# CHAPTER 9

## FUNDAMENTALS OF TRIGONOMETRY

**Introduction:**

**1.** Word trigonometry is made by combining three Greek words

Trei $\rightarrow$ three

Goni $\rightarrow$ angles

Metron $\rightarrow$ measurement

Which means measurement of triangle.

> **SHORT QUESTION**
>
> ***Define trigonometry? RWP 2023, SWL 2023***

**2.** It is an important branch of mathematics and base of calculus.

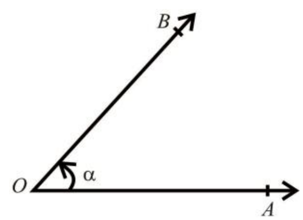
**3.** It is used in Business, Engineering, Surveying, Navigation, Astronomy, Physical and Social Sciences.

**Concept of an Angle:**

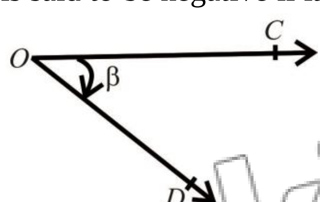
Two rays with a common starting point form an angle. One of the rays of angle is called initial side and other as terminal side.

An angle may be negative or positive due to direction of rotation.

An angle is said to be positive if its direction is anti-clock wise (counter clockwise)




An angle is said to be negative if its direction is clock wise.




Common starting point is called **vertex** of angle.

> **SHORT QUESTION**
>
> ***Define degree and radian? SGD* 2022**

Usually angle is denoted by $\alpha, \beta, \gamma, \theta, \varphi$ etc.

**Systems of Measure of Angles:**

**1.** Sexagesimal System.

**2.** Circular System.

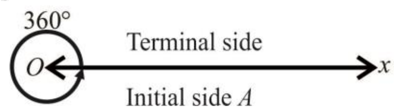
**1.** **Sexagesimal System** $(D^\circ M' S'')$.

In this system angle is measured in degree, minutes and seconds.

**Degree:**

If we divide the circumference of a circle into 360 equal arcs then central angle formed by each arc is called degree.

If the initial side rotates in anti-clock wise direction in such a way that it coincides with itself, the angle so formed is said to be of $360^\circ$.




So

One rotation (anti-clock wise) = $360^\circ$

$\frac{1}{2}$ rotation (anti-clock wise) = $180^\circ$ (Straight angle)

$\frac{1}{4}$ rotation (anti-clock wise) = $90^\circ$ (Right angle)

Also 1 minutes is equal to $60^{th}$ part of $1^\circ$

$$1' = \left(\frac{1}{60}\right)^\circ \Rightarrow 1^\circ = 60'$$

1 second is equal to $60^{th}$ part of 1 minute

$$1'' = \left(\frac{1}{60}\right)' \Rightarrow 1' = 60''$$

$$1'' = \left(\frac{1}{3600}\right)^\circ \Rightarrow 1^\circ = 3600''$$

**2.** **Circular System:**

In this system angle is measured in radians.

Radian is the measure of angle subtended at the centre of circle by an arc whose length is equal to its radius.

**Relation between the Length of an arc of a Circle and the Circular of its Central Angle:**

$$\ell = r\theta$$

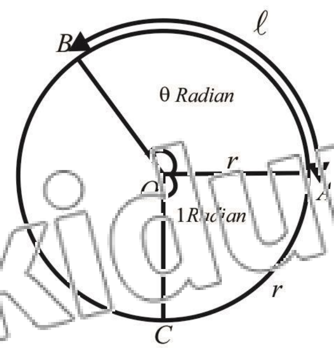
where $\ell$ is arc length $r$ is radius of circle $\theta$ is angle in radians

**Proof:**

Consider a circle with centre 'O' and radius '$r$'.

Suppose that $AB = \ell$ and central angle $m\angle AOB = \theta$ radian

Take an arc AC of length = r




By definition $m\angle AOC = 1$ radian
We know that from elementary geometry that measures of central angles of the arcs of a circle are proportional to the lengths of their arcs.

$$\frac{m\angle AOB}{m\angle AOC} = \frac{mAB}{mAC}$$

$$\frac{\theta\, rad}{1\, rad} = \frac{\ell}{r}$$

$$\theta = \frac{\ell}{r}$$

$$\Rightarrow \qquad \ell = r\theta$$

**Remember:**

In $\theta = \frac{\ell}{r}$ , $\ell$ and $r$ have same units so $\theta$ is unit-less i.e. a real number.

**Relationship between Degree and Radians:**

We know that circumference of a circle with radius $r = 2\pi r$
Let $\ell = 2\pi r$
and angle formed by one complete revolution $= \theta$ rad
So

$$\theta = \frac{\ell}{r}$$

$$\theta = \frac{2\pi r}{r}$$

$$\theta = 2\pi \ radians$$

we can write

$$2\pi \ radian = 360^o$$

$$\boxed{\pi \ rad = 180^\circ}$$

**Deductions:**

**(i)**

$$1\, rad = \frac{180^o}{\pi}$$

$$1 rad \approx \frac{180^o}{3.1416}$$

$$\boxed{1\, rad \approx 57.296^o}$$

**(ii)**

$$180^o = \pi \ rad$$

$$1^o = \frac{\pi}{180} \ rad$$

$$= \frac{3.1416}{180} \ rad$$

$$\boxed{1^\circ \approx 0.01745\, rad}$$

## EXERCISE 9.1

**Q.1 Explain the following measure of angles in radians**

**(i) 30°**

**Solution:** As $1^\circ = \frac{\pi}{180}$ radians

So $30^\circ = 30 \times \frac{\pi}{180}$

$= \frac{\pi}{6}$ rad.

**(ii) 45°**

**Solution:** As $1^\circ = \frac{\pi}{180}$ radians

So $45^\circ = 45 \times \frac{\pi}{180}$

$= \frac{\pi}{4}$ rad.

**(iii) 60°**

**Solution:** As $1^\circ = \frac{\pi}{180}$ radians

So $60^\circ = 60 \times \frac{\pi}{180}$

$= \frac{\pi}{3}$ rad.

**(iv) 75°**

**Solution:** As $1^\circ = \frac{\pi}{180}$ radians

So $75^\circ = 75 \times \frac{\pi}{180}$

$= \frac{5\pi}{12}$ rad.

**(v) 90°**

**Solution:** As $1^\circ = \frac{\pi}{180}$ radians

So $90^\circ = 90 \times \frac{\pi}{180}$

$= \frac{\pi}{2}$ rad.

**(vi) 105°**

**Solution:** As $1^\circ = \frac{\pi}{180}$ radians

So $105^\circ = 105 \times \frac{\pi}{180}$

$= \frac{7\pi}{12}$ rad.

**(vii) 120°**

**Solution:** As $1^\circ = \frac{\pi}{180}$ radians

So $120^\circ = 120 \times \frac{\pi}{180}$

$= \frac{2\pi}{3}$ rad.

**(viii) 135°**

**Solution:** As $1^\circ = \frac{\pi}{180}$ radians

So $135^\circ = 135 \times \frac{\pi}{180}$

$= \frac{3\pi}{4}$ rad.

**(ix) 150°**

**Solution:** As $1^\circ = \frac{\pi}{180}$ radians

So $150^\circ = 150 \times \frac{\pi}{180}$

$= \frac{5\pi}{6}$ rad.

**(x) 10°15′**

**Solution:** $10^\circ 15' = \left(10 + \frac{15}{60}\right)^\circ$

$= \left(10 + \frac{1}{4}\right)^\circ$

$= \left(\frac{40+1}{4}\right)^\circ$

$= \left(\frac{41}{4}\right)^\circ$

As $1^\circ = \frac{\pi}{180}$ radians

So $\left(\frac{41}{4}\right)^\circ = \frac{41}{4} \times \frac{\pi}{180}$

$= \frac{41\pi}{720}$ rad.

**(xi) 35º 20′**

**Solution:** $35°20' = \left(35 + \frac{20}{60}\right)^\circ$

$= \left(35 + \frac{1}{3}\right)^\circ$

$= \left(\frac{105+1}{3}\right)^\circ$

$= \left(\frac{106}{3}\right)^\circ$

As $1° = \frac{\pi}{180}$ radians

So $\left(\frac{106}{3}\right)^\circ = \frac{106}{3} \times \frac{\pi}{180}$

$= \frac{53\pi}{270}$ rad.

**(xii) 75º 6′30″** ***LHR* 2022**

**Solution:**

$75°6'30'' = \left(75 + \frac{6}{60} + \frac{30}{3600}\right)^\circ$

$= \left(75 + \frac{1}{10} + \frac{1}{120}\right)^\circ$

$= \left(\frac{9000+12+1}{120}\right)^\circ$

$= \left(\frac{9013}{120}\right)^\circ$

As $1° = \frac{\pi}{180}$ radians

So $\left(\frac{9013}{120}\right)^\circ = \frac{9013}{120} \times \frac{\pi}{180}$

$= \frac{9013\pi}{21600}$ rad.

**(xiii) 120′40″**

**Solution:**

$120'40'' = \left(\frac{120}{60} + \frac{40}{3600}\right)^\circ$

$= \left(2 + \frac{1}{90}\right)^\circ$

$= \left(\frac{180+1}{90}\right)^\circ$

$= \left(\frac{181}{90}\right)^\circ$

As $1° = \frac{\pi}{180}$ radians

So $\left(\frac{181}{90}\right)^\circ = \frac{181}{90} \times \frac{\pi}{180}$

$= \frac{181\pi}{16200}$ rad.

**(xiv) 154º 20″**

**Solution:**

$154°20'' = \left(154 + \frac{20}{3600}\right)^\circ$

$= \left(154 + \frac{1}{180}\right)^\circ$

$= \left(\frac{27720+1}{180}\right)^\circ$

$= \left(\frac{27721}{180}\right)^\circ$

As $1° = \frac{\pi}{180}$ radians

So $\left(\frac{27721}{180}\right)^\circ = \frac{27721}{180} \times \frac{\pi}{180}$

$= \frac{27721\pi}{32400}$ rad.

**(xv) 0º**

**Solution:** As $1° = \frac{\pi}{180}$ radians

So $0° = 0 \times \frac{\pi}{180}$ radian

$= 0$ rad.

**(xvi) 3″**

**Solution:** $3'' = \left(\frac{3}{3600}\right)^\circ$

$= \left(\frac{1}{1200}\right)^\circ$

As $1° = \frac{\pi}{180}$ radians

So $\left(\frac{1}{1200}\right)^\circ = \frac{1}{1200} \times \frac{\pi}{180}$

$= \frac{\pi}{216000}$ rad.

