CHAPTER 4

QUADRATIC EQUATIONS

Quadratic Equation:

A quadratic equation in one variable x is an equation of the form $ax^2+bx+c=0$ where *a*, *b* and *c* are real numbers and $a\neq 0$.

Example:

$4x^2+5x+3=0$, $x^2-7x+10=0$ are quadratic equations.

Note:

(i) $ax^2+bx+c=0$ is called **standard form** of quadratic equation.

(ii) Another name for quadratic equation is **Second Degree Polynomial in** x.

Solution of Quadratic Equation:

There are three techniques for solving a quadratic equation.

(i) **By Factorization:**

In this method we factorize the polynomial ax^2+bx+c into two linear factors.
We use the fact that, if $ab=0$ then either $a=0$ or $b=0$

(ii) **By Completing Square:**

In this method we form, by construction, the perfect square of a linear term involving variables. Then solution is found by extracting square roots.
This method is used, when quadratic equation is not easily factorable.

(iii) **By Quadratic Formula:**

In this method we can find the solution by applying a formula known as "Quadratic Formula"

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Every quadratic equation can be solved by using quadratic formula.

Derivation of Quadratic Formula:

For given quadratic equation $ax^2+bx+c=0$; $a\neq 0$

Step#1:

Divide both sides of equation by *a*

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

Step#2:

Shift constant term to right side of equation

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

Step#3:

Add $\left(\frac{b}{2a}\right)^2$ (the square of half of the coefficient of x) to both sides.

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$

$$(x)^2+2(x)\left(\frac{b}{2a}\right)+\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Quadratic formula: $\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$

Note: The solutions of an equation are also called its roots.

Exercise 4.1

Solve the following equations by Factorization:

Q.1 $3x^2+4x+1=0$

Solution:

$3x^2+4x+1=0$

$3x^2+3x+x+1=0$

$3x(x+1)+1(x+1)=0$

$(x+1)(3x+1)=0$

Either $x+1=0$ or $3x+1=0$

$x=-1$; $3x=-1$

; $x=-\frac{1}{3}$

$\therefore$ Solution Set$=\left\{\frac{-1}{3},-1\right\}$

Q.2 $x^2+7x+12=0$

Solution:

$x^2+7x+12=0$

$x^2+4x+3x+12=0$

$x(x+4)+3(x+4)=0$

$(x+4)(x+3)=0$

Either $x+4=0$ or $x+3=0$

$x=-4$; $x=-3$

$\therefore$ Solution Set $=\{-4,-3\}$

Q.3 $9x^2-12x-5=0$

Solution:

$9x^2-12x-5=0$

$9x^2-15x+3x-5=0$

$3x(3x-5)+1(3x-5)=0$

$(3x-5)(3x+1)=0$

Either $3x-5=0$ or $3x+1=0$

$3x=5$; $3x=-1$

$x=\frac{5}{3}$; $x=\frac{-1}{3}$

$\therefore$ Solution Set is $\left\{\frac{-1}{3},\frac{5}{3}\right\}$

Q.4 $x^2-x=2$

Solution:

$x^2-x=2$

$x^2-x-2=0$

$x^2-2x+x-2=0$

$x(x-2)+1(x-2)=0$

$(x-2)(x+1)=0$

Either $x-2=0$ or $x+1=0$

$x=2$; $x=-1$

$\therefore$ Solution Set$=\{-1,2\}$

Q.5 $x(x+7)=(2x-1)(x+4)$

Solution:

$x(x+7)=(2x-1)(x+4)$

$x^2+7x=2x^2+8x-x-4$

$2x^2+8x-x-4-x^2-7x=0$

$x^2-4=0$

$(x)^2-(2)^2=0$

$(x-2)(x+2)=0$

Either $x-2=0$ or $x+2=0$

$x=2$; $x=-2$

$\therefore$ Solution Set$=\{-2,2\}$

Q.6 $\frac{x}{x+1}+\frac{x+1}{x}=\frac{5}{2}, x\neq -1, 0$

Solution:

$\frac{x}{x+1}+\frac{x+1}{x}=\frac{5}{2}$

Multiplying both sides by '$2x(x+1)$'

$$2x^2+2(x+1)^2=5x(x+1)$$
$$2x^2+2(x^2+1+2x)=5x^2+5x$$
$$2x^2+2x^2+2+4x=5x^2+5x$$
$$5x^2+5x-2x^2-2x^2-4x-2=0$$
$$x^2+x-2=0$$
$$x^2+2x-x-2=0$$
$$x(x+2)-1(x+2)=0$$
$$(x+2)(x-1)=0$$

Either $x+2=0$ or $x-1=0$

$x=-2$; $x=1$

$\therefore$ Solution Set $=\{-2,1\}$

Q.7 $\dfrac{1}{x+1}+\dfrac{2}{x+2}=\dfrac{7}{x+5}, x\neq -1,-2,-5$

Solution:

$$\frac{1}{x+1}+\frac{2}{x+2}=\frac{7}{x+5}$$

Multiplying Both sides by $(x+1)(x+2)(x+5)$

$$(x+2)(x+5)+2(x+1)(x+5)=7(x+1)(x+2)$$
$$x^2+7x+10+2(x^2+6x+5)=7(x^2+3x+2)$$
$$x^2+7x+10+2x^2+12x+10=7x^2+21x+14$$
$$7x^2-3x^2+21x-19x+14-20=0$$
$$4x^2+2x-6=0$$

$2x^2+x-3=0$ $\because$ Dividing both sides by 2

$$2x^2+3x-2x-3=0$$
$$x(2x+3)-1(2x+3)=0$$
$$(2x+3)(x-1)=0$$

Either $2x+3=0$ or $x-1=0$

$x=\dfrac{-3}{2}$; $x=1$

$\therefore$ Solution Set $=\left\{\dfrac{-3}{2},1\right\}$

Q.8 $\dfrac{a}{ax-1}+\dfrac{b}{bx-1}=a+b, x\neq\dfrac{1}{a},\dfrac{1}{b}$

Solution:

$$\frac{a}{ax-1}+\frac{b}{bx-1}=a+b$$

$$\frac{a}{ax-1}-b+\frac{b}{bx-1}-a=0$$

$$\frac{a-b(ax-1)}{ax-1}+\frac{b-a(bx-1)}{bx-1}=0$$

$$\frac{a-abx+b}{ax-1}+\frac{b-abx+a}{bx-1}=0$$

$$\frac{(bx-1)[a-abx+b]+(ax-1)[b-abx+a]}{(ax-1)(bx-1)}=0$$

$$\frac{(a+b-abx)[bx-1+ax-1]}{(ax-1)(bx-1)}=0$$

$$(a+b-abx)(bx+ax-2)=0$$

Either $a+b-abx=0$ or $bx+ax-2=0$

$abx=a+b$; $x(a+b)=2$

$x=\dfrac{a+b}{ab}$; $x=\dfrac{2}{a+b}$

$\therefore$ Solution set $=\left\{\dfrac{a+b}{ab},\dfrac{2}{a+b}\right\}$

Solve the following Equations by completing square:

Q.9 $x^2-2x-899=0$

Solution:

$$x^2-2x-899=0$$

$$x^2-2x=899$$

Adding 1^2 on both sides

$$x^2-2x+1^2=899+1^2$$

$$(x-1)^2=900$$

Taking square root on both sides

$$x-1=\pm 30$$

Either $x-1=30$ or $x-1=-30$

$x=31$; $x=-29$

$\therefore$ Solution Set = $\{-29,31\}$

Q.10 $x^2+4x-1085=0$

Solution:

$x^2+4x-1085=0$

$x^2+4x=1085$

Adding $(2)^2$ on both sides

$x^2+4x+(2)^2=1085+(2)^2$

$(x+2)^2=1089$

Taking square root on both sides

$x+2=\pm33$

Either $x+2=33$ or $x+2=-33$

$x=31$; $x=-35$

$\therefore$ Solution Set = $\{-35,31\}$

Q.11 $x^2+6x-567=0$

Solution:

$x^2+6x-567=0$

$x^2+6x=567$

Adding $(3)^2$ on both sides

$x^2+6x+(3)^2=567+(3)^2$

$(x+3)^2=576$

Taking square root on both sides

$(x+3)=\pm24$

Either $x+3=24$ or $x+3=-24$

$x=21$; $x=-27$

$\therefore$ Solution Set = $\{-27,21\}$

Q.12 $x^2-3x-648=0$

Solution:

$x^2-3x-648=0$

$x^2-3x=648$

Adding $\left(\frac{3}{2}\right)^2$ on both sides

$$x^2-3x+\left(\frac{3}{2}\right)^2=648+\left(\frac{3}{2}\right)^2$$

$$\left(x-\frac{3}{2}\right)^2=648+\frac{9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=\frac{2592+9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=\frac{2601}{4}$$

Taking square root on both sides

$$x-\frac{3}{2}=\pm\frac{51}{2}$$

Either $x-\frac{3}{2}=\frac{51}{2}$ or $x-\frac{3}{2}=\frac{-51}{2}$

$x=\frac{51}{2}+\frac{3}{2}=\frac{54}{2}$; $x=\frac{-51}{2}+\frac{3}{2}=-\frac{48}{2}$

$x=27$; $x=-24$

$\therefore$ Solution Set $=\{-24,27\}$

Q.13 $\mathbf{x^2-x-1806=0}$

Solution:

$$x^2-x-1806=0$$

$$x^2-x=1806$$

Adding $\left(\frac{1}{2}\right)^2$ on both sides

$$x^2-x+\left(\frac{1}{2}\right)^2=1806+\left(\frac{1}{2}\right)^2$$

$$\left(x-\frac{1}{2}\right)^2=\frac{7224+1}{4}$$

$$\left(x-\frac{1}{2}\right)^2=\frac{7225}{4}$$

$$x-\frac{1}{2}=\pm\frac{85}{2}$$

Either $x-\frac{1}{2}=\frac{85}{2}$ or $x-\frac{1}{2}=\frac{-85}{2}$

$x=\frac{85}{2}+\frac{1}{2}=\frac{86}{2}$; $x=\frac{-85}{2}+\frac{1}{2}=-\frac{84}{2}$

$x=43$; $x=-42$

$\therefore$ Solution Set is $=\{-42,43\}$

Q.14 $2x^2+12x-110=0$

Solution:

$2x^2+12x-110=0$

Dividing both sides by 2

$x^2+6x-55=0$

$x^2+6x=55$

Adding "$(3)^2$" on both sides

$x^2+6x+(3)^2=55+(3)^2$

$(x+3)^2=64$

Taking square root on both sides

$x+3=\pm 8$

Either $x+3=8$ or $x+3=-8$

$x=5$; $x=-11$

$\therefore$ Solution Set $=\{-11,5\}$

Find roots of the following equation by using quadratic formula:

Q.15 $5x^2-13x+6=0$

Solution:

$5x^2-13x+6=0$

By Comparing with $ax^2+bx+c=0$, we get,

$a=5$, $b=-13$, $c=6$

By quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-13)\pm\sqrt{(-13)^2-4(5)(6)}}{2(5)}$$

$$x=\frac{13\pm\sqrt{169-120}}{10}$$

$$x=\frac{13\pm\sqrt{49}}{10}$$

$$x=\frac{13\pm 7}{10}$$

Either $x=\frac{13+7}{10}=\frac{20}{10}$ or $x=\frac{13-7}{10}=\frac{6}{10}$

$x=2$; $x=\frac{3}{5}$

$\therefore$ Solution Set $=\left\{\frac{3}{5},2\right\}$

Q.16 $\mathbf{4x^2+7x-1=0}$

Solution:

$4x^2+7x-1=0$

By Comparing with $ax^2+bx+c=0$, we get,

$a=4,\ b=7,\ c=-1$

By quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-7\pm\sqrt{(7)^2-4(4)(-1)}}{2(4)}$$

$$x=\frac{-7\pm\sqrt{49+16}}{8}$$

$$x=\frac{-7\pm\sqrt{65}}{8}$$

Either $x=\frac{-7+\sqrt{65}}{8}$ or $x=\frac{-7-\sqrt{65}}{8}$

$\therefore$ Solution Set $=\left\{\frac{-7-\sqrt{65}}{8},\frac{-7+\sqrt{65}}{8}\right\}$

Q.17 $\mathbf{15x^2+2ax-a^2=0}$

Solution:

$15x^2+2ax-a^2=0$

By Comparing with $Ax^2+Bx+C=0$, we get,

$A=15,\ B=2a,\ C=-a^2$

By quadratic formula

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{-2a\pm\sqrt{(2a)^2-4(15)(-a^2)}}{2(15)}$$

$$x=\frac{-2a\pm\sqrt{4a^2+60a^2}}{30}$$

$$x=\frac{-2a\pm\sqrt{64a^2}}{30}$$

$$x=\frac{-2a\pm 8a}{30}$$

Either $x=\frac{-2a+8a}{30}=\frac{6a}{30}$ or $x=\frac{-2a-8a}{30}=-\frac{10a}{30}$

$x=\frac{a}{5}$; $x=\frac{-a}{3}$

$\therefore$ Solution Set$=\left\{\frac{-a}{3},\frac{a}{5}\right\}$

Q.18 $16x^2+8x+1=0$

Solution:

$$16x^2+8x+1=0$$

By Comparing with $ax^2+bx+c=0$, we get,

$a=16$, $b=8$, $c=1$

By quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-8\pm\sqrt{(8)^2-4(16)(1)}}{2(16)}$$

$$x=\frac{-8\pm\sqrt{64-64}}{32}$$

$$x=\frac{-8}{32}$$

$$x=\frac{-1}{4}$$

$\therefore$ Solution Set $=\left\{\frac{-1}{4}\right\}$

Q.19 $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$

Solution:

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$$

$$x^2-ax-bx+ab+x^2-bx-cx+bc+x^2-ax-cx+ac=0$$

$$3x^2-2ax-2bx-2cx+ab+bc+ca=0$$

$$3x^2-2(a+b+c)x+ab+bc+ca=0$$

By Comparing with $Ax^2+Bx+C=0$, we get,

$A=3$, $B=-2(a+b+c)$, $C=ab+bc+ca$

By quadratic formula

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$=\frac{-\left[-2(a+b+c)\right]\pm\sqrt{\left(-2(a+b+c)\right)^2-4(3)(ab+bc+ca)}}{2(3)}$$

$$=\frac{2(a+b+c)\pm\sqrt{4\left[a^2+b^2+c^2+2ab+2bc+2ca-3ab-3bc-3ca\right]}}{6}$$

$$=\frac{2(a+b+c)\pm2\sqrt{a^2+b^2+c^2-ab-bc-ca}}{6}$$

$$= \frac{\cancel{2}\left[(a+b+c) \pm \sqrt{a^2+b^2+c^2-ab-bc-ca}\right]}{\cancel{6}^{3}}$$

$$x = \frac{(a+b+c) \pm \sqrt{a^2+b^2+c^2-ab-bc-ca}}{3}$$

$$\therefore \text{ the Solution Set } = \left\{ \frac{(a+b+c) \pm \sqrt{a^2+b^2+c^2-ab-bc-ca}}{3} \right\}$$

Q.20 $\boldsymbol{(a+b)x^2 + (a+2b+c)x + b + c = 0}$

Solution:

$(a+b)x^2 + (a+2b+c)x + b + c = 0$

By Comparing with $Ax^2 + Bx + C = 0$, we get,

$A = a+b$, $B = a+2b+c$, $C = b+c$

By using quadratic formula

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$= \frac{-(a+2b+c) \pm \sqrt{(a+2b+c)^2 - 4(a+b)(b+c)}}{2(a+b)}$$

$$= \frac{-(a+2b+c) \pm \sqrt{a^2+4b^2+c^2+4ab+4bc+2ac-4(ab+ac+b^2+bc)}}{2(a+b)}$$

$$= \frac{-(a+2b+c) \pm \sqrt{a^2+\cancel{4b^2}+c^2+\cancel{4ab}+\cancel{4bc}+2ca-\cancel{4ab}-4ac-\cancel{4b^2}-\cancel{4bc}}}{2(a+b)}$$

$$= \frac{-(a+2b+c) \pm \sqrt{(a-c)^2}}{2(a+b)}$$

$$= \frac{-(a+2b+c) \pm (a-c)}{2(a+b)}$$

Either $x = \dfrac{-(a+2b+c)+(a-c)}{2(a+b)}$ or $x = \dfrac{-(a+2b+c)-(a-c)}{2(a+b)}$

$x = \dfrac{-2b-2c}{2(a+b)} = \dfrac{-\cancel{2}(b+c)}{\cancel{2}(a+b)}$ or $x = \dfrac{-2a-2b}{2(a+b)} = -\dfrac{\cancel{2(a+b)}}{\cancel{2(a+b)}}$

$x = -\dfrac{b+c}{a+b}$ or $x = -1$

$$\therefore \text{ Solution Set } = \left\{-\frac{b+c}{a+b}, -1\right\}$$

Solution of Equations Reducible to the Quadratic Equation:-

There are certain types of equations, which do not look to be of degree 2, but they can be reduced to the quadratic form. We shall discuss the solutions of such five types of the equations one by one.

Type-1:

The equations of the form $ax^{2n}+bx^n+c=0$; $a\neq 0$ can be converted into quadratic equation by substituting $y=x^n$ and $y^2=x^{2n}$.

Type-2:

The equation of the form $(x+a)(x+b)(x+c)(x+d)=k$, where $a+b=c+d$,substitute $x^2+(a+b)x=y$ and $x^2+(c+d)x=y$ $\left(Since \quad y=x^2+(a+b)x=x^2+(c+d)x\right)$

Type-3:

Exponential Equations:

Equations, in which the variable occurs in exponent, are called exponential equations.

For example.

$a^{2x}+b.a^x+c=0$, Substitute $a^x=y$

Type-4:

Reciprocal equations:

There are the equations which remain unchanged even when x is replaced by $\frac{1}{x}$.

If equation written in order (ascending or descending) the coefficients of terms equidistant from beginning and end are equal in magnitude. For example

$ax^2+bx+c+\frac{b}{x}+\frac{a}{x^2}=0$, Substitute $x+\frac{1}{x}=y$ and $x^2+\frac{1}{x^2}=y^2-2$

Exercise 4.2

Q.1 $x^4-6x^2+8=0$

Solution:

$x^4-6x^2+8=0$ (i)

Put $x^2=y \Rightarrow x^4=y^2$

Equation (i) becomes

$y^2-6y+8=0$

$y^2-4y-2y+8=0$

$y(y-4)-2(y-4)=0$

$(y-4)(y-2)=0$

Either $y-4=0$ or $y-2=0$

As $y=x^2$; As $y=x^2$

$x^2=4$; $x^2=2$

$x=\pm 2$; $x=\pm\sqrt{2}$

$\therefore$ Solution Set $=\{-2,-\sqrt{2},\sqrt{2},2\}$

Q.2 $x^{-2}-10=3x^{-1}$

Solution:

$x^{-2}-10=3x^{-1}$ (i)

Put $x^{-1}=y \Rightarrow x^{-2}=y^2$

Equation (i) becomes

$y^2-3y-10=0$

$y^2-5y+2y-10=0$

$y(y-5)+2(y-5)=0$

$(y-5)(y+2)=0$

Either $y-5=0$ or $y+2=0$

$y=5$; $y=-2$

As $y=x^{-1}$; As $y=x^{-1}$

$x^{-1}=5$; $x^{-1}=-2$

$x=\frac{1}{5}$; $x=-\frac{1}{2}$

$\therefore$ Solution Set $=\left\{-\frac{1}{2},\frac{1}{5}\right\}$

Q.3 $\boldsymbol{x^6 - 9x^3 + 8 = 0}$

Solution:

$x^6 - 9x^3 + 8 = 0$ (i)

