

EXERCISE 4.4

Q.1 Find the point of intersection of the lines

(i) $x - 2y + 1 = 0$ and $2x - y + 2 = 0$

Solution:

$$x - 2y + 1 = 0 \Rightarrow x = 2y - 1 \dots (i)$$

$$2x - y + 2 = 0$$

$$2x = y - 2$$

$$x = \frac{y - 2}{2} \dots (ii)$$

Comparing (i) and (ii)

$$2y - 1 = \frac{y - 2}{2}$$

$$4y - 2 = y - 2$$

$$3y = 0$$

$$y = 0$$

Put $y = 0$ in (i)

$$x = 2(0) - 1$$

$$x = -1$$

So point of intersection is $(-1, 0)$

(ii) $3x + y + 12 = 0$ and $x + 2y - 1 = 0$

Solution:

$$x + 2y - 1 = 0 \Rightarrow x = -2y + 1 \dots (i)$$

$$3x + y + 12 = 0$$

$$3x = -y - 12$$

$$x = \frac{-y - 12}{3} \dots (ii)$$

Comparing (i) and (ii)

$$\frac{-y - 12}{3} = -2y + 1$$

$$-y - 12 = -6y + 3$$

$$5y = 15$$

$$y = 3$$

Put $y = 3$ in (i)

$$x = -2(3) + 1$$

$$x = -6 + 1$$

$$x = -5$$

So point of intersection is $(-5, 3)$.

(ii) $x + 4y - 12 = 0$ and $x - 3y + 3 = 0$

Solution:

$$x + 4y - 12 = 0 \Rightarrow x = -4y + 12 \dots (i)$$

$$x - 3y + 3 = 0$$

$$x = 3y - 3 \dots (ii)$$

Comparing (i) and (ii)

$$-4y + 12 = 3y - 3$$

$$7y = 15$$

$$y = \frac{15}{7}$$

Put $y = \frac{15}{7}$ in (ii)

$$x = 3\left(\frac{15}{7}\right) - 3$$

$$x = \frac{45 - 21}{7} = \frac{24}{7}$$

So point of intersection is $\left(\frac{24}{7}, \frac{15}{7}\right)$

Q.2 Find an equation of the line through

(i) The point $(2, -9)$ and the intersection of the lines

$$2x + 5y - 8 = 0 \text{ and } 3x - 4y - 6 = 0$$

Solution:

$$2x + 5y - 8 = 0$$

$$2x = -5y + 8$$

$$x = \frac{-5y + 8}{2} \dots (i)$$

$$3x - 4y - 6 = 0$$

$$3x = 4y + 6$$

$$x = \frac{4y + 6}{3} \dots (ii)$$

Comparing (i) and (ii)

$$\frac{-5y + 8}{2} = \frac{4y + 6}{3}$$

$$-15y + 24 = 8y + 12$$

$$23y = 12$$

$$y = \frac{12}{23}$$

Put $y = \frac{12}{23}$ in (ii)

$$x = \frac{4\left(\frac{12}{23}\right) + 6}{3}$$

$$x = \frac{48 + 138}{3 \times 23}$$

$$x = \frac{186}{3 \times 23}$$

$$x = \frac{62}{23}$$

So point of intersection is $\left(\frac{62}{23}, \frac{12}{23}\right)$

So equation of line through $(2, -9)$ and

$\left(\frac{62}{23}, \frac{12}{23}\right)$ is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-9) = \frac{\frac{12}{23} - (-9)}{\frac{62}{23} - 2}(x - 2)$$

$$y - (-9) = \frac{219}{16}(x - 2)$$

$$16y + 144 = 219x - 438$$

$$\boxed{219x - 16y - 582 = 0}$$

(ii) The intersection of the lines

$$x - y - 4 = 0 \text{ and } 7x + y + 20 = 0$$

(a) Parallel to the line $6x + y - 14 = 0$

Solution:

$$x - y - 4 = 0 \Rightarrow y = x - 4 \dots (i)$$

$$7x + y + 20 = 0 \Rightarrow y = -7x - 20 \dots (ii)$$

Comparing (i) and (ii)

$$x - 4 = -7x - 20$$

$$8x = -16$$

$$x = -2$$

Put $x = -2$ in (i)

$$y = -2 - 4$$

$$y = -6$$

So point of intersection is $(-2, -6)$

Given line is $6x + y - 14 = 0$

$$\text{Slope of given line} = m_1 = \frac{-a}{b} = \frac{-6}{1} = -6$$

As slope of parallel lines are equal so

slope of required line = $m = -6$

Using point slope form

$$y - y_1 = m(x - x_1)$$

$$y - (-6) = -6(x - (-2))$$

$$y + 6 = -6(x + 2)$$

$$y + 6 = -6x - 12$$

$$\boxed{6x + y + 18 = 0}$$

(b) Perpendicular to the line

$$6x + y - 14 = 0$$

Solution:

$$\text{Slope of given line} = m_1 = -6$$

$$\text{Slope of required line} = m = -\frac{1}{m_1} = \frac{1}{6}$$

Using point slope form

$$y - y_1 = m(x - x_1)$$

$$y - (-6) = \frac{1}{6}(x - (-2))$$

$$6y + 36 = x + 2$$

$$\boxed{x - 6y - 34 = 0}$$

(iii) The intersection of the lines

$$x + 2y + 3 = 0; 3x + 4y + 7 = 0 \text{ and}$$

making equal intercepts on the axes

Solution:

$$x + 2y + 3 = 0 \Rightarrow x = -2y - 3 \dots (i)$$

$$3x + 4y + 7 = 0$$

$$3x = -4y - 7$$

$$x = \frac{-4y - 7}{3} \dots (ii)$$

Comparing (i) and (ii)

$$\frac{-4y - 7}{3} = -2y - 3$$

$$-4y - 7 = -6y - 9$$

$$2y = -2$$

$$y = -1$$

Put $y = -1$ in (i)

$$x = -2(-1) - 3$$

$$x = 2 - 3$$

$$x = -1$$

So the point of intersection is $(-1, -1)$

Equation of line in two intercepts form

$$\frac{x}{a} + \frac{y}{b} = 1$$

As line is making equal intercepts i.e.

$$a = b, \text{ so}$$

$$\frac{x}{a} + \frac{y}{a} = 1$$

$$x + y = a \dots \text{(iii)}$$

As the line passes through $(-1, -1)$ so

$$-1 + (-1) = a$$

$$a = -2$$

So, (iii) becomes

$$x + y = -2$$

$$\boxed{x + y + 2 = 0}$$

- Q.3** Find an equation of the line through the intersection of $16x - 10y - 33 = 0$ and $12x + 14y + 29 = 0$ and the intersection of $x - y + 4 = 0$ and $x - 7y + 2 = 0$.

