CHAPTER 1
NUMBER SYSTEMS

Rational Number:

A number which can be written in the form of $\frac{p}{q}$ (where $p,q \in \mathbb{Z}$ and $q \neq 0$) and $\frac{p}{q}$ is in its lowest form. $\mathbb{Q} = \left\{ x \mid x = \frac{p}{q}, p,q \in \mathbb{Z} \wedge q \neq 0 \right\}$.

For example. $\frac{3}{5}, \frac{11}{19}, \frac{21}{2}$ are rational numbers.

Irrational Number:

A number which cannot be written in the form of $\frac{p}{q}$, where $p,q \in \mathbb{Z}$ and $q \neq 0$.

$\mathbb{Q}' = \left\{ x \mid x \neq \frac{p}{q}, p,q \in \mathbb{Z} \wedge q \neq 0 \right\}$.

For example. $\sqrt{2}, \sqrt{3}, \frac{3\sqrt{3}}{2}, \sqrt{\frac{17}{11}}$ are irrational numbers.

SHORT QUESTION

What are terminating and recurring decimal called?

Terminating Decimals:

A decimal in which there are finite number of digits in its decimal part is called a terminating decimal.

For example. $1.23, 0.2, 7.95$ are terminating decimals.

Recurring Decimals:

A decimal in which one or more digits repeat indefinitely.

For example:

$\frac{1}{3} = 0.333..., 7.2323..., 1.325325...$ are recurring decimals

Non-Terminating, Non-Recurring Decimal:

A decimal which neither terminates nor it is recurring.

For example: $0.1257..., 3.391...$

$3.141628732...$ is an important irrational number, which is the value of $\pi\ (Pi)$. It denotes the constant ratio of the circumference of any circle to the length of its diameter.

$$\pi = \frac{\text{Circumference of any circle}}{\text{length of its diameter}}$$

NOTE:

(i) *Every recurring decimal represents a rational number.*

(ii) *An approximate value of π is $\frac{22}{7}$ and a better approximation is $\frac{355}{113}$.*

(iii) *Every non-terminating, Non-recurring decimal represents an irrational number.*

Real Number:

The union of rational and irrational numbers form real numbers. $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}'$

PROPERTIES OF REAL NUMBERS

Binary Operation:

A binary operation in a set A is a rule usually denoted by $*$ that assigns to any pair of elements of A, taken in a definite order, to another element of A.

Now we are discussing two important binary operations addition $(+)$ and multiplication $(\cdot)$ defined in the set of real numbers.

Addition Laws:	Multiplication Laws:
(i) Closure Law: $\forall a,b \in \mathbb{R}$ such that $a+b \in \mathbb{R}$	**(i) Closure Law:** $\forall a,b \in \mathbb{R}$ such that $a.b \in \mathbb{R}$
(ii) Associative Law: $\forall a,b,c \in \mathbb{R}$ such that $a+(b+c)=(a+b)+c$	**(ii) Associative Law:** $\forall a,b,c \in \mathbb{R}$ such that $a(bc)=(ab)c$
(iii) Additive identity: $\forall a \in \mathbb{R}\ \exists\ 0 \in \mathbb{R}$ such that $a+0=0+a=a$ **Note:** Zero is called additive identity of real numbers.	**(iii) Multiplicative identity:** $\forall a \in \mathbb{R}\ \exists 1 \in \mathbb{R}$ such that $a.1=1.a=a$ **Note:** "1" is called the multiplicative identity of real numbers.
(iv) Additive inverse: $\forall a \in \mathbb{R}\ \exists -a \in \mathbb{R}$ such that $a+(-a)=(-a)+a=0 \in \mathbb{R}$ **Note:** For additive inverse just change sign.	**(iv) Multiplicative inverse:** $\forall a(\neq 0) \in \mathbb{R}\ \exists\ \frac{1}{a} \in \mathbb{R}$ such that $a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \in \mathbb{R}$ **Note:** For multiplicative inverse just take reciprocal.
(v) Commutative Law: $\forall a,b \in \mathbb{R}$ $a+b=b+a$	**(v) Commutative Law:** $\forall a,b \in \mathbb{R}$ $a \cdot b = b \cdot a$

Distributive property of multiplication over addition:

$\forall a,b,c \in \mathbb{R}$

(i) $a(b+c)=ab+ac$ Left distributive law

(ii) $(a+b)c=ac+bc$ Right distributive law

Note: If any set possesses the above all properties then that set is called field. In sets $\mathbb{Q}$ (Rational Numbers), $\mathbb{R}$ (Real Numbers) and $\mathbb{C}$ (Complex Numbers) are known as fields.

PROPERTIES OF EQUALITY:	PROPERTIES OF INEQUALITIES: (order properties)
Equality of numbers denotes by "=" possesses the following properties. **(i) Reflexive property:** $\forall a \in \mathbb{R}$	**(i) Trichotomy Property:** $\forall a,b \in \mathbb{R}$ either $a=b$ or $a>b$ or $a<b$

$a=a$	
(ii) **Symmetric property**: $\forall a,b\in\mathbb{R}$ $a=b\Rightarrow b=a$	**(ii)** **Transitive Property**: $\forall a,b,c\in\mathbb{R}$ $a>b\wedge b>c\Rightarrow a>c$ $a<b\wedge b<c\Rightarrow a<c$
(iii) **Transitive property**: $\forall a,b,c\in\mathbb{R}$ $a=b\wedge b=c\Rightarrow a=c$ **(iv)** **Additive Property**: $\forall a,b,c\in\mathbb{R}$ $a=b\Rightarrow a+c=b+c$	**(iii)** **Additive Property**: $\forall a,b,c,d\in\mathbb{R}$ (a) (i) $a>b\Rightarrow a+c>b+c$ (ii) $a<b\Rightarrow a+c<b+c$ (b) (i) $a>b\wedge c>d\Rightarrow a+c>b+d$ (ii) $a<b\wedge c<d\Rightarrow a+c<b+d$
(v) **Multiplicative Property**: $\forall a,b,c\in\mathbb{R}$ $a=b\Rightarrow ac=bc\wedge ca=cb$ **(vi)** **Cancellation Property w.r.t Addition**: $\forall a,b,c\in\mathbb{R}$ $a+c=b+c\Rightarrow a=b$ **(vii)** **Cancellation Property w.r.t Multiplication**: $\forall a,b,c\in\mathbb{R}$ $ac=bc\Rightarrow a=b, c\neq 0$	**(iv)** **Multiplicative Property**: (a) $\forall a,b,c\in\mathbb{R}$ and $c>0$ (i) $a>b\Rightarrow ac>bc$ (ii) $a<b\Rightarrow ac<bc$ (b) $\forall a,b,c\in\mathbb{R}$ and $c<0$ (i) $a>b\Rightarrow ac<bc$ (ii) $a<b\Rightarrow ac>bc$ (c) $\forall a,b,c,d\in\mathbb{R}$ and a,b,c,d are all positive (i) $a>b\wedge c>d\Rightarrow ac>bd$ (ii) $a<b\wedge c<d\Rightarrow ac<bd$

EXERCISE 1.1

Q.1 Which of the following sets have closure property w.r.t addition and multiplications?

(i) $\{0\}$

Solution:

Let $G=\{0\}$

As $0+0=0\in G$

Hence, G possess closure property w.r.t addition

As $0\times 0=0\in G$

Hence, G possess closure property w.r.t multiplications

(ii) $\{1\}$ ***FSD 2023***

Solution:

Let $G=\{1\}$

As $1+1=2\notin G$

Hence, G does not possess closure property w.r.t addition

As $1\times 1=1\in G$

Hence, G possess closure property w.r.t multiplication.