Put $x^3 = y \Rightarrow x^6 = y^2$

Equation (i) becomes

$y^2 - 9y + 8 = 0$

$y^2 - 8y - y + 8 = 0$

$y(y-8) - 1(y-8) = 0$

$(y-8)(y-1) = 0$

Either

$y - 8 = 0$	or	$y - 1 = 0$
$y = 8$	;	$y = 1$
As $y = x^3$	;	As $y = x^3$
$x^3 = 8 = (2)^3$	;	$x^3 = 1 = (1)^3$
$(x)^3 - (2)^3 = 0$	;	$(x)^3 - (1)^3 = 0$
$(x-2)(x^2+2x+4) = 0$	;	$(x-1)(x^2+x+1) = 0$
Either		Either
$x - 2 = 0$ or $x^2 + 2x + 4 = 0$	;	$x - 1 = 0$ or $x^2 + x + 1 = 0$
$\boxed{x = 2}$ or $a = 1, b = 2, c = 4$	;	$\boxed{x = 1}$ or $a = 1, b = 1, c = 1$
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	;	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$	;	$x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$
$x = \dfrac{-2 \pm \sqrt{4-16}}{2}$	;	$x = \dfrac{-1 \pm \sqrt{-3}}{2}$
$x = \dfrac{-2 \pm \sqrt{-12}}{2}$	;	$\boxed{x = \dfrac{-1 \pm \sqrt{3}i}{2}}$

$$x = \frac{-2 \pm \sqrt{4 \times (-3)}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{-3}}{2}$$

$$x = \frac{\cancel{2}\left(-1 \pm \sqrt{-3}\right)}{\cancel{2}}$$

$$\boxed{x = -1 \pm \sqrt{3}i}$$

$$\boxed{\therefore \text{ Solution Set } = \left\{1, 2, -1 \pm \sqrt{3}i, \frac{-1 \pm \sqrt{3}i}{2}\right\}}$$

Q.4 $\mathbf{8x^6 - 19x^3 - 27 = 0}$

Solution:

$8x^6 - 19x^3 - 27 = 0$ ... (i)

Put $x^3 = y \Rightarrow x^6 = y^2$

Equation (i) becomes

$8y^2 - 19y - 27 = 0$

$8y^2 - 27y + 8y - 27 = 0$

$y(8y - 27) + 1(8y - 27) = 0$

$(8y - 27)(y + 1) = 0$

Either

$8y - 27 = 0$ or $y + 1 = 0$

$8y = 27$; $y + 1 = 0$

$y = \frac{27}{8}$; $y = -1$

As $y = x^3$; As $y = x^3$

$x^3 = \frac{27}{8} = \left(\frac{3}{2}\right)^3$; $x^3 = -1 \Rightarrow x^3 + 1 = 0 \Rightarrow (x)^3 + (1)^3 = 0$

$(x)^3 - \left(\frac{3}{2}\right)^3 = 0$; $(x+1)(x^2 - x + 1) = 0$

; $x + 1 = 0$ or $x^2 - x + 1 = 0$

$\left(x - \frac{3}{2}\right)\left(x^2 + \frac{3}{2}x + \frac{9}{4}\right) = 0$; $\boxed{x = -1}$ or $a = 1, b = -1, c = 1$

$x - \frac{3}{2} = 0$ or $x^2 + \frac{3}{2}x + \frac{9}{4} = 0$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\boxed{x = \frac{3}{2}}$ or $4x^2 + 6x + 9 = 0$; $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$a = 4, b = 6, c = 9$; $x = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{-3}}{2}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $\boxed{x = \frac{1 \pm \sqrt{3}i}{2}}$

$$x=\frac{-6\pm\sqrt{(6)^2-4(4)(9)}}{2(4)}$$

$$x=\frac{-6\pm\sqrt{36(1-4)}}{8}$$

$$x=\frac{-6\pm\sqrt{36(-3)}}{8}$$

$$x=\frac{-6\pm6\sqrt{3}i}{8}$$

$$x=\frac{\not{2}\left(-3\pm3\sqrt{3}i\right)}{\not{8}}$$

$$\boxed{x=\frac{-3\pm3\sqrt{3}i}{4}}$$

$$\boxed{\therefore\ \text{Solution Set}=\left\{-1,\frac{3}{2},\frac{-3\pm3\sqrt{3}i}{4},\frac{1\pm\sqrt{3}i}{2}\right\}}$$

Q.5 $\boldsymbol{x^{\frac{2}{5}}+8=6x^{\frac{1}{5}}}$

Solution:

$$x^{\frac{2}{5}}+8=6x^{\frac{1}{5}}$$

$$x^{\frac{2}{5}}-6x^{\frac{1}{5}}+8=0 \qquad \text{(i)}$$

Put $y=x^{\frac{1}{5}}\Rightarrow y^2=x^{\frac{2}{5}}$

Equation (i) becomes

$$y^2-6y+8=0$$

$$y^2-4y-2y+8=0$$

$$y(y-4)-2(y-4)=0$$

$$(y-4)(y-2)=0$$

Either

$y-4=0$ or $y-2=0$

$y=4$; $y=2$

As $y=x^{\frac{1}{5}}$; As $y=x^{\frac{1}{5}}$

$x^{\frac{1}{5}} = 4$; $x^{\frac{1}{5}} = 2$

$x = (4)^5$; $x = (2)^5$

$x = 1024$; $x = 32$

$\therefore$ Solution Set $= \{32, 1024\}$

Q.6 $(x+1)(x+2)(x+3)(x+4) = 24$

Solution:

$(x+1)(x+2)(x+3)(x+4) = 24$

Re-arranging the equation

$[(x+1)(x+4)][(x+3)(x+2)] = 24$ $(\because 1+4 = 3+2 = 5)$

$(x^2+5x+4)(x^2+5x+6) = 24$

Put $x^2+5x = y$

Then above equation becomes

$(y+4)(y+6) = 24$

$y^2+10y+\cancel{24} = \cancel{24}$

$y(y+10) = 0$

Either

$y = 0$ or $y+10 = 0$

As $x^2+5x = y$; As $y = x^2+5x$

$x^2+5x = 0$; $x^2+5x+10 = 0$

$x(x+5) = 0$; $a = 1, b = 5, c = 10$

$\boxed{x = 0}$ or $x+5 = 0$; $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\boxed{x = -5}$ $x = \dfrac{-5 \pm \sqrt{(5)^2 - 4(1)(10)}}{2(1)}$

$x = \dfrac{-5 \pm \sqrt{25-40}}{2}$

$\boxed{x = \dfrac{-5 \pm \sqrt{15}i}{2}}$

$\therefore$ Soultion Set $= \left\{-5, 0, \dfrac{-5 \pm \sqrt{15}i}{2}\right\}$

Q.7 $\mathbf{(x-1)(x+5)(x+8)(x+2)-880=0}$

Solution:

$$(x-1)(x+5)(x+8)(x+2)-880=0$$

Re-arranging the equation

$$\left[(x-1)(x+8)\right]\left[(x+5)(x+2)\right]=880 \qquad (\because -1+8=5+2=7)$$

$$(x^2+7x-8)(x^2+7x+10)=880$$

Put $x^2+7x=y$

$$(y-8)(y+10)=880$$

$$y^2+2y-80-880=0$$

$$y^2+2y-960=0$$

$$y^2+32y-30y-960=0$$

$$y(y+32)-30(y+32)=0$$

$$(y+32)(y-30)=0$$

Either

$y+32=0$	or $y-30=0$
As $y=x^2+7x$;	As $y=x^2+7x$
$x^2+7x+32=0$;	$x^2+7x-30=0$
$a=1, b=7, c=32$;	$x^2+10x-3x-30=0$
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$;	$x(x+10)-3(x+10)=0$
$x=\dfrac{-7\pm\sqrt{(7)^2-4(1)(32)}}{2(1)}$;	$(x+10)(x-3)=0$
$x=\dfrac{-7\pm\sqrt{49-128}}{2}$;	$x+10=0$ or $x-3=0$
$x=\dfrac{-7\pm\sqrt{-79}}{2}$;	$\boxed{x=-10}$ or $\boxed{x=3}$
$\boxed{x=\dfrac{-7\pm\sqrt{79}i}{2}}$	

$$\boxed{\therefore \text{ Solution Set }=\left\{-10, 3, \frac{-7\pm\sqrt{79}i}{2}\right\}}$$

