CHAPTER 5

PARTIAL FRACTIONS

Rational Fractions:

A fraction $\frac{P(x)}{Q(x)}$ where both $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$ for all values of x is called Rational Fraction.

It has two types

Proper Rational Fraction	Improper Rational Fraction
A fraction is proper if degree of the polynomial in numerator is less than the degree of the polynomial in the denominator.	A fraction is improper if degree of the polynomial in numerator is greater than or equal to the degree of the polynomial in the denominator.

Partial Fractions:

"Partial Fractions" are rational fractions obtained by decomposing a single rational fraction into sum of two or more proper rational fractions.

Partial Fraction Resolution:

The process of decomposing a single rational fraction into sum of two or more proper rational fractions is called the "Partial Fraction Resolution".

Note: There are two types of equations

Conditional Equation	Identity
An equation in which two algebraic expressions are equal for some particular value / values of the variable is called "Conditional equation".	An equation in which two algebraic expressions are equal for all values of the variable is called the "Identity".

Note:

(i) For simplicity conditional equations will be called "equations" here.

(ii) For both "equations" and "identities" "=" symbol will be used.

(iii) Any improper fraction can be converted into a mixed form. Which consist of sum of a polynomial and a proper "rational fraction".

(iv) When a rational fraction is resolved and represented into partial fractions, the result is an identity.

Theorem:

Two polynomials are equal for all values of the variable if and only if these have the same degree and equal co-efficients for all like terms.

How to resolve a rational fraction $\frac{P(x)}{Q(x)}$:

(i) If $\frac{P(x)}{Q(x)}$ is not a "proper fraction, then first write $\frac{P(x)}{Q(x)}$ into mixed form.

(ii) Identify the type of given rational fraction.

(iii) Write an identity according to the type, it will consist of some unknown constant.

(iv) Simplify and equate the co-efficients of like terms on both sides of identity.

(v) Solved the resulting equations to get value of unknown constants.

Case-I:

When $Q(x)$ has only non-repeated linear factors:

The polynomial $Q(x)$ may be written as:

$Q(x)=(x-a_1)(x-a_2)...(x-a_n)$ where $a_1 \neq a_2 \neq ... \neq a_n$

$\therefore$ $\frac{P(x)}{Q(x)}=\frac{A_1}{x-a_1}+\frac{A_2}{x-a_2}+...+\frac{A_n}{x-a_n}$ is an identity.

Where, the coefficients $A_1, A_2, ..., A_n$ are unknown constants to be found.

Example:

Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fractions.

Solution:

Suppose

$$\frac{7x+25}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4} \qquad \text{(i)}$$

Multiplying both sides $(x+3)(x+4)$

$\Rightarrow \quad 7x+25 = A(x+4) + B(x+3)$ (ii)

As two sides of the identity are equal for all values of x,

Let us put $x+3=0 \Rightarrow x=-3$ in equation (ii)

We get $-21+25 = A(-3+4)$

$\Rightarrow \quad A = 4$

Putting $x+4=0 \Rightarrow x=-4$ in equation (ii)

We get $-28+25 = B(-4+3)$

$\Rightarrow \quad B = 3$

Hence the partial fractions are:

$$\boxed{\frac{7x+25}{(x+3)(x+4)} = \frac{4}{x+3} + \frac{3}{x+4}}.$$

EXERCISE 5.1

Resolve the following into partial fractions.

Q.1 $\dfrac{1}{x^2-1}$

Solution: $\dfrac{1}{x^2-1}=\dfrac{1}{(x+1)(x-1)}$

Let, $\dfrac{1}{(x+1)(x-1)}=\dfrac{A}{x+1}+\dfrac{B}{x-1}$ (i)

$\because$ Multiplying both sides by $(x+1)(x-1)$, we get

$$\frac{1}{(x+1)(x-1)}\times(x+1)(x-1)=\left(\frac{A}{x+1}\right)(x+1)(x-1)+\left(\frac{B}{x-1}\right)(x+1)(x-1)$$

$1=A(x-1)+B(x+1)$ (ii)

As two sides of identity are equal for all values of x, let us put

$x-1=0 \Rightarrow x=1$ in equation (ii), we get

$1=A(0)+B(2)$

$\Rightarrow \boxed{B=\dfrac{1}{2}}$

$\because$ Putting $x+1=0 \Rightarrow x=-1$ in equation (ii), we get

$1=A(-1-1)+B(-1+1)$

$1=A(-2)+0$

$\Rightarrow \boxed{A=-\dfrac{1}{2}}$

Putting the values of A and B in equation (i)

So we have,

$$\frac{1}{(x+1)(x-1)}=\frac{-\frac{1}{2}}{x+1}+\frac{\frac{1}{2}}{x-1}$$

Hence the partial fractions are:

$$\boxed{\frac{1}{(x+1)(x-1)}=\frac{1}{2(x-1)}-\frac{1}{2(x+1)}}$$

Q.2 $\dfrac{x^2+1}{(x+1)(x-1)}$

Solution:

Since the degree of polynomial of numerator and denominator is equal so, it is improper Rational Fraction. First transform it into mixed form by division.

$\dfrac{x^2+1}{x^2-1}$

$$\begin{array}{r|l} & 1 \\ \hline x^2-1 & x^2+1 \\ & -x^2 \mp 1 \\ \hline & 2 \end{array}$$

Dividing x^2+1 by x^2-1, we have quotient = 1 and remainder = 2, therefore

$$\frac{x^2+1}{x^2-1}=1+\frac{2}{x^2-1}=1+\frac{2}{(x+1)(x-1)} \qquad \text{(i)}$$

Let, $\dfrac{2}{(x+1)(x-1)}=\dfrac{A}{x+1}+\dfrac{B}{x-1}$ (ii)

Multiplying by $(x+1)(x-1)$ on both sides of equation (ii), we get

$$2=A(x-1)+B(x+1) \qquad \text{(iii)}$$

Which is identity equation in *x*.

Putting $x-1=0 \Rightarrow x=-1$ in equation (iii)

$2=A(0)+B(1+1)$

$2=2B$

$\Rightarrow \boxed{B=1}$

Putting $x+1=0 \Rightarrow x=-1$ in equation (iii)

$2=A(-1-1)+B(0)$

$2=-2A$

$\Rightarrow \boxed{A=-1}$

Putting the values of *A* and *B* in equation (ii), we get

$$\frac{2}{(x+1)(x-1)}=\frac{-1}{x+1}+\frac{1}{x-1}$$

By using equation (i)

Hence the partial fraction are:

$$\boxed{\frac{x^2+1}{x^2-1}=1+\frac{1}{x-1}-\frac{1}{x+1}}$$

Q.3 $\dfrac{2x+1}{(x-1)(x+2)(x+3)}$

Solution: Let,

$$\frac{2x+1}{(x-1)(x+2)(x+3)}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x+3} \quad \text{(i)}$$

Multiplying by $(x-1)(x+2)(x+3)$ on both sides of equation (i), we get

$$(2x+1)=A(x+2)(x+3)+B(x-1)(x+3)+C(x-1)(x+2) \quad \text{(ii)}$$

Which is an identity in x

Putting $x-1=0 \Rightarrow x=1$ in equation (ii)

$$2(1)+1=A(1+2)(1+3)+B(0)(x+3)+C(0)(x+2)$$

$$3=A(3)(4)+0+0$$

$$\Rightarrow 12A=3$$

$$\Rightarrow A=\frac{3}{12}$$

$$\boxed{A=\frac{1}{4}}$$

Putting $x+2=0 \Rightarrow x=-2$ in equation (ii)

$$2(-2)+1=A(0)(x+3)+B(-2-1)(-2+3)+C(-2-1)(0)$$

$$-4+1=0+B(-3)(1)+0$$

$$-3=-3B$$

$$\Rightarrow \boxed{B=1}$$

Putting $x+3=0 \Rightarrow x=-3$ in equation (ii)

$$2(-3)+1=A(-3+2)(0)+B(-3-1)(0)+C(-3-1)(-3+2)$$

$$-6+1=0+0+C(-4)(-1)$$

$$-5=4C$$

$$\Rightarrow \boxed{C=-\frac{5}{4}}$$

Putting values of A, B and C in equation (i)

$$\frac{2x+1}{(x-1)(x+2)(x+3)}=\frac{\frac{1}{4}}{x-1}+\frac{1}{x+2}+\frac{-\frac{5}{4}}{x+3}$$

