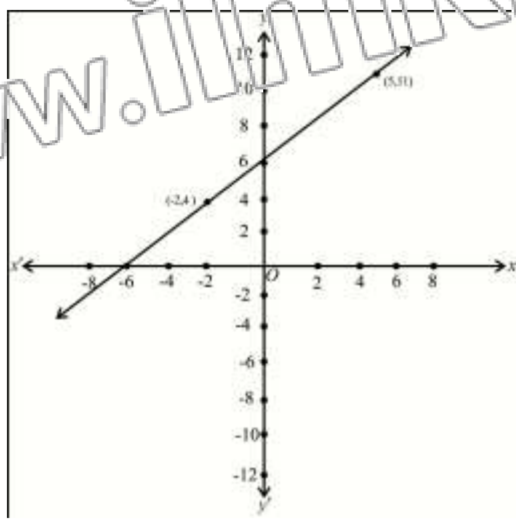
EXERCISE 4.3

Q.1 Find the slope and inclination of the line joining the points. Sketch each line in the plane

(i) $(-2,4);(5,11)$

Solution:



$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{11-4}{5-(-2)} = \frac{7}{7}$$

$$\boxed{m = 1}$$

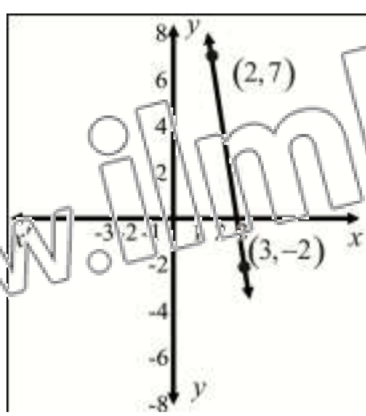
$$m = \tan\alpha = 1$$

$$\alpha = \tan^{-1}(1)$$

$$\boxed{\alpha = 45^\circ}$$

(ii) $(3,-2);(2,7)$

Solution:



$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7-(-2)}{2-3} = \frac{9}{-1}$$

$$\boxed{m = -9}$$

$$m = \tan\alpha = -9$$

$$\alpha = \tan^{-1}(-9)$$

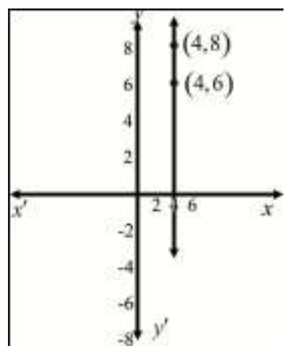
$$= 180^\circ - \tan^{-1}(9)$$

$$= 180^\circ - 83.66^\circ$$

$$\boxed{\alpha = 96.34^\circ}$$

(iii) $(4,6);(4,8)$

Solution:



$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8-6}{4-4} = \infty$$

$$\boxed{m = \infty}$$

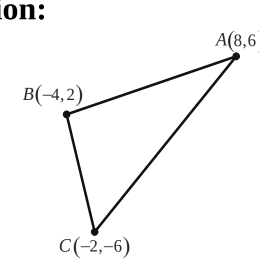
$$m = \tan\alpha = \infty$$

$$\Rightarrow \quad \boxed{\alpha = 90^\circ}$$

Q.2 In the triangle $A(8,6)$, $B(-4,2)$ and $C(-2,-6)$, find the slope of

(a) Each side of the triangle

Solution:



Slope of side $\overline{AB} = m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{2-6}{-4-8}$$

$$= \frac{-4}{-12} = \frac{1}{3}$$

Slope of side $\overline{BC} = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-6-2}{-2-(-4)}$$

$$= \frac{-8}{2} = -4$$

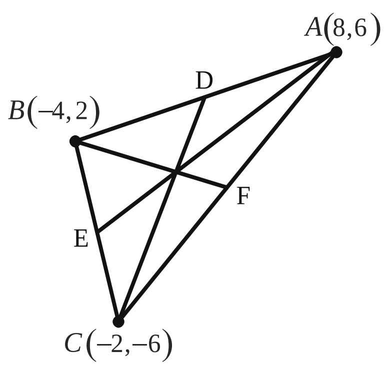
Slope of side $\overline{AC} = m_3 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-6-6}{-2-8}$$

$$= \frac{-12}{-10} = \frac{6}{5}$$

(b) Each median of the triangle

Solution:



Let D, E, F be the mid points of sides $\overline{AB}, \overline{BC}$ and $\overline{AC}$ respectively

Using mid-point formula on $A(8,6)$ and $B(-4,2)$

$$D\left(\frac{8+(-4)}{2}, \frac{6+2}{2}\right) = D(2,4)$$

Using mid-point formula on $B(-4,2)$ and $C(-2,-6)$

$$E\left(\frac{-4+(-2)}{2}, \frac{2+(-6)}{2}\right) = E(-3,-2)$$

Using mid-point formula on $A(8,6)$ and $C(-2,-6)$

$$F\left(\frac{8+(-2)}{2}, \frac{6+(-6)}{2}\right) = F(3,0)$$

Slope of median $\overline{CD} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{4-(-6)}{2-(-2)}$$

$$= \frac{10}{4} = \frac{5}{2}$$

Slope of median $\overline{BF} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{0-2}{3+4} = \frac{-2}{7}$$

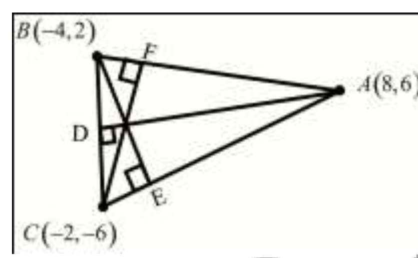
Slope of median $\overline{AE} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-2-6}{-3-8}$$

$$= \frac{-8}{-11} = \frac{8}{11}$$

(c) Each altitude of the triangle

Solution:



Slope of side $\overline{AB} = m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-6}{-4-8}$

$$= \frac{-4}{-12} = \frac{1}{3}$$

Slope of side $\overline{BC} = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-6-2}{-2-(-4)}$$

$$= \frac{-8}{2} = -4$$

Slope of side $\overline{AC} = m_3 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-6-6}{-2-8}$

$= \dfrac{-12}{-10} = \dfrac{6}{5}$

$\overline{AD}$ is altitude on side $\overline{BC}$. As $\overline{AD} \perp \overline{BC}$ so,

$\left(\text{Slope of } \overline{AD}\right)\left(\text{Slope of } \overline{BC}\right) = -1$

$\left(\text{Slope of } \overline{AD}\right)(-4) = -1$

$\text{Slope of } \overline{AD} = \dfrac{1}{4}$

$\overline{BE}$ is altitude on side $\overline{AC}$. As $\overline{BE} \perp \overline{AC}$ so,

$\left(\text{Slope of } \overline{BE}\right)\left(\text{Slope of } \overline{AC}\right) = -1$

$\left(\text{Slope of } \overline{BE}\right)\left(\dfrac{6}{5}\right) = -1$

Slope of $\overline{BE} = \dfrac{-5}{6}$

$\overline{CF}$ is altitude on side $\overline{AB}$.

As $\overline{CF} \perp \overline{AB}$ so,

$\left(\text{Slope of } \overline{CF}\right)\left(\text{Slope of } \overline{AB}\right) = -1$

$\left(\text{Slope of } \overline{CF}\right)\left(\dfrac{1}{3}\right) = -1$

Slope of $\overline{CF} = -3$

Q.3 By means of slopes, show that the following points lie on the same line:

(a) $(-1,-3);(1,5);(2,9)$

Solution:

Let the points be $A(-1,-3)$, $B(1,5)$ and $C(2,9)$

Slope of $\overline{AB} = m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{5-(-3)}{1-(-1)}$

$m_1 = \dfrac{8}{2} = 4$

Slope of $\overline{BC} = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{9-5}{2-1}$

$m_2 = 4$

As $m_1 = m_2$ so points $A(-1,-3), B(1,5)$ and $C(2,9)$ lie on same line

(b) $(4,-5);(7,5);(10,15)$

Solution:

Let the points be $A(4,-5), B(7,5)$ and $C(10,15)$

Slope of $\overline{AB} = m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{5-(-5)}{7-4}$

$m_1 = \dfrac{10}{3}$

Slope of $\overline{BC} = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{15-5}{10-7}$

$m_2 = \dfrac{10}{3}$

As $m_1 = m_2$ so points $A(4,-5), B(7,5)$ and $C(10,15)$ lie on the same line.