(iii) $\{0,-1\}$

Solution:

Let $G=\{0,-1\}$

$0+0=0$

$0+(-1)=(-1)+0=-1$

$-1+(-1)=-2\notin G$

Hence, G does not possess a closure property w.r.t addition because all the sums do not belong to the set G

$0\times 0=0$

$0\times(-1)=(-1)\times 0=0$

$(-1)\times(-1)=1\notin G$

Hence, G does not possess a closure property w.r.t multiplication because all the multiplications do not belong to the set G

(iv) $\{1,-1\}$

***LHR* 2022, *GRW* 2021-23, *DGK* 2022, *RWP* 2022, *FSD* 2021**

Solution:

Let $G=\{1,-1\}$

$1+1=2\notin G$

$1+(-1)=(-1)+1=0\notin G$

$-1+(-1)=-2\notin G$

Hence, G does not possess a closure property w.r.t addition, because all the sums do not belongs to the set G.

As $1\times1=1\in G$

$1\times(-1)=(-1)\times1=-1\in G$

$(-1)\times(-1)=1\in G$

Hence, G possess a closure property w.r.t multiplications because all the products belong to the set G.

Q.2 Name the properties used in the following equations. (Letters, where used, represent real numbers).

(i) $4+9=9+4$ ***FSD* 2019**

Solution:

[Commutative Property w.r.t addition]

(ii) $(a+1)+\frac{3}{4}=a+\left(1+\frac{3}{4}\right)$

Solution:

[Associative Property w.r.t addition.]

(iii) $\left(\sqrt{3}+\sqrt{5}\right)+\sqrt{7}=\sqrt{3}+\left(\sqrt{5}+\sqrt{7}\right)$

Solution:

[Associative Property w.r.t addition.]

(iv) $100+0=100$

Solution:

[Additive Identity]

(v) $1000\times1=1000$ ***FSD* 2019, *RWP* 2023**

Solution:

[Multiplicative Identity]

(v) $4.1+(-4.1)=0$

Solution:

[Additive Inverse]

(vi) $a-a=0$

Solution:

[Additive Inverse]

(vii) $\sqrt{2}\times\sqrt{5}=\sqrt{5}\times\sqrt{2}$

Solution:

[Commutative property w.r.t multiplication.]

(viii) $a(b-c)=ab-ac$

Solution:

[Distributivity of multiplication over subtraction

(ix) $(x-y)z=xz-yz$

Solution:

[Distributivity of multiplication over subtraction]

(x) $4\times(5\times8)=(4\times5)\times8$

Solution:

[Associative property w.r.t multiplication.]

(xi) $a(b+c-d)=ab+ac-ad$

Solution:

[Distributivity of multiplication over addition and subtraction.]

Q.3 Name the properties used in the following Inequalities:

(i) $-3<-2\Rightarrow0<1$

Solution:

$-3<-2$

By adding 3 on both sides

$-3+(3)<-2+3$

$0<1$

Additive property of inequalities

(ii) $-5<-4\Rightarrow20>16$

Solution:

$-5<-4$

By multiplying –4 on both sides

$(-4)(-5)>(-4)(-4)$

$20>16$

Multiplicative property of inequalities

(iii) $1>-1\Rightarrow-3>-5$

Solution:

$1>-1$

By adding (– 4) on both sides

$1+(-4)>-1+(-4)$

$-3>-5$

Additive property of inequalities

(iv) $a<0 \Rightarrow -a>0$

Solution:

$a<0$

By multiplying -1 on both sides

$(-1)a>(-1)0$

$-a>0$

Multiplicative property of inequalities

(v) $a>b \Rightarrow \frac{1}{a}<\frac{1}{b}$

Solution:

$a>b$

Multiplying both sides by $\frac{1}{ab}$

$\frac{1}{ab}a>\frac{1}{ab}b$

$\frac{1}{b}\left(\frac{1}{a}a\right)>\frac{1}{a}\left(\frac{1}{b}b\right)$

$\frac{1}{b}(1)>\frac{1}{a}.(1)$

By multiplicative Inverse Law

$\frac{1}{b}>\frac{1}{a}$

$\Rightarrow \frac{1}{a}<\frac{1}{b}$

Multiplicative property of inequalities

(vi) $a>b \Rightarrow -a<-b$

Solution:

$a>b$

By multiplying both sides by -1

$(-1)a<(-1)b$

$-a<-b$

Q.4 Prove the following rules of addition.

(i) $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$

***LHR* 2019, *GRW* 2021, *RWP* 2021**

Proof:

L.H.S $=\frac{a}{c}+\frac{b}{c}$

$=a.\frac{1}{c}+b.\frac{1}{c}$ **[Rule for product of fractions]**

$=(a+b)\frac{1}{c}$ **[Distributive Law]**

$=\frac{a+b}{c}$ **[Rule for product of fractions]**

= R.H.S

(ii) $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

Proof:

L.H.S $=\frac{a}{b}+\frac{c}{d}$

$=\frac{ad}{bd}+\frac{bc}{bd}$ **[By Golden rule of fraction]**

$=ad.\frac{1}{bd}+bc.\frac{1}{bd}$

[Rule for product of fractions]

$=(ad+bc).\frac{1}{bd}$ **[Distributive Law]**

$=\frac{ad+bc}{bd}$ **[Rule for product of fractions]**

= R.H.S

Q.5 Prove that $-\frac{7}{12}-\frac{5}{18}=\frac{-21-10}{36}$

***SGD* 2021**

Proof:

L.H.S $=-\frac{7}{12}-\frac{5}{18}$

$=-\frac{7\times 3}{12\times 3}-\frac{5\times 2}{18\times 2}$

[Golden rule of fraction]

$=-\frac{21}{36}-\frac{10}{36}$

$=-21\times\frac{1}{36}-10\times\frac{1}{36}$

[Rule for product of fraction]

$=(-21-10)\times\frac{1}{36}$ **[Distributive Law]**

$=\frac{-21-10}{36}$ **[Rule for product of Fraction]**

= R.H.S

Q.6 Simplify by justifying each step:

(i) $\frac{4+16x}{4}$

Solution:

$\frac{4+16x}{4}$

$=\frac{1}{4}.(4+16x)$

[Rule for product of Fraction]

$$=\frac{1}{4}.\left[(4)(1)+(4)(4x)\right]$$

$$=\frac{1}{4}.4\left[1+4x\right] \quad \textbf{[Distributive Law]}$$

$$=1\left[1+4x\right] \quad \textbf{[Multiplicative inverse]}$$

$$=1+4x \quad \textbf{[Multiplicative Identity]}$$

(ii) $\dfrac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}$ **(GRW 2018, RWP 2019, SHW, 2022)**

Solution:

$$\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}$$

$$=\frac{\frac{1\times5}{4\times5}+\frac{1\times4}{5\times4}}{\frac{1\times5}{4\times5}-\frac{1\times4}{5\times4}} \quad \textbf{[Golden rule of fraction]}$$

$$=\frac{\frac{5}{20}+\frac{4}{20}}{\frac{5}{20}-\frac{4}{20}} \quad \textbf{[Closure Law]}$$

$$=\frac{5\times\frac{1}{20}+4\times\frac{1}{20}}{5\times\frac{1}{20}-4\times\frac{1}{20}}$$

[Rule for product of fractions]

$$=\frac{(5+4)\ \frac{1}{20}}{(5-4)\ \frac{1}{20}} \quad \textbf{[Distributive law]}$$

$$=\frac{(5+4)\ \frac{1}{20}\cdot20}{(5-4)\ \frac{1}{20}\cdot20} \quad \textbf{[Golden rule of fraction]}$$

$$=\frac{(5+4).1}{(5-4).1} \quad \textbf{[Multiplicative Inverse]}$$

$$=\frac{9}{1}=9 \quad \textbf{[Closure Law]}$$

(iii) $\dfrac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}$ ***FSD* 2018, *DGK* 2022, *SGD* 2023**