**Q.2 Convert the following into the measures of sexagesimal system.**

**(i)** $\frac{\pi}{8}$

**Solution:**

As 1 radian $= \left(\frac{180}{\pi}\right)^\circ$

So $\frac{\pi}{8} = \frac{\pi}{8} \times \left(\frac{180}{\pi}\right)^\circ$

$= (22.5)^\circ$

$= 22^\circ 30'$

**(ii)** $\frac{\pi}{6}$

**Solution:**

As 1 radian $= \left(\frac{180}{\pi}\right)^\circ$

So $\frac{\pi}{6} = \frac{\pi}{6} \times \left(\frac{180}{\pi}\right)^\circ$

$= 30^\circ$

**(iii)** $\frac{\pi}{4}$

**Solution:**

As 1 radian $= \left(\frac{180}{\pi}\right)^\circ$

So $\frac{\pi}{4} = \frac{\pi}{4} \times \left(\frac{180}{\pi}\right)^\circ$

$= 45^\circ$

**(iv)** $\frac{\pi}{3}$

**Solution:**

As 1 radian $= \left(\frac{180}{\pi}\right)^\circ$

So $\frac{\pi}{3} = \frac{\pi}{3} \times \left(\frac{180}{\pi}\right)^\circ$

$= 60^\circ$

**SHORT QUESTION**

***Convert 3 radian into degree. SWL 2023***

**(v)** $\frac{\pi}{2}$

**Solution:**

As 1 radian $= \left(\frac{180}{\pi}\right)^\circ$

So $\frac{\pi}{2} = \frac{\pi}{2} \times \left(\frac{180}{\pi}\right)^\circ$

$= 90^\circ$

**(vi)** $\frac{2\pi}{3}$

**Solution:**

As 1 radian $= \left(\frac{180}{\pi}\right)^\circ$

So $\frac{2\pi}{3} = \frac{2\pi}{3} \times \left(\frac{180}{\pi}\right)^\circ$

$= 120^\circ$

**(vii)** $\frac{3\pi}{4}$

**Solution:**

As 1 radian $= \left(\frac{180}{\pi}\right)^\circ$

So $\frac{3\pi}{4} = \frac{3\pi}{4} \times \left(\frac{180}{\pi}\right)^\circ$

$= 135^\circ$

**(viii)** $\frac{5\pi}{6}$

**Solution:**

As 1 radian $= \left(\frac{180}{\pi}\right)^\circ$

So $\frac{5\pi}{6}=\frac{5\pi}{6}\times\left(\frac{180}{\pi}\right)^\circ$

$=150^\circ$

**(ix)** $\frac{7\pi}{12}$

**Solution:**

As 1 radian $=\left(\frac{180}{\pi}\right)^\circ$

So $\frac{7\pi}{12}=\frac{7\pi}{12}\times\left(\frac{180}{\pi}\right)^\circ$

$=105^\circ$

**(x)** $\frac{9\pi}{5}$

**Solution:**

As 1 radian $=\left(\frac{180}{\pi}\right)^\circ$

So $\frac{9\pi}{5}=\frac{9\pi}{5}\times\left(\frac{180}{\pi}\right)^\circ$

$=324^\circ$

**(xi)** $\frac{11\pi}{27}$

**Solution:**

As 1 radian $=\left(\frac{180}{\pi}\right)^\circ$

So $\frac{11\pi}{27}=\frac{11\pi}{27}\times\left(\frac{180}{\pi}\right)^\circ$

$=\left(\frac{220}{3}\right)^\circ$

$=\left(73\frac{1}{3}\right)^\circ$

$=73^\circ\left(\frac{1}{3}\times60\right)'$

$=73^\circ20'$

**(xii)** $\frac{13\pi}{16}$

**Solution:**

As 1 radian $=\left(\frac{180}{\pi}\right)^\circ$

So $\frac{13\pi}{16}=\frac{13\pi}{16}\times\left(\frac{180}{\pi}\right)^\circ$

$=\left(\frac{2340}{16}\right)^\circ$

$=\left(146\frac{1}{4}\right)^\circ$

$=146^\circ\left(\frac{1}{4}\times60\right)'$

$=146^\circ15'$

**(xiii)** $\frac{17\pi}{24}$

**Solution:**

As 1 radian $=\left(\frac{180}{\pi}\right)^\circ$

$\frac{17\pi}{24}=\frac{17\pi}{24}\times\left(\frac{180}{\pi}\right)^\circ$

$=\left(\frac{255}{2}\right)^\circ$

$=\left(127\frac{1}{2}\right)^\circ$

$=127^\circ\left(\frac{1}{2}\times60\right)'$

$=127^\circ30'$

**(xiv)** $\frac{25\pi}{36}$

**Solution:**

As 1 radian $=\left(\frac{180}{\pi}\right)^\circ$

So $\frac{25\pi}{36}=\frac{25\pi}{36}\times\left(\frac{180}{\pi}\right)^\circ$

$=125^\circ$

**(xv)** $\frac{19\pi}{32}$

**Solution:**

As 1 radian $=\left(\frac{180}{\pi}\right)^\circ$

So $\frac{19\pi}{32}=\frac{19\pi}{32}\times\left(\frac{180}{\pi}\right)^\circ$

$=\left(106\frac{7}{8}\right)^\circ$

$=106^\circ\left(\frac{7}{8}\times 60\right)'$

$=106^\circ\left(\frac{105}{2}\right)'$

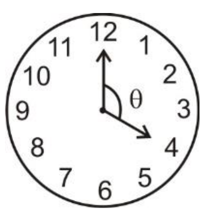
$=106^\circ\left(52\frac{1}{2}\right)'$

$=106^\circ 52'\left(\frac{1}{2}\times 60\right)''$

$=106^\circ 52' 30''$

**Q.3 What is the circular measure of the angle between the hands of a clock at 4 O' clock?** ***SGD* 2021**

**Solution:**




By fixing minute hand at 12

Angle measured in 12 hours $=2\pi$ rad

Angle measured in 1 hour $=\frac{2\pi}{12}=\frac{\pi}{6}$

Angle measured at 4 O' clock $=4\times\frac{\pi}{6}=\frac{2\pi}{3}$

**Q.4 Find θ when**

**(i)** ***l*** **= 1.5cm,** ***r*** **= 2.5cm** ***FSD* 2021, *RWP* 2023**

**Solution:**

$\ell=1.5cm$, $r=2.5cm$

Using $\theta=\frac{\ell}{r}$

$=\frac{1.5}{2.5}$

$=0.6$ radians.

**(ii)** ***l*** **= 3.2m ,** ***r*** **= 2m** ***FSD* 2022**

**Solution:**

$\ell=3.2m$, $r=2m$

Using $\theta=\frac{\ell}{r}$

$=\frac{3.2}{2}$

$=1.6$ rad.

**Q.5 Find *l* when**

**(i) θ = π,** ***r*** **= 6cm**

**Solution:**

$\theta=\pi$, $r=6cm$

Using $\theta=\frac{\ell}{r}$

$\ell=r\theta$

$=6\pi$

$=6(3.1416)$

$=18.86cm$

**(ii) θ = 65°20′** ***r*** **= 18mm** ***GRW* 2022, *BWP* 2022, *FSD* 2023**

**Solution:**

$r=18mm$

$\theta=65^\circ 20'$

$=\left(65+\frac{20}{60}\right)^\circ$

$=\left(65+\frac{1}{3}\right)^\circ$

$$=\left(\frac{195+1}{3}\right)^{\circ}$$

$$=\left(\frac{196}{3}\right)^{\circ}$$

$$=\frac{196}{3}\times\frac{\pi}{180}$$

$$=\frac{49\pi}{135}$$

Using $\theta=\frac{\ell}{r}$

$$\ell=r\theta$$

$$=(18)\left(\frac{49\pi}{135}\right)$$

$$\ell\approx 20.52mm$$

**Q.6 Find *r* when**

**(i)** $l = 5cm,\ \theta=\frac{1}{2}$ radian

***LHR* 2022, *MTN* 2022**

**Solution:**

$$\ell=5cm,\ \theta=\frac{1}{2}\text{ radian}$$

Using $\theta=\frac{\ell}{r}$

$$r=\frac{l}{\theta}$$

$$r=\frac{5}{\frac{1}{2}}$$

$$=10cm$$

**(ii)** $l = 56\text{cm},\quad \theta = 45^{\circ}$

***GRW* 2022, *MTN* 2022, *RWP* 2022**

**Solution:**

$$\ell=56cm$$

$$\theta=45^{\circ}$$

$$=45\times\frac{\pi}{180}$$

$$=\frac{\pi}{4}$$

Using $\theta=\frac{\ell}{r}$

$$r=\frac{\ell}{\theta}$$

$$=\frac{56}{\frac{\pi}{4}}$$

$$=\frac{224}{\pi}$$

$$r=71.30cm$$

**Q.7 What is the length of the arc intercepted on a circle of radius 14cm by the arms of central angle of 45°.** ***GRW* 2023, *MTN* 2023**

**Solution:**

$$r=14cm,\ \ell=?$$

$$\theta=45^{\circ}$$

$$=45\times\frac{\pi}{180}$$

$$=\frac{\pi}{4}$$

Using $\theta=\frac{\ell}{r}$

$$\ell=r\theta$$

$$=14\times\frac{\pi}{4}$$

$$\ell\approx 11cm$$

**Q.8 Find the radius of the circle, in which the arms of a central angle of measure 1 radian cut off an arc of length 35cm.** ***DGK* 2022**

**Solution:**

$$\ell=35cm,\ r=?$$

$$\theta=1\text{ rad}$$

Using

$$\theta = \frac{\ell}{r}$$
$$r = \frac{\ell}{\theta}$$
$$r = \frac{35}{1}$$
$$r = 35cm$$

**Q.9 A railway train is running on a circular track of radius 500m at the rate of 30km/h. through what angle will it turn in 10 sec?**

***RWP* 2023, *BWP* 2023**

**Solution:**

$$r = 500m$$
$$\text{speed of train} = 30kmh^{-1}$$
$$= \frac{30 \times 1000}{3600} ms^{-1}$$
$$= \frac{25}{3} ms^{-1}$$

and

$$\ell = s = r\theta$$
$$\ell = \frac{25}{3} \times 10$$
$$= \frac{250}{3} m$$

Using

$$\theta = \frac{\ell}{r}$$
$$\theta = \frac{\frac{250}{3}}{500}$$
$$= \frac{1}{6} \text{rad}$$

**Q.10 A horse is tethered to a peg by a rope of $9m$ length and it can move in a circle with the peg as centre. If the horse moves along the circumference of the circle, keeping the rope tight how far will it have gone when the rope has turned through angle of $70°$?**

**Solution:**

Here

$$\theta = 70°$$
$$= 70 \times \frac{\pi}{180}$$
$$= \frac{7\pi}{18}$$

And

$$r = 9m$$

Using

$$\theta = \frac{\ell}{r}$$
$$\ell = r\theta$$
$$= (9)\left(\frac{7\pi}{18}\right)$$
$$= \frac{7\pi}{2}$$
$$\ell \approx 11m$$

**Q.11 The pendulum of a clock is 20cm long and it swings through an angle of 20° each second. How far does tip of the pendulum move in 1 sec?** ***SWL* 2022**

**Solution:**

Here

$$\theta = 20°$$
$$= 20 \times \frac{\pi}{180}$$
$$= \frac{\pi}{9}$$

And

$$r = 20cm$$

Using

$\theta = \frac{\ell}{r}$

$\ell = r\theta$

$= (20)\left(\frac{\pi}{9}\right)$

$\ell \approx 6.98cm$

**Q.12 Assuming the average distance of the earth from the sun to be $148 \times 10^6$ km and the angle subtended by the sun at the eye of a person on the earth is of measure $9.3 \times 10^{-3}$ radians. Find the diameter of the sun.**

**Solution:**

Let O be the eye of observer on earth, r be the distance between earth and sun. $\theta$ is the angle subtended by the sun on the earth.


