CHAPTER 6

SEQUENCE AND SERIES

Sequence:

Sequences also called progressions, are used to represent ordered lists of numbers. A sequence is a function whose domain is the subset of natural numbers N or W (in some cases). If a natural number n belongs to the domain of sequence $\{a_n\}$ then the corresponding elements in its range are denoted by a_n. The elements in the range of sequence $\{a_n\}$ are called its terms. A special notation a_n is used for n^{th} term of the sequence.

Finite and Infinite Sequence:

A sequence having finite terms is called finite sequence e.g. $1,3,5,...,11$ is a finite sequence. Whereas, a sequence with infinite number of terms is called an infinite sequence. e.g., $3,7,11,...$ is an infinite sequence. An infinite sequence has no last term.

Real Sequence:

If all members of a sequence are real numbers, then it is called a real sequence.

EXERCISE 6.1

Q.1 Write the first four terms of the following sequences, if

(i) $a_n = 2n-3$

Solution:

As given $a_n = 2n-3$

Put $n=1 \Rightarrow a_1 = 2(1)-3 = -1$

Put $n=2 \Rightarrow a_2 = 2(2)-3 = 1$

Put $n=3 \Rightarrow a_3 = 2(3)-3 = 3$

Put $n=4 \Rightarrow a_4 = 2(4)-3 = 5$

So first four terms are

$-1, 1, 3, 5$

(ii) $a_n = (-1)^n n^2$

Solution:

As given $a_n = (-1)^n n^2$

Put $n=1 \Rightarrow a_1 = (-1)^1(1)^2 = -1\times 1 = -1$

Put $n=2 \Rightarrow a_2 = (-1)^2(2)^2 = 1\times 4 = 4$

Put $n=3 \Rightarrow a_3 = (-1)^3(3)^2 = -1\times 9 = -9$

Put $n=4 \Rightarrow a_4 = (-1)^4(4)^2 = 1\times 16 = 16$

So first four terms are

$-1, 4, -9, 16$

(iii) $a_n = (-1)^n(2n-3)$

Solution:

As given $a_n = (-1)^n(2n-3)$

Put $n=1 \Rightarrow a_1 = (-1)^1(2(1)-3) = -1(-1) = 1$

Put $n=2 \Rightarrow a_2 = (-1)^2(2(2)-3) = 1(4-3) = 1$

Put $n=3 \Rightarrow a_3 = (-1)^3(2(3)-3) = (-1)(6-3) = (-1)(3) = -3$

Put $n=4 \Rightarrow a_4 = (-1)^4(2(4)-3) = 1(8-3) = 5$

So first four terms are

$1, 1, -3, 5$

(iv) $\mathbf{a_n = 3n - 5}$

Solution:

As given $a_n = 3n - 5$

Put $n = 1 \Rightarrow a_1 = 3(1) - 5 = 3 - 5 = -2$

Put $n = 2 \Rightarrow a_2 = 3(2) - 5 = 6 - 5 = 1$

Put $n = 3 \Rightarrow a_3 = 3(3) - 5 = 9 - 5 = 4$

Put

$n = 4 \Rightarrow a_4 = 3(4) - 5 = 12 - 5 = 7$

So first four terms are

$-2, 1, 4, 7$

(v) $\mathbf{a_n = \dfrac{n}{2n+1}}$

Solution:

As given $a_n = \dfrac{n}{2n+1}$

Put $n = 1 \Rightarrow a_1 = \dfrac{1}{2(1)+1} = \dfrac{1}{3}$

Put $n = 2 \Rightarrow a_2 = \dfrac{2}{2(2)+1} = \dfrac{2}{4+1} = \dfrac{2}{5}$

Put $n = 3 \Rightarrow a_3 = \dfrac{3}{2(3)+1} = \dfrac{3}{6+1} = \dfrac{3}{7}$

Put $n = 4 \Rightarrow a_4 = \dfrac{4}{2(4)+1} = \dfrac{4}{8+1} = \dfrac{4}{9}$

So first four terms are

$\dfrac{1}{3}, \dfrac{2}{5}, \dfrac{3}{7}, \dfrac{4}{9}$

(vi) $\mathbf{a_n = \dfrac{1}{2^n}}$

Solution:

As given $a_n = \dfrac{1}{2^n}$

Put $n = 1 \Rightarrow a_1 = \dfrac{1}{2^1} = \dfrac{1}{2}$

Put $n = 2 \Rightarrow a_2 = \dfrac{1}{2^2} = \dfrac{1}{4}$

Put $n = 3 \Rightarrow a_3 = \dfrac{1}{2^3} = \dfrac{1}{8}$

Put $n = 4 \Rightarrow a_4 = \dfrac{1}{2^4} = \dfrac{1}{16}$

So first four terms are

$\dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}$

(vii) $\mathbf{a_n - a_{n-1} = n + 2,\ a_1 = 2}$

Solution:

As given $a_n - a_{n-1} = n + 2$ (i)

and $a_1 = 2$

Put $n = 2$ in equation (i)

$a_2 - a_{2-1} = 2 + 2$

$a_2 - a_1 = 4$

$a_2 - 2 = 4$

$a_2 = 4 + 2$

$a_2 = 6$

Put $n = 3$ in equation (i)

$a_3 - a_{3-1} = 3 + 2$

$a_3 - a_2 = 5$

$a_3 - 6 = 5$

$a_3 = 5 + 6$

$a_3 = 11$

Put $n = 4$ in equation (i)

$a_4 - a_{4-1} = 4 + 2$

$a_4 - a_3 = 6$

$a_4 - 11 = 6$

$a_4 = 6 + 11$

$a_4 = 17$

So first four terms are

$2, 6, 11, 17$

(viii) $\mathbf{a_n = na_{n-1},\ a_1 = 1}$

Solution:

As given $a_n = na_{n-1}$ (i)

and $a_1 = 1$

Put $n = 2$ in eq. (i)

$a_2 = 2a_{2-1} = 2a_1 = 2(1) = 2$

Put $n = 3$ in eq. (i)

$a_3 = 3a_{3-1} = 3a_2 = 3(2) = 6$

Put $n = 4$ in eq. (i)

$a_4 = 4a_{4-1} = 4a_3 = 4(6) = 24$

So first four terms are

1, 2, 6, 24

(ix) $\mathbf{a_n = (n+1)a_{n-1},\ a_1 = 1}$

Solution:

$a_n = (n+1)a_{n-1}$ (i)

and $a_1 = 1$

Put $n = 2$ in eq. (i)

$a_2 = (2+1)a_{n-1} = 3a_1 = 3(1) = 3$

Put $n = 3$ in eq. (i)

$a_3 = (3+1)a_{3-1} = 4a_2 = 4(3) = 12$

Put $n = 4$ in eq. (i)

$a_4 = (4+1)a_{4-1} = 5a_3 = 5(12) = 60$

So first four terms are

1, 3, 12, 60

(x) $\mathbf{a_n = \dfrac{1}{a+(n-1)d}}$

Solution:

As given $a_n = \dfrac{1}{a+(n-1)d}$

Put $n = 1 \Rightarrow a_1 = \dfrac{1}{a+(1-1)d} = \dfrac{1}{a+0(d)} = \dfrac{1}{a+0} = \dfrac{1}{a}$

Put $n = 2 \Rightarrow a_2 = \dfrac{1}{a+(2-1)d} = \dfrac{1}{a+(1)d} = \dfrac{1}{a+d}$

Put $n = 3 \Rightarrow a_3 = \dfrac{1}{a+(3-1)d} = \dfrac{1}{a+2d}$

Put $n = 4 \Rightarrow a_4 = \dfrac{1}{a+(4-1)d} = \dfrac{1}{a+3d}$

So first four terms are

$\dfrac{1}{a}, \dfrac{1}{a+d}, \dfrac{1}{a+2d}, \dfrac{1}{a+3d}$

Q.2 Find the indicated terms of the following sequences.

(i) $2,6,11,17,...a_7$

Solution:

$2,6,11,17,...a_7$

Here

$a_1 = 2$

$a_2 = 2+4 = 6$

$a_3 = 6+5 = 11$

$a_4 = 11+6 = 17$

$a_5 = 17+7 = 24$

$a_6 = 24+8 = 32$

$a_7 = 32+9 = 41$

So $\boxed{a_7 = 41}$

(ii) 1, 3, 12, 60,......., a_6

Solution:

1, 3, 12, 60,......., a_6

Here

$a_1 = 1$

$a_2 = 1\times 3 = 3$

$a_3 = 3\times 4 = 12$

$a_4 = 12\times 5 = 60$

$a_5 = 60\times 6 = 360$

$a_6 = 360\times 7 = 2520$

So $\boxed{a_6 = 2520}$

(iii) $1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8},.......a_7$

Solution:

$1, \frac{3}{2}, \frac{5}{4}, \frac{7}{8},.......a_7$

Here

$a_1 = \frac{1}{1} = 1$

$a_2 = \frac{1+2}{1\times 2} = \frac{3}{2}$

$a_3 = \frac{3+2}{2\times 2} = \frac{5}{4}$

$a_4 = \frac{5+2}{4\times 2} = \frac{7}{8}$

$a_5 = \frac{7+2}{8\times 2} = \frac{9}{16}$

$a_6 = \frac{9+2}{16\times 2} = \frac{11}{32}$

$a_7 = \frac{11+2}{32\times 2} = \frac{13}{64}$

So, $\boxed{a_7 = \frac{13}{64}}$

(iv) $1,1,-3,5,-7,9,.....a_8$

Solution:

$1,1,-3,5,-7,9,.....a_8$

$a_1 = 1$

$a_3 = 1-4 = -3$

$a_5 = -3-4 = -7$

$a_7 = -7-4 = -11$

$a_2 = 1$

$a_4 = 1+4 = 5$

$a_6 = 5+4 = 9$

$a_8 = 9+4 = 13$

So, $\boxed{a_8 = 13}$

(v) $1,-3,5,-7,9,-11.....a_8$

Solution:

$1,-3,5,-7,9,-11.....a_8$

$a_1 = 1$

$a_3 = 1+4 = 5$

$a_5 = 5+4 = 9$

$a_7 = 9+4 = 13$

$a_2 = -3$

$a_4 = -3-4 = -7$

$a_6 = -7-4 = -11$

$a_8 = -11-4 = -15$

So, $\boxed{a_8 = -15}$

Q.3 Find the next two terms of the following sequences:

(i) $7,9,12,16,......$

Solution:

$7,9,12,16,......$

Here

$a_1 = 7$

$a_2 = 7+2 = 9$

$a_3 = 9+3 = 12$

$a_4 = 12+4 = 16$

$a_5 = 16+5 = 21$

$a_6 = 21+6 = 27$

So next two terms are

$21, 27$

(ii) $1,3,7,15,31,..........$

Solution:

$1,3,7,15,31,..........$

Here

$a_1 = 1$

$a_2 = 1+2 = 3$

$a_3 = 3+4 = 7$

$a_4 = 7+8 = 15$

$a_5 = 15+16 = 31$

$a_6 = 31+32 = 63$

$a_7 = 63+64 = 127$

So next two terms are

$63,127$

(iii) $-1,2,12,40,.........$

Solution:

$-1,2,12,40,.........$

Here

$a_1 = -1\times 1 = -1$

$a_2 = (-1+2)(1\times 2) = 1\times 2 = 2$

$a_3 = (1+2)(2\times 2) = 3\times 4 = 12$

$a_4 = (3+2)(4\times 2) = 5\times 8 = 40$

$a_5 = (5+2)(8\times 2) = 7\times 16 = 112$

$a_6 = (7+2)(16\times 2) = 9\times 32 = 288$

So next two terms are

$112, 288$

(iv) $1,-3,5,-7,9,-11,\ldots$

Solution:

$1,-3,5,-7,9,-11,\ldots$

$a_1=1$	$a_2=-3$
$a_3=1+4=5$	$a_4=-3-4=-7$
$a_5=5+4=9$	$a_6=-7-4=-11$
$a_7=9+4=13$	$a_8=-11-4=-15$