Hence the partial fractions are:

$$\boxed{\frac{2x+1}{(x-1)(x+2)(x+3)}=\frac{1}{4(x-1)}+\frac{1}{x+2}-\frac{5}{4(x+3)}}$$

Q.4 $\dfrac{3x^2 - 4x - 5}{(x-2)(x^2+7x+10)}$

Solution: The factor $x^2+7x+10$ in the denominator can be factorized, as

Consider $x^2+7x+10 = x^2+5x+2x+10$

$x(x+5)+2(x+5) = (x+5)(x+2)$

So,

$$\frac{3x^2-4x-5}{(x-2)(x^2+7x+10)} = \frac{3x^2-4x-5}{(x-2)(x+2)(x+5)}$$

Let

$$\frac{3x^2-4x-5}{(x-2)(x+2)(x+5)} = \frac{A}{x-2}+\frac{B}{x+2}+\frac{C}{x+5} \qquad \text{(i)}$$

Multiplying $(x-2)(x+2)(x+5)$ on both sides of equation (i), we get

$$3x^2-4x-5 = A(x+2)(x+5)+B(x-2)(x+5)+C(x-2)(x+2) \qquad \text{(ii)}$$

Which is identity in x

Putting $x-2=0 \Rightarrow x=2$ in equation (ii)

$$3(2)^2-4(2)-5 = A(2+2)(2+5)+B(0)(2+5)+C(0)(2+2)$$

$$3(4)-8-5 = A(4)(7)+0+0$$

$$-1 = 28A$$

$$\Rightarrow \boxed{A = \frac{-1}{28}}$$

Putting $x+2=0 \Rightarrow x=-2$ in equation (ii)

$$3(-2)^2-4(-2)-5 = A(0)(-2+5)+B(-2-2)(-2+5)+C(-2-2)(0)$$

$$12+8-5 = 0+B(-4)(3)+0$$

$$15 = -12B$$

$$\Rightarrow B = \frac{-15}{12}$$

$$\boxed{B = -\frac{5}{4}}$$

Putting $x+5=0 \Rightarrow x=-5$ in equation (ii)

$$3(-5)^2-4(-5)-5 = A(-5+2)(0)+B(-5-2)(0)+C(-5-2)(-5+2)$$

$$75+20-5 = 0+0+C(-7)(-3)$$

$90 = 21C$

$\Rightarrow C = \dfrac{90}{21}$

$\boxed{C = \dfrac{30}{7}}$

Putting the values of A, B and C in equation (i)

$$\frac{3x^2-4x-5}{(x-2)(x+2)(x+5)} = \frac{-\frac{1}{28}}{x-2} + \frac{-\frac{5}{4}}{x+2} + \frac{\frac{30}{7}}{x+5}$$

Hence the partial fraction are:

$$\boxed{\frac{3x^2-4x-5}{(x-2)(x+2)(x+5)} = \frac{30}{7(x+5)} - \frac{1}{28(x-2)} - \frac{5}{4(x+2)}}$$

Q.5 $\quad \dfrac{\mathbf{1}}{(\boldsymbol{x}\,\mathbf{-1})(\mathbf{2}\boldsymbol{x}\,\mathbf{-1})(\mathbf{3}\boldsymbol{x}\,\mathbf{-1})}$

Solution:

Let $\dfrac{1}{(x-1)(2x-1)(3x-1)} = \dfrac{A}{x-1} + \dfrac{B}{2x-1} + \dfrac{C}{3x-1}$ (i)

Multiplying by $(x-1)(2x-1)(3x-1)$ on both sides of equation (i), we get

$1 = A(2x-1)(3x-1) + B(x-1)(3x-1) + C(x-1)(2x-1)$ (ii)

Which is identity in x.

Putting $x-1=0 \Rightarrow x=1$ in equation (ii)

$1 = A(2(1)-1)(3(1)-1) + B(0)(3(1)-1) + C(0)(2(1)-1)$

$1 = \mathrm{A}(1)(2) + 0 + 0$

$\boxed{\Rightarrow A = \dfrac{1}{2}}$

Putting $2x-1=0 \Rightarrow 2x=1 \Rightarrow x = \dfrac{1}{2}$ in equation (ii)

$1 = \mathrm{A}(0)\left[3\left(\dfrac{1}{2}\right)-1\right] + \mathrm{B}\left(\dfrac{1}{2}-1\right)\left[3\left(\dfrac{1}{2}\right)-1\right] + C\left(\dfrac{1}{2}-1\right)(0)$

$1 = 0 + B\left(\dfrac{-1}{2}\right)\left(\dfrac{3-2}{2}\right) + 0$

$1 = B\left(\dfrac{-1}{2}\right)\left(\dfrac{1}{2}\right)$

$1 = B\left(\dfrac{-1}{4}\right)$

$\Rightarrow \boxed{B = -4}$

Putting $3x-1=0 \Rightarrow 3x=1 \Rightarrow x=\dfrac{1}{3}$ in equation (ii), we get

$1 = A\left[2\left(\dfrac{1}{3}\right)-1\right](0) + B\left(\dfrac{1}{3}-1\right)(0) + C\left(\dfrac{1}{3}-1\right)\left[2\left(\dfrac{1}{3}\right)-1\right]$

$1 = 0 + 0 + C\left(\dfrac{1-3}{3}\right)\left(\dfrac{2-3}{3}\right)$

$1 = C\left(\dfrac{-2}{3}\right)\left(\dfrac{-1}{3}\right)$

$1 = C\left(\dfrac{2}{9}\right)$

$\Rightarrow \boxed{C = \dfrac{9}{2}}$

Putting the values of A,B and C in equation (i), we get

$$\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{\frac{1}{2}}{x-1} + \frac{-4}{2x-1} + \frac{\frac{9}{2}}{3x-1}$$

Hence the partial fractions are:

$$\boxed{\frac{1}{(x-1)(2x-1)(3x-1)} = \frac{1}{2(x-1)} - \frac{4}{2x-1} + \frac{9}{2(3x-1)}}$$

Q.6 $\quad \boldsymbol{\dfrac{x}{(x-a)(x-b)(x-c)}}$

Solution: Let,

$$\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c} \qquad \text{(i)}$$

Multiplying $(x-a)(x-b)(x-c)$ on both sides of equation (i), we get

Which is identity in x

$$x = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b) \qquad \text{(ii)}$$

Putting $x-a=0 \Rightarrow x=a$ in equation (ii)

$a = A(a-b)(a-c) + B(0)(a-c) + C(0)(a-b)$

$a = A(a-b)(a-c) + 0 + 0$

$$\Rightarrow \boxed{A=\frac{a}{(a-b)(a-c)}}$$

Putting $x-b=0 \Rightarrow x=b$ in equation (ii), we get

$$b=A(0)(b-c)+B(b-a)(b-c)+C(b-a)(0)$$

$$b=0+B(b-a)(b-c)+0$$

$$\Rightarrow \boxed{B=\frac{b}{(b-a)(b-c)}}$$

Putting $x-c=0 \Rightarrow x=c$ in equation (ii), we get

$$c=A(c-b)(0)+B(c-a)(0)+C(c-a)(c-b)$$

$$c=0+0+C(c-a)(c-b)$$

$$c=C(c-a)(c-b)$$

$$\Rightarrow \boxed{C=\frac{c}{(c-a)(c-b)}}$$

Putting the values of *A*, *B* and *C* in equation (i), we get

$$\frac{x}{(x-a)(x-b)(x-c)}=\frac{\frac{a}{(a-b)(a-c)}}{x-a}+\frac{\frac{b}{(b-a)(b-c)}}{x-b}+\frac{\frac{c}{(c-a)(c-b)}}{x-c}$$