Here

$r = 148 \times 10^6 km$

$\theta = 9.3 \times 10^{-3}$ rad

Using

$\theta = \frac{\ell}{r}$

So

$\ell = r\theta$

$\ell = (148 \times 10^6)(9.3 \times 10^{-3})$

$= 1.37 \times 10^6 km$

But here $\ell \approx d$ (diameter of the sun)

**Q.13 A circular wire of radius 6cm is cut straightened and then bent so as to lie along the circumference of a hoop of radius 24cm. Find the measure of the angle which it subtends at the centre of the hoop.**

**Solution:**

Here

$r = 6$

Circumference of hoop:

$C = \ell = 2\pi r = 2\pi(6) = 12\pi$

Radius of hoop

$r = 24cm$

Using

$\theta = \frac{\ell}{r}$

$= \frac{12\pi}{24}$

$\theta = \frac{\pi}{2}$

**Q.14 Show that the area of a sector of a circular region of radius *r* is $\frac{1}{2}r^2\theta$, where $\theta$ is the circular measure of the central angle of sector.** ***DGK* 2023**

**Solution:**

From elementary geometry, we know that

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{angle subtended by sector}}{\text{angle subtended by circle}}$$

$$\frac{\text{area of sector}}{\pi r^2}=\frac{\theta}{2\pi}$$

$$\text{area of sector}=\frac{1}{2}r^2\theta$$

**Q.15 Two cities A and B lie on the equator such that their longitudes are 45°E and 25°W respectively. Find the distance between the two cities, taking radius of earth as 6400km.** ***LHR* 2021**

**Solution:**

Here Radius of earth, $r=6400Km$

Angel between city $A$ and $B$: $\theta=45^\circ+25^\circ$

$$\theta=70^\circ$$

$$=70\times\frac{\pi}{180}\text{rad}$$

$$=\frac{7\pi}{18}\text{rad}$$

Let $\ell$ be the distance between city $A$ and $B$

Using $\theta=\dfrac{\ell}{r}$

$$\ell=r\theta$$

$$=6400\times\frac{7\pi}{18}$$

$$\ell\approx 7819km$$

But $\ell$ is an approximate distance between two cities, so $|AB|\approx 7819km$

**Q.16 The moon subtends an angle of 0.5° at the eye of an observer on earth. The distance of the moon from earth is $3.844\times10^5$km. What is the length of the diameter of the moon?**

**Solution:**


Let O be the eye of observer on earth, r be the distance between earth and moon.

$\theta$ be the angle subtended by moon on the earth.

Here

$$\theta=0.5^\circ$$

$$=\frac{1}{2}\times\frac{\pi}{180}$$

$$\theta=\frac{\pi}{360}$$

Using $\theta=\dfrac{\ell}{r}$

$$\ell = r\theta$$

$$= 3.844 \times 10^5 \times \frac{\pi}{360}$$

$$\ell \approx 3354.31 km$$

**Q.17 The angle subtended by the earth at the eye of a space man, landed on the moon is $1°54'$. The radius of the earth is 6400km. Find the approximate distance between the moon and earth.**

**Solution:**

Let O be the eye of observer on moon, $\theta$ be the angle subtended by earth on moon.

$l$ be the diameter of earth and r be the distance between moon and earth.

Here $\ell$ = diameter of earth $= 2 \times 6400 = 12800 km$


$$\theta = \left(1 + \frac{54}{60}\right)^{\circ}$$

$$= \left(1 + \frac{9}{10}\right)^{\circ}$$

$$= \left(\frac{19}{10}\right)^{\circ}$$

$$\theta = \frac{19}{10} \times \frac{\pi}{180} \text{ rad}$$

$$= \frac{19\pi}{1800}$$

Using $\theta = \frac{\ell}{r}$

$$r = \frac{\ell}{\theta}$$

$$r \approx \frac{12800}{\frac{19\pi}{1800}}$$

$$r \approx 385993 km$$

So the distance between earth and moon is approximately $3.85993 \times 10^5$ km.

## General Angle (Co-terminal Angles):

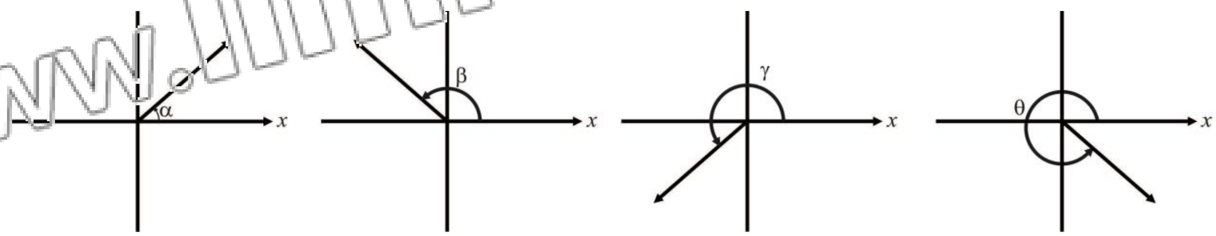
Angles with the same initial and terminal sides are called co-terminal angles.

General angle is $\theta + 2k\pi, k \in Z$ . Where $0 \le \theta \le 2\pi$

## Angle in Standard Position:

An angle is said to be in standard position if its vertex lies at the origin of a rectangular coordinate system and its initial side along positive $x-axis$ .

An angle in standard position is said to lie in a quadrant due to its terminal side as




Angle $\alpha$ lies in I Quadrant as its terminal side lies in I Quadrant

Angle $\beta$ lies in II Quadrant as its terminal side lies in II Quadrant

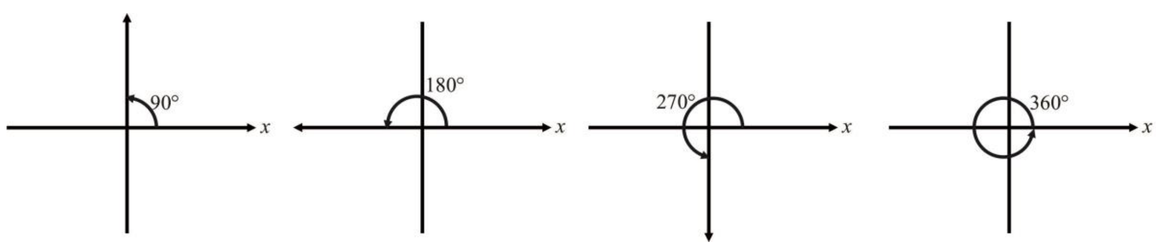
Angle $\gamma$ lies in III Quadrant as its terminal side lies in III Quadrant

Angle $\theta$ lies in IV Quadrant as its terminal side lies in IV Quadrant

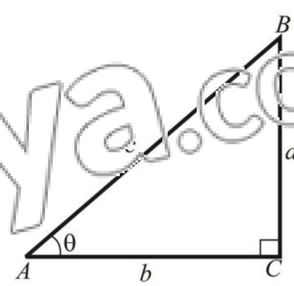
## Quadrantal angle:

An angle in standard position is said to be quadrantal angle if its terminal side lies along $x-axis$ or $y-axis$ .

Such as $0°, 90°, 180°, 270°, 360°, 450°$......




## Trigonometric Functions:




If $m\angle BAC = \theta$ is fixed angle in right angle triangle $ABC$

And $m\overline{AB} = c \quad m\overline{BC} = a \quad m\overline{AC} = b$

Six ratios can be formed by using all three sides of a triangle as $\frac{a}{b}, \frac{a}{c}, \frac{b}{c}, \frac{b}{a}, \frac{c}{a}, \frac{c}{b}$

These ratios are called **trigonometric functions** of angle $\theta$ and are defined as below

$\sin\theta = \frac{a}{c} = \frac{\text{opp}}{\text{hyp}}$ , $\text{cosec}\,\theta = \frac{c}{a} = \frac{\text{hyp}}{\text{opp}}$

$\cos\theta = \frac{b}{c} = \frac{\text{adj}}{\text{hyp}}$ , $\sec\theta = \frac{c}{b} = \frac{\text{hyp}}{\text{adj}}$

$\tan\theta = \frac{a}{b} = \frac{\text{opp}}{\text{adj}}$ , $\cot\theta = \frac{b}{a} = \frac{\text{adj}}{\text{opp}}$

We also have relations between these six trigonometric function as follows:

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$ ; $\sin\theta = \frac{1}{\text{cosec}\,\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ ; $\cos\theta = \frac{1}{\sec\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ ; $\tan\theta = \frac{1}{\cot\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$ ; $\tan\theta = \frac{\sin\theta}{\cos\theta}$

**Fundamental Identities:**

**(1)** $\mathbf{\sin^2\theta + \cos^2\theta = 1}$

**(2)** $\mathbf{1 + \tan^2\theta = \sec^2\theta}$

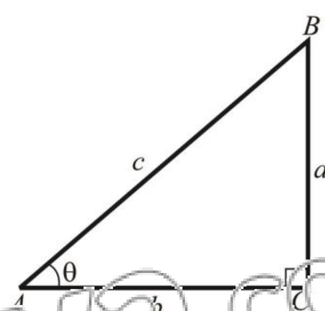
**(3)** $\mathbf{1 + \cot^2\theta = cosec^2\theta}$

**Proof:**

**(1)** Refer to right angle triangle $ABC$

by Pythagoras theorem $a^2 + b^2 = c^2$ (i)

divide by "$c^2$" both sides of equation (i)




$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$$

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

Hence $\sin^2\theta + \cos^2\theta = 1$

**(2)** Dividing by '$b^2$' both sides of equation (i)

$$\frac{a^2}{b^2} + \frac{b^2}{b^2} = \frac{c^2}{b^2}$$

$$\left(\frac{a}{b}\right)^2 + 1 = \left(\frac{c}{b}\right)^2$$

$\tan^2\theta + 1 = \sec^2\theta$

Hence $1 + \tan^2\theta = \sec^2\theta$

**(3)** Dividing by '$a^2$' both sides of equation (i)

$$\frac{a^2}{a^2}+\frac{b^2}{a^2}=\frac{c^2}{a^2}$$

$$1+\left(\frac{b}{a}\right)^2=\left(\frac{c}{a}\right)^2$$

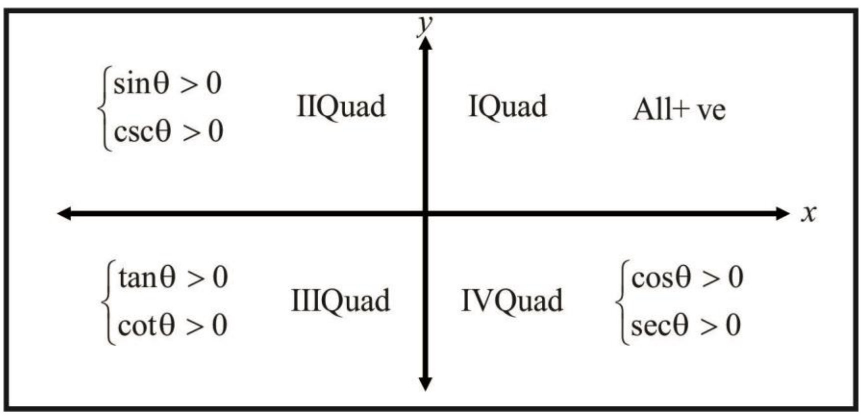
Hence $1+\cot^2\theta=\operatorname{cosec}^2\theta$

**Note:**

$(\sin\theta)^2=\sin^2\theta, (\cos\theta)^2=\cos^2\theta$ and $(\tan\theta)^2=\tan^2\theta$ etc.

