CHAPTER 14

SOLUTIONS OF TRIGONOMETRIC EQUATIONS

Trigonometric Equations:

The equations, containing at least one trigonometric function, are called Trigonometric Equations e.g. each of the following is a trigonometric equation:

$\sin x = \frac{2}{5}, \sec x = \tan x$ and $\sin^2 x - \sec x + 1 = \frac{3}{4}$

Note:

Trigonometric equations have infinite number of solutions due to the periodicity of the trigonometric functions.

For Example:

If $\sin\theta = 0$ then $\theta = 0, \pm\pi, \pm 2\pi, ...$

which can be written as $\theta = n\pi$; when $n \in Z$

Reference Angle:

The reference angle is the smallest angle between the terminal side and $x - axis$

General Solution:

A general solution to a trigonometric equation is a set of expressions that represents all possible solutions.

Method of Solving Trigonometric Equations:

In solving trigonometric equations, first we find the solution over the interval whose length is equal to its period and then find the general solution.

Solution of General Trigonometric Equations:

When a trigonometric equation contains more than one trigonometric function, trigonometric identities and algebraic formulae are used to transform such trigonometric equation to an equivalent equation that contains only one trigonometric function.

Note:

(i) In solving the equations of the form $\sin kx = c$, we first find the solution of $\sin u = c$ (where $kx = u$) and then required solution is obtained by dividing each term of this solution set by k.

For Example:

$$\sin x\cos x=\frac{\sqrt{3}}{4}$$

$$\Rightarrow \frac{1}{2}(2\sin x\cos x)=\frac{\sqrt{3}}{4}$$

$$\Rightarrow \sin 2x=\frac{\sqrt{3}}{2}$$

$\because$ $\sin 2x$ is positive in I and II quadrants with the reference angle $2x=\frac{\pi}{3}$

$\therefore$ $2x=\frac{\pi}{3}$ and $2x=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$ are two solutions in $[0,2\pi]$

As 2π is the period of $\sin 2x$.

$\therefore$ General values of $2x$ are $\frac{\pi}{3}+2n\pi$ and $\frac{2\pi}{3}+2n\pi$, $n\in Z$

$\Rightarrow$ General values of x are $\frac{\pi}{6}+n\pi$ and $\frac{\pi}{3}+n\pi$, $n\in Z$

Hence solution set $=\left\{\frac{\pi}{6}+n\pi\right\}\cup\left\{\frac{\pi}{3}+n\pi\right\}$, $n\in Z$

(ii) Sometimes it is necessary to square both sides of a trigonometric equation. In such a case, extraneous roots can occur which are to be discarded. So each value of x must be check by substituting it in the given equation.

For Example:

$$\cos ecx=\sqrt{3}+\cot x$$

$$\frac{1}{\sin x}=\sqrt{3}+\frac{\cos x}{\sin x}$$

$$1=\sqrt{3}\sin x+\cos x$$

$$1-\cos x=\sqrt{3}\sin x$$

$$(1-\cos x)^2=\left(\sqrt{3}\sin x\right)^2$$

$$1-2\cos x+\cos^2 x=3\sin^2 x$$

$$1-2\cos x+\cos^2 x=3\left(1-\cos^2 x\right)$$

$4\cos^2 x - 2\cos x - 2 = 0$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = 1$

If $\cos x = -\frac{1}{2}$

Since $\cos x$ is negative in II and III quadrant with the reference angle $x = \frac{\pi}{3}$

$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ and $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$ where $x \in [0, 2\pi]$

Now $x = \frac{4\pi}{3}$ does not satisfy the given equation (i)

$\therefore$ $x = \frac{4\pi}{3}$ is not admissible and so $x = \frac{2\pi}{3}$ is the only solution.

Since 2π is the period of $\cos x$

$\therefore$ General value of x is $\frac{2\pi}{3} + 2n\pi$, $n \in Z$

If $\cos x = 1$

$x = 0$ and $x = 2\pi$ where $x \in [0, 2\pi]$

Now both $\operatorname{cosec} x$ and $\cot x$ are not defined for $x = 0$ and $x = 2\pi$

$\therefore$ $x = 0$ and $x = 2\pi$ are not admissible.

Hence solution set $= \left\{\frac{2\pi}{3} + 2n\pi\right\}$, $n \in Z$

EXERCISE 14

Q.1 Find the solutions of the following equations which lies in $[0, 2\pi]$

(i) $\sin x = \frac{-\sqrt{3}}{2}$

Solution:

$$\sin x = \frac{-\sqrt{3}}{2}$$

$\sin x$ is negative in III and IV Quadrant with reference angle $x = \frac{\pi}{3}$

In III Quadrant	In IV Quadrant
$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$	$x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$\text{Solution Set } = \left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$$

(ii) $\text{cosec}\,\theta = 2$

Solution:

$$\text{cosec}\,\theta = 2$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2}$$

$\sin\theta$ is positive in I and II Quadrant with the reference angle $\theta = \frac{\pi}{6}$

In I Quadrant	In II Quadrant
$\theta = \frac{\pi}{6}$	$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$$\text{Solution Set } = \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$$

(iii) $\sec x = -2$

Solution:

$$\sec x = -2$$

$$\Rightarrow \quad \cos x = \frac{-1}{2}$$

$\cos x$ is negative in II and III Quadrant with reference angle $x = \frac{\pi}{3}$

