right)$.

Fig.10.22(a): Two reservoirs at different temperature where heat flows from hot to cold reservoir performs work.

Thus change in entropy $= \frac{Q}{T_2} - \frac{Q}{T_1}$

When the temperature of both reservoirs becomes equal as shown in Fig.10.22(b), flow of heat stops and no work is possible. It means the heat energy which is present in the system but not available for useful work. Hence entropy can also be defined as, "the unavailability of energy for useful work".

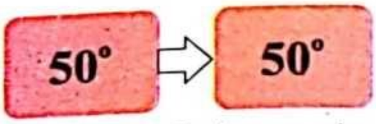


Fig.10.22(b): Both reservoirs have same temperature. There is no flow of heat and no work performs.

Entropy of reversible and irreversible process

Let a system undergoes a reversible process absorbs heat Q_1 at temperature T_1 and releases heat Q_2 at temperature T_2. Then its total entropy as shown in Fig. 10.23 is given as.

$$S = S_1 + S_2 + S_3 + S_4$$

$$S = \frac{Q_1}{T_1} + 0 - \frac{Q_2}{T_2} + 0$$

$$S = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

As $$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

Therefore, $$S = 0$$

> Entropy differs from energy, which does not obey law of conservation of energy.

This shows that entropy of a system under reversible process remains constant.

When the system undergoes an irreversible process at temperatures T_1 and T_2then according to Carnot theorem the efficiency of irreversible is less than that of the efficiency of the reversible i.e.

$$\eta_{irrev} < \eta_{rev}$$

$$1 - \frac{Q_2}{Q_1} < 1 - \frac{T_2}{T_1}$$

$$\frac{Q_2}{Q_1} > \frac{T_2}{T_1}$$

$$\frac{Q_2}{T_2} > \frac{Q_1}{T_1}$$

$$\frac{Q_2}{T_2} - \frac{Q_1}{T_1} > 0$$

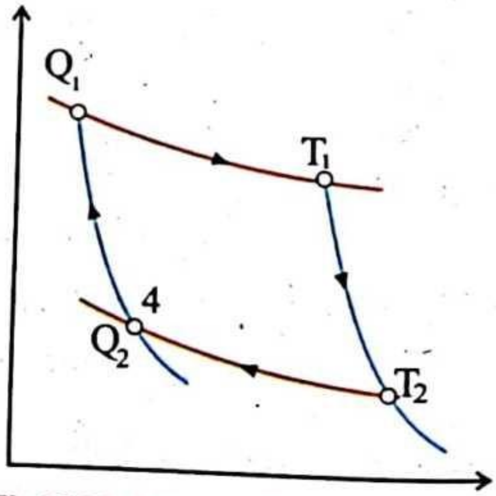


Fig.10.23: Entropy of a cycle reversible

The above results show that entropy of a system under irreversible process increases. By applying these two results of reversible and irreversible process to the universe or a natural process we conclude the entropy of the universe either remains constant or increases. This is called law of increase of entropy and this is another statement of second law of thermodynamics which is stated as **"in any natural process the entropy increases and the available energy for doing work decreases"**.

Entropy and Heat Death

It is a universal truth that hot places are becoming cold and cold places are becoming hot. This is according to law of nature. This process is continued till the temperatures will become same everywhere. At this stage, where the temperature difference for useful work is not available and entropy will be maximum. This is termed as heat death i.e., the available energy could not be brought in its respective function i.e. it could not be used for useful work.

Example 10.6

Calculate the entropy change when 1 kg of ice at 0°C melts into water at 0°C. Latent heat of fusion of ice is $3.36 \times 10^5\ J \cdot Kg^{-1}$.

Solution:

Mass = m = 1 Kg

Temperature = T = 0°C = 273 K

Latent heat of fusion = $L_f = 3.36 \times 10^5\ J \cdot Kg^{-1}$

Entropy = ΔS=?

$$\Delta S = \frac{\Delta Q}{T} = \frac{mL_f}{T}$$

$$\Delta S = \frac{1 \times 3.36 \times 10^5}{273}$$

$$\Delta S = 1.23 \times 10^3\ JK^{-1}$$

SUMMARY

- **Thermal equilibrium**: The condition of a system in which the flow of heat between the bodies is zero called thermal equilibrium.
- **Internal energy:** The sum of all forms of molecular energies of a substance is known as its internal energy.
- **First law of thermodynamics:** This law states that "When heat energy is converted into other form of energies or other form of energies are converted into heat energy but total energy remains constant" Mathematically $\Delta U = \Delta Q + \Delta W$.