**Signs of Trigonometric Functions:**

It $\theta$ is in standard position and not a quadrantal angle then it will lie in a quadrant and sign of trigonometric functions in different quadrants are shown in diagram




**Results:**

$\sin(-\theta)=-\sin\theta$ ; $\operatorname{cosec}(-\theta)=-\operatorname{cosec}\theta$

$\cos(-\theta)=\cos\theta$ ; $\sec(-\theta)=\sec\theta$

$\tan(-\theta)=-\tan\theta$ ; $\cot(-\theta)=-\cot\theta$

## EXERCISE 9.2

**Q.1 Find the signs of the following.**

**(i) $\sin 160°$**

**Solution:** Positive

$(\because 160°$ lies in IInd quadrant$)$

**(ii) $\cos 190°$**

**Solution:** Negative

$(\because 190°$ lies in IIIrd quadrant$)$

**(iii) $\tan 115°$**

**Solution:** Negative

$(\because 115°$ lies in IIrd quadrant$)$

**(iv) $\sec 245°$**

**Solution:** Negative

$(\because 245°$ lies in IIIrd quadrant$)$

**(v) $\cot 80°$**

**Solution:** Positive

$(\because 80°$ lies in Ist quadrant$)$

**(vi) $\operatorname{cosec} 297°$**

**Solution:** Negative

$(\because 297°$lies in IVth quadrant$)$

**Q.2 Fill up the blanks in the following**

**(i) $\sin(-310°) =$**

**Solution:** $\sin(-310°) = -\sin 310°$

**(ii) $\tan(-182°) =$**

**Solution:** $\tan(-182°) = -\tan 182°$

**(iii) $\cos(-75°) =$**

**Solution:** $\cos(-75°) = +\cos 75°$

**(iv) $\cot(-137°) =$**

**Solution:** $\cot(-137°) = -\cot 137°$

**(v) $\sec(-216°) =$**

**Solution:** $\sec(-216°) = +\sec 216°$

**(vi) $\operatorname{cosec}(-15°) =$**

**Solution:** $\operatorname{cosec}(-15°) = -\operatorname{cosec} 15°$

**Q.3 In which quadrant are the terminal arms of the angle lie when.**

**(i) $\sin\theta < 0$ and $\cos\theta > 0$**

**Solution:**

As $\sin\theta < 0 \Rightarrow \theta$ lies in quad III or IV

$\cos\theta > 0 \Rightarrow \theta$ lies in quad I or IV

Hence $\theta$ lies in quad IV

**(ii) $\cot\theta > 0$ and $\operatorname{cosec}\theta > 0$**

**Solution:**

As $\cot\theta > 0 \Rightarrow \theta$ lies in quad I or III

$\operatorname{cosec}\theta > 0 \Rightarrow \theta$ lies in quad I or II

Hence $\theta$ lies in quad I

**(iii) $\tan\theta < 0$ and $\cos\theta > 0$**

**Solution:**

As $\tan\theta < 0 \Rightarrow \theta$ lies in quad II or IV

$\cos\theta > 0 \Rightarrow \theta$ lies in quad I or IV

Hence $\theta$ lies in quad IV

**(iv) $\sec\theta < 0$ and $\sin\theta < 0$**

**Solution:**

As $\sec\theta < 0 \Rightarrow \theta$ lies in quad II or III

$\sin\theta < 0 \Rightarrow \theta$ lies in quad III or IV

Hence $\theta$ lies in quad III

**(v) $\cot\theta > 0$ and $\sin\theta < 0$**

**Solution:**

As $\cot\theta > 0 \Rightarrow \theta$ lies in quad I or III

$\sin\theta < 0 \Rightarrow \theta$ lies in quad III or IV

Hence $\theta$ lies in quad III

**(vi) $\cos\theta < 0$ and $\tan\theta < 0$**

**Solution:**

As $\cos\theta < 0 \Rightarrow \theta$ lies in quad II or III

$\tan\theta < 0 \Rightarrow \theta$ lies in quad II or IV

Hence $\theta$ lies in quad II

**Q.4 Find the values of the remaining trigonometric functions if,**

**(i) $\sin\theta = \frac{12}{13}$ and terminal arm of θ is in 1st quadrant.**

**Solution:**

Given that $\sin\theta = \frac{12}{13}$ so

$$\operatorname{cosec}\theta = \frac{13}{12}$$

Now $\cos^2\theta = 1 - \sin^2\theta$

$$= 1 - \left(\frac{12}{13}\right)^2$$

$$= 1 - \frac{144}{169}$$

$$= \frac{169 - 144}{169}$$

$$\cos^2\theta = \frac{25}{169}$$

$$\cos\theta = \pm\frac{5}{13}$$

But θ lies in 1st quadrant, so

$$\cos\theta = \frac{5}{13} \text{ and } \sec\theta = \frac{13}{5}$$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\tan\theta = \frac{\frac{12}{13}}{\frac{5}{13}}$$

$$\tan\theta = \frac{12}{5} \text{ and } \cot\theta = \frac{5}{12}$$

**(ii) $\cos\theta = \frac{9}{41}$ and terminal arm of the angle θ is in 4th quadrant**

**Solution:**

$$\cos\theta = \frac{9}{41} \text{ and } \sec\theta = \frac{41}{9}$$

Now $\sin^2\theta = 1 - \cos^2\theta$

$$= 1 - \left(\frac{9}{41}\right)^2$$

$$= 1 - \frac{81}{1681}$$

$$= \frac{1681 - 81}{1681}$$

$$\sin^2\theta = \frac{1600}{1681}$$

$$\sin\theta = \pm\frac{40}{41}$$

But θ lies in 4th quadrant,

$$\sin\theta = \frac{-40}{41} \text{ and } \operatorname{cosec}\theta = \frac{-41}{40}$$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$= \frac{-\frac{40}{41}}{\frac{9}{41}}$$

$$\tan\theta = \frac{-40}{9} \quad \text{and} \quad \cot\theta = \frac{-9}{40}$$

**Alternate Solution:**

$$\cos\theta = \frac{9}{41},\ \sec\theta = \frac{41}{9}$$

Consider a right triangle, by using Pythagoras theorem

$$x^2 + 9^2 = 41^2$$

$$x^2 = 1681 - 81$$

$$x^2 = 1600$$

$$x = 40$$

Since $\theta$ lies in 4th quadrant,

$\sin\theta = -\frac{40}{41}$, $\text{cosec}\,\theta = -\frac{41}{40}$

$\tan\theta = -\frac{40}{9}$, $\cot\theta = -\frac{9}{40}$

**(iii)** $\cos\theta = -\frac{\sqrt{3}}{2}$ **and the terminal arm of the angle is in quad. III**

**Solution:**

Given that $\cos\theta = -\frac{\sqrt{3}}{2}$ so

$\sec\theta = -\frac{2}{\sqrt{3}}$

Now $\sin^2\theta = 1 - \cos^2\theta$

$= 1 - \left(\frac{-\sqrt{3}}{2}\right)^2$

$= 1 - \frac{3}{4} = \frac{4-3}{4}$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

But θ lies in 3rd quadrant so

$\sin\theta = \frac{-1}{2}$ and $\text{cosec}\,\theta = -2$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$\tan\theta = \frac{1}{\sqrt{3}}$ and $\cot\theta = \sqrt{3}$

**Alternate Solution:**

$\cos\theta = -\frac{\sqrt{3}}{2}$, $\sec\theta = -\frac{2}{\sqrt{3}}$

Consider a right triangle

Using Pythagoras theorem

$x^2 + \left(\sqrt{3}\right)^2 = (2)^2$

$x^2 + 3 = 4$

$x^2 = 4 - 3 = 1$

$x = 1$

Since $\theta$ lies in 3rd quadrant

So

$\sin\theta = -\frac{1}{2}$, $\text{cosec}\,\theta = -2$

$\tan\theta = \frac{1}{\sqrt{3}}$, $\cot\theta = \sqrt{3}$

**(iv)** $\tan\theta = \frac{-1}{3}$ **and the terminal arm of the angle is in quad. II**

**Solution:**

Given that

$\tan\theta = \frac{-1}{3}$ so $\cot\theta = -3$

Now $\sec^2\theta = 1 + \tan^2\theta$

$= 1 + \left(\frac{-1}{3}\right)^2$

$= 1 + \frac{1}{9}$

$\sec^2\theta = \frac{10}{9}$

$\sec\theta = \pm\frac{\sqrt{10}}{3}$

But θ lies in 2nd quadrant, so

$\sec\theta = \frac{-\sqrt{10}}{3}$ and $\cos\theta = \frac{-3}{\sqrt{10}}$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin\theta = \tan\theta.\cot\theta$

$\sin\theta = \left(-\frac{1}{3}\right)\left(\frac{-3}{\sqrt{10}}\right)$

$\sin\theta = \frac{1}{\sqrt{10}}$

and $\operatorname{cosec}\theta = \sqrt{10}$

**(v) $\sin\theta = \frac{-1}{\sqrt{2}}$ and terminal arm of θ is not in 3rd quadrant.**

**Solution:**

Given that $\sin\theta = \frac{-1}{\sqrt{2}}$ and

$\operatorname{cosec}\theta = -\sqrt{2}$

Now $\cos^2\theta = 1 - \sin^2\theta$

$= 1 - \left(\frac{-1}{\sqrt{2}}\right)^2$

$= 1 - \frac{1}{2}$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$

As terminal arm of $\theta$ is not in quad-III

So $\theta$ lies in 4th quadrant so

$\cos\theta = \frac{1}{\sqrt{2}}$ and $\sec\theta = \sqrt{2}$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

