

78. Find unit vector perpendicular to the plane of  $\underline{a}$  and  $\underline{b}$  if  $\underline{a} = -\underline{i} - \underline{j} - \underline{k}$ ,  $\underline{b} = 2\underline{i} - 3\underline{j} + 4\underline{k}$ .
79. Find vertices and equation of directrices of hyperbola  $x^2 - y^2 = 9$ . 17Grp11,
80. Find  $\alpha$  so that  $\underline{u} = \alpha\underline{i} + 2\underline{a}\underline{j} - \underline{k}$  and  $\underline{v} = \underline{i} + \alpha\underline{j} + 3\underline{k}$  are perpendicular.
81. Find  $a$ , so that  $|\underline{a}\underline{i} + (a+1)\underline{j} + 2\underline{k}| = 3$ .
82. Find the value  $3\underline{j} \cdot \underline{k} \times \underline{i}$ .
83. If  $\overline{AB} = \overline{CD}$ , find coordinates of points A. If B, C, D are (1, 2), (-2, 5), (4, 1)
84. If  $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$  and  $\underline{b} = \underline{i} - \underline{j} + \underline{k}$  find the cross product  $\underline{a} \times \underline{b}$
85. If  $\underline{u} = 3\underline{i} + \underline{j} - \underline{k}$  and  $\underline{v} = 2\underline{i} - \underline{j} + \underline{k}$ , find the cosines of the angle  $\theta$  between  $\underline{u}$  and  $\underline{v}$
86. If O is the origin and  $\overrightarrow{OP} = \overline{AB}$ , find the point P when A and B are (-3, 7) and (1, 0) respectively
87. Prove that if  $\underline{a} + \underline{b} + \underline{c} = \underline{0}$  then  $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$
88. Prove that  $\underline{a} \times (\underline{b} + \underline{c}) + \underline{b} \times (\underline{c} + \underline{a}) + \underline{c} \times (\underline{a} + \underline{b}) = \underline{0}$ .
89. Prove that if the lines are perpendicular, then product of their slopes = -1
90. Show that the points A(3, 1), B(-2, -3) and C(2, 2) are vertices of an isosceles triangle.
91. Show that the points A(-1, 2), B(7, 5) and C(2, -6) are vertices of a right triangle.
92. Show that the triangle with vertices A(1, 1), B(4, 5) and C(12, -5) is right triangle.
93. Show that vectors  $3\underline{i} - 2\underline{j} + \underline{k}$ ,  $\underline{i} - 3\underline{j} + 5\underline{k}$  and  $2\underline{i} + \underline{j} - 4\underline{k}$  form a right triangle.
94. Transform  $5x - 12y + 39 = 0$  into two intercept form. 15 Grp II,
95. Two lines  $l_1$  and  $l_2$  with respective slopes  $m_1$  and  $m_2$  are parallel if  $m_1 = m_2$ .
96. Write an equation of parabola with focus (-1, 0), vertex (-1, 2).
97. Write direction cosine of  $\overline{PQ}$ , if P(2, 1, 5), Q(1, 3, 1).
98. Write down the equation of straight line with x-intercept (2, 0) and y-intercept (0, -4)
99. Find the mid-point of line segment joining the points A  $(-\sqrt{5}, -\frac{1}{3})$  and  $(-3\sqrt{5}, 5)$ .
100. Find the slope and inclination of the line joining the points (-2, 4) and (5, 11).
101. Find equation of tangent to the circle  $x^2 + y^2 = 25$  at (4, 3).
102. Find the vertex and directrix of parabola  $x^2 = 4(y - 1)$ .
103. Find the centre and vertices of the ellipse  $9x^2 + y^3 = 18$ .
104. Find the sum of vectors  $\overline{AB}$  and  $\overline{CD}$ , given the four points A(1, -1), B(2, 0), C(-1, 3) and D(-2, 2).
105. Find a vector perpendicular to each of the vectors  $\underline{a} = 2\underline{i} + \underline{j} + \underline{k}$  and  $\underline{b} = 4\underline{i} + 2\underline{j} - \underline{k}$ .
106. Prove that the vectors  $\underline{i} - 2\underline{j} + 3\underline{k}$ ,  $2\underline{i} + 3\underline{j} - 4\underline{k}$  and  $\underline{i} - 3\underline{j} + 5\underline{k}$  are co-planar.
107. Find equation of a line through (-4, 7) and parallel to the line  $2x - 7y + 4 = 0$ .
108. Find equation of a line through (-6, 5) having slope = 7
109. Find distance from the point P (6, -1) to the line  $6x - 14y + 9 = 0$
110. Find area of triangular region whose vertices are A (5, 3), B (-2, 2), C (4, 2).
111. Find the equation of tangent to the circle  $x^2 + y^2 = 25$  at (4, 3). 14 Grp I,
112. Find the equation of parabola whose focus is (2, 5) and directrix is  $y = 1$
113. Find foci and eccentricity of ellipse
114. Find vector from A to origin whose  $\overline{AB} = 4\underline{i} - 2\underline{j}$  and B (-2, 5).
115. Find a vector whose magnitude is 2 and is parallel to  $\underline{i} + \underline{j} + \underline{k}$ .
116. Find  $\alpha$  so that the vectors  $2\underline{i} + \alpha\underline{j} + 5\underline{k}$  and  $3\underline{i} + \underline{j} + \alpha\underline{k}$  are perpendicular.
117. 129. Find  $\alpha$  so that  $\alpha\underline{i} + \underline{j}$ ,  $\underline{i} + \underline{j} + \alpha\underline{k}$ ,  $2\underline{i} + \underline{j} - 2\underline{k}$  are co-planar