In II Quadrant	In III Quadrant
$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$	$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$$\text{Solution Set} = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$$

(iv) $\cot\theta = \frac{1}{\sqrt{3}}$

Solution:

$\cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\tan\theta$ is positive in I and III Quadrant with reference angle $\theta = \frac{\pi}{3}$

In I Quadrant	In III Quadrant
$\theta = \frac{\pi}{3}$	$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$$\text{Solution Set } = \left\{\frac{\pi}{3}, \frac{4\pi}{3}\right\}$$

Q.2 Solve the following trigonometric equations:

(i) $\tan^2\theta = \frac{1}{3}$

Solution:

$\tan^2\theta = \frac{1}{3}$

Taking square root on both sides

$\Rightarrow \quad \tan\theta = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$	$\Rightarrow \quad \tan\theta = \frac{-1}{\sqrt{3}}$
$\tan\theta$ is positive in I and III Quadrant with reference angle $\theta = \frac{\pi}{6}$	$\tan\theta$ is negative in II and IV Quadrant with reference angle $\theta = \frac{\pi}{6}$

In I Quadrant	In III Quadrant	In II Quadrant	In IV Quadrant
$\theta = \frac{\pi}{6}$	$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$	$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$	$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$$\text{Solution Set } = \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$$

(ii) $\mathbf{cosec^2\theta = \frac{4}{3}}$

Solution:

$$\text{cosec}^2\theta = \frac{4}{3}$$

$$\Rightarrow \quad \sin^2\theta = \frac{3}{4}$$

Taking square root on both sides

$$\Rightarrow \quad \sin\theta = \pm\frac{\sqrt{3}}{2}$$

$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}$	$\Rightarrow \quad \sin\theta = -\frac{\sqrt{3}}{2}$
$\sin\theta$ is positive in I and II Quadrant with reference angle $\theta = \frac{\pi}{3}$	$\sin\theta$ is negative in III and IV Quadrant with reference angle $\theta = \frac{\pi}{3}$

In I Quadrant	In II Quadrant	In III Quadrant	In IV Quadrant
$\theta = \frac{\pi}{3}$	$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$	$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$	$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$$\text{Solution Set} = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$$

(iii) $\mathbf{sec^2\theta = \frac{4}{3}}$

Solution:

$$\sec^2\theta = \frac{4}{3}$$

$$\Rightarrow \quad \cos^2\theta = \frac{3}{4}$$

Taking square root on both sides

$$\Rightarrow \quad \cos\theta = \pm\frac{\sqrt{3}}{2}$$

$\Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{2}$	$\Rightarrow \quad \cos\theta = -\frac{\sqrt{3}}{2}$
$\cos\theta$ is positive in I and IV Quadrant with reference angle $\theta = \frac{\pi}{6}$	$\cos\theta$ is negative in II and III Quadrant with reference angle $\theta = \frac{\pi}{6}$

In I Quadrant	In IV Quadrant	In II Quadrant	In III Quadrant
$\theta = \frac{\pi}{6}$	$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$	$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$	$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$$\text{Solution Set} = \left\{\frac{\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}\right\}$$

(iv) $\cot^2\theta = \dfrac{1}{3}$

Solution:

$$\cot^2\theta = \frac{1}{3}$$

$\Rightarrow \quad \tan^2\theta = 3$

Taking square root on both sides

$\Rightarrow \quad \tan\theta = \pm\sqrt{3}$

$\Rightarrow \quad \tan\theta = \sqrt{3}$

$\tan\theta$ is positive in I and III Quadrant with reference angle $\theta = \dfrac{\pi}{3}$

In I Quadrant

$$\theta = \frac{\pi}{3}$$

In III Quadrant

$$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$\Rightarrow \quad \tan\theta = -\sqrt{3}$

$\tan\theta$ is negative in II and IV Quadrant with reference angle $\theta = \dfrac{\pi}{3}$

In II Quadrant

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

In IV Quadrant

$$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$

$$\text{Solution Set } = \left\{\frac{\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}\right\}$$

Find the values of θ satisfying the following equations:

Q.3 $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$

Solution:

$$3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$$

Using Quadratic Formula

$\Rightarrow \quad \tan\theta = \dfrac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(3)(1)}}{2\times 3}$

$\Rightarrow \quad \tan\theta = \dfrac{-2\sqrt{3} \pm \sqrt{12-12}}{6}$

$\Rightarrow \quad \tan\theta = \dfrac{-2\sqrt{3} \pm \sqrt{0}}{6}$

$\Rightarrow \quad \tan\theta = \dfrac{-2\sqrt{3}}{6}$

$\Rightarrow \quad \tan\theta = \dfrac{-\sqrt{3}}{3}$

$\Rightarrow \quad \tan\theta = \dfrac{-1}{\sqrt{3}}$

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$\tan\theta$ is negative in II and IV Quadrant with reference angle $\theta = \frac{\pi}{6}$

In II Quadrant	In IV Quadrant
$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$	$\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\therefore \quad \theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

As π is the period of $\tan\theta$.