$\tan\theta = -1$ and $\cot\theta = -1$

**Q.5 If $\cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in quadrant I, find the values of cosθ and cosecθ.**

**Solution:**

Give that $\cot\theta = \frac{15}{8}$ so $\tan\theta = \frac{8}{15}$

Now $\sec^2\theta = 1 + \tan^2\theta$

$= 1 + \left(\frac{8}{15}\right)^2$

$= 1 + \frac{64}{225}$

$= \frac{225+64}{225}$

$\sec^2\theta = \frac{289}{225}$

$\sec\theta = \pm\frac{17}{15}$

As terminal arm of $\theta$ is not in quad. I

So $\theta$ lies in 3rd quadrant, so

$\sec\theta = -\frac{17}{15}$ and $\cos\theta = \frac{-15}{17}$

and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin\theta = \tan\theta.\cos\theta$

$= \left(\frac{8}{15}\right)\left(\frac{-15}{17}\right)$

$\sin\theta = -\frac{8}{17}$

So $\operatorname{cosec}\theta = \frac{-17}{8}$

**Q.6 If $\operatorname{cosec}\theta = \dfrac{m^2+1}{2m}$ and $m>0, \left(0<\theta<\dfrac{\pi}{2}\right)$, find the values of remaining trigonometric ratios.**

**Solution:**

Given that $\operatorname{cosec}\theta = \dfrac{m^2+1}{2m}$ so

$$\sin\theta = \frac{2m}{1+m^2}$$

Now $\cos^2\theta = 1-\sin^2\theta$

$$= 1-\left(\frac{2m}{1+m^2}\right)^2$$

$$= 1-\frac{4m^2}{1+m^4+2m^2}$$

$$= \frac{1+m^4+2m^2-4m^2}{1+m^4+2m^2}$$

$$\cos^2\theta = \frac{1+m^4-2m^2}{1+m^4+2m^2}$$

$$\cos^2\theta = \frac{(1-m^2)^2}{(1+m^2)^2}$$

$$\cos\theta = \pm\frac{1-m^2}{1+m^2}$$

But θ lies in 1st quadrant, so

$$\cos\theta = \frac{1-m^2}{1+m^2} \text{ and } \sec\theta = \frac{1+m^2}{1-m^2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\frac{2m}{1+m^2}}{\frac{1-m^2}{1+m^2}}$$

$$\tan\theta = \frac{2m}{1-m^2} \text{ and } \cot\theta = \frac{1-m^2}{2m}$$

**Q.7 If $\tan\theta = \dfrac{1}{\sqrt{7}}$ and the terminal arm of θ is not in the 3rd quadrant, find the value of $\dfrac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta}$**

**Solution:**

Given that $\tan\theta = \dfrac{1}{\sqrt{7}}$

and θ does not lie in 3rd quadrant then it must lies in 1st quadrant.

Now $\sec^2\theta = 1+\tan^2\theta$

$$= 1+\left(\frac{1}{\sqrt{7}}\right)^2$$

$$= 1+\frac{1}{7}$$

$$\sec^2\theta = \frac{8}{7}$$

Also, $\operatorname{cosec}^2\theta = 1+\cot^2\theta$

$$= 1+\left(\sqrt{7}\right)^2$$

$$= 1+7$$

$$\operatorname{cosec}^2\theta = 8$$

Now $\dfrac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta} = \dfrac{8-\frac{8}{7}}{8+\frac{8}{7}}$

$$= \frac{\frac{56-8}{7}}{\frac{56+8}{7}}$$

$$= \frac{\frac{48}{7}}{\frac{64}{7}}$$

$$= \frac{48}{64}$$

$$= \frac{3}{4}$$

**Alternate Solution:**

$\tan\theta = \dfrac{1}{\sqrt{7}}$, $\cot\theta = \sqrt{7}$

As θ does not lie in 3rd quadrant, so it must lies in 1st quadrant.

Take $\dfrac{\operatorname{cosec}^2\theta-\sec^2\theta}{\operatorname{cosec}^2\theta+\sec^2\theta}$

$$= \frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$$

$$=\frac{7-\frac{1}{7}}{2+7+\frac{1}{7}}$$

$$=\frac{\frac{49-1}{7}}{9+\frac{1}{7}}$$

$$=\frac{\frac{48}{7}}{\frac{64}{7}}$$

$$=\frac{48}{64}$$

$$=\frac{3}{4}$$

**Q.8 If $\cot\theta=\frac{5}{2}$ and the terminal arm of θ is in the first quadrant, find the value of $\frac{3\sin\theta+4\cos\theta}{\cos\theta-\sin\theta}$**

**Solution:**

Given that $\cot\theta=\frac{5}{2}$ so $\tan\theta=\frac{2}{5}$

Now $\sec^2\theta=1+\tan^2\theta$

$$=1+\left(\frac{2}{5}\right)^2$$

$$=1+\frac{4}{25}$$

$$=\frac{25+4}{25}$$

$$\sec^2\theta=\frac{29}{25}$$

As $\theta$ lies in 1st quadrant

So $\sec\theta=\frac{\sqrt{29}}{5}$ and $\cos\theta=\frac{5}{\sqrt{29}}$

Now $\tan\theta=\frac{\sin\theta}{\cos\theta}$

$$\sin\theta=\tan\theta.\cos\theta$$

$$=\left(\frac{2}{5}\right)\left(\frac{5}{\sqrt{29}}\right)$$

$$\sin\theta=\frac{2}{\sqrt{29}}$$

So

$$\frac{3\sin\theta+4\cos\theta}{\cos\theta-\sin\theta}=\frac{3\left(\frac{2}{\sqrt{29}}\right)+4\left(\frac{5}{\sqrt{29}}\right)}{\frac{5}{\sqrt{29}}-\frac{2}{\sqrt{29}}}$$

$$=\frac{\frac{6+20}{\sqrt{29}}}{\frac{5-2}{\sqrt{29}}}$$

$$=\frac{\frac{26}{\sqrt{29}}}{\frac{3}{\sqrt{29}}}$$

$$=\frac{26}{3}$$

**Alternate Solution:**

$$\cot\theta=\frac{5}{2}$$

**Take** $\frac{3\sin\theta+4\cos\theta}{\cos\theta-\sin\theta}$

**Dividing up and down by sin$\theta$**

$$=\frac{\frac{3\sin\theta+4\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\sin\theta}}$$

$$=\frac{3+4\cot\theta}{\cot\theta-1}$$

$$=\frac{3+4\left(\frac{5}{2}\right)}{\frac{5}{2}-1}$$

$$=\frac{\frac{6+20}{2}}{\frac{5-2}{2}}$$

$$=\frac{\frac{26}{2}}{\frac{3}{2}}$$

$$=\frac{26}{3}$$

**The values of trigonometric functions of acute angles:**

| $\theta$ | 30º | 45º | 60º |
|---|---|---|---|
| $\sin\theta$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ |
| $\cos\theta$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ |
| $\tan\theta$ | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ |
| $\cot\theta$ | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ |
| $\sec\theta$ | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 |
| $\text{cosec}\,\theta$ | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ |

**The values of trigonometric function of angles** $0^o, 90^o, 180^o, 270^o, 360^o$**:**

| $\theta$ | 0º | 90º | 180º | 270º | 360º |
|---|---|---|---|---|---|
| $\sin\theta$ | 0 | 1 | 0 | – 1 | 0 |
| $\cos\theta$ | 1 | 0 | –1 | 0 | 1 |
| $\tan\theta$ | 0 | ∞ | 0 | – ∞ | 0 |
| $\cot\theta$ | ∞ | 0 | ∞ | 0 | ∞ |
| $\sec\theta$ | 1 | ∞ | – 1 | ∞ | 1 |
| $\text{cosec}\,\theta$ | ∞ | 1 | ∞ | – 1 | ∞ |

## Exercise 9.3

**Q.1 Verify the following.**

**(i)** $\mathbf{\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ}$

**Solution:**

L.H.S

$= \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

$= \frac{1}{2} = \sin 30^\circ = \text{R.H.S.}$

**(ii)** $\mathbf{\sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4} = 2}$

**Solution:**

L.H.S. $= \sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4}$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$

$= \frac{1}{4} + \frac{3}{4} + 1$

$= \frac{1+3+4}{4}$

$= \frac{8}{4}$

$= 2 =$ R.H.S.

**(iii)** $\mathbf{2\sin 45^\circ + \frac{1}{2}\ \text{cosec}\, 45^\circ = \frac{3}{\sqrt{2}}}$

**Solution:**

L.H.S. $= 2\sin 45^\circ + \frac{1}{2}\text{cosec}\, 45^\circ$

$= 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}\left(\sqrt{2}\right)$

$= \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2}$

$= \sqrt{2} + \frac{1}{\sqrt{2}}$

$= \frac{2+1}{\sqrt{2}}$

$= \frac{3}{\sqrt{2}} = \text{R.H.S}$

**(iv)** $\mathbf{\sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2} = 1:2:3:4}$

**Solution:**

L.H.S. $= \sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2}$

$= \left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2 : (1)^2$

$= \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$

Multiplying by 4, we get

$= 1:2:3:4 = \text{R.H.S}$

**Q.2 Evaluate the following**

**(i)** $\mathbf{\dfrac{\tan\frac{\pi}{3} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{6}}}$

**Solution:**

$\dfrac{\tan\frac{\pi}{3} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{6}} = \dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \left(\sqrt{3}\right)\left(\frac{1}{\sqrt{3}}\right)}$

$= \dfrac{\frac{3-1}{\sqrt{3}}}{1+1}$

$= \dfrac{\frac{2}{\sqrt{3}}}{2}$

$= \frac{1}{\sqrt{3}}$

**(ii)** $\dfrac{1-\tan^2\dfrac{\pi}{3}}{1+\tan^2\dfrac{\pi}{3}}$

**Solution:**

$$\frac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}}=\frac{1-\left(\sqrt{3}\right)^2}{1+\left(\sqrt{3}\right)^2}$$

$$=\frac{1-3}{1+3}$$

$$=\frac{-2}{4}$$

$$=\frac{-1}{2}$$

**Q.3 Verify the following when** $\theta=30°, 45°$

**(i)** $\mathbf{\sin 2\theta = 2\sin\theta\cos\theta}$

**(a)** When $\theta=30°$

**L.H.S.** $=\sin 2\theta$
$=\sin 2(30°)$
$=\sin 60°$
$=\dfrac{\sqrt{3}}{2}$

**R.H.S.** $=2\sin\theta\cos\theta$
$=2\sin 30°\cos 30°$
$=2\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}\right)$
$=\dfrac{\sqrt{3}}{2}$

**(b)** When $\theta=45°$

**L.H.S.** $=\sin 2\theta$
$=\sin 2(45°)$
$=\sin 90°$
$=1$

**R.H.S.** $=2\sin\theta\cos\theta$
$=2\sin 45°\cos 45°$
$=2\left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{1}{\sqrt{2}}\right)$
$=1$