Hence the partial fractions are:

$$\boxed{\frac{x}{(x-a)(x-b)(x-c)}=\frac{a}{(x-a)(a-b)(a-c)}+\frac{b}{(x-b)(b-a)(b-c)}+\frac{c}{(x-c)(c-a)(c-b)}}$$

Q.7 $\frac{6x^3+5x^2-7}{2x^2-x-1}$

Solution:

It is improper rational fraction so, transform it into mixed form.

$$\begin{array}{r|l} & 3x+4 \\ 2x^2-x-1 & 6x^3+5x^2-7 \\ & \underline{6x^3-3x^2-3x} \\ & 8x^2+3x-7 \\ & \underline{8x^2-4x-4} \\ & 7x-3 \end{array}$$

Dividing $6x^3+5x^2-7$ by $2x^2-x-1$, we have

Quotient $=3x+4$ and Remainder $7x-3$.

$$\therefore \frac{6x^3+5x^2-7}{2x^2-x-1}=3x+4+\frac{7x-3}{2x^2-x-1} \qquad \text{(i)}$$

Consider, $2x^2-x-1$

$$\Rightarrow 2x^2-2x+x-1$$

$\Rightarrow 2x(x-1)+1(x-1) = (x-1)(2x+1)$

So (i) becomes

$$\frac{6x^3+5x^2-7}{2x^2-x-1} = 3x+4+\frac{7x-3}{(x-1)(2x+1)} \quad \text{(ii)}$$

Let,

$$\frac{7x-3}{(x-1)(2x+1)} = \frac{A}{x-1}+\frac{B}{2x+1} \quad \text{(iii)}$$

Multiplying $(x-1)(2x+1)$ on both sides of equation (iii), we get

$$7x-3 = A(2x+1)+B(x-1) \quad \text{(iv)}$$

Which is identity in *x*.

Putting $x-1=0 \Rightarrow x=1$ in equation (iv)

$7(1)-3 = A(2(1)+1)+B(0)$

$4 = 3A$

$\Rightarrow \boxed{A=\frac{4}{3}}$

Putting $2x+1=0 \Rightarrow x=-\frac{1}{2}$ in equation (iv), we get

$7\left(\frac{-1}{2}\right)-3 = A(0)+B\left(\frac{-1}{2}-1\right)$

$\frac{-7-6}{2} = B\left(\frac{-3}{2}\right)$

$\frac{-13}{2} = B\left(\frac{-3}{2}\right)$

$\Rightarrow B = \left(\frac{-13}{2}\right)\left(\frac{-2}{3}\right)$

$\boxed{B=\frac{13}{3}}$

Putting the values of *A* and *B* in equation (iii), we get

$$\frac{7x-3}{(x-1)(2x+1)} = \frac{\frac{4}{3}}{x-1}+\frac{\frac{13}{3}}{2x+1}$$

By using equation (ii), we have

$$\boxed{\frac{6x^3+5x^2-7}{2x^2-x-1} = 3x+4+\frac{4}{3(x-1)}+\frac{13}{3(2x+1)}}$$

Q.8 $\mathbf{\dfrac{2x^3+x^2-5x+3}{2x^3+x^2-3x}}$

Solution:

It is improper rational fraction first transform it into mixed form.

$$2x^3+x^2-3x\overline{\smash{\big)}\,2x^3+x^2-5x+3\,}\quad 1$$
$$\underline{2x^3+x^2-3x}$$
$$-2x+3$$

Dividing $2x^3+x^2-5x+3$ by $2x^3+x^2-3x$, we get

Quotient = 1 and Remainder = $-2x+3$

$$\therefore \frac{2x^3+x^2-5x+3}{2x^3+x^2-3x}=1+\frac{-2x+3}{2x^3+x^2-3x}$$

$$=1-\frac{2x-3}{x(2x^2+x-3)} \qquad \text{(i)}$$

The factor in the denominator $2x^2+x-3$ can be factorized as,

$2x^2+3x-2x-3$

$\Rightarrow x(2x+3)-1(2x+3)$

$\Rightarrow (2x+3)(x-1)$

So (i) becomes,

$$\frac{2x^3+x^2-5x+3}{2x^3+x^2-3x}=1-\frac{2x-3}{x(x-1)(2x+3)} \qquad \text{(ii)}$$

Let, $\dfrac{2x-3}{x(x-1)(2x+3)}=\dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{2x+3}$ (iii)

Multiplying by $x(x-1)(2x+3)$ on both sides of equation (iii), we get

$$2x-3=A(x-1)(2x+3)+B(2x+3)(x)+C(x)(x-1) \qquad \text{(iv)}$$

Which is identity in x

Putting $x=0$ in equation (iv)

$2(0)-3=A(0-1)(2(0)+3)+B(2(0)+3)(0)+C(0)(0-1)$

$-3=A(-1)(3)+0+0$

$\Rightarrow -3A=-3$

$\boxed{A=1}$

Putting $x-1=0\Rightarrow x=1$ in equation (iv)

$2(1)-3=A(0)(2(1)+3)+B(1)(2(1)+3)+C(1)(0)$

$-1=0+5B+0$

$\Rightarrow 5B=-1$

$\boxed{B=-\frac{1}{5}}$

Putting $2x+3=0 \Rightarrow 2x=-3 \Rightarrow x=-\frac{3}{2}$ in equation (iv), we get

$$2\left(\frac{-3}{2}\right)-3=A\left(\frac{-3}{2}-1\right)(0)+B(0)(-3/2)+C\left(\frac{-3}{2}\right)\left(\frac{-3}{2}-1\right)$$

$$-6=0+0+C\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)$$

$$-6=\frac{15}{2}C$$

$$\Rightarrow C=(-6)\left(\frac{2}{15}\right)$$

$$\boxed{C=\frac{-4}{5}}$$

Putting the values of A, B and C in equation (iii), we get

$$\frac{2x-3}{x(x-1)(2x+3)}=\frac{1}{x}+\frac{-\frac{1}{5}}{x-1}+\frac{-\frac{4}{5}}{2x+3}$$ by using equation (i), we get

$$\frac{2x^3+x^2-5x+3}{2x^3+x^2-3x}=1-\left[\frac{1}{x}-\frac{1}{5(x-1)}-\frac{4}{5(2x+3)}\right]$$

Hence the partial fractions are:

$$\boxed{\frac{2x^3+x^2-5x+3}{2x^3+x^2-3x}=1-\frac{1}{x}+\frac{1}{5(x-1)}+\frac{4}{5(2x+3)}}$$

Q.9 $\dfrac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}$

Solution: It is improper rational fraction first transform it into mixed form.

$$\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}=\frac{(x-1)(x^2-8x+15)}{(x-2)(x^2-10x+24)}$$

$$=\frac{x^3-8x^2+15x-x^2+8x-15}{x^3-10x^2+24x-2x^2+20x-48}$$

$$=\frac{x^3-9x^2+23x-15}{x^3-12x^2+44x-48}$$

$$\begin{array}{r|l} & 1 \\ x^3-12x^2+44x-48 & x^3-9x^2+23x-15 \\ & \underline{\cancel{x^3}-12x^2+44x-48} \\ & 3x^2-21x+33 \end{array}$$

So,

Dividing $x^3-9x^2+23x-15$ by $x^3-12x^2+44x-48,$ we have

Quotient = 1 and Remainder = $3x^2-21x+33$

$$\frac{x^3-9x^2+23x-15}{x^3-12x^2+44x-48}=1+\frac{3x^2-21x+33}{x^3-12x^2+44x-48}=1+\frac{3x^2-21x+33}{(x-2)(x-4)(x-6)} \qquad \text{(i)}$$

