DERIVATIVE OF EXPONENTIAL FUNCTION:

(1) Let $y = e^x$

$$y + \delta y = e^{x+\delta x}$$

$$\delta y = e^{x+\delta x} - y$$

$$\delta y = e^{x}.e^{\delta x} - e^{x}$$

$$\delta y = e^{x}(e^{\delta x} - 1)$$

$$\frac{\delta y}{\delta x} = e^{x}\left(\frac{e^{\delta x} - 1}{\delta x}\right)$$

$$\lim_{\delta x \to 0}\frac{\delta y}{\delta x} = e^{x}\left(\lim_{\delta x \to 0}\frac{e^{\delta x} - 1}{\delta x}\right)$$

$$\frac{dy}{dx} = e^{x} \times \ln e$$

$$= e^{x} \times 1$$

$$\boxed{\frac{d}{dx}\left(e^{x}\right) = e^{x}}$$

(2) Let $y = a^x$

$$y + \delta y = a^{x+\delta x}$$

$$\delta y = a^{x+\delta x} - y$$

$$\delta y = a^{x}.a^{\delta x} - a^{x}$$

$$= a^{x}\left(a^{\delta x} - 1\right)$$

$$\frac{\delta y}{\delta x} = a^{x}\left(\frac{a^{\delta x} - 1}{\delta x}\right)$$

$$\lim_{\delta x \to 0}\frac{\delta y}{\delta x} = a^{x}\left(\lim_{\delta x \to 0}\frac{a^{\delta x} - 1}{\delta x}\right) \qquad \because \lim_{\delta x \to 0}\frac{a^{\delta x} - 1}{\delta x} = \ln a$$

$$\frac{dy}{dx} = a^{x}.\ln a$$

$$\boxed{\frac{d}{dx}\left(a^{x}\right) = a^{x}\ln a}$$

DERIVATIVE OF THE LOGARITHMIC FUNCTION:

(3) Let $y = \ln x$

$$y + \delta y = \ln(x + \delta x)$$

$$\delta y = \ln(x + \delta x) - \mathrm{y}$$

$$\delta y = \ln\left(x + \delta x\right) - \ln x$$

$$\delta y=\ln\left(\frac{x+\delta x}{x}\right) \qquad \because \ell og m-\ell og n=\ell og\frac{m}{n}$$

$$\delta y=\ln\left(1+\frac{\delta x}{x}\right)$$

$$\frac{\delta y}{\delta x}=\frac{1}{\delta x}\ln\left(1+\frac{\delta x}{x}\right)$$

$$\frac{\delta y}{\delta x}=\frac{1}{x}.\frac{x}{\delta x}\ln\left(1+\frac{\delta x}{x}\right)$$

$$\frac{\delta y}{\delta x}=\frac{1}{x}\ln\left(1+\frac{\delta x}{x}\right)^{\frac{x}{\delta x}}$$

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\frac{1}{x}\ln\left[\lim_{\delta x\to 0}\left(1+\frac{\delta x}{x}\right)^{\frac{x}{\delta x}}\right] \qquad \because \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e$$

$$\frac{dy}{dx}=\frac{1}{x}\ln e \qquad \because \ell ne=1$$

$$\boxed{\frac{d}{dx}\left(\ln x\right)=\frac{1}{x}}$$

(4) **Let** $y=\log_a x$

$$y+\delta y=\log_a\left(x+\delta x\right)$$

$$\delta y=\log_a\left(x+\delta x\right)-y$$

$$\delta y=\log_a\left(x+\delta x\right)-\log_a x$$

$$\delta y=\log_a\left(\frac{x+\delta x}{x}\right)$$

$$\frac{\delta y}{\delta x}=\frac{1}{\delta x}\log_a\left(1+\frac{\delta x}{x}\right)$$

$$\frac{\delta y}{\delta x}=\frac{1}{x}.\frac{x}{\delta x}\log_a\left(1+\frac{\delta x}{x}\right)$$

$$\frac{\delta y}{\delta x}=\frac{1}{x}\log_a\left(1+\frac{\delta x}{x}\right)^{\frac{x}{\delta x}}$$

$$\lim_{\delta x\to 0}\frac{\delta y}{\delta x}=\frac{1}{x}\log_a\left[\lim_{\delta x\to 0}\left(1+\frac{\delta x}{x}\right)^{\frac{x}{\delta x}}\right]$$

$$\frac{dy}{dx}=\frac{1}{x}\log_a e$$

$$\frac{dy}{dx}=\frac{1}{x}\frac{1}{\log_e a}=\frac{1}{x\ln a}$$

$$\boxed{\frac{d}{dx}\left(\log_a x\right)=\frac{1}{x\ln a}}$$

DERIVATIVE OF HYPERBOLIC FUNCTIONS:

(1) $\frac{d}{dx}\left(\sinh x\right)=\cosh x$

(2) $\frac{d}{dx}\left(\cosh x\right)=\sinh x$

(3) $\frac{d}{dx}\left(\tanh x\right)=\text{sech}^2 x$

(4) $\frac{d}{dx}\left(\coth x\right)=-\text{cosech}^2 x$

(5) $\frac{d}{dx}\left(\text{sech}\, x\right)=-\text{sech}\, x\tanh x$

(6) $\frac{d}{dx}\left(\text{cosech}\, x\right)=-\text{cosech}\, x\coth x$

Proof:

(1) **Let** $y=\sinh x$

$$y=\frac{e^x-e^{-x}}{2}$$

Differentiate w.r.t ‘x’

$$\frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}\left(e^x-e^{-x}\right)$$

$$=\frac{1}{2}\left[\frac{d}{dx}\left(e^x\right)-\frac{d}{dx}\left(e^{-x}\right)\right]$$

$$=\frac{1}{2}\left(e^x+e^{-x}\right)$$

$$\boxed{\frac{d}{dx}\left(\sinh x\right)=\cosh x}$$

(2) **Let** $y=\cosh x$

$$y=\frac{e^x+e^{-x}}{2}$$

Differentiate w.r.t ‘x’

$$\frac{dy}{dx}=\frac{1}{2}\left[\frac{d}{dx}\left(e^x+e^{-x}\right)\right]$$

$$=\frac{1}{2}\left[\frac{d}{dx}\left(e^x\right)+\frac{d}{dx}\left(e^{-x}\right)\right]$$

$$=\frac{1}{2}\left(e^x-e^{-x}\right)$$

$$\boxed{\frac{d}{dx}\left(\cosh x\right)=\sinh x}$$

(3) **Let** $y=\tanh x$

$$y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

Differentiate w.r.t 'x'

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)$$

$$=\frac{\left(e^x+e^{-x}\right)\frac{d}{dx}\left(e^x-e^{-x}\right)-\left(e^x-e^{-x}\right)\frac{d}{dx}\left(e^x+e^{-x}\right)}{\left(e^x+e^{-x}\right)^2}=\frac{\left(e^x+e^{-x}\right)\left(e^x+e^{-x}\right)-\left(e^x-e^x\right)\left(e^x-e^{-x}\right)}{\left(e^x+e^{-x}\right)^2}$$

$$=\frac{\left(e^x+e^{-x}\right)^2-\left(e^x-e^{-x}\right)^2}{\left(e^x+e^{-x}\right)^2}$$

$$=\frac{e^{2x}+e^{-2x}+2-\left(e^{2x}+e^{-2x}-2\right)}{\left(e^x+e^{-x}\right)^2}$$

$$=\frac{\cancel{e}^{2x}+\cancel{e}^{-2x}+2-\cancel{e}^{2x}-\cancel{e}^{-2x}+2}{\left(e^x+e^{-x}\right)^2}$$

$$=\frac{4}{\left(e^x+e^{-x}\right)^2}$$

$$=\left(\frac{2}{e^x+e^{-x}}\right)^2$$

$$\boxed{\frac{d}{dx}\left(\tanh x\right)=\operatorname{sech}^2 x}$$

(4) **Let** $y=\coth x$

$$y=\frac{e^x+e^{-x}}{e^x-e^{-x}}$$

Differentiate w.r.t 'x'

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right)$$

$$=\frac{\left(e^x-e^{-x}\right)\frac{d}{dx}\left(e^x+e^{-x}\right)-\left(e^x+e^{-x}\right)\frac{d}{dx}\left(e^x-e^{-x}\right)}{\left(e^x-e^{-x}\right)^2}$$

$$=\frac{\left(e^x-e^{-x}\right)\left(e^x-e^{-x}\right)-\left(e^x+e^{-x}\right)\left(e^x+e^{-x}\right)}{\left(e^x-e^{-x}\right)^2}$$

$$=\frac{\left(e^x-e^{-x}\right)^2-\left(e^x+e^{-x}\right)^2}{\left(e^x-e^{-x}\right)^2}$$

$$= \frac{e^{2x} + e^{-2x} - 2 - \left(e^{2x} + e^{-2x} + 2\right)}{\left(e^{x} - e^{-x}\right)^{2}}$$

$$= \frac{e^{2x} + e^{-2x} - 2 - e^{2x} - e^{-2x} - 2}{\left(e^{x} - e^{-x}\right)^{2}}$$

$$= -\frac{4}{\left(e^{x} - e^{-x}\right)^{2}}$$

$$= -\left(\frac{2}{e^{x} - e^{-x}}\right)^{2}$$

$$\boxed{\frac{d}{dx}\left(\coth x\right) = -\text{cosech}^{2}\, x}$$

(5) **Let** $y = \text{sech}\, x$

$$y = \frac{2}{e^{x} + e^{-x}}$$

Differentiate w.r.t 'x'

$$\frac{dy}{dx} = 2\frac{d}{dx}\left(\frac{1}{e^{x} + e^{-x}}\right)$$

$$\frac{dy}{dx} = 2\left[\frac{\left(e^{x} + e^{-x}\right)\frac{d}{dx}(1) - 1\frac{d}{dx}\left(e^{x} + e^{-x}\right)}{\left(e^{x} + e^{-x}\right)^{2}}\right]$$

$$\frac{dy}{dx} = 2\left[\frac{0 - \left(e^{x} - e^{-x}\right)}{\left(e^{x} + e^{-x}\right)^{2}}\right]$$

$$= -2\frac{1}{\left(e^{x} + e^{-x}\right)^{2}}\left(e^{x} - e^{-x}\right)$$

$$= -\left(\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right)\frac{2}{\left(e^{x} + e^{-x}\right)}$$