**(ii)** $\mathbf{\cos 2\theta = \cos^2\theta - \sin^2\theta}$

**(a) When** $\theta=30°$

**L.H.S.** $=\cos 2\theta$
$=\cos 2(30°)$
$=\cos 60°$
$=\dfrac{1}{2}$

**R.H.S.** $=\cos^2\theta-\sin^2\theta$
$=\cos^2 30°-\sin^2 30°$
$=\left(\dfrac{\sqrt{3}}{2}\right)^2-\left(\dfrac{1}{2}\right)^2$
$=\dfrac{3}{4}-\dfrac{1}{4}$
$=\dfrac{3-1}{4}$
$=\dfrac{2}{4}$
$=\dfrac{1}{2}$

**(b) When** $\theta = 45°$

| L.H.S. | R.H.S. |
|---|---|
| $= \cos 2\theta$ | $= \cos^2\theta - \sin^2\theta$ |
| $= \cos 2(45°)$ | $= \cos^2 45° - \sin^2 45°$ |
| $= \cos 90°$ | $= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$ |
| $= 0$ | $= \frac{1}{2} - \frac{1}{2}$ |
| | $= 0$ |

**(iii)** $\cos 2\theta = 2\cos^2\theta - 1$

**(a) When** $\theta = 30°$

| L.H.S. | R.H.S. |
|---|---|
| $= \cos 2\theta$ | $= 2\cos^2\theta - 1$ |
| $= \cos 2(30°)$ | $= 2(\cos 30°)^2 - 1$ |
| $= \cos 60°$ | $= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$ |
| $= \frac{1}{2}$ | $= 2\left(\frac{3}{4}\right) - 1$ |
| | $= \frac{3}{2} - 1$ |
| | $= \frac{3-2}{2}$ |
| | $= \frac{1}{2}$ |

**(b) When** $= \theta = 45°$

| L.H.S. | R.H.S. |
|---|---|
| $= \cos 2\theta$ | $= 2\cos^2\theta - 1$ |
| $= \cos 2(45°)$ | $= 2\cos^2 45° - 1$ |
| $= \cos 90°$ | $= 2\left(\frac{1}{2}\right) - 1$ |
| $= 0$ | $= 0$ |

**(iv)** $\cos 2\theta = 1 - 2\sin^2\theta$

**(a) When** $\theta = 30°$

| L.H.S. | R.H.S. |
|---|---|
| $= \cos 2\theta$ | $= 1 - 2\sin^2\theta$ |
| $= \cos 2(30°)$ | $= 1 - 2\sin^2 30°$ |
| $= \cos 60°$ | $= 1 - 2\left(\frac{1}{4}\right) = 1 - \frac{1}{2}$ |
| $= \frac{1}{2}$ | $= \frac{1}{2}$ |

**(b) When $\theta = 45°$**

**L.H.S.** $= \cos 2\theta$
$= \cos 2(45°)$
$= \cos 90°$
$= 0$

**R.H.S.** $= 1 - 2\sin^2\theta$
$= 1 - 2\sin^2 45°$
$= 1 - 2\left(\frac{1}{2}\right)$
$= 0$

**(v)** $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

**(a) When $\theta = 30°$**

**L.H.S.** $= \tan 2\theta$
$= \tan 2(30°)$
$= \tan 60°$
$= \sqrt{3}$

**R.H.S.** $= \dfrac{2\tan\theta}{1-\tan^2\theta}$
$= \dfrac{2\tan 30°}{1-\tan^2 30°}$
$= \dfrac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\frac{1}{3}} = \dfrac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}}$
$= \dfrac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \dfrac{3}{\sqrt{3}}$
$= \sqrt{3}$

**(b) When $\theta = 45°$**

**L.H.S.** $= \tan 2\theta$
$= \tan 2(45°)$
$= \tan 90°$
= undefined

**R.H.S.** $= \dfrac{2\tan\theta}{1-\tan^2\theta}$
$= \dfrac{2\tan 45°}{1-\tan^2(45°)}$
$= \dfrac{2(1)}{1-1}$
= undefined

**Q.4 Find the values of the trigonometric functions of the following quadrantal angles.**

**(i)** $-\pi$

**Solution:**

$$-\pi = -\pi + 2\pi = \pi$$

So the values of trigonometric functions at $-\pi$ are same as that of $\pi$

| | |
|---|---|
| $\sin(-\pi) = \sin\pi = 0$ | $\operatorname{cosec}(-\pi) = \operatorname{cosec}\pi = \text{undefined}$ |
| $\cos(-\pi) = \cos\pi = -1$ | $\sec(-\pi) = \sec\pi = -1$ |
| $\tan(-\pi) = \tan\pi = 0$ | $\cot(-\pi) = \cot\pi = \text{undefined}$ |

**(ii)** $-3\pi$

**Solution:**

$$-3\pi = -3\pi + 2(2\pi) = -3\pi + 4\pi = \pi$$

So the values of trigonometric function at $-3\pi$ are same as that of $\pi$ .

| | |
|---|---|
| $\sin\pi = 0$ | $\operatorname{cosec}\pi = \text{undefined}$ |
| $\cos\pi = -1$ | $\sec\pi = -1$ |
| $\tan\pi = 0$ | $\cot\pi = \text{undefined}$ |

**(iii)** $\frac{5\pi}{2}$

**Solution:**

$$\frac{5\pi}{2} = \frac{5\pi}{2} + (-1)(2\pi) = \frac{5\pi}{2} - 2\pi$$

$$= \frac{5\pi - 4\pi}{2} = \frac{\pi}{2}$$

So the values of trigonometric function at $\frac{5\pi}{2}$ are same as that of $\frac{\pi}{2}$.

| | |
|---|---|
| $\sin\frac{\pi}{2} = 1$ | $\cos ec\frac{\pi}{2} = 1$ |
| $\cos\frac{\pi}{2} = 0$ | $\sec\frac{\pi}{2} = \infty$ |
| $\tan\frac{\pi}{2} = \infty$ | $\cot\frac{\pi}{2} = 0$ |

**(iv)** $-\frac{9\pi}{2}$

**Solution:**

$$-\frac{9\pi}{2}=-\frac{9\pi}{2}+3(2\pi)=-\frac{9\pi}{2}+6\pi$$
$$=\frac{-9\pi+12\pi}{2}=\frac{3\pi}{2}$$

So the values of trigonometric function at $\frac{-9\pi}{2}$ are same as that of $\frac{3\pi}{2}$.

| | |
|---|---|
| $\sin\frac{3\pi}{2}=-1$ | $\text{cosec}\frac{3\pi}{2}=-1$ |
| $\cos\frac{3\pi}{2}=0$ | $\sec\frac{3\pi}{2}=\infty$ |
| $\tan\frac{3\pi}{2}=\infty$ | $\text{cosec}\frac{3\pi}{2}=-1$ |

**(v)** $-15\pi$

**Solution:**

$$-15\pi=-15\pi+8(2\pi)=-15\pi+16\pi=\pi$$

So the values of trigonometric function at $-15\pi$ are same as that of $\pi$.

| | |
|---|---|
| $\sin\pi=0$ | $\text{cosec}\,\pi=\infty$ |
| $\cos\pi=-1$ | $\sec\pi=-1$ |
| $\tan\pi=0$ | $\cot\pi=\infty$ |

**(vi)** 1530°

**Solution:**

$$1530^o=1530^o+(-4)(360^o)=1530^o-1440^o=90^o$$

So the values of trigonometric function at $1530^o$ are same as that of $90^o$.

| | |
|---|---|
| $\sin 90^o=1$ | $\text{cosec}\,90^o=1$ |
| $\cos 90^o=0$ | $\sec 90^o=\infty$ |
| $\tan 90^o=\infty$ | $\cot 90^o=0$ |

**(vii)** –2430°

**Solution:**

$$-2430^o=-2430^o+7\left(360^o\right)$$
$$=-2430^o+2520^o$$
$$=90^o$$

So the values of trigonometric function at $-2430^o$ are same as that of $90^o$.

| | |
|---|---|
| $\sin 90^o=1$ | $\text{cosec}\,90^o=1$ |
| $\cos 90^o=0$ | $\sec 90^o=\infty$ |
| $\tan 90^o=\infty$ | $\cot 90^o=0$ |

**(viii)** $\frac{235\pi}{2}$

**Solution:**

$$\frac{235\pi}{2}=\frac{235\pi}{2}+(-58)(2\pi)=\frac{235\pi}{2}-116\pi$$

$$=\frac{235\pi-232\pi}{2}$$

$$=\frac{3\pi}{2}$$

So the values of trigonometric function at $\frac{235\pi}{2}$ are same as that of $\frac{3\pi}{2}$.

| | |
|---|---|
| $\sin\frac{3\pi}{2}=-1$ | $\operatorname{cosec}\frac{3\pi}{2}=-1$ |
| $\cos\frac{3\pi}{2}=0$ | $\sec\frac{3\pi}{2}=\infty$ |
| $\tan\frac{3\pi}{2}=\infty$ | $\cot\frac{3\pi}{2}=0$ |

**(ix)** $\frac{407\pi}{2}$

**Solution:**

$$\frac{407\pi}{2}=\frac{407\pi}{2}+(-101)(2\pi)=\frac{407\pi}{2}-202\pi$$

$$=\frac{407\pi-404\pi}{2}$$

$$=\frac{3\pi}{2}$$

So the values of trigonometric function at $\frac{407\pi}{2}$ are same as that of $\frac{3\pi}{2}$.

| | |
|---|---|
| $\sin\frac{3\pi}{2}=-1$ | $\operatorname{cosec}\frac{3\pi}{2}=-1$ |
| $\cos\frac{3\pi}{2}=0$ | $\sec\frac{3\pi}{2}=\infty$ |
| $\tan\frac{3\pi}{2}=\infty$ | $\cot\frac{3\pi}{2}=0$ |

**Q.5 Find the values of trigonometric functions of the following angles.**

**(i) 390°**

**Solution:**

$390° = 390° + (-1)(360°) = 390° - 360° = 30°$

So the values of trigonometric function at $390°$ are same as that of $30°$.

| | |
|---|---|
| $\sin 30° = \frac{1}{2}$ | $\operatorname{cosec} 30° = 2$ |
| $\cos 30° = \frac{\sqrt{3}}{2}$ | $\sec 30° = \frac{2}{\sqrt{3}}$ |
| $\tan 30° = \frac{1}{\sqrt{3}}$ | $\cot 30° = \sqrt{3}$ |

**(ii) –330°**

**Solution:**

$-330° = -330° + 360° = 30°$

So the values of trigonometric function at $-330°$ are same as that of $30°$.

| | |
|---|---|
| $\sin 30° = \frac{1}{2}$ | $\operatorname{cosec} 30° = 2$ |
| $\cos 30° = \frac{\sqrt{3}}{2}$ | $\sec 30° = \frac{2}{\sqrt{3}}$ |
| $\tan 30° = \frac{1}{\sqrt{3}}$ | $\cot 30° = \sqrt{3}$ |

**(iii) 765°**

**Solution:**

$765° = 765° + (-2)(360°) = 765° - 720° = 45°$

So the values of trigonometric function at $765°$ are same as that of $45°$.