So, next two terms are 13 and –15

Arithmetic progression:

A sequence $\{a_n\}$ is an Arithmetic sequence or arithmetic progression (A.P), if a_n-a_{n-1} is the same number $\forall\ n\in N$ and $n>1$. The difference $a_n-a_{n-1}(n>1)$ i.e., the difference of two consecutive terms of an A.P., is called the **common difference** and is usually denoted by ***d.***

Rule for the *n*th term of A.P:

We know that $a_n-a_{n-1}=d \quad (n>1)$

Which implies $a_n=a_{n-1}+d \quad (n>1)$ (i)

Putting $n=2,3,4,\ldots$ in (i) we get

$a_2=a_1+d=a_1+(2-1)d$

$a_3=a_2+d=(a_1+d)+d$

$a_3=a_1+2d=a_1+(3-1)d$

$a_4=a_3+d=(a_1+2d)+d$

$a_4=a_1+3d=a_1+(4-1)d$

Thus we conclude that

$a_n=a_1+(n-1)d$

Where $a_n=n^{\text{th}}$ term or general term,

$a_1=1^{\text{st}}$ term , $n=$Number of terms , $d=$Common difference

EXERCISE 6.2

Q.1 Write the first four terms of the following arithmetic sequence. If

(i) $\mathbf{a_1 = 5}$ **and other three consecutive terms are** $\mathbf{23, 26, 29}$

Solution:

Three consecutive terms are

$23, 26, 29$

Common difference

$= d = 26 - 23 = 3$

$a_2 = a_1 + d = 5 + 3 = 8$

$a_3 = a_1 + 2d = 5 + 2(3) = 11$

$a_4 = a_1 + 3d = 5 + 3(3) = 14$

So first four terms are

$5, 8, 11, 14$

(ii) $\mathbf{a_5 = 17}$ and $\mathbf{a_9 = 37}$

Solution:

$a_5 = 17$

$\Rightarrow a_1 + 4d = 17 \qquad \text{(i)}$

and $a_9 = 37$

$\Rightarrow a_1 + 8d = 37 \qquad \text{(ii)}$

Equation (ii) – equation (i)

$a_1 + 8d = 37$

$\pm a_1 \pm 4d = \pm 17$

$4d = 20$

$d = 5$ Put in (i)

$a_1 + 4(5) = 17$

$a_1 + 20 = 17$

$a_1 = -3$

$a_2 = a_1 + d = -3 + 5 = 2$

$a_3 = a_1 + 2d = -3 + 2(5) = 7$

$a_4 = a_1 + 3d = -3 + 3(5) = 12$

So first four terms are

$-3, 2, 7, 12, \ldots\ldots$

(iii) $\mathbf{3a_7 = 7a_4}$ and $\mathbf{a_{10} = 33}$

Solution:

$3a_7 = 7a_4$

We know that

$a_4 = a_1 + 3d \;;\; a_7 = a_1 + 6d$

$3(a_1 + 6d) = 7(a_1 + 3d)$

$3a_1 + 18d = 7a_1 + 21d$

$3a_1 - 7a_1 + 18d - 21d = 0$

$-4a_1 - 3d = 0$

$-(4a_1 + 3d) = 0$

$4a_1 + 3d = 0 \qquad \text{(i)}$

and $a_{10} = 33$

$\Rightarrow a_1 + 9d = 33 \qquad \text{(ii)}$

Equation (i) multiply by 3, then subtract equation (ii) from equation (i)

$12a_1 + 9d = 0$

$\pm a_1 \pm 9d = \pm 33$

$11a_1 = -33$

$a_1 = -3$

Put in (i)

$4(-3) + 3d = 0$

$-12 + 3d = 0$

$3d = 12$

$d = 4$

Now $a_2 = a_1 + d = -3 + 4 = 1$

$a_3 = a_1 + 2d = -3 + 2(4) = -3 + 8 = 5$

$a_4 = a_1 + 3d = -3 + 3(4) = -3 + 12 = 9$

So first four terms are

$-3, 1, 5, 9, \ldots\ldots.$

Q.2 If $a_{n-3} = 2n - 5$, find the nth term of the sequence.

Solution:

$a_{n-3} = 2n - 5$

Put $n = 4 \Rightarrow a_{4-3} = 2(4) - 5 \Rightarrow a_1 = 3$

Put $n = 5 \Rightarrow a_{5-3} = 2(5) - 5 \Rightarrow a_2 = 5$

Put $n = 6 \Rightarrow a_{6-3} = 2(6) - 5 \Rightarrow a_3 = 7$

Common difference

$d = 5 - 3 = 2$; $d = 7 - 5 = 2$

As difference is same so it is arithmetic sequence. So

$a_n = a_1 + (n-1)d$

$= 3 + (n-1)2$

$= 3 + 2n - 2$

$a_n = 2n + 1$

Alternative Method:

$a_{n-3} = 2n - 5$

Replace n by $n+3$

$a_{n+3-3} = 2(n+3) - 5$

$a_n = 2n + 6 - 5$

$a_n = 2n + 1$

Q.3 If the 5th term of an A.P is 16 and the 20th term is 46, what is its 12th term?

Solution:

$a_5 = 16$, $a_{20} = 46$, $a_{12} = ?$

$a_1 + 4d = 16 \rightarrow (\text{i})$; $a_1 + 19d = 46 \rightarrow (\text{ii})$ $\because a_n = a_1 + (n-1)d$

$Eq\ i) - Eq\ ii)$

$a_1 + 4d = 16$

$\pm a_1 \pm 19d = \pm 46$

$-15d = -30$

$d = 2$

Put in (i)

$a_1 + 4(2) = 16$

$a_1 + 8 = 16$

$a_1 = 8$

Now $a_{12} = a_1 + 11d = 8 + 11(2) = 30$

Q.4 Find the 13th term of the sequence $x, 1, 2-x, 3-2x, ...$

Solution:

$x, 1, 2-x, 3-2x, ...$

$a_1 = x \quad, n = 13 \,, a_{13} = ?$

$d = a_2 - a_1 = 1 - x \quad ; \quad d = a_3 - a_2 = (2-x) - 1 = 2 - x - 1 = 1 - x$

As $a_n = a_1 + (n-1)d$

$a_{13} = a_1 + 12d$

$= x + 12(1-x)$

$= x + 12 - 12x$

$a_{13} = 12 - 11x$

Q.5 Find the 18 term of the A.P. if its 6th term is 19 and the 9th term is 31.

Solution:

$a_6 = 19 \quad , \quad a_{18} = ?$

$\Rightarrow a_1 + 5d = 19 \qquad \text{(i)}$

and $a_9 = 31$

$\Rightarrow a_1 + 8d = 31 \qquad \text{(ii)}$

$Eq\ i) - Eq\ ii)$

$$\begin{array}{r} a_1 + 5d = 19 \\ \pm a_1 \pm 8d = \pm 31 \\ \hline -3d = -12 \end{array}$$

$d = 4$

Put in (i)

$a_1 + 5(4) = 19$

$a_1 + 20 = 19$

$a_1 = -1$

Now $a_{18} = a_1 + 17d$

$= -1 + 17(4)$

$= -1 + 68$

$a_{18} = 67$

Q.6 Which term of the A.P $5, 2, -1, ...$ is -85?

Solution:

$5, 2, -1, ...\ -85$

Here $a_1 = 5, d = a_2 - a_1 = 2 - 5 = -3,$

$a_n = -85$

As we know

$a_n = a_1 + (n-1)d$

$-85 = 5 + (n-1)(-3)$

$-90 = (n-1)(-3)$

$30 = n - 1$

$31 = n$

So, -85 is the 31^{th} term of the given A.P.

Q.7 Which term of the A.P $-2, 4, 10, ...$ is 148?

Solution:

$-2, 4, 10, ...148$

Here $a_1 = -2, d = a_2 - a_1$

$= 4 - (-2) = 6, \ a_n = 148$

As we know

$a_n = a_1 + (n-1)d$

$148 = -2 + (n-1)6$

$150 = (n-1)6$

$25 = n-1$

$26 = n$

So 148 is the 26th term of the given A.P.

Q.8 How many terms are there in the A.P in which $a_1 = 11$; $a_n = 68$; $d = 3$?

Solution:

Given that:

$a_1 = 11$; $a_n = 68$; $d = 3$, n=?

As we know

$a_n = a_1 + (n-1)d$

$68 = 11 + (n-1)(3)$

$68 - 11 = (n-1)3$

$\frac{57}{3} = n-1$

$19 = n-1$

$19 + 1 = n$

$20 = n$

The number of terms in A.P are 20.

Q.9 If the *n*th term of the A.P is $3n-1$, find the A.P.

Solution:

Given that: $a_n = 3n-1$

Put $n = 1 \Rightarrow a_1 = 3(1) - 1 = 2$

Put $n = 2 \Rightarrow a_2 = 3(2) - 1 = 5$

Put $n = 3 \Rightarrow a_3 = 3(3) - 1 = 8$

Put $n = 4 \Rightarrow a_4 = 3(4) - 1 = 11$

So the A.P. is 2,5,8,11,.........

Q.10 Determine whether i) -19 ii) 2 are the terms of the A.P 17,13,9,...... or not?

(i) For -19

Solution:

Let $a_n = -19$

$17, 13, 9, \ldots\ldots -19$

$a_1 = 17$; $d = 13 - 17 = -4$

As $a_n = a_1 + (n-1)d$

$-19 = 17 + (n-1)(-4)$

$-19 - 17 = (n-1)(-4)$

$-36 = (n-1)(-4)$

$\frac{-36}{-4} = (n-1)$

$9 = n-1$

$9 + 1 = n$

$10 = n$

So, -19 is the term of A.P

(ii) For 2

Solution:

Let $a_n = 2$

$17, 13, 9, \ldots\ldots 2$

$a_1 = 17$; $d = 13 - 17 = -4$

As $a_n = a_1 + (n-1)d$

$2 = 17 + (n-1)(-4)$

$2 - 17 = (n-1)(-4)$

$-15 = (n-1)(-4)$

$\frac{-15}{-4} = n-1$

$\frac{15}{4} = n-1$

$\frac{15}{4} + 1 = n$

$\frac{19}{4} = n$ Not Possible

So, 2 is not the term of A.P

Q.11 If l, m, n are the p^{th}, q^{th}, and r^{th} terms of an A.P. show that

(i) $l(q-r)+m(r-p)+n(p-q)=0$

Solution:

Given that l, m, n are the p^{th}, q^{th}, and r^{th} terms of an A.P.