Solution:

$$\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}$$

$$=\frac{\frac{ad}{bd}+\frac{bc}{bd}}{\frac{ad}{bd}-\frac{bc}{bd}} \quad \textbf{[Golden rule of fraction]}$$

$$=\frac{ad.\frac{1}{bd}+bc.\frac{1}{bd}}{ad.\frac{1}{bd}-bc.\frac{1}{bd}}$$

[Rule for product of fractions]

$$=\frac{(ad+bc)\ .\frac{1}{bd}}{(ad-bc)\ .\frac{1}{bd}} \quad \textbf{[Distributive Law]}$$

$$=\frac{(ad+bc)\cdot\frac{1}{bd}\cdot\text{bd}}{(ad-bc)\cdot\frac{1}{bd}\cdot bd}$$

[Golden rule of fraction]

$$=\frac{(ad+bc)\cdot1}{(ad-bc)\cdot1} \quad \textbf{[Multiplicative Inverse]}$$

$$=\frac{ad+bc}{ad-bc} \quad \textbf{[Multiplicative Identity]}$$

(iv) $\dfrac{\frac{1}{a}-\frac{1}{b}}{1-\frac{1}{a}.\frac{1}{b}}$ **(RWP 2017)**

Solution:

$$\frac{\frac{1}{a}-\frac{1}{b}}{1-\frac{1}{a}.\frac{1}{b}}$$

$$=\frac{\frac{1\times b}{a\times b}-\frac{a\times1}{a\times b}}{\frac{1\times ab}{1\times ab}-\frac{1}{ab}} \quad \textbf{[Golden rule of fraction]}$$

$$=\frac{\frac{b}{ab}-\frac{a}{ab}}{\frac{ab}{ab}-\frac{1}{ab}}$$ **[Closure Law]**

$$=\frac{b\times\frac{1}{ab}-a\times\frac{1}{ab}}{ab\times\frac{1}{ab}-1\times\frac{1}{ab}}$$ **[Rule for product of fractions]**

$$=\frac{(b-a)\,\frac{1}{ab}}{(ab-1)\,\frac{1}{ab}}$$ **[By Distributive law]**

$$=\frac{(b-a)\cdot\frac{1}{ab}\cdot ab}{(ab-1)\cdot\frac{1}{ab}\cdot ab}$$ **[Golden rule of fraction]**

$$=\frac{(b-a)\cdot 1}{(ab-1)\cdot 1}$$ **[Multiplicative inverse]**

$$=\frac{b-a}{ab-1}$$ **[Multiplicative identity]**

EXERCISE 1.2

Q.1 Verify the addition properties of complex numbers

(i) Closure Property: $\forall z_1, z_2 \in \mathbb{C}$

Solution:

Let $z_1 = a+bi,\ z_2 = c+di$

$z_1 + z_2 = (a+bi)+(c+di)$

$= (a+c)+(b+d)i$

$\Rightarrow z_1 + z_2 \in \mathbb{C}$

Hence, Closure property holds in complex numbers

(ii) Associative Property: $\forall z_1, z_2, z_3 \in \mathbb{C}$

Solution:

Let $z_1 = a+bi, z_2 = c+di, z_3 = e+fi$

$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$

L.H.S $= z_1 + (z_2 + z_3)$

$=(a+bi)+\left[(c+di)+(e+fi)\right]$

$=(a+bi)+\left[(c+e)+(d+f)i\right]$

$=\left[a+(c+e)\right]+\left[b+(d+f)\right]i$

$=\left[(a+c)+e\right]+\left[(b+d)+f\right]i$

$=\left[(a+c)+(b+d)i\right]+(e+fi)$

$=\left[(a+bi)+(c+di)\right]+(e+fi)$

$=(z_1+z_2)+z_3$

= R.H.S

(iii) Identify Element: $\forall z \in \mathbb{C}, \exists O \in \mathbb{C}$

Solution:

Let $z = a+bi$

$(a+bi)+(0+i0)=(0+i0)+(a+bi)=a+bi$

(iv) Inverse element: $\forall z \in \mathbb{C}, \exists -z \in \mathbb{C}$

Solution:

Let $z = a+bi \Rightarrow -z = -a-bi$

$(a+bi)+(-a-bi)=(-a-bi)+(a+bi)=0$

(v) Commutative Property: $\forall z_1, z_2 \in \mathbb{C}$

Solution:

Let $z_1 = a+bi,\ z_2 = c+di$

$z_1 + z_2 = (a+bi)+(c+di)$

$=(a+c)+(b+d)i$

$=(c+a)+(d+b)i$

$=(c+di)+(a+bi)$

$= z_2 + z_1$

Q.2 Verify the multiplication properties of Complex numbers.

(i) Closure Property: $\forall z_1, z_2 \in \mathbb{C}$

Solution:

Let $z_1 = a+bi, z_2 = c+di$

$z_1.z_2 = (a+bi)(c+di)$

$= ac + adi + bci + bdi^2$

$= ac + (ad+bc)i - bd$

$= (ac-bd)+(ad+bc)i$

$\Rightarrow z_1.z_2 \in \mathbb{C}$

(ii) Associative Property: $\forall z_1, z_2, z_3 \in \mathbb{C}$

Solution:

$z_1 = a+bi, z_2 = c+di, z_3 = e+fi$

$z_1.(z_2.z_3) = (z_1.z_2).z_3$

L.H.S $= z_1.(z_2.z_3)$

$=(a+bi).\left[(c+di).(e+fi)\right]$

$=(a+bi).\left[ce+cfi+edi+dfi^2\right]\quad \because i^2=-1$

$=(a+bi).\left[ce+(cf+ed)i-df\right]$

$=(a+bi).\left[(ce-df)+(cf+ed)i\right]$

$=a(ce-df)+a(cf+ed)i+b(ce-df)i+b(cf+ed)i^2$

$=a(ce-df)+a(cf+ed)i+b(ce-df)i-b(cf+ed)$

$=\left[a(ce-df)-b(cf+ed)\right]+\left[a(cf+ed)+b(ce-df)\right]i$

$=(ace-adf-bcf-bde)+(acf+ade+bce-bdf)i$ (i)

R.H.S $=(z_1.z_2).z_3$

$=\left[(a+bi)(c+di)\right](e+fi)$

$=\left[ac+adi+bci+bdi^2\right].(e+fi)\quad \because i^2=-1$

$=\left[ac+(ad+bc)i-bd\right].(e+fi)$

$=\left[(ac-bd)+(ad+bc)i\right].(e+fi)$

$=\left[(ac-bd)e-(ad+bc)f\right]+\left[(ac-bd)f+(ad+bc)e\right]i$

$=(ace-bde-adf-bcf)+(acf-bdf+ade+bce)i$

$=(ace-adf-bcf-bde)+(acf+ade+bce-bdf)i$ (ii)

From equation (i) and equation (ii)

$z_1.(z_2.z_3)=(z_1.z_2).z_3$

(iii) Identity Element: $\forall z\in\mathbb{C}, \exists\, 1+0i\in\mathbb{C}$

Solution:

Let $z=a+bi$

$(a+bi)(1+0i)=(1+0i)(a+bi)=a+bi$

Hence, $1+0i$ is the multiplicative identity of complex number.