(c) $(-4,6);(3,8);(10,10)$

Solution:

Let the points be $A(-4,6)$, $B(3,8)$ and $C(10,10)$

Slope of $\overline{AB} = m_1 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \dfrac{8-6}{3-(-4)} = \dfrac{2}{7}$

Slope of $\overline{BC} = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m_2 = \dfrac{10-8}{10-3} = \dfrac{2}{7}$

As $m_1 = m_2$ so points $A(-4,6), B(3,8)$ and $C(10,10)$ lie on the same line.

(d) $(a,2b);(c,a+b);(2c-a,2a)$

Solution:

Let the points be $A(a,2b)$,

$B(c,a+b)$ and $C(2c-a,2a)$

Slope of $\overline{AB} = m_1 = \dfrac{a+b-2b}{c-a}$

$$m_1 = \frac{a-b}{c-a}$$

Slope of $\overline{BC} = m_2 = \dfrac{y_2-y_1}{x_2-x_1}$

$$= \frac{2a-(a+b)}{2c-a-c}$$

$$m_2 = \frac{a-b}{c-a}$$

As $m_1 = m_2$ so points

$A(a,2b), B(c,a+b)$ and

$C(2c-a,2a)$ lie on the same line

Q.4 Find k so that the line joining $A(7,3);B(k,-6)$ and the line joining $C(-4,5);D(-6,4)$ are (i) parallel (ii) perpendicular.

(i)

Solution:

Slope of $\overline{AB} = m_1 = \dfrac{y_2-y_1}{x_2-x_1}$

$$= \frac{-6-3}{k-7} = \frac{-9}{k-7}$$

Slope of $\overline{CD} = m_2 = \dfrac{y_2-y_1}{x_2-x_1}$

$$= \frac{4-5}{-6-(-4)}$$

$$m_2 = \frac{-1}{-2} = \frac{1}{2}$$

As lines are parallel so,

$$m_1 = m_2$$

$$\frac{-9}{k-7} = \frac{1}{2}$$

$$-18 = k-7$$

$$k = -18+7$$

$$\boxed{k=-11}$$

(ii)

Solution:

As the lines are perpendicular, so

$$m_1m_2 = -1$$

$$\left(\frac{-9}{k-7}\right)\left(\frac{1}{2}\right) = -1$$

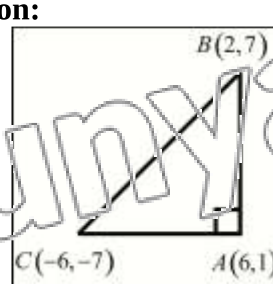
$$9 = 2k-14$$

$$2k = 23$$

$$\boxed{k=\frac{23}{2}}$$

Q.5 Using slopes, show that the triangle with its vertices $A(6,1), B(2,7)$ and $C(-6,-7)$ is a right triangle.

Solution:



Slope of $\overline{AB} = m_1 = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{7-1}{2-6}$

$$m_1 = \frac{6}{-4} = \frac{-3}{2}$$

Slope of $\overline{BC} = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-7}{-6-2}$

$$m_2 = \frac{-14}{-8} = \frac{7}{4}$$

Slope of $\overline{AC} = m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-1}{-6-6}$

$$m_3 = \frac{-8}{-12} = \frac{2}{3}$$

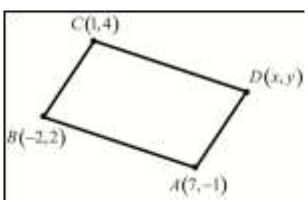
Consider $m_1 m_3 = \left(\frac{-3}{2}\right)\left(\frac{2}{3}\right)$

$$m_1 m_3 = -1$$

Since the product of slopes of two sides is '–1' so the sides are perpendicular and triangle is right triangle.

Q.6 The three points $A(7,-1), B(-2,2)$ and $C(1,4)$ are consecutive vertices of a parallelogram. Find the fourth vertex.

Solution:



Let the fourth vertex be $D(x,y)$

Since the diagonals of parallelogram bisect each other.

So mid-point of diagonal

$$AC = \left(\frac{7+1}{2}, \frac{-1+4}{2}\right)$$

$$= \left(4, \frac{3}{2}\right)$$

Mid-point of diagonal $BD = \left(4, \frac{3}{2}\right)$

$$\left(\frac{-2+x}{2}, \frac{2+y}{2}\right) = \left(4, \frac{3}{2}\right)$$

$$\frac{-2+x}{2} = 4 \text{ and } \frac{2+y}{2} = \frac{3}{2}$$

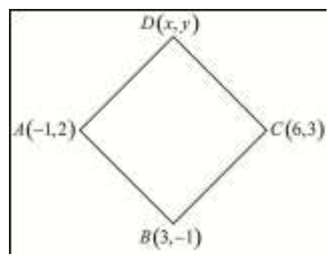
$$-2+x = 8\ ,\ 2+y = 3$$

$$x = 10\ ,\ y = 1$$

So fourth vertex is $D(10,1)$

Q.7 The points $A(-1,2), B(3,-1)$ and $C(6,3)$ are consecutive vertices of a rhombus. Find the fourth vertex and show that the diagonals of the rhombus are perpendicular to each other.

Solution:



Let the fourth vertex be $D(x,y)$.

Slope of $\overline{AD} = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y-2}{x+1}$

Slope of $\overline{BC} = m_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{3-(-1)}{6-3} = \frac{4}{3}$$

As sides $\overline{AD}$ and $\overline{BC}$ are parallel so

$$m_1 = m_2$$

$$\frac{y-2}{x+1} = \frac{4}{3}$$

$$3y - 6 = 4x + 4$$

$$4x = 3y - 10$$

$$x = \frac{3y-10}{4} \quad \ldots\text{(i)}$$

Slope of $\overline{AB} = m_3 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-1-2}{3-(-1)} = \frac{-3}{4}$$

Slope of $\overline{CD} = m_4 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y-3}{x-6}$

As sides $\overline{AB}$ and $\overline{CD}$ are parallel, so

$$m_3 = m_4$$

$$\frac{-3}{4} = \frac{y-3}{x-6}$$

$$-3x + 18 = 4y - 12$$

$$30 - 4y = 3x$$

$$x = \frac{30-4y}{3} \quad \ldots\text{(ii)}$$

Comparing (i) and (ii)

$$\frac{3y-10}{4} = \frac{30-4y}{3}$$

$$9y - 30 = 120 - 16y$$

$$25y = 150$$

$$y = 6$$

Put $y = 6$ in (i)

$$x = \frac{3(6)-10}{4} = \frac{8}{4} = 2$$

So the fourth vertex is $D(2,6)$.

Now

Slope of diagonal $\overline{AC} = m_5 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{3-2}{6-(-1)} = \frac{1}{7}$$

Slope of diagonal $\overline{BD} = m_6 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{6-(-1)}{2-3}$$

$$m_6 = \frac{7}{-1} = -7$$

Consider

$$m_5 m_6 = \left(\frac{1}{7}\right)(-7)$$

$$m_5 m_6 = -1$$

As product of slope of $\overline{AC}$ and slope of $\overline{BD}$ is '–1' so diagonals of rhombus are perpendicular.

Q.8 Two pairs of points are given. Find whether the two lines determined by these points are:
(i) Parallel (ii) Perpendicular
(iii) None

(a) $(1,-2),(2,4)$ **and** $(4,1),(-8,2)$

Solution:

Let the points be $A(1,-2), B(2,4)$ and $C(4,1), D(-8,2)$

Slope of $\overline{AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{4-(-2)}{2-1} = 6$$

Slope of $\overline{CD} = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{-8-4}$

$$= \frac{1}{-12}$$

As $m_1 \neq m_2$ and $m_1 m_2 \neq -1$ so the lines are neither parallel nor perpendicular.