$\therefore$ General value of θ is $\frac{5\pi}{6} + n\pi$, $n \in Z$

Q.4 $\tan^2\theta - \sec\theta - 1 = 0$

Solution:

$\tan^2\theta - \sec\theta - 1 = 0$

$\Rightarrow \quad (\sec^2\theta - 1) - \sec\theta - 1 = 0 \qquad \because 1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \quad \sec^2\theta - \sec\theta - 2 = 0$

$\Rightarrow \quad \sec^2\theta - 2\sec\theta + \sec\theta - 2 = 0 \qquad$ (By factorization)

$\Rightarrow \quad \sec\theta(\sec\theta - 2) + 1(\sec\theta - 2) = 0$

$\Rightarrow \quad (\sec\theta - 2)(\sec\theta + 1) = 0$

Either

$\Rightarrow \sec\theta - 2 = 0$

$\Rightarrow \sec\theta = 2$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\cos\theta$ is positive in I and IV Quadrant with reference angle $\theta = \frac{\pi}{3}$

In I Quadrant	In IV Quadrant
$\theta = \frac{\pi}{3}$	$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Or

$\Rightarrow \sec\theta + 1 = 0$

$\Rightarrow \sec\theta = -1$

$\Rightarrow \cos\theta = -1$

As $\cos\theta$ is –1 so there is only one solution i.e. $\theta = \pi$

$\therefore \quad \theta = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$

As 2π is the period of $\cos\theta$

$\therefore$ General values of θ are $\frac{\pi}{3} + 2n\pi$ and $\pi + 2n\pi$, $n \in Z$

Q.5 $2\sin\theta+\cos^2\theta-1=0$

Solution:

$2\sin\theta+\cos^2\theta-1=0$

$\Rightarrow \quad 2\sin\theta+1-\sin^2\theta-1=0 \qquad \because \sin^2\theta+\cos^2\theta=1$

$\Rightarrow \quad -\sin^2\theta+2\sin\theta=0$

$\Rightarrow \quad -\sin\theta(\sin\theta-2)=0$

$\Rightarrow \quad \sin\theta(\sin\theta-2)=0$

Either	Or
$\Rightarrow \quad \sin\theta=0$	$\Rightarrow \quad \sin\theta-2=0$
$\theta=0,\pi$	$\Rightarrow \quad \sin\theta=2$
	Not Possible

$\therefore \quad \theta=0,\pi$

As 2π is the period of $\sin\theta$

$\therefore$ General values of θ are $2n\pi,\ \pi+2n\pi, n\in Z$

Q.6 $2\sin^2\theta-\sin\theta=0$

Solution:

$2\sin^2\theta-\sin\theta=0$

$\Rightarrow \quad \sin\theta(2\sin\theta-1)=0$

Either

$\Rightarrow \quad \sin\theta=0$

$\theta=0,\pi$

Or

$\Rightarrow \quad 2\sin\theta-1=0$

$\Rightarrow \quad 2\sin\theta=1$

$\Rightarrow \quad \sin\theta=\dfrac{1}{2}$

$\sin\theta$ is +ve in I and II Quadrant with reference angle $\dfrac{\pi}{6}$

In I Quadrant	In II Quadrant
$\theta=\dfrac{\pi}{6}$	$\theta=\pi-\dfrac{\pi}{6}=\dfrac{5\pi}{6}$

$\theta=0,\pi,\dfrac{\pi}{6},\dfrac{5\pi}{6}$

As 2π is the period of $\sin\theta$

$\therefore$ General values of θ are $2n\pi,\ \pi+2n\pi,\ \dfrac{\pi}{6}+2n\pi,\ \dfrac{5\pi}{6}+2n\pi$

Q.7 $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

Solution:

$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

Divided both sides by $\cos^2\theta$

$\Rightarrow \quad \frac{3\cos^2\theta}{\cos^2\theta} - \frac{2\sqrt{3}\sin\theta.\cos\theta}{\cos^2\theta} - \frac{3\sin^2\theta}{\cos^2\theta} = \frac{0}{\cos^2\theta}$

$\Rightarrow \quad 3 - 2\sqrt{3}\frac{\sin\theta}{\cos\theta} - 3\frac{\sin^2\theta}{\cos^2\theta} = 0$

$\Rightarrow \quad 3 - 2\sqrt{3}\tan\theta - 3\tan^2\theta = 0$

$\Rightarrow \quad -3\tan^2\theta - 2\sqrt{3}\tan\theta + 3 = 0$

$\Rightarrow \quad 3\tan^2\theta + 2\sqrt{3}\tan\theta - 3 = 0$

Using Quadratic Formula,

$\Rightarrow \quad \tan\theta = \frac{-2\sqrt{3} \pm \sqrt{\left(2\sqrt{3}\right)^2 - 4(3)(-3)}}{2\times 3}$

$\Rightarrow \quad \tan\theta = \frac{-2\sqrt{3} \pm \sqrt{12+36}}{6}$

$\Rightarrow \quad \tan\theta = \frac{-2\sqrt{3} \pm \sqrt{48}}{6}$

$\Rightarrow \quad \tan\theta = \frac{-2\sqrt{3} \pm 4\sqrt{3}}{6}$

$\Rightarrow \quad \tan\theta = \frac{2\left(-\sqrt{3} \pm 2\sqrt{3}\right)}{6}$

$\Rightarrow \quad \tan\theta = \frac{-\sqrt{3} \pm 2\sqrt{3}}{3}$

Either	Or
$\Rightarrow \quad \tan\theta = \frac{\left(-\sqrt{3} + 2\sqrt{3}\right)}{3}$	$\Rightarrow \quad \tan\theta = \frac{\left(-\sqrt{3} - 2\sqrt{3}\right)}{3}$
$\Rightarrow \quad \tan\theta = \frac{\sqrt{3}}{3}$	$\Rightarrow \quad \tan\theta = \frac{-3\sqrt{3}}{3}$
$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$	$\Rightarrow \quad \tan\theta = -\sqrt{3}$
$\tan\theta$ is positive in I and III Quadrant with reference angle $\theta = \frac{\pi}{6}$	$\tan\theta$ is negative in II and IV Quadrant with reference angle $\theta = \frac{\pi}{3}$