(iv) Inverse Element: $\forall z\in\mathbb{C}, \exists \frac{1}{z}\in\mathbb{C}$ **with** $(z\neq 0)$

Solution:

Let $z=a+bi$

$\frac{1}{z}=\frac{1}{a+bi}$

$=\frac{1}{a+bi}\times\frac{a-bi}{a-bi}$

$=\frac{a-bi}{a^2-(bi)^2}\quad \because i^2=-1$

$=\frac{a-bi}{a^2-b^2(-1)}$

$=\frac{a-bi}{a^2+b^2}$

$\frac{1}{z}=\left(\frac{a}{a^2+b^2}\right)+\left(\frac{-b}{a^2+b^2}\right)i$

Now, we will be show that

$z.\frac{1}{z}=\frac{1}{z}.z=1$

$z.\frac{1}{z}=(a+bi).\left(\frac{a}{a^2+b^2}-\frac{b}{a^2+b^2}i\right)$

$=\frac{(a)^2-(bi)^2}{a^2+b^2}\quad \because i^2=-1$

$=\frac{a^2-b^2(-1)}{a^2+b^2}$

$=\frac{a^2+b^2}{a^2+b^2}$

$=1$

Similarly, $\frac{1}{z}.z=1.$

(v) Commutative Property: $\forall z_1,z_2\in\mathbb{C}$

Solution:

Let $z_1=a+bi,\ z_2=c+di$

$z_1.z_2=z_2.z_1$

L.H.S $=z_1.z_2$

$=(a+bi)(c+di)$

$=ac+adi+bci+bdi^2\quad \because i^2=-1$

$=(ac-bd)+(ad+bc)i$ (i)

R.H.S $=z_2.z_1$

$=(c+di)(a+bi)$

$=ca+cbi+dai+dbi^2\quad \because i^2=-1$

$=ac+bci+adi+bd(-1)$

$=(ac-bd)+(ad+bc)i$ (ii)

By equation (i) and equation (ii) L.H.S = R.H.S

Q.3 Verify the distributive law of complex numbers

$(a,b)\left[(c,d)+(e,f)\right]=(a,b).(c,d)+(a,b)(e,f)$

Proof:

L.H.S $=(a,b)\left[(c,d)+(e,f)\right]$

$=(a,b).(c+e,d+f)$

$\therefore (a,b)(c,d)=(ac-bd,ad+bc)$

$=\left(a(c+e)-b(d+f),a(d+f)+b(c+e)\right)$

$=(ac+ae+bd-bf,ad+af+bc+be)$ (i)

R.H.S $=(a,b)(c,d)+(a,b)(e,f)$
$=(ac-bd,ad+bc)+(ae-bf,af+be)$
$=(ac-bd+ae-bf,ad+bc+af+be)$ (ii)

By equation (i) and equation (ii) L.H.S = R. H .S

Q.4 Simplify the following.

(i) i^9 *RWP 2021*

Solution:

i^9
$=i^8.i$
$=(i^2)^4.i$
$=(-1)^4.i$
$=1.i$
$=i$ Answer

(ii) i^{14}

Solution:

i^{14}
$=(i^2)^7$
$=(-1)^7$
$=-1$ Answer

(iii) $(-i)^{19}$

Solution:

$(-i)^{19}$
$=(-1\times i)^{19}$
$=(-1)^{19}i^{19}$
$=(-1)i^{18}.i$
$=(-1)(i^2)^9.i$
$=(-1)(-1)^9.i$
$=(-1)(-1)i$
$=i$ Answer

(iv) $(-1)^{\frac{-21}{2}}$

***LHR* 2018, *FSD* 21**

Solution:

$(-1)^{\frac{-21}{2}}$
$=[i^2]^{-\frac{21}{2}}$
$=i^{-21}$
$=\frac{1}{i^{21}}$
$=\frac{1}{i^{20}.i}$
$=\frac{1}{(i^2)^{10}i}$
$=\frac{1}{(-1)^{10}i}$
$=\frac{1}{i}$
$=-i$ Answer

Q.5 Write in terms of i.

(i) $\sqrt{-1}\,b$

Solution:

$=\sqrt{-1}\,b$
$=ib$

(ii) $\sqrt{-5}$

Solution:

$=\sqrt{-5}$
$=\sqrt{-1\times 5}$
$=\sqrt{-1}\,\sqrt{5}$
$=i\sqrt{5}$ Answer

(iii) $\sqrt{\frac{-16}{25}}$

Solution:

$=\sqrt{\frac{-16}{25}}$
$=\sqrt{\frac{-1\times 16}{25}}$
$=\frac{\sqrt{-1}\,\sqrt{16}}{\sqrt{25}}$
$=\frac{4}{5}i$ Answer

(iv) $\sqrt{\frac{1}{-4}}$

Solution:

$= \sqrt{\dfrac{1}{-4}}$

$= \dfrac{1}{\sqrt{-1\times 4}}$

$= \dfrac{1}{\sqrt{-1}\,\sqrt{4}}$

$= \dfrac{1}{2\,i}$

$= \dfrac{1}{2\,i} \times \dfrac{i}{i}$

$= \dfrac{i}{2\,i^2}$

$= \dfrac{i}{2(-1)}$

$= \dfrac{-i}{2}$ Answer

Simplify the following:

Q.6 $(7,9)+(3,-5)$

Solution:

$=(7,9)+(3,-5)$

$=(7+3, 9+(-5))$

$=(10,4)$ Answer

Q.7 $(8,-5)-(-7,4)$

Solution:

$=(8,-5)-(-7,4)$

$=(8-(-7), -5-4)$

$=(15,-9)$ Answer

Q.8 $(2,6)(3,7)$

Solution:

$=(2,6)(3,7)$

$=(2+6i)(3+7i)$

$=6+14i+18i+42i^2$

$=6+32i+42(-1)$

$=6-42+32i$

$=-36+32i$

$=(-36,32)$ Answer

Q.9 $(5,-4)(-3,-2)$ ***LHR* 2022, *MTN* 2023, *GRW* 2023, *SGD* 2021**

Solution:

$=(5,-4)(-3,-2)$

$=(5-4i)(-3-2i)$

$=-15-10i+12i+8i^2$

$=-15+2i+8(-1)$

$=-15+2i-8$

$=-23+2i$

$=(-23,2)$ Answer

Q.10 $(0,3)(0,5)$

Solution:

$=(0,3)(0,5)$

$=(0+3i)(0+5i)$

$=(3i)(5i)$

$=15i^2$

$=15(-1)$

$=-15+0i$

$=(-15,0)$ Answer

Q.11 $(2,6)\div(3,7)$ ***GRW* 2022, *MTN* 2022**

Solution:

$=(2,6)\div(3,7)$

$= \dfrac{2+6i}{3+7i}$

$= \dfrac{2+6i}{3+7i}\times\dfrac{3-7i}{3-7i}$

$= \dfrac{(2+6i)(3-7i)}{(3+7i)(3-7i)}$

$= \dfrac{6-14i+18i-42i^2}{3^2-(7i)^2}$

$= \dfrac{6+4i-42(-1)}{9-49(-1)}$

$= \dfrac{6+4i+42}{9+49}$

$= \dfrac{48+4i}{58}$

$= \dfrac{48}{58}+\dfrac{4}{58}\,i$

$$= \frac{24}{29} + \frac{2}{29}i$$

$$= \left(\frac{24}{29}, \frac{2}{29}\right) \text{ Answer}$$

Q.12 $(5,-4) \div (-3,-8)$ ***LHR* 2021**

Solution:

$$= (5,-4) \div (-3,-8)$$

$$= \frac{5-4i}{-3-8i}$$

$$= \frac{5-4i}{-3-8i} \times \frac{-3+8i}{-3+8i}$$

$$= \frac{-15+40i+12i-32i^2}{(-3)^2-(8i)^2}$$

$$= \frac{-15+52i-32(-1)}{9-64(-1)}$$

$$= \frac{-15+52i+32}{9+64}$$

$$= \frac{17+52i}{73}$$

$$= \frac{17}{73} + \frac{52}{73}i$$

$$= \left(\frac{17}{73}, \frac{52}{73}\right) \text{ Answer}$$

Q.13 Prove that the sum as well as the product of any two conjugate complex numbers is a real number. ***FSD* 2022, *GRW* 2019**

Proof:

Let $z = x + iy$ and $\bar{z} = x - iy$

$$z + \bar{z} = x + iy + x - iy$$

$= 2x$ is a real number

$$z.\bar{z} = (x+iy)(x-iy)$$

$$= (x)^2 - (iy)^2 \quad \because a^2 - b^2 = (a+b)(a-b)$$

$$= x^2 - i^2y^2$$

$$= x^2 - (-1)y^2$$

$= x^2 + y^2$ is a real number.