(b) $(-3,4),(6,2)$ **and** $(4,5),(-2,-7)$

Solution:

Let the points be $A(-3,4), B(6,2)$ and $C(4,5), D(-2,-7)$

Slope of $\overline{AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{2-4}{6-(-3)} = \frac{-2}{9}$$

Slope of $\overline{CD} = m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-7-5}{-2-4}$

$$m_2 = \frac{-12}{-6} = 2$$

As $m_1 \neq m_2$ and $m_1 m_2 \neq -1$ so the lines are neither parallel nor perpendicular.

Q.9 Find an equation of

(a) The horizontal line through $(7,-9)$

Solution:

As line is horizontal so slope = m = 0

$(x_1, y_1) = (7,-9)$

Using point slope form

$y - y_1 = m(x - x_1)$

$y - (-9) = 0(x - 7)$

$\boxed{y + 9 = 0}$

(b) The vertical line through $(-5,3)$

Solution:

As vertical line does not intersect y-axis, so the equation is

$x = -5$

$\boxed{x + 5 = 0}$

(c) The line bisecting the first and third quadrants.

Solution:

As line bisecting first and third quadrants makes an angle of $45°$ with positive x-axis and passes through origin $(0,0)$, so

Slope $= m = \tan\alpha = \tan 45° = 1$

Also $(x_1, y_1) = (0,0)$

Using point slope form

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 0)$

$\boxed{y = x}$

(d) The line bisecting the second and fourth quadrants.

Solution:

As line bisecting 2nd and 4th quadrants makes an angle of $135°$ with positive x-axis and also passes through origin $(0,0)$, so

Slope $= m = \tan\alpha = \tan 135° = -1$

Also $(x_1, y_1) = (0,0)$

Using point slope form

$y - y_1 = m(x - x_1)$

$y - 0 = (-1)(x - 0)$

$\boxed{y = -x}$

Q.10 Find an equation of the line

(a) Through $A(-6,5)$ having slope 7

Solution:

Here

$x_1 = -6, y_1 = 5, m = 7$

Using point slope form

$y - y_1 = m(x - x_1)$

$y - 5 = 7(x - (-6))$

$y - 5 = 7x + 42$

$7x - y + 42 + 5 = 0$

$\boxed{7x - y + 47 = 0}$

(b) Through $(8,-3)$ having slope 0

Solution:

Here

$x_1 = 8, y_1 = -3, m = 0$

Using point slope form

$y - y_1 = m(x - x_1)$

$y - (-3) = 0(x - 8)$

$\boxed{y + 3 = 0}$

(c) Through $(-8,5)$ having slope undefined

Solution:

As slope is undefined so line is vertical which only intersects x-axis. So its equation is

$x = -8$

$\boxed{x + 8 = 0}$

(d) Through $(-5,-3)$ and $(9,-1)$

Solution:

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Slope} = m = \frac{-1-(-3)}{9-(-5)}$$

$$\text{Slope} = m = \frac{2}{14} = \frac{1}{7}$$

Using point $(-5,-3)$ and slope $= \frac{1}{7}$,

required equation is

$$y - y_1 = m(x - x_1)$$

$$y-(-3) = \frac{1}{7}(x-(-5))$$

$$y+3 = \frac{1}{7}(x+5)$$

$$7y+21 = x+5$$

$$\boxed{x-7y-16=0}$$

(e) *y*-intercept: -7 and slope: -5

Solution:

Here

$c=-7, m=-5$

Using slope intercept form

$$y = mx + c$$

$$y = -5x - 7$$

$$\boxed{5x+y+7=0}$$

(f) *x*-intercept: -3 and *y*-intercept: 4

Solution:

Here

$a=-3, b=4$

Using two intercepts form

$$\frac{x}{a}+\frac{y}{b}=1$$

$$\frac{x}{-3}+\frac{y}{4}=1$$

$$\frac{4x-3y}{-12}=1$$

$$4x-3y=-12$$

$$\boxed{4x-3y+12=0}$$

(g) *x*-intercept: -9 and slope: -4

Solution:

As x-intercept is -9, so

$(x_1, y_1) = (-9, 0)$

Also $m=-4$

Using point slope form

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -4(x-(-9))$$

$$y = -4(x+9)$$

$$y = -4x - 36$$

$$\boxed{4x+y+36=0}$$

Q.11 Find an equation of the perpendicular bisector of the segment joining the points $A(3,5)$ and $B(9,8)$.

Solution:

$$\text{Slope of } \overline{AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-5}{9-3}$$

$$m_1 = \frac{3}{6} = \frac{1}{2}$$

Mid-point of segment $\overline{AB}$

$$= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{3+9}{2}, \frac{8+5}{2}\right)$$

$$= \left(6, \frac{13}{2}\right)$$

Slope of perpendicular bisector

$$= m = -\frac{1}{m_1}$$

$$m = -\frac{1}{\left(\frac{1}{2}\right)} = -2$$

Also $(x_1, y_1) = \left(6, \frac{13}{2}\right)$

Using point slope form

$$y-y_1=m(x-x_1)$$

$$y-\frac{13}{2}=-2(x-6)$$

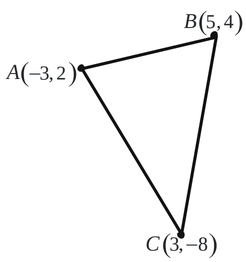
$$2y-13=-4(x-6)$$

$$2y-13=-4x+24$$

$$\boxed{4x+2y-37=0}$$

Q.12 Find equations of the sides, altitudes and medians of the triangle whose vertices are $A(-3,2), B(5,4)$ and $C(3,-8)$.

Solution:



Slope of side $\overline{AB}=m_1=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{4-2}{5-(-3)}$$

$$m_1=\frac{2}{8}=\frac{1}{4}$$

Using $A(-3,2)$ and slope $=\frac{1}{4}$,

equation of side $\overline{AB}$ is:

$$y-y_1=m(x-x_1)$$

$$y-2=\frac{1}{4}(x-(-3))$$

$$4y-8=x+3$$

$$\boxed{x-4y+11=0}$$

Slope of $\overline{AC}=m_2=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{-8-2}{3-(-3)}$$

$$m_2=\frac{-10}{6}=\frac{-5}{3}$$

Using $A(-3,2)$ and Slope $=-\frac{5}{3}$,

equation of side $\overline{AC}$ is

$$y-y_1=m(x-x_1)$$

$$y-2=\frac{-5}{3}(x-(-3))$$

$$3y-6=-5(x+3)$$

$$3y-6=-5x-15$$

$$\boxed{5x+3y+9=0}$$

Slope of $\overline{BC}=m_3=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{-8-4}{3-5}$$

$$m_3=\frac{-12}{-2}=6$$

Using $B(5,4)$ and slope $=6$,

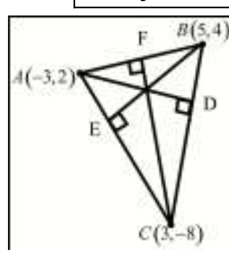
equation of side $\overline{BC}$ is

$$y-y_1=m(x-x_1)$$

$$y-4=6(x-5)$$

$$y-4=6x-30$$

$$\boxed{6x-y-26=0}$$



Slope of $\overline{BC}=m_3=\frac{y_2-y_1}{x_2-x_1}$

$$m_3=6$$

Slope of altitude

$$\overline{AD}=m_4=\frac{-1}{\text{slope of }\overline{BC}}$$

$$m_4=-\frac{1}{6}$$

Using $A(-3,2)$ and slope $=-\frac{1}{6}$,

equation of $\overline{AD}$ is

$$y-y_1=m(x-x_1)$$

$$y-2=\frac{-1}{6}\left(x-(-3)\right)$$
$$6y-12=-(x+3)$$
$$6y-12=-x-3$$
$$\boxed{x+6y-9=0}$$

Slope of $\overline{AC}=m_2=\frac{y_2-y_1}{x_2-x_1}$

$$m_2=-\frac{5}{3}$$

Slope of $\overline{BE}=m_5=\frac{-1}{\text{slope of }\overline{AC}}=\frac{3}{5}$

