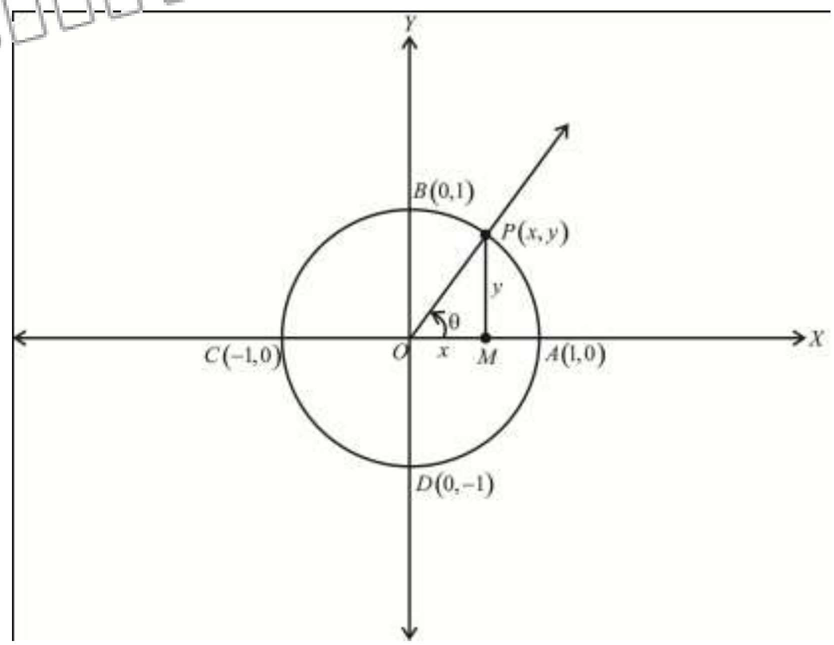
CHAPTER 11
TRIGONOMETRIC FUNCTIONS AND THEIR GRAPHS

Domain and Range of Sine Function:



Let $P(x, y)$ be any point on the unit circle with centre at the origin such that $\angle XOP = \theta$ is in standard position, then in right triangle OMP,

$$\sin\theta = \frac{|\overline{PM}|}{|\overline{OP}|}$$

$$\sin\theta = \frac{y}{1}$$

$$\sin\theta = y$$

$\Rightarrow$ For any real number θ there is one and only one value of y i.e. of $\sin\theta$.

Hence $\sin\theta$ is the function of θ and its domain is $\mathbb{R}$, a set of real numbers.

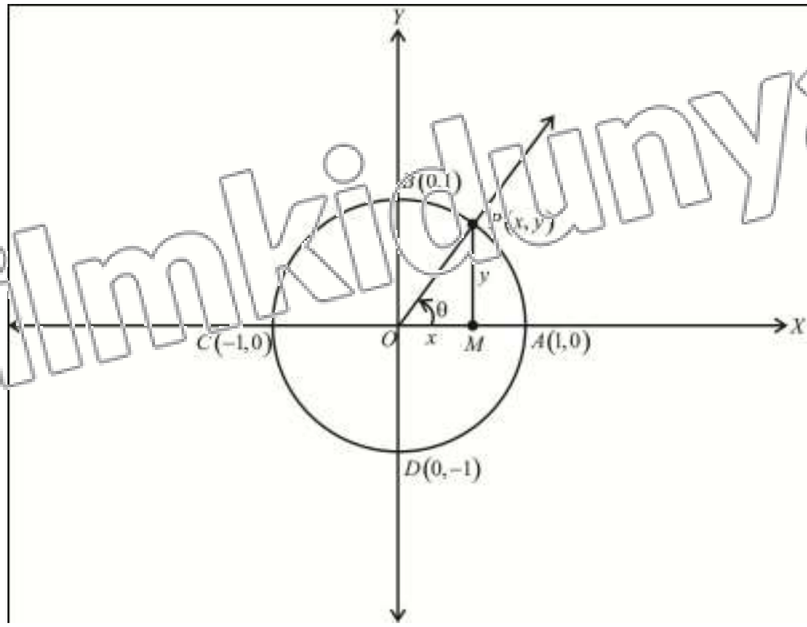
Since $P(x, y)$ is a point on the unit circle with centre at the origin O.

$\therefore \quad -1 \le y \le 1$

$\Rightarrow \quad -1 \le \sin\theta \le 1$

Thus the range of the sine function is $[-1,1]$.

Domain and Range of Cosine Function:



Let $P(x,y)$ be any point on the unit circle with centre at the origin such that $\angle XOP = \theta$ is in standard position, then in right triangle OMP,

$$\cos\theta = \frac{|\overline{OM}|}{|\overline{OP}|}$$

$$\cos\theta = \frac{x}{1}$$

$$\cos\theta = x$$

$\Rightarrow$ For any real number θ there is one and only one value of x i.e. of $\cos\theta$.

Hence $\cos\theta$ is the function of θ and its domain is $\mathbb{R}$, a set of real numbers.

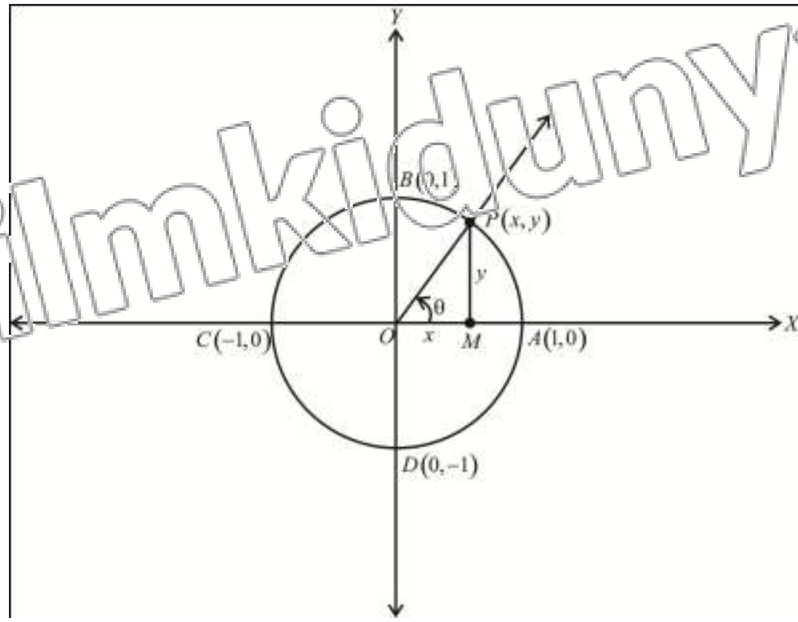
Since $P(x,y)$ is a point on the unit circle with centre at the origin O.

$\therefore \quad -1 \le x \le 1$

$\Rightarrow \quad -1 \le \cos\theta \le 1$

Thus the range of the cosine function is $[-1,1]$.