$a_p = l \quad \Rightarrow \quad a_1+(p-1)d = l$ (i)

$a_q = m \quad \Rightarrow \quad a_1+(q-1)d = m$ (ii)

$a_r = n \quad \Rightarrow \quad a_1+(r-1)d = n$ (iii)

Subtract (ii) from (i)

$$\left[a_1+(p-1)d\right]-\left[a_1+(q-1)d\right]=l-m$$

$$(a_1+pd-d)-(a_1+qd-d)=l-m$$

$$(p-q)d=l-m$$

$$p-q=\frac{l-m}{d} \quad \text{(iv)}$$

Subtract (iii) from (ii)

$$\left[a_1+(q-1)d\right]-\left[a_1+(r-1)d\right]=m-n$$

$$(a_1+qd-d)-(a_1+rd-d)=m-n$$

$$(q-r)d=m-n$$

$$q-r=\frac{m-n}{d} \quad \text{(v)}$$

Subtract (i) from (iii)

$$\left[a_1+(r-1)d\right]-\left[a_1+(p-1)d\right]=n-l$$

$$(a_1+rd-d)-(a_1+pd-d)=n-l$$

$$(r-p)d=n-l$$

$$r-p=\frac{n-l}{d} \quad \text{(vi)}$$

L.H.S: $l(q-r)+m(r-p)+n(p-q)$

$$=l\left(\frac{m-n}{d}\right)+m\left(\frac{n-l}{d}\right)+n\left(\frac{l-m}{d}\right)$$

$$=\frac{1}{d}(lm-ln+mn-ml+ln-nm)$$

$$=\frac{1}{d}(0)$$

$$=0 \text{ R.H.S}$$

So, $l(q-r)+m(r-p)+n(p-q)=0$

(ii) $\mathbf{p(m-n)+q(n-l)+r(l-m)=0}$

Solution:

L.H.S: $p(m-n)+q(n-l)+r(l-m)$

From equations (iv) , (v) , (vi)

$p-q=\frac{l-m}{d}$	$q-r=\frac{m-n}{d}$	$r-p=\frac{n-l}{d}$
$l-m=d(p-q)$	$m-n=d(q-r)$	$n-l=d(r-p)$

Put values of $(l-m),(m-n),(n-l)$

$=p(q-r)d+q(r-p)d+r(p-q)d$

$=pqd-prd+qrd-pqd+prd-qrd$

$=0$ R.H.S

So, $p(m-n)+q(n-l)+r(l-m)=0$

Q.12 Find the nth term of the sequence $\left(\frac{4}{3}\right)^2,\left(\frac{7}{3}\right)^2,\left(\frac{10}{3}\right)^2,..........$

Solution:

Given sequence is:

$\left(\frac{4}{3}\right)^2,\left(\frac{7}{3}\right)^2,\left(\frac{10}{3}\right)^2,..........$

Consider the A. P:

$4,7,10,......$

$a_1=4$; $d=7-4=3$

As we know

$a_n=a_1+(n-1)d$

$a_n=4+(n-1)3$

$a_n=4+3n-3$

$a_n=3n+1$

Hence, the required nth term of the given sequence is:

$a_n=\left(\frac{3n+1}{3}\right)^2$

Q.13 If $\frac{1}{a},\frac{1}{b}$ and $\frac{1}{c}$ are in A.P. show that $b=\frac{2ac}{a+c}$

Solution:

Given that:

$\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

$d=\frac{1}{b}-\frac{1}{a}$ (i)

$d=\frac{1}{c}-\frac{1}{b}$ (ii)

Common difference is same in A.P.
So comparing (i) and (ii)

$\frac{1}{b}-\frac{1}{a}=\frac{1}{c}-\frac{1}{b}$

$\frac{1}{b}+\frac{1}{b}=\frac{1}{c}+\frac{1}{a}$

$\frac{2}{b}=\frac{a+c}{ac}$

$\frac{1}{b}=\frac{a+c}{2ac}$

$b=\frac{2ac}{a+c}$

Hence proved that: $b=\frac{2ac}{a+c}$

Q.14 If $\frac{1}{a},\frac{1}{b}$ and $\frac{1}{c}$ are in A.P. show that the common difference is $\frac{a-c}{2ac}$

Solution:

Given that:

$\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P

$$d=\frac{1}{b}-\frac{1}{a} \qquad \text{(i)}$$

$$d=\frac{1}{c}-\frac{1}{b} \qquad \text{(ii)}$$

Adding (i) and (ii)

$$d+d=\frac{1}{b}-\frac{1}{a}+\frac{1}{c}-\frac{1}{b}$$

$$2d=\frac{1}{c}-\frac{1}{a}$$

$$2d=\frac{a-c}{ac}$$

$$d=\frac{a-c}{2ac}$$

Hence prove that common difference is $\frac{a-c}{2ac}$.

Arithmetic Mean (A.M.):

A number A is said to be the A.M. between the two numbers a and b if a, A, b are in A.P.

Then

$$A-a=b-A$$
$$A+A=b+a$$
$$2A=a+b$$
$$A=\frac{a+b}{2}$$

In general, we can say that a_n is the A.M between a_{n-1} and a_{n+1} i.e.,

$$a_n=\frac{a_{n-1}+a_{n+1}}{2}$$

***n* Arithmetic means between *a* and *b*:**

Let $A_1, A_2, A_3, ...A_n$ be n arithmetic means between a and b.

Then $a, A_1, A_2, A_3, ...A_n, b$ are in A. P

$$\Rightarrow a_1=a$$

$$a_{n+2}=b$$

$$a_1+(n+2-1)d=b \qquad \text{By using} \quad a_n=a_1+(n-1)d$$

$$a+(n+1)d=b$$

$$d=\frac{b-a}{n+1}$$

Thus, $A_1=a+d=a+\frac{b-a}{n+1}=\frac{na+b}{n+1}$

$$A_2=a+2d=a+2\left(\frac{b-a}{n+1}\right)=\frac{(n-1)a+2b}{n+1}$$

$$A_3=a+3d=a+3\left(\frac{b-a}{n+1}\right)=\frac{(n-2)a+3b}{n+1}$$

Similarly,

$$A_n=a+nd=a+n\left(\frac{b-a}{n+1}\right)=\frac{a+nb}{n+1}$$

EXERCISE 6.3

Q.1 Find A.M between

(i) $3\sqrt{5}$ and $5\sqrt{5}$

Solution:

$a = 3\sqrt{5}$; $b = 5\sqrt{5}$

A.M. $= \frac{a+b}{2}$

$= \frac{3\sqrt{5}+5\sqrt{5}}{2}$

$= \frac{8\sqrt{5}}{2}$

A.M. $= 4\sqrt{5}$

(ii) $x-3$ and $x+5$

Solution:

$a = x-3$; $b = x+5$

A.M. $= \frac{a+b}{2}$

$= \frac{x-3+x+5}{2}$

$= \frac{2x+2}{2}$

$= \frac{2(x+1)}{2}$

A.M. $= x+1$

(iii) $1-x+x^2$ and $1+x+x^2$

Solution:

$a = 1-x+x^2$; $b = 1+x+x^2$

A.M. $= \frac{a+b}{2}$

$= \frac{1-x+x^2+1+x+x^2}{2}$

$= \frac{2+2x^2}{2}$

$= \frac{2(1+x^2)}{2}$

A.M. $= 1+x^2$

Q.2 If 5,8 are two A.Ms between *a* and *b*. Find *a* and *b*.

Solution:

As 5,8 are two A.Ms between a and b

so, $a,5,8,b$ are in A.P

$5-a = 8-5$, $8-5 = b-8$

$5-a = 3$, $3 = b-8$

$5-3 = a$, $3+8 = b$

$a = 2$, $b = 11$

Alternative Method:

$a,5,8,b$ are in A.P

5 is A.M between a and 8

$5 = \frac{a+8}{2}$

$10 = a+8$

$10-8 = a$

$2 = a$

8 is A.M between 5 and b

$8 = \frac{5+b}{2}$

$16 = 5+b$

$16-5 = b$

$11 = b$

Q.3 Find 6 A.Ms between 2 and 5

Solution:

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be 6 A.Ms between 2 and 5

Then $2, A_1, A_2, A_3, A_4, A_5, A_6, 5$ are in A. P

$a_1 = 2$, $a_8 = 5$

$$a_1 + 7d = 5$$
$$2 + 7d = 5$$
$$7d = 5 - 2$$
$$7d = 3$$
$$d = \frac{3}{7}$$

$A_1 = a_2 = a_1 + d$	$A_2 = a_3 = a_2 + d$	$A_3 = a_4 = a_3 + d$	$A_4 = a_5 = a_4 + d$	$A_5 = a_6 = a_5 + d$	$A_6 = a_7 = a_6 + d$
$= 2 + \frac{3}{7}$	$= \frac{17}{7} + \frac{3}{7}$	$= \frac{20}{7} + \frac{3}{7}$	$= \frac{23}{7} + \frac{3}{7}$	$= \frac{26}{7} + \frac{3}{7}$	$= \frac{29}{7} + \frac{3}{7}$
$= \frac{17}{7}$	$= \frac{20}{7}$	$= \frac{23}{7}$	$= \frac{26}{7}$	$= \frac{29}{7}$	$= \frac{32}{7}$

$\frac{17}{7}, \frac{20}{7}, \frac{23}{7}, \frac{26}{7}, \frac{29}{7}, \frac{32}{7}$ are 6 A.Ms, between 2 and 5

Q.4 Find four A.Ms between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$.

Solution:

Let A_1, A_2, A_3, A_4 be four A.Ms between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$

So, $\sqrt{2}, A_1, A_2, A_3, A_4, \frac{12}{\sqrt{2}}$ are in A.P.

$a_1 = \sqrt{2}$, $a_6 = \frac{12}{\sqrt{2}}$

$$a_1 + 5d = \frac{12}{\sqrt{2}} \qquad \text{Q}\ a_n = a_1 + (n-1)d$$

$$\sqrt{2} + 5d = \frac{12}{\sqrt{2}}$$

$$5d = \frac{12}{\sqrt{2}} - \sqrt{2}$$

$$5d = \frac{12-2}{\sqrt{2}}$$

$$5d = \frac{10}{\sqrt{2}}$$

$$d = \frac{10}{5\sqrt{2}}$$

$$d = \frac{2}{\sqrt{2}}$$

$$d = \sqrt{2}$$

$A_1 = a_2 = a_1 + d$	$A_2 = a_3 = a_2 + d$	$A_3 = a_4 = a_3 + d$	$A_4 = a_5 = a_4 + d$
$= \sqrt{2} + \sqrt{2}$	$= 2\sqrt{2} + \sqrt{2}$	$= \sqrt{2} + 3\sqrt{2}$	$= 4\sqrt{2} + \sqrt{2}$
$A_1 = 2\sqrt{2}$	$A_2 = 3\sqrt{2}$	$A_3 = 4\sqrt{2}$	$A_4 = 5\sqrt{2}$

$2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}$ are four A.Ms between $\sqrt{2}$ and $\frac{12}{\sqrt{2}}$

Q.5 Insert 7 A.Ms between 4 and 8.

Solution:

Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ be 7 A.Ms between 4 and 8.

So, $4, A_1, A_2, A_3, A_4, A_5, A_6, A_7, 8$ are in A.P.

$a_1 = 4$, $a_9 = 8$

$a_1 + 8d = 8$ $\quad\quad \because a_n = a_1 + (n-1)d$

$4 + 8d = 8$

$8d = 4$

$d = \frac{1}{2}$

$A_1 = a_1 + d$	$A_2 = A_1 + d$	$A_3 = A_2 + d$	$A_4 = A_3 + d$	$A_5 = A_4 + d$	$A_6 = A_5 + d$	$A_7 = A_6 + d$
$= 4 + \frac{1}{2}$	$= \frac{9}{2} + \frac{1}{2}$	$= \frac{10}{2} + \frac{1}{2}$	$= \frac{11}{2} + \frac{1}{2}$	$= \frac{12}{2} + \frac{1}{2}$	$= \frac{13}{2} + \frac{1}{2}$	$= \frac{14}{2} + \frac{1}{2}$
$A_1 = \frac{9}{2}$	$A_2 = 5$	$A_3 = \frac{11}{2}$	$= \frac{12}{2}$	$A_5 = \frac{13}{2}$	$= \frac{14}{2}$	$A_7 = \frac{15}{2}$
			$A_4 = 6$		$A_6 = 7$	

$\frac{9}{2}, 5, \frac{11}{2}, 6, \frac{13}{2}, 7, \frac{15}{2}$ are 7 A.Ms between 4 and 8

Q.6 Find three A.Ms between 3 and 11.

Solution:

Let A_1, A_2, A_3 be 3 A.Ms between 3 and 11

So, $3, A_1, A_2, A_3, 11$ are in A.P.

$a_1 = 3$ & $a_5 = 11$

$a_1 + 4d = 11$ $\qquad \because a_n = a_1 + (n-1)d$

$3 + 4d = 11$

$4d = 8$

$d = 2$

$A_1 = a_2 = a_1 + d$	$A_2 = a_3 = A_1 + d$	$A_3 = a_4 = A_2 + d$
$= 3 + 2$	$= 5 + 2$	$= 7 + 2$
$A_1 = 5$	$A_2 = 7$	$A_3 = 9$

$5, 7, 9$ are three A.Ms between 3 and 11

Q.7 Find *n* so that $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ may be the A.M between *a* and *b*.

Solution:

Let $\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}}$ be the A.M between a and b.