## Long Questions

### 1. Chapter No. 1 (Functions and Limits)

- Evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$  17Grp I,
- Evaluate  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

3. Evaluate  $\lim_{\theta \rightarrow 0} \frac{(1 - \cos p\theta)}{(1 - \cos q\theta)}$
4. Evaluate  $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$
5. Find the values of  $m$  and  $n$ , so that given function  $f$  is continuous at  $x = 3$ .
6. If  $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$
7. Discuss the continuity of  $f(x)$  at  $x = 2$  and  $x = -2$ .
8. If  $f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$
9. If  $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2 \\ k & x = 2 \end{cases}$
10. Find the value of  $k$  so that  $f$  is continuous at  $x = 2$ .
11. Let  $f(x) = \frac{2x+1}{x-1}$ ;  $x \neq 1$ , find  $f^{-1}(x)$  and verify  $f \circ f^{-1}(x) = x$
12. Prove  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^n = e$  14 Grp II, 10. Prove that  $\lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x}\right) = \log_e a$
13. Prove that  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

## Chapter No. 2 (Differentiation)

1. Differentiate  $\frac{x^2+1}{x^2-1}$  w.r.t.  $\frac{x-1}{x+1}$
2. Differentiate  $x^2 + \frac{1}{x^2}$  w.r.t.  $x - \frac{1}{x}$
3. Differentiate  $\cos \sqrt{x}$  from the first principle.
4. Differentiate  $\sin \sqrt{\frac{1+2x}{1+x}}$  w.r.t  $x$
5. Find  $\frac{dy}{dx}$  if  $x = a(\cos t + \sin t)$ ,  $y = a(\sin t - t \cos t)$
6. Find two positive integers whose sum is 9 and the product of one with the square of the other will be maximum.
7. If  $x = \sin \theta$ ,  $y = \sin m\theta$ , Show that  $(1 - x^2)y_2 - xy_1 + m^2y = 0$
8. If  $y = (\cos^{-1} x)^2$ , prove that  $(1 - x^2)y_2 - xy_1 - 2 = 0$
9. If  $y = e^x \cdot \sin x$ , then prove that  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
10. Prove that  $y \frac{dy}{dx} + x = 0$  if  $x = \frac{1-t^2}{1+t^2}$ ,  $y = \frac{2t}{1+t^2}$ .
11. Show that  $\cos(x+h) = \cos x - h \sin x - \frac{h^2}{2} \cos x + \frac{h^3}{3} \sin x + \dots$  And evaluate  $\cos 61^\circ$
12. Show that  $\frac{dy}{dx} = \frac{y}{x}$  if  $\frac{y}{x} = \tan^{-1} \frac{y}{x}$ .
13. Show that  $y = \frac{n!}{x}$  has maximum value at  $x = e$ .
14. Show that  $y = x^x$  has a maximum value at  $= \frac{1}{e}$

## Chapter No. 3 (Integration)

1. Evaluate  $\int \frac{(1 - \sin x)}{(1 - \cos x)} e^x dx$
2. Evaluate  $\int \frac{(1 - \sin x)}{(1 - \cos x)} e^x dx$
3. Evaluate  $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$