Domain and Range of Tangent Function:



Let $P(x,y)$ be any point on the unit circle with centre at the origin such that $\angle XOP = \theta$ is in standard position, then in right triangle OMP,

$$\tan\theta = \frac{|\overline{PM}|}{|\overline{OM}|}$$

$$\tan\theta = \frac{y}{x},\ \ x \neq 0$$

$\Rightarrow$ Terminal side $\overrightarrow{OP}$ should not coincide with OY or OY' (i.e. $Y-axis$)

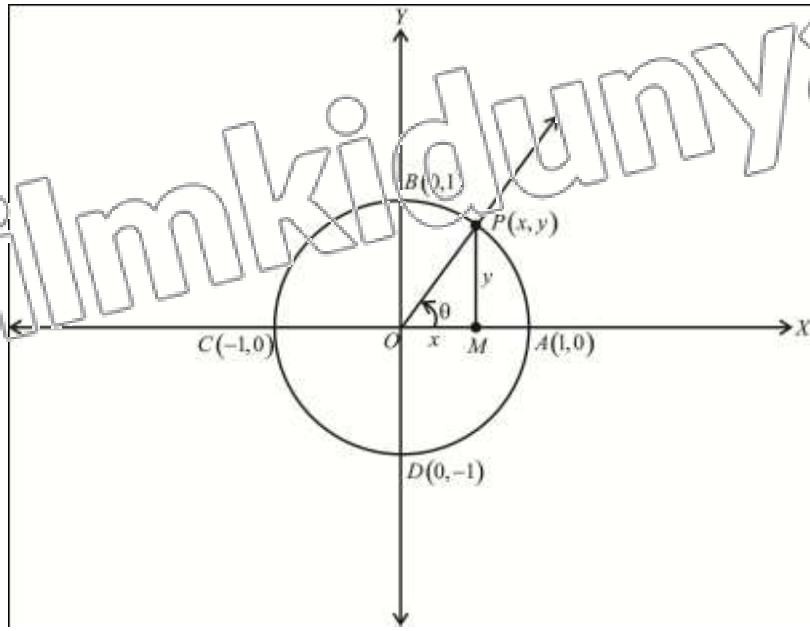
$$\Rightarrow \theta \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$$

$$\Rightarrow \theta \neq (2n+1)\frac{\pi}{2}, \text{where } n \in Z$$

$\therefore$ Domain of tangent function $= \mathbb{R} - \left\{x \mid x = (2n+1)\frac{\pi}{2}, n \in Z\right\}$

Range of tangent function $= \mathbb{R} =$ set of real numbers

Domain and Range of Cotangent Function:



Let $P(x, y)$ be any point on the unit circle with centre at the origin such that $\angle XOP = \theta$ is in standard position, then in right triangle OMP,

$$\cot\theta = \frac{|\overline{OM}|}{|\overline{PM}|}$$

$$\cot\theta = \frac{x}{y}, \quad y \neq 0$$

$\Rightarrow$ Terminal side $\overrightarrow{OP}$ should not coincide with OX or OX' (i.e. $X - axis$)

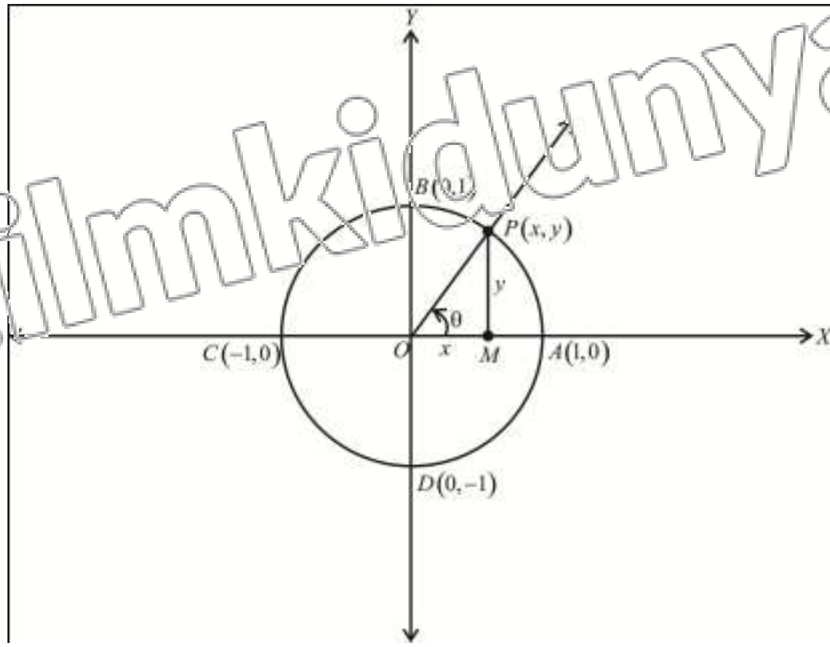
$\Rightarrow \theta \neq 0, \pm\pi, \pm 2\pi, \ldots$

$\Rightarrow \theta \neq n\pi,$ Where $n \in Z$

$\therefore$ Domain of cotangent function $= \mathbb{R} - \{x \mid x = n\pi,\ n \in Z\}$

Range of cotangent function $= \mathbb{R}$ = set of real numbers

Domain and Range of Secant Function:



Let $P(x, y)$ be any point on the unit circle with centre at the origin such that $\angle XOP = \theta$ is in standard position, then in right triangle OMP,

$$\sec\theta = \frac{|\overline{OP}|}{|\overline{OM}|}$$

$$\sec\theta = \frac{1}{x}, \quad x \neq 0$$

$\Rightarrow$ Terminal side $\overrightarrow{OP}$ should not coincide with OY or OY' (i.e. $Y-axis$)

$$\Rightarrow \theta \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$$

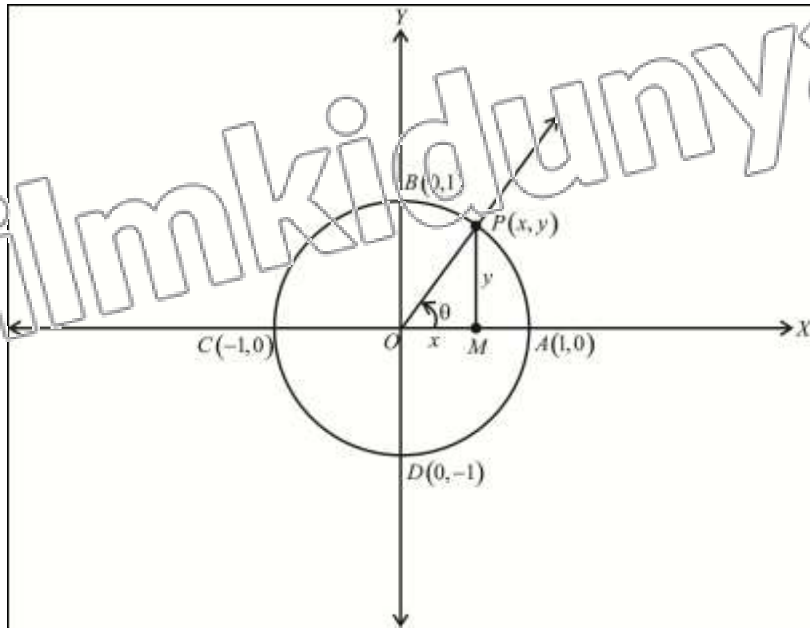
$$\Rightarrow \theta \neq (2n+1)\frac{\pi}{2}, \text{Where } n \in Z$$

$\therefore$ Domain of secant function $= \mathbb{R} - \left\{x \mid x = (2n+1)\frac{\pi}{2},\ n \in Z\right\}$

As $\sec\theta$ attains all real values except those between -1 and 1.

$\therefore$ Range of secant function $= \mathbb{R} - \{x \mid -1 < x < 1\}$

Domain and Range of Cosecant Function:



Let $P(x,y)$ be any point on the unit circle with centre at the origin such that $\angle XOP=\theta$ is in standard position, then in right triangle OMP,

$$\csc\theta=\frac{|\overline{OP}|}{|\overline{PM}|}$$

$$\csc\theta=\frac{1}{y}\ ,\quad y\neq 0$$

$\Rightarrow$ Terminal side $\overrightarrow{OP}$ should not coincide with OX or OX' (i.e. $X-axis$)

$\Rightarrow \theta\neq 0,\pm\pi,\pm 2\pi,...$

$\Rightarrow \theta\neq n\pi,$ Where $n\in Z$

$\therefore$ Domain of cosecant function $=\mathbb{R}-\{x\,|\,x=n\pi,\ n\in Z\}$ As $\csc\theta$ attains all values except those between –1 and 1.