| | |
|---|---|
| $\sin 45° = \frac{1}{\sqrt{2}}$ | $\operatorname{cosec} 45° = \sqrt{2}$ |
| $\cos 45° = \frac{1}{\sqrt{2}}$ | $\sec 45° = \sqrt{2}$ |
| $\tan 45° = 1$ | $\cot 45° = 1$ |

**(iv) –675°**

**Solution:**

$-675° = -675° + 2(360°) = -675° + 720° = 45°$

So the values of trigonometric function at $-675°$ are same as that of $45°$.

| | |
|---|---|
| $\sin 45^\circ = \frac{1}{\sqrt{2}}$ | $\operatorname{cosec} 45^\circ = \sqrt{2}$ |
| $\cos 45^\circ = \frac{1}{\sqrt{2}}$ | $\sec 45^\circ = \sqrt{2}$ |
| $\tan 45^\circ = 1$ | $\cot 45^\circ = 1$ |

**(v)** $-\frac{17\pi}{3}$

**Solution:**

$$-\frac{17\pi}{3} = \frac{-17\pi}{3} + 3(2\pi) = \frac{-17\pi}{3} + 6\pi = \frac{-17\pi + 18\pi}{3} = \frac{\pi}{3}$$

So the values of trigonometric function at $-\frac{17\pi}{3}$ are same as that of $\frac{\pi}{3}$.

| | |
|---|---|
| $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ | $\operatorname{cosec}\frac{\pi}{3} = \frac{2}{\sqrt{3}}$ |
| $\cos\frac{\pi}{3} = \frac{1}{2}$ | $\sec\frac{\pi}{3} = 2$ |
| $\tan\frac{\pi}{3} = \sqrt{3}$ | $\cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$ |

**(vi)** $\frac{13\pi}{3}$

**Solution:**

$$\frac{13\pi}{3} = \frac{13\pi}{3} + (-2)(2\pi) = \frac{13\pi}{3} - 4\pi = \frac{13\pi - 12\pi}{3} = \frac{\pi}{3}$$

So the values of trigonometric function at $\frac{13\pi}{3}$ are same as that of $\frac{\pi}{3}$.

| | |
|---|---|
| $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ | $\operatorname{cosec}\frac{\pi}{3} = \frac{2}{\sqrt{3}}$ |
| $\cos\frac{\pi}{3} = \frac{1}{2}$ | $\sec\frac{\pi}{3} = 2$ |
| $\tan\frac{\pi}{3} = \sqrt{3}$ | $\cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$ |

**(vii)** $\mathbf{\frac{25\pi}{6}}$

**Solution:**

$$\frac{25\pi}{6}=\frac{25\pi}{6}+(-2)(2\pi)=\frac{25\pi}{6}-4\pi=\frac{25\pi-24\pi}{6}=\frac{\pi}{6}$$

So the values of trigonometric function at $\frac{25\pi}{6}$ are same as that of $\frac{\pi}{6}$.

| | |
|---|---|
| $\sin\frac{\pi}{6}=\frac{1}{2}$ | $\operatorname{cosec}\frac{\pi}{6}=2$ |
| $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ | $\sec\frac{\pi}{6}=\frac{2}{\sqrt{3}}$ |
| $\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$ | $\cot\frac{\pi}{6}=\sqrt{3}$ |

**(viii)** $\mathbf{\frac{-71\pi}{6}}$

**Solution:**

$$\frac{-71\pi}{6}=\frac{-71\pi}{6}+6(2\pi)=\frac{-71\pi}{6}+12\pi=\frac{-71\pi+72\pi}{6}=\frac{\pi}{6}$$

So the values of trigonometric function at $\frac{-71\pi}{6}$ are same as that of $\frac{\pi}{6}$.

| | |
|---|---|
| $\sin\frac{\pi}{6}=\frac{1}{2}$ | $\operatorname{cosec}\frac{\pi}{6}=2$ |
| $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ | $\sec\frac{\pi}{6}=\frac{2}{\sqrt{3}}$ |
| $\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$ | $\cot\frac{\pi}{6}=\sqrt{3}$ |

**(ix)** $\mathbf{-1035^\circ}$

**Solution:**

$$-1035^\circ=-1035^\circ+3(360^\circ)=-1035^\circ+1080^\circ=45^\circ$$

So the values of trigonometric function at $-1035^\circ$ are same as that of $45^\circ$.

| | |
|---|---|
| $\sin 45^\circ=\frac{1}{\sqrt{2}}$ | $\operatorname{cosec}45^\circ=\sqrt{2}$ |
| $\cos 45^\circ=\frac{1}{\sqrt{2}}$ | $\sec 45^\circ=\sqrt{2}$ |
| $\tan 45^\circ=1$ | $\cot 45^\circ=1$ |

**Domains of "Trigonometric Functions and Fundamental Identities".**

We list the trigonometric functions and fundamental identities, learnt so far mentioning their domains as follows.

**(i)** $\sin\theta$ , for all $\theta \in \mathbb{R}$

**(ii)** $\cos\theta$ , for all $\theta \in \mathbb{R}$

**(iii)** $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ , for all $\theta \in \mathbb{R}$ but $\theta \neq n\pi$ $n \in Z$

**(iv)** $\sec\theta = \dfrac{1}{\cos\theta}$ , for all $\theta \in \mathbb{R}$ but $\theta \neq (2n+1)\dfrac{\pi}{2}$ $n \in Z$

**(v)** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ , for all $\theta \in \mathbb{R}$ but $\theta \neq (2n+1)\dfrac{\pi}{2}$ $n \in Z$

**(vi)** $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ , for all $\theta \in \mathbb{R}$ but $\theta \neq n\pi$ $n \in Z$

**(vii)** $\sin^2\theta + \cos^2\theta = 1$ , for all $\theta \in \mathbb{R}$

**(viii)** $1 + \tan^2\theta = \sec^2\theta$ , for all $\theta \in \mathbb{R}$ but $\theta \neq (2n+1)\dfrac{\pi}{2}$ $n \in Z$

**(ix)** $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ , for all $\theta \in \mathbb{R}$ but $\theta \neq n\pi$ $n \in Z$

## Exercise 9.4

**Prove the following identities, state the domain of θ in each case.**

**Q.1 $\tan\theta+\cot\theta=\operatorname{cosec}\theta\sec\theta$**

**Solution:**

L.H.S. $=\tan\theta+\cot\theta$

$=\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$

$=\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$

$=\dfrac{1}{\cos\theta}.\dfrac{1}{\sin\theta}$

$=\operatorname{cosec}\theta.\sec\theta=$ R.H.S.

Domain of θ : $\theta\in\mathbb{R}$, but $\theta\neq\dfrac{n\pi}{2}, n\in\mathbb{Z}$

**Q.2 $\sec\theta\operatorname{cosec}\theta\sin\theta\cos\theta=1$**

**Solution:**

L.H.S. $=\sec\theta\operatorname{cosec}\theta\sin\theta\cos\theta$

$=\dfrac{1}{\sin\theta}.\dfrac{1}{\cos\theta}.\sin\theta\cos\theta$

$=1=$ R.H.S.

Domain of $\theta$ : $\theta\in\mathbb{R}$, but $\theta\neq\dfrac{n\pi}{2}, n\in\mathbb{Z}$

**Q.3 $\cos\theta+\tan\theta\sin\theta=\sec\theta$**

**Solution:**

L.H.S. $=\cos\theta+\tan\theta.\sin\theta$

$=\cos\theta+\dfrac{\sin\theta}{\cos\theta}.\sin\theta$

$=\dfrac{\cos^2\theta+\sin^2\theta}{\cos\theta}$

$=\dfrac{1}{\cos\theta}$

$=\sec\theta=$ R.H.S.

Domain of

$\theta:\theta\in\mathbb{R}$, but $\theta\neq(2n+1)\dfrac{\pi}{2}, n\in\mathbb{Z}$

**Q.4 $\operatorname{cosec}\theta+\tan\theta\sec\theta=\operatorname{cosec}\theta\sec^2\theta$**

**Solution:**

L.H.S. $=\operatorname{cosec}\theta+\tan\theta\sec\theta$

$=\dfrac{1}{\sin\theta}+\dfrac{\sin\theta}{\cos\theta}.\dfrac{1}{\cos\theta}$

$=\dfrac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos^2\theta}$

$=\dfrac{1}{\sin\theta}.\dfrac{1}{\cos^2\theta}$

$=\operatorname{cosec}\theta.\sec^2\theta=$ R.H.S.

Domain of $\theta$ : $\theta\in\mathbb{R}$, but $\theta\neq\dfrac{n\pi}{2}, n\in\mathbb{Z}$

**Q.5 $\sec^2\theta-\operatorname{cosec}^2\theta=\tan^2\theta-\cot^2\theta$**

**Solution:**

L.H.S. $=\sec^2\theta-\operatorname{cosec}^2\theta$

$=1+\tan^2\theta-\left(1+\cot^2\theta\right)$

$\left(\because \sec^2\theta=1+\tan^2\theta \text{ and } \operatorname{cosec}^2\theta=1+\cot^2\theta\right)$

$=1+\tan^2\theta-1-\cot^2\theta$

$=\tan^2\theta-\cot^2\theta$

= R.H.S.

Domain of $\theta$ : $\theta\in\mathbb{R}$, but $\theta\neq\dfrac{n\pi}{2}, n\in\mathbb{Z}$

**Q.6 $\cot^2\theta-\cos^2\theta=\cot^2\theta.\cos^2\theta$**

**Solution:**

L.H.S. $=\cot^2\theta-\cos^2\theta$

$=\dfrac{\cos^2\theta}{\sin^2\theta}-\cos^2\theta$

$=\cos^2\theta\left(\dfrac{1}{\sin^2\theta}-1\right)$

$=\cos^2\theta\left(\dfrac{1-\sin^2\theta}{\sin^2\theta}\right)$

$$=\left(\frac{\cos^2\theta}{\sin^2\theta}\right).\left(\cos^2\theta\right)$$

$$=\cot^2\theta.\cos^2\theta = \text{ R.H.S.}$$

Domain of $\theta$ : $\theta\in\mathbb{R}$, but $\theta\neq n\pi, n\in\mathbb{Z}$

**Q.7** $\quad(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=1$

**Solution:**

L.H.S. $=(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)$

$$=\sec^2\theta-\tan^2\theta$$

$$=\sec^2\theta-\left(\sec^2\theta-1\right)$$

$$=\sec^2\theta-\sec^2\theta+1$$

$$=1= \text{ R.H.S.}$$

Domain of $\theta$

$:\theta\in\mathbb{R}$, but $\theta\neq(2n+1)\frac{\pi}{2}, n\in\mathbb{Z}$

**Q.8** $\quad 2\cos^2\theta-1=1-2\sin^2\theta$

**Solution:**

L.H.S. $=2\cos^2\theta-1$

$$=2\left(1-\sin^2\theta\right)-1$$

$$=2-2\sin^2\theta-1$$

$$=1-2\sin^2\theta$$

= R.H.S.