$$\boxed{\frac{d}{dx}\left(\text{sech}\, x\right) = -\tanh x\ \text{sech}\, x}$$

(6) **Let** $y = \text{cosech}\, x$

$$y = \frac{2}{e^{x} - e^{-x}}$$

Differentiate w.r.t 'x'

$$\frac{dy}{dx} = 2\frac{d}{dx}\left(e^{x} - e^{-x}\right)^{-1}$$

$$= -2\left(e^{x} - e^{-x}\right)^{-2}\frac{d}{dx}\left(e^{x} - e^{-x}\right)$$

$$= -2\frac{e^x + e^{-x}}{\left(e^x - e^{-x}\right)^2}$$

$$= -\frac{e^x + e^{-x}}{e^x - e^{-x}} \cdot \frac{2}{e^x - e^{-x}}$$

$$\frac{d}{dx}(\operatorname{cosech} x) = -\coth x \operatorname{cosech} x$$

Inverse Hyperbolic Functions

(1) $\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$

(2) $\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$

(1) **Let** $y = \sinh^{-1} x$ for $x, y \in R$ then

$$\sinh y = x$$

$$x = \sinh y$$

$$x = \frac{e^y - e^{-y}}{2}$$

$$2x = e^y - e^{-y}$$

$$2x = e^y - \frac{1}{e^y}$$

$$2x = \frac{e^{2y} - 1}{e^y}$$

$$2e^y x = e^{2y} - 1$$

$$\left(e^y\right)^2 - 2x\left(e^y\right) - 1 = 0$$

Which is quadratic in e^y. Now we have

$$e^y = \frac{-(-2x) \pm \sqrt{(-2x)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2x \pm \sqrt{4x^2 + 4}}{2}$$

$$= \frac{2x \pm 2\sqrt{x^2 + 1}}{2}$$

$$e^y = x \pm \sqrt{x^2 + 1}$$

As e^y is positive for $y \in R$

So $e^y = x + \sqrt{x^2 + 1}$

$$\ln\left(e^y\right) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$y = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

(2) **Let** $y=\cosh^{-1}x$ $\left(\text{for } x\in(1,\infty),\ y\in(0,\infty)\right)$

$\cosh y = x$

$x=\cosh y$

$$x=\frac{e^{y}+e^{-y}}{2}$$

$$2x=e^{y}+e^{-y}$$

$$2x=e^{y}+\frac{1}{e^{y}}$$

$$2x=\frac{e^{2y}+1}{e^{y}}$$

$$2xe^{y}=e^{2y}+1$$

$$e^{2y}-2xe^{y}+1=0$$

$$\left(e^{y}\right)^{2}-2x\left(e^{y}\right)+1=0$$

Which is quadratic in e^{y}, then we have

$$e^{y}=\frac{-(-2x)\pm\sqrt{(-2x)^{2}-4(1)(1)}}{2(1)}$$

$$=\frac{2x\pm\sqrt{4x^{2}-4}}{2}$$

$$=\frac{2x\pm2\sqrt{x^{2}-1}}{2}$$

$$=x\pm\sqrt{x^{2}-1}$$

As e^{y} is positive

So $e^{y}=x+\sqrt{x^{2}-1}$

$$\ln\left(e^{y}\right)=\ln\left(x+\sqrt{x^{2}-1}\right)$$

$$y=\ln\left(x+\sqrt{x^{2}-1}\right)$$

$$\cosh^{-1}x=\ln\left(x+\sqrt{x^{2}-1}\right)$$

DERIVATIVE OF INVERSE HYPERBOLIC FUNCTIONS:

(1) $\frac{d}{dx}\left(\sinh^{-1}x\right)=\frac{1}{\sqrt{1+x^{2}}}$

(2) $\frac{d}{dx}\left(\cosh^{-1}x\right)=\frac{1}{\sqrt{x^{2}-1}}$

(3) $\frac{d}{dx}\left(\tanh^{-1}x\right)=\frac{1}{1-x^{2}}$

(4) $\frac{d}{dx}\left(\coth^{-1}x\right)=\frac{1}{1-x^{2}}$

(5) $\frac{d}{dx}\left(\operatorname{sech}^{-1}x\right)=\frac{-1}{x\sqrt{1-x^2}}$

(6) $\frac{d}{dx}\left(\operatorname{cosech}^{-1}x\right)=\frac{-1}{x\sqrt{1+x^2}}$

Proof:

(1) **Let** $y=\sinh^{-1}x$

$\sinh y=x$

Differentiate w.r.t 'x'

$$\frac{d}{dx}\left(\sinh y\right)=\frac{d}{dx}\left(x\right)$$

$$\cosh y\frac{dy}{dx}=1$$

$$\frac{dy}{dx}=\frac{1}{\cosh y} \qquad \left(\cosh^2 y=1+\sinh^2 y\right)$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{1+\sinh^2 y}}$$

$$\boxed{\frac{dy}{dx}=\frac{1}{\sqrt{1+x^2}}}$$

(2) **Let** $y=\cosh^{-1}x$

$\cosh y=x$

Differentiate w.r.t 'x'

$$\frac{d}{dx}\left(\cosh y\right)=\frac{d}{dx}\left(x\right)$$

$$\sinh y\frac{dy}{dx}=1$$

$$\frac{dy}{dx}=\frac{1}{\sinh y}$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{\cosh^2 y-1}} \qquad \left(\sinh^2 y=\cosh^2 y-1\right)$$