Using $B(5,4)$ and slope $=\frac{3}{5}$,

equation of $\overline{BE}$ is

$$y-y_1=m(x-x_1)$$
$$y-4=\frac{3}{5}(x-5)$$
$$5y-20=3x-15$$
$$0=3x-15-5y+20$$
$$\boxed{3x-5y+5=0}$$

Slope of side $\overline{AB}=m_1=\frac{y_2-y_1}{x_2-x_1}$

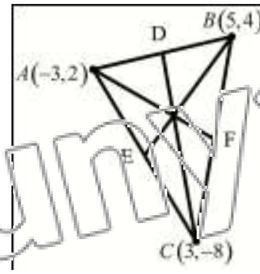
$$m_1=\frac{1}{4}$$

Slope of $\overline{CF}=m_6=\frac{-1}{\text{slope of }\overline{AB}}=-4$

Using $C(3,-8)$ and slope $=-4$,

equation of $\overline{CF}$ is

$$y-y_1=m(x-x_1)$$
$$y-(-8)=-4(x-3)$$
$$y+8=-4x+12$$
$$\boxed{4x+y-4=0}$$



Let D, E and F are mid points of the sides $\overline{AB},\overline{AC}$ and $\overline{BC}$ respectively.

Mid point of $\overline{AB}=D\left(\frac{-3+5}{2},\frac{2+4}{2}\right)$

$$=D(1,3)$$

Mid point of $\overline{AC}=E\left(\frac{3-3}{2},\frac{-8+2}{2}\right)$

$$=E(0,-3)$$

Mid point of $\overline{BC}=F\left(\frac{5+3}{2},\frac{4+(-8)}{2}\right)$

$$=F(4,-2)$$

Slope of $\overline{AF}=m_1'=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{-2-2}{4-(-3)}=\frac{-4}{7}$$

Using $A(-3,2)$ and slope $=\frac{-4}{7}$,

equation of median $\overline{AF}$ is:

$$y-y_1=m(x-x_1)$$
$$y-2=\frac{-4}{7}\left(x-(-3)\right)$$
$$7(y-2)=-4(x+3)$$
$$7y-14=-4x-12$$
$$\boxed{4x+7y-2=0}$$

Slope of $\overline{BE}=m_2'=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{-3-4}{0-5}=\frac{7}{5}$$

Using $B(5,4)$ and slope $=\frac{7}{5}$,

equation of median $\overline{BE}$ is

$$y-y_1=m(x-x_1)$$

$$y-4=\frac{7}{5}(x-5)$$
$$5y-20=7x-35$$
$$\boxed{7x-5y-15=0}$$

Slope of $\overline{CD}=m_3'=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{3-(-8)}{1-3}=-\frac{11}{2}$$

Using $C(3,-8)$ and slope $=-\frac{11}{2}$,

equation of median $\overline{CD}$ is

$$y-y_1=m(x-x_1)$$
$$y-(-8)=\frac{-11}{2}(x-3)$$
$$2y+16=-11x+33$$
$$\boxed{11x+2y-17=0}$$

Q.13 Find an equation of line through $(-4,-6)$ and perpendicular to a line having slope $\frac{-3}{2}$.

Solution:

Slope of given line $=m_1=-\frac{3}{2}$

Slope of required line $=m=-\frac{1}{m_1}=\frac{2}{3}$

Using point $(-4,-6)$ and slope $=\frac{2}{3}$,

required equation is

$$y-y_1=m(x-x_1)$$
$$y-(-6)=\frac{2}{3}(x-(-4))$$
$$y+6=\frac{2}{3}(x+4)$$
$$3y+18=2x+8$$
$$\boxed{2x-3y-10=0}$$

Q.14 Find an equation of the line through $(11,-5)$ and parallel to a line with slope -24.

Solution:

Slope of given line $=m_1=-24$

Slope of required line $=m=m_1=-24$

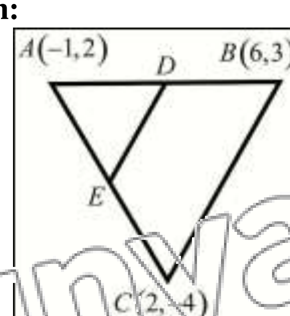
Using point $(11,-5)$ and slope $=-24$

required equation is

$$y-y_1=m(x-x_1)$$
$$y-(-5)=-24(x-11)$$
$$y+5=-24x+264$$
$$\boxed{24x+y-259=0}$$

Q.15 The points $A(-1,2)$, $B(6,3)$ and $C(2,-4)$ are vertices of a triangle. Show that the line joining the mid point *D* of *AB* and the mid point *E* of *AC* is parallel to *BC* and $DE=\frac{1}{2}BC$.

Solution:



Mid point of $\overline{AC}=E\left(\frac{-1+2}{2},\frac{2+(-4)}{2}\right)$

$$=E\left(\frac{1}{2},-1\right)$$

Mid point of $\overline{AB}=D\left(\frac{6+(-1)}{2},\frac{3+2}{2}\right)$

$$= D\left(\frac{5}{2}, \frac{5}{2}\right)$$

Slope of $\overline{ED} = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\frac{5}{2} - (-1)}{\frac{5}{2} - \frac{1}{2}}$

$$m_1 = \frac{\frac{5+2}{2}}{\frac{5-1}{2}} = \frac{7}{4}$$

Slope of $\overline{BC} = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-4)}{6-2}$

$$= \frac{7}{4}$$

As slope of $\overline{BC}$ = slope of $\overline{ED}$, so $\overline{DE}$ is parallel to $\overline{BC}$.

$$\left|\overline{BC}\right| = \sqrt{(6-2)^2 + (3-(-4))^2}$$

$$\left|\overline{BC}\right| = \sqrt{16+49}$$

$$\left|\overline{BC}\right| = \sqrt{65} \ldots \text{(i)}$$

$$\left|\overline{DE}\right| = \sqrt{\left(\frac{5}{2} - \frac{1}{2}\right)^2 + \left(\frac{5}{2} - (-1)\right)^2}$$

$$\left|\overline{DE}\right| = \sqrt{\left(\frac{5-1}{2}\right)^2 + \left(\frac{5}{2} + 1\right)^2} = \sqrt{\left(\frac{4}{2}\right)^2 + \left(\frac{7}{2}\right)^2}$$

$$\left|\overline{DE}\right| = \sqrt{\left(\frac{16}{4} + \frac{49}{4}\right)}$$

$$\left|\overline{DE}\right| = \sqrt{\frac{65}{4}}$$

$$\left|\overline{DE}\right| = \frac{\sqrt{65}}{2} \ldots \text{(ii)}$$

Comparing (i) and (ii)

$$\overline{DE} = \frac{1}{2}\overline{BC}$$

Q.16 A milkman can sell 560 litres of milk at Rs. 12.50 per litre and 700 litres of milk at Rs. 12.00 per litre. Assuming the graph of the sale price and the milk sold to be a straight line, find the number of litres of milk that the milkman can sell at Rs. 12.25 per litre.