In I Quadrant	In III Quadrant	In II Quadrant	In IV Quadrant
$\theta = \frac{\pi}{6}$	$\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$	$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$	$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\therefore \quad \theta = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{2\pi}{3}, \frac{5\pi}{3}$

As π is the period of $\tan\theta$

$\therefore$ General values of θ are $\frac{\pi}{6} + n\pi, \frac{2\pi}{3} + n\pi, \quad n \in Z$

Q.8 $4\sin^2\theta - 8\cos\theta + 1 = 0$

Solution:

$4\sin^2\theta - 8\cos\theta + 1 = 0$

$\Rightarrow \quad 4(1-\cos^2\theta) - 8\cos\theta + 1 = 0 \qquad \because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad 4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$

$\Rightarrow \quad -4\cos^2\theta - 8\cos\theta + 5 = 0$

$\Rightarrow \quad 4\cos^2\theta + 8\cos\theta - 5 = 0$

$\Rightarrow \quad 4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$ By factorization

$\Rightarrow \quad 2\cos\theta(2\cos\theta + 5) - 1(2\cos\theta + 5) = 0$

$\Rightarrow \quad (2\cos\theta - 1)(2\cos\theta + 5) = 0$

Either

$\Rightarrow \quad 2\cos\theta - 1 = 0$

$\Rightarrow \quad 2\cos\theta = 1$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\cos\theta$ is positive in I and IV Quadrant with reference angle $\theta = \frac{\pi}{3}$

Or

$\Rightarrow \quad 2\cos\theta + 5 = 0$

$\Rightarrow \quad 2\cos\theta = -5$

$\Rightarrow \quad \cos\theta = \frac{-5}{2}$

Not Possible

In I Quadrant

$\theta = \frac{\pi}{3}$

In IV Quadrant

$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\therefore \quad \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

As 2π is the period of $\cos\theta$

$\therefore$ General values of θ are $\frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, \quad n \in Z$

Find the solution sets of the following equations:

Q.9 $\sqrt{3}\tan x - \sec x - 1 = 0$

Solution:

$\sqrt{3}\tan x - \sec x - 1 = 0$

$\Rightarrow \sqrt{3}\tan x = \sec x + 1$

Taking square on both side

$\Rightarrow (\sqrt{3}\tan x)^2 = (\sec x + 1)^2$

$\Rightarrow 3\tan^2 x = \sec^2 x + 1 + 2\sec x$ $\quad\because 1 + \tan^2 x = \sec^2 x$

$\Rightarrow 3(\sec^2 x - 1) = \sec^2 x + 2\sec x + 1$

$\Rightarrow 3\sec^2 x - 3 = \sec^2 x + 2\sec x + 1$

$\Rightarrow 3\sec^2 x - \sec^2 x - 2\sec x - 3 - 1 = 0$

$\Rightarrow 2\sec^2 x - 2\sec x - 4 = 0$

$\Rightarrow 2(\sec^2 x - \sec x - 2) = 0$

$\Rightarrow \sec^2 x - \sec x - 2 = 0$

$\Rightarrow \sec^2 x - 2\sec x + \sec x - 2 = 0$ $\quad$ By Factorization

$\Rightarrow \sec x(\sec x - 2) + 1(\sec x - 2) = 0$

$\Rightarrow (\sec x + 1)(\sec x - 2) = 0$

Either

$\Rightarrow \sec x - 2 = 0$

$\Rightarrow \sec x = 2$

$\Rightarrow \cos x = \frac{1}{2}$

$\cos x$ is positive in I and IV Quadrant with reference angle $x = \frac{\pi}{3}$

In I Quadrant

$x = \frac{\pi}{3}$

As 2π is the period of cosine

General Value of x is

$\frac{\pi}{3} + 2n\pi$, $n \in Z$

In IV Quadrant

$x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Not Satisfied

Or

$\Rightarrow \sec x + 1 = 0$

$\Rightarrow \sec x = -1$

$\Rightarrow \cos x = -1$

As $\cos x = -1$ so there is only one solution i.e.

$x = \pi$

As 2π is the Period of $\cos x$.

General value of x is $\pi + 2n\pi$ $n \in Z$

Hence solution set $= \{\pi + 2n\pi\} \cup \left\{\frac{\pi}{3} + 2n\pi\right\}$ $n \in Z$