$$\boxed{\frac{d}{dx}\left(\cosh^{-1}x\right)=\frac{1}{\sqrt{x^2-1}}}$$

(3) **Let** $y=\tanh^{-1}x$

$\tanh y=x$

Differentiate w.r.t 'x'

$$\frac{d}{dx}\left(\tanh y\right)=\frac{d}{dx}\left(x\right)$$

$$\operatorname{sech}^2 y\frac{dy}{dx}=1$$

$$\frac{dy}{dx}=\frac{1}{\operatorname{sech}^2 y} \qquad \because \operatorname{sech}^2 y=1-\tanh^2 y$$

$$\frac{dy}{dx}=\frac{1}{1-\tanh^2 y}$$

$$\boxed{\frac{d}{dx}\left(\tanh^{-1}x\right)=\frac{1}{1-x^2}}$$

(4) **Let** $y=\coth^{-1}x$

$\coth y = x$

Differentiate w.r.t 'x'

$$\frac{d}{dx}(\coth y)=\frac{d}{dx}(x)$$

$$-\operatorname{cosech}^2 y\frac{dy}{dx}=1$$

$$\frac{dy}{dx}=\frac{1}{-\operatorname{cosech}^2 y} \qquad \because \operatorname{cosech}^2 y=\coth^2 y-1$$

$$\frac{dy}{dx}=\frac{-1}{\coth^2 y-1}$$

$$=\frac{1}{1-\coth^2 y}$$

$$\boxed{\frac{d}{dx}\left(\coth^{-1}x\right)=\frac{1}{1-x^2}}$$

(5) **Let** $y=\operatorname{sech}^{-1}x$

$\operatorname{sech} y = x$

Differentiate w.r.t 'x'

$$\frac{d}{dx}(\operatorname{sech} y)=\frac{d}{dx}(x)$$

$$-\operatorname{sech} y\ \tanh y\frac{dy}{dx}=1$$

$$\frac{dy}{dx}=-\frac{1}{\operatorname{sech} y\tanh y} \qquad \because \tanh^2 y=1-\operatorname{sech}^2 y$$

So $$\frac{dy}{dx}=-\frac{1}{\operatorname{sech} y\sqrt{1-\operatorname{sech}^2 y}}$$

$$\boxed{\frac{d}{dx}\left(\operatorname{sech}^{-1}x\right)=-\frac{1}{x\sqrt{1-x^2}}}$$

(6) **Let** $y=\operatorname{cosech}^{-1}x$

$\operatorname{cosech} y = x$

Differentiate w.r.t 'x'

$$\frac{d}{dx}(\operatorname{cosech} y)=\frac{d}{dx}(x)$$

$$-\operatorname{cosech} y\ \coth y\ \frac{dy}{dx}=1$$

$$\frac{dy}{dx}=-\frac{1}{\operatorname{cosech} y\coth y} \qquad \because \coth^2 y=1+\operatorname{cosech}^2 y$$

$$\frac{d}{dx}\left(\operatorname{cosech}^{-1}x\right)=-\frac{1}{\operatorname{cosech} y\sqrt{1+\operatorname{cosech}^2 y}}$$

$$\boxed{\frac{d}{dx}\left(\operatorname{cosech}^{-1}x\right)=-\frac{1}{x\sqrt{1+x^2}}}$$

EXERCISE 2.6

Q.1 Find $f'(x)$ if

(i) $f(x)=e^{\sqrt{x}-1}$

Solution:

$$f(x)=e^{\sqrt{x}-1}$$

Differentiate w.r.t "*x*"

$$f'(x)=e^{\sqrt{x}-1}\cdot\frac{d}{dx}\left(\sqrt{x}-1\right)$$

$$=e^{\sqrt{x}-1}\cdot\frac{1}{2\sqrt{x}}$$

$$\boxed{f'(x)=\frac{e^{\sqrt{x}-1}}{2\sqrt{x}}}$$

(ii) $f(x)=x^3e^{\frac{1}{x}}$

Solution:

$$f(x)=x^3e^{\frac{1}{x}}$$

Differentiate w.r.t "*x*"

$$f'(x)=x^3\frac{d}{dx}\left(e^{\frac{1}{x}}\right)+e^{\frac{1}{x}}\frac{d}{dx}\left(x^3\right)$$

$$=x^3e^{\frac{1}{x}}\left(\frac{-1}{x^2}\right)+e^{\frac{1}{x}}\left(3x^2\right)$$

$$=-xe^{\frac{1}{x}}+3x^2e^{\frac{1}{x}}$$

$$\boxed{f'(x)=xe^{\frac{1}{x}}\left(3x-1\right)}$$

(iii) $f(x)=e^x\left(1+\ln x\right)$

Solution:

$$f(x)=e^x\left(1+\ln x\right)$$

Differentiate w.r.t "*x*"

$$f'(x)=e^x\frac{d}{dx}\left(1+\ln x\right)+\left(1+\ln x\right)\frac{d}{dx}\left(e^x\right)$$

$$=e^x\left(\frac{1}{x}\right)+\left(1+\ln x\right)e^x$$

$$\boxed{f'(x)=e^x\left[\frac{1}{x}+1+\ln x\right]}$$

(iv) $f(x)=\frac{e^x}{e^{-x}+1}$

Solution:

$$f(x)=\frac{e^x}{e^{-x}+1}$$

Differentiate w.r.t "x"

$$f'(x)=\frac{(e^{-x}+1)\frac{d}{dx}(e^x)-(e^x)\frac{d}{dx}(e^{-x}+1)}{(e^{-x}+1)^2}$$

$$=\frac{(e^{-x}+1)e^x+(e^x)(e^{-x})}{(e^{-x}+1)^2}$$

$$=\frac{1+e^x+1}{(e^{-x}+1)^2}$$

$$\boxed{f'(x)=\frac{2+e^x}{(e^{-x}+1)^2}}$$

(v) $f(x)=\ln(e^x+e^{-x})$

Solution:

$$f(x)=\ln(e^x+e^{-x})$$

Differentiate w.r.t "x"

$$f'(x)=\frac{1}{e^x+e^{-x}}\frac{d}{dx}(e^x+e^{-x})$$

$$=\frac{1}{e^x+e^{-x}}\left[e^x-e^{-x}\right]$$

$$\boxed{f'(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}}$$

(vi) $f(x)=\frac{e^{ax}-e^{-ax}}{e^{ax}+e^{-ax}}$

Solution:

$$f(x)=\frac{e^{ax}-e^{-ax}}{e^{ax}+e^{-ax}}$$

Differentiate w.r.t "x"

$$f'(x)=\frac{(e^{ax}+e^{-ax})\frac{d}{dx}(e^{ax}-e^{-ax})-(e^{ax}-e^{-ax})\frac{d}{dx}(e^{ax}+e^{-ax})}{(e^{ax}+e^{-ax})^2}$$

$$f'(x)=\frac{\left(e^{ax}+e^{-ax}\right)\left(ae^{ax}+ae^{-ax}\right)-\left(e^{ax}-e^{-ax}\right)\left(ae^{ax}-ae^{-ax}\right)}{\left(e^{ax}+e^{-ax}\right)^2}$$

$$=\frac{a\left(e^{ax}+e^{-ax}\right)^2-a\left(e^{ax}-e^{-ax}\right)^2}{\left(e^{ax}+e^{-ax}\right)^2}$$

$$=\frac{a\left(e^{2ax}+e^{-2ax}+2\right)-a\left(e^{2ax}+e^{-2ax}-2\right)}{\left(e^{ax}+e^{-ax}\right)^2}$$

$$\boxed{f'(x)=\frac{4a}{\left(e^{ax}+e^{-ax}\right)^2}}$$

(vii) $f(x)=\sqrt{\ln\left(e^{2x}+e^{-2x}\right)}$

Solution:

$$f(x)=\sqrt{\ln\left(e^{2x}+e^{-2x}\right)}$$

Differentiate w.r.t "*x*"

$$f'(x)=\frac{1}{2\sqrt{\ln\left(e^{2x}+e^{-2x}\right)}}\frac{d}{dx}\ln\left[e^{2x}+e^{-2x}\right]$$

$$=\frac{1}{2\sqrt{\ln\left(e^{2x}+e^{-2x}\right)}\left(e^{2x}+e^{-2x}\right)}\left[2e^{2x}-2e^{-2x}\right]$$

$$=\frac{1}{2\sqrt{\ln\left(e^{2x}+e^{-2x}\right)}\left(e^{2x}+e^{-2x}\right)}2\left[e^{2x}-e^{-2x}\right]$$

$$\boxed{f'(x)=\frac{e^{2x}-e^{-2x}}{\left(e^{2x}+e^{-2x}\right)\sqrt{\ln\left(e^{2x}+e^{-2x}\right)}}}$$

(viii) $f(x)=\ln\sqrt{e^{2x}+e^{-2x}}$

Solution:

$$f(x)=\ln\sqrt{e^{2x}+e^{-2x}}$$

Differentiate w.r.t "*x*"

$$f'(x)=\frac{1}{\sqrt{e^{2x}+e^{-2x}}}\frac{d}{dx}\sqrt{e^{2x}+e^{-2x}}$$

$$=\frac{1}{\sqrt{e^{2x}+e^{-2x}}}\cdot\frac{1}{2\sqrt{e^{2x}+e^{-2x}}}\left[2e^{2x}-2e^{-2x}\right]$$

$$f'(x)=\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$$

$$\boxed{f'(x)=\tanh 2x}$$

Q.2 Find $\frac{dy}{dx}$ if

(i) $y=x^2\ln\sqrt{x}$

Solution:

$$y=x^2\ln\sqrt{x}$$

Differentiate w.r.t “x’

$$\frac{dy}{dx}=x^2\frac{d}{dx}\left(\ln\sqrt{x}\right)+\ln\sqrt{x}\frac{d}{dx}\left(x^2\right)$$

$$=x^2\frac{1}{\sqrt{x}}\frac{1}{2\sqrt{x}}+\ln\sqrt{x}.(2x)$$

$$=\frac{x^2}{2x}+2x\ln\sqrt{x}$$

$$=2x\ln\sqrt{x}+\frac{x}{2}$$

$$\boxed{\frac{dy}{dx}=x\left(2\ln\sqrt{x}+\frac{1}{2}\right)}$$

(ii) $y=x\sqrt{\ln x}$

Solution:

$$y=x\sqrt{\ln x}$$

Differentiate w.r.t “x”

$$\frac{dy}{dx}=x\frac{d}{dx}\sqrt{\ln x}+\sqrt{\ln x}\frac{d}{dx}(x)$$

$$=x\frac{1}{2\sqrt{\ln x}}.\frac{1}{x}+\sqrt{\ln x}$$

$$=\frac{x}{2\sqrt{\ln x}}.\frac{1}{x}+\sqrt{\ln x}$$

$$=\frac{1}{2\sqrt{\ln x}}+\sqrt{\ln x}$$

$$\boxed{\frac{dy}{dx}=\frac{1+2\ln x}{2\sqrt{\ln x}}}$$

(iii) $y=\frac{x}{\ln x}$

Solution:

$$y=\frac{x}{\ln x}$$

Differentiate w.r.t “x”

$$\frac{dy}{dx}=\frac{\ln x\frac{d}{dx}\left(x\right)-x\frac{d}{dx}\left(\ln x\right)}{\left(\ln x\right)^{2}}$$

$$=\frac{\ln x-x\left(\frac{1}{x}\right)}{\left(\ln x\right)^{2}}$$

$$\boxed{\frac{dy}{dx}=\frac{\ln x-1}{\left(\ln x\right)^{2}}}$$

(iv) $y=x^{2}\ln\frac{1}{x}$

Solution:

$$y=x^{2}\ln\frac{1}{x}$$

$$=x^{2}\left[\ln 1-\ln x\right]$$

$$=x^{2}\left[0-\ln x\right]$$

$$y=-x^{2}\ln x$$

Differentiate w.r.t "x"

$$\frac{dy}{dx}=-\left[x^{2}.\frac{1}{x}+\ln x\left(2x\right)\right]$$

$$\boxed{\frac{dy}{dx}=-\left[x+2x\ln x\right]}$$

(v) $y=\ln\sqrt{\frac{x^{2}-1}{x^{2}+1}}$

Solution:

$$y=\ln\sqrt{\frac{x^{2}-1}{x^{2}+1}}$$

$$y=\ln\frac{\sqrt{x^{2}-1}}{\sqrt{x^{2}+1}}$$

$$y=\ln\sqrt{x^{2}-1}-\ln\sqrt{x^{2}+1}$$

$$y=\frac{1}{2}\ln\left(x^{2}-1\right)-\frac{1}{2}\ln\left(x^{2}+1\right)\qquad\because \log m^{n}=n\log m$$

Differentiate w.r.t "x"

$$\frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}\ln\left(x^{2}-1\right)-\frac{1}{2}\frac{d}{dx}\ln\left(x^{2}+1\right)$$

$$=\frac{1}{2}.\frac{1}{x^{2}-1}\left(2x\right)-\frac{1}{2}\frac{\left(2x\right)}{x^{2}+1}$$

$$= \frac{x}{x^2-1} - \frac{x}{x^2+1}$$

$$= \frac{x\left(x^2+1\right) - x\left(x^2-1\right)}{\left(x^2-1\right)\left(x^2+1\right)}$$

$$= \frac{\cancel{x^3} + x - \cancel{x^3} + x}{x^4-1}$$

$$\boxed{\frac{dy}{dx} = \frac{2x}{x^4-1}}$$

(vi) $y = \ln\left(x + \sqrt{x^2+1}\right)$

Solution:

$$y = \ln\left(x + \sqrt{x^2+1}\right)$$

Differentiate w.r.t "x"

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left[1 + \frac{1(2x)}{2\sqrt{x^2+1}}\right]$$

$$= \frac{1}{x+\sqrt{x^2+1}}\left[\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right]$$

$$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}}$$

(vii) $y = \ln\left(9 - x^2\right)$

Solution:

$$y = \ln\left(9 - x^2\right)$$

Differentiate w.r.t "x"

$$\frac{dy}{dx} = \frac{1}{9-x^2}\frac{d}{dx}\left(9-x^2\right)$$

$$= \frac{1}{9-x^2}(-2x)$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{9-x^2}}$$

(viii) $y = e^{-2x}\sin 2x$

Solution:

$$y = e^{-2x}\sin 2x$$

Differentiate w.r.t "x"

$$\frac{dy}{dx} = e^{-2x}\frac{d}{dx}(\sin 2x) + \sin 2x\frac{d}{dx}\left(e^{-2x}\right)$$

$$= e^{-2x}\cos 2x.2 + \sin 2x\left(-2e^{-2x}\right)$$

$$\boxed{\frac{dy}{dx} = 2e^{-2x}\left[\cos 2x - \sin 2x\right]}$$

(ix) $y = e^{-x}\left(x^3 + 2x^2 + 1\right)$

Solution:

$$y = e^{-x}\left(x^3 + 2x^2 + 1\right)$$

Differentiate w.r.t "*x*"

$$\frac{dy}{dx} = e^{-x}\frac{d}{dx}\left(x^3 + 2x^2 + 1\right) + \left(x^3 + 2x^2 + 1\right)\frac{d}{dx}\left(e^{-x}\right)$$

$$\frac{dy}{dx} = e^{-x}\left[3x^2 + 4x\right] + \left(x^3 + 2x^2 + 1\right)\left(e^{-x}\right)(-1)$$