4. Evaluate  $\int \frac{e^x(1+\sin x)}{(1+\cos x)} dx$
5. Evaluate  $\int \frac{1}{x(x^3-1)} dx$
6. Evaluate  $\int \cos^3 x \sqrt{\sin x} dx$ , ( $\sin x > 0$ )
7. Evaluate  $\int \operatorname{cosec}^3 x dx$
8. Evaluate  $\int \frac{\cos x}{\sin x \ln \sin x} dx$
9. Evaluate  $\int \frac{dx}{\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x}$
10. Evaluate  $\int_0^{\pi/2} 2^x \cos 3x dx$
11. Evaluate  $\int \tan^3 x \sec x dx$
12. Evaluate  $\int_0^{\pi/6} \frac{\cos x}{\sin x(2+\sin x)} dx$
13. Evaluate  $\int_0^{\pi/4} \cos^4 t dt$
14. Evaluate  $\int_0^{\pi/6} \cos^3 \theta d\theta$
15. Evaluate  $\int_0^{\pi/4} \frac{\sin x - 1}{\cos^2 x} dx$
16. Evaluate  $\int_0^{\pi/4} \frac{\sec \theta}{\sec \theta + \cos \theta} d\theta$
17. Evaluate  $\int_{-1}^2 (x + |x|) dx$
18. Evaluate  $\int_2^3 \frac{3x^2 - 2x + 1}{(x-1)(x^2+1)} dx$
19. Evaluate  $\int_2^3 \left(x - \frac{1}{x}\right)^2 dx$
20. Evaluate the indefinite integral  $\int \sqrt{a^2 - x^2} dx$
21. Find the area between the  $x$ -axis and the curve  $y = \sqrt{2ax - x^2}$ ;  $a > 0$
22. Find the area bounded by the curve  $y = x^3 - 4x$  and  $x$ -axis
23. Show that  $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln(x + \sqrt{x^2 - a^2}) + c$
24. Solve the differential equation  $(x^2 - yx^2) \frac{dy}{dx} + y^2 + xy^2 = 0$
25. Solve the following differential equation  $(x^2 - yx^2) \frac{dy}{dx} + y^2 + xy^2 = 0$
26. Solve the following differential equation  $1 + \cos x \tan y \frac{dy}{dx} = 0$
27. Solve the following differential equation  $x dy + y(x - 1) dx$
28. Use differentials to approximate the values of  $(31)^{1/5}$
29.  $y = \sqrt{2ax - x^2}$  when  $a > 0$ .

## Chapter No. 4 (Intro. to Analytic Geometry)

1. Find a joint equation of the straight lines through the origin perpendicular to the lines represented by  $x^2 + xy - 6y^2 = 0$
2. Find an equation of the perpendicular bisector joining the points  $A(3, 5)$  and  $B(9, 8)$
3. Find an equation of the perpendicular bisector of the segment joining the points  $A(3, 5)$  and  $B(9, 8)$
4. Find equations of the sides, altitudes and medians of the triangle whose vertices are  $A(-3, 2)$ ,  $B(5, 4)$  and  $C(3, -8)$ .
5. Find equations of two parallel lines perpendicular to  $2x - y + 3 = 0$  such that the product of the  $x$ -intercept and  $y$ -intercept of each is 3.
6. Find  $h$  such that the points  $A(\sqrt{3}, -1)$ ,  $B(0, 2)$ ,  $C(h, -2)$  are the vertices of a right triangle with right angle at the vertex  $A$ .
7. Find interior angles of a triangle whose vertices are  $A(6, 1)$ ,  $B(2, 7)$  and  $C(-6, 7)$ .
8. Find the condition that the line  $y = mx + c$  touches the circle  $x^2 + y^2 = a^2$  at a single point.
9. Find the condition that the lines  $y = m_1x + c_1$ ;  $m_2x + c_2$ ;  $y = m_3x + c_3$  are concurrent.
10. Find the distance between the given parallel lines. Also find equation of parallel lying midway between them.  $3x - 4y + 3 = 0$  and  $3x - 4y + 7 = 0$

11. Find the equations of altitudes of  $\triangle ABC$  whose vertices are  $A(-3, 2)$ ,  $B(5, 4)$  and  $C(3, -8)$
12. Find the interior angles of a triangle whose vertices are  $A(6, 1)$ ,  $B(2, 7)$ ,  $C(-6, -7)$ .
13. Find the length of the chord cut off from the line  $2x + 3y = 13$  by the circle  $x^2 + y^2 = 26$ .
14. Find the lines represented by each of the following and also find measure of the angle between them  $x^2 + 2xy \sec \alpha + y^2 = 0$
15. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long.
16. Prove that the line segments joining the mid-points of sides of quadrilateral taken in order form a parallelogram.
17. Prove that the midpoint of the hypotenuse of a right triangle is the circumcenter of the triangle. 11 Grp II,
18. The points  $A(-1, 2)$ ,  $B(6, 3)$  and  $C(2, -4)$  are vertices of a triangle. Show the line joining the midpoint  $D$  of  $AB$  and the midpoint  $E$  of  $AC$  is parallel to  $BC$  and  $DE = \frac{1}{2} BC$
19. The three points  $A(7, -1)$ ,  $B(-2, 2)$  and  $C(1, 4)$  are consecutive vertices of a parallelogram, find the fourth vertex.
20. The vertices of a triangle are  $A(-2, 3)$ ,  $B(-4, 1)$  and  $C(3, 5)$ . Find the circumcircle of the triangle.