Q.8 $(x-5)(x-7)(x+6)(x+4)-504=0$

Solution:

$(x-5)(x-7)(x+6)(x+4)-504=0$

Re-arranging the equation

$[(x-5)(x+4)][(x-7)(x+6)]=504$ $(\because -5+4=-7+6=-1)$

$(x^2-x-20)(x^2-x-42)=504$

Put $x^2-x=y$

$(y-20)(y-42)=504$

$y^2-62y+840-504=0$

$y^2-62y+336=0$

$y^2-56y-6y+336=0$

$y(y-56)-6(y-56)=0$

$(y-56)(y-6)=0$

Either

$y-56=0$		or	$y-6=0$		
$y=56$		;	$y=6$		
As $x^2-x=y$		;	As $x^2-x=y$		
As $x^2-x-56=0$		;	$x^2-x-6=0$		
$x^2-8x+7x-56=0$		;	$x^2-3x+2x-6=0$		
$x(x-8)+7(x-8)=0$		;	$x(x-3)+2(x-3)=0$		
$(x-8)(x+7)=0$		;	$(x-3)(x+2)=0$		
$x-8=0$ or	$x+7=0$	;	$x-3=0$ or	$x+2=0$	
$x=8$;	$x=-7$	;	$x=3$;	$x=-2$	

$\therefore$ solution Set $=\{-7,-2,3,8\}$

Q.9 $(x-1)(x-2)(x-8)(x+5)+360=0$

Solution:

$(x-1)(x-2)(x-8)(x+5)+360=0$

$[(x-1)(x-2)][(x-8)(x+5)]+360=0$ $(\because -1-2=-8+5=-3)$

$\left(x^2-3x+2\right)\left(x^2-3x-40\right)+360=0$

Put $x^2-3x=y$

$(y+2)(y-40)+360=0$

$y^2-38y-80+360=0$

$y^2-38y+280=0$

$y^2-28y-10y+280=0$

$y(y-28)-10(y-28)=0$

$(y-28)(y-10)=0$

Either

$y-28=0$			or	$y-10=0$		
As $x^2-3x=y$			;	As $x^2-3x=y$		
$x^2-3x-28=0$			;	$x^2-3x-10=0$		
$x^2-7x+4x-28=0$			;	$x^2-5x+2x-10=0$		
$x(x-7)+4(x-7)=0$			;	$x(x-5)+2(x-5)=0$		
$(x-7)(x+4)=0$			;	$(x-5)(x+2)=0$		
$x-7=0$	or	$x+4=0$	;	$x-5=0$	or	$x+2=0$
$\boxed{x=7}$	;	$\boxed{x=-4}$	;	$\boxed{x=5}$	;	$\boxed{x=-2}$

$\boxed{\therefore \text{ solution Set} =\{-4,-2,5,7\}}$

Q.10 $\mathbf{(x+1)(2x+3)(2x+5)(x+3)=945}$

Solution:

$(x+1)(2x+3)(2x+5)(x+3)=945$

Re-arranging the equation

$[(x+1)(x+3)][(2x+3)(2x+5)]=945 \qquad \left(\because 1+3=4, \frac{3}{2}+\frac{5}{2}=\frac{8}{2}=4\right)$

$\left(x^2+4x+3\right)\left(4x^2+10x+6x+15\right)=945$

$\left(x^2+4x+3\right)\left(4x^2+16x+15\right)=945$

$\left(x^2+4x+3\right)\left(4\left(x^2+4x\right)+15\right)=945$

Put $x^2+4x=y$

$(y+3)(4y+15)=945$

$4y^2+27y+45-945=0$

$4y^2+27y-900=0$

$4y^2+75y-48y-900=0$

$y(4y+75)-12(4y+75)=0$

$(y-12)(4y+75)=0$

$y-12=0$ or $4y+75=0$

As $x^2+4x=y$	;	As $x^2+4x=y$
$x^2+4x-12=0$	;	$4(x^2+4x)+75=0$
$x^2+6x-2x-12=0$	;	$4x^2+16x+75=0$
$x(x+6)-2(x+6)=0$	;	$a=4, b=16, c=75$
$(x+6)(x-2)=0$	;	$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$
Either		
$x+6=0$ or $x-2=0$	;	$x=\dfrac{-16\pm\sqrt{16^2-4(4)(75)}}{2(4)}$
$\boxed{x=-6}$ or $\boxed{x=2}$	;	$x=\dfrac{-16\pm\sqrt{256-16\times 75}}{8}$

$$x=\frac{-16\pm 4\sqrt{-59}}{8}$$

$$x=\frac{\cancel{4}\left(-4\pm\sqrt{59}i\right)}{\cancel{8}}$$

$$\boxed{x=\frac{-4\pm\sqrt{59}i}{2}}$$

$$\boxed{\therefore\ \text{Solution Set}=\left\{-6, 2, \frac{-4\pm\sqrt{59}i}{2}\right\}}$$

Q.11 $(2x-7)(x^2-9)(2x+5)-91=0$

Solution:

$(2x-7)(x^2-9)(2x+5)-91=0$

$(2x-7)(x-3)(x+3)(2x+5)-91=0$

Re-arranging the equation

$[(2x-7)(x+3)][(x-3)(2x+5)]-91=0$ $\left(\because \frac{-7}{2}+3=\frac{5}{2}-3=\frac{-1}{2}\right)$

$(2x^2-x-21)(2x^2-x-15)-91=0$

Put $2x^2-x=y$

$(y-21)(y-15)-91=0$

$y^2-36y+315-91=0$

$y^2-36y+224=0$

$y^2-28y-8y+224=0$

$y(y-28)-8(y-28)=0$

$(y-28)(y-8)=0$

Either

$y-28=0$	or	$y-8=0$
As $y=2x^2-x$	;	As $y=2x^2-x$
$2x^2-x-28=0$	;	$2x^2-x-8=0$
$2x^2-8x+7x-28=0$	;	$a=2, b=-1, c=-8$
$2x(x-4)+7(x-4)=0$	;	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$(x-4)(2x+7)=0$	;	$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(2)(-8)}}{2(2)}$
$x-4=0$ or $2x+7=0$	;	$x=\frac{1\pm\sqrt{1+64}}{4}$
$\boxed{x=4}$; $2x=-7$	;	$\boxed{x=\frac{1\pm\sqrt{65}}{4}}$
; $\boxed{x=\frac{-7}{2}}$		

$$\boxed{\therefore \text{ Solution Set}=\left\{\frac{-7}{2}, 4, \frac{1\pm\sqrt{65}}{4}\right\}}$$