$\dfrac{a^n + b^n}{a^{n-1} + b^{n-1}} = \dfrac{a+b}{2}$ $\qquad \because \text{A.M} = \dfrac{a+b}{2}$

$2\left(a^n + b^n\right) = (a+b)\left(a^{n-1} + b^{n-1}\right)$

$2a^n + 2b^n = a.a^{n-1} + a.b^{n-1} + b.a^{n-1} + b.b^{n-1}$

$2a^n + 2b^n = a^n + ab^{n-1} + ba^{n-1} + b^n$

$2a^n - a^n - a^{n-1}b = ab^{n-1} + b^n - 2b^n$

$a^n - a^{n-1}b = ab^{n-1} - b^n$

$a^{n-1}(a-b) = b^{n-1}(a-b)$

$\dfrac{a^{n-1}}{b^{n-1}} = \dfrac{a-b}{a-b}$ $\qquad$ Where $a - b \neq 0$

$$\frac{a^{n-1}}{b^{n-1}}=1$$

$$\left(\frac{a}{b}\right)^{n-1}=\left(\frac{a}{b}\right)^{0}$$

$$n-1=0$$

$$n=1$$

Q.8 Show that the sum of *n* A.Ms between *a* and *b* is equal to *n* times of their A.M.

Solution.

Let $A_1, A_2, A_3, ..., A_n$ be *n* A.Ms between *a* and *b*.

Then $a, A_1, A_2, A_3......, A_n,$ b are in A.P

Let *d* be common difference

$\therefore A_1=a+d \quad , \quad b=A_n+d \Rightarrow A_n=b-d$

Let S_n be the sum of *n* A.Ms between *a* and *b*

$$\therefore S_n=\frac{n}{2}\left[A_1+A_n\right]$$

$$A_1+A_2+...+A_n=\frac{n}{2}\left[a+\cancel{d}+b-\cancel{d}\right]$$

$$=n\frac{(a+b)}{2}$$

Hence proved that sum of *n* A.Ms between *a* and *b* is equal to the n times their A.M.

Series:

The sum of an indicated number of terms in a sequence is called a series. For example, the sum of first seven terms of the sequence $\{n^2\}$ is the series, $1+4+9+16+25+36+49$

Arithmetic Series:

The sum of an indicated number of terms in an A.P is called Arithmetic series.

e.g., $a_1+(a_1+d)+(a_1+2d)+...+a_1+(n-1)d$ is called an arithmetic series.

Sum of first *n* Terms of an Arithmetic Series:

For any sequence $\{a_n\}$, we have,

$$S_n=a_1+a_2+a_3+...+a_n$$

If $\{a_n\}$ is an A.P., then S_n can be written with usual notations as:

$$S_n=a_1+(a_1+d)+(a_1+2d)+...+(a_n-2d)+(a_n-d)+a_n \rightarrow (i)$$

If we write the terms of the series in the reverse order, the sum of *n* terms remains the same, that is:

$$S_n=a_n+(a_n-d)+(a_n-2d)+...+(a_1+2d)+(a_1+d)+a_1 \rightarrow (ii)$$

Adding (i) and (ii), we have

$$2S_n=(a_1+a_n)+(a_1+a_n)+(a_1+a_n)+.....+(a_1+a_n)+(a_1+a_n)+(a_1+a_n)$$
$$=(a_1+a_n)+(a_1+a_n)+(a_1+a_n)+..........\text{to } n \text{ terms}$$
$$2S_n=n(a_1+a_n)$$

$$S_n=\frac{n}{2}(a_1+a_n)$$
$$=\frac{n}{2}\left[a_1+a_1+(n-1)d\right] \qquad \because a_n=a_1+(n-1)d$$
$$\boxed{S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]}$$

EXERCISE 6.4

Q.1 Find the sum of all the integral multiples of 3 between 4 and 97.

Solution:

Required sum is

$$S_n = 6+9+12+\ldots+96$$

$$= 3(2+3+4+\ldots+32)$$

Using $S_n = \frac{n}{2}(a_1+a_n)$, $a_1 = 2$, $a_n = 32$, $n = 31$

$$S_{31} = 3\left[\frac{31}{2}(2+32)\right]$$

$$= 3\left[\frac{31}{2}(34)\right]$$

$$= 3(31\times 17)$$

$$S_{31} = 1581$$

Q.2 Sum the series

(i) $(-3)+(-1)+1+3+5+\ldots\ldots+a_{16}$

Solution:

Given series is:

$$(-3)+(-1)+1+3+5+\ldots\ldots+a_{16}$$

$a_1 = -3$, $d = -1-(-3) = -1+3 = 2$, $n = 16$

Using formula,

$$S_n = \frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$S_{16} = \frac{16}{2}\left[2\times(-3)+(16-1)2\right]$$

$$= 8(-6+30)$$

$$= 8(24)$$

$$S_{16} = 192$$

(ii) $\mathbf{\frac{3}{\sqrt{2}}+2\sqrt{2}+\frac{5}{\sqrt{2}}+.........+a_{13}}$

Solution:

Given series is:

$$\frac{3}{\sqrt{2}}+2\sqrt{2}+\frac{5}{\sqrt{2}}+.........+a_{13}$$

$$a_1=\frac{3}{\sqrt{2}} \quad , \quad n=13 \quad , \quad d=2\sqrt{2}-\frac{3}{\sqrt{2}}=\frac{2(2)-3}{\sqrt{2}}=\frac{4-3}{\sqrt{2}}=\frac{1}{\sqrt{2}}$$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$S_{13}=\frac{13}{2}\left[2\left(\frac{3}{\sqrt{2}}\right)+(13-1)\frac{1}{\sqrt{2}}\right]$$

$$=\frac{13}{2}\left(\frac{6}{\sqrt{2}}+\frac{12}{\sqrt{2}}\right)$$

$$=\frac{13}{2}\left(\frac{18}{\sqrt{2}}\right)$$

$$S_{13}=\frac{117}{\sqrt{2}}$$

(iii) $\mathbf{1.11+1.41+1.71+..........a_{10}}$

Solution:

Given series is:

$$1.11+1.41+1.71+..........a_{10}$$

$$a_1=1.11 \quad , \quad n=10 \quad , \quad d=1.41-1.11=0.3$$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$S_{10}=\frac{10}{2}\left[2(1.11)+(10-1)0.3\right]$$

$$=5(2.22+2.7)$$

$$=5(4.92)$$

$$S_{10}=24.6$$

(iv) $-8-3\frac{1}{2}+1+........+a_{11}$

Solution:

Given series is:

$$-8-3\frac{1}{2}+1+........+a_{11}$$

$$a_1=-8 \quad , \quad n=11 \quad , \quad d=-3\frac{1}{2}-(-8)=-\frac{7}{2}+8=\frac{-7+16}{2}=\frac{9}{2}$$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$S_{11}=\frac{11}{2}\left[2(-8)+(11-1)\left(\frac{9}{2}\right)\right]$$

$$=\frac{11}{2}\left(-16+10\left(\frac{9}{2}\right)\right)$$

$$=\frac{11}{2}(-16+45)$$

$$=\frac{11}{2}(29)$$

$$=11\times 14.5$$

$$S_{11}=159.5$$

(v) $(x-a)+(x+a)+(x+3a)+........$ **+ to n terms**

Solution:

Given series is:

$$(x-a)+(x+a)+(x+3a)+........+\text{ to } n \text{ terms}$$

$$a_1=x-a \quad , \quad n=n \quad , \quad d=x+a-(x-a)=x+a-x+a=2a$$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$=\frac{n}{2}\left[2(x-a)+(n-1)2a\right]$$

$$=\frac{n}{2}(2x-2a+2na-2a)$$

$$=\frac{n}{2}\left(2x+2na-4a\right)$$

$$=\frac{n}{2}\times 2\left(x+na-2a\right)$$

$$S_n=n\left[x+\left(n-2\right)a\right]$$

(vi) $\frac{1}{1-\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1+\sqrt{x}}+........+$ **to *n* terms**

Solution:

Given series is:

$$\frac{1}{1-\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1+\sqrt{x}}+........+\text{ to } n \text{ terms}$$

$$a_1=\frac{1}{1-\sqrt{x}}=\frac{1+\sqrt{x}}{\left(1-\sqrt{x}\right)\left(1+\sqrt{x}\right)}=\frac{1+\sqrt{x}}{1-x}$$

$$d=\frac{1}{1-x}-\frac{1+\sqrt{x}}{1-x}$$

$$d=\frac{-\sqrt{x}}{1-x}$$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+\left(n-1\right)d\right]$$

$$=\frac{n}{2}\left[2\left(\frac{1+\sqrt{x}}{1-x}\right)+\left(n-1\right)\left(\frac{-\sqrt{x}}{1-x}\right)\right]$$

$$=\frac{n}{2}\left[\frac{2\left(1+\sqrt{x}\right)+\left(n-1\right)\left(-\sqrt{x}\right)}{1-x}\right]$$

$$=\frac{n}{2}\left[\frac{2+2\sqrt{x}-n\sqrt{x}+\sqrt{x}}{1-x}\right]$$

$$=\frac{n}{2}\left[\frac{2+3\sqrt{x}-n\sqrt{x}}{1-x}\right]$$

$$S_n=\frac{n}{2}\left[\frac{2+\left(3-n\right)\sqrt{x}}{1-x}\right]$$

(vii) $\frac{1}{1+\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1-\sqrt{x}}+.......+$ **to n terms**

Solution:

Given series is:

$$\frac{1}{1+\sqrt{x}}+\frac{1}{1-x}+\frac{1}{1-\sqrt{x}}+.......+\text{to } n \text{ terms}$$

$$a_1=\frac{1}{1+\sqrt{x}}=\frac{1-\sqrt{x}}{\left(1+\sqrt{x}\right)\left(1-\sqrt{x}\right)}=\frac{1-\sqrt{x}}{1-x}$$

$$a_1=\frac{1-\sqrt{x}}{1-x}$$

$$d=\frac{1}{1-x}-\frac{1-\sqrt{x}}{1-x}=\frac{1-1+\sqrt{x}}{1-x}$$

$$d=\frac{\sqrt{x}}{1-x}$$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$=\frac{n}{2}\left[2\left(\frac{1-\sqrt{x}}{1-x}\right)+(n-1)\left(\frac{\sqrt{x}}{1-x}\right)\right]$$

$$=\frac{n}{2}\left[\frac{2\left(1-\sqrt{x}\right)+(n-1)\sqrt{x}}{1-x}\right]$$

$$=\frac{n}{2}\left[\frac{2-2\sqrt{x}+n\sqrt{x}-\sqrt{x}}{1-x}\right]$$

$$=\frac{n}{2}\left[\frac{2+n\sqrt{x}-3\sqrt{x}}{1-x}\right]$$

$$S_n=\frac{n}{2}\left[\frac{2+(n-3)\sqrt{x}}{1-x}\right]$$

Q.3 How many terms of the series

(i) $-7+(-5)+(-3)+.........$ **amount to 65?**

Solution:

$a_1=-7$, $S_n=65$, $d=-5-(-7)=-5+7=2$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$65=\frac{n}{2}\left[2(-7)+(n-1)2\right]$$

$$65=\frac{n}{2}(-14+2n-2)$$

$$65=\frac{n}{2}(2n-16)$$

$$65=\frac{n}{\not 2}\not 2(n-8)$$

$$65=n(n-8)$$

$$65=n^2-8n$$

$$n^2-8n-65=0$$

$$n^2-13n+5n-65=0$$

$$n(n-13)+5(n-13)=0$$

$$(n-13)(n+5)=0$$

Either	Or
$n-13=0$	$n+5=0$
$n=13$	$n=-5$ Not Possible

So, $n=13$

(ii) $-7+(-4)+(-1)+.........$ **amount to 114?**

Solution:

Given series is:

$-7+(-4)+(-1)+.........$ amount to 114

$a_1=-7$, $S_n=114$, $d=-4-(-7)=-4+7=3$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$114=\frac{n}{2}\left[2\times(-7)+(n-1)3\right]$$

$$114=\frac{n}{2}(-14+3n-3)$$

$$228=n(3n-17)$$

$$228=3n^2-17n$$

$$3n^2-17n-228=0$$

$$3n^2-36n+19n-228=0$$

$$3n(n-12)+19(n-12)=0$$

$$(n-12)(3n+19)=0$$

Either	Or
$n-12=0$	$3n+19=0$
$n=12$	$n=-\frac{19}{3}$ Not Possible

So, $n=12$

Q.4 Sum the series

(i) $3+5-7+9+11-13+15+17-19+..........$ to $3n$ terms

Solution:

$3+5-7+9+11-13+15+17-19+..........$ to $3n$ terms

$(3+5-7)+(9+11-13)+(15+17-19)+........$ to n terms

$1+7+13+.........$ to n terms

$a_1=1$, $d=7-1=6$, $d=13-7=6$, $n=n$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$=\frac{n}{2}\left[2\times1+(n-1)6\right]$$

$$=\frac{n}{2}(2+6n-6)$$

$$=\frac{n}{2}(6n-4)$$

$$=\frac{n}{\cancel{2}}\cancel{2}(3n-2)$$

$$S_n=n(3n-2)$$

(ii) $1+4-7+10+13-16+19+22-25+........$ to $3n$ terms

Solution:

Given series is:

$1+4-7+10+13-16+19+22-25+........$ to $3n$ terms

$(1+4-7)+(10+13-16)+(19+22-25)+........$ to n terms

$-2+7+16+.....$ to n terms

$a_1=-2$, $d=7-(-2)=7+2=9$, $d=16-9=7$, $n=n$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$S_n=\frac{n}{2}\left[2(-2)+(n-1)9\right]$$

$$=\frac{n}{2}(-4+9n-9)$$

$$S_n=\frac{n}{2}(9n-13)$$

Q.5 Find the sum of 20 terms of the series whose rth term is $3r+1$.

Solution:

Given that:

$a_r=3r+1$ (i)

Put $r=1$, $a_1=3(1)+1=3+1=4$

Put $r=2$, $a_2=3(2)+1=6+1=7$

Put $r=3$, $a_3=3(3)+1=9+1=10$

Put $r=4$, $a_4=3(4)+1=12+1=13$

The series is:

$4+7+10+13+.........$ to 20 terms

$a_1=4$, $d=7-4=3$, $d=10-7=3$, $n=20$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$S_{20}=\frac{20}{2}\left[2\times 4+(20-1)3\right]$$

$$=10(8+19\times 3)$$

$$=10(65)$$

$$S_{20}=650$$

Q.6 If $S_n = n(2n-1)$ then find the series.

Solution:

As given that:

$S_n = n(2n-1)$

Put $n=1$, $S_1 = 1(2(1)-1) = 2-1 = 1$

Put $n=2$, $S_2 = 2(2(2)-1) = 2(4-1) = 2(3) = 6$

Put $n=3$, $S_3 = 3(2(3)-1) = 3(6-1) = 3(5) = 15$

Put $n=4$, $S_4 = 4(2(4)-1) = 4(8-1) = 4(7) = 28$

As we know that,

$a_1 = S_1 = 1$

$a_2 = S_2 - S_1 = 6-1 = 5$

$a_3 = S_3 - S_2 = 15-6 = 9$

$a_4 = S_4 - S_3 = 28-15 = 13$

Thus the series is:

$1+5+9+13+\ldots\ldots\ldots\ldots$

Q.7 The ratio of the sums of n terms of two series in A.P is $3n+2:n+1$. Find the ratio of their 8th terms.

Solution:

Let a_1, a_1' be the first terms and d, d' be the common differences of two series in A.P. respectively. Let S_n and S_n' be the sums of n terms of the two series then:

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right] \quad , \quad S_n' = \frac{n}{2}\left[2a_1' + (n-1)d'\right]$$

As given that:

$S_n : S_n' = 3n+2 : n+1$

$$\frac{S_n}{S_n'} = \frac{3n+2}{n+1}$$

$$\frac{\frac{n}{2}\left[2a_1 + (n-1)d\right]}{\frac{n}{2}\left[2a_1' + (n-1)d'\right]} = \frac{3n+2}{n+1}$$

$$\frac{2a_1 + (n-1)d}{2a_1' + (n-1)d'} = \frac{3n+2}{n+1}$$

$$\frac{2\left[a_1+\left(\frac{n-1}{2}\right)d\right]}{2\left[a_1'+\left(\frac{n-1}{2}\right)d'\right]}=\frac{3n+2}{n+1}$$

$$\frac{a_1+\left(\frac{n-1}{2}\right)d}{a_1'+\left(\frac{n-1}{2}\right)d'}=\frac{3n+2}{n+1} \qquad \text{(i)}$$

For the ratio of 8^{th} terms,

Put $\frac{n-1}{2}=7$

$n-1=14$

$n=15$

Put $n=15$ in equation (i)

$$\frac{a_1+\left(\frac{15-1}{2}\right)d}{a_1'+\left(\frac{15-1}{2}\right)d'}=\frac{3(15)+2}{15+1}$$

$$\frac{a_1+7d}{a_1'+7d'}=\frac{45+2}{16}$$

$$\frac{a_8}{a_8'}=\frac{47}{16}$$

So the ratio of their 8^{th} terms is: $a_8:a_8'=47:16$

Q.8 If S_2, S_3, S_5 are the sum of $2n, 3n, 5n$ terms of an A.P. show that $S_5=5(S-S_2)$

Solution:

As S_2, S_3, S_5 are the sums of $2n, 3n, 5n$ terms of an A.P. So,

$$S_2=\frac{2n}{2}\left[2a_1+(2n-1)d\right] \qquad \because S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$S_3=\frac{3n}{2}\left[2a_1+(3n-1)d\right]$$

$$S_5=\frac{5n}{2}\left[2a_1+(5n-1)d\right]$$

We have to prove that

$S_5=5(S_3-S_2)$

R.H.S: $=5(S_3-S_2)$

$$=5\left[\frac{3n}{2}\left[2a_1+(3n-1)d\right]-\frac{2n}{2}\left[2a_1+(2n-1)d\right]\right]$$

$$=5\left[\frac{3n}{2}\left[2a_1+3nd-d\right]-\frac{2n}{2}\left[2a_1+2nd-d\right]\right]$$

Taking common $\frac{n}{2}$

$$=\frac{5n}{2}\left[3(2a_1+3nd-d)-2(2a_1+2nd-d)\right]$$

$$=\frac{5n}{2}(6a_1+9nd-3d-4a_1-4nd+2d)$$

$$=\frac{5n}{2}(2a_1+5nd-d)$$

$$=\frac{5n}{2}\left[2a_1+(5n-1)d\right]$$

$$=S_5$$

Hence, proved that: $S_5=5(S_3-S_2)$

Q.9 Obtain the sum of all integers in the first 1000 integers which are neither divisible by 5 nor by 2.

Solution:

The series of the first 1000 integers which are neither divisible by 5 nor by 2 is:

$$=1+3+7+9+11+13+17+19+21+23+27+29+......+9991+993+997+999$$

$$=(1+3+7+9)+(11+13+17+19)+(21+23+27+29)+.........(991+993+997+999)$$

$$=20+60+100+.......+3980$$

$a_1=20$, $d=60-20=40$, $d=100-60=40$, $a_n=3980$

As we know that:

$$a_n=a_1+(n-1)d$$

$$3980=20+(n-1)40$$

$$3980-20=(n-1)40$$

$$3960=(n-1)40$$

$$\frac{3960}{40}=n-1$$

$$99=n-1$$

$$n=100$$

Using formula,

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$S_{100} = \frac{100}{2}\left[2\times 20 + (100-1)40\right]$$

$$= 50(40 + 99\times 40)$$

$$= 50(40 + 3960)$$

$$= 50(4000)$$

$$S_{100} = 200000$$

Q.10 S_8 and S_9 are the sums of the first eight and nine terms of an A.P. Find S_9 if $50S_9 = 63S_8$ and $a_1 = 2$

Solution:

Given that:

$50S_9 = 63S_8$ and $a_1 = 2$

$$S_8 = \frac{8}{2}\left[2a_1 + (8-1)d\right] \qquad \because S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$S_8 = 4(2a_1 + 7d)$$

Similarly,

$$S_9 = \frac{9}{2}\left[2a_1 + (9-1)d\right]$$

$$= \frac{9}{2}(2a_1 + 8d)$$

$$= \frac{9}{2}\times 2(a_1 + 4d)$$

$$S_9 = 9(a_1 + 4d)$$

Now,

$$50\,S_9 = 63\,S_8$$

$$50\left[9(a_1 + 4d)\right] = 63\left[4(2a_1 + 7d)\right]$$

As given that $a_1 = 2$

$$450(2 + 4d) = 252(4 + 7d)$$

$$900 + 1800d = 1008 + 1764d$$

$$1800d - 1764d = 1008 - 900$$

$36d = 108$

$d = \frac{108}{36}$

$d = 3$

Now,

$S_9 = 9(a_1 + 4d)$

Put $a_1 = 2$ and $d = 3$

$= 9(2 + 4\times 3)$

$= 9(2+12)$

$= 9(14)$

$S_9 = 126$

Q.11 The sum of 9 terms of an A.P is 171 and its eight term is 31. Find the series.

Solution:

Given that:

$a_8 = 31$

$\Rightarrow a_1 + 7d = 31$ (i)

and $S_9 = 171$

As we know that,

$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

$\Rightarrow \frac{9}{2}\left[2a_1 + (9-1)d\right] = 171$

$\frac{9}{2}(2a_1 + 8d) = 171$

$\frac{9}{2}\times 2(a_1 + 4d) = 171$

$9(a_1 + 4d) = 171$

$a_1 + 4d = 19$ (ii)

Equation (i) – equation (ii)

$a_1 + 7d = 31$

$\underline{\pm a_1 \pm 4d = \pm 19}$

$3d = 12$

$d = 4$

Put $d = 4$ in (i)

$a_1 + 7(4) = 31$

$a_1 + 28 = 31$

$a_1 = 31 - 28$

$a_1 = 3$

Now,

$a_2 = a_1 + d$	$a_3 = a_2 + d$	$a_4 = a_3 + d$
$a_2 = 3 + 4$	$a_3 = 7 + 4$	$a_4 = 11 + 4$
$a_2 = 7$	$a_3 = 11$	$a_4 = 15$

Thus the series

$3 + 7 + 11 + 15 + ...$

Q.12 The sum of S_9 and S_7 is 203 and $S_9 - S_7 = 49$. S_7 and S_9 being the sums of the first 7 and 9 terms of an A.P. respectively. Determine the series.

Solution:

Given that

$S_9 + S_7 = 203$ (i)

$S_9 - S_7 = 49$ (ii)

Adding (i) and (ii)

$S_9 + S_7 = 203$

$\underline{S_9 - S_7 = 49}$

$2S_9 = 252$

$S_9 = 126$

Put $S_9 = 126$ in (i)

$126 + S_7 = 203$

$S_7 = 203 - 126$

$S_7 = 77$

Now,

$S_9 = 126$

$\frac{9}{2}\left[2a_1 + (9-1)d\right] = 126$

$\frac{9}{2}(2a_1 + 8d) = 126$

$\frac{9}{2} \times 2(a_1 + 4d) = 126$

$9(a_1 + 4d) = 126$

$a_1 + 4d = 14$ (iii)

and $S_7 = 77$

$\frac{7}{2}\left[2a_1 + (7-1)d\right] = 77$

$\frac{7}{2}(2a_1 + 6d) = 77$

$a_1 + 3d = 11$ (iv)

Equation (iii) – equation (iv)

$a_1 + 4d = 14$

$\underline{\pm a_1 \pm 3d = \pm 11}$

$d = 3$

Put $d = 3$ in (iii)

$a_1 + 4(3) = 14$

$a_1 + 12 = 14$

$a_1 = 2$

Now,

$a_2 = a_1 + d$	$a_3 = a_2 + d$	$a_4 = a_3 + d$
$a_2 = 2 + 3$	$a_3 = 5 + 3$	$a_4 = 8 + 3$
$a_2 = 5$	$a_3 = 8$	$a_4 = 11$

Thus the series;

$2 + 5 + 8 + 11 + ...$

Q.13 **S_7 and S_9 are the sums of the first 7 and 9 terms of an A.P respectively.**

If $\frac{S_9}{S_7} = \frac{18}{11}$ and $a_7 = 20$. Find the series.