Domain of $\theta$ : $\theta\in\mathbb{R}$

**Q.9** $\quad\cos^2\theta-\sin^2\theta=\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

**Solution:**

R.H.S. $=\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$$=\frac{1-\tan^2\theta}{\sec^2\theta}$$

$$=\cos^2\theta\left(1-\frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$=\cos^2\theta\left(\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\right)$$

$$=\cos^2\theta-\sin^2\theta$$

= L.H.S.

Domain of $\theta$ :

$\theta\in\mathbb{R}$, but $\theta\neq(2n+1)\frac{\pi}{2}, n\in\mathbb{Z}$

**Q.10** $\quad\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\dfrac{\cot\theta-1}{\cot\theta+1}$

**Solution:**

R.H.S. $=\dfrac{\cot\theta-1}{\cot\theta+1}$

$$=\frac{\frac{\cos\theta}{\sin\theta}-1}{\frac{\cos\theta}{\sin\theta}+1}$$

$$=\frac{\frac{\cos\theta-\sin\theta}{\sin\theta}}{\frac{\cos\theta+\sin\theta}{\sin\theta}}$$

$$=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}= \text{ L.H.S.}$$

Domain of $\theta$ : $\theta\in\mathbb{R}$, but $\theta\neq n\pi, n\in\mathbb{Z}$

**Q.11** $\quad\dfrac{\sin\theta}{1+\cos\theta}+\cot\theta=\operatorname{cosec}\theta$

**Solution:**

L.H.S. $=\dfrac{\sin\theta}{1+\cos\theta}+\cot\theta$

$$=\frac{\sin\theta}{1+\cos\theta}+\frac{\cos\theta}{\sin\theta}$$

$$=\frac{\sin^2\theta+\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$=\frac{\sin^2\theta+\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$=\frac{1+\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$=\frac{1}{\sin\theta}$$

$$=\operatorname{cosec}\theta = \text{R.H.S.}$$

Domain of $\theta$ ; $\theta\in\mathbb{R}$, but $\theta\neq n\pi, n\in\mathbb{Z}$

**Q.12** $\mathbf{\frac{\cot^2\theta-1}{1+\cot^2\theta}=2\cos^2\theta-1}$

**Solution:**

$$\text{L.H.S.} =\frac{\cot^2\theta-1}{1+\cot^2\theta}$$

$$=\frac{\cot^2\theta-1}{\operatorname{cosec}^2\theta}$$

$$=\sin^2\theta\left(\frac{\cos^2\theta}{\sin^2\theta}-1\right)$$

$$=\sin^2\theta\left(\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}\right)$$

$$=\cos^2\theta-\sin^2\theta$$

$$=\cos^2\theta-\left(1-\cos^2\theta\right)$$

$$=\cos^2\theta-1+\cos^2\theta$$

$$=2\cos^2\theta-1=\text{R.H.S.}$$

Domain of $\theta$ : $\theta\in\mathbb{R}$, but $\theta\neq n\pi, n\in\mathbb{Z}$

**Q.13** $\mathbf{\frac{1+\cos\theta}{1-\cos\theta}=(\operatorname{cosec}\theta+\cot\theta)^2}$

**Solution:**

$$\text{R.H.S.} =(\operatorname{cosec}\theta+\cot\theta)^2$$

$$=\left(\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\right)^2$$

$$=\left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$=\frac{(1+\cos\theta)^2}{\sin^2\theta}$$

$$=\frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

$$=\frac{(1+\cos\theta)\cancel{(1+\cos\theta)}}{(1-\cos\theta)\cancel{(1+\cos\theta)}}$$

$$=\frac{1+\cos\theta}{1-\cos\theta}=\text{L.H.S.}$$

Domain of $\theta$ : $\theta\in\mathbb{R}$, but $\theta\neq n\pi, n\in\mathbb{Z}$

**Q.14** $\mathbf{(\sec\theta-\tan\theta)^2=\frac{1-\sin\theta}{1+\sin\theta}}$

**Solution:**

$$\text{L.H.S.} =(\sec\theta-\tan\theta)^2$$

$$=\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)^2$$

$$=\left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$=\frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$=\frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$=\frac{\cancel{(1-\sin\theta)}(1-\sin\theta)}{\cancel{(1-\sin\theta)}(1+\sin\theta)}$$

$$=\frac{1-\sin\theta}{1+\sin\theta}=\text{R.H.S.}$$

Domain of $\theta$ :

$\theta\in\mathbb{R}$, but $\theta\neq(2n+1)\frac{\pi}{2}, n\in\mathbb{Z}$

**Q.15** $\mathbf{\frac{2\tan\theta}{1+\tan^2\theta}=2\sin\theta\cos\theta}$

**Solution:**

$$\text{L.H.S.} =\frac{2\tan\theta}{1+\tan^2\theta}$$

$$=\frac{2\tan\theta}{\sec^2\theta}$$

$$= \frac{2\sin\theta}{\cos\theta}.\cos^2\theta$$

$$= 2\sin\theta\cos\theta$$

= R.H.S.

Domain of $\theta$ :

$\theta \in \mathbb{R}$, but $\theta \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$

**Q.16** $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$

**Solution:**

$$\text{L.H.S.} = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}.\frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{(1-\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos\theta}{1+\sin\theta} = \text{R.H.S.}$$

Domain of $\theta$ :

$\theta \in \mathbb{R}$, but $\theta \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$

**Q.17** $(\tan\theta + \cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$

**Solution:**

$$\text{L.H.S.} = (\tan\theta + \cot\theta)^2$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta\sin\theta}\right)^2$$

$$= \frac{1}{\cos^2\theta}.\frac{1}{\sin^2\theta}$$

$$= \sec^2\theta \operatorname{cosec}^2\theta = \text{R.H.S}$$

Domain of $\theta$ : $\theta \in \mathbb{R}$, but $\theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$

**Q.18** $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$

**Solution:**

$$\text{L.H.S.} = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)[1 - \sec\theta + \tan\theta]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)(\tan\theta - \sec\theta + 1)}{\tan\theta - \sec\theta + 1}$$

$$= \tan\theta + \sec\theta = \text{R.H.S.}$$

Domain of $\theta$ :

$\theta \in \mathbb{R}$, but $\theta \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$

**Q.19** $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$

$$= \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

**Solution:**

We are to prove that

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} + \frac{1}{\operatorname{cosec}\theta + \cot\theta} = \frac{2}{\sin\theta}$$

$$\text{L.H.S} = \frac{1}{\operatorname{cosec}\theta - \cot\theta} + \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

$$=\frac{\operatorname{cosec}\theta+\cot\theta+\operatorname{cosec}\theta-\cot\theta}{(\operatorname{cosec}\theta-\cot\theta)(\operatorname{cosec}\theta+\cot\theta)}$$

$$=\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta-\cot^2\theta}$$

$$=\frac{2\operatorname{cosec}\theta}{1+\cot^2\theta-\cot^2\theta}$$

$$=\frac{2\operatorname{cosec}\theta}{1}$$

$$=\frac{2}{\sin\theta}=\text{R.H.S}$$

**Q.20**

$$\mathbf{\sin^3\theta-\cos^3\theta=(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)}$$

**Solution:**

L.H.S. $=\sin^3\theta-\cos^3\theta$

$$=(\sin\theta)^3-(\cos\theta)^3$$

$$=(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)$$

$$=(\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$$

$=$ R.H.S.

Domain of $\theta$ : $\theta=\mathbb{R}$

**Q.21** $\mathbf{\sin^6\theta-\cos^6\theta=\left(\sin^2\theta-\cos^2\theta\right)\left(1-\sin^2\theta\cos^2\theta\right)}$

**Solution:**

L.H.S. $=\sin^6\theta-\cos^6\theta$

$$=\left(\sin^2\theta\right)^3-\left(\cos^2\theta\right)^3$$

$$=\left(\sin^2\theta-\cos^2\theta\right)\left(\sin^4\theta+\cos^4\theta+\sin^2\theta\cos^2\theta\right)$$

$$=\left(\sin^2\theta-\cos^2\theta\right)\left[\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2+2\sin^2\theta\cos^2\theta-\sin^2\theta\cos^2\theta\right]$$

$$=\left(\sin^2\theta-\cos^2\theta\right)\left[\left(\sin^2\theta+\cos^2\theta\right)^2-\sin^2\theta\cos^2\theta\right]$$

$$=\left(\sin^2\theta-\cos^2\theta\right)\left(1-\sin^2\theta\cos^2\theta\right)$$

= R.H.S.

Domain of $\theta$ : $\theta\in\mathbb{R}$

**Q:22** $\mathbf{\sin^6\theta+\cos^6\theta=1-3\sin^2\theta\cos^2\theta}$

**Solution:**

L.H.S. $=\sin^6\theta+\cos^6\theta$

$$=\left(\sin^2\theta\right)^3+\left(\cos^2\theta\right)^3$$

$$=\left(\sin^2\theta+\cos^2\theta\right)\left[\sin^4\theta+\cos^4\theta-\sin^2\theta\cos^2\theta\right]$$

$$=(1)\left[\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2+2\sin^2\theta\cos^2\theta-3\sin^2\theta\cos^2\theta\right]$$

$$=\left(\sin^2\theta+\cos^2\theta\right)^2-3\sin^2\theta\cos^2\theta$$

$$=1-3\sin^2\theta\cos^2\theta=\text{ R.H.S.}$$

Domain of $\theta$ : $\theta\in\mathbb{R}$

**Q:23** $\mathbf{\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}=2\sec^2\theta}$

**Solution:**

$$\text{L.H.S.} = \frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$$

$$= \frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta}$$

$$= \frac{2}{\cos^2\theta}$$

$$= 2\sec^2\theta = \text{R.H.S.}$$

Domain of $\theta$ **:** $\theta \in \mathbb{R}$, but $\theta \neq (2n+1)\dfrac{\pi}{2}, n \in \mathbb{Z}$

**Q:24** $\mathbf{\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\dfrac{2}{1-2\sin^2\theta}}$

**Solution:**

$$\text{L.H.S.} = \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

$$= \frac{(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$$

$$= \frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta+\cos^2\theta+\sin^2\theta-2\cos\theta\sin\theta}{\cos^2\theta-\sin^2\theta}$$

$$= \frac{2(\cos^2\theta+\sin^2\theta)}{\cos^2\theta-\sin^2\theta}$$

$$= \frac{2(\cos^2\theta+\sin^2\theta)}{1-\sin^2\theta-\sin^2\theta}$$

$$= \frac{2}{1-\sin^2\theta-\sin^2\theta}$$

$$= \frac{2}{1-2\sin^2\theta}$$

$$= \text{R.H.S.}$$

Domain of $\theta$ : $\theta \in \mathbb{R}$, but $\theta \neq (2n+1)\dfrac{\pi}{4}, n \in \mathbb{Z}$