Q.12 $\left(x^2+6x+8\right)\left(x^2+14x+48\right)=105$

Solution:

$\left(x^2+6x+8\right)\left(x^2+14x+48\right)=105$

$\left(x^2+4x+2x+8\right)\left(x^2+8x+6x+48\right)=105$

$\{x(x+4)+2(x+4)\}\{x(x+8)+6(x+8)\}=105$

$(x+4)(x+2)(x+6)(x+8)=105$

Re-arranging the equation

$[(x+4)(x+6)][(x+2)(x+8)]=105 \qquad (\because 4+6=2+8=10)$

$\left(x^2+10x+24\right)\left(x^2+10x+16\right)=105$

Put $x^2+10x=y$

$(y+24)(y+16)=105$

$y^2+40y+384-105=0$

$y^2+40y+279=0$

$y^2+31y+9y+279=0$

$y(y+31)+9(y+31)=0$

$(y+31)(y+9)=0$

Either

$y+31=0$	or	$y+9=0$
As $y=x^2+10x$	;	As $y=x^2+10x$
$x^2+10x+31=0$	;	$x^2+10x+9=0$
$a=1, b=10, c=31$	;	$x^2+9x+x+9=0$
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	;	$x(x+9)+1(x+9)=0$
$x=\dfrac{-10\pm\sqrt{(10)^2-4(1)(31)}}{2(1)}$	;	$(x+9)(x+1)=0$
$x=\dfrac{-10\pm\sqrt{100-124}}{2}$	;	$x+9=0$ or $x+1=0$
$x=\dfrac{-10\pm\sqrt{-24}}{2}$	;	$\boxed{x=-9}$; $\boxed{x=-1}$

$$x=\frac{-10\pm\sqrt{4\times(-6)}}{2}$$

$$x=\frac{-10\pm2\sqrt{6}i}{2}$$

$$x=\frac{2\left(-5\pm\sqrt{6}i\right)}{2}$$

$$\boxed{x=-5\pm\sqrt{6}i}$$

$$\boxed{\therefore \text{ Solution Set } = \left\{-9,-1,-5\pm\sqrt{6}i\right\}}$$

Q.13 $\left(x^2+6x-27\right)\left(x^2-2x-35\right)=385$

Solution:

$$\left(x^2+6x-27\right)\left(x^2-2x-35\right)=385$$

$$\left(x^2+9x-3x-27\right)\left(x^2-7x+5x-35\right)=385$$

$$\left\{x(x+9)-3(x+9)\right\}\left\{x(x-7)+5(x-7)\right\}=385$$

$$(x+9)(x-3)(x-7)(x+5)=385$$

Re-arranging the equation

$$\left[(x+9)(x-7)\right]\left[(x+5)(x-3)\right]=385 \qquad (\because 9-7=5-3=2)$$

$$\left(x^2+2x-63\right)\left(x^2+2x-15\right)=385$$

Put $x^2+2x=y$

$$(y-63)(y-15)=385$$

$$y^2-78y+945-385=0$$

$$y^2-78y+560=0$$

$$y^2-70y-8y+560=0$$

$$y(y-70)-8(y-70)=0$$

$$(y-70)(y-8)=0$$

Either

$y-70=0$ or $y-8=0$

As $x^2+2x=y$; As $x^2+2x=y$

$x^2+2x-70=0$; $x^2+2x-8=0$

$a = 1, b = 2, c = -70$; $x^2 + 4x - 2x - 8 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $x(x+4) - 2(x+4) = 0$

$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(-70)}}{2(1)}$; $(x+4)(x-2) = 0$

$x = \dfrac{-2 \pm \sqrt{4 + 280}}{2}$; $x + 4 = 0$ or $x - 2 = 0$

$x = \dfrac{-2 \pm \sqrt{4(1+70)}}{2}$; $\boxed{x = -4}$; $\boxed{x = 2}$

$x = \dfrac{-2 \pm 2\sqrt{71}}{2}$

$\boxed{x = -1 \pm \sqrt{71}}$

$\boxed{\therefore \text{ Solution Set is } = \{-4, 2, -1 \pm \sqrt{71}\}}$

Q.14 $\mathbf{4.2^{2x+1} - 9.2^x + 1 = 0}$

Solution:

$4.2^{2x+1} - 9.2^x + 1 = 0$

$4.2.2^{2x} - 9.2^x + 1 = 0$

$8.2^{2x} - 9.2^x + 1 = 0$ (i)

Put $y = 2^x \Rightarrow y^2 = 2^{2x}$

Equation (i) becomes

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$(y-1)(8y-1) = 0$

Either $y - 1 = 0$ or $8y - 1 = 0$

As $y = 2^x$; $y = 2^x$

$2^x - 1 = 0$; $8.2^x - 1 = 0$

$2^x = 1$; $2^x = \dfrac{1}{8}$

$2^x = 2^0$; $2^x = 2^{-3}$

$\boxed{x = 0}$; $\boxed{x = -3}$

$\boxed{\therefore \text{ Solution Set} = \{-3, 0\}}$

Q.15 $\mathbf{2^x + 2^{-x+6} - 20 = 0}$

Solution:

$$2^x + 2^{-x+6} - 20 = 0$$

$$2^x + \frac{2^6}{2^x} - 20 = 0$$

$$2^{2x} + 64 - 20.2^x = 0 \qquad \text{(i)}$$

Put $2^x = y \Rightarrow 2^{2x} = y^2$

Equation (i) becomes

$$y^2 - 20y + 64 = 0$$

$$y^2 - 16y - 4y + 64 = 0$$

$$y(y-16) - 4(y-16) = 0$$

$$(y-16)(y-4) = 0$$

Either $y - 16 = 0$ or $y - 4 = 0$

As $y = 2^x$	;	As $y = 2^x$
$2^x - 16 = 0$	;	$2^x - 4 = 0$
$2^x = 16$	;	$2^x = 4$
$2^x = 2^4$	;	$2^x = 2^2$
$\boxed{x = 4}$	;	$\boxed{x = 2}$

$\therefore$ Solution Set $= \{2, 4\}$

Q.16 $\mathbf{4^x - 3.2^{x+3} + 128 = 0}$

Solution:

$$4^x - 3.2^{x+3} + 128 = 0$$

$$(2^2)^x - 3.2^3.2^x + 128 = 0$$

$$2^{2x} - 24.2^x + 128 = 0 \qquad \text{(i)}$$

Put $y = 2^x \Rightarrow y^2 = 2^{2x}$

Equation (i) becomes

$$y^2 - 24y + 128 = 0$$

$$y^2 - 16y - 8y + 128 = 0$$

$$y(y-16) - 8(y-16) = 0$$

$$(y-16)(y-8) = 0$$

Either $y - 16 = 0$ or $y - 8 = 0$

$y = 16$	;	$y = 8$
As $y = 2^x$	;	As $y = 2^x$
$2^x = 2^4$	;	$2^x = 2^3$
$\boxed{x = 4}$	;	$\boxed{x = 3}$