Solution:

Here

$$16x - 10y - 33 = 0 \quad \text{(i)}$$

$$12x + 14y + 29 = 0 \quad \text{(ii)}$$

From (i)

$$16x = 10y + 33$$

$$x = \frac{10y + 33}{16} \quad \text{(iii)}$$

From (ii)

$$12x = -14y - 29$$

$$x = \frac{-14y - 29}{12} \quad \text{(iv)}$$

Comparing (iii) and (iv)

$$\frac{10y + 33}{16} = \frac{-14y - 29}{12}$$

$$120y + 396 = -224y - 464$$

$$120y + 224y = -464 - 396$$

$$344y = -860$$

$$y = \frac{-860}{344}$$

$$y = \frac{-5}{2}$$

Put $y = \frac{-5}{2}$ in (iii)

$$x = \frac{10\left(\frac{-5}{2}\right) + 33}{16}$$

$$= \frac{-25 + 33}{16}$$

$$= \frac{8}{16}$$

$$= \frac{1}{2}$$

So point of intersection is $\left(\frac{1}{2}, \frac{-5}{2}\right)$

Also

$$x - y + 4 = 0 \quad \text{(v)}$$

$$x - 7y + 2 = 0$$

(vi)

From (v)

$$x = y - 4$$

(vii)

Put this in (vi)

$$-6y - 2 = 0$$

$$y - 4 - 7y + 2 = 0$$

$$y = \frac{-2}{6} = \frac{-1}{3}$$

Put $y = \frac{-1}{3}$ in (vii)

$$x = \frac{-1}{3} - 4 = \frac{-1 - 12}{3}$$

$$x = \frac{-13}{3}$$

So point of intersection is $\left(\frac{-13}{3}, \frac{-1}{3}\right)$

$$\text{Slope of line} = m = \frac{-1 - \left(\frac{-5}{2}\right)}{\frac{3}{2} - \frac{1}{2}} = \frac{-2 + 5}{3 - 1} = \frac{3}{2}$$

Using point $\left(\frac{1}{2}, \frac{-5}{2}\right)$ and

Slope = $\frac{-13}{29}$, equation of the line is

$$y - y_1 = m(x - x_1)$$

$$y - \left(\frac{-5}{2}\right) = \frac{-13}{29} \left(x - \frac{1}{2}\right)$$

$$\frac{2y + 5}{2} = \frac{-13}{29} \left(\frac{2x - 1}{2}\right)$$

$$29(2y + 5) = -13(2x - 1)$$

$$58y + 145 = -26x + 13$$

$$26x - 13 + 58y + 145 = 0$$

$$26x + 58y + 132 = 0$$

Divide by 2

$$13x + 29y + 66 = 0$$

Q.4 Find the condition that the lines

$$y = m_1x + c_1; y = m_2x + c_2 \text{ and}$$

$$y = m_3x + c_3 \text{ are concurrent.}$$

Solution:

We can write

$$m_1x - y + c_1 = 0$$

$$m_2x - y + c_2 = 0$$

$$m_3x - y + c_3 = 0$$

Applying condition of concurrency

$$\begin{vmatrix} m_1 & -1 & c_1 \\ m_2 & -1 & c_2 \\ m_3 & -1 & c_3 \end{vmatrix} = 0$$

$$\text{By } R_2 - R_1, R_3 - R_1$$

$$\begin{vmatrix} m_1 & -1 & c_1 \\ m_2 - m_1 & 0 & c_2 - c_1 \\ m_3 - m_1 & 0 & c_3 - c_1 \end{vmatrix} = 0$$

Expanding by C_2

$$-(-1) \begin{vmatrix} m_2 - m_1 & c_2 - c_1 \\ m_3 - m_1 & c_3 - c_1 \end{vmatrix} + 0 - 0 = 0$$

$$(m_2 - m_1)(c_3 - c_1) - (m_3 - m_1)(c_2 - c_1) = 0$$

$$\boxed{(m_2 - m_1)(c_3 - c_1) = (m_3 - m_1)(c_2 - c_1)}$$

Q.5 Determine the value of p such that the lines $2x - 3y - 1 = 0$, $3x - y - 5 = 0$ and $3x + py + 8 = 0$ meet at a point.

Solution:

$$2x - 3y - 1 = 0$$

$$3x - y - 5 = 0$$

$$3x + py + 8 = 0$$

As lines meet at a point so

$$\begin{vmatrix} 2 & -3 & -1 \\ 3 & -1 & -5 \\ 3 & p & 8 \end{vmatrix} = 0$$

Expanding by R_1

$$2 \begin{vmatrix} -1 & -5 \\ p & 8 \end{vmatrix} - (-3) \begin{vmatrix} 3 & -5 \\ 3 & 8 \end{vmatrix} + (-1) \begin{vmatrix} 3 & -1 \\ 3 & p \end{vmatrix} = 0$$

$$2(-8 + 5p) + 3(24 + 15) - 1(3p + 3) = 0$$

$$-16 + 10p + 117 - 3p - 3 = 0$$

$$7p + 98 = 0$$

$$7p = -98$$

$$\boxed{p = -14}$$

Q.6 Show that the lines

$$4x - 3y - 8 = 0, 3x - 4y - 6 = 0 \text{ and}$$

$x - y - 2 = 0$ are concurrent and the third line bisects the angle formed by the first two lines.

Solution:

Let

$$\ell_1: 4x - 3y - 8 = 0$$

$$\ell_2: 3x - 4y - 6 = 0$$

$$\ell_3: x - y - 2 = 0$$

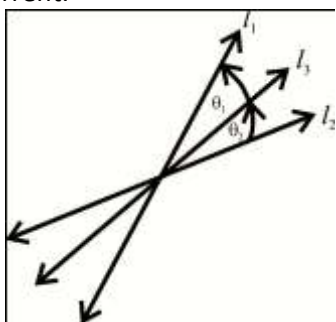
Consider

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$

Expanding by R_1

$$\begin{aligned} &= 4 \begin{vmatrix} -4 & -6 \\ -1 & -2 \end{vmatrix} - (-3) \begin{vmatrix} 3 & -6 \\ 1 & -2 \end{vmatrix} + (-8) \begin{vmatrix} 3 & -4 \\ 1 & -1 \end{vmatrix} \\ &= 4(3-6) - 3(-6+6) - 8(-3+4) \\ &= 8+0-8=0 \end{aligned}$$

As determinant is zero so lines are concurrent.