$\therefore$ Range of cosecant function $=\mathbb{R}-\{x\,|\,-1<x<1\}$

The following table summarize the domain and ranges of the trigonometric functions:

Trigonometric Function	Domain	Range
$y=\sin x$	$-\infty<x<\infty$	$-1\leq y\leq 1$
$y=\cos x$	$-\infty<x<\infty$	$-1\leq y\leq 1$
$y=\tan x$	$-\infty<x<\infty$ But $x\neq(2n+1)\frac{\pi}{2},\ n\in Z$	$-\infty<y<\infty$
$y=\cot x$	$-\infty<x<\infty$ But $x\neq n\pi\ ,\ n\in Z$	$-\infty<y<\infty$
$y=\sec x$	$-\infty<x<\infty$ But $x\neq(2n+1)\frac{\pi}{2}, n\in Z$	$y\leq -1$ or $y\geq 1$
$y=\csc x$	$-\infty<x<\infty$ But $x\neq n\pi,\ \ n\in Z$	$y\leq -1$ or $y\geq 1$

Periodicity:

All the six trigonometric functions repeat their values for each increase or decrease of 2π in θ i.e. the values of trigonometric functions for θ and $\theta \pm 2n\pi$, where $\theta \in \mathbb{R}$ and $n \in Z$, are the same.

This behavior of trigonometric functions is called periodicity.

Period of Trigonometric Functions:

Period of a trigonometric function is the smallest positive number which, when added to the original circular measure of the angle, gives the same value of the function.

Let us now discover the periods of trigonometric functions.

Theorem:

Sine is a periodic function and its period is 2π.

Proof:

Suppose p is the period of sine function such that

$$\sin(\theta + p) = \sin\theta \quad \forall \theta \in \mathbb{R} \qquad \text{(i)}$$

Put $\theta = 0$, we have

$$\sin(0 + p) = \sin 0$$

$$\Rightarrow \sin p = 0$$

$$p = \pi, 2\pi, 3\pi, ...$$ (p cannot take zero and negative values)

Checking $p = \pi$ in equation (i)

L.H.S: $\sin(\theta + \pi) = -\sin\theta \neq$ R.H.S

$\therefore$ π is not the period of $\sin\theta$

Checking $p = 2\pi$ in equation (i)

L.H.S: $\sin(\theta + 2\pi) = \sin\theta =$ R.H.S

As 2π is the smallest positive real number for which

$$\sin(\theta + 2\pi) = \sin\theta$$

$\therefore$ 2π is the period of $\sin\theta$.

Theorem:

Cosine is a periodic function and its period is 2π.

Proof:

Suppose p is the period of cosine function such that

$$\cos(\theta + p) = \cos\theta \quad \forall \theta \in \mathbb{R} \qquad \text{(i)}$$

Put $\theta = 0$

$$\cos(0 + p) = \cos 0$$

$$\cos p = 1$$

$p = 2\pi, 4\pi, 6\pi, ...$ (p cannot take zero and negative values)

Checking $p = 2\pi$ in equation (i)

L.H.S: $\cos(\theta + 2\pi) = \cos\theta =$ R.H.S

As 2π is the smallest positive real number for which

$\cos(\theta + 2\pi) = \cos\theta$

$\therefore$ 2π is the period of $\cos\theta$

Theorem:

Tangent is a periodic function and its period is π.

Proof:

Suppose p is the period of tangent function such that

$\tan(\theta + p) = \tan\theta \quad \forall \theta \in \mathbb{R}$ (i)

Put $\theta = 0$

$\tan(0 + p) = \tan 0$

$\tan p = 0$

$p = \pi, 2\pi, 3\pi, ...$ (p cannot take zero and negative values)

Checking $p = \pi$ in equation (i)

L.H.S: $\tan(\theta + \pi) = \tan\theta$ =R.H.S

As π is the smallest positive real number for which

$\tan(\theta + \pi) = \tan\theta$

$\therefore \pi$ is the period of $\tan\theta$.

Theorem:

Cotangent is a periodic function and its period is π.

Proof:

Suppose p is the period of cotangent function such that

$\cot(\theta + p) = \cot\theta \quad \forall \theta \in \mathbb{R}$ (i)

Put $\theta = 0$

$\cot(0 + p) = \cot 0$

$\cot p =$ undefined

$p = \pi, 2\pi, 3\pi.....$ (p cannot take zero and negative values)

Checking $p = \pi$ in equation (i)

L.H.S: $\cot(\theta + \pi) = \cot\theta$ =R.H.S

As π is the smallest positive real number for which

$\cot(\theta+\pi)=\cot\theta$

$\therefore \pi$ is the period of $\cot\theta$.

Theorem:

Secant is a periodic function and its period is 2π.

Proof:

Suppose p is the period of secant function such that

$\sec(\theta+p)=\sec\theta \qquad \forall\theta\in\mathbb{R}$ (i)

Put $\theta=0$

$\sec(0+p)=\sec 0$

$\sec p=1$

$p=2\pi,4\pi,6\pi.....$ (p cannot take zero and negative values)

Checking $p=2\pi$ in equation (i)

L.H.S: $\sec(\theta+2\pi)=\sec\theta$ =R.H.S

As 2π is the smallest positive real number for which

$\sec(\theta+2\pi)=\sec\theta$

$\therefore 2\pi$ is the period of $\sec\theta$

Theorem:

Cosecant is a periodic function and its period is 2π.

Proof:

Suppose p is the period of cosecant function such that

$\text{cosec}(\theta+p)=\text{cosec}\,\theta \qquad \forall\theta\in\mathbb{R}$ (i)

Put $\theta=0$

$\text{cosec}(0+p)=\text{cosec}\,0$

$\text{cosec}\,p=\text{undefined}$

$p=\pi,2\pi,3\pi,.....$ (p cannot take zero and negative values)

Checking $p=\pi$ in equation (i)

L.H.S: $\text{cosec}(\theta+\pi)=-\text{cosec}\,\theta\neq$ R.H.S

$\therefore \pi$ is not the period of $\text{cosec}\,\theta$

Checking $p=2\pi$ in equation (i)

L.H.S: $\text{cosec}(\theta+2\pi)=\text{cosec}\,\theta$ =R.H.S

As 2π is the smallest positive real number for which

$\text{cosec}(\theta + 2\pi) = \text{cosec}\,\theta$

$\therefore 2\pi$ is the period of $\text{cosec}\,\theta$.

The following table shows the periods of trigonometric functions.

Trigonometric Function	Period
Sine	2π
Cosine	2π
Secant	2π
cosecant	2π
tangent	π
cotangent	π

EXERCISE 11.1

Find the periods of the following functions:

Q.1 $\sin 3x$

Solution:

$\sin 3x$

We know that the period of sine is 2π.

$\therefore \sin(3x+2\pi)=\sin 3x$

$\Rightarrow \sin 3\left(x+\frac{2\pi}{3}\right)=\sin 3x$

It means that the value of $\sin 3x$ repeats when x is increased by $\frac{2\pi}{3}$.

Hence $\frac{2\pi}{3}$ is the period of $\sin 3x$.

Q.2 $\cos 2x$

Solution:

$\cos 2x$

We know that the period of cosine is 2π.

$\therefore \cos(2x+2\pi)=\cos 2x$

$\Rightarrow \cos 2(x+\pi)=\cos 2x$

It means that the value of $\cos 2x$ repeats when x is increased by π.

Hence π is the period of $\cos 2x$.

Q.3 $\tan 4x$

Solution:

$\tan 4x$

We know that the period of tangent is π

$\therefore \tan(4x+\pi)=\tan 4x$

$\Rightarrow \tan 4\left(x+\frac{\pi}{4}\right)=\tan 4x$

It means that the value of $\tan 4x$ repeats when x is increased by $\frac{\pi}{4}$.

Hence $\frac{\pi}{4}$ is the period of $\tan 4x$.