$$= e^{-x}\left(3x^2 + 4x\right) - e^{-x}\left(x^3 + 2x^2 + 1\right)$$

$$= e^{-x}\left[3x^2 + 4x - x^3 - 2x^2 - 1\right]$$

$$\boxed{\frac{dy}{dx} = -e^{-x}\left(x^3 - x^2 - 4x + 1\right)}$$

(x) $y = xe^{\sin x}$

Solution:

$$y = xe^{\sin x}$$

Differentiate w.r.t "*x*"

$$\frac{dy}{dx} = x\frac{d}{dx}\left(e^{\sin x}\right) + e^{\sin x}\frac{d}{dx}(x)$$

$$= xe^{\sin x}.\cos x + e^{\sin x}.1$$

$$\boxed{\frac{dy}{dx} = e^{\sin x}\left(x\cos x + 1\right)}$$

(xi) $y = 5e^{3x-4}$

Solution:

$$y = 5e^{3x-4}$$

Differentiate w.r.t "*x*"

$$\frac{dy}{dx} = 5\frac{d}{dx}\left(e^{3x-4}\right) = 5e^{3x-4}\frac{d}{dx}(3x-4)$$

$$= 5e^{3x-4}.(3)$$

$$\boxed{\frac{dy}{dx} = 15e^{3x-4}}$$

(xii) $y = (x+1)^x$

Solution:

$$y = (x+1)^x$$

Taking natural log on both sides,

$$\ln y = \ln(x+1)^x$$

$$\ln y = x.\ln(x+1)$$

Differentiate w.r.t "*x*"

$$\frac{d}{dx}(\ln y)=x\frac{d}{dx}\ln(x+1)+\ln(x+1)\frac{d}{dx}(x)$$

$$\frac{1}{y}\frac{dy}{dx}=x.\frac{1}{x+1}+\ln(x+1).1$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{x}{x+1}+\ln(x+1)$$

$$\frac{dy}{dx}=y\left[\frac{x+(x+1)\ln(x+1)}{x+1}\right]$$

$$\boxed{\frac{dy}{dx}=(x+1)^{x}\left[\frac{x+(x+1)\ln(x+1)}{x+1}\right]}$$

Or

$$\boxed{\frac{dy}{dx}=(x+1)^{x}\left[\frac{x}{x+1}+\ln(x+1)\right]}$$

(xiii) $y=(\ln x)^{\ln x}$

Solution:

$$y=(\ln x)^{\ln x}$$

Taking natural log on both sides,

$$\ln y=\ln(\ln x)^{\ln x}$$

$$\ln y=\ln x.\ln(\ln x)$$

Differentiate w.r.t "*x*"

$$\frac{d}{dx}(\ln y)=\ln x\frac{d}{dx}(\ln(\ln x))+\ln(\ln x)\frac{d}{dx}(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx}=\cancel{\ln x}.\frac{1}{\cancel{\ln x}}.\frac{1}{x}+\ln(\ln x).\frac{1}{x}$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{x}+\frac{1}{x}\ln(\ln x)$$

$$\frac{dy}{dx}=y\left[\frac{1+\ln(\ln x)}{x}\right]$$

$$\boxed{\frac{dy}{dx}=(\ln x)^{\ln x}\left[\frac{1+\ln(\ln x)}{x}\right]}$$

(xiv) $y=\dfrac{\sqrt{x^2-1}\left(x+1\right)}{\left(x^3+1\right)^{\frac{3}{2}}}$

Solution:

$$y=\frac{\sqrt{x^2-1}\left(x+1\right)}{\left(x^3+1\right)^{\frac{3}{2}}}$$

Taking natural log on both sides,

$$\ln y=\ln\frac{\left(x^2-1\right)^{\frac{1}{2}}\left(x+1\right)}{\left(x^3+1\right)^{\frac{3}{2}}}$$

$$\ln y=\ln\left(x^2-1\right)^{\frac{1}{2}}+\ln\left(x+1\right)-\ln\left(x^3+1\right)^{\frac{3}{2}}$$

$$\ln y=\frac{1}{2}\ln\left(x^2-1\right)+\ln\left(x+1\right)-\frac{3}{2}\ln\left(x^3+1\right)$$

Differentiate w.r.t "*x*"

$$\frac{d}{dx}(\ln y)=\frac{1}{2}\frac{d}{dx}\left(x^2-1\right)+\frac{d}{dx}\ln\left(x+1\right)-\frac{3}{2}\frac{d}{dx}\ln\left(x^3+1\right)$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{2}\frac{1}{x^2-1}(2x)+\frac{1}{x+1}-\frac{3}{2}.\frac{1\left(3x^2\right)}{x^3+1}$$

$$=\frac{x}{x^2-1}+\frac{1}{x+1}-\frac{9x^2}{2\left(x^3+1\right)}$$

$$=\frac{x}{(x-1)(x+1)}+\frac{1}{x+1}-\frac{9x^2}{2\left(x+1\right)\left(x^2-x+1\right)}$$

$$=\frac{x(2)\left(x^2-x+1\right)+(2)(x-1)\left(x^2-x+1\right)-9x^2(x-1)}{2(x-1)(x+1)\left(x^2-x+1\right)}$$