Q.10 $\cos 2x = \sin 3x$

Solution:

$$\cos 2x = \sin 3x$$

$$\Rightarrow \quad 1-2\sin^2 x = 3\sin x - 4\sin^3 x \qquad \left(\because \cos 2x = 1-2\sin^2 x \text{ and } \sin 3x = 3\sin x - 4\sin^3 x\right)$$

$$\Rightarrow \quad 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$$

Using Synthetic Division

1	4	-2	-3	1
		4	2	-1
	4	2	-1	0

$$\Rightarrow \quad \sin x = 1$$

$$x = \frac{\pi}{2}$$

As 2π is the Period of $\sin x$

General values of x is

$$\frac{\pi}{2} + 2n\pi \qquad n \in Z$$

$$\Rightarrow \quad 4\sin^2 x + 2\sin x - 1 = 0$$

Using Quadratic Formula

$$\Rightarrow \quad \sin x = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-1)}}{2\times 4}$$

$$\Rightarrow \quad \sin x = \frac{-2 \pm \sqrt{4+16}}{8}$$

$$\Rightarrow \quad \sin x = \frac{-2 \pm \sqrt{20}}{8}$$

Either

$$\Rightarrow \quad \sin x = \frac{-2+\sqrt{20}}{8}$$

$$\Rightarrow \quad \sin x = 0{\cdot}3090$$

$\sin x$ is positive in I and II quadrant with reference angle $x = 18° = \frac{\pi}{10}$

Or

$$\Rightarrow \quad \sin x = \frac{-2-\sqrt{20}}{8}$$

$$\Rightarrow \quad \sin x = -0{\cdot}8090$$

$\sin x$ is negative in III and IV quadrant with reference angle $x = 54° = \frac{3\pi}{10}$

In I Quadrant	In II Quadrant	In III Quadrant	In IV Quadrant
$x = \frac{\pi}{10}$	$x = \pi - \frac{\pi}{10} = \frac{9\pi}{10}$	$x = \pi + \frac{3\pi}{10} = \frac{13\pi}{10}$	$x = 2\pi - \frac{3\pi}{10} = \frac{17\pi}{10}$
As 2π is the period of $\sin x$	As 2π is the period of $\sin x$	As 2π is the period of $\sin x$	As 2π is the period of $\sin x$
General values of x is	General values of x is	General values of x is	General values of x is
$\frac{\pi}{10} + 2n\pi \quad n \in Z$	$\frac{9\pi}{10} + 2n\pi \quad n \in Z$	$\frac{13\pi}{10} + 2n\pi \quad n \in Z$	$\frac{17\pi}{10} + 2n\pi \quad n \in Z$

Hence Solution Set $= \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{\pi}{10} + 2n\pi\right\} \cup \left\{\frac{9\pi}{10} + 2n\pi\right\} \cup \left\{\frac{13\pi}{10} + 2n\pi\right\} \cup \left\{\frac{17\pi}{10} + 2n\pi\right\} \quad n \in Z$

Q.11 $\sec 3\theta = \sec\theta$

Solution:

$$\sec 3\theta = \sec\theta$$

$$\Rightarrow \frac{1}{\cos 3\theta} = \frac{1}{\cos\theta}$$

$$\Rightarrow \cos\theta = \cos 3\theta \qquad \therefore \cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\Rightarrow \cos\theta = 4\cos^3\theta - 3\cos\theta$$

$$\Rightarrow 4\cos^3\theta - 3\cos\theta - \cos\theta = 0$$

$$\Rightarrow 4\cos^3\theta - 4\cos\theta = 0$$

$$\Rightarrow 4\cos\theta(\cos^2\theta - 1) = 0$$

$$\Rightarrow \cos\theta(\cos^2\theta - 1) = 0$$

Either

$$\Rightarrow \cos\theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

As at $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$ given equation is not defined so these values of θ will not include in solution set.

Or

$$\Rightarrow \cos^2\theta - 1 = 0$$

$$\Rightarrow \cos^2\theta = 1$$

Taking square root on both sides

$$\cos\theta = \pm 1$$

$$\Rightarrow \cos\theta = 1$$

$$\theta = 0, \pi$$

As 2π is the period of $\cos x$

So General value of x is

$2n\pi$, $\pi + 2n\pi$ $\quad n \in Z$

$$\Rightarrow \cos\theta = -1$$

As $\cos\theta = -1$ so there is only one solution i.e $\theta = \pi$

As 2π is the period of $\cos x$

So General value of x is

$\pi + 2n\pi$ $\quad n \in Z$

Hence the Solution Set $= \{\pi + 2n\pi\} \cup \{2n\pi\} \quad n \in Z$

Q.12 $\tan 2\theta + \cot\theta = 0$

Solution:

$$\tan 2\theta + \cot\theta = 0$$

$$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} + \frac{\cos\theta}{\sin\theta} = 0$$

$\Rightarrow \quad \dfrac{\sin 2\theta \sin\theta + \cos\theta \cos 2\theta}{\cos 2\theta \sin\theta} = 0$

$\Rightarrow \quad \cos 2\theta \cos\theta + \sin 2\theta \sin\theta = 0(\cos 2\theta \sin\theta)$

$\Rightarrow \quad \cos 2\theta \cos\theta + \sin 2\theta \sin\theta = 0$

$\Rightarrow \quad \cos(2\theta - \theta) = 0 \qquad \because \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\Rightarrow \quad \cos\theta = 0$

$\theta = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

As 2π is the period of $\cos\theta$

General value of θ is $\dfrac{\pi}{2} + 2n\pi, \dfrac{3\pi}{2} + 2n\pi$

Hence the solution Set $= \left\{\dfrac{\pi}{2} + 2n\pi\right\} \cup \left\{\dfrac{3\pi}{2} + 2n\pi\right\} \quad n \in Z$

Q.13 $\sin 2x + \sin x = 0$

Solution:

$\sin 2x + \sin x = 0$

$\Rightarrow \quad 2\sin x \cos x + \sin x = 0 \qquad \because \sin 2\theta = 2\sin\theta.\cos\theta$