Solution:

Let number of litres of milk be denoted by 'l' and price per litre by p then

$A(l_1, p_1) = (560, 12.5)$, $B(l_2, p_2) = (700, 12)$

Slope of $\overline{AB} = m = \frac{p_2 - p_1}{l_2 - l_1}$

$$m = \frac{12 - 12.5}{700 - 560}$$

$$m = \frac{-0.5}{140}$$

$$m = \frac{-1}{280}$$

Using point slope form

$$y - y_1 = m(x - x_1)$$

$$p - p_1 = m(l - l_1)$$

$$p - 12.5 = \frac{-1}{280}(l - 560)$$

$$280p - 3500 = -l + 560$$

$$l + 280p - 4060 = 0$$

To find number of litres of milk at Rs 12.25 per litre, put $p = 12.25$

$$l + 280(12.25) - 4060 = 0$$

$$l + 3430 - 4060 = 0$$

$$\boxed{l = 630} \text{ litres}$$

Q.17 The population of Pakistan to the nearest million was 60 million in 1961 and 95 million in 1981. Using 't' as the number of years after 1961, find an equation of the line that gives the population is terms of t. Use this equation to find the population in

(a) 1947 (b) 1997

Solution:

Let population be denoted by 'p', then

$$A(p_1, t_1) = A(60, 1961-1961) = A(60, 0)$$

$$B(p_2, t_2) = B(95, 1981-1961) = B(95, 20)$$

Slope of $\overline{AB} = m = \frac{t_2 - t_1}{p_2 - p_1}$

$$m = \frac{20-0}{95-60}$$

$$m = \frac{20}{35}$$

$m = \frac{4}{7}$

Using point slope form

$y - y_1 = m(x - x_1)$

$t - t_1 = m(p - p_1)$

$t - 0 = \frac{4}{7}(p - 60)$

$7t - 0 = 4p - 240$

$4p = 7t + 240$

$p = \frac{7}{4}t + 60 \ldots$(i)

(a) Population in 1947

Put $t = 1947 - 1961 = -14$ in (i)

$p = \frac{7}{4}(-14) + 60$

$p = -24.5 + 60$

$\boxed{p = 35.5}$ Million

(b) Population in 1997

Put $t = 1997 - 1961 = 36$ in (i)

$p = \frac{7}{4}(36) + 60$

$p = 63 + 60$

$\boxed{p = 123}$ Million

Q.18 A house was purchased for Rs. 1 million in 1980. It is worth Rs. 4 million in 1996. Assuming that the value increased by the same amount each year, find an equation that gives the value of the house after *t* years of the date of purchase. What was its value in 1990?

Solution:

Let the price of the house be denoted by *p*, then

$A(p_1, t_1) = A(1, 1980 - 1980) = A(1, 0)$

$B(p_2, t_2) = B(4, 1996 - 1980) = B(4, 16)$

Slope of $\overline{AB} = m = \frac{t_2 - t_1}{p_2 - p_1}$

$m = \frac{16 - 0}{4 - 1} = \frac{16}{3}$

Using point slope form

$y - y_1 = m(x - x_1)$

$t - t_1 = m(p - p_1)$

$t - 0 = \frac{16}{3}(p - 1)$

$\frac{3}{16}t = p - 1$

$p = \frac{3}{16}t + 1 \ldots$(i)

To find price of house in 1990, put *t* = 1990 – 1980 =10 in (i)

$p = \frac{3}{16}(10) + 1$

$p = \frac{30 + 16}{16} = \frac{46}{16}$

$\boxed{p = 2.875}$ Million

Q.19 Plot the Celsius (C) and Fahrenheit (F)temperature scales on the horizontal axis and the vertical axis respectively. Draw the line joining the freezing point and the boiling point of water. Find an equation giving F temperature in terms of C.

Solution:

Since

Freezing point of water

$= 0°C = 32°F$

Boiling point of water

$= 100°C = 212°F$, So

$A(C_1, F_1) = A(0, 32)$

$B(C_2, F_2) = B(100, 212)$

Slope of $\overline{AB} = m = \frac{F_2 - F_1}{C_2 - C_1}$

$= \frac{212 - 32}{100 - 0}$

$m = \frac{180}{100} = \frac{9}{5}$

Using point slope form

$y - y_1 = m(x - x_1)$

$F - F_1 = m(C - C_1)$

$F - 32 = \frac{9}{5}(C - 0)$

$\boxed{F = \frac{9}{5}C + 32}$

Q.20 The average entry test score of engineering candidates was 592 in the year 1998 while the score was 564 in 2002. Assuming that the relationship between time and score is linear, find the average score for 2006.

Solution:

Let test score be denoted by *s* and time by *t*, then

$A(s_1, t_1) = A(592, 1998)$

$B(s_2, t_2) = B(564, 2002)$

Slope of $\overline{AB} = m = \frac{2002 - 1998}{564 - 592}$

$m = -\frac{4}{28} = -\frac{1}{7}$

Using point slope form

$y - y_1 = m(x - x_1)$

$t - t_1 = m(s - s_1)$

$t - 1998 = \frac{-1}{7}(s - 592)$

$7(t - 1998) = -s + 592$

$\boxed{s = 592 - 7(t - 1998)}$

To find, average score for 2006, put *t* = 2006

$s = 592 - 7(2006 - 1998)$

$s = 592 - 7(8)$

$s = 592 - 56$

$\boxed{s = 536}$

Q.21 Convert each of the following equation into:
(i) Slope intercept form
(ii) Two intercepts form
(iii) Normal form

(a) $2x - 4y + 11 = 0$

(b) $4x + 7y - 2 = 0$

(c) $15y - 8x + 3 = 0$

Also find the length of the perpendicular from $(0,0)$ to each line.

(i) Slope intercept form

(a) $2x - 4y + 11 = 0$

Solution:

$4y = 2x + 11$

$y = \frac{2}{4}x + \frac{11}{4}$

$\boxed{y = \frac{1}{2}x + \frac{11}{4}}$

(b) $4x + 7y - 2 = 0$

Solution:

$7y = -4x + 2$

$\boxed{y = -\frac{4}{7}x + \frac{2}{7}}$

(c) $15y - 8x + 3 = 0$

Solution:

$15y = 8x - 3$

$y = \frac{8}{15}x - \frac{3}{15}$

$y = \frac{8}{15}x - \frac{1}{5}$

$\boxed{y = \frac{8}{15}x - \frac{1}{5}}$

(ii) Two intercepts form

(a) $2x - 4y + 11 = 0$

Solution:

$2x - 4y = -11$

$$\frac{2x}{-11}-\frac{4y}{-11}=\frac{-11}{-11}$$

$$\boxed{\frac{x}{\frac{-11}{2}}+\frac{y}{\frac{11}{4}}=1}$$

(b) $4x+7y-2=0$

Solution:

$$4x+7y=2$$

$$\frac{4x}{2}+\frac{7y}{2}=\frac{2}{2}$$

$$2x+\frac{7y}{2}=1$$

$$\boxed{\frac{x}{\frac{1}{2}}+\frac{y}{\frac{2}{7}}=1}$$

(c) $15y-8x+3=0$

Solution:

$$8x-15y=3$$

$$\frac{8}{3}x-\frac{15}{3}y=\frac{3}{3}$$

$$\frac{8}{3}x-5y=1$$

$$\frac{x}{\frac{3}{8}}+\frac{y}{\frac{-1}{5}}=1$$

$$\boxed{\frac{x}{\frac{3}{8}}+\frac{y}{\frac{-1}{5}}=1}$$

(iii) Normal form

(a) $2x-4y+11=0$

Solution:

$$-2x+4y=11$$

Dividing both sides by $\sqrt{(2)^2+(-4)^2}=\sqrt{20}$

$$\frac{-2x}{\sqrt{20}}+\frac{4y}{\sqrt{20}}=\frac{11}{\sqrt{20}}$$

$$\frac{-2x}{2\sqrt{5}}+\frac{4y}{2\sqrt{5}}=\frac{11}{2\sqrt{5}}$$

$$\frac{-x}{\sqrt{5}}+\frac{2y}{\sqrt{5}}=\frac{11}{2\sqrt{5}}$$

Where $\cos\alpha=-\frac{1}{\sqrt{5}}$, $\sin\alpha=\frac{2}{\sqrt{5}}$

$$\Rightarrow \tan\alpha=\frac{\frac{2}{\sqrt{5}}}{-\frac{1}{\sqrt{5}}}=-2$$

$$\Rightarrow \alpha=\tan^{-1}(-2)$$

$$\phi=\tan^{-1}(2)=63.43^\circ$$

$$\Rightarrow \alpha=180^\circ-\phi$$

$$=180^\circ-63.43^\circ$$

$$\alpha=116.57^\circ$$

So the required normal form is

$$\boxed{x\cos116.57^\circ+y\sin116.57^\circ=\frac{11}{2\sqrt{5}}}$$

Length of perpendicular from $(0,0)$ to the line $=\frac{11}{2\sqrt{5}}$

(b) $4x+7y-2=0$

Solution:

$$4x+7y=2$$

Dividing both sides by $\sqrt{4^2+7^2}=\sqrt{65}$

$$\frac{4}{\sqrt{65}}x+\frac{7}{\sqrt{65}}y=\frac{2}{\sqrt{65}}$$

Where $\cos\alpha=\frac{4}{\sqrt{65}}$, $\sin\alpha=\frac{7}{\sqrt{65}}$

$$\Rightarrow \tan\alpha=\frac{\frac{7}{\sqrt{65}}}{\frac{4}{\sqrt{65}}}=\frac{7}{4}$$

$$\Rightarrow \alpha=\tan^{-1}\left(\frac{7}{4}\right)$$

$$\alpha=60.26^\circ$$

So the required normal form is

$$x\cos 60.26° + y\sin 60.26° = \frac{2}{\sqrt{65}}$$

Length of perpendicular from origin $(0,0)$ to the line $= \frac{2}{\sqrt{65}}$

(c) $15y - 8x + 3 = 0$

Solution:

$8x - 15y = 3$

Dividing both sides by $\sqrt{8^2 + (-15)^2} = 17$

$\frac{8}{17}x - \frac{15}{17}y = \frac{3}{17}$

Where $\cos\alpha = \frac{8}{17}$, $\sin\alpha = -\frac{15}{17}$

$\Rightarrow \tan\alpha = \frac{-\frac{15}{17}}{\frac{8}{17}} = -\frac{15}{8}$

$\Rightarrow \phi = \tan^{-1}\left(\frac{15}{8}\right) = 61.93°$

$\Rightarrow \alpha = 360° - \phi$

$= 360° - 61.93°$

$\alpha = 298.07°$

So the required normal form is

$$x\cos 298.07° + y\sin 298.07° = \frac{3}{17}$$

Length of perpendicular from origin (0,0) to the line $= \frac{3}{17}$

Q.22 In each of the following check whether the two lines are

(i) Parallel

(ii) Perpendicular

(iii) Neither parallel nor perpendicular

(a) $2x + y - 3 = 0$ **;** $4x + 2y + 5 = 0$

Solution:

$2x + y - 3 = 0$

$m_1 = -\frac{a}{b}$

$= -\frac{2}{1} = -2$

$4x + 2y + 5 = 0$

$m_2 = -\frac{a}{b}$

$= -\frac{4}{2} = -2$

As $m_1 = m_2$ so the lines are parallel.

(b) $3y = 2x + 5$ **;** $3x + 2y - 8 = 0$

Solution:

$3y = 2x + 5$

$m_1 = -\frac{a}{b} = \frac{-2}{-3} = \frac{2}{3}$

$3x + 2y - 8 = 0$

$m_2 = \frac{-a}{b} = \frac{-3}{2}$

Consider $m_1 m_2 = \left(\frac{2}{3}\right)\left(\frac{-3}{2}\right) = -1$

As product of slopes is '–1' so the lines are perpendicular.

(c) $4y + 2x - 1 = 0$ **;** $x - 2y - 7 = 0$

Solution:

$4y + 2x - 1 = 0$

$m_1 = \frac{-a}{b} = \frac{-2}{4} = \frac{-1}{2}$

$x - 2y - 7 = 0$

$m_2 = \frac{-a}{b} = \frac{-1}{-2} = \frac{1}{2}$

As $m_1 \neq m_2$ and $m_1 m_2 \neq -1$ so the lines are neither parallel nor perpendicular.

(d) $4x - y + 2 = 0$ **;** $12x - 3y + 1 = 0$

Solution:

$4x - y + 2 = 0$

$m_1 = \frac{-a}{b} = \frac{-4}{-1} = 4$

$12x - 3y + 1 = 0$

$m_2 = \frac{-a}{b} = \frac{-12}{-3} = 4$

As $m_1 = m_2$ so the lines are parallel.

(e) $12x + 35y - 7 = 0$ **;** $105x - 36y + 11 = 0$

Solution:

$12x+35y-7=0$

$m_1=\frac{-a}{b}=\frac{-12}{35}$

$105x-36y+11=0$

$m_2=\frac{-a}{b}=\frac{-105}{-36}=\frac{35}{12}$

Consider $m_1m_2=\left(\frac{-12}{35}\right)\left(\frac{35}{12}\right)$

$m_1m_2=-1$

As product of slopes is '–1' so the lines are perpendicular.

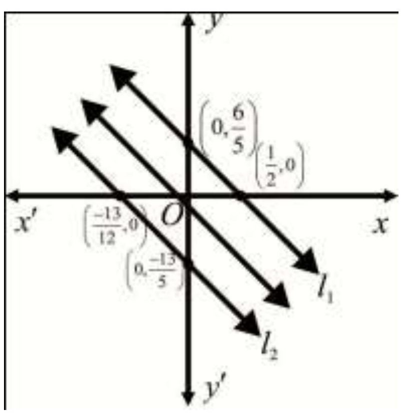
Q.23 Find the distance between the given parallel lines. Sketch the lines. Also find an equation of parallel line lying mid-way between them.

(a) $3x-4y+3=0$; $3x-4y+7=0$

Solution:

For Sketch

$\ell_1: 3x-4y+3=0$



For x-intercept

Let $y=0\Rightarrow x=-1$

For y-intercept

Let $x=0\Rightarrow y=\frac{3}{4}$

So points are $A(-1,0)$, $B\left(0,\frac{3}{4}\right)$

$\ell_2: 3x-4y+7=0$

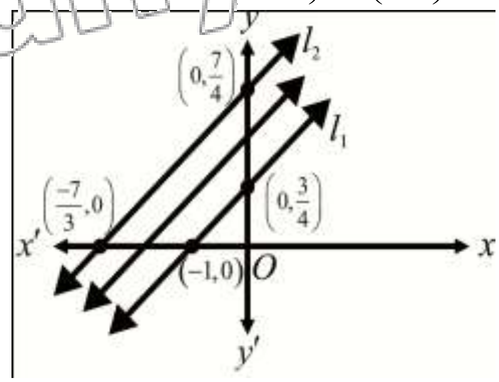
For x-intercept

Let $y=0\Rightarrow x=\frac{-7}{3}$

For y-intercept

Let $x=0\Rightarrow y=\frac{7}{4}$

So points are $C\left(\frac{-7}{3},0\right)$, $D\left(0,\frac{7}{4}\right)$



Distance between parallel lines= Distance of $A(-1,0)$ from

$3x-4y+7=0$

$d=\frac{|3(-1)+(-4)(0)+7|}{\sqrt{3^2+(-4)^2}}=\frac{|-3+0+7|}{\sqrt{9+16}}$

$=\frac{4}{\sqrt{25}}=\frac{4}{5}$

y-intercept of line lying midway between parallel lines is average of y–intercepts of l_1 and l_2 so

$c=\frac{\frac{7}{4}+\frac{3}{4}}{2}$

$c=\frac{\frac{7+3}{4}}{2}=\frac{10^{5}}{2\times 4}$

$c=\frac{5}{4}$

As slopes of parallel lines are same

so $m=-\frac{a}{b}=-\frac{3}{-4}=\frac{3}{4}$

Using slope intercept form

$y=mx+c$

$y=\frac{3}{4}x+\frac{5}{4}$

$4y = 3x + 5$

$\boxed{3x - 4y + 5 = 0}$

(b) $12x + 5y - 6 = 0$ **;** $12x + 5y + 13 = 0$

Solution:

For Sketch

$\ell_1 : 12x + 5y - 6 = 0$

For x-intercept

Let $y = 0 \Rightarrow x = \dfrac{1}{2}$

For y-intercept

Let $x = 0 \Rightarrow y = \dfrac{6}{5}$

So points are $A\left(\dfrac{1}{2}, 0\right),\ B\left(0, \dfrac{6}{5}\right)$

$\ell_2 : 12x + 5y + 13 = 0$

For x-intercept

Let $y = 0 \Rightarrow x = \dfrac{-13}{12}$

For y-intercept

Let $x = 0 \Rightarrow y = \dfrac{-13}{5}$

So points are

$C\left(\dfrac{-13}{12}, 0\right),\ D\left(0, \dfrac{-13}{5}\right)$

Distance between parallel lines =

Distance of $A\left(\dfrac{1}{2}, 0\right)$ from

$12x + 5y + 13 = 0$

$$d = \frac{\left|12\left(\frac{1}{2}\right) + 5(0) + 13\right|}{\sqrt{(12)^2 + (5)^2}} = \frac{|6 + 0 + 13|}{\sqrt{144 + 25}} = \frac{19}{\sqrt{169}}$$

$= \dfrac{19}{13}$

y-intercept of line lying midway between parallel lines is average of y-intercepts of 'l_1' and 'l_2' so

$$c = \frac{\frac{6}{5} + \left(\frac{-13}{5}\right)}{2}$$

$$c = \frac{\frac{-7}{5}}{2} = \frac{-7}{10}$$

As slopes of parallel lines are same so $m = -\dfrac{a}{b} = -\dfrac{12}{5}$.