Q.14 Find the multiplicative inverse of each of the following numbers:

(i) $(-4,7)$

***LHR* 2018, *SGD* 2021-22, *RWP* 2023**

Solution:

Let $z = (-4,7)$

$$z = -4 + 7i$$

$$\frac{1}{z} = \frac{1}{-4+7i}$$

$$= \frac{1}{-4+7i} \times \frac{-4-7i}{-4-7i}$$

$$= \frac{-4-7i}{(-4)^2-(7i)^2}$$

$$= \frac{-4-7i}{16-49(-1)}$$

$$= \frac{-4-7i}{16+49}$$

$$= \frac{-4-7i}{65}$$

$$= \frac{-4}{65} - \frac{7}{65}i$$

$$\frac{1}{z} = \left(\frac{-4}{65}, \frac{-7}{65}\right) \text{ Answer}$$

(ii) $\left(\sqrt{2}, -\sqrt{5}\right)$

***GRW* 2021-22, *LHR* 2019-22, *FSD* 2021**

Solution:

Let $z = \left(\sqrt{2}, -\sqrt{5}\right)$

$$z = \sqrt{2} - \sqrt{5}i$$

$$\frac{1}{z} = \frac{1}{\sqrt{2} - \sqrt{5}i}$$

$$= \frac{1}{\sqrt{2}-\sqrt{5}i} \times \frac{\sqrt{2}+\sqrt{5}i}{\sqrt{2}+\sqrt{5}i}$$

$$= \frac{\sqrt{2}+\sqrt{5}i}{(\sqrt{2})^2-(\sqrt{5}i)^2}$$

$$\frac{1}{z} = \frac{\sqrt{2}+\sqrt{5}i}{2-5(-1)}$$

$$= \frac{\sqrt{2}+\sqrt{5}i}{2+5}$$

$$= \frac{\sqrt{2}+\sqrt{5}i}{7}$$

$$=\frac{\sqrt{2}}{7}+\frac{\sqrt{5}}{7}i$$

$$\frac{1}{z}=\left(\frac{\sqrt{2}}{7},\frac{\sqrt{5}}{7}\right) \text{ Answer}$$

(iii) $(1,0)$

Solution:

Let $z=(1,0)$

$$z=1+0i$$

$$\frac{1}{z}=\frac{1}{1+0i}$$

$$\frac{1}{z}=\frac{1}{1+0i}\times\frac{1-0i}{1-0i}$$

$$=\frac{1-0i}{(1)^2-(0i)^2}$$

$$=\frac{1}{1}$$

$$\frac{1}{z}=1$$

$$\frac{1}{z}=(1,0) \text{ Answer}$$

Q.15 Factorize the following

(i) a^2+4b^2

***LHR* 2019, *GRW* 2021, *SGD* 2018**

Solution:

$$=a^2+4b^2$$

$$=a^2-\left(-4b^2\right)$$

$$=a^2-(-1)4b^2$$

$$=a^2-i^2 4b^2$$

$$=a^2-(i2b)^2$$

$$=(a-i2b)(a+i2b) \text{ Answer}$$

(ii) $9a^2+16b^2$

***FSD* 2018-22, *LHR* 2022, *DGK* 2022, *GRW* 2023**

Solution:

$$=9a^2+16b^2$$

$$=9a^2-(-16b^2)$$

$$=9a^2-(-1)16b^2$$

$$=9a^2-i^2 16b^2$$

$$=(3a)^2-(i\ 4b)^2$$

$$=(3a-i4b)(3a+i4b) \text{ Answer}$$

(iii) $3x^2+3y^2$ ***SHW* 2023**

Solution:

$$=3x^2+3y^2$$

$$=3[x^2+y^2]$$

$$=3[x^2-(-y^2)]$$

$$=3[x^2-(-1)y^2]$$

$$=3[x^2-i^2y^2]$$

$$=3[x^2-(iy)^2]$$

$$=3(x-iy)(x+iy) \text{ Answer}$$

Q.16 Separate into real and imaginary parts (Write as a simple complex number):

(i) $\frac{2-7i}{4+5i}$ ***LHR* 2021, *SGD* 2022**

Solution:

$$=\frac{2-7i}{4+5i}$$

$$=\frac{2-7i}{4+5i}\times\frac{4-5i}{4-5i}$$

$$=\frac{(2-7i)(4-5i)}{(4)^2-(5i)^2}$$

$$=\frac{8-10i-28i+35i^2}{16-25(-1)}$$

$$=\frac{8-38i+35(-1)}{16+25}$$

$$=\frac{8-38i-35}{41}$$

$$=\frac{-27-38i}{41}$$

$$=\frac{-27}{41}-\frac{38}{41}i$$

$$\text{Real Part}=\frac{-27}{41}, \quad \text{Imaginary Part}=\frac{-38}{41}$$

(ii) $\frac{(-2+3i)^2}{1+i}$ ***DGK* 2022**

Solution:

$$=\frac{(-2+3i)^2}{1+i}$$

$$=\frac{(-2)^2+(3i)^2+2(-2)(3i)}{1+i}$$
$$=\frac{4-9-12i}{1+i}$$
$$=\frac{-5-12i}{1+i}$$
$$=\frac{-5-12i}{1+i}\times\frac{1-i}{1-i}$$
$$=\frac{(-5-12i)(1-i)}{(1+i)(1-i)}$$
$$=\frac{-5+5i-12i+12i^2}{1^2-i^2}$$
$$=\frac{-5-7i+12(-1)}{1-(-1)}$$
$$=\frac{-5-7i-12}{1+1}$$
$$=\frac{-17-7i}{2}$$
$$=\frac{-17}{2}-\frac{7}{2}i$$

Real Part $=\frac{-17}{2}$, Imaginary Part $=\frac{-7}{2}$

(iii) $\frac{i}{1+i}$

***GRW* 2019, *RWP* 2018-23, *MTN* 2022-23, *SHW* 2022)**

Solution:

$$=\frac{i}{1+i}$$
$$=\frac{i}{1+i}\times\frac{1-i}{1-i}$$
$$=\frac{i(1-i)}{1^2-i^2}$$
$$=\frac{i-i^2}{1-(-1)}$$
$$=\frac{i-(-1)}{2}$$
$$=\frac{1+i}{2}$$
$$=\frac{1}{2}+\frac{1}{2}i$$

Real Part $=\frac{1}{2}$, Imaginary Part $=\frac{1}{2}$

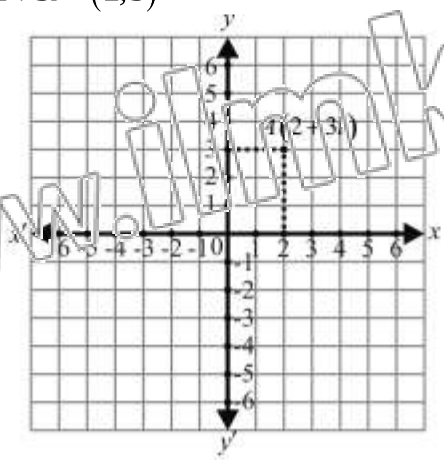
EXERCISE 1.3

Q.1 Graph the following numbers in the complex plane.