Q.4 $\cot\frac{x}{2}$

Solution:

$\cot\frac{x}{2}$

We know that the period of cotangent is π.

$\therefore \cot\left(\frac{x}{2}+\pi\right)=\cot\frac{x}{2}$

$\Rightarrow \cot\frac{1}{2}(x+2\pi)=\cot\frac{x}{2}$

It means that the value of $\cot\frac{x}{2}$ repeats when x is increased by 2π.

Hence 2π is the period of $\cot\frac{x}{2}$.

Q.5 $\sin\frac{x}{3}$

Solution:

$\sin\frac{x}{3}$

We know that the period of sine is 2π

$\therefore \sin\left(\frac{x}{3}+2\pi\right)=\sin\frac{x}{3}$

$\Rightarrow \sin\frac{1}{3}(x+6\pi)=\sin\frac{x}{3}$

It means that the value of $\sin\frac{x}{3}$ repeats when x is increased by 6π.

Hence 6π is the period of $\sin\frac{x}{3}$.

Q.6 **$\text{cosec}\dfrac{x}{4}$**

Solution:

$\text{cosec}\dfrac{x}{4}$

We know that the period of cosecant is 2π.

$\therefore \text{cosec}\left(\dfrac{x}{4}+2\pi\right)=\text{cosec}\dfrac{x}{4}$

$\Rightarrow \text{cosec}\dfrac{1}{4}(x+8\pi)=\text{cosec}\dfrac{x}{4}$

It mean that the value of $\text{cosec}\dfrac{x}{4}$ repeats when x is increased by 8π.

Hence 8π is the period of $\text{cosec}\dfrac{x}{4}$.

Q.7 **$\sin\dfrac{x}{5}$**

Solution:

$\sin\dfrac{x}{5}$

We know that the period of sine is 2π.

$\therefore \sin\left(\dfrac{x}{5}+2\pi\right)=\sin\dfrac{x}{5}$

$\Rightarrow \sin\dfrac{1}{5}(x+10\pi)=\sin\dfrac{x}{5}$

It means that the value of $\sin\dfrac{x}{5}$ repeats when x is increased by 10π.

Hence 10π is the period of $\sin\dfrac{x}{5}$.

Q.8 **$\cos\dfrac{x}{6}$**

Solution:

$\cos\dfrac{x}{6}$

We know that the period of cosine is 2π .

$\because \cos\left(\dfrac{x}{6}+2\pi\right)=\cos\dfrac{x}{6}$

$\Rightarrow \cos\dfrac{1}{6}(x+12\pi)=\cos\dfrac{x}{6}$

It means that the value of $\cos\dfrac{x}{6}$ repeats when x is increased by 12π.

Hence 12π is the period of $\cos\dfrac{x}{6}$.

Q.9 **$\tan\dfrac{x}{7}$**

Solution:

$\tan\dfrac{x}{7}$

We know that the period of tangent is π.

$\therefore \tan\left(\dfrac{x}{7}+\pi\right)=\tan\dfrac{x}{7}$

$\Rightarrow \tan\dfrac{1}{7}(x+7\pi)=\tan\dfrac{x}{7}$

It means that the value of $\tan\dfrac{x}{7}$ repeats when x is increased by 7π.

Hence 7π is the period of $\tan\dfrac{x}{7}$.

Q.10 **$\cot 8x$**

Solution:

$\cot 8x$

We know that the period of cotangent is π.

$\therefore \cot(8x+\pi)=\cot 8x$

$\Rightarrow \cot 8\left(x+\dfrac{\pi}{8}\right)=\cot 8x$

It means that the value of $\cot 8x$ repeats when x is increased by $\frac{\pi}{8}$.

Hence $\frac{\pi}{8}$ is the period of cot8x.

Q.11 sec9*x*

Solution:

$\sec 9x$

We know that the period of secant is 2π.

$\therefore \sec(9x+2\pi)=\sec 9x$

$\Rightarrow \sec 9\left(x+\frac{2\pi}{9}\right)=\sec 9x$

It means that the value of sec9x repeats when x is increased by $\frac{2\pi}{9}$.

Hence $\frac{2\pi}{9}$ is the period of sec9x.

Q.12 cosec10*x*

Solution:

$\operatorname{cosec}10x$

We know that the period of cosecant is 2π.

$\therefore \operatorname{cosec}(10x+2\pi)=\operatorname{cosec}10x$

$\Rightarrow \operatorname{cosec}10\left(x+\frac{\pi}{5}\right)=\operatorname{cosec}10x$

It means that the value of cosec10x repeats when x is increased by $\frac{\pi}{5}$.

Hence $\frac{\pi}{5}$ is the period of cosec10x.

Q.13 3sin *x*

Solution:

$3\sin x$

We know that the period of sine is 2π.

$\therefore 3\sin(x+2\pi)=3\sin x$

It means that the value of 3sin$x$ repeats when x is increased by 2π.

Hence 2π is the period of 3sin$x$.

Q.14 2cos *x*

Solution:

$2\cos x$

We know that the period of cosine is 2π

$\therefore 2\cos(x+2\pi)=2\cos x$

It means that the value of 2cos$x$ repeats when x is increased by 2π.

Hence 2π is the period of 2cos$x$.

Q.15 $3\cos\frac{x}{5}$

Solution:

$3\cos\frac{x}{5}$

We know that the period of cosine is 2π.

$\therefore 3\cos\left(\frac{x}{5}+2\pi\right)=3\cos\frac{x}{5}$

$\Rightarrow 3\cos\frac{1}{5}(x+10\pi)=3\cos\frac{x}{5}$

It means that the value of $3\cos\frac{x}{5}$ repeats when x is increased by 10π.

Hence 10π is the period of $3\cos\frac{x}{3}$.

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Graphs of Trigonometric Function:

We shall now learn the methods of drawing the graphs of all the six trigonometric functions.

These graphs are used very often in calculus and social sciences. The following procedure is adopted to draw the graphs of the trigonometric functions:

(i) Table of ordered Pairs $P(x, y)$ is constructed, when x is the measure of the angle and y is the value of the trigonometric ratio for the angle of measure x.

(ii) The measures of the angles are taken along the $x-axis$.

(iii) The values of the trigonometric functions are taken along the $y-axis$.

(iv) The points corresponding to the ordered pairs are plotted on the graph paper.

(v) These points are joined with the help of smooth curves.

Note:

(i) The graphs of trigonometric functions will be smooth curves.

(ii) none of them graph will be line segments or will have sharp corners or breaks within their domains.

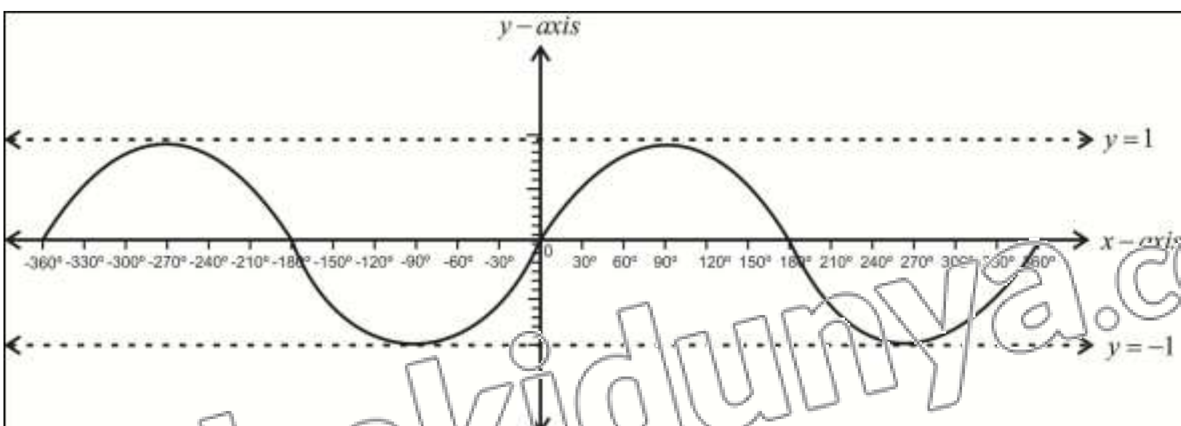
(iii) This behavior of the curves is called continuity.