Let,

$$\frac{3x^2-21x+33}{(x-2)(x-4)(x-6)}=\frac{A}{x-2}+\frac{B}{x-4}+\frac{C}{x-6} \qquad \text{(ii)}$$

Multiplying by $(x-2)(x-4)(x-6)$ to equation (ii)

$$3x^2-21x+33=A(x-4)(x-6)+B(x-2)(x-6)+C(x-2)(x-4) \qquad \text{(iii)}$$

Putting $x-2=0 \Rightarrow x=2$ in equation (iii), we have

$3(2)^2-21(2)+33=A(2-4)(2-6)+B(0)(2-6)+C(0)(2-4)$

$12-42+33 = A(-2)(-4)+0+0$

$3=8A$

$\Rightarrow \boxed{A=\frac{3}{8}}$

Putting $x-4=0$ $x=4$ in equation (iii)

$3(4)^2-21(4)+33=A(0)(4-6)+B(4-2)(4-6)+C(4-2)(0)$

$48-84+33=0+B(2)(-2)+0$

$-3=-4B$

$\Rightarrow B=\frac{-3}{-4}$

$\boxed{B=\frac{3}{4}}$

Putting $x-6=\Rightarrow x=6$ in equation (iii)

$3(6)^2-21(6)+33=A(6-4)(0)+B(6-2)(0)+C(6-2)(6-4)$

$3(36)-126+33=0+0+C(4)(2)$

$108-126+33 = 8C$

$15=8C$

$\Rightarrow \boxed{C=\frac{15}{8}}$

Putting the values of A,B and C in equation (ii), we get

$$\frac{3x^2-21x+33}{(x-2)(x-4)(x-6)}=\frac{\frac{3}{8}}{x-2}+\frac{\frac{3}{4}}{x-4}+\frac{\frac{15}{8}}{x-6}$$

By using (i), partial fractions are:

$$\boxed{\frac{(x-1)(x-3)(x-5)}{(x-2)(x-4)(x-6)}=1+\frac{3}{8(x-2)}+\frac{3}{4(x-4)}+\frac{15}{8(x-6)}}$$

Q.10 $\frac{1}{(1-ax)(1-bx)(1-cx)}$

Solution: Let

$$\frac{1}{(1-ax)(1-bx)(1-cx)}=\frac{A}{1-ax}+\frac{B}{1-bx}+\frac{C}{1-cx} \qquad \text{(i)}$$

Multiplying by $1-ax)(1-bx)(1-cx)$ on both sides of equation (i), we get

$$1 = A(1-bx)(1-cx)+B(1-ax)(1-cx)+C(1-ax)(1-bx) \qquad \text{(ii)}$$

Putting $1-ax=0 \Rightarrow ax=1 \Rightarrow x=\frac{1}{a}$ in equation (ii)

$$1 = A\left[1-b\left(\frac{1}{a}\right)\right]\left[1-c\left(\frac{1}{a}\right)\right]+B(0)\left[1-c\left(\frac{1}{a}\right)\right]+C(0)\left[1-b\left(\frac{1}{a}\right)\right]$$

$$1 = A\left(1-\frac{b}{a}\right)\left(1-\frac{c}{a}\right)+0+0$$

$$1 = A\left(\frac{a-b}{a}\right)\left(\frac{a-c}{a}\right)$$

$$\Rightarrow \boxed{A=\frac{a^2}{(a-b)(a-c)}}$$

Putting $1-bx = 0 \Rightarrow bx =1 \Rightarrow x=\frac{1}{b}$ in equation (ii)

$$1 = A\left[1-c\left(\frac{1}{b}\right)\right](0)+B\left[1-a\left(\frac{1}{b}\right)\right]\left[1-c\left(\frac{1}{b}\right)\right]+c\left[1-a\left(\frac{1}{b}\right)\right](0)$$

$$1 = 0+B\left(1-\frac{a}{b}\right)\left(1-\frac{c}{b}\right)+0$$

$$1 = B\left(\frac{b-a}{b}\right)\left(\frac{b-c}{b}\right)$$

$$\Rightarrow \boxed{B=\frac{b^2}{(b-a)(b-c)}}$$

Putting $1-cx=0 \Rightarrow cx=1 \Rightarrow x=\frac{1}{c}$ in equation (ii)

$$1= A\left[1-b\left(\frac{1}{c}\right)\right](0)+B\left[1-a\left(\frac{1}{c}\right)\right](0)+C\left[1-a\left(\frac{1}{c}\right)\right]\left[1-b\left(\frac{1}{c}\right)\right]$$

$$1= 0+0+C\left(1-\frac{a}{c}\right)\left(1-\frac{b}{c}\right)$$

$$1=C\left(\frac{c-a}{c}\right)\left(\frac{c-b}{c}\right)$$

$$\Rightarrow \boxed{C=\frac{c^2}{(c-a)(c-b)}}$$

Putting the values of A,B and C in equation (i)

$$\frac{1}{(1-ax)(1-bx)(1-cx)}=\frac{a^2/(a-b)(a-c)}{1-ax}+\frac{b^2/(b-a)(b-c)}{1-bx}+\frac{c^2/(c-a)(c-b)}{1-cx}$$

Hence the partial fractions are:

$$\boxed{\frac{1}{(1-ax)(1-bx)(1-cx)}=\frac{a^2}{(a-b)(a-c)(1-ax)}+\frac{b^2}{(b-a)(b-c)(1-bx)}+\frac{c^2}{(c-a)(c-b)(1-cx)}}$$

Q.11 $\dfrac{x^2+a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)}$

Solution: Let,

$x^2=y$, then

$$\frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)}$$

Let,

$$\frac{y+a^2}{(y+b^2)(y+c^2)(y+d^2)}=\frac{A}{y+b^2}+\frac{B}{y+c^2}+\frac{C}{y+d^2} \qquad \text{(i)}$$

Multiplying by $(y+b^2)(y+c^2)(y+d^2)$ on both sides of equation (i) we get

$$y+a^2 = A(y+c^2)(y+d^2)+B(y+b^2)(y+d^2)+C(y+b^2)(y+c^2) \qquad \text{(ii)}$$

Putting $y+b^2=0 \quad \Rightarrow \quad y=-b^2$ in equation (ii)

$$-b^2+a^2 = A(-b^2+c^2)(-b^2+d^2)+B(0)(-b^2+d^2)+C(0)(-b^2+c^2)$$

$$a^2-b^2 = A(c^2-b^2)(d^2-b^2)+0+0$$

$$\Rightarrow \boxed{A=\frac{a^2-b^2}{(c^2-b^2)(d^2-b^2)}}$$

Putting $y+c^2=0 \Rightarrow y=-c^2$ in equation (ii)

$$-c^2+a^2 = A(0)(-c^2+d^2)+B(-c^2+b^2)(-c^2+d^2)+C(-c^2+b^2)(0)$$

$$a^2-c^2 = 0+B(b^2-c^2)(d^2-c^2)+0$$

$$\Rightarrow \boxed{B=\frac{a^2-c^2}{(b^2-c^2)(d^2-c^2)}}$$

Putting $y+d^2=0 \Rightarrow y=-d^2$ in equation (ii), we get

$$-d^2+a^2 = A(-d^2+c^2)(0)+B(-d^2+b^2)(0)+C(-d^2+b^2)(-d^2+c^2)$$

$$a^2 - d^2 = C(b^2 - d^2)(c^2 - d^2)$$

$$\Rightarrow \boxed{C = \frac{a^2 - d^2}{(b^2 - d^2)(c^2 - d^2)}}$$

Putting the values of A, B and C in equation (i), we get

$$\frac{y + a^2}{(y + b^2)(y + c^2)(y + d^2)} = \frac{(a^2 - b^2)/(c^2 - b^2)(d^2 - b^2)}{y + b^2} + \frac{(a^2 - c^2)/(b^2 - c^2)(d^2 - c^2)}{y + c^2}$$
$$+ \frac{(a^2 - d^2)/(b^2 - d^2)(c^2 - d^2)}{y + d^2}$$