(i) $2+3i$

Solution:

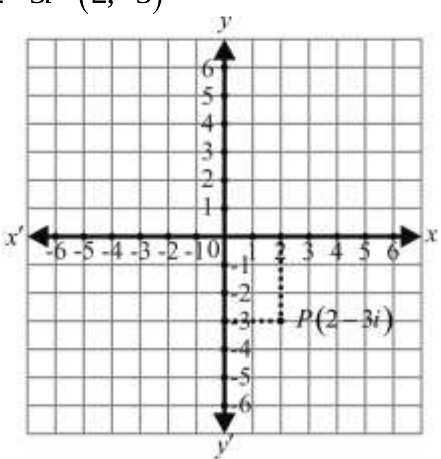
As $2+3i=(2,3)$



(ii) $2-3i$

Solution:

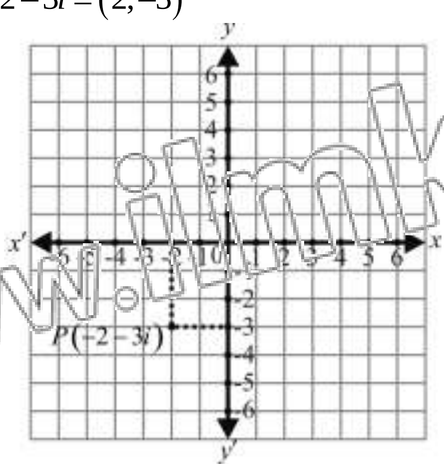
As $2-3i=(2,-3)$



(iii) $-2-3i$

Solution:

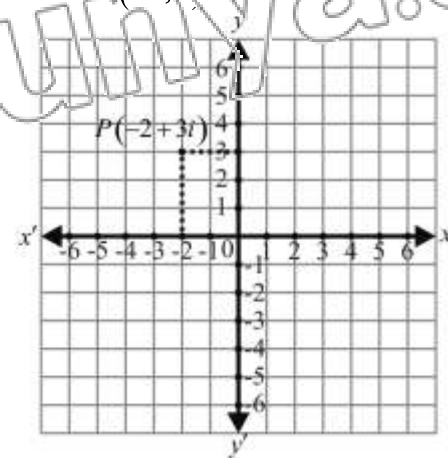
As $2-3i=(2,-3)$



(iv) $-2+3i$

Solution:

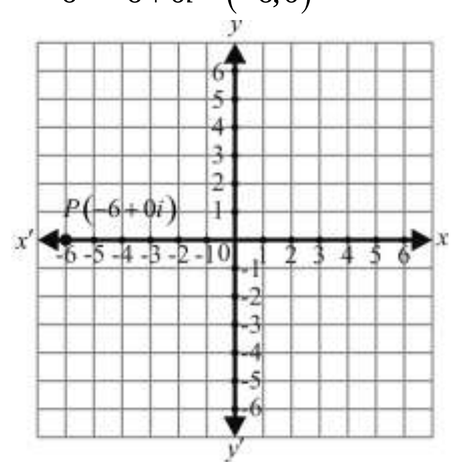
As $-2+3i=(-2,3)$



(v) -6

Solution:

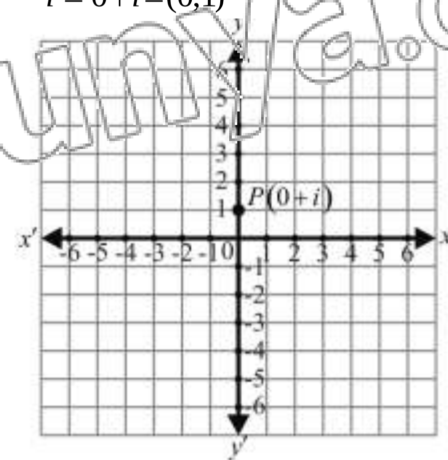
$-6=-6+0i=(-6,0)$



(vi) i

Solution:

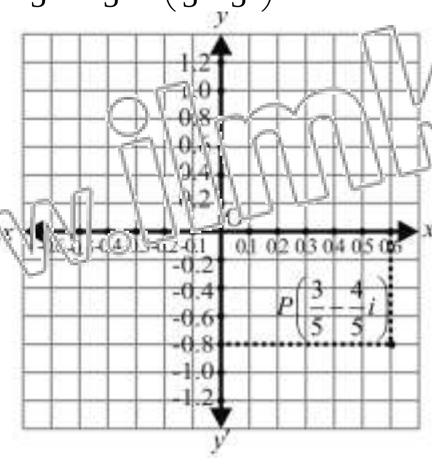
$i=0+i=(0,1)$



(vii) $\frac{3}{5} - \frac{4}{5}i$

Solution:

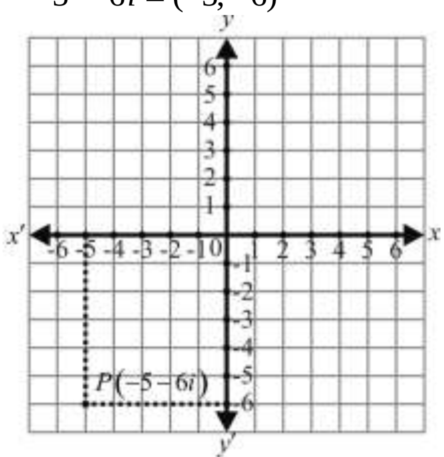
$$\frac{3}{5} - \frac{4}{5}i = \left(\frac{3}{5}, \frac{-4}{5}\right)$$



(viii) $-5-6i$

Solution:

$$-5 - 6i = (-5, -6)$$



Q.2 Find the multiplicative inverse of each of the following numbers:

(i) $-3i$

Solution:

Let $z = -3i$

$$\frac{1}{z} = \frac{1}{-3i}$$

$$= \frac{1}{-3i} \times \frac{3i}{3i}$$

$$= \frac{3i}{-9i^2}$$

$$= \frac{3i}{-9(-1)}$$

$$= \frac{3i}{9}$$

$$\frac{1}{z} = \frac{1}{3}i \text{ Answer}$$

(ii) $1-2i$

Solution:

Let $z = 1-2i$

$$\frac{1}{z} = \frac{1}{1-2i}$$

$$= \frac{1}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+2i}{(1)^2-(2i)^2}$$

$$\because a^2-b^2 = (a+b)(a-b)$$

$$= \frac{1+2i}{1-4(-1)}$$

$$\frac{1}{z} = \frac{1+2i}{1+4}$$

$$\frac{1}{z} = \frac{1}{5} + \frac{2}{5}i \text{ Answer}$$

(iii) $-3-5i$

Solution:

Let $z = -3-5i$

$$\frac{1}{z} = \frac{1}{-3-5i}$$

$$\frac{1}{z} = \frac{1}{-3-5i} \times \frac{-3+5i}{-3+5i}$$

$$= \frac{-3+5i}{(-3)^2-(5i)^2}$$

$$\because a^2-b^2 = (a+b)(a-b)$$

$$= \frac{-3+5i}{9-25i^2}$$

$$= \frac{-3+5i}{9-25(-1)}$$

$$= \frac{-3+5i}{9+25}$$

$$= \frac{-3+5i}{34}$$

$$\frac{1}{z} = \frac{-3}{34} + \frac{5}{34}i \text{ Answer}$$

(iv) $(1, 2)$

Solution:

Let $z = (1, 2)$

$z = 1 + 2i$

$$\frac{1}{z} = \frac{1}{1+2i}$$

$$= \frac{1}{1+2i} \times \frac{1-2i}{1-2i}$$

$$= \frac{1-2i}{(1)^2-(2i)^2}$$

$\because a^2 - b^2 = (a+b)(a-b)$

$$= \frac{1-2i}{1-4(-1)}$$

$$= \frac{1-2i}{1+4}$$

$$= \frac{1-2i}{5}$$

$$= \frac{1}{5} - \frac{2}{5}i$$

$$\frac{1}{z} = \left(\frac{1}{5}, \frac{-2}{5}\right) \text{ Answer}$$

Q.3 Simplify

(i) i^{101} ***GRW* 2019**

Solution:

$= i^{101}$

$= i^{100}.i$

$= (i^2)^{50}.i$

$= (-1)^{50}.i$

$= 1.i$

$= i$ Answer

(ii) $(-ai)^4$, $a \in \mathbb{R}$

Solution:

$= (-ai)^4$

$= (-a \times i)^4$

$= (-a)^4\, i^4$

$= a^4(i^2)^2$

$= a^4(-1)^2$

$= a^4 \times 1$

$= a^4$ Answer

(iii) i^{-3}

Solution:

$= i^{-3}$

$$= \frac{1}{i^3}$$

$$= \frac{1}{i^2 . i}$$

$$= \frac{1}{-i}$$

$$= \frac{1}{-i} \times \frac{i}{i}$$

$$= \frac{i}{-i^2}$$

$$= \frac{i}{-(-1)}$$

$= i$ Answer

(iv) i^{-10}

Solution:

$= i^{-10}$

$$= \frac{1}{i^{10}}$$

$$= \frac{1}{(i^2)^5}$$

$$= \frac{1}{(-1)^5}$$

$$= \frac{1}{-1}$$

$= -1$ Answer

Q.4 Prove that $z = \bar{z}$ if z is real.