- **Molar specific heat:** It is the amount of heat required to raise the temperature of one mole of a gas through one Kelvin.
  Molar specific heat at constant volume is the amount of heat required to raise the temperature of one mole of a gas through one Kelvin keeping volume constant.
  Molar specific heat at constant pressure is the amount of heat required to raise the temperature of one mole of a gas through one Kelvin keeping volume constant.
- **Reversible process:** A process in which the system is in equilibrium at any instant due to its reverse direction is a reversible process.
- **Irreversible process:** A process which cannot be retraced in the reverse direction and the system does not remain in equilibrium is an irreversible process.
- **Heat engine**: It is a device which converts heat energy into mechanical energy.
- **Second law of thermodynamics**: This law is stated as "There is no heat engine which takes heat and converts it completely into mechanical work".
- **Carnot cycle**: It is a reversible cycle which is completed under four steps, two for expansion and two for compression.
- **Carnot theorem**: According to this theorem there is no heat engine which is more efficient than reversible engine working between the same two temperatures.
- **Refrigerator**: It is a device to maintain the temperature of a body below than the temperature of its surrounding.
- **Entropy:** Measurement of disorder is called entropy.
- **Degradation of energy**: It is the transfer of heat energy from hot reservoir to cold reservoir.

EXERCISE

Multiple choice questions.

1. When two bodies are made at thermal contact having the same temperature then they are at:
   (a) Physical equilibrium (b) Thermal equilibrium
   (c) Mechanical equilibrium (d) Chemical equilibrium
2. Normal temperature of a human body is 98.6°F while its atmosphere temperature is 84.6°F. What will be the temperature of the dead body in such atmosphere
   (a) 84.6°F (b) 98.6°F (c) 92.5°F (d) 185°F
3. When the system is expanded by adding heat energy then the work done is:

(a) Positive and on the system (b) Negative and on the system
(c) Positive and by the system (d) Negative and by the system

4. Which substance possesses the largest internal energy at t°C
(a) Solid (b) Liquid (c) Gas (d) All of these

5. Internal energy of a substance is defined in terms of
(a) P and V (b) P and T (c) T and V (d) P, V and T

6. The ratio between work done and heat energy is equal to:
(a) Adiabatic constant (b) Joul's constant
(c) Specific heat constant (d) Real gas constant

7. A system which transfers neither mass not energy is called;
(a) Open system (b) Close system
(c) Isolated system (d) Non-cyclic system

8. First law of thermodynamics is based upon law of conservation of;
(a) Mass (b) Energy (c) Momentum (d) Charges

9. A process in which all the heat energy is used for increasing internal energy of the system is known as:
(a) Isobaric (b) Isochoric (c) Isothermal (d) Adiabatic

10. In which process the internal energy is used for doing work:
(a) Isobaric (b) Isochoric (c) Isothermal (d) Adiabatic

11. Specific heat of a gas in an isothermal process is:
(a) Zero (b) Remains constant
(c) Negative (d) Infinite

12. A process in which the system remains at thermal equilibrium is known as:
(a) Isobaric (b) Isochoric (c) Isothermal (d) Adiabatic

13. The value of adiabatic constant for mono-atomic gas is;
(a) 1.40 (b) 1.44 (c) 1.60 (d) 1.66

14. The efficiency of a heat engine will be 100% when
(a) Engine takes huge amount of heat from source
(b) Engine exhaust a very small amount of heat from sink
(c) The temperature of cold reservoir is 0°C
(d) The temperature of cold reservoir is 0 K

15. Second law of thermodynamics provides the proper direction of;
(a) Temperature (b) Force (c) Pressure (d) Flow of heat

16. A device which converts mechanical energy into heat energy is known as:
(a) Heat engine (b) Carnot Engine (c) Refrigerator (d) Turbine

17. Entropy of a system in a reversible process;
(a) Decreases (b) Increases (c) Infinite (d) Zero

18. Entropy remains constant in the process of;
(a) Isochoric (b) Isobaric (c) Isothermal (d) Adiabatic

SHORT QUESTIONS

Give the short answers of the following questions.