Since $y = x^2$

So,

Partial fractions are

$$\boxed{\frac{x^2 + a^2}{(x^2 + b^2)(x^2 + c^2)(x^2 + d^2)} = \frac{a^2 - b^2}{(x^2 + b^2)(c^2 - b^2)(d^2 - b^2)} + \frac{a^2 - c^2}{(x^2 + c^2)(b^2 - c^2)(d^2 - c^2)} + \frac{a^2 - d^2}{(x^2 + d^2)(b^2 - d^2)(c^2 - d^2)}}$$

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Case-II:

When $Q(x)$ has repeated linear factors:

If the polynomial has a factor $(x-a)^n, n \ge 2$ and n is a positive integer, then $\frac{P(x)}{Q(x)}$ may be written as the following identity:

$$\therefore \quad \frac{P(x)}{Q(x)} = \frac{A_1}{(x-a_1)} + \frac{A_2}{(x-a_2)^2} + \dots + \frac{A_n}{(x-a_n)^n}$$

Where, the coefficients $A_1, A_2, \dots, A_n$ are unknown constants to be found.

Example:

Resolve $\frac{x^2+x-1}{(x+2)^3}$ into partial fractions.

Solution:

Suppose

$$\frac{x^2+x-1}{(x+2)^3} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} \qquad \text{(i)}$$

Multiplying both sides $(x+2)^3$

$$\Rightarrow \quad x^2+x-1 = A(x+2)^2 + B(x+2) + C \qquad \text{(ii)}$$

$$\Rightarrow \quad x^2+x-1 = A(x^2+4x+4) + B(x+2) + C \qquad \text{(iii)}$$

Let us put $x+2=0 \Rightarrow x=-2$ in equation (ii)

We get $(-2)^2 + (-2) - 1 = A(0) + B(0) + C$

$\Rightarrow \quad 1 = C$

Comparing the coefficients of x^2 in equation (iii)

We get $A=1$ and

Comparing the coefficients of x in equation (iii)

We get $1 = 4A + B$

$\Rightarrow \quad 1 = 4 + B$

$\Rightarrow \quad B = -3$

Hence the partial fractions are:

$$\boxed{\frac{x^2+x-1}{(x+2)^3} = \frac{1}{x+2} - \frac{3}{(x+2)^2} + \frac{1}{(x+2)^3}}.$$

EXERCISE 5.2

Q:1 Resolve $\dfrac{2x^2-3x+4}{(x-1)^3}$ into Partial Fraction .

Solution:

Let $\dfrac{2x^2-3x+4}{(x-1)^3} = \dfrac{A}{(x-1)}+\dfrac{B}{(x-1)^2}+\dfrac{C}{(x-1)^3}$ (i)

Multiplying by $(x-1)^3$ on both sides of equation (i), we get

$2x^2-3x+4 = A(x-1)^2+B(x-1)+C$ (ii)

Putting $x-1=0 \Rightarrow x=1$ in equation (ii), we get

$2(1)^2-3(1)+4 = A(0)^2+B(0)+C$

$2-3+4 = 0+0+C$

$\boxed{C=3}$

Expanding equation (ii) ,we get

$2x^2-3x+4 = A(x^2+1-2x)+B(x-1)+C$

$2x^2-3x+4 = Ax^2+A-2Ax+Bx-B+C$

$2x^2-3x+4 = Ax^2+(B-2A)x+A-B+C$ (iii)

Comparing the coefficients of x^2 from equation (iii), we get

$\boxed{A=2}$

Similarly comparing coefficients of x from equation (iii), we get

$-3=B-2A$

$2A-3=B$

$\Rightarrow B=2A-3$

$B=2(2)-3 \quad \because A=2$

$\boxed{B=1}$

Putting the values of A , B and C in equation (i), we get

$$\boxed{\dfrac{2x^2-3x+4}{(x-1)^3} = \dfrac{2}{x-1}+\dfrac{1}{(x-1)^2}+\dfrac{3}{(x-1)^3}}$$

Q:2 Resolve, $\frac{5x^2-2x+3}{(x+2)^3}$ into partial fractions.

Solution:

Let $\frac{5x^2-2x+3}{(x+2)^3} = \frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{(x+2)^3}$ (i)

Multiplying by $(x+2)^3$ on both sides of equation (i), we get

$5x^2-2x+3 = A(x+2)^2+B(x+2)+C$ (ii)

Putting $x+2=0 \Rightarrow x=-2$ in equation (ii), we get

$5(-2)^2-2(-2)+3 = A(0)^2+B(0)+C$

$20+4+3=0+0+C$

$\Rightarrow \boxed{C=27}$

Expanding equation (ii), we get

$5x^2-2x+3 = A(x^2+4+4x)+B(x+2)+C$

$5x^2-2x+3 = Ax^2+4A+4Ax+Bx+2B+C$

$5x^2-2x+3 = Ax^2+(4A+B)x+4A+2B+C$ (iii)

Comparing Coefficients of x^2 from equation (iii), we get

$5=\text{A}$

$\Rightarrow \boxed{A=5}$

Comparing Coefficients of x from equation (iii), we get

$-2=4A+B$

$B=-2-4A$

$B=-2-4(5)$ $\because A=5$

$\boxed{B=-22}$

Putting the values of A, B and C in equation (i), we get

$$\boxed{\frac{5x^2-2x+3}{(x+2)^3}=\frac{5}{x+2}-\frac{22}{(x+2)^2}+\frac{27}{(x+2)^3}}$$

Q:3 Resolve, $\dfrac{4x}{(x+1)^2(x-1)}$ **into Partial Fractions.**

Solution:

Let $\dfrac{4x}{(x+1)^2(x-1)} = \dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{C}{x-1}$ (i)

Multiplying by $(x+1)^2(x-1)$ on both sides of equation (i), we get

$4x = A(x+1)(x-1)+B(x-1)+C(x+1)^2$ (ii)

Putting $x+1=0 \Rightarrow x=-1$ in equation (ii), we get

$4(-1) = A(0)(-1-1)+B(-1-1)+C(0)^2$

$-4 = 0 + B(-2) + 0$

$\Rightarrow -2B = -4$

$\boxed{B=2}$

Putting $x-1 = 0 \Rightarrow x = 1$ in equation (ii), we get

$4(1) = A(1+1)(0)+B(0)+C(1+1)^2$

$4 = 0 + 0 + 4C$

$\Rightarrow 4C = 4$

$C = \dfrac{4}{4}$

$\boxed{C=1}$

Expanding equation (ii), we get

$4x = A(x^2-1)+B(x-1)+C(x^2+2x+1)$

$4x = Ax^2-A+Bx-B+Cx^2+2Cx+C$

$4x = Ax^2+Cx^2+2Cx+Bx-A+C$ (iii)

Comparing Coefficients of x^2 from equation (iii), we get

$A+C = 0$

$A = -C$

$\boxed{A=-1}$ $\because C=1$

Putting the values of A, B and C in equation (i), we get

$\dfrac{4x}{(x+1)^2(x-1)} = \dfrac{-1}{x+1}+\dfrac{2}{(x+1)^2}+\dfrac{1}{x-1}$

$$\boxed{\dfrac{4x}{(x+1)^2(x-1)} = \dfrac{1}{x-1}-\dfrac{1}{x+1}+\dfrac{2}{(x+1)^2}}$$

Q:4 Resolve, $\dfrac{9}{(x+2)^2(x-1)}$ **into Partial Fractions.**

Solution:

Let $\dfrac{9}{(x+2)^2(x-1)} = \dfrac{A}{(x+2)}+\dfrac{B}{(x+2)^2}+\dfrac{C}{(x-1)}$ (i)