## Chapter No. 5 (Linear Inequalities and Linear Programming)

1. Graph the feasible region of system of linear inequalities and find the corner points.
2.  $2x + 3y \leq 18$ ,  $x + 4y \leq 12$ ,  $3x + y \leq 12$ ,  $x \geq 0$ ,  $y \geq 0$
3. Graph the feasible region of system of linear inequalities and find the corner points.
4.  $3x + 7y \leq 21$ ,  $2x - y \leq -3$ ,  $y \geq 0$
5. Shade the feasible region and also find the corner points of:  $2x - 3y \leq 6$ ,  $2x + 3y \leq 12$ ,  $x \geq 0$ ,  $y \geq 0$
6. Minimize  $z = 2x + y$  subject to the constraints.  $x + y \geq 3$ ;  $7x + 5y \leq 35$ ;  $x \geq 0$ ;  $y \geq 0$
7. Graph the feasible region of system of linear inequalities and find the corner points.
8.  $x + y \leq 5$ ;  $-2x + y \leq 2$ ;  $y \geq 0$
9. Graph the feasible region of system of linear inequalities and find the corner points.
10.  $2x - 3y \leq 6$ ;  $2x + y \geq 2$ ;  $y \geq 0$ ,  $x \geq 0$
11. Minimize  $f(x, y) = x + 3y$  subject to constraint.
12.  $2x + 5y \leq 30$ ;  $5x + 4y \leq 20$ ;  $x \geq 0$ ,  $y \geq 0$
13. Minimize  $f(x, y) = 2x + 3y$  subject to constraint.
14.  $2x + y \leq 8$ ;  $x + 2y \leq 14$ ;  $x \geq 0$ ,  $y \geq 0$
15. Find the minimum value of  $\phi(x, y) = 4x + 6y$  under the constraints:  $2x - 3y \leq 6$ ,  $2x + y \geq 2$ ,  $2x + 3y \leq 12$ ,  $x \geq 0$ ,  $y \geq 0$
16. Minimize the function  $z = 3x + y$  subject to the constraints:  $3x + 5y \geq 6$ ,  $x + 6y \geq 9$ ,  $x \geq 0$ ,  $y \geq 0$

## Chapter No. 6 (Conic Sections)

1. Find an equation of parabola having its focus at the origin and directrix parallel to  $y$ -axis.
2. Find the centre, foci, eccentricity, vertices and equation of directrices of  $\frac{y^2}{4} - x^2 = 1$ .
3. Find  $x$  so that points  $A(1, -1, 0)$ ,  $B(-2, 2, 1)$  and  $C(0, 2, x)$  form triangle with right angle at  $C$ .
4. Find the coordinates of the points of intersection of the line  $2x + y + 5 = 0$  and the circle  $x^2 + y^2 + 2x - 9 = 0$ . Also find the length of intercepted chord.
5. Find equation of parabola with elements directrix :  $x = -2$ , focus  $(2, 2)$ .
6. Find an equation of parabola whose focus is  $F(-3, 4)$ , directrix line is  $3x - 4y + 5 = 0$ .
7. Find the focus, vertex and the directrix of the parabola  $x^2 - 4x - 8y + 4 = 0$ .
8. Write an equation of the parabola with axis  $y = 0$  and passing through  $(2, 1)$  and  $(11, -2)$ .
9. Show that the line  $3x - 2y = 0$  and  $2x + 3y - 13 = 0$  are tangents to the circle  $x^2 + y^2 + 6x - 4y = 0$ . 17
10. Show that the equation  $9x^2 - 18x + 4y^2 + 8y - 23 = 0$  represent an ellipse. Find its elements (foci, vertices and directrices)