$$=\frac{2x^3-2x^2+2x+2\left(x^3-2x^2+2x-1\right)-9x^3+9x^2}{2(x-1)(x+1)\left(x^2-x+1\right)}$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{2x^3-2x^2+2x+2x^3-4x^2+4x-2-9x^3+9x^2}{2(x-1)(x+1)\left(x^2-x+1\right)}$$

$$\frac{1}{y}\frac{dy}{dx}=\frac{-5x^3+3x^2+6x-2}{2(x-1)(x+1)\left(x^2-x+1\right)}$$

$$\frac{dy}{dx}=\frac{(x-1)^{\frac{1}{2}}(x+1)^{\frac{1}{2}}\left(\cancel{x+1}\right)}{(x-1)^{\frac{3}{2}}\left(x^2-x+1\right)^{\frac{3}{2}}}\left[\frac{-5x^3+3x^2+6x-2}{2(x-1)\left(\cancel{x+1}\right)\left(x^2-x+1\right)}\right]$$

$$\frac{dy}{dx}=\frac{\left(-5x^3+3x^2+6x-2\right)\left(\cancel{x+1}\right)}{2(x+1)\left(x^2-x+1\right)^{\frac{5}{2}}(x-1)^{\frac{1}{2}}\left(\cancel{x+1}\right)}$$

$$\boxed{\frac{dy}{dx}=-\frac{5x^3-3x^2-6x+2}{2\sqrt{x-1}(x+1)\left(x^2-x+1\right)^{\frac{5}{2}}}}$$

$$\boxed{\frac{dy}{dx}=-\frac{5x^2-8x+2}{2\sqrt{x-1}\left(x^2-x+1\right)^{\frac{5}{2}}}}$$

$$\boxed{\begin{array}{r}
5x^2-8x+2\\
\because x+1\overline{\smash{\big)}\,5x^3-3x^2-6x+2}\\
\underline{5x^3\pm5x^2\qquad\qquad}\\
-8x^2-6x\qquad\\
\underline{-8x^2\mp8x\qquad}\\
2x+2\\
\underline{2x+2}\\
0\;\;
\end{array}}$$

Q.3 Find $\frac{dy}{dx}$ if

(i) $y=\cos h2x$

Solution:

$y=\cos h2x$

Differentiate w.r.t "x"

$$\frac{dy}{dx}=\sin h2x\frac{d}{dx}(2x)$$

$$=\sin h2x.2$$

$$\boxed{\frac{dy}{dx}=2\sin h2x}$$

(ii) $y=\sinh 3x$

Solution:

$y=\sinh 3x$

Differentiate w.r.t "x"

$$\frac{dy}{dx}=\cosh 3x\frac{d}{dx}(3x)$$

$$\boxed{\frac{dy}{dx}=3\cos h3x}$$

(iii) $y=\tanh^{-1}(\sin x)$

Solution:

$y=\tanh^{-1}(\sin x)$

Differentiate w.r.t "x"

$$\frac{dy}{dx}=\frac{d}{dx}\left(\tanh^{-1}\left(\sin x\right)\right)$$

$$=\frac{1}{1-\sin^2 x}\frac{d}{dx}\left(\sin x\right)$$

$$=\frac{1}{\cos^2 x}\cos x$$

$$=\frac{1}{\cos x}$$

$$\boxed{\frac{dy}{dx}=\sec x}$$

(iv) $y=\sinh^{-1}\left(x^3\right)$

Solution:

$$y=\sinh^{-1}\left(x^3\right)$$

Differentiate w.r.t "x"

$$\frac{dy}{dx}=\frac{d}{dx}\sinh^{-1}\left(x^3\right)$$

$$=\frac{1}{\sqrt{1+\left(x^3\right)^2}}\frac{d}{dx}\left(x^3\right)$$

$$=\frac{1}{\sqrt{1+x^6}}\left(3x^2\right)$$

$$\boxed{\frac{dy}{dx}=\frac{3x^2}{\sqrt{1+x^6}}}$$

(v) $y=\ln\tanh x$

Solution:

$$y=\ln\tanh x$$

Differentiate w.r.t "x"

$$\frac{dy}{dx}=\frac{1}{\tan hx}.\sec h^2 x$$

$$=\coth x\operatorname{sech}^2 x$$

$$=\frac{\cosh x}{\sinh x}\frac{1}{\cosh^2 x}$$

$$=\frac{1}{\sinh x\cosh x}$$

$$=\frac{2}{2\sinh x\cosh x}$$

$$\boxed{\frac{dy}{dx}=2\cos ec\,h2x}$$

(vi) $y=\sinh^{-1}\left(\frac{x}{2}\right)$

Solution:

$$y=\sinh^{-1}\left(\frac{x}{2}\right)$$

Differentiate w.r.t "x"

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sinh^{-1}\left(\frac{x}{2}\right)\right)$$

$$=\frac{1}{\sqrt{1+\frac{x^2}{4}}}\frac{d}{dx}\left(\frac{x}{2}\right)$$

$$=\frac{1}{\sqrt{\frac{4+x^2}{4}}}\frac{1}{2}$$

$$=\frac{1}{\frac{\sqrt{4+x^2}}{2}}\frac{1}{2}$$

$$\boxed{\frac{dy}{dx}=\frac{1}{\sqrt{4+x^2}}}$$