Multiplying by $(x+2)^2(x-1)$ on both sides of equation (i), we get

$9 = A(x+2)(x-1)+B(x-1)+C(x+2)^2$ (ii)

Putting $x+2=0 \Rightarrow x=-2$ in equation (ii), we get

$9 = A(0)(-2-1)+B(-2-1)+C(0)^2$

$9 = 0-3B+0$

$\Rightarrow -3B = 9 \Rightarrow B = -\dfrac{9}{3}$

$\boxed{B=-3}$

Putting $x-1=0 \Rightarrow x=1$ in equation (ii), we get

$9 = A(1+2)(0) + B(0) + C(1+2)^2$

$9 = 0+0 +9C$

$\Rightarrow 9C = 9 \Rightarrow C = \dfrac{9}{9}$

$\boxed{C=1}$

Expanding equation (ii), we get

$9 = A(x^2+x-2)+B(x-1)+C(x^2+4x+4)$

$9 = Ax^2+Ax-2A+Bx-B+Cx^2+4Cx+4C$

$0x^2+0x+9 = (A+C)x^2+(A+B+4C)x-2A-B+4C$ (iii)

Comparing Coefficients of x^2 from equation (iii), we get

$0 = A + C$

$\Rightarrow A + C = 0$

$A = -C$

$\boxed{A=-1}$ $\because C=1$

Putting the values of A , B and C in equation (i), we get

$\dfrac{9}{(x+2)^2(x-1)} = \dfrac{-1}{x+2}+\dfrac{-3}{(x+2)^2}+\dfrac{1}{x-1}$

$\boxed{\dfrac{9}{(x+2)^2(x-1)} = \dfrac{1}{x-1}-\dfrac{1}{x+2}-\dfrac{3}{(x+2)^2}}$

Q:5 Resolve, $\dfrac{1}{(x-3)^2(x+1)}$ into partial fractions.

Solution:

Let $\dfrac{1}{(x-3)^2(x+1)}=\dfrac{A}{x-3}+\dfrac{B}{(x-3)^2}+\dfrac{C}{x+1}$ (i)

Multiplying by $(x-3)^2(x+1)$ on both side of equation (i), we get

$1=A(x-3)(x+1)+B(x+1)+C(x-3)^2$ (ii)

Putting $x-3=0\Rightarrow x=3$ in equation (ii), we get

$1=A(0)(3+1)+B(3+1)+C(0)$

$1=0+4B+0$

$\Rightarrow 4B=1$

$\boxed{B=\dfrac{1}{4}}$

Putting $x+1=0\Rightarrow x=-1$ in equation (ii), we get

$1=A(-1-3)(0)+B(0)+C(-1-3)^2$

$1=0+0+16C$

$\Rightarrow 16C=1$

$\boxed{C=\dfrac{1}{16}}$

By expanding equation (ii), we get

$1=A(x^2+x-3x-3)+B(x+1)+C(x^2+9-6x)$

$1=A(x^2-2x-3)+B(x+1)+C(x^2-6x+9)$

$1=Ax^2-2Ax-3A+Bx+B+Cx^2-6Cx+9C$

$1=(A+C)x^2+(B-2A-6C)x-3A+B+9C$

Comparing Coefficient of x^2, we get

$0=A+C$

$A=-C$

$\boxed{A=-\dfrac{1}{16}}$ $\quad \therefore C=\dfrac{1}{16}$

Putting the values of *A*, *B* and *C* in equation (i), we get

$$\frac{1}{(x-3)^2(x+1)}=\frac{-\frac{1}{16}}{x-3}+\frac{\frac{1}{4}}{(x-3)^2}+\frac{\frac{1}{16}}{x+1}$$

$$\boxed{\frac{1}{(x-3)^2(x+1)}=\frac{1}{16(x+1)}-\frac{1}{16(x-3)}+\frac{1}{4(x-3)^2}}$$

Q:6 Resolve, $\dfrac{x^2}{(x-2)(x-1)^2}$ into partial fractions.

Solution:

Let $\dfrac{x^2}{(x-2)(x-1)^2}=\dfrac{A}{x-2}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ (i)

Multiplying by $(x-2)(x-1)^2$ on both sides of equation (i), we get

$x^2=A(x-1)^2+B(x-2)(x-1)+C(x-2)$ (ii)

Putting $x-2=0 \Rightarrow x=2$ in equation (ii), we get

$(2)^2=A(2-1)+B(0)(2-1)+C(0)$

$4=A(1)+0+0$

$\Rightarrow \boxed{A=4}$

Putting $x-1=0 \Rightarrow x=1$ in equation (ii), we get

$(1)^2=A(1-1)^2+B(1-2)(0)+C(1-2)$

$1=-C$

$\Rightarrow \boxed{C=-1}$

By expanding equation (ii), we get

$x^2=A(x^2+1-2x)+B(x^2-x-2x+2)+C(x-2)$

$x^2=A(x^2-2x+1)+B(x^2-3x+2)+C(x-2)$

$x^2=Ax^2-2Ax+A+Bx^2-3Bx+2B+Cx-2C$

$x^2+0x+0=(A+B)x^2+(C-2A-3B)x+A+2B-2C$

Comparing coefficient of x^2, we get

$1=A+B$

$\Rightarrow B=1-A$

$B=1-4$ $\therefore A=4$

$\boxed{B=-3}$

Putting the values of A, B and C in equation (i), we get

$\dfrac{x^2}{(x-2)(x-1)^2}=\dfrac{4}{x-2}+\dfrac{-3}{x-1}+\dfrac{-1}{(x-1)^2}$

$\boxed{\dfrac{x^2}{(x-2)(x-1)^2}=\dfrac{4}{x-2}-\dfrac{3}{x-1}-\dfrac{1}{(x-1)^2}}$

Q:7 Resolve, $\dfrac{1}{(x-1)^2(x+1)}$ into partial Fractions.

Solution:

Let $\dfrac{1}{(x-1)^2(x+1)} = \dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{x+1}$ (i)

Multiplying by $(x-1)^2(x+1)$ on both sides of equation (i), we get

$1 = A(x-1)(x+1)+B(x+1)+C(x-1)^2$ (ii)

Putting $x-1=0 \Rightarrow x=1$ in equation (ii), we get

$1 = A(0)(1+1)+B(1+1)+C(0)^2$

$1 = 0 + 2B + 0$

$\Rightarrow 2B = 1$

$\boxed{B=\dfrac{1}{2}}$

Putting $x+1 = 0 \quad \Rightarrow \quad x=-1$ in equation (ii), we get

$1 = A(-1-1)(0)+B(0)+C(-1-1)^2$

$1 = 0 + 0 + 4C$

$\Rightarrow \quad 4C = 1$

$\boxed{C=\dfrac{1}{4}}$

Expanding equation (ii), we get

$1 = A(x^2-1)+B(x+1)+C(x^2+1-2x)$

$1 = Ax^2-A+Bx+B+Cx^2-2Cx+C$

$0x^2+0x+1=(A+C)x^2+(B-2C)x-A+B+C$ (iii)

Comparing Coefficients of x^2 from equation (iii), we get

$0 = A+C$

$\Rightarrow A+C = 0$

$A = -C$

$\boxed{A = -\dfrac{1}{4}} \qquad \because C=\dfrac{1}{4}$

Putting the values of *A*, *B* and *C* in equation (i), we get

$$\frac{1}{(x-1)^2(x+1)} = \frac{-\frac{1}{4}}{x-1}+\frac{\frac{1}{2}}{(x-1)^2}+\frac{\frac{1}{4}}{x+1}$$

$$\boxed{\frac{1}{(x-1)^2(x+1)} = \frac{1}{4(x+1)}-\frac{1}{4(x-1)}+\frac{1}{2(x-1)^2}}$$

Q.8 Resolve, $\dfrac{x^2}{(x-1)^3(x+1)}$ into Partial Fraction .