(iv) The graphs of trigonometric functions are continuous, wherever they are defined.

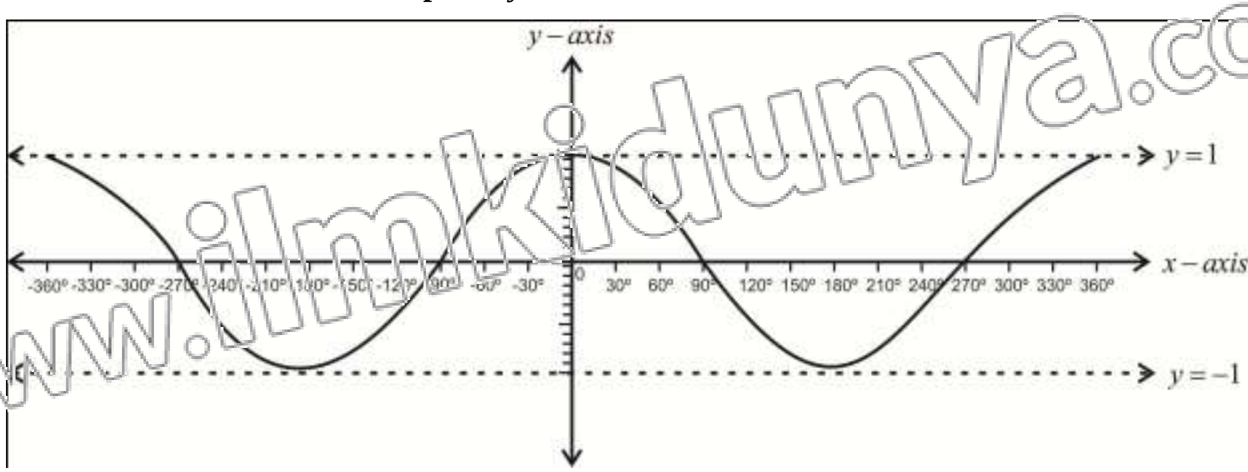
(v) As trigonometric functions are periodic so their curves repeat after a fixed interval.

Graph:

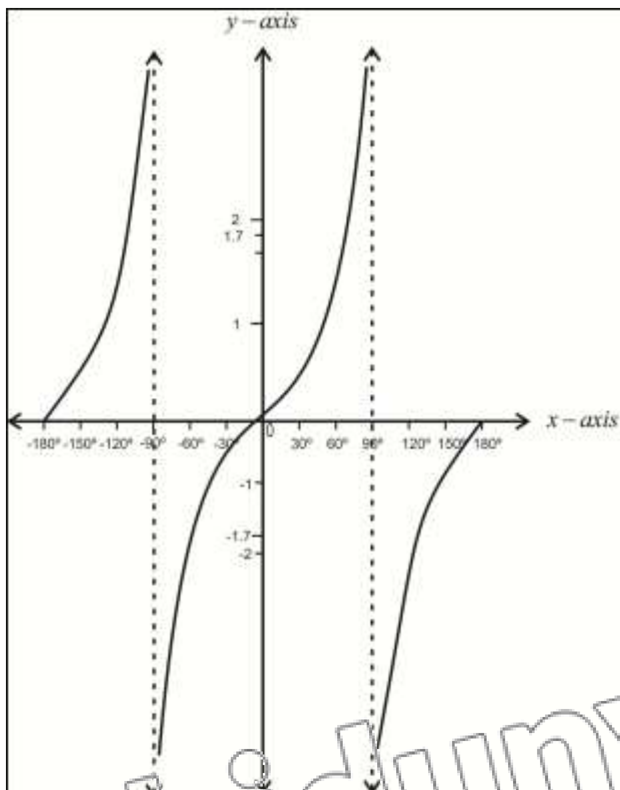
Graph of $y = \sin x$ **from** -360^o **to** 360^o



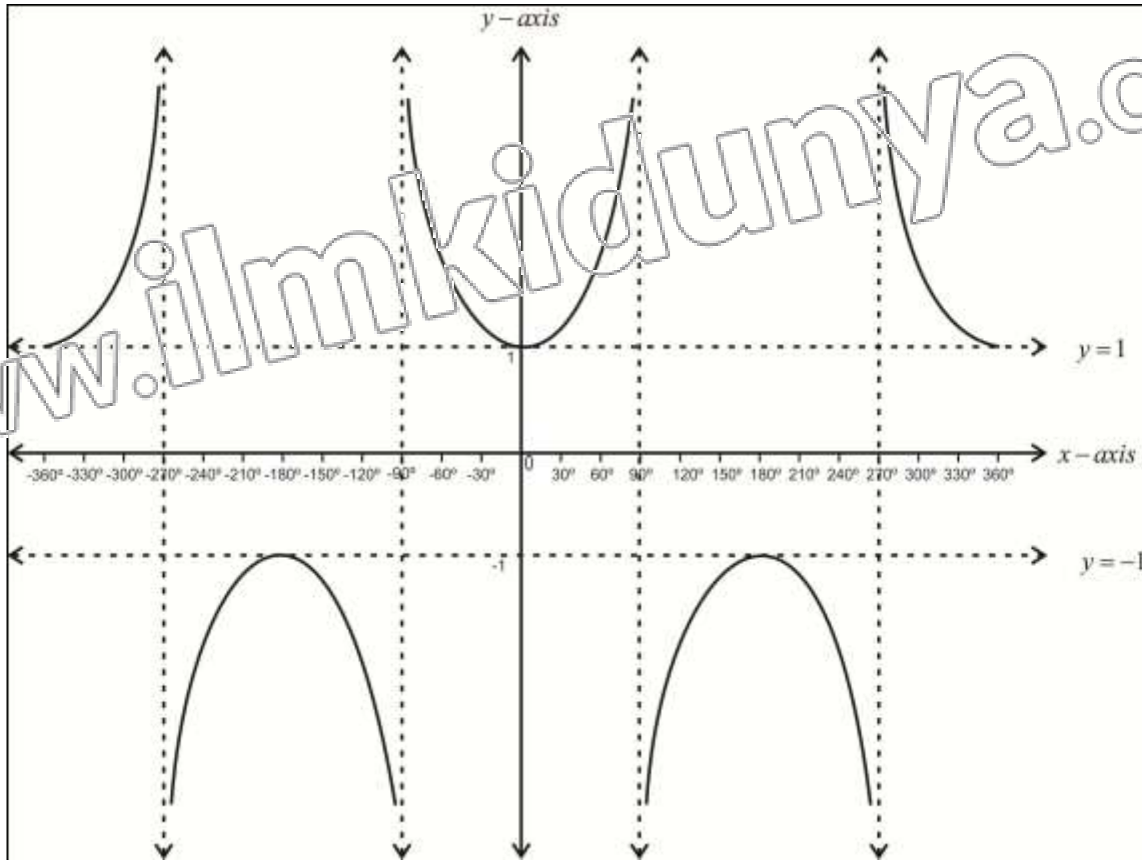
Graph of $y = \cos x$ **from** -360° **to** 360°