$\therefore$ Solution Set $= \{3, 4\}$

Q.17 $3^{2x-1}-12.3^{x}+81=0$

Solution:

$$3^{2x-1}-12.3^{x}+81=0$$

$$\frac{3^{2x}}{3}-12.3^{x}+81=0$$

$$3^{2x}-36.3^{x}+243=0 \qquad \text{(i)}$$

Put $y=3^{x} \Rightarrow y^{2}=3^{2x}$

$$y^{2}-36y+243=0$$

$$y^{2}-27y-9y+243=0$$

$$y(y-27)-9(y-27)=0$$

$$(y-27)(y-9)=0$$

Either $y-27=0$ or $y-9=0$

As $y=3^{x}$; As $y=3^{x}$

$3^{x}=3^{3}$; $3^{x}=3^{2}$

$\boxed{x=3}$; $\boxed{x=2}$

$\boxed{\therefore \text{ Solution Set } = \{2,3\}}$

Q.18 $\left(x+\frac{1}{x}\right)^{2}-3\left(x+\frac{1}{x}\right)-4=0$

Solution:

$$\left(x+\frac{1}{x}\right)^{2}-3\left(x+\frac{1}{x}\right)-4=0 \qquad \text{(i)}$$

Put $x+\frac{1}{x}=y$

Equation (i) becomes

$$y^{2}-3y-4=0$$

$$y^{2}-4y+y-4=0$$

$$y(y-4)+1(y-4)=0$$

$$(y-4)(y+1)=0$$

Either

$y-4=0$ or $y+1=0$

As $y=x+\frac{1}{x}$; As $y=x+\frac{1}{x}$

$x+\frac{1}{x}-4=0$; $x+\frac{1}{x}+1=0$

$x^2-4x+1=0$; $x^2+x+1=0$

$a=1, b=-4, c=1$; $a=1, b=1, c=1$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$; $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$; $x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$

$x=\frac{4\pm\sqrt{12}}{2}$; $x=\frac{-1\pm\sqrt{-3}}{2}$

$x=\frac{4\pm2\sqrt{3}}{2}$; $\boxed{x=\frac{-1\pm\sqrt{3}i}{2}}$

$\boxed{x=2\pm\sqrt{3}}$

$$\boxed{\therefore\ \text{Solution Set}=\left\{2\pm\sqrt{3},\frac{-1\pm\sqrt{3}i}{2}\right\}}$$

Q.19 $\boldsymbol{x^2+x-4+\frac{1}{x}+\frac{1}{x^2}=0}$

Solution:

$$x^2+x-4+\frac{1}{x}+\frac{1}{x^2}=0$$

$$x^2+\frac{1}{x^2}+x+\frac{1}{x}-4=0$$

$$x^2+\frac{1}{x^2}+2+x+\frac{1}{x}-4-2=0$$

$$\left(x+\frac{1}{x}\right)^2+\left(x+\frac{1}{x}\right)-6=0 \qquad \text{(i)}$$

Put $y=x+\frac{1}{x}$ $\Rightarrow$ $y^2=\left(x+\frac{1}{x}\right)^2$

Equation (i) becomes

$y^2+y-6=0$

$y^2+3y-2y-6=0$

$y(y+3)-2(y+3)=0$

$(y+3)(y-2)=0$

Either

$y+3=0$	or	$y-2=0$
As $y=x+\frac{1}{x}$	;	As $y=x+\frac{1}{x}$
$x+\frac{1}{x}+3=0$	;	$x+\frac{1}{x}-2=0$
$x^2+3x+1=0$	;	$x^2-2x+1=0$
$a=1, b=3, c=1$	;	$(x-1)^2=0$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	;	$x-1=0$
$x=\frac{-3\pm\sqrt{(3)^2-4(1)(1)}}{2(1)}$	;	$\boxed{x=1}$

$x=\frac{-3\pm\sqrt{9-4}}{2}$

$\boxed{x=\frac{-3\pm\sqrt{5}}{2}}$

$\boxed{\therefore \text{ Solution Set }=\left\{1, \frac{-3\pm\sqrt{5}}{2}\right\}}$

Q.20 $\left(x-\frac{1}{x}\right)^2+3\left(x+\frac{1}{x}\right)=0$

Solution:

$\left(x-\frac{1}{x}\right)^2+3\left(x+\frac{1}{x}\right)=0$

$x^2+\frac{1}{x^2}-2+3\left(x+\frac{1}{x}\right)=0$

$x^2+\frac{1}{x^2}+2+3\left(x+\frac{1}{x}\right)-2-2=0$

$$\left(x+\frac{1}{x}\right)^2+3\left(x+\frac{1}{x}\right)-4=0 \qquad (i)$$

Put $x+\frac{1}{x}=y \Rightarrow \left(x+\frac{1}{x}\right)^2=y^2$

Equation (i) becomes

$y^2+3y-4=0$

$y^2+4y-y-4=0$

$y(y+4)-1(y+4)=0$

$(y+4)(y-1)=0$

Either

$y+4=0$	or	$y-1=0$
As $y=x+\frac{1}{x}$	;	As $y=x+\frac{1}{x}$
$x+\frac{1}{x}+4=0$	;	$x+\frac{1}{x}-1=0$
$x^2+4x+1=0$	;	$x^2-x+1=0$
$a=1, b=4, c=1$	;	$a=1, b=-1, c=1$
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	;	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x=\frac{-4\pm\sqrt{(4)^2-4(1)(1)}}{2(1)}$	;	$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$
$x=\frac{-4\pm\sqrt{12}}{2}$	;	$x=\frac{1\pm\sqrt{-3}}{2}$
$x=\frac{-4\pm2\sqrt{3}}{2}$	;	$x=\frac{1\pm\sqrt{3}i}{2}$
$\boxed{x=-2\pm\sqrt{3}}$	;	$\boxed{x=\frac{1\pm\sqrt{3}i}{2}}$

$$\boxed{\therefore \text{ Solution Set} = \left\{-2\pm\sqrt{3}, \frac{1\pm\sqrt{3}i}{2}\right\}}$$

Q.21 $\mathbf{2x^4-3x^3-x^2-3x+2=0}$

Solution:

$2x^4-3x^3-x^2-3x+2=0$

Dividing by 'x^2'

$2x^2-3x-1-\frac{3}{x}+\frac{2}{x^2}=0$

$2\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)-1=0$

$2\left(x^2+\frac{1}{x^2}+2\right)-3\left(x+\frac{1}{x}\right)-1-4=0$

$2\left(x+\frac{1}{x}\right)^2-3\left(x+\frac{1}{x}\right)-5=0$ (i)