Solution:

Let $\dfrac{x^2}{(x-1)^3(x+1)} = \dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}+\dfrac{C}{(x-1)^3}+\dfrac{D}{x+1}$ (i)

Multiplying by $(x-1)^3(x+1)$ on both sides of equation (i), we get.

$x^2 = A(x-1)^2(x+1)+B(x-1)(x+1)+C(x+1)+D(x-1)^3$ (ii)

Putting $x-1=0 \Rightarrow x=1$ in equation (ii), we get

$(1)^2 = A(0)^2(1+1)+B(0)(1+1)+C(1+1)+D(0)^3$

$1 = 0 + 0 + 2C + 0$

$\Rightarrow 2C = 1$

$\boxed{C=\dfrac{1}{2}}$

Putting $x+1=0 \Rightarrow x=-1$ in equation (ii), we get

$(-1)^2 = A(-1-1)^2(0)+B(-1-1)(0)+C(0)+D(-1-1)^3$

$1 = 0 + 0 + 0 - 8D$

$\Rightarrow -8D = 1$

$\boxed{D=-\dfrac{1}{8}}$

Expanding equation (ii), we get

$x^2 = A(x^2+1-2x)(x+1)+B(x^2-1)+C(x+1)+D\left[x^3-1-3x(x-1)\right]$

$x^2 = A(x^3+x^2+x+1-2x^2-2x)+B(x^2-1)+C(x+1)+D(x^3-1-3x^2+3x)$

$x^2 = A(x^3-x^2-x+1)+B(x^2-1)+C(x+1)+D(x^3-3x^2+3x-1)$

$0x^3+x^2+0x+0 = (A+D)x^3+(B-A-3D)x^2+(C-A+3D)x+(A-B+C-D)$ (iii)

Comparing Coefficients of x^3 from equation (iii), we get

$0 = A+D$

$\Rightarrow A+D=0 \Rightarrow A=-D$

$A=-\left(-\dfrac{1}{8}\right) \quad \because D=-\dfrac{1}{8}$

$A=\dfrac{1}{8}$

Comparing coefficients of x^2 from equation (iii), we get

$1 = B-A-3D$

$B = 1+A+3D$

$$B = 1 + \frac{1}{8} - \frac{3}{8} \qquad \because A = \frac{1}{8},\ D = -\frac{1}{8}$$

$$B = \frac{8+1-3}{8}$$

$$B = \frac{6}{8}$$

$$\boxed{B = \frac{3}{4}}$$

Putting the values of A , B , C and D in equation (i), we get

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{\frac{1}{8}}{x-1} + \frac{\frac{3}{4}}{(x-1)^2} + \frac{\frac{1}{2}}{(x-1)^3} + \frac{-\frac{1}{8}}{x+1}$$

$$\boxed{\frac{x^2}{(x-1)^3(x+1)} = \frac{1}{8(x-1)} + \frac{3}{4(x-1)^2} + \frac{1}{2(x-1)^3} - \frac{1}{8(x+1)}}$$

Q:9 Resolve, $\frac{x-1}{(x-2)(x+1)^3}$ into Partial Fractions.

Solution:

Let $$\frac{x-1}{(x-2)(x+1)^3} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} \qquad \text{(i)}$$

Multiplying by $(x-2)(x+1)^3$ on both sides of equation (i), we get

$$x-1 = A(x+1)^3 + B(x-2)(x+1)^2 + C(x-2)(x+1) + D(x-2) \qquad \text{(ii)}$$

Putting $x-2 = 0 \Rightarrow x = 2$ in equation (ii), we get

$$2-1 = A(2+1)^3 + B(0)(2+1)^2 + C(0)(2+1) + D(0)$$

$$1 = A(27) + 0 + 0 + 0$$

$$\Rightarrow 27A = 1$$

$$\boxed{A = \frac{1}{27}}$$

Putting $x+1 = 0 \Rightarrow x = -1$ in equation (ii), we get

$$-1-1 = A(0)^3 + B(-1-2)(0)^2 + C(-1-2)(0) + D(-1-2)$$

$$-2 = 0 + 0 + 0 + D(-3)$$

$$\Rightarrow -3D = -2$$

$D=\frac{-2}{-3}$

$\boxed{D=\frac{2}{3}}$

By Expanding equation (ii), we get

$x-1=A\left(x^3+1+3x^2+3x\right)+B\left(x-2\right)\left(x^2+2x+1\right)+C\left(x^2+x-2x-2\right)+D\left(x-2\right)$

$x-1=A\left(x^3+3x^2+3x+1\right)+B\left(x^3+2x^2+x-2x^2-4x-2\right)+C\left(x^2-x-2\right)+D\left(x-2\right)$

$x-1=A\left(x^3+3x^2+3x+1\right)+B\left(x^3-3x-2\right)+C\left(x^2-x-2\right)+D\left(x-2\right)$

$x-1=Ax^3+3Ax^2+3Ax+A+Bx^3-3Bx-2B+Cx^2-Cx-2C+Dx-2D$

Arranging terms of x^3, x^2, x and constant, we have

$0x^3+0x^2+x-1=\left(A+B\right)x^3+\left(3A+C\right)x^2+\left(3A\text{-}3B\text{-}C\text{+}D\right)x+A\text{-}2B\text{-}2C\text{-}2D$ (iii)

Comparing the coefficient x^3 we have

$0 = A+B$

$\Rightarrow B=-A$

$\boxed{B=-\frac{1}{27}}$ $\therefore A=\frac{1}{27}$

Comparing Coefficients of x^2 from equation (iii) ,we get

$0=3A+C$

$\Rightarrow C=-3A$

$C=-3\left(\frac{1}{27}\right)$

$\boxed{C=-\frac{1}{9}}$

Putting the values of A , B , C and D in equation (i), we get

$$\frac{x-1}{(x-2)(x+1)^3}=\frac{\frac{1}{27}}{x-2}+\frac{-\frac{1}{27}}{x+1}+\frac{-\frac{1}{9}}{(x+1)^2}+\frac{\frac{2}{3}}{(x+1)^3}$$

$$\boxed{\frac{x-1}{(x-2)(x+1)^3}=\frac{1}{27(x-2)}-\frac{1}{27(x+1)}-\frac{1}{9(x+1)^2}+\frac{2}{3(x+1)^3}}$$

Q:10 Resolve, $\frac{4x^3}{(x^2-1)(x+1)^2}$ into Partial Fractions.

Solution:

$$\frac{4x^3}{\left(x^2-1\right)\left(x+1\right)^3}=\frac{4x^3}{(x+1)(x-1)(x+1)^2}$$

Let $\dfrac{4x^3}{(x-1)(x+1)^3} = \dfrac{A}{x-1}+\dfrac{B}{x+1}+\dfrac{C}{(x+1)^2}+\dfrac{D}{(x+1)^3}$ (i)

Multiplying by $(x-1)(x+1)^3$ on both sides of equation (i), we get

$4x^3 = A(x+1)^3 + B(x-1)(x+1)^2 + C(x-1)(x+1) + D(x-1)$ (ii)

Putting $x+1=0 \Rightarrow x=-1$ in equation (ii), we get

$4(-1)^3 = A(0)^3 + B(-1-1)(0)^2 + C(-1-1)(0) + D(-1-1)$

$-4 = 0+0+0+D(-2)$

$\Rightarrow -2D = -4$

$D = \dfrac{-4}{-2}$

$\boxed{D = 2}$

Putting $x-1=0 \Rightarrow x=1$ in equation (ii), we get

$4(1)^3 = A(1+1)^3 + B(0)(1+1)^2 + C(0)(1+1) + D(0)$

$4 = 8A + 0+0+0$

$\Rightarrow 8A = 4$

$A = \dfrac{4}{8}$

$\boxed{A = \dfrac{1}{2}}$

Expanding equation (ii), we get

$4x^3 = A(x^3+1+3x^2+3x) + B(x-1)(x^2+2x+1) + C(x^2-1) + D(x-1)$

$4x^3 = A(x^3+1+3x^2+3x) + B(x^3+x^2-x-1) + C(x^2-1) + D(x-1)$

$4x^3+0x^2+0x+0 = (A+B)x^3 + (3A+B+C)x^2 + (3A-B+D)x + A-B-C-D$ (iii)