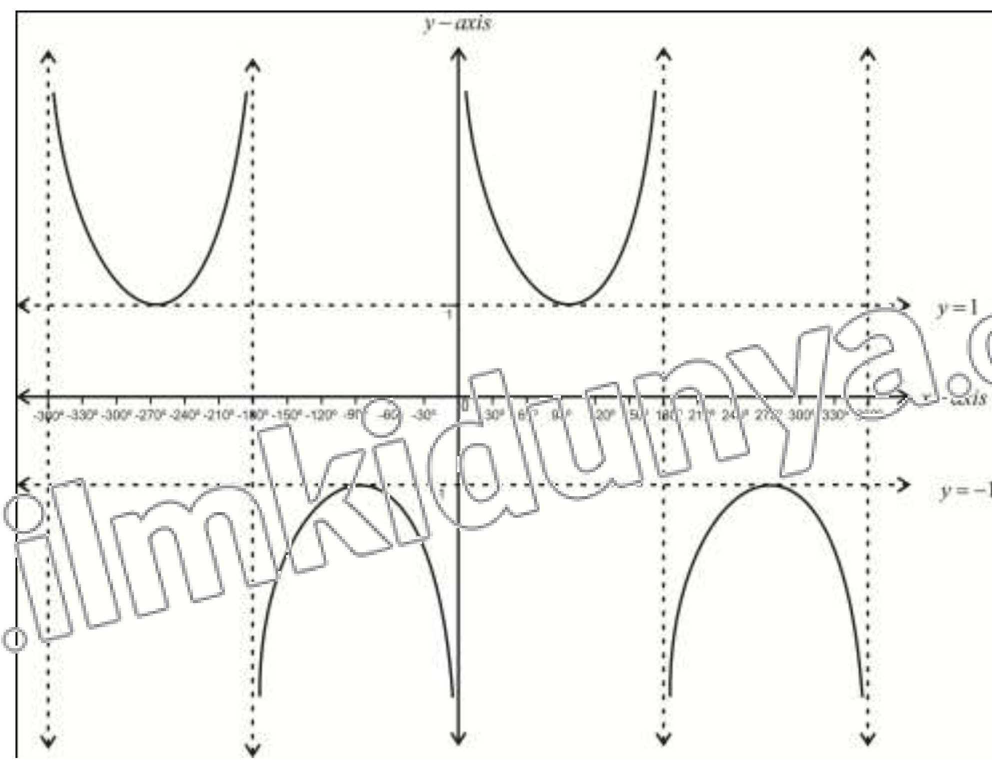
Graph of $y = \tan x$ **from** -180° **to** 180°



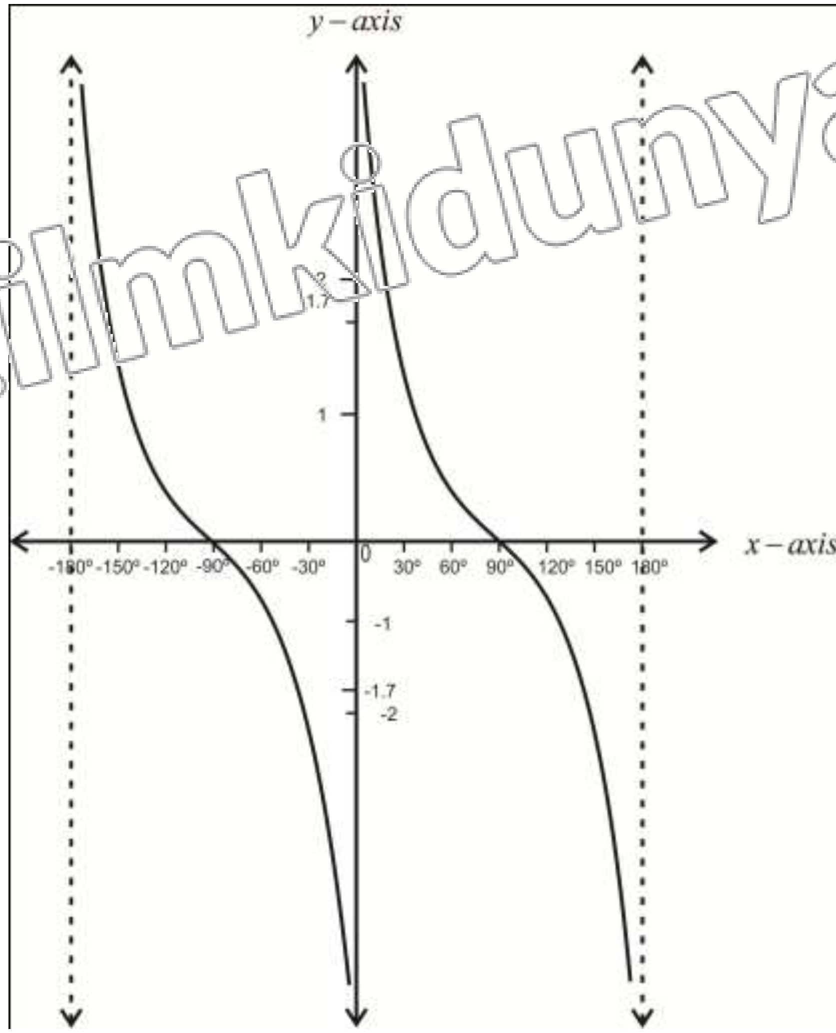
Graph of $y = \sec x$ **from** -360° **to** 360°



Graph of $y = \operatorname{cosec} x$ **from** -360^{o} **to** 360^{o}



Graph of $y = \cot x$ **from** -180^{o} **to** 180^{o}



Note:

(i) From the graphs of trigonometric function we can check their domains and ranges.

(ii) By making use of the periodic property, each one of these graphs can be extended on the left as well as on the right side of $x-axis$ depending upon the period of the functions.

(iii) The dashes lines are vertical asymptotes in the graphs of $\tan x, \cot x, \sec x$ *and* $\csc x$.

EXERCISE 11.2

Q.1 Draw the graph of each of the following function for the intervals mentioned against each:

(i) $y = -\sin x, \quad x \in [-2\pi, 2\pi]$

(ii) $y = 2\cos x, \quad x \in [0, 2\pi]$

(iii) $y = \tan 2x, \quad x \in [-\pi, \pi]$

(iv) $y = \tan x, \quad x \in [-2\pi, 2\pi]$

(v) $y = \sin\frac{x}{2}, \quad x \in [0, 2\pi]$

(vi) $y = \cos\frac{x}{2}, \quad x \in [-\pi, \pi]$

Solution:

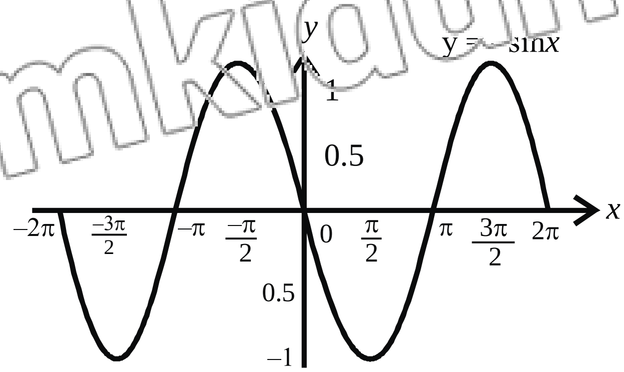
(i) $y = -\sin x, \quad x \in [-2\pi, 2\pi]$

Take the subintervals of the given interval $[-2\pi, 2\pi]$, each of length $\frac{\pi}{6}$, we form the following table of values:

x	-2π	$-\frac{11\pi}{6}$	$-\frac{5\pi}{3}$	$-\frac{3\pi}{2}$	$-\frac{4\pi}{3}$	$-\frac{7\pi}{6}$	$-\pi$	$-\frac{5\pi}{6}$	$-\frac{2\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0
$y = -\sin x$	0	–0.5	–0.87	–1	–0.87	–0.5	0	0.5	0.87	1	0.87	0.5	0

x	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$y = -\sin x$	–0.5	–0.87	–1	–0.87	–0.5	0	0.5	0.87	1	0.87	0.5	0

Taking some suitable scale along x-axis and y-axis, we draw the following graph of $y = -\sin x$ in the interval $[-2\pi, 2\pi]$.