Put $y=x+\frac{1}{x}\Rightarrow y^2=\left(x+\frac{1}{x}\right)^2$

Equation (i) becomes

$2y^2-3y-5=0$

$2y^2-5y+2y-5=0$

$y(2y-5)+1(2y-5)=0$

$(2y-5)(y+1)=0$

Either

$2y-5=0$	or	$y+1=0$
As $y=x+\frac{1}{x}$	;	As $y=x+\frac{1}{x}$
$2\left(x+\frac{1}{x}\right)-5=0$	;	$x+\frac{1}{x}+1=0$
$2x+\frac{2}{x}-5=0$	;	$x^2+x+1=0$
$2x^2-5x+2=0$	;	$a=1,b=1,c=1$
$2x^2-4x-x+2=0$	;	$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$2x(x-2)-1(x-2)=0$	;	$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$
$(x-2)(2x-1)=0$	;	$x=\frac{-1\pm\sqrt{-3}}{2}$
$x-2=0, 2x-1=0$	;	$x=\frac{-1\pm\sqrt{3}i}{2}$
$x=2, x=\frac{1}{2}$		

$\therefore$ Solution Set $=\left\{\frac{1}{2},2,\frac{-1\pm\sqrt{3}i}{2}\right\}$

Q.22 $\mathbf{2x^4+3x^3-4x^2-3x+2=0}$

Solution:

$2x^4+3x^3-4x^2-3x+2=0$

Dividing by 'x^2'

$2x^2+3x-4-\frac{3}{x}+\frac{2}{x^2}=0$

$2\left(x^2+\frac{1}{x^2}\right)+3\left(x-\frac{1}{x}\right)-4=0$

$2\left(x^2+\frac{1}{x^2}-2\right)+3\left(x-\frac{1}{x}\right)=0$

$2\left(x-\frac{1}{x}\right)^2+3\left(x-\frac{1}{x}\right)=0$ (i)

Put $x-\frac{1}{x}=y \Rightarrow \left(x-\frac{1}{x}\right)^2=y^2$

Equation (i) becomes

$2y^2+3y=0$

$y(2y+3)=0$

Either

$y=0$ or $2y+3=0$

As $x-\frac{1}{x}=y$; As $y=x-\frac{1}{x}$

$x-\frac{1}{x}=0$; $2\left(x-\frac{1}{x}\right)+3=0$

$x^2-1=0$; $2x-\frac{2}{x}+3=0$

$x^2=1$; $2x^2+3x-2=0$

$x=\pm1$; $2x^2+4x-x-2=0$

; $2x(x+2)-1(x+2)=0$

; $(x+2)(2x-1)=0$

; $x+2=0$ or $2x-1=0$

; $\boxed{x=-2}$; $\boxed{x=\frac{1}{2}}$

$\boxed{\therefore \text{ Solution Set }=\left\{-1,1,-2,\frac{1}{2}\right\}}$

Q.23 $6x^4-35x^3+62x^2-35x+6=0$

Solution:

$$6x^4-35x^3+62x^2-35x+6=0$$

Dividing by 'x^2'

$$6x^2-35x+62-\frac{35}{x}+\frac{6}{x^2}=0$$

$$6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0$$

$$6\left(x^2+\frac{1}{x^2}+2\right)-35\left(x+\frac{1}{x}\right)+62-12=0$$

$$6\left(x+\frac{1}{x}\right)^2-35\left(x+\frac{1}{x}\right)+50=0 \qquad \text{(i)}$$

Put $x+\frac{1}{x}=y \Rightarrow \left(x+\frac{1}{x}\right)^2=y^2$

Equation (i) becomes

$$6y^2-35y+50=0$$

$$6y^2-20y-15y+50=0$$

$$2y(3y-10)-5(3y-10)=0$$

$$(3y-10)(2y-5)=0$$

Either

$3y-10=0$ or $2y-5=0$

As $x+\frac{1}{x}=y$; As $x+\frac{1}{x}=y$

$3\left(x+\frac{1}{x}\right)-10=0$; $2\left(x+\frac{1}{x}\right)-5=0$

$3x+\frac{3}{x}-10=0$; $2x+\frac{2}{x}-5=0$

$3x^2+3-10x=0$; $2x^2-5x+2=0$

$3x^2-10x+3=0$; $2x^2-4x-x+2=0$

$3x^2-9x-x+3=0$; $2x(x-2)-1(x-2)=0$

$3x(x-3)-1(x-3)=0$; $(x-2)(2x-1)=0$

$(x-3)(3x-1)=0$

$x-3=0$ or $3x-1=0$; $x-2=0$ or $2x-1=0$

$\boxed{x=3}$ $\boxed{x=\frac{1}{3}}$; $\boxed{x=2}$ $\boxed{x=\frac{1}{2}}$

$$\boxed{\therefore \text{ Solution Set}=\left\{\frac{1}{3},\frac{1}{2},2,3\right\}}$$

Q.24 $x^4 - 6x^2 + 10 - \frac{6}{x^2} + \frac{1}{x^4} = 0$

Solution:

$$x^4 - 6x^2 + 10 - \frac{6}{x^2} + \frac{1}{x^4} = 0$$

$$x^4 + \frac{1}{x^4} - 6\left(x^2 + \frac{1}{x^2}\right) + 10 = 0$$

$$x^4 + \frac{1}{x^4} + 2 - 6\left(x^2 + \frac{1}{x^2}\right) + 10 - 2 = 0$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 - 6\left(x^2 + \frac{1}{x^2}\right) + 8 = 0 \qquad \text{(i)}$$

Put $x^2 + \frac{1}{x^2} = y \Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = y^2$

Equation (i) becomes

$$y^2 - 6y + 8 = 0$$

$$y^2 - 4y - 2y + 8 = 0$$

$$y(y-4) - 2(y-4) = 0$$

$$(y-4)(y-2) = 0$$

Either

$y - 4 = 0$ or $y - 2 = 0$

As $y = x^2 + \frac{1}{x^2}$; $y = x^2 + \frac{1}{x^2}$

$x^2 + \frac{1}{x^2} - 4 = 0$; $x^2 + \frac{1}{x^2} - 2 = 0$

$x^4 - 4x^2 + 1 = 0$; $x^4 - 2x^2 + 1 = 0$

Put $x^2 = t \Rightarrow x^4 = t^2$

$t^2 - 4t + 1 = 0$

$a = 1, b = -4, c = 1$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$

$t = \dfrac{4 \pm \sqrt{16-4}}{2} = \dfrac{4 \pm \sqrt{12}}{2}$

$t = \dfrac{4 \pm 2\sqrt{3}}{2}$

$t = 2 \pm \sqrt{3}$

As $t = x^2$

$x^2 = 2 \pm \sqrt{3}$

$\boxed{x = \pm\sqrt{2 \pm \sqrt{3}}}$

Put $x^2 = t \Rightarrow x^4 = t^2$

$t^2 - 2t + 1 = 0$

$(t-1)^2 = 0$

$t - 1 = 0$

$t = 1$

As $t = x^2$

$x^2 = 1$

$\boxed{x = \pm 1}$

$\boxed{\therefore \text{ Solution Set} = \left\{\pm 1, \pm\sqrt{2 \pm \sqrt{3}}\right\}}$