Solution:

Given that:

$a_7 = 20$

$\Rightarrow a_1 + 6d = 20$ (i)

$\frac{S_9}{S_7} = \frac{18}{11}$

As we know that,

$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

$$\Rightarrow \frac{\frac{9}{2}\left[2a_1 + (9-1)d\right]}{\frac{7}{2}\left[2a_1 + (7-1)d\right]} = \frac{18}{11}$$

$$\frac{9(2a_1 + 8d)}{7(2a_1 + 6d)} = \frac{18}{11}$$

$$\frac{9 \times 2(a_1 + 4d)}{7 \times 2(a_1 + 3d)} = \frac{18}{11}$$

$$\frac{\cancel{9}(a_1 + 4d)}{7(a_1 + 3d)} = \frac{\cancel{18}^{2}}{11}$$

$11(a_1 + 4d) = 2 \times 7(a_1 + 3d)$

$11a_1 + 44d = 14a_1 + 42d$

$44d - 42d = 14a_1 - 11a_1$

$2d = 3a_1$

$d = \frac{3}{2}a_1$ (ii)

Put $d = \frac{3}{2}a_1$ in (i)

$a_1 + \cancel{6}\left(\frac{3}{\cancel{2}}\right)a_1 = 20$

$a_1 + 9a_1 = 20$

$10a_1 = 20$

$a_1 = 2$

Put $a_1 = 2$ in (ii)

$d = \frac{3}{\cancel{2}} \times \cancel{2}$

$d = 3$

Now,

$a_2 = a_1 + d$	$a_3 = a_2 + d$	$a_4 = a_3 + d$
$a_2 = 2 + 3$	$a_3 = 5 + 3$	$a_4 = 8 + 3$
$a_2 = 5$	$a_3 = 8$	$a_4 = 11$

Thus the series is

$2 + 5 + 8 + 11 + ...$

Q.14 The sum of three numbers in an A.P is 24 and their product is 440. Find the numbers.

Solution:

Let $a-d, a, a+d$ are the three numbers in A.P.

Sum = 24

$a-d+a+a+d=24$

$3a=24$

$a=8$

Product = 440

$(a-d)(a)(a+d)=440$

Put $a=8$,

$(8-d)(8)(8+d)=440$

$(8)^2-(d)^2=55$

$64-d^2=55$

$64-55=d^2$

$9=d^2$

$d=\pm 3$

When $a=8$ and $d=3$,	When $a=8$ and $d=-3$
the numbers are: ,	the numbers are:
$a-d , a , a+d$,	$a-d , a , a+d$
$8-3 , 8 , 8+3$,	$8-(-3) , 8 , 8+(-3)$
$5 , 8 , 11$,	$8+3 , 8 , 8-3$
	$11 , 8 , 5$

So the required numbers are $5 , 8 , 11$ or $11 , 8 , 5$.

Q.15 Find four numbers in A.P whose sum is 32 and the sum of whose squares is 276.

Solution:

Let $a-3d, a-d, a+d, a+3d$ are the four numbers in A.P

According to given condition that the sum of numbers is 32

$a-3d+a-d+a+d+a+3d=32$

$4a=32$

$a=8$ (i)

According to given condition that the sum of squares of numbers is 276

$(a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2=276$

$a^2+9d^2-\cancel{6ad}+a^2+d^2-\cancel{2ad}+a^2+d^2+\cancel{2ad}+a^2+9d^2+\cancel{6ad}=276$

$4a^2+20d^2=276$

$4(a^2+5d^2)=276$

$a^2+5d^2=69$

Put $a=8$ from (i)

$8^2+5d^2=69$

$64+5d^2=69$

$5d^2=69-64$

$5d^2=5$

$d^2=1$

$d=\pm1$

When $a=8$ and $d=1$	,	When $a=8$ and $d=-1$
the numbers are:	,	the numbers are:
$a-3d,a-d,a+d,a+3d$		$a-3d,a-d,a+d,a+3d$
$8-3(1),8-1,8+1,8+3(1)$	,	$8-3(-1),8-(-1),8+(-1),8+3(-1)$
$8-3,7,9,8+3$		$8+3,8+1,8-1,8-3$
$5,7,9,11$		$11,9,7,5$

So required numbers are $5,7,9,11$ or $11,9,7,5$.

Q.16 Find the five numbers in A.P whose sum is 25 and the sum of whose squares is 135

Solution:

Let $a-2d,a-d,a,a+d,a+2d$ are the five numbers in A.P

According to given condition that the sum of numbers is 25

$a-2d+a-d+a+a+d+a+2d=25$

$5a=25$

$a=5$ (i)

According to given condition that the sum of squares of numbers is 135

$(a-2d)^2+(a-d)^2+(a)^2+(a+d)^2+(a+2d)^2=135$

$a^2+4d^2-\cancel{4ad}+a^2+d^2-\cancel{2ad}+a^2+a^2+d^2+\cancel{2ad}+a^2+4d^2+\cancel{4ad}=135$

$5a^2+10d^2=135$

$5(a^2+2d^2)=135$

$a^2+2d^2=27$

Put $a=5$ from (i)

$5^2+2d^2=27$

$25+2d^2=27$

$2d^2=2$

$d^2=1$

$d=\pm1$

When $a=5$ and $d=1$	,	When $a=5$ and $d=-1$
the numbers are:	,	the numbers are:
$a-2d, a-d, a, a+d, a+2d$		$a-2d, a-d, a, a+d, a+2d$
$5-2(1), 5-1, 5, 5+1, 5+2(1)$	,	$5-2(-1), 5-(-1), 5, 5+(-1), 5+2(-1)$
$5-2, 4, 5, 6, 5+2$		$5+2, 5+1, 5, 5-1, 5-2$
$3, 4, 5, 6, 7$		$7, 6, 5, 4, 3$

So required numbers are $3,4,5,6,7$ or $7,6,5,4,3$.

Q.17 The sum of the 6th and 8th terms of an A.P is 40 and the product of 4th and 7th terms is 220. Find the A.P.

Solution:

$a_6+a_8=40$

$\Rightarrow a_1+5d+a_1+7d=40$

$2a_1+12d=40$

$2(a_1+6d)=40$

$a_1+6d=20$ (i)

and $(a_4)(a_7)=220$

$\Rightarrow (a_1+3d)(a_1+6d)=220$

Put $a_1+6d=20$ from (i)

$(a_1+3d)(20)=220$

$a_1+3d=11$ (ii)

Equation (i) – equation (ii)

$$\begin{array}{r} a_1+6d=20 \\ \pm a_1\pm 3d=\pm 11 \\ \hline 3d=9 \end{array}$$

$d=3$

Put $d=3$ in (i)

$a_1+6(3)=20$

$a_1+18=20$

$a_1=2$

Now,

$a_2=a_1+d$	$a_3=a_2+d$	$a_4=a_3+d$
$a_2=2+3$	$a_3=5+3$	$a_4=8+3$
$a_2=5$	$a_3=8$	$a_4=11$

Thus the A.P is

$2,5,8,11,......$

Q.18 If a^2, b^2 and c^2 are in A.P, show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

Solution:

a^2, b^2, c^2 are in A.P. So,

$b^2 - a^2 = c^2 - b^2$

$(b+a)(b-a) = (c+b)(c-b)$ (i)

We have to show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P. then:

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{b+c-(c+a)}{(c+a)(b+c)} = \frac{c+a-(a+b)}{(c+a)(a+b)}$$

$$\frac{b+\cancel{c}-\cancel{c}-a}{\cancel{(c+a)}(b+c)} = \frac{c+\cancel{a}-\cancel{a}-b}{\cancel{(c+a)}(a+b)}$$

$$\frac{(b-a)}{(b+c)} = \frac{(c-b)}{(a+b)}$$

$(b+a)(b-a) = (c+b)(c-b)$ (ii)

By comparing (i) and (ii) it is proved that, $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

EXERCISE 6.5

Q.1 An man deposits in a bank Rs.10 in the first month; Rs 15 in the second month; Rs 20 in the third month and so on. Find how much he will have deposited in the bank by the 9th month.

Solution:

Series of the deposited amount is:

$10+15+20+......$ to 9 terms

$a_1=10\ ,\ d=15-10=5,\ \ n=9$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$S_9=\frac{9}{2}[2(10)+(9-1)5]$$

$$S_9=\frac{9}{2}(20+40)$$

$$S_9=\frac{9}{2}(60)$$

$$S_9=9\times 30$$

$$S_9=270$$

So the deposited amount by the 9th month is 270.

Q.2 378 trees are planted in rows in the shape of an isosceles triangle, the number in successive rows decreasing by one from the base to the top. How many trees are there in the row which forms the base of the triangle?

Solution:

The series of trees from top to bottom is:

$1+2+3+.....$ n terms

$a_1=1\ ,\ d=2-1=1\ ,\ n=n\ ,\ S_n=378$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$378=\frac{n}{2}\left[2(1)+(n-1)(1)\right]$$

$$378=\frac{n}{2}(2+n-1)$$

$$378=\frac{n(n+1)}{2}$$

$756 = n^2 + n$

$n^2 + n - 756 = 0$

$n^2 + 28n - 27n - 756 = 0$

$n(n+28) - 27(n+28) = 0$

$(n-27)(n+28) = 0$

Either	Or
$n - 27 = 0$	$n + 28 = 0$
$n = 27$	$n = -28$ Not Possible

The number of trees in the base row, in the triangle is:

$a_n = a_1 + (n-1)d$

$a_{27} = 1 + (27-1) \times 1$

$= 1 + 26 \times 1$

$a_{27} = 27$

Q.3 A man borrows Rs 1100 and agree to repay with a total interest of Rs 230 in 14 installments, each installment being less than the preceding by Rs 10. What should be his first installment?

Solution:

Let the first installment is x, so the sequence is:

$x, x-10, x-20, \ldots\ldots$ to 14 terms

S_{14} = total amount to pay = 1100+270=1330

$a_1 = x$, $d = x - 10 - x = -10$,

Using formula,

$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$

$S_{14} = \frac{14}{2}\left[2x + (14-1)(-10)\right]$

$1330 = 7(2x - 130)$

$\frac{1330}{7} = 2x - 130$

$190 = 2x - 130$

$190 + 130 = 2x$

$2x = 320$

$x = \frac{320}{2}$

$x = 160$

Thus the first installment is 160.

Q.4 A clock strikes once when its hour hand is at one. Twice when it is at two and so on. How many times does the clock strike in twelve hours?

Solution:

The strikes of clock form the sequence.

$1, 2, 3, ...$ to 12 terms

$a_1 = 1 \quad , \quad d = 2 - 1 = 1 \quad , \quad n = 12$

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2\times1 + (12-1)1\right]$$

$$= 6(2+11)$$

$$= 6(13)$$

$$S_{12} = 78$$

So the clock strikes 78 times in twelve hours.

Q.5 A student saves Rs. 12 at the end of the first week and goes on increasing his saving Rs. 4 weekly. After how many weeks will he be able to save Rs 2100?