Using slope intercept form

$y = mx + c$

$y = \dfrac{-12}{5}x - \dfrac{7}{10}$

$10y = -24x - 7$

$24x + 10y + 7 = 0$

$\boxed{12x + 5y + \dfrac{7}{2} = 0}$

(c) $x + 2y - 5 = 0$ **;** $2x + 4y = 1$

Solution:

For Sketch

$\ell_1 : x + 2y - 5 = 0$

For x-intercept

Let $y = 0 \Rightarrow x = 5$

For y-intercept

Let $x = 0 \Rightarrow y = \dfrac{5}{2}$

So points are $A(5, 0),\ B\left(0, \dfrac{5}{2}\right)$

$\ell_2 : 2x + 4y - 1 = 0$

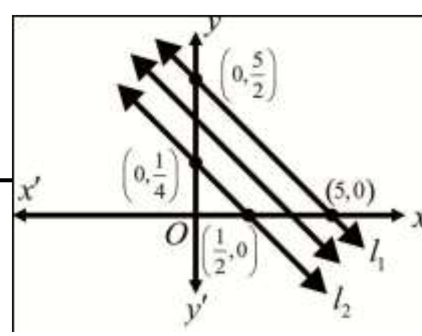
For x-intercept

Let $y = 0 \Rightarrow x = \dfrac{1}{2}$

For y-intercept

Let $x = 0 \Rightarrow y = \dfrac{1}{4}$

So points are $C\left(\dfrac{1}{2}, 0\right),\ D\left(0, \dfrac{1}{4}\right)$



Distance between parallel lines=
Distance of $A(5,0)$ from

$2x+4y-1=0$

$$d=\frac{|2(5)+4(0)-1|}{\sqrt{(2)^2+(4)^2}}=\frac{|10+0-1|}{\sqrt{4+16}}=\frac{9}{\sqrt{20}}$$

$$=\frac{9}{2\sqrt{5}}$$

y-intercept of line lying midway between parallel lines is average of y-intercepts of parallel lines so

$$c=\frac{\frac{5}{2}+\frac{1}{4}}{2}$$

$$c=\frac{\frac{10+1}{4}}{2}=\frac{11}{8}$$

As slopes of parallel lines are same so $m=-\frac{a}{b}=-\frac{1}{2}$

Using slope intercept form

$y=mx+c$

$$y=-\frac{1}{2}x+\frac{11}{8}$$

$8y=-4x+11$

$$\boxed{4x+8y-11=0}$$

Q.24 Find an equation of the line through $(-4,7)$ and parallel to the line $2x-7y+4=0$

Solution:

Slope of given line

$$=m_1=-\frac{a}{b}=\frac{-2}{-7}=\frac{2}{7}$$

As slopes of parallel lines are same so

Slope of required line $=m=m_1=\frac{2}{7}$

Also $(x_1,y_1)=(-4,7)$

Using point slope form

$y-y_1=m(x-x_1)$

$$y-7=\frac{2}{7}(x-(-4))$$

$7y-49=2x+8$

$$\boxed{2x-7y+57=0}$$

Q.25 Find an equation of the line through $(5,-8)$ and perpendicular to the join of $A(-15,-8), B(10,7)$

Solution:

Slope of $\overline{AB}=m_1=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{7-(-8)}{10-(-15)}$$

$$m_1=\frac{15}{25}=\frac{3}{5}$$

Slope of required line $=m$

$$=-\frac{1}{m_1}=-\frac{5}{3}$$

Also $(x_1,y_1)=(5,-8)$

Using point slope form

$y-y_1=m(x-x_1)$

$$y-(-8)=\frac{-5}{3}(x-5)$$

$3y+24=-5x+25$

$$\boxed{5x+3y-1=0}$$

Q.26 Find equations of two parallel lines perpendicular to $2x-y+3=0$ such that the product of x and y-intercepts of each is 3.

Solution:

Equation of any line perpendicular to $2x-y+3=0$ is

$x+2y+c=0$ ... (i)

For x-intercept	For y-intercept
Let $y=0$	Let $x=0$
$\Rightarrow x=-c$	$\Rightarrow y=-\frac{c}{2}$

According to the given condition
(x-intercept) (y-intercept) = 3

$$(-c)\left(\frac{-c}{2}\right)=3$$

$$c^2=6$$

$$c=\pm\sqrt{6}$$

Put value of c in (i)

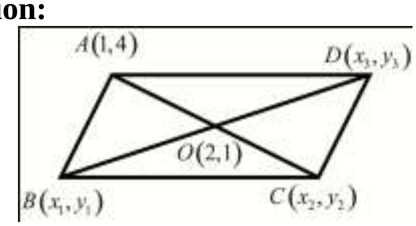
$x+2y\pm\sqrt{6}=0$

Hence required lines are

$x+2y+\sqrt{6}=0$ and $x+2y-\sqrt{6}=0$

Q.27 One vertex of a parallelogram is $(1,4)$; the diagonals intersect at $(2,1)$ and the sides have slopes 1 and $\frac{-1}{7}$. Find the other three vertices.

Solution:



Let $B(x_1,y_1), C(x_2,y_2)$ and $D(x_3,y_3)$ be other there vertices of parallelogram. 'O' is the point of intersection of diagonals.

As $(2,1)$ is the midpoint of $A(1,4)$ and $C(x_2,y_2)$ so

$2=\frac{1+x_2}{2}$; $1=\frac{4+y_2}{2}$

$x_2+1=4$; $4+y_2=2$

$x_2=3$; $y_2=-2$

So $C(x_2,y_2)=C(3,-2)$

Let slope of $\overline{BC}=\frac{-1}{7}$

$\frac{y_1-y_2}{x_1-x_2}=-\frac{1}{7}$

$\frac{y_1+2}{x_1-3}=-\frac{1}{7}$

$-x_1+3=7y_1+14$

$x_1=-7y_1-11\ldots$(i)

Let Slope of $\overline{AB}=1$

$\frac{y_1-4}{x_1-1}=1$

$x_1-1=y_1-4$

$x_1=y_1-3\ldots$(ii)

Comparing (i) and (ii)

$-7y_1-11=y_1-3$

$8y_1=-8$

$y_1=-1$

Put $y_1=-1$ in (ii)

$x_1=-1-3$

$x_1=-4$

So $B(x_1,y_1)=B(-4,-1)$

As $(2,1)$ is the mid-point of $\overline{BD}$, So

$2=\frac{x_1+x_3}{2}$, $1=\frac{y_1+y_3}{2}$

$2=\frac{-4+x_3}{2}$, $1=\frac{-1+y_3}{2}$

$4=-4+x_3$, $2=-1+y_3$

$x_3=8$, $y_3=3$

So $D(x_3,y_3)=D(8,3)$

Q.28 Find whether the given point lies above or below the given line

(a) $(5,8); 2x-3y+6=0$

Solution:

$2x-3y+6=0$

Here $b=-3$ is negative

Put $(5,8)$ in L.H.S. of Equation

$3y-2x-6=0$

$=2(5)-3(8)+6$

$=10-24+6$

$=16-24=-8<0$

As b and (ax_1+by_1+c) have same signs so point (5,8) lies above the line.