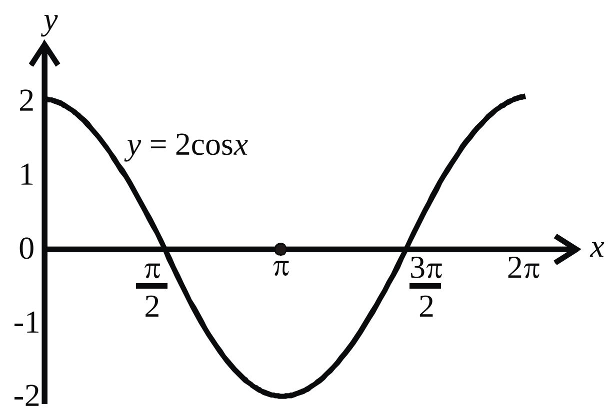


(ii) $y = 2\cos x, \quad x \in [0, 2\pi]$

Take the subintervals of the given interval $[0, 2\pi]$, each of length $\frac{\pi}{6}$, we form the following table of values:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$y = \cos x$	1	0.87	0.5	0	–0.5	–0.87	–1	–0.87	–0.5	0	0.5	0.87	1
$y = 2\cos x$	2	1.7	1	0	–1	–1.7	–2	–1.7	–1	0	1	1.7	2

Taking some suitable scale along x-axis and y-axis, we draw the graph of $y = 2\cos x$ in the interval $[0, 2\pi]$.

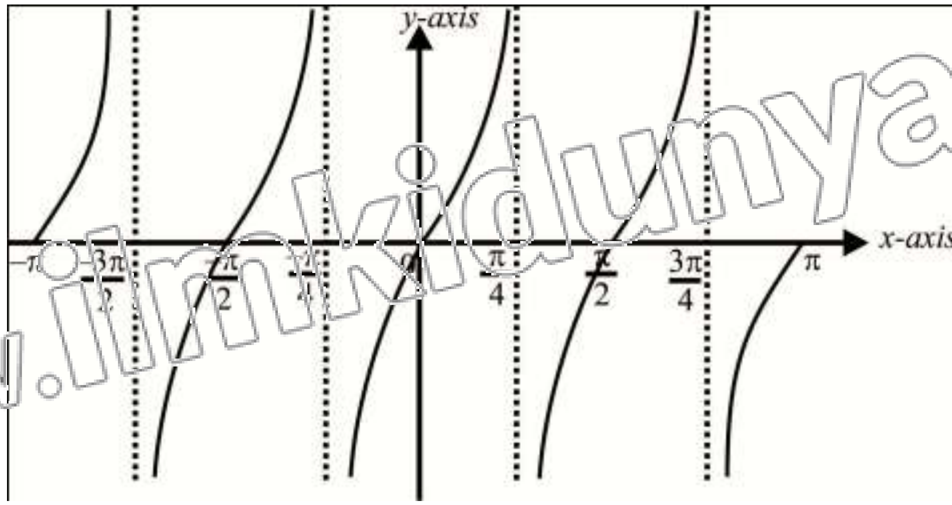


(iii) $y = \tan 2x, \quad x \in [-\pi, \pi]$

Take the subintervals of the given interval $[-\pi, \pi]$, each of length $\frac{\pi}{6}$, we form the following table of values:

x	$-\pi$	$-\frac{5\pi}{6}$	$-\frac{2\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$2x$	-2π	$-\frac{5\pi}{3}$	$-\frac{4\pi}{3}$	$-\pi$	$-\frac{2\pi}{3}$	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$y = \tan 2x$	0	1.7	–1.7	0	1.7	–1.7	0	1.7	–1.7	0	1.7	– 1.7	0

Taking some suitable scale along x-axis and y-axis, we draw the following graph of $y = \tan 2x$ in the interval $[-\pi, \pi]$.

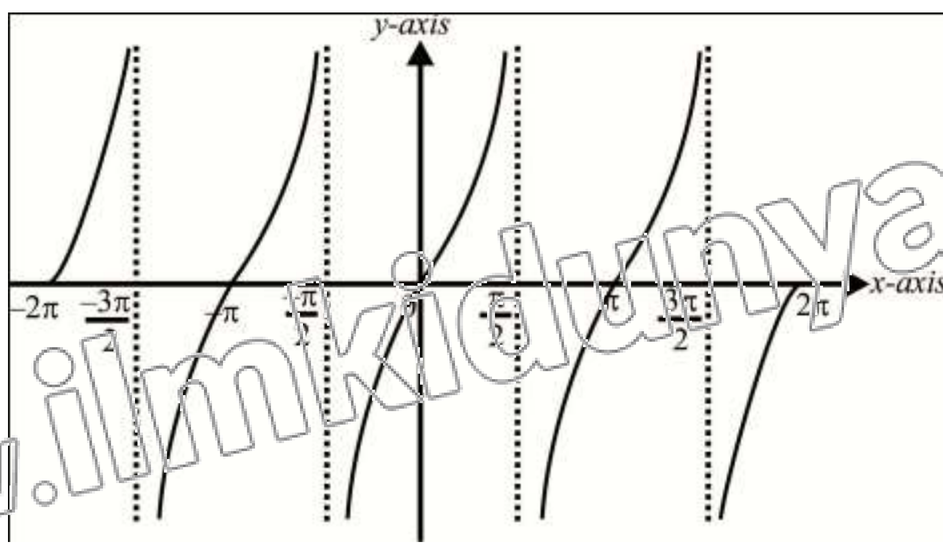


(iv) $y = \tan x$, $x \in [-2\pi, 2\pi]$

Take the subintervals of the given interval $[-2\pi, 2\pi]$, each of length $\frac{\pi}{3}$, we form the following table of values:

x	-2π	$-\frac{5\pi}{3}$	$-\frac{4\pi}{3}$	$-\pi$	$-\frac{2\pi}{3}$	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$y = \tan x$	0	1.7	–1.7	0	1.7	–1.7	0	1.7	–1.7	0	1.7	–1.7	0

Taking some suitable scale along *x*-axis and *y*-axis, we draw the following graph of $y = \tan x$ in the interval $[-2\pi, 2\pi]$.

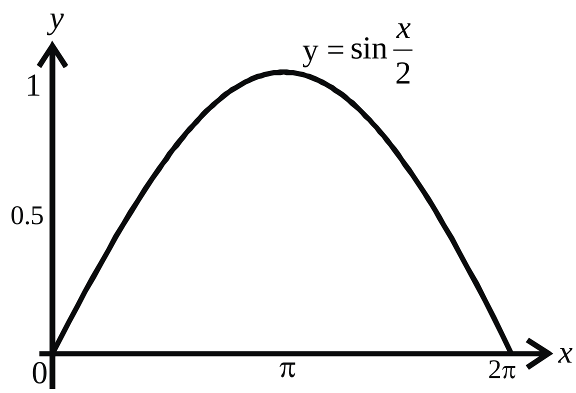


(v) $y=\sin\frac{x}{2}, \quad x\in[0,2\pi]$

Take the subintervals of the given interval $[0,2\pi]$, each of length $\frac{\pi}{6}$, we form the following table of values:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$\frac{x}{2}$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
$y=\sin\frac{x}{2}$	0	0.26	0.5	0.71	0.87	0.97	1	0.97	0.87	0.71	0.5	0.26	0

Taking some suitable scale along *x*-axis and y-axis, we draw the following graph of $y=\sin\frac{x}{2}$ in the interval $[0,2\pi]$.