Comparing coefficients of x^3 from equation (iii), we get

$4 = A+B$

$\Rightarrow B = 4-A$

$B = 4-\dfrac{1}{2}$ $\quad\because A = \dfrac{1}{2}$

$B = \dfrac{8-1}{2}$

$$\boxed{B = \frac{7}{2}}$$

Comparing Coefficients of x^2 from equation (iii), we get

$0 = 3A + B + C$

$\Rightarrow C = -3A - B$

$C = -3\left(\frac{1}{2}\right) - \frac{7}{2}$

$C = \frac{-3}{2} + \frac{-7}{2}$

$C = \frac{-10}{2}$

$$\boxed{C = -5}$$

Putting the values of *A*, *B* , *C* and *D* in equation (i)

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{\frac{1}{2}}{x-1} + \frac{\frac{7}{2}}{x+1} + \frac{-5}{(x+1)^2} + \frac{2}{(x+1)^2}$$

$$\boxed{\frac{4x^3}{(x-1)(x+1)^3} = \frac{1}{2(x-1)} + \frac{7}{2(x+1)} - \frac{5}{(x+1)^2} + \frac{2}{(x+1)^3}}$$

Q:11 Resolve, $\frac{2x+1}{(x+3)(x-1)(x+2)^2}$ into Partial Fraction .

Solution:

Let $$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{x+2} + \frac{D}{(x+2)^2} \qquad \text{(i)}$$

Multiplying by $(x+3)(x-1)(x+2)^2$ on both sides of equation (i)

$$2x+1 = A(x-1)(x+2)^2 + B(x+3)(x+2)^2 + C(x+3)(x-1)(x+2) + D(x+3)(x-1) \qquad \text{(ii)}$$

Putting $x + 3 = 0 \quad \Rightarrow x = -3$ in equation (ii)

$2(-3)+1 = A(-3-1)(-3+2)^2 + B(0)(-3+2)^2 + C(0)(-3-1)(-3+2) + D(0)(-3-1)$

$-6+1 = A(-4)(1) + 0 + 0 + 0$

$-5 = -4A$

$\Rightarrow A = \frac{-5}{-4}$

$\boxed{A = \frac{5}{4}}$

Putting $x-1=0 \quad \Rightarrow \quad x=1$ in equation (ii)

$$2(1)+1 = A(0)(1+2)^2 + B(1+3)(1+2)^2 + C(1+3)(0)(1+2) + D(1+3)(0)$$

$$3 = 0 + B(4)(9) + 0 + 0$$

$$3 = 36B$$

$$\Rightarrow B = \frac{3}{36}$$

$$\Rightarrow \boxed{B = \frac{1}{12}}$$

Putting $x+2=0 \quad \Rightarrow \quad x=-2$ in equation (ii) , we get

$$2(-2)+1 = A(-2-1)(0)^2 + B(-2+3)(0)^2 + C(-2+3)(-2-1)(0) + D(-2+3)(-2-1)$$

$$-3 = 0+0+0+D(1)(-3)$$

$$\Rightarrow -3D = -3$$

$$D = \frac{-3}{-3}$$

$\boxed{D = 1}$

Expanding equation (ii), we get

$$2x+1 = A(x-1)(x^2+4x+4) + B(x+3)(x^2+4x+4) + C(x+3)(x^2+x-2) + D(x^2+2x-3)$$

$$2x+1 = A(x^3+3x^2-4) + B(x^3+7x^2+16x+12) + C(x^3+4x^2+x-6) + D(x^2+2x-3)$$

$$0x^3+0x^2+2x+1 = (A+B+C)x^3 + (3A+7B+4C+D)x^2 + (16B+C+2D)x - 4A + 12B - 6C - 3D \quad \text{(iii)}$$

Comparing Coefficients of x^3

$$0 = A+B+C$$

$$\Rightarrow C = -A-B$$

$$C = -\frac{5}{4} - \frac{1}{12} \qquad \because A = \frac{5}{4}, B = \frac{1}{2}$$

$$C = \frac{-15-1}{12}$$

$$C = \frac{-16}{12}$$

$$\boxed{C = \frac{-4}{3}}$$

Putting the values of A, B , C and D in equation (i) , we get

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{\frac{5}{4}}{x+3} + \frac{\frac{1}{12}}{x-1} + \frac{\frac{-4}{3}}{x+2} + \frac{1}{(x+2)^2}$$

$$\boxed{\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{5}{4(x+3)} + \frac{1}{12(x-1)} - \frac{4}{3(x+2)} + \frac{1}{(x+2)^2}}$$

Q:12 Resolve $\frac{2x^4}{(x-3)(x+2)^2}$ into Partial Fraction.

Solution:

It is improper rational Fraction, so first we transform it into mixed form.

$$\frac{2x^4}{(x-3)(x^2+4x+4)} = \frac{2x^4}{x^3+x^2-8x-12}$$

$$\begin{array}{r|l} & 2x-2 \\ \hline x^3+x^2-8x-12 & 2x^4 \\ & -2x^4 \pm 2x^3 \mp 16x^2 \mp 24x \\ \hline & -2x^3+16x^2+24x \\ & \mp 2x^3 \mp 2x^2 \pm 16x \pm 24 \\ \hline & 18x^2+8x-24 \end{array}$$

So

$$\frac{2x^4}{(x-3)(x+2)^2} = 2x-2+\frac{18x^2+8x-24}{(x-3)(x+2)^2} \quad \text{(i)}$$

Let, $\frac{18x^2+8x-24}{(x-3)(x+2)^2} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ (ii)

Multiplying by $(x-3)(x+2)^2$ on both sides of equation (ii)

$18x^2+8x-24 = A(x+2)^2 + B(x-3)(x+2) + C(x-3)$ (iii)

Putting $x-3=0 \quad \Rightarrow x=3$ in equation (iii)

$18(3)^2+8(3)-24 \quad = A(3+2)^2 + B(0)(3+2) + C(0)$

$162 = A(25) + 0 + 0$

$25A = 162$

$$\boxed{\Rightarrow A = \frac{162}{25}}$$

Putting $x + 2 = 0 \Rightarrow x = -2$ in equation (iii)

$$18(-2)^2 + 8(-2) - 24 = A(0)^2 + B(-2-3)(0) + C(-2-3)$$

$$18(4) - 16 - 24 = 0 + 0 - 5C$$

$$32 = -5C$$

$$\Rightarrow \boxed{C = \frac{-32}{5}}$$

Expanding equation (iii) , we get.

$$18x^2 + 8x - 24 = A\left(x^2 + 4x + 4\right) + B\left(x^2 - x - 6\right) + C(x-3)$$

$$18x^2 + 8x - 24 = (A+B)x^2 + (4A - B + C)x + 4A - 6B - 3C \qquad \text{(iv)}$$

Comparing Coefficient of x^2 from equation (iv)

$$18 = A + B$$

$$\Rightarrow \text{B} = 18 - \text{A}$$

$$B = 18 - \frac{162}{25}$$

$$B = \frac{450 - 162}{25}$$

$$\boxed{B = \frac{288}{25}}$$

Putting the values of A , B and C in equation (ii) , we get

$$\frac{18x^2 + 8x - 24}{(x-3)(x+2)^2} = \frac{\frac{162}{25}}{x-3} + \frac{\frac{288}{25}}{x+2} + \frac{\frac{-32}{5}}{(x+2)^2}$$

Now by using equation (i), partial fractions are:

$$\boxed{\frac{2x^4}{(x-3)(x+2)^2} = 2x - 2 + \frac{162}{25(x-3)} + \frac{288}{25(x+2)} - \frac{32}{5(x+2)^2}}$$