1. What is the condition of perfect thermal equilibrium?
2. Why is the earth not in thermal equilibrium with the sun?
3. What is the difference between the work done on the system and work done by the system?
4. In which process the internal energy of the system remains constant?
5. Which variable remains constant in adiabatic process?
6. Why the curve of adiabatic is steeper than isothermal process?
7. Why the measurement of molar specific heat is being preferred than specific heat?
8. Why molar specific heat at constant pressure is greater than molar specific heat at constant volume?
9. What is the difference between reversible and irreversible process?
10. Why the construction of a heat engine with 100% efficiency is impossible?
11. What is the difference between heat engine and refrigerator?
12. How can a Carnot cycle be completed?
13. What do you know about the heat death?
14. What will be the work done by a system when its hot and cold reservoirs are at same temperature?

COMPREHENSIVE QUESTIONS

1. What is thermal equilibrium? Discuss the condition of perfect thermal equilibrium.
2. Define internal energy of the given system in terms of the kinetic energies of the molecules of the gas.
3. State and explain thermodynamics system with all its kinds such as; open system, closed system and isolated system.
4. Explain that how did Rumford observe the relation between work and heat? Also show the graphical representation of the work done.
5. State and explain first law of thermodynamics and discuss its four applications.
6. Compare specific heat and molar specific heat. Also prove that $C_P - C_V = R$.
7. State and prove adiabatic equation, $PV^{\gamma} =$ Constant.
8. Explain reversible and irreversible processes with examples.
9. What is heat engine? Show the working principle and efficiency of a heat engine.
10. State and explain the second law of thermodynamics with examples.

11. Define Carnot engine, Carnot cycle, efficiency of Carnot engine and Carnot theorem.
12. What is refrigerator? Explain the working principle and co-efficient of performance of a refrigerator.
13. State and explain entropy. Calculate entropy of reversible and irreversible process. Also discuss the condition of heat death.

NUMERICAL PROBLEMS

1. How much work is done by an ideal gas in expanding from a volume of 3 litres to a volume of 33 litres at constant pressure of 2.5 atm.? **(7.6 kJ)**
2. A sample of gas is compressed to one half of its initial volume at constant pressure of 1.25×10^5 Nm^{-2}. During the compression, 100 J of work is done on the gas; determine the final volume of the gas. **(8×10^{-4} m^3)**
3. An ideal gas undergoes an isobaric expansion at 2.5 kPa. If the volume increases from $2m^3$ to $5m^3$ and 13 kJ of energy is transferred to the gas by heat, what is the change in its internal energy. **(5.5 kJ)**
4. The temperature of 2 kg metal block is raised from 15°C to 90°C by absorbing heat energy 86 kJ. Calculate the specific heat of the metal block. **(5735 J/kg °C)**
5. A mechanical engineer develops an engine, working between 327°C and 27°C and claims to have an efficiency of 50%. Does he claim correctly? Explain. **(Yes)**
6. A heat engine has a power output of 6 kW and an efficiency of 30%. Assume that the engine exhausts 8 kJ of heat energy in each cycle. Find (a) energy absorbed in each cycle and (b) the time for each cycle. **(11.4 kJ, 0.55s)**
7. In a refrigerator, heat from inside at 277 K is transferred to a room at 300 K. How much joule of heat will be delivered to the room for each joule of electrical energy consumed ideally? **(12 J)**
8. A Carnot engine utilizes an ideal gas. The source temperature is 227°C and the sink temperature is 127°C. Find the efficiency of the engine. Also find the heat input from the source and heat rejected to the sink when 10000 J of work is done. **(20%, 5×10^4 J, 4×10^4 J)**
9. How much work does on ideal Carnot refrigerator require to remove 1 J of energy from liquid helium at 4K and rejects this thermal energy to a room temperature 293 K environment. **(72.3 J)**
10. 336 J of energy is required to melt 1 g of ice at 0°C. What is the change in entropy of 30 g of water at 0°C as it is changed to ice at 0°C by a refrigerator? **(–36.8 J K^{-1})**