(b) $(-7,6); 4x+3y-9=0 \ldots$(i)

Solution:

$4x+3y-9=0$

As $b=3$ is positive

Putting $(-7,6)$ in L.H.S. of (i)

$= 4(-7)+3(6)-9$

$=-28+18-9$

$=-28+9$

$=-19<0$

As b and (ax_1+by_1+c) have opposite signs, so point $(-7,6)$ lies below the line.

Q.29 Check whether the given points are on the same or opposite sides of the given line.

(a) $(0,0)$ **and** $(-4,7); 6x-7y+70=0$

Solution:

$6x-7y+70=0 \ldots$(i)

Put $(0,0)$ in L.H.S. of (i)

$6(0)-7(0)+70$

$=70>0$

Put $(-4,7)$ in L.H.S. of (i)

$6(-4)-7(7)+70$

$=-24-49+70$

$=-73+70$

$=-3<0$

As both results have opposite signs, so the points are on opposite sides of the line.

(b) $(2,3)$ and $(-2,3); 3x-5y+8=0$

Solution:

$3x-5y+8=0 \ldots$(i)

Put $(2,3)$ in L.H.S. of (i)

$=3(2)-5(3)+8$

$=6-15+8$

$=14-15=-1<0$

Put $(-2,3)$ in L.H.S. of (i)

$=3(-2)-5(3)+8$

$=-6-15+8$

$=2-15=-13<0$

As both results have same signs, so the points are on same sides of the line.

Q.30 Find the distance from the point $P(6,-1)$ to the line $6x-4y+9=0$

Solution:

Distance of a point $P(x_1,y_1)$ from line $ax+by+c=0$ is given by

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

$$d=\frac{|6(6)-4(-1)+9|}{\sqrt{(6)^2+(-4)^2}}$$

$$d=\frac{|36+4+9|}{\sqrt{36+16}}$$

$$\boxed{d=\frac{49}{\sqrt{52}}}$$

Q.31 Find the area of triangular region whose vertices are $A(5,3), B(-2,2), C(4,2)$.

Solution:

If three vertices are given then area of triangular region is

$$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Delta=\frac{1}{2}\begin{vmatrix} 5 & 3 & 1 \\ -2 & 2 & 1 \\ 4 & 2 & 1 \end{vmatrix}$$

Expanding by R_1

$$\Delta=\frac{1}{2}\left[5\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix}-3\begin{vmatrix} -2 & 1 \\ 4 & 1 \end{vmatrix}+1\begin{vmatrix} -2 & 2 \\ 4 & 2 \end{vmatrix}\right]$$

$$\Delta=\frac{1}{2}\left[5(2-2)-3(-2-4)+1(-4-8)\right]$$

$$\Delta=\frac{1}{2}[0+18-12]$$

$$\Delta=\frac{1}{2}(6)=3 \text{ square units}$$

Q.32 The coordinates of three points are $A(2,3), B(-1,1)$ and $C(4,-5)$. By computing the area bounded by ABC check whether the points are collinear.

Solution:

Area of triangle is given by

$$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Delta=\frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 1 \\ 4 & -5 & 1 \end{vmatrix}$$

Expanding by R_1

$$\Delta=\frac{1}{2}\left[2\begin{vmatrix} 1 & 1 \\ -5 & 1 \end{vmatrix}-3\begin{vmatrix} -1 & 1 \\ 4 & 1 \end{vmatrix}+1\begin{vmatrix} -1 & 1 \\ 4 & -5 \end{vmatrix}\right]$$

$$\Delta=\frac{1}{2}\left[2(1+5)-3(-1-4)+1(5-4)\right]$$

$$\Delta=\frac{1}{2}[12+15+1]$$

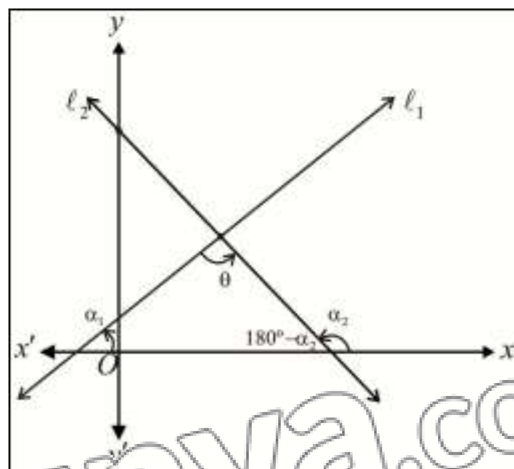
$$\Delta=\frac{1}{2}[28]$$

$\Delta=14$ square units

As area of triangle is non-zero so given points are non-collinear

<u>Angles between two lines</u>:

If l_1 and l_2 are two non-vertical lines such that they are not perpendicular to each other. If m_1 and m_2 are the slopes of l_1 and l_2 respectively, then the angle θ from l_1 to l_2 is given by $\tan\theta=\frac{m_2-m_1}{1+m_1m_2}$



Proof:

From the figure

$$\alpha_1+180^\circ-\alpha_2+\theta=180^\circ$$

$$\Rightarrow \theta=\alpha_2-\alpha_1$$

$$\tan\theta=\tan(\alpha_2-\alpha_1)$$

$$\tan\theta=\frac{\tan\alpha_2-\tan\alpha_1}{1+\tan\alpha_1\tan\alpha_2}$$

$$\Rightarrow \tan\theta=\frac{m_2-m_1}{1+m_1m_2}$$

<u>Two important results</u>:

(i) $\ell_1 \parallel \ell_2$ if and only if $m_1=m_2$

$\Leftrightarrow \quad \tan 0^\circ = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

$\Leftrightarrow \quad 0 = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

$\Leftrightarrow m_1 = m_2$

(ii) $l_1 \perp l_2$ if and only if $1 + m_2 m_1 = 0$

$\Leftrightarrow \quad \tan 90^\circ = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

$\Leftrightarrow \quad \infty = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

$\Leftrightarrow \quad 1 + m_2 m_1 = 0$

$\Leftrightarrow \quad m_1 m_2 = -1$

<u>Two and three straight lines</u>:

For any two distinct lines l_1, l_2.

$l_1 : a_1 x + b_1 y + c_1 = 0$ and $l_2 : a_2 x + b_2 y + c_2 = 0$, one and only one of the following holds:

(i) $l_1 \parallel l_2$ **(ii)** $l_1 \perp l_2$ **(iii)** l_1 and l_2 are not related as (i) or (ii)

(i) $l_1 \parallel l_2 \Leftrightarrow$ slope of $l_1(m_1) =$ slope of $l_2(m_2)$

$\Leftrightarrow -\dfrac{a_1}{b_1} = -\dfrac{a_2}{b_2} \quad \Leftrightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \quad \Leftrightarrow a_1 b_2 - b_1 a_2 = 0$

(ii) $l_1 \perp l_2 \quad \Leftrightarrow m_1 m_2 = -1$

$\Leftrightarrow \left(-\dfrac{a_1}{b_1}\right)\left(-\dfrac{a_2}{b_2}\right) = -1 \quad \Leftrightarrow a_1 a_2 + b_1 b_2 = 0$

(iii) If l_1 and l_2 are not related as in (i) and (ii) then there is no simple relation of the above forms

<u>The Point of Intersection of two Straight Lines</u>:

Let $l_1 : a_1 x + b_1 y + c_1 = 0$ (i)

and $l_2 : a_2 x + b_2 y + c_2 = 0$ (ii)

be two non-parallel lines. Then $a_1 b_2 - b_1 a_2 \neq 0$

Let $P(x_1, y_1)$ be the point of intersection of l_1 and l_2. Then solving (i) and (ii) simultaneously, we have

$$\frac{x_1}{b_1c_2 - b_2c_1} = \frac{y_1}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x_1 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and } y_1 = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

Is the required point of intersection?

Condition of Concurrency of Three Straight Lines:

Three non-parallel lines

$l_1 : a_1x + b_1y + c_1 = 0$

$l_2 : a_2x + b_2y + c_2 = 0$

$l_3 : a_3x + b_3y + c_3 = 0$

Are concurrent iff $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$