Solution:

The sequence of his saving at the end of n weeks is:

$12, 16, 20, ...$ to n terms

$S_n = 2100 \quad , \quad a_1 = 12 \quad , \quad d = 16 - 12 = 4 \quad , \quad n = n$

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$2100 = \frac{n}{2}\left[2\times12 + (n-1)4\right]$$

$$2100 = \frac{n}{2}(24 + 4n - 4)$$

$$2100 = \frac{n}{2}(4n + 20)$$

$$2100 = \frac{n}{2}\times4(n+5)$$

$$2100 = 2n(n+5)$$

$$\frac{2100}{2} = n(n+5)$$

$$1050 = n(n+5)$$

$$1050 = n^2 + 5n$$

$n^2+5n-1050=0$

$n^2+35n-30n-1050=0$

$n(n+35)-30(n+35)=0$

$(n-30)(n+35)=0$

Either | Or

$n-30=0$ | $n+35=0$

$n=30$ | $n=-35$ Not possible

Thus the student save Rs. 2100 in 30 weeks.

Q.6 An object falling from rest, falls 9 meters during the first second, 27 meters during the next second, 45 meters during the third second and so on.

(i) How far will it fall during the fifth second?

(ii) How far will it fall up to the fifth second?

Solution:

The sequence of the fall is

$9, 27, 45, ...$

$a_1=9$, $d=27-9=18$

(i) To calculate how far will it fall during the fifth second.

$n=5$

As we know

$a_n=a_1+(n-1)d$

$a_5=9+(5-1)18$

$a_5=9+(4)18$

$a_5=9+72$

$a_5=81$

So the object will fall a distance of 81 meters during the fifth second.

(ii) To calculate how far will it fall up to the fifth second.

$S_5=?$, $a_1=9$, $d=18$, $n=5$

Using formula,

$$S_n=\frac{n}{2}\left[2a_1+(n-1)d\right]$$

$$S_5=\frac{5}{2}\left[2\times9+(5-1)18\right]$$

$$S_5 = \frac{5}{2}(18 + 4 \times 18)$$

$$S_5 = \frac{5}{2}(18 + 72)$$

$$S_5 = \frac{5}{2} \times 90$$

$$S_5 = 5 \times 45$$

$$S_5 = 225$$

So the object will fall a distance of 225 meters upto the fifth second.

Q.7 An investor earned Rs. 6000 for year 1980 and Rs. 12000 for year 1990 on the same investment. If his earning has increased by the same amount each year. How much income he has received from the investment over the past eleven years?

Solution:

The first earned amount = 6000

The final earned amount = 12000

Total no. of years = 11

$a_1 = 6000$, $a_n = 12000$, $n = 11$, $S_{11} = ?$

Using formula,

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{11} = \frac{11}{2}(6000 + 12000)$$

$$= \frac{11}{2}(18000)$$

$$= 11 \times 9000$$

$$S_{11} = 99000$$

The income he received over the past eleven years is 99000.

Q.8 The sum of interior angles of polygons having sides 3,4,5,……etc. from an A.P. Find the sum of the interior angles for a 16 sided polygon.

Solution:

Sum of the interior angles of 3 – sided polygon $= \pi$

Sum of interior angles of 4 – sided polygon $= 2\pi$

Sum of interior angles of 5 – sided polygon $= 3\pi$

Sum of interior angles of 16 – sided polygon $= ?$

So the sequence of sums of interior angles is:

$\pi, 2\pi, 3\pi, ...$

$a_1 = \pi$, $d = 2\pi - \pi = \pi$

$n = 14$ (For polygon having 16 – sides)

Using formula,

$a_n = a_1 + (n-1)d$

$a_{14} = \pi + (14-1)\pi$

$a_{14} = \pi + 13\pi$

$a_{14} = 14\pi$

Sum of interior angles of polygon having 16–sides is 14π.

Q.9 The prize money Rs. 60,000 will be distributed among the eight teams according to their positions determined in the match-series. The award increases by the same amount of each higher position. If the last place team is given Rs. 4000, how much will be awarded to the first place team?

Solution:

Total amount $= S_8 = 60,000$, $n = 8$, $a_1 = 4000$, $a_8 = ?$

Using formula,

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_8 = \frac{8}{2}(4000 + a_8)$$

$$60,000 = 4(4000 + a_8)$$

$$\frac{60000}{4} = 4000 + a_8$$

$$15000 = 4000 + a_8$$

$$a_8 = 15000 - 4000$$

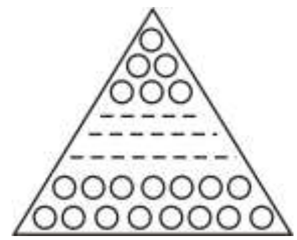
$$a_8 = 11000$$

The prize money awarded to the first place team is 11000.

Q.10 A equilateral triangular base is filled by placing eight ball in the first row, 7 balls in the second row and so on with one ball in the last row. After this base layer, second layer is formed by placing 7 balls in its first row, 6 balls in its second row and so on with one ball in its last row. Continuing this process a pyramid of balls is formed with one ball on top. How balls are there in the pyramid?

Solution:

Balls in the first layer are
$8+7+6+5+4+3+2+1=36$
Balls in the 2nd layer are
$7+6+5+4+3+2+1=28$
Balls in the 3rd layer are
$6+5+4+3+2+1=21$
Balls in the 4th layer are
$5+4+3+2+1=15$
Ball in the 5th layer are
$4+3+2+1=10$
Balls in the 6th layer are
$3+2+1=6$
Balls in the 7th layer are
$2+1=3$
Balls in the 8th layer = 1
The number of balls $=36+28+21+15+10+6+3+1=120$



Geometric progression (G.P):

A sequence $\{a_n\}$ is a geometric sequence or geometric progression if $\frac{a_n}{a_{n-1}}$ is the same non-zero number for all $n \in N$ and $n>1$. The quotient $\frac{a_n}{a_{n-1}}$ is usually denoted by r and is called common ratio of the G.P. The common ration $r=\frac{a_n}{a_{n-1}}$ is defined only if $a_{n-1} \neq 0$, i.e., no term of the geometric sequence is zero.

Rule for *n*th term of a G.P:

In G.P each term after the first term is an r multiple of its preceding term. Thus we have,

$a_2 = a_1 r = a_1 r^{2-1}$

$a_3 = a_2 r = (a_1 r) r = a_1 r^2 = a_1 r^{3-1}$

$a_4 = a_3 r = (a_1 r^2) r = a_1 r^3 = a_1 r^{4-1}$

Similarly we have,

$a_n = a_1 r^{n-1}$ which is the general term of a G.P

EXERCISE 6.6

Q.1 Find the 5th term of the G.P $3, 6, 12, ...$

Solution:

$$a_1 = 3 \quad , \quad r = \frac{6}{3} = 2 \quad , \quad a_5 = ?$$

Using formula,

$$a_n = a_1 r^{n-1}$$

$$a_5 = a_1 r^{5-1}$$

$$a_5 = 3(2)^{5-1}$$

$$= 3(2)^4$$

$$a_5 = 48$$

Q.2 Find the 11th term of the sequence $1+i, 2, \frac{4}{1+i}, ...$

Solution:

$$a_1 = 1+i \quad , \quad n = 11$$

$$r = \frac{2}{1+i} = \frac{2}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{2\times(1-i)}{1-i^2} = \frac{2\times(1-i)}{1-(-1)}$$

$$r = \frac{2\times(1-i)}{2}$$

$$r = 1-i$$

Using formula,

$$a_n = a_1 r^{n-1}$$

$$a_{11} = (1+i)(1-i)^{11-1}$$

$$= (1+i)(1-i)^{10}$$

$$= (1+i)\left[(1-i)^2\right]^5$$

$$= (1+i)(-2i)^5$$

$$= (1+i)(-32i^5)$$

$$= (1+i)(-32i)$$

$$= -32i(1+i)$$

$$= -32(i+i^2)$$

$$= -32(-1+i)$$

$$a_{11} = 32(1-i)$$

Q.3 Find the 12th term of $1+i, 2i, -2+2i, ...$

Solution:

$$a_1 = 1+i \quad , \quad n = 12$$

$$r = \frac{2i}{1+i} = \frac{2i}{1+i} \times \frac{1-i}{1-i} = \frac{2(i-i^2)}{1-i^2}$$

$$= \frac{2(i+1)}{1+1} = \frac{2(1+i)}{2}$$

$$r = 1+i$$

Using formula,

$$a_n = a_1 r^{n-1}$$

$$a_{12} = a_1 r^{11}$$

$$= (1+i)(1+i)^{11}$$

$$= (1+i)^{12}$$

$$= \left[(1+i)^2\right]^6$$

$$= (2i)^6$$

$$= 64 i^6$$

$$= 64(i^2)^3$$

$$= 64(-1)^3$$

$$a_{12} = -64$$

Q.4 Find the 11th term of the sequence $1+i, 2, 2(1-i), ...$

Solution:

$$a_1 = 1+i \quad , \quad n = 11$$

$$r = \frac{2(1-i)}{2}$$

$$r = 1-i$$

Using formula,

$$a_n = a_1 r^{n-1}$$

$$a_{11} = a_1 r^{10}$$

$$= (1+i)(1-i)^{10}$$

$$= (1+i)\left((1-i)^2\right)^5$$

$$=(1+i)(-2i)^5 \qquad (1-i)^2=1+i^2-2i=1-1-2i=-2i$$

$$=(1+i)(-32i^5)$$

$$=(1+i)(-32i) \qquad \because i^5=i.i^4=i(i^2)^2=i(-1)^2=i$$

$$=-32(i+i^2)$$

$$=-32(i-1)$$

$$a_{11}=32(1-i)$$

Q.5 If an automobile depreciates in value 5% every year, at the end of 4 years what is the value of the automobile purchased for Rs 12,000?

Solution:

$$a_1=12000 \quad , \quad r=1-5\%=1-\frac{5}{100}=1-0.05=0.95$$

At the end of 4 years $n=5$

Using formula,

$$a_n=a_1r^{n-1}$$

$$a_5=a_1r^4$$

$$=12000(0.95)^4$$

$$=12000(0.8145)$$

$$a_5=9774 \text{ (Approximately)}$$

Q.6 Which term of the sequence: $\mathbf{x^2-y^2, x+y, \frac{x+y}{x-y}, ...}$ is $\mathbf{\frac{x+y}{(x-y)^9}}$?

Solution:

$$a_1=x^2-y^2=(x+y)(x-y) \quad , \quad a_n=\frac{x+y}{(x-y)^9}$$

$$r=\frac{x+y}{x-y}\times\frac{1}{x+y}$$

$$r=\frac{1}{x-y}$$

Using formula,

$$a_n=a_1r^{n-1}$$

$$\frac{(x+y)}{(x-y)^9}=(x-y)(x+y)\left(\frac{1}{x-y}\right)^{n-1}$$

$$\frac{1}{(x-y)^9}=(x-y)\times\left(\frac{1}{x-y}\right)^{n-1}$$

$$\frac{1}{(x-y)^9\times(x-y)}=\left(\frac{1}{x-y}\right)^{n-1}$$

$$\left(\frac{1}{x-y}\right)^{10}=\left(\frac{1}{x-y}\right)^{n-1}$$

$n-1=10$

$n=10+1$

$n=11$

Q.7 If a,b,c,d are in G.P, prove that

(i) $a-b,b-c,c-d$ are in G.P

(ii) a^2-b^2,b^2-c^2,c^2-d^2 are in G.P

(iii) a^2+b^2,b^2+c^2,c^2+d^2 are in G.P

Solution:

As a,b,c,d are in G.P

$r=\frac{b}{a}$, $r=\frac{c}{b}$, $r=\frac{d}{c}$

$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$

$\frac{b}{a}=\frac{c}{b}$, $\frac{c}{b}=\frac{d}{c}$, $\frac{b}{a}=\frac{d}{c}$

$b^2=ac$ (i)

$c^2=bd$ (ii)

$bc=ad$ (iii)

(i) $a-b,b-c,c-d$ are in G.P.