$\Rightarrow \quad \sin x(2\cos x + 1) = 0$

Either

$\Rightarrow \quad \sin x = 0$

$x = 0, \pi$

As 2π is the period of $\sin x$

So General value of x is

$2n\pi$ and $\pi + 2n\pi \quad n \in Z$

Or

$\Rightarrow \quad 2\cos x + 1 = 0$

$\Rightarrow \quad 2\cos x = -1$

$\Rightarrow \quad \cos x = \dfrac{-1}{2}$

As $\cos x$ is negative in II and III Quadrant with reference angle

$x = \dfrac{\pi}{3}$

In II Quadrant

$x = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

As 2π is the period of $\cos x$

So General value of x is

$\dfrac{2\pi}{3} + 2n\pi \qquad n \in Z$

In III Quadrant

$x = \pi + \dfrac{\pi}{3} = \dfrac{4\pi}{3}$

As 2π is the period of $\cos x$

So General value of x is

$\dfrac{4\pi}{3} + 2n\pi \qquad n \in Z$

Hence the Solution Set $= \{n\pi\} \cup \left\{\dfrac{2\pi}{3} + 2n\pi\right\} \cup \left\{\dfrac{4\pi}{3} + 2n\pi\right\} \qquad n \in Z$

Q.14 $\sin 4x - \sin 2x = \cos 3x$

Solution:

$\sin 4x - \sin 2x = \cos 3x$

Using formula to convert difference into product

$\Rightarrow \quad 2\cos\left(\frac{4x+2x}{2}\right)\sin\left(\frac{4x-2x}{2}\right) = \cos 3x \qquad \because \sin P - \sin Q = 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$

$\Rightarrow \quad 2\cos 3x \sin x = \cos 3x$

$\Rightarrow \quad 2\cos 3x \sin x - \cos 3x = 0$

$\Rightarrow \quad \cos 3x(2\sin x - 1) = 0$

$\Rightarrow \quad \cos 3x(2\sin x - 1) = 0$

Either

$\Rightarrow \quad \cos 3x = 0$

$3x = \frac{\pi}{2}, \frac{3\pi}{2}$

As 2π is the period of $\cos x$

So General value of x is

$3x = \frac{\pi}{2} + 2n\pi \,,\; 3x = \frac{3\pi}{2} + 2n\pi,\; n \in Z$

$x = \frac{\pi}{6} + \frac{2n\pi}{3} \,,\; x = \frac{\pi}{2} + \frac{2n\pi}{3}$

Or

$\Rightarrow \quad 2\sin x - 1 = 0$

$\Rightarrow \quad 2\sin x = 1$

$\Rightarrow \quad \sin x = \frac{1}{2}$

$\sin x$ is positive in I and II

Quadrant with reference angle $x = \frac{\pi}{6}$

In I Quadrant

$x = \frac{\pi}{6}$

As 2π is the period of $\sin x$

So General value of x is

$\frac{\pi}{6} + 2n\pi \qquad n \in Z$

In II Quadrant

$x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

As 2π is the period of $\sin x$

So General value of x is

$\frac{5\pi}{6} + 2n\pi \qquad n \in Z$

Hence Solution Set $= \left\{\frac{\pi}{6} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{2} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\} \qquad n \in Z$

Q.15 $\sin x + \cos 3x = \cos 5x$

Solution:

$\sin x + \cos 3x = \cos 5x$

$\Rightarrow \quad \sin x = \cos 5x - \cos 3x$

Using formula to convert difference into product

$\Rightarrow \quad \sin x = -2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right) \qquad \because \cos P - \cos Q = -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$

$\Rightarrow \quad \sin x = -2\sin 4x \sin x$

$\Rightarrow \quad \sin x + 2\sin 4x \sin x = 0$

$\Rightarrow \quad \sin x(1+2\sin 4x) = 0$

Either

$\Rightarrow \quad \sin x = 0$

$x = 0, \pi$

As 2π is the period of $\sin x$

So General value of x is

$\pi + 2n\pi$ and $2n\pi \qquad n \in Z$

Or

$\Rightarrow \quad 1+2\sin 4x = 0$

$\Rightarrow \quad 2\sin 4x = -1$

$\Rightarrow \quad \sin 4x = \frac{-1}{2}$

$\sin 4x$ is negative in III and IV Quadrant with reference

angle $4x = \frac{\pi}{6}$

In III Quadrant

$4x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

As 2π is the period of $\sin x$

So General value of x is

$4x = \frac{7\pi}{6} + 2n\pi \qquad n \in Z$

$x = \frac{7\pi}{24} + \frac{n\pi}{2}$

In IV Quadrant

$4x = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

As 2π is the period of $\sin x$

So General value of x is

$4x = \frac{11\pi}{6} + 2n\pi \qquad n \in Z$

$x = \frac{11\pi}{24} + \frac{n\pi}{2}$

Hence the Solution Set $= \{2n\pi\} \cup \{\pi + 2n\pi\} \cup \left\{\frac{7\pi}{24} + \frac{n\pi}{2}\right\} \cup \left\{\frac{11\pi}{24} + \frac{n\pi}{2}\right\}, n \in Z$

Q.16 $\sin 3x + \sin 2x + \sin x = 0$

Solution:

$\sin 3x + \sin 2x + \sin x = 0$

$\Rightarrow \quad \sin 3x + \sin x + \sin 2x = 0$

Using formula to convert sum into product

$\Rightarrow \quad 2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) + \sin 2x = 0 \qquad \because \sin P + \sin Q = 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$

$\Rightarrow \quad 2\sin 2x \cos x + \sin 2x = 0$

$\Rightarrow \quad \sin 2x(2\cos x + 1) = 0$

Either

$\Rightarrow \quad \sin 2x = 0$

$2x = 0, \pi$

As 2π is the period of $\sin x$

So General value of x is

$2x = \pi + 2n\pi$, $2x = 2n\pi \qquad n \in Z$

$x = \frac{\pi}{2} + n\pi$, $x = n\pi$

Or

$\Rightarrow \quad 2\cos x + 1 = 0$

$\Rightarrow \quad 2\cos x = -1$

$\cos x$ is negative in II and III Quadrant with reference angle $x = \frac{\pi}{3}$

In II Quadrant

$x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

As 2π is the period of $\cos x$

So General value of x is

$x = \frac{2\pi}{3} + 2n\pi \qquad n \in Z$

In III Quadrant

$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

As 2π is the period of $\cos x$

So General value of x is

$x = \frac{4\pi}{3} + 2n\pi \qquad n \in Z$

Hence Solution Set $= \{n\pi\} \cup \left\{\frac{\pi}{2} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\} \quad n \in Z$

Q.17 $\quad \sin 7x - \sin x = \sin 3x$

Solution:

$\sin 7x - \sin x = \sin 3x$

Using formula to convert difference into product

$$\Rightarrow \quad 2\cos\left(\frac{7x + x}{2}\right)\sin\left(\frac{7x - x}{2}\right) = \sin 3x \quad \because \sin P - \sin Q = 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$$

$\Rightarrow \quad 2\cos 4x \sin 3x = \sin 3x$

$\Rightarrow \quad 2\cos 4x \sin 3x - \sin 3x = 0$

$\Rightarrow \quad \sin 3x(2\cos 4x - 1) = 0$

Either

$\Rightarrow \quad \sin 3x = 0$

$3x = 0, \pi$

As 2π is the period of $\sin x$

So General value of x is

$3x = \pi + 2n\pi$, $3x = 2n\pi \qquad n \in Z$

$x = \frac{\pi}{3} + \frac{2n\pi}{3}$, $x = \frac{2n\pi}{3}$

Or

$\Rightarrow \quad 2\cos 4x - 1 = 0$

$\Rightarrow \quad 2\cos 4x = 1$

$\Rightarrow \quad \cos 4x = \frac{1}{2}$

$\cos 4x$ is positive in the I and IV Quadrant with reference angle $4x = \frac{\pi}{3}$

In I Quadrant

$4x = \frac{\pi}{3}$

As 2π is the period of $\cos x$

So General value of x is

$4x = \frac{\pi}{3} + 2n\pi \qquad n \in Z$

$x = \frac{\pi}{12} + \frac{n\pi}{2}$

In IV Quadrant

$4x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

As 2π is the period of $\cos x$

So General value of x is

$4x = \frac{5\pi}{3} + 2n\pi \qquad n \in Z$

$x = \frac{5\pi}{12} + \frac{n\pi}{2}$

Hence Solution Set $= \left\{\frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{12} + \frac{n\pi}{2}\right\} \cup \left\{\frac{5\pi}{12} + \frac{n\pi}{2}\right\} \quad n \in Z$

Q.18 $\sin x+\sin 3x+\sin 5x=0$

Solution:

$$\sin x+\sin 3x+\sin 5x=0$$

$$\Rightarrow \quad (\sin 5x+\sin x)+\sin 3x=0$$

Using formula to convert sum into product

$$\Rightarrow \quad 2\sin\left(\frac{5x+x}{2}\right)\cos\left(\frac{5x-x}{2}\right)+\sin 3x=0 \qquad \because \sin P+\sin Q=2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$$

$$\Rightarrow \quad 2\sin 3x\cos 2x+\sin 3x=0$$

$$\Rightarrow \quad \sin 3x(2\cos 2x+1)=0$$

Either

$$\Rightarrow \quad \sin 3x=0$$

$$3x=0,\pi$$

As 2π is the period of $\sin x$

So General value of x is

$$3x=\pi+2n\pi\ ,\ 3x=2n\pi \qquad n\in Z$$

$$x=\frac{\pi}{3}+\frac{2n\pi}{3}\ ,\ x=\frac{2n\pi}{3}$$

Or

$$\Rightarrow \quad 2\cos 2x+1=0$$

$$\Rightarrow \quad 2\cos 2x=-1$$

$$\Rightarrow \quad \cos 2x=\frac{-1}{2}$$

$\cos 2x$ is negative in the II and III Quadrant with

reference angle $2x=\frac{\pi}{3}$

In II Quadrant

$$2x=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$$

As 2π is the period of $\cos x$

So General value of x is

$$2x=\frac{2\pi}{3}+2n\pi \qquad n\in Z$$

$$x=\frac{\pi}{3}+n\pi$$

In III Quadrant

$$2x=\pi+\frac{\pi}{3}=\frac{4\pi}{3}$$

As 2π is the period of $\cos x$

So General value of x is

$$2x=\frac{4\pi}{3}+2n\pi \qquad n\in Z$$

$$x=\frac{2\pi}{3}+n\pi$$

The Solution Set $=\left\{\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{3}+\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{3}+n\pi\right\}\cup\left\{\frac{2\pi}{3}+n\pi\right\}\quad n\in Z$