$$\text{Slope of } l_1 = m_1 = \frac{-a}{b} = -\left(\frac{4}{-3}\right) = \frac{4}{3}$$

$$\text{Slope of } l_2 = m_2 = \frac{-a}{b} = -\frac{3}{(-4)} = \frac{3}{4}$$

$$\text{Slope of } l_3 = m_3 = \frac{-a}{b} = -\left(\frac{1}{-1}\right) = 1$$

Let the angle from l_3 and l_1 be θ_1 , then

$$\begin{aligned} \tan \theta_1 &= \frac{m_1 - m_3}{1 + m_1 m_3} \\ &= \frac{\frac{4}{3} - 1}{1 + \left(\frac{4}{3}\right)(1)} = \frac{\frac{4-3}{3}}{\frac{3+4}{3}} \end{aligned}$$

$$\begin{aligned} \tan \theta_1 &= \frac{1}{7} \\ \theta_1 &= \tan^{-1}\left(\frac{1}{7}\right) \dots (i) \end{aligned}$$

Let the angle from l_2 and l_3 be θ_2 , then

$$\begin{aligned} \tan \theta_2 &= \frac{m_3 - m_2}{1 + m_3 m_2} = \frac{1 - \frac{3}{4}}{1 + (1)\left(\frac{3}{4}\right)} \\ &= \frac{4-3}{4+3} = \frac{1}{7} \end{aligned}$$

$$\theta_2 = \tan^{-1}\left(\frac{1}{7}\right) \dots (ii)$$

Comparing (i) and (ii)

$$\theta_1 = \theta_2$$

As $\theta_1 = \theta_2$ so third line bisects the angle formed by first two lines.

Q.7 The vertices of a triangle are $A(-2,3)$, $B(-4,1)$ and $C(3,5)$.

Find coordinates of the

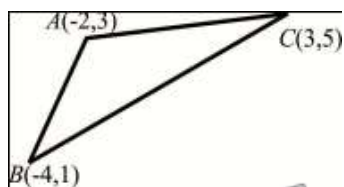
- (i) Centroid
 - (ii) Orthocentre
 - (iii) Circumcentre of the triangle
- Are these three points collinear?

Solution:

- (i) If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of triangle, then

$$\begin{aligned} \text{Centroid} &= C\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) \\ &= C\left(\frac{-2 + (-4) + 3}{3}, \frac{3 + 1 + 5}{3}\right) \\ &= C\left(\frac{-3}{3}, \frac{9}{3}\right) \\ &= C(-1, 3) \end{aligned}$$

- (ii) **Orthocentre**
Orthocenter is the point of intersection of altitudes.



$$\begin{aligned} \text{Slope of side } \overline{AB} = m_1 &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - 1}{-2 - (-4)} \\ &= \frac{2}{2} = 1 \end{aligned}$$

$$\begin{aligned} \text{Slope of line perpendicular to side } \overline{AB} \\ = m_2 &= \frac{-1}{m_1} = -1 \end{aligned}$$

Using point $C(3,5)$ and slope $= -1$, the equation of altitude on \overline{AB} from 'C' is

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -1(x - 3)$$

$$y - 5 = -x + 3$$

$$x + y - 8 = 0 \dots (i)$$

$$\text{Slope of side } \overline{BC} = m_3 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 1}{3 - (-4)} = \frac{4}{7}$$

Slope of side perpendicular to

$$\overline{BC} = m_4 = -\frac{1}{m_3} = -\frac{1}{\left(\frac{4}{7}\right)} = -\frac{7}{4}$$

Using point $A(-2,3)$ and slope $= -\frac{7}{4}$,

the equation of altitude on \overline{BC} from A is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{7}{4}(x - (-2))$$

$$4y - 12 = -7x - 14$$

$$7x + 4y + 2 = 0 \dots (ii)$$

As orthocentre is point of intersection of (i) and (ii) so

From (i)

$$y = 8 - x \dots (iii)$$

Put $y = 8 - x$ in (ii)

$$7x + 4(8 - x) + 2 = 0$$

$$7x + 32 - 4x + 2 = 0$$

$$3x + 34 = 0$$

$$x = \frac{-34}{3}$$

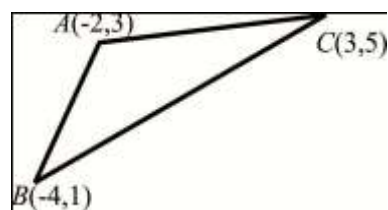
Put $x = \frac{-34}{3}$ in (iii)

$$y = 8 - \left(\frac{-34}{3}\right) = \frac{24 + 34}{3}$$

$$y = \frac{58}{3}$$

So orthocentre is $\left(\frac{-34}{3}, \frac{58}{3}\right)$

(iii) Circumcentre of the triangle



Circumcentre is the point of intersection of right bisectors of a triangle.

$$\begin{aligned} \text{Mid-point of } \overline{AB} &= \left(\frac{-2 + (-4)}{2}, \frac{3 + 1}{2}\right) \\ &= (-3, 2) \end{aligned}$$

$$\begin{aligned} \text{Slope of side } \overline{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - 3}{-4 - (-2)} = \frac{-2}{-2} = 1 \end{aligned}$$

Slope of line perpendicular to

$$\overline{AB} = \frac{-1}{1} = -1$$

Using point $(-3, 2)$ and slope $= -1$, the

equation of right bisector of \overline{AB} is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -1(x - (-3))$$

$$y - 2 = -x - 3$$

$$x + y + 1 = 0 \dots (i)$$

$$\begin{aligned} \text{Mid point of side } \overline{BC} &= \left(\frac{-4+3}{2}, \frac{1+5}{2} \right) \\ &= \left(\frac{-1}{2}, 3 \right) \end{aligned}$$

$$\begin{aligned} \text{Slope of side } \overline{BC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 1}{3 - (-4)} = \frac{4}{7} \end{aligned}$$

Slope of line perpendicular to

$$\overline{BC} = \frac{-1}{4} = \frac{-7}{4}$$

Using point $\left(\frac{-1}{2}, 3\right)$ and slope $= \frac{-7}{4}$,

the equation of right bisector of \overline{BC} is

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{-7}{4} \left(x - \left(\frac{-1}{2} \right) \right)$$

$$4y - 12 = -7 \left(\frac{2x + 1}{2} \right)$$

$$8y - 24 = -14x - 7$$

$$14x + 8y - 17 = 0 \dots (ii)$$

As circumcentre is point of intersection of (i) and (ii), so

From (i)

$$y = -x - 1 \dots (iii)$$

Put $y = -x - 1$ in (ii)

$$14x + 8(-x - 1) - 17 = 0$$

$$14x - 8x - 8 - 17 = 0$$

$$6x - 25 = 0$$

$$x = \frac{25}{6}$$

Put $x = \frac{25}{6}$ in (iii)

$$y = -\left(\frac{25}{6}\right) - 1$$

$$y = \frac{-25 - 6}{6} = \frac{-31}{6}$$

So circumcentre is $\left(\frac{25}{6}, \frac{-31}{6}\right)$.

To check whether centroid $(-1, 3)$,

orthocenter $\left(\frac{-34}{3}, \frac{58}{3}\right)$ and

circumcentre $\left(\frac{25}{6}, \frac{-31}{6}\right)$ are

collinear, consider

$$\begin{vmatrix} -1 & 3 & 1 \\ -34 & 58 & 1 \\ \frac{25}{6} & \frac{-31}{6} & 1 \end{vmatrix}$$

Expanding by R_1

$$= -1 \begin{vmatrix} \frac{58}{3} & 1 \\ -31 & 1 \end{vmatrix} - 3 \begin{vmatrix} -34 & 1 \\ \frac{25}{6} & 1 \end{vmatrix} + 1 \begin{vmatrix} -34 & \frac{58}{3} \\ \frac{25}{6} & \frac{-31}{6} \end{vmatrix}$$

$$= -1 \left(\frac{58}{3} + 31 \right) - 3 \left(\frac{-34}{3} - \frac{25}{6} \right) + 1 \left(\frac{1054}{18} - \frac{1450}{18} \right)$$

$$= - \left(\frac{116 + 31}{6} \right) + 3 \left(\frac{68 + 25}{6} \right) + \left(\frac{1054 - 1450}{18} \right)$$

$$\begin{aligned} &= -\frac{147}{6} + \frac{279}{6} - \frac{396}{18} \\ &= \frac{-441 + 837 - 396}{18} \\ &= \frac{-837 + 837}{18} = \frac{0}{18} = 0 \end{aligned}$$

As determinant is zero so these points are collinear.

Q.8 Check whether the lines

$$4x - 3y - 8 = 0; 3x - 4y - 6 = 0;$$

$$x - y - 2 = 0 \text{ are concurrent. If so, find}$$

the point where they meet.

Solution:

$$4x - 3y - 8 = 0 \dots(i)$$

$$3x - 4y - 6 = 0 \dots(ii)$$

$$x - y - 2 = 0 \dots(iii)$$

Applying condition of concurrency

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$

Expanding by R_1

$$= 4 \begin{vmatrix} -4 & -6 \\ -1 & -2 \end{vmatrix} - (-3) \begin{vmatrix} 3 & -6 \\ 1 & -2 \end{vmatrix} + (-8) \begin{vmatrix} 3 & -4 \\ 1 & -1 \end{vmatrix}$$

$$= 4(8 - 6) + 3(-6 + 6) - 8(-3 + 4)$$

$$= 8 + 0 - 8$$

$$= 0$$

As determinant is zero so given lines are concurrent.

From (iii)

$$y = x - 2 \dots(iv)$$

Put $y = x - 2$ in (ii)

$$3x - 4(x - 2) - 6 = 0$$

$$3x - 4x + 8 - 6 = 0$$

$$-x + 2 = 0$$

$$x = 2$$

Put $x = 2$ in (iv)

$$y = 2 - 2$$

$$y = 0$$

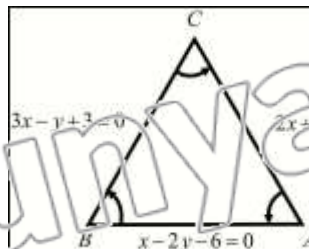
So lines meet at $(2, 0)$

Q.9 Find the coordinates of the vertices of the triangle formed by the lines

$$x - 2y - 6 = 0; 3x - y + 3 = 0;$$

$$2x + y - 4 = 0 \text{ Also find measures of the angles of the triangle.}$$

Solution:



$$l_1: x - 2y - 6 = 0 \dots(i)$$

$$l_2: 3x - y + 3 = 0 \dots(ii)$$

$$l_3: 2x + y - 4 = 0 \dots(iii)$$

Solving (i) and (ii)

From (i)

$$x = 2y + 6 \dots(iv)$$

Put $x = 2y + 6$ in (ii)

$$3(2y + 6) - y + 3 = 0$$

$$6y + 18 - y + 3 = 0$$

$$5y + 21 = 0$$

$$y = \frac{-21}{5}$$

Put $y = \frac{-21}{5}$ in (iv)

$$x = 2\left(\frac{-21}{5}\right) + 6$$

$$x = \frac{-42 + 30}{5}$$

$$x = \frac{-12}{5}$$

So $B\left(\frac{-12}{5}, \frac{-21}{5}\right)$ is the point ofintersection of l_1 and l_2

Solving (i) and (iii)

From (i)

$$x = 2y + 6 \dots(iv)$$

Put $x = 2y + 6$ in (iii)

$$2(2y + 6) + y - 4 = 0$$

$$4y + 12 + y - 4 = 0$$

$$5y + 8 = 0$$

$$y = \frac{-8}{5}$$

$$\text{Put } y = \frac{-8}{5} \text{ in (iv)}$$

$$x = 2\left(\frac{-8}{5}\right) + 6$$

$$x = \frac{-16 + 30}{5}$$

$$x = \frac{14}{5}$$

So $A\left(\frac{14}{5}, \frac{-8}{5}\right)$ is the point of

intersection of ℓ_1 and ℓ_3

Solving (ii) and (iii)

From (ii)

$$y = 3x + 3 \dots \text{(v)}$$

Put $y = 3x + 3$ in (iii)

$$2x + 3x + 3 - 4 = 0$$

$$5x - 1 = 0$$

$$x = \frac{1}{5}$$

Put $x = \frac{1}{5}$ in (v)

$$y = 3\left(\frac{1}{5}\right) + 3$$

$$y = \frac{3 + 15}{5} = \frac{18}{5}$$

So $C\left(\frac{1}{5}, \frac{18}{5}\right)$ is the point of

intersection of ℓ_2 and ℓ_3

$$\text{Slope of } \overline{AB} = m_1 = \frac{-a}{b} = \frac{-1}{-2} = \frac{1}{2}$$

$$\text{Slope of } \overline{BC} = m_2 = \frac{-a}{b} = \frac{-3}{-1} = 3$$

$$\text{Slope of } \overline{AC} = m_3 = \frac{-a}{b} = \frac{-2}{1} = -2$$

$$\tan(m\angle A) = \frac{m_1 - m_3}{1 + m_1 m_3}$$

$$= \frac{\frac{1}{2} - (-2)}{1 + (-2)\left(\frac{1}{2}\right)}$$

$$\tan(m\angle A) = \frac{4 + 1}{0}$$

$$\tan(m\angle A) = \infty$$

$$m\angle A = 90^\circ$$

$$\tan(m\angle B) = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{3 - \frac{1}{2}}{1 + \left(\frac{1}{2}\right)(3)}$$

$$\tan(m\angle B) = \frac{6 - 1}{2 + 3} = \frac{5}{5} = 1$$

$$m\angle B = \tan^{-1}(1)$$

$$m\angle B = 45^\circ$$

Also

$$m\angle C = 180^\circ - (m\angle A + m\angle B)$$

$$m\angle C = 180^\circ - (90^\circ + 45^\circ)$$

$$m\angle C = 45^\circ$$

Q.10 Find the angle measured from the line l_1 to the line l_2 where

l_1 : Joining (2,7) and (7,10)

(i) l_2 : Joining (1,1) and (-5,3)

Also find the acute angle in each case.

Solution:

$$\text{Slope of the line } l_1 = m_1 = \frac{10 - 7}{7 - 2} = \frac{3}{5}$$

$$\text{Slope of the line } l_2 = m_2 = \frac{3 - 1}{-5 - 1} = \frac{-1}{3}$$

If θ is the angle from line l_1 to l_2 then

$$\begin{aligned} \tan \theta &= \frac{m_2 - m_1}{1 + m_1 m_2} \\ &= \frac{-\frac{1}{3} - \frac{3}{5}}{1 + \left(\frac{-1}{3}\right)\left(\frac{3}{5}\right)} \end{aligned}$$

$$\begin{aligned} &= \frac{-5-9}{15-3} = \frac{-14}{12} \\ &= \frac{-7}{6} \\ \theta &= \tan^{-1}\left(\frac{-7}{6}\right) \\ &= 180^\circ - \tan^{-1}\left(\frac{7}{6}\right) \\ &= 180^\circ - 49.4^\circ \\ \theta &= 130.6^\circ \end{aligned}$$

$$\text{Acute angle} = 180^\circ - 130.6^\circ = 49.4^\circ$$

(ii) ℓ_1 : Joining (3, -1) and (5, 7)

ℓ_2 : Joining (2, 4) and (-8, 2)

Solution:

$$\text{Slope of line } \ell_1 = m_1 = \frac{7 - (-1)}{5 - 3} = 4$$

$$\text{Slope of line } \ell_2 = m_2 = \frac{2 - 4}{-8 - 2} = \frac{1}{5}$$

If θ is the angle from line ℓ_1 to ℓ_2 , then

$$\begin{aligned} \tan \theta &= \frac{m_2 - m_1}{1 + m_1 m_2} \\ &= \frac{\frac{1}{5} - 4}{1 + \left(\frac{1}{5}\right)(4)} \\ &= \frac{1 - 20}{5 + 4} = \frac{-19}{9} \end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{-19}{9}\right)$$

$$\theta = 180^\circ - \tan^{-1}\left(\frac{19}{9}\right)$$

$$\theta = 180^\circ - 64.65^\circ$$

$$\theta = 115.35^\circ$$

$$\text{Acute angle} = 180^\circ - 115.35^\circ$$

$$= 64.65^\circ$$

(iii) ℓ_1 : Joining (1, -7) and (6, -4)

ℓ_2 : Joining (-1, 2) and (-6, -1)

Solution:

$$\text{Slope of line } \ell_1 = m_1 = \frac{-4 + 7}{6 - 1} = \frac{3}{5}$$

$$\text{Slope of line } \ell_2 = m_2 = \frac{-1 - 2}{-6 + 1} = \frac{3}{5}$$

If θ is the angle from line ℓ_1 to ℓ_2 , then

$$\begin{aligned} \tan \theta &= \frac{m_2 - m_1}{1 + m_2 m_1} \\ &= \frac{\frac{3}{5} - \frac{3}{5}}{1 + \left(\frac{3}{5}\right)\left(\frac{3}{5}\right)} \end{aligned}$$

$$\tan \theta = 0$$

$$\theta = 0$$

(iv) ℓ_1 : Joining (-9, -1) and (3, -5)

ℓ_2 : Joining (2, 7) and (-6, -7)

Solution:

$$\text{Slope of line } \ell_1 = m_1 = \frac{-5 + 1}{3 + 9} = \frac{-1}{3}$$

$$\text{Slope of line } \ell_2 = m_2 = \frac{-7 - 7}{-6 - 2} = \frac{7}{4}$$

If θ is the angle from line ℓ_1 to ℓ_2 then

$$\begin{aligned} \tan \theta &= \frac{m_2 - m_1}{1 + m_2 m_1} \\ &= \frac{\frac{7}{4} - \left(-\frac{1}{3}\right)}{1 + \left(\frac{7}{4}\right)\left(-\frac{1}{3}\right)} \\ &= \frac{21 + 4}{12 - 7} = \frac{25}{5} \\ &= \frac{25}{5} \end{aligned}$$

$$\tan \theta = 5$$

$$\theta = \tan^{-1}(5)$$

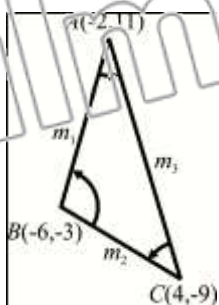
$$\theta = 78.69^\circ$$

Q.11 Find the interior angles of the triangle whose vertices are

(i) $A(-2,11), B(-6,-3), C(4,-9)$

Solution:

$A(-2,11), B(-6,-3), C(4,-9)$



$$\text{Slope of } \overline{AB} = m_1 = \frac{11 - (-3)}{-2 - (-6)} = \frac{7}{2}$$

$$\text{Slope of } \overline{BC} = m_2 = \frac{-9 - (-3)}{4 - (-6)} = \frac{-3}{5}$$

$$\text{Slope of } \overline{AC} = m_3 = \frac{-9 - 11}{4 - (-2)} = \frac{-10}{3}$$

$$\begin{aligned} \tan(m\angle B) &= \frac{m_1 - m_2}{1 + m_1 m_2} \\ &= \frac{\frac{7}{2} - \left(\frac{-3}{5}\right)}{1 + \left(\frac{7}{2}\right)\left(\frac{-3}{5}\right)} \end{aligned}$$

$$= \frac{\frac{35 + 6}{10}}{\frac{10 - 21}{10}}$$

$$= \frac{41}{-11}$$

$$m\angle B = \tan^{-1}\left(\frac{41}{11}\right)$$

$$= 180^\circ - \tan^{-1}\left(\frac{41}{11}\right)$$

$$= 180^\circ - 74.98^\circ$$

$$m\angle B = 105.02^\circ$$

$$\begin{aligned} \tan(m\angle A) &= \frac{m_3 - m_1}{1 + m_3 m_1} \\ &= \frac{\frac{-10}{3} - \frac{7}{2}}{1 + \left(\frac{-10}{3}\right)\left(\frac{7}{2}\right)} \\ &= \frac{\frac{-20 - 21}{6}}{\frac{6 - 70}{6}} \\ &= \frac{-41}{-64} = \frac{41}{64} \end{aligned}$$

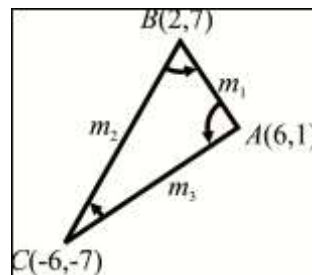
$$m\angle A = \tan^{-1}\left(\frac{41}{64}\right) = 32.64^\circ$$

Also

$$\begin{aligned} m\angle C &= 180^\circ - m\angle A - m\angle B \\ &= 180^\circ - 32.64^\circ - 105.02^\circ \\ &= 42.34^\circ \end{aligned}$$

(ii) $A(6,1), B(2,7), C(-6,-7)$

Solution:



$$\text{Slope of } \overline{AB} = m_1 = \frac{7 - 1}{2 - 6} = \frac{6}{-4} = \frac{-3}{2}$$

$$\text{Slope of } \overline{BC} = m_2 = \frac{-7 - 7}{-6 - 2} = \frac{-14}{-8} = \frac{7}{4}$$

$$\text{Slope of } \overline{AC} = m_3 = \frac{-7 - 1}{-6 - 6} = \frac{-8}{-12} = \frac{2}{3}$$

$$\begin{aligned} \tan(m\angle B) &= \frac{m_1 - m_2}{1 + m_1 m_2} \\ &= \frac{\frac{-3}{2} - \frac{7}{4}}{1 + \left(\frac{-3}{2}\right)\left(\frac{7}{4}\right)} \end{aligned}$$

$$\begin{aligned} &= \frac{-6-7}{8-21} \\ &= \frac{4}{16} \\ &= \frac{-13}{4} \times \frac{8}{-13} = 2 \end{aligned}$$

$$m\angle B = \tan^{-1}(2)$$

$$= 63.43^\circ$$

$$\tan(m\angle C) = \frac{m_2 - m_3}{1 + m_2 m_3}$$

$$\begin{aligned} &= \frac{\frac{7}{4} - \frac{2}{3}}{1 + \left(\frac{7}{4}\right)\left(\frac{2}{3}\right)} \\ &= \frac{\frac{21-8}{12}}{\frac{12+14}{12}} = \frac{13}{26} = \frac{1}{2} \end{aligned}$$

$$m\angle C = \tan^{-1}\left(\frac{1}{2}\right)$$

$$= 26.57^\circ$$

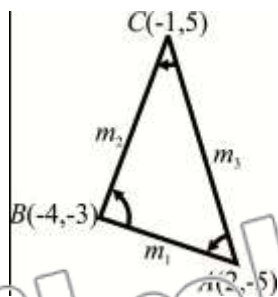
$$m\angle A = 180^\circ - m\angle B - m\angle C$$

$$= 180^\circ - 63.43^\circ - 26.57^\circ$$

$$= 90^\circ$$

(iii) $A(2, -5), B(-4, -3), C(-1, 5)$

Solution:



$$\text{Slope of } \overline{AB} = m_1 = \frac{-5+3}{2+4} = \frac{-2}{6} = \frac{-1}{3}$$

$$\text{Slope of } \overline{BC} = m_2 = \frac{5+3}{-1+4} = \frac{8}{3}$$

$$\text{Slope of } \overline{AC} = m_3 = \frac{5+5}{-1-2} = \frac{-10}{3}$$

$$\tan(m\angle B) = \frac{m_2 - m_1}{1 + m_2 m_1}$$

$$\begin{aligned} &= \frac{\frac{8}{3} - \left(\frac{-1}{3}\right)}{1 + \left(\frac{8}{3}\right)\left(\frac{-1}{3}\right)} \\ &= \frac{\frac{8+1}{3}}{\frac{9-8}{9}} = \frac{9}{1} = 9 \end{aligned}$$

$$\begin{aligned} &= \frac{8+1}{9-8} = \frac{9}{1} = 9 \end{aligned}$$

$$m\angle B = \tan^{-1}(9)$$

$$= 87.88^\circ$$

$$\tan(m\angle C) = \frac{m_3 - m_2}{1 + m_3 m_2}$$

$$\begin{aligned} &= \frac{\frac{-10}{3} - \frac{8}{3}}{1 + \left(\frac{-10}{3}\right)\left(\frac{8}{3}\right)} \\ &= \frac{\frac{-10-8}{3}}{\frac{9-80}{9}} = \frac{-18}{3} \times \frac{9}{-71} = \frac{54}{71} \end{aligned}$$

$$\begin{aligned} &= \frac{-10-8}{9-80} \\ &= \frac{-18}{9-80} \end{aligned}$$

$$= \frac{-18}{3} \times \frac{9}{-71} = \frac{54}{71}$$

$$m\angle C = \tan^{-1}\left(\frac{54}{71}\right)$$

$$= 37.25^\circ$$

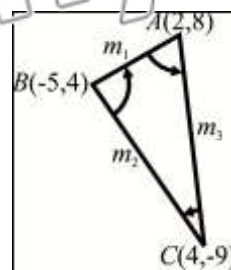
$$m\angle A = 180^\circ - m\angle B - m\angle C$$

$$= 180^\circ - 87.88^\circ - 37.25^\circ$$

$$= 54.87^\circ$$

(iv) $A(2, 8), B(-5, 4), C(4, -9)$

Solution



$$\text{Slope of side } \overline{AB} = m_1 = \frac{8-4}{2+5} = \frac{4}{7}$$

$$\text{Slope of side } \overline{BC} = m_2 = \frac{4+9}{-5-4} = -\frac{13}{9}$$

$$\text{Slope of side } \overline{AC} = m_3 = \frac{8-(-9)}{2-4} = -\frac{17}{2}$$

$$\begin{aligned} \tan(m\angle B) &= \frac{m_1 - m_2}{1 + m_1 m_2} \\ &= \frac{\frac{4}{7} - \left(-\frac{13}{9}\right)}{1 + \left(\frac{4}{7}\right)\left(-\frac{13}{9}\right)} \\ &= \frac{\frac{36+91}{63}}{\frac{63-52}{63}} = \frac{127}{11} \end{aligned}$$

$$\begin{aligned} m\angle B &= \tan^{-1}\left(\frac{127}{11}\right) \\ &= 85.05^\circ \end{aligned}$$

$$\begin{aligned} \tan(m\angle C) &= \frac{m_2 - m_3}{1 + m_2 m_3} \\ &= \frac{\frac{-13}{9} - \left(-\frac{17}{2}\right)}{1 + \left(\frac{-13}{9}\right)\left(-\frac{17}{2}\right)} \\ &= \frac{\frac{-26+153}{18}}{\frac{18+221}{18}} = \frac{127}{239} \end{aligned}$$

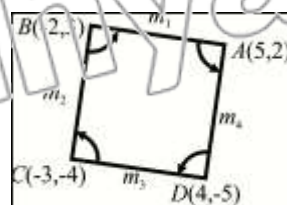
$$\begin{aligned} m\angle C &= \tan^{-1}\left(\frac{127}{239}\right) \\ &= 27.99^\circ \end{aligned}$$

$$\begin{aligned} m\angle A &= 180^\circ - m\angle B - m\angle C \\ &= 180^\circ - 85.05^\circ - 27.99^\circ \\ &= 66.96^\circ \end{aligned}$$

Q.12 Find the interior angles of the quadrilateral whose vertices are

$A(5,2), B(-2,3), C(-3,-4)$ and $D(4,-5)$.

Solution:



$$\text{Slope of side } \overline{AB} = m_1 = \frac{2-3}{5-(-2)} = \frac{-1}{7}$$

$$\text{Slope of side } \overline{BC} = m_2 = \frac{3+4}{-2+3} = 7$$

$$\text{Slope of side } \overline{CD} = m_3 = \frac{-5+4}{4+3} = \frac{-1}{7}$$

$$\text{Slope of side } \overline{AD} = m_4 = \frac{2+5}{5-4} = 7$$

$$\begin{aligned} \tan(m\angle B) &= \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{-1}{7} - 7}{1 + \left(\frac{-1}{7}\right)(7)} \\ &= \frac{\frac{-1-7}{7}}{0} = \infty \end{aligned}$$

$$m\angle B = 90^\circ$$

$$\begin{aligned} \tan(m\angle A) &= \frac{m_4 - m_1}{1 + m_4 m_1} \\ &= \frac{7 - \left(\frac{-1}{7}\right)}{1 + (7)\left(\frac{-1}{7}\right)} \\ &= \frac{7 + \frac{1}{7}}{0} = \infty \end{aligned}$$

$$m\angle A = 90^\circ$$

As $m_1 = m_3$ and $m_2 = m_4$ so $ABCD$ is parallelogram, hence its opposite angles are equal i.e.

$$m\angle D = m\angle B = 90^\circ \text{ and}$$

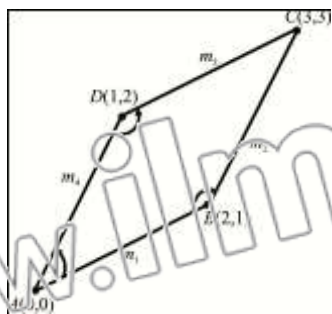
$$m\angle C = m\angle A = 90^\circ$$

Q.13 Show that the points

$A(0,0), B(2,1), C(3,3)$ and $D(1,2)$

are the vertices of a rhombus. Find its interior angles.

Solution:



$$\text{Slope of } \overline{AB} = m_1 = \frac{1-0}{2-0} = \frac{1}{2}$$

$$\text{Slope of } \overline{BC} = m_2 = \frac{3-1}{3-2} = 2$$

$$\text{Slope of } \overline{CD} = m_3 = \frac{3-2}{3-1} = \frac{1}{2}$$

$$\text{Slope of } \overline{AD} = m_4 = \frac{2-0}{1-0} = 2$$

As $m_1 = m_3$ and $m_2 = m_4$ so opposite sides are parallel.

Using distance formula

$$|\overline{AB}| = \sqrt{(2-0)^2 + (1-0)^2} = \sqrt{4+1} = \sqrt{5}$$

$$|\overline{BC}| = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{1+4} = \sqrt{5}$$

$$|\overline{CD}| = \sqrt{(3-1)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$$

$$|\overline{AD}| = \sqrt{(2-0)^2 + (1-0)^2} = \sqrt{5}$$

As length of all sides are equal so it is a rhombus.

$$\tan(m\angle B) = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{\frac{1}{2} - 2}{1 + \left(\frac{1}{2}\right)(2)}$$

$$\frac{1-4}{2+2} = \frac{-3}{4}$$

$$m\angle B = \tan^{-1}\left(\frac{-3}{4}\right)$$

$$= 180^\circ - \tan^{-1}\left(\frac{3}{4}\right)$$

$$= 180^\circ - 36.9^\circ$$

$$= 143.1^\circ$$

Since opposite angles are equal, so

$$m\angle D = m\angle B = 143.1^\circ$$

$$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$$

$$m\angle A + m\angle C + 143.1^\circ + 143.1^\circ = 360^\circ$$

$$m\angle A + m\angle C = 73.8^\circ$$

Since opposite angles are equal, so

$$m\angle A = m\angle C = 36.9^\circ$$

Q.14 Find the area of the region bounded by the triangle whose sides are

$$7x - y - 10 = 0; 10x + y - 41 = 0;$$

$$3x + 2y + 3 = 0$$

Solution:

$$\ell_1: 7x - y - 10 = 0 \Rightarrow y = 7x - 10 \dots (i)$$

$$\ell_2: 10x + y - 41 = 0 \dots (ii)$$

Put $y = 7x - 10$ in (ii)

$$10x + 7x - 10 - 41 = 0$$

$$17x - 51 = 0$$

$$17x = 51$$

$$x = 3$$

Put $x = 3$ in (i)

$$y = 7(3) - 10$$

$$y = 21 - 10$$

$$y = 11$$

Point of intersection of ℓ_1 and ℓ_2 is (3,11)

$$\ell_3: 3x + 2y + 3 = 0 \dots (iii)$$

Put $y = 7x - 10$ in (iii)

$$3x + 2(7x - 10) + 3 = 0$$

$$3x + 14x - 20 + 3 = 0$$

$$17x - 17 = 0$$

$$x = 1$$

Put $x = 1$ in (i)

$$y = 7(1) - 10$$

$$y = -3$$

Point of intersection of l_1 and l_3 is

$$(1, -3)$$

$$l_2 : 10x + y - 41 = 0 \dots (ii)$$

$$y = 41 - 10x \dots (iv)$$

Put $y = 41 - 10x$ in (iii)

$$3x + 2(41 - 10x) + 3 = 0$$

$$3x + 82 - 20x + 3 = 0$$

$$-17x + 85 = 0$$

$$17x = 85$$

$$x = 5$$

Put $x = 5$ in... (iv)

$$y = 41 - 10(5)$$

$$y = 41 - 50$$

$$y = -9$$

Point of intersection of l_2 and l_3 is

$$(5, -9)$$

Hence three vertices of triangle are

$$(3, 11), (1, -3), (5, -9)$$

Area of triangle is given by

$$\Delta = \frac{1}{2} \begin{vmatrix} 3 & 11 & 1 \\ 1 & -3 & 1 \\ 5 & -9 & 1 \end{vmatrix}$$

Expanding by R_1

$$= \frac{1}{2} \left[3 \begin{vmatrix} -3 & 1 \\ -9 & 1 \end{vmatrix} - 11 \begin{vmatrix} 1 & 1 \\ 5 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & -3 \\ 5 & -9 \end{vmatrix} \right]$$

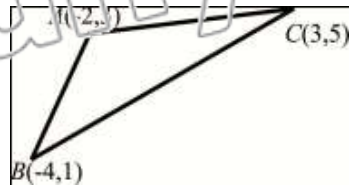
$$= \frac{1}{2} [3(-3+9) - 11(1-5) + 1(-9+15)]$$

$$= \frac{1}{2} [18 + 44 - 6]$$

$$= \frac{1}{2} (68) = 34 \text{ square unit}$$

Q.15 The vertices of a triangle are $A(-2, 3)$, $B(-4, 1)$ and $C(3, 5)$. Find the centre of the circumcircle of the triangle.

Solution:



$$\text{Slope of } \overline{AB} = m_1 = \frac{3-1}{-2+4} = \frac{2}{2} = 1$$

$$\text{Slope of } \overline{BC} = m_2 = \frac{5-1}{3+4} = \frac{4}{7}$$

$$\text{Slope of } \overline{AC} = m_3 = \frac{5-3}{3+2} = \frac{2}{5}$$

Slope of line perpendicular to

$$\overline{AB} = m_4 = \frac{-1}{m_1} = \frac{-1}{1} = -1$$

$$\begin{aligned} \text{Mid point of } \overline{AB} &= \left(\frac{-2+(-4)}{2}, \frac{3+1}{2} \right) \\ &= (-3, 2) \end{aligned}$$

Equation of line passing through

$(-3, 2)$ having slope -1 is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -1(x - (-3))$$

$$y - 2 = -1(x + 3)$$

$$y - 2 = -x - 3$$

$$x + y + 1 = 0 \dots (i)$$

Slope of line perpendicular to

$$\overline{BC} = m_5 = \frac{-1}{m_2} = \frac{-1}{\left(\frac{4}{7}\right)} = -\frac{7}{4}$$

$$\text{Mid point of } \overline{BC} = \left(\frac{-4+3}{2}, \frac{1+5}{2} \right)$$

$$= \left(\frac{-1}{2}, 3 \right)$$

Equation of line passing through

$$\left(\frac{-1}{2}, 3 \right) \text{ and having slope } -\frac{7}{4} \text{ is}$$

$$y - y_1 = m(x - x_1)$$

$$y-3 = \frac{-7}{4} \left(x - \left(\frac{-1}{2} \right) \right)$$

$$4y-12 = -7 \left(\frac{2x+1}{2} \right)$$

$$8y-24 = -14x-7$$

$$14x+8y-17=0 \dots (ii)$$

Circumcentre is the point of the intersection of (i) and (ii)

From (i)

$$y = -x-1 \dots (iii)$$

Put $y = -x-1$ in (ii)

$$14x+8(-x-1)-17=0$$

$$14x-8x-8-17=0$$

$$6x-25=0$$

$$x = \frac{25}{6}$$

Put $x = \frac{25}{6}$ in (iii)

$$y = \frac{-25}{6} - 1 = \frac{-25-6}{6} = \frac{-31}{6}$$

So circumcentre is $\left(\frac{25}{6}, \frac{-31}{6} \right)$

Q.16 Express the given system of equations in matrix form. Find in each case whether the lines are concurrent.

(i) $x+3y-2=0$; $2x-y+4=0$; $x-11y+14=0$

Solution:

$$l_1: x+3y-2=0$$

$$l_2: 2x-y+4=0$$

$$l_3: x-11y+14=0$$

The matrix equation of the system is

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Determinant of matrix of co-efficients is

$$\begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{vmatrix}$$

Expanding by R_1

$$= 1 \begin{vmatrix} -1 & 4 \\ -11 & 14 \end{vmatrix} - 3 \begin{vmatrix} 2 & 4 \\ 1 & 14 \end{vmatrix} + (-2) \begin{vmatrix} 2 & -1 \\ 1 & -11 \end{vmatrix}$$

$$= 1(-14+44) - 3(28-4) - 2(-22+1)$$

$$= 30 - 3(24) - 2(-21)$$

$$= 30 - 72 + 42$$

$$= 0$$

So given lines are concurrent

(ii) $2x-3y+4=0$; $x-2y-3=0$; $3x+y-8=0$

Solution:

$$l_1: 2x+3y+4=0$$

$$l_2: x-2y-3=0$$

$$l_3: 3x+y-8=0$$

The matrix equation of the system is

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Determinant of matrix of co-efficients is

$$\begin{vmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{vmatrix}$$

Expanding by R_1

$$= 2 \begin{vmatrix} -2 & -3 \\ 1 & -8 \end{vmatrix} - 3 \begin{vmatrix} 1 & -3 \\ 3 & -8 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix}$$

$$= 2(16+3) - 3(-8+9) + 4(1+6)$$

$$= 38 - 3 + 28$$

$$= 63 \neq 0$$

So, given lines are not concurrent

(iii) $3x-4y-2=0$; $x+2y-4=0$;
 $3x-2y+5=0$

Solution:

$$l_1: 3x-4y-2=0$$

$$l_2: x+2y-4=0$$

$$l_3: 3x-2y+5=0$$

The matrix equation of the system is

$$\begin{bmatrix} 3 & -4 & -2 \\ 1 & 2 & -4 \\ 3 & -2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Determinant of matrix of co-efficients is

$$\begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & -4 \\ 3 & -2 & 5 \end{vmatrix}$$

Expanding by R_1

$$\begin{aligned} &= 3 \begin{vmatrix} 2 & -4 \\ -2 & 5 \end{vmatrix} - (-4) \begin{vmatrix} 1 & -4 \\ 3 & 5 \end{vmatrix} + (-2) \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} \\ &= 3(10 - 8) + 4(5 + 12) - 2(-2 - 6) \\ &= 6 + 68 + 16 \\ &= 90 \neq 0 \end{aligned}$$

So the lines are not concurrent

Q.17 Find a system of linear equations corresponding to the given matrix form. Check whether the lines represented by the system are concurrent.

Solution:

$$(a) \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x + 0y - 1 \\ 2x + 0y + 1 \\ 0x - y + 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So required system of equations is

$$\begin{aligned} x - 1 &= 0 \\ 2x + 1 &= 0 \\ -y + 2 &= 0 \end{aligned}$$

Determinant of matrix of co-efficients is

$$\begin{vmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{vmatrix}$$

Expanding by R_1

$$\begin{aligned} &= 1 \begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} - 0 + (-1) \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} \\ &= 1(0 + 1) + (-1)(-2 - 0) \\ &= 1 + 2 = 3 \neq 0 \end{aligned}$$

So the lines are not concurrent

$$(b) \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} x + y + 2 \\ 2x + 4y - 3 \\ 3x + 6y - 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So required system of equations is

$$x + y + 2 = 0$$

$$2x + 4y - 3 = 0$$

$$3x + 6y - 5 = 0$$

Determinant of matrix of co-efficients is

$$\begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{vmatrix}$$

Expanding by R_1

$$\begin{aligned} &= 1 \begin{vmatrix} 4 & -3 \\ 6 & -5 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 3 & -5 \end{vmatrix} + 2 \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} \\ &= 1(-20 + 18) - 1(-10 + 9) + 2(12 - 