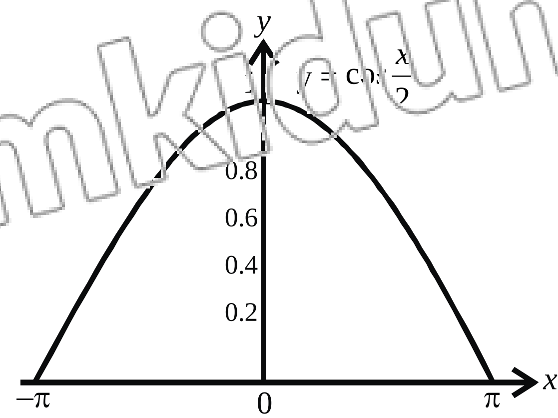


(vi) $y=\cos\frac{x}{2}, \quad x\in[-\pi,\pi]$

Take the subintervals of the given interval $[-\pi,\pi]$, each of length $\frac{\pi}{6}$, we form the following table of values:

x	$-\pi$	$-\frac{5\pi}{6}$	$-\frac{2\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$\frac{x}{2}$	$-\frac{\pi}{2}$	$-\frac{5\pi}{12}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$
$y=\cos\frac{x}{2}$	0	0.26	0.5	0.71	0.87	0.97	1	0.97	0.87	0.71	0.5	0.26	0

Taking some suitable scale along x-axis and y-axis, we draw the following graph of $y=\cos\frac{x}{2}$ in the interval $[-\pi,\pi]$.



Q.2 On the same axes and to the same scale, draw the graphs of the following function for their complete period:

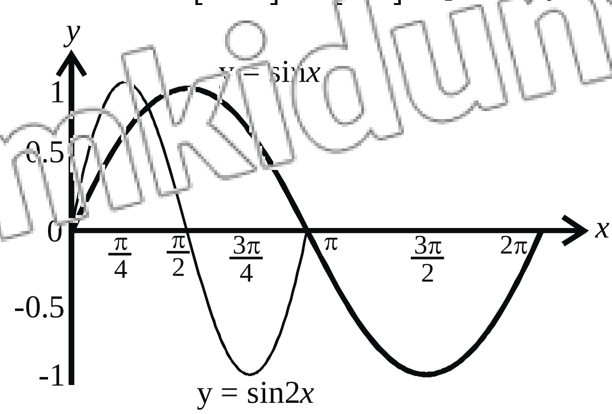
(i) $y=\sin x$ **and** $y=\sin 2x$

Solution:

The period of $\sin x$ is 2π and the period of $\sin 2x$ is π, so considering the subintervals of the interval $[0,2\pi]$, each of length $\frac{\pi}{6}$, we form the following table of values:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$2x$	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π	$\frac{7\pi}{3}$					
$y=\sin x$	0	0.5	0.87	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0
$y=\sin 2x$	0	0.87	0.87	0	−0.87	−0.87	0						

Taking some suitable scale along x-axis and y-axis, we draw the graphs of $\sin x$ and $\sin 2x$ in the intervals $[0,2\pi]$ and $[0,\pi]$ respectively.



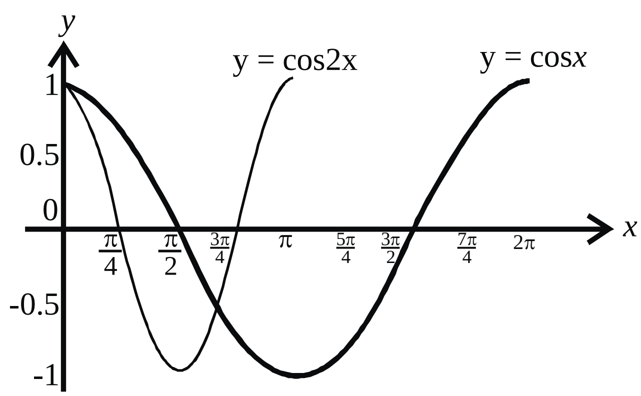
(ii) $y=\cos x$ **and** $y=\cos 2x$

Solution:

The period of $\cos x$ is 2π and the period of $\cos 2x$ is π, so considering the subintervals of the interval $[0, 2\pi]$, each of length $\frac{\pi}{6}$, we form the following table of values.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$2x$	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π						
$y=\cos x$	1	0.87	0.5	0	–0.5	–0.87	–1	–0.87	–0.5	0	0.5	0.87	1
$y=\cos 2x$	1	0.5	–0.5	–1	–0.5	0.5	1						

Taking some suitable scale along x-axis and y-axis, we draw the graphs of $\cos x$ and $\cos 2x$ in the intervals $[0, 2\pi]$ and $[0, \pi]$ respectively.



Q.3 Solve graphically:

(i) $\sin x = \cos x, \quad x \in [0, \pi]$

(ii) $\sin x = x, \quad x \in [0, \pi]$

Solution:

(i) $\sin x = \cos x, \quad x \in [0, \pi]$

Take subintervals of the interval $[0, \pi]$, each of length $\frac{\pi}{6}$, we form the following table of values of $\sin x$ and $\cos x$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$y=\sin x$	0	0.5	0.87	1	0.87	0.5	0
$y=\cos x$	1	0.87	0.5	0	–0.5	–0.87	–1

Taking some suitable scale along x-axis and y-axis, we draw the graphs of $\sin x$ and $\cos x$ in the interval $[0, \pi]$.

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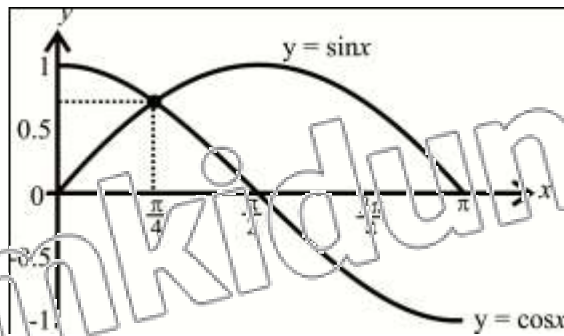


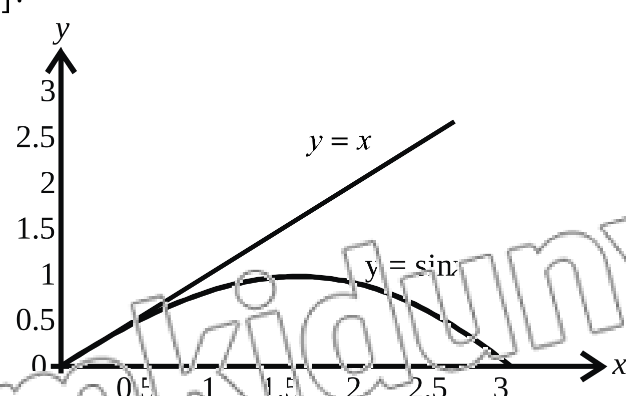
In the above figure, the two curves intersect each other at a point where $x = \frac{\pi}{4}$. The point of intersection of these curves is $\frac{1}{\sqrt{2}}$. Thus the solution of the equation $\sin x = \cos x$ in the interval $[0, \pi]$ is $x = \frac{\pi}{4}$.

(ii) $\sin x = x, \quad x \in [0, \pi]$

Take the subintervals of the intervals $[0, \pi]$, each of length $\frac{\pi}{6}$, we form the following table of values of $\sin x$ and x.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$y = \sin x$	0	0.5	0.87	1	0.87	0.5	0
$y = x$	0	$\frac{\pi}{6} = 0.52$	$\frac{\pi}{3} = 1.05$	$\frac{\pi}{2} = 1.57$	$\frac{2\pi}{3} = 2.10$	$\frac{5\pi}{6} = 2.62$	$\pi = 3.14$

Taking some suitable scale along x-axis and y-axis, we draw the graphs of $\sin x$ and x in the interval $[0, \pi]$.



The graph of $y = x$ is a straight line. The solution of equation $\sin x = x$ is the point of intersection of the curve $y = \sin x$ and the line $y = x$. The line and the curve intersect each other at $x = 0$. Hence solution is $x = 0$.