***FSD* 2019, *GRW* 2022, *MTN* 2022, *RWP* 2022-23, *SHW* 2023**

Proof:

Let $z = x + iy$ (i)

Suppose that $z = \bar{z}$

Now we have to prove z is real

As $z = \bar{z}$

$x + iy = x - iy$

$x + iy - x + iy = 0$

$2iy = 0$

As $2i \neq 0,\ y = 0$

Put in a equation (i)

$z = x + i(0)$

$z = x$

Hence z is a real number.

Conversely,

Suppose z is a real number

Now we have to prove that $z = \bar{z}$

Let $z = x + 0i = x$

$\bar{z} = x - 0i = x$

Hence

$z = \bar{z}$

Q.5 Simplify by expressing in the form $a+bi$

(i) $5+2\sqrt{-4}$

Solution:

$= 5+2\sqrt{-4}$

$= 5+2\sqrt{-1\times 4}$

$= 5+2\sqrt{-1}\sqrt{4}$

$= 5+2(i)2$

$= 5+4i$ Answer

(ii) $\left(2+\sqrt{-3}\right)\left(3+\sqrt{-3}\right)$ ***GRW* 2021**

Solution:

$= \left(2+\sqrt{-3}\right)\left(3+\sqrt{-3}\right)$

$= \left(2+\sqrt{-1\times 3}\right)\left(3+\sqrt{-1\times 3}\right)$

$= \left(2+\sqrt{-1}\sqrt{3}\right)\left(3+\sqrt{-1}\sqrt{3}\right)$

$= \left(2+i\sqrt{3}\right)\left(3+i\sqrt{3}\right)$

$= 6+i\,2\sqrt{3}+i3\sqrt{3}+i^2(\sqrt{3})^2$

$= 6+i\,5\sqrt{3}+(-1)(3)$

$= 6+i\,5\sqrt{3}-3$

$= 3+i5\sqrt{3}$ Answer

(iii) $\dfrac{2}{\sqrt{5}+\sqrt{-8}}$

Solution:

$= \dfrac{2}{\sqrt{5}+\sqrt{-8}}$

$= \dfrac{2}{\sqrt{5}+\sqrt{-1\times 8}}$

$= \dfrac{2}{\sqrt{5}+\sqrt{-1}\sqrt{8}}$

$= \dfrac{2}{\sqrt{5}+i\sqrt{8}}$

$= \dfrac{2}{\sqrt{5}+i\sqrt{8}}\times\dfrac{\sqrt{5}-i\sqrt{8}}{\sqrt{5}-i\sqrt{8}}$

$= \dfrac{2\left[\sqrt{5}-i\sqrt{8}\right]}{\left(\sqrt{5}\right)^2-\left(i\sqrt{8}\right)^2}$

$\because a^2-b^2=(a+b)(a-b)$

$= \dfrac{2\left[\sqrt{5}-i\sqrt{8}\right]}{5-(-1)8}$

$= \dfrac{2\left[\sqrt{5}-i\sqrt{8}\right]}{13}$

$= \dfrac{2\sqrt{5}}{13}-\dfrac{2\sqrt{8}}{13}i$

$= \dfrac{2\sqrt{5}}{13}-\dfrac{2\times 2\sqrt{2}}{13}i$

$= \dfrac{2\sqrt{5}}{13}-\dfrac{4\sqrt{2}}{13}i$ Answer

(iv) $\dfrac{3}{\sqrt{6}-\sqrt{-12}}$

Solution:

$= \dfrac{3}{\sqrt{6}-\sqrt{-12}}$

$= \dfrac{3}{\sqrt{6}-\sqrt{-1\times 12}}$

$= \dfrac{3}{\sqrt{6}-i\sqrt{12}}$

$= \dfrac{3}{\sqrt{6}-i\sqrt{12}}\times\dfrac{\sqrt{6}+i\sqrt{12}}{\sqrt{6}+i\sqrt{12}}$

$= \dfrac{3\left[\sqrt{6}+i\sqrt{12}\right]}{\left(\sqrt{6}\right)^2-\left(i\sqrt{12}\right)^2}$

$\because a^2-b^2=(a+b)(a-b)$

$= \dfrac{3\left[\sqrt{6}+i\sqrt{12}\right]}{6-(-1)\,12}$

$$= \frac{3\left[\sqrt{6} + i\sqrt{12}\right]}{18}$$

$$= \frac{\sqrt{6} + i\sqrt{12}}{6}$$

$$= \frac{\sqrt{6}}{6} + i\frac{\sqrt{6}\sqrt{2}}{6}$$

$$= \frac{1}{\sqrt{6}} + i\frac{\sqrt{2}}{\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}} + i\frac{\sqrt{2}}{\sqrt{3}\sqrt{2}}$$

$$= \frac{1}{\sqrt{6}} + i\frac{1}{\sqrt{3}} \text{ Answer}$$

Q.6 Show that $\forall\ z \in \mathbb{C}$

(i) $z^2 + \bar{z}^2$ **is a real number.**

***GRW* 2019-21, *FSD* 2022, *BWP* 2023, *LHR* 2023**

Solution:

Let $z = x + iy$

$\bar{z} = x - iy$

$$z^2 + \bar{z}^2 = (x + iy)^2 + (x - iy)^2$$

$$= x^2 + (iy)^2 + 2x(iy) + x^2 + (iy)^2 - 2x(iy)$$

$$= x^2 + (-1)y^2 + x^2 + (-1)y^2$$

$$= 2x^2 - 2y^2 \text{ is a real number}$$

(ii) $(z - \bar{z})^2$

***GRW* 2018, *RWP* 2017, *LHR* 2022, *MTN* 2022**

Solution:

Let $z = x + iy$

$z = x - iy$

$$(z - \bar{z}) = (x + iy) - (x - iy)$$

$$= x + iy - x + iy$$

$$z - \bar{z} = 2iy$$

By taking square on both sides:

$$(z - \bar{z})^2 = (2iy)^2$$

$$= 4(-1)y^2$$

$$= -4y^2 \text{ is a real number}$$

Q.7 Simplify the following:

(i) $\left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right)^3$

Solution:

$$= \left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right)^3$$

$$= \left(\frac{-1 + \sqrt{3}i}{2}\right)^3$$

$$= \frac{(-1 + \sqrt{3}i)^3}{2^3}$$

$$= \frac{1}{8}\left[-1 + \sqrt{3}i\right]^3$$

$$= \frac{1}{8}\left[(-1)^3 + 3(-1)^2(\sqrt{3}i) + 3(-1)(\sqrt{3}i)^2 + (\sqrt{3}i)^3\right]$$

$$= \frac{1}{8}\left[-1 + 3\sqrt{3}i - 3\times 3(-1) + (\sqrt{3})^3 i^3\right]$$

$$= \frac{1}{8}\left[-1 + 3\sqrt{3}i + 9 + (\sqrt{3})^2\sqrt{3}(-i)\right]$$

$$= \frac{1}{8}\left[-1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i\right]$$

$$= \frac{1}{8}[8]$$

$$= 1 \text{ Answer}$$

(ii) $\left(\frac{-1}{2} - \frac{\sqrt{3}}{2}i\right)^3$ ***RWP* 2018, *FSD* 2022**