Solution:

If $a-b,b-c,c-d$ are in G.P, then:

$$\frac{b-c}{a-b}=\frac{c-d}{b-c}$$

$$(b-c)^2=(a-b)(c-d)$$

L.H.S $=(b-c)^2$

$=b^2+c^2-2bc$

$=b^2+c^2-bc-bc$

$=b^2-bc-bc+c^2$

Using (i), (ii), (iii)

$=ac-bc-ad+bd$

$=c(a-b)-d(a-b)$

$=(a-b)(c-d)$

=R.H.S

Hence proved that $a-b,b-c,c-d$ are in G.P

(ii) $\mathbf{a^2-b^2, b^2-c^2, c^2-d^2}$ **are in G.P**

Solution:

If $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P, then

$$\frac{b^2-c^2}{a^2-b^2}=\frac{c^2-d^2}{b^2-c^2}$$

$$\left(b^2-c^2\right)^2=\left(a^2-b^2\right)\left(c^2-d^2\right)$$

L.H.S. $=\left(b^2-c^2\right)^2$

$=b^4+c^4-2b^2c^2$

$=\left(b^2\right)^2-b^2c^2-b^2c^2+\left(c^2\right)^2$

Using (i), (ii) and (iii)

$=\left(ac\right)^2-b^2c^2-\left(ad\right)^2+\left(bd\right)^2$

$=a^2c^2-b^2c^2-a^2d^2+b^2d^2$

$=c^2\left(a^2-b^2\right)-d^2\left(a^2-b^2\right)$

$=\left(c^2-d^2\right)\left(a^2-b^2\right)$

= R.H.S.

Hence proved that $a^2-b^2, b^2-c^2, c^2-d^2$ are in G.P

(iii) $\mathbf{a^2+b^2, b^2+c^2, c^2+d^2}$ **are in G.P**

Solution:

If $a^2+b^2, b^2+c^2, c^2+d^2$ are in G.P, then

$$\frac{b^2+c^2}{a^2+b^2}=\frac{c^2+d^2}{b^2+c^2}$$

$$\left(b^2+c^2\right)^2=\left(a^2+b^2\right)\left(c^2+d^2\right)$$

L.H.S $=\left(b^2+c^2\right)^2$

$=\left(b^2\right)^2+\left(c^2\right)^2+2b^2c^2$

$=\left(b^2\right)^2+\left(c^2\right)^2+b^2c^2+b^2c^2$

$=\left(ac\right)^2+\left(bd\right)^2+\left(ad\right)^2+b^2c^2$

$=a^2c^2+b^2c^2+a^2d^2+b^2d^2$

$=c^2\left(a^2+b^2\right)+d^2\left(a^2+b^2\right)$

$=\left(a^2+b^2\right)\left(c^2+d^2\right)$

= R.H.S

Hence proved that $a^2+b^2, b^2+c^2, c^2+d^2$ are in G.P

Q.8 Show that the reciprocals of the terms of the geometric sequence $a_1, a_1r^2, a_1r^4, \ldots$ form another geometric sequence.

Solution:

As $a_1, a_1r^2, a_1r^4, \ldots\ldots$ are in G.P

Now reciprocals of the terms of G.P are

$$\frac{1}{a_1}, \frac{1}{a_1r^2}, \frac{1}{a_1r^4}, \ldots$$

$$\text{Common ratio} = \frac{\frac{1}{a_1r^2}}{\frac{1}{a_1}}$$

$$= \frac{1}{a_1r^2} \times a_1$$

$$= \frac{1}{r^2}$$

$$\text{Common ratio} = \frac{\frac{1}{a_1r^4}}{\frac{1}{a_1r^2}}$$

$$= \frac{1}{a_1r^4} \times a_1r^2$$

$$= \frac{1}{r^2}$$

As common ratio is same, so it is proved that $\frac{1}{a_1}, \frac{1}{a_1r^2}, \frac{1}{a^1r^4}, \ldots\ldots$ are in G.P

Q.9 Find the *n*th term of the geometric sequence if: $\mathbf{\frac{a_5}{a_3} = \frac{4}{9}}$ and $\mathbf{a_2 = \frac{4}{9}}$

Solution:

$$a_2 = \frac{4}{9}$$

$$\Rightarrow a_1r = \frac{4}{9} \qquad \text{(i)}$$

$$\frac{a_5}{a_3} = \frac{4}{9}$$

$$\frac{a_1r^4}{a_1r^2} = \frac{4}{9} \quad \Rightarrow \quad r^2 = \frac{4}{9} \quad \Rightarrow r = \pm\frac{2}{3}$$

Put $r = \frac{2}{3}$ in (i)

$$a_1\left(\frac{2}{3}\right) = \frac{4}{9}$$

$$a_1 = \frac{4}{9} \times \frac{3}{2}$$

$a_1 = \frac{2}{3}$

Using Formula,

$a_n = a_1 r^{n-1}$

$a_n = \left(\frac{2}{3}\right)^1 \left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^{1+n-1} = \left(\frac{2}{3}\right)^n$

Put $r = -\frac{2}{3}$ in (i)

$a_1 \left(\frac{-2}{3}\right) = \frac{4}{9}$

$a_1 = \frac{4}{9}\left(\frac{-3}{2}\right)$

$a_1 = -\frac{2}{3}$

Using Formula,

$$a_n = a_1 r^{n-1}$$

$$= \left(\frac{-2}{3}\right)^1 \left(\frac{-2}{3}\right)^{n-1} = \left(\frac{-2}{3}\right)^{1+n-1} = \left(\frac{-2}{3}\right)^n$$

$$a_n = (-1)^n \left(\frac{2}{3}\right)^n$$

Q.10 Find three consecutive numbers in G.P whose sum is 26 and their product is 216.

Solution:

Let $\frac{a}{r}$, a, ar be the three consecutive numbers of G.P.

Their product $\frac{a}{r} \times a \times ar = 216$

$a^3 = 216$

$(a)^3 = (6)^3$

$a = 6$

And sum $\frac{a}{r} + a + ar = 26$

Put $a = 6$

$\frac{6}{r} + 6 + 6r = 26$

$$\frac{6}{r}+6r=26-6$$

$$\frac{6+6r^2}{r}=20$$

$$6+6r^2=20r$$

$$6r^2-20r+6=0$$

$$3r^2-10r+3=0$$

$$3r^2-9r-r+3=0$$

$$3r(r-3)-1(r-3)=0$$

$$(r-3)(3r-1)=0$$

Either $r-3=0$

$r=3$

When $a=6$ and $r=3$, the numbers are:

$$\frac{a}{r},a,ar$$

$$\frac{6}{3},6,6\times3$$

$$2,6,18$$

or $3r-1=0$

$$r=\frac{1}{3}$$

When $a=6$ and $r=\frac{1}{3}$, the numbers are:

$$\frac{a}{r},a,ar$$

$$\frac{6}{\left(\frac{1}{3}\right)},6,6\left(\frac{1}{3}\right)$$

$$6\times3,6,2$$

$$18,6,2$$

So, the required numbers are $2,6,18$ or $18,6,2$.

Q.11 If the sum of four consecutive terms of a G.P is 80 and A.M of the second and the fourth of them is 30. Find the terms.

Solution:

Let $a_1, a_1r, a_1r^2, a_1r^3$ are four consecutive terms of G.P.

Their Sum $a_1+a_1r+a_1r^2+a_1r^3=80$

$$a_1\left(1+r+r^2+r^3\right)=80$$

$$a_1\left[1(1+r)+r^2(1+r)\right]=80$$

$$a_1(1+r)\left(1+r^2\right)=80$$

$$a_1\left(1+r^2\right)=\frac{80}{1+r} \qquad \text{(i)}$$

Also A.M. of 2nd and 4th term

$$\frac{a_2+a_4}{2}=30$$

$$\frac{a_1r+a_1r^3}{2}=30$$

$$a_1r\left(1+r^2\right)=60$$

$$a_1\left(1+r^2\right)=\frac{60}{r} \qquad \text{(ii)}$$

$$\frac{80}{1+r}=\frac{60}{r} \qquad \text{from equation (i)}$$

$$80r=60(1+r)$$

$$80r=60+60r$$

$$80r-60r=60$$

$$20r=60$$

$$r=3$$

Put $r=3$ in (ii)

$$a_1\left(1+3^2\right)=\frac{60}{3}$$

$$a_1(10)=20$$

$$a_1=2$$

When $a_1=2$ and $r=3$ then the required numbers are:

$$a_1, a_1r, a_1r^2, a_1r^3$$

$$2, 2\times3, 2\times3^2, 2\times3^3$$

$$2, 6, 2\times9, 2\times27$$

$$2, 6, 18, 54$$

Q.12 If $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in G.P. show that the common ratio is $\pm\sqrt{\frac{a}{c}}$

Solution:

If $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in G.P, then

$$r=\frac{\frac{1}{b}}{\frac{1}{a}}$$

$$r=\frac{1}{b}\times a$$

$$r=\frac{a}{b} \qquad \text{(i)}$$

$$r=\frac{\frac{1}{c}}{\frac{1}{b}}$$

$$r=\frac{1}{c}\times b$$

$$r=\frac{b}{c} \qquad \text{(ii)}$$

Multiply (i) and (ii)

$$r.r=\frac{a}{b}\times\frac{b}{c}$$

$$r^2=\frac{a}{c}$$

$$r=\pm\sqrt{\frac{a}{c}}$$

Q.13 If the number 1,4 and 3 are subtracted from three consecutive terms of an A.P. the resulting numbers are in G.P. Find the numbers if their sum is 21.

Solution:

Let $a-d,\ a,\ a+d$ be the three consecutive terms of an A.P

Their sum $\quad a-d+a+a+d=21$

$$3a=21$$

$$a=7$$

Again according to given condition:

$a-d-1,\ a-4,\ a+d-3$ are in G.P

Put $a=7$

$$7-d-1,\ 7-4,\ 7+d-3$$

$6-d, 3, 4+d$ are in G.P , so ratio will be same

$$\frac{3}{6-d}=\frac{4+d}{3}$$

$$(3)^2=(6-d)(4+d)$$

$$9 = 24 + 6d - 4d - d^2$$
$$9 = 24 + 2d - d^2$$
$$d^2 - 2d + 9 - 24 = 0$$
$$d^2 - 2d - 15 = 0$$
$$d^2 - 5d + 3d - 15 = 0$$
$$d(d-5) + 3(d-5) = 0$$
$$(d+3)(d-5) = 0$$

Either

$d + 3 = 0$

$d = -3$

When $d = -3$ and $a = 7$, the numbers are:

$a-d, a, a+d$

$7-(-3), 7, 7-3$

$7+3, 7, 4$

$10, 7, 4$

Or

$d - 5 = 0$

$d = 5$

When $d = 5$ and $a = 7$, the numbers are:

$a-d, a, a+d$

$7-5, 7, 7+5$

$2, 7, 12$

So the required numbers are 10,7,4 or 2,7,12

Q.14 If three consecutive numbers in A.P are increased by 1,4,15 respectively, the resulting numbers are in G.P. Find the original numbers if their sum is 6.

Solution:

Let $a-d,\ a,\ a+d$ be the three consecutive numbers of an A.P.

Their sum $a - d + a + a + d = 6$

$3a = 6$

$a = 2$

Again according to the given condition:

$a-d+1, a+4, a+d+15$ are in G.P

Put $a = 2$

$2-d+1, 2+4, 2+d+15$

$3-d, 6, 17+d$ are in G.P so ratio will be same

$$\frac{6}{3-d} = \frac{17+d}{6}$$

$$(6)^2 = (3-d)(17+d)$$

$36 = 51 + 3d - 17d - d^2$

$36 = 51 - 14d - d^2$

$d^2 + 14d + 36 - 51 = 0$

$d^2 + 14d - 15 = 0$

$d^2 + 15d - d - 15 = 0$

$d(d + 15) - 1(d + 15) = 0$

$(d - 1)(d + 15) = 0$

Either

$d - 1 = 0$

$d = 1$

When $a = 2$ and $d = 1$, the numbers are:

$a - d, a, a + d$

$2 - 1, 2, 2 + 1$

$1, 2, 3$

Or

$d + 15 = 0$

$d = -15$

When $a = 2$ and $d = -15$, the numbers are:

$a - d, a, a + d$

$2 - (-15), 2, 2 + (-15)$

$2 + 15, 2, 2 - 15$

$17, 2, -13$

So the required numbers are 1,2,3 or $17, 2, -13$