11. Show that the equation  $x^2 + 16x + 4y^2 - 16y + 76 = 0$  represent an ellipse. Find its foci eccentricity, vertices and directrices.
12. Write equations of tangent lines to the circle  $x^2 + y^2 + 4x + 2y = 0$  down from the point  $P(-1, 2)$ . Also find the tangential distance.
13. Prove that in any triangle ABC by vector method  $a^2 = b^2 + c^2 - 2bc \cos A$
14. Find equation of ellipse having vertices  $(0, \pm 5)$  and eccentricity  $\frac{3}{5}$
15. Find an equation of the circle passing through the point  $(-2, -5)$  and touching the line  $3x + 4y - 24 = 0$  at the point  $(4, 3)$
16. Find the foci, vertex and directrix of the parabola  $y = 6x^2 - 1$ ,
17. Find equations of the tangents to the circle  $x^2 + y^2 = 2$
18. Find an equation of an ellipse with Foci  $(-3\sqrt{3}, 0)$  and vertices  $(\pm 6, 0)$
19. Find equation of the circle passing through  $A(a, 0)$ ,  $B(0, b)$  and  $C(0, 0)$
20. Find an equation of the parabola with focus  $(1, 2)$  and vertex  $(3, 2)$ ,
21. Write an equation of the circle that passes through the point  $A(a, 0)$ ,  $B(0, b)$ ,  $C(0, 0)$ ,
22. Write an equation of the circle that passes through the points  $A(4, 5)$ ,  $B(-4, -3)$ , and  $C(8, -3)$ .

## Chapter No. 7 (Vectors)

1. Find the value of  $\alpha$ , in the coplanar vectors  $\alpha \underline{i} + \underline{j}$ ,  $\underline{i} + \underline{j} + 3\underline{k}$ , and  $2\underline{i} + \underline{j} - 2\underline{k}$ .
2. If  $\underline{a} = 3\underline{i} - \underline{j} - 4\underline{k}$ ;  $\underline{b} = -2\underline{i} - 4\underline{j} - 3\underline{k}$  and  $\underline{c} = \underline{i} + 2\underline{j} - \underline{k}$ , then find a unit vector parallel to  $-3\underline{a} - 2\underline{b} + 4\underline{c}$ , 16
3. (Example) Find the volume of the tetrahedron whose vertices are  $A(2, 1, 8)$ ,  $B(3, 2, 9)$ ,  $C(2, 1, 4)$  and  $D(3, 3, 10)$ .
4. Prove that  $\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \sin \beta$  by method of vectors.
5. Find the volume of the tetrahedron with the vertices of  $A(0, 1, 2)$ ,  $B(3, 2, 1)$ ,  $C(1, 2, 1)$  and  $D(5, 5, 6)$
6. Find the constant  $a$  such that the vectors are coplanar  $\underline{i} - \underline{j} + \underline{k}$ ,  $\underline{i} - 2\underline{j} - 3\underline{k}$ , and  $3\underline{i} - a\underline{j} + 5\underline{k}$ .
7. The position vectors of the points A, B, C and D are  $2\underline{i} - \underline{j} + \underline{k}$ ,  $3\underline{i} + \underline{j}$ ,  $2\underline{i} + 4\underline{j} - 2\underline{k}$  and  $-\underline{i} + 2\underline{j} + \underline{k}$  respectively. Show that AB is parallel to CD.
8. A force of magnitude 6 units acting parallel to  $2\underline{i} - 2\underline{j} + \underline{k}$  displaces the point of application from  $(1, 2, 3)$  to  $(5, 3, 7)$ . Find the work done.
9. Prove by using vectors that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and half as long.
10. If  $\underline{a} + \underline{b} + \underline{c} = \underline{0}$  then prove that  $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$
11. A force  $\underline{F} = 4\underline{i} - 3\underline{k}$  passes through the point  $A(2, -2, 5)$ . Find the moment of the force about the point  $B(1, -3, 1)$
12. Find a unit vector perpendicular to both vectors  $\underline{a}$  and  $\underline{b}$  where  $\underline{a} = -\underline{i} - \underline{j} - \underline{k}$  and  $\underline{b} = 2\underline{i} - 3\underline{j} + 4\underline{k}$ ,
13. If  $\underline{a} = 3\underline{i} - \underline{j} - 4\underline{k}$ ,  $\underline{b} = -2\underline{i} - 4\underline{j} - 3\underline{k}$  and  $\underline{c} = \underline{i} + 2\underline{j} - \underline{k}$  find a unit vector parallel to  $3\underline{a} - 2\underline{b} + 4\underline{c}$ .
24. Find equation of the circle of radius 2 and tangent to the line  $x - y - 4 = 0$  at  $A(1, -3)$