Q.19 $\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta=0$

Solution:

$$\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta=0$$

$$(\sin 7\theta+\sin\theta)+(\sin 5\theta+\sin 3\theta)=0$$

Using formula to convert sum into product

$\Rightarrow \left(2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)\right)+\left(2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)\right)=0 \quad \because \sin P+\sin Q=2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$

$\Rightarrow \quad 2\sin 4\theta\cos 3\theta+2\sin 4\theta\cos\theta=0$

$\Rightarrow \quad 2\sin 4\theta(\cos 3\theta+\cos\theta)=0$

$\Rightarrow \quad \sin 4\theta(\cos 3\theta+\cos\theta)=0$

Either

$\Rightarrow \quad \sin 4\theta=0$

$4\theta=0,\pi$

As 2π is the period of $\sin\theta$

So General value of x is

$4\theta=\pi+2n\pi$, $4\theta=2n\pi \quad n\in Z$

$\theta=\frac{\pi}{4}+\frac{n\pi}{2}$, $\theta=\frac{n\pi}{2}$

Or

$\Rightarrow \quad \cos 3\theta+\cos\theta=0$

Using formula to convert sum into product

$\Rightarrow \quad 2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)=0 \quad \because \cos P+\cos Q=2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$

$\Rightarrow \quad 2\cos 2\theta\cos\theta=0$

$\Rightarrow \quad \cos 2\theta\cos\theta=0$

Either

$\Rightarrow \quad \cos 2\theta=0$

$2\theta=\frac{\pi}{2},\ \frac{3\pi}{2}$

As 2π is the period of $\cos\theta$

So General value of θ is

$2\theta=\frac{\pi}{2}+2n\pi$, $2\theta=\frac{3\pi}{2}+2n\pi \quad n\in Z$

$\theta=\frac{\pi}{4}+n\pi$, $\theta=\frac{3\pi}{4}+n\pi$

Or

$\Rightarrow \quad \cos\theta=0$

$\theta=\frac{\pi}{2},\ \frac{3\pi}{2}$

As 2π is the period of $\cos\theta$

So General value of θ is

$\theta=\frac{\pi}{2}+2n\pi$, $\theta=\frac{3\pi}{2}+2n\pi$

The Solution Set $=\left\{\frac{n\pi}{2}\right\}\cup\left\{\frac{\pi}{4}+\frac{n\pi}{2}\right\}\cup\left\{\frac{\pi}{4}+n\pi\right\}\cup\left\{\frac{3\pi}{4}+n\pi\right\}\cup\left\{\frac{\pi}{2}+2n\pi\right\}\cup\left\{\frac{3\pi}{2}+2n\pi\right\} \quad n\in Z$

Q.20 $\mathbf{\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0}$

Solution:

$\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0$

$\Rightarrow \quad (\cos 7\theta+\cos\theta)+(\cos 5\theta+\cos 3\theta)=0$

Using formula to convert sum into product

$\Rightarrow \quad 2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)+2\cos\theta\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)=0 \quad \because \cos P+\cos Q=2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$

$\Rightarrow \quad 2\cos 4\theta\cos 3\theta+2\cos 4\theta\cos\theta=0$

$\Rightarrow \quad 2\cos 4\theta(\cos 3\theta+\cos\theta)=0$

$\Rightarrow \quad \cos 4\theta(\cos 3\theta + \cos\theta) = 0$

Either

$\Rightarrow \quad \cos 4\theta = 0$

$4\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

As 2π is the period of $\cos\theta$

So General value of θ is

$4\theta = \frac{\pi}{2} + 2n\pi, \; 4\theta = \frac{3\pi}{2} + 2n\pi \quad n \in Z$

$\theta = \frac{\pi}{8} + \frac{n\pi}{2}, \; \theta = \frac{3\pi}{8} + \frac{n\pi}{2}$

Or

$\Rightarrow \quad \cos 3\theta + \cos\theta = 0$

Using formula to convert sum into product

$\Rightarrow \quad 2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) = 0 \qquad \because \cos P + \cos Q = 2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$

$\Rightarrow \quad 2\cos 2\theta\cos\theta = 0$

$\Rightarrow \quad \cos 2\theta\cos\theta = 0$

Either

$\Rightarrow \quad \cos 2\theta = 0$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

As 2π is the period of $\cos\theta$

So General value of θ is

$2\theta = \frac{\pi}{2} + 2n\pi, \; 2\theta = \frac{3\pi}{2} + 2n\pi \quad n \in Z$

$\theta = \frac{\pi}{4} + n\pi, \; \theta = \frac{3\pi}{4} + n\pi$

Or

$\Rightarrow \quad \cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

As 2π is the period of $\cos\theta$

So General value of θ is

$\theta = \frac{\pi}{2} + 2n\pi, \; \theta = \frac{3\pi}{2} + 2n\pi \quad n \in Z$

Hence the Solution Set $= \left\{\frac{\pi}{8} + \frac{n\pi}{2}\right\} \cup \left\{\frac{3\pi}{8} + \frac{n\pi}{2}\right\} \cup \left\{\frac{\pi}{4} + n\pi\right\} \cup \left\{\frac{3\pi}{4} + n\pi\right\} \cup \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{3\pi}{2} + 2n\pi\right\} n \in Z$