Solution:

$$= \left(\frac{-1}{2} - \frac{\sqrt{3}}{2}i\right)^3$$

$$= \left(\frac{-1 - \sqrt{3}i}{2}\right)^3$$

$$= \frac{(-1 - \sqrt{3}i)^3}{2^3}$$

$$= \frac{1}{8}\left[-1 - \sqrt{3}i\right]^3$$

$$= \frac{1}{8}\left[(-1)^3 - 3(-1)^2(\sqrt{3}i) + 3(-1)(\sqrt{3}i)^2 - (\sqrt{3}i)^3\right]$$

$$= \frac{1}{8}\left[-1 - 3\sqrt{3}i - 3\times 3(-1) - (\sqrt{3})^3 i^3\right]$$

$$= \frac{1}{8}\left[-1 - 3\sqrt{3}i + 9 - (\sqrt{3})^2\sqrt{3}(-i)\right]$$

$=\frac{1}{8}\left[-1-3\sqrt{3}\,i+9+3\sqrt{3}\,i\right]$

$=\frac{1}{8}[8]$

$=1$ Answer

(iii) $\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2}\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)$

Solution:

$=\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2}\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)$

$=\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{-2+1}$

$=\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^{-1}$

$=\left(\frac{-1-\sqrt{3}i}{2}\right)^{-1}$

$=\frac{2}{-1-\sqrt{3}i}$

$=\frac{2}{-1-\sqrt{3}i}\times\frac{-1+\sqrt{3}i}{-1+\sqrt{3}i}$

$=\frac{2\left[-1+\sqrt{3}i\right]}{(-1)^2-(\sqrt{3}i)^2}$

$=\frac{2\left[-1+\sqrt{3}\,i\right]}{1-3(-1)}$

$=\frac{2\left[-1+\sqrt{3}\,i\right]}{4}$

$=\frac{-1+\sqrt{3}i}{2}$ Answer

(iv) $(a+bi)^2$

Solution:

$=(a+bi)^2$

$=a^2+(bi)^2+2(a)(bi)$

$=a^2+b^2(-1)+2abi$

$=a^2-b^2+2abi$ Answer

(v) $(a+bi)^{-2}$

Solution:

$=(a+bi)^{-2}$

$=\frac{1}{(a+bi)^2}$

$=\frac{1}{(a+bi)^2}\times\frac{(a-bi)^2}{(a-bi)^2}$

$=\frac{(a-bi)^2}{(a+bi)^2\,(a-bi)^2}$

$=\frac{a^2+(bi)^2-2(a)(bi)}{\left[a^2-(bi)^2\right]^2}$

$=\frac{a^2+b^2(-1)-2abi}{\left[a^2-b^2(-1)\right]^2}$

$=\frac{a^2-b^2-2abi}{(a^2+b^2)^2}$

$=\frac{a^2-b^2}{(a^2+b^2)^2}-\frac{2ab}{(a^2+b^2)^2}i$ Answer

(vi) $(a+bi)^3$ ***BWP* 2022**

Solution:

$=(a+bi)^3$

$=a^3+3(a)^2(bi)+3a(bi)^2+(bi)^3$

$=a^3+3a^2bi+3ab^2(-1)+b^3i^3$

$=a^3+3a^2bi-3ab^2+b^3i^2.i$

$=a^3+3a^2bi-3ab^2+b^3(-1).i$

$=a^3-3ab^2+3a^2bi-b^3\,i$

$=(a^3-3ab^2)+(3a^2b-b^3)\,i$ Answer

(vii) $(a-bi)^3$

Solution:

$=(a-bi)^3$

$=a^3-3(a)^2(bi)+3a(bi)^2-(bi)^3$

$=a^3-3a^2bi+3ab^2(-1)-b^3i^3$

$=a^3-3a^2bi-3ab^2-b^3i^2.i$

$=a^3-3a^2bi-3ab^2-b^3(-1).i$

$=a^3-3a^2bi-3ab^2+b^3\,i$

$=(a^3-3ab^2)+(-3a^2b+b^3)\,i$ Answer

(viii) $\left(3-\sqrt{-4}\right)^{-3}$

Solution:

$=\left(3-\sqrt{-4}\right)^{-3}$

$$= \left(3 - \sqrt{-1}\sqrt{4}\right)^{-3}$$
$$= \left(3 - i\,2\right)^{-3}$$
$$= \frac{1}{(3-2i)^3}$$
$$= \frac{1}{(3-2i)^3} \times \frac{(3+2i)^3}{(3+2i)^3}$$
$$= \frac{(3+2i)^3}{\left[(3-2i)(3+2i)\right]^3}$$
$$= \frac{3^3 + 3(3)^2(2i) + 3(3)(2i)^2 + (2i)^3}{\left[3^2 - (2i)^2\right]^3}$$
$$= \frac{27 + 54i + 9\times 4(-1) + 8i^3}{\left[9 - 4(-1)\right]^3}$$
$$= \frac{27 + 54i - 36 + 8i^2 i}{(13)^3}$$
$$= \frac{27 + 54i - 36 + 8(-1)i}{13^3}$$
$$= \frac{-9 + 54i - 8i}{2197}$$
$$= \frac{-9 + 46i}{2197}$$
$$= \frac{-9}{2197} + \frac{46}{2197}i \text{ Answer}$$

SELF IMPROVEMENT TEST (SIT)

Q.No.1 Multiple Choice Questions.

1. 3.141592 is a/an__________ number:

Ⓐ Natural Ⓑ Whole Ⓒ Rational Ⓓ Irrational

2. If $x=y \wedge y=z \Rightarrow x=z, \forall x,y,z \in \mathbb{R}$ this property is called:

Ⓐ Symmetric Ⓑ Transitive Ⓒ Additive Ⓓ Trichotomy

3. The imaginary part of $z=2+\sqrt{-4}i$ is: **(CONCEPTUAL)**

Ⓐ 0 Ⓑ 2 Ⓒ $\sqrt{-4}$ Ⓓ 4

4. The multiplicative inverse of $z=-3-i^{11}$ is: **(CONCEPTUAL)**

Ⓐ $\left(\frac{3}{10},\frac{1}{10}\right)$ Ⓑ $\left(-\frac{3}{10},\frac{1}{10}\right)$ Ⓒ $\left(-\frac{3}{10},-\frac{1}{10}\right)$ Ⓓ $\left(\frac{3}{10},-\frac{1}{10}\right)$

5. The modulus of $\frac{1+i}{1-i}$ is equal to: **(CONCEPTUAL)**

Ⓐ 1 Ⓑ 2 Ⓒ –1 Ⓓ 0

6. If $z=1+i\sqrt{3}$ then argument of z is:

Ⓐ 30° Ⓑ 45° Ⓒ 60° Ⓓ 90°

7. The complex number $z=\frac{3-4i}{5}$ lies in___________ quadrant:

Ⓐ 1^{st} Ⓑ 2^{nd} Ⓒ 3^{rd} Ⓓ 4^{th}

Q.No.2 Short Questions.

(i) Prove by rules of addition $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

(ii) Express $\frac{2-7i}{4+5i}$ in the form of $a+bi$.

(iii) Factorize: $3x^2+5y^2$

(iv) If $z=\text{Cos}\,\theta+i\sin\theta$ then find the value of $z-\frac{1}{z}$ **(CONCEPTUAL)**

Q.No.3 Long Questions.

(a) Prove that $\sqrt{3}$ is an irrational number.

(b) Simplify: $\left(\frac{i+\sqrt{3}}{2}\right)^3+\left(\frac{i-\sqrt{3}}{2}\right)^3$ **(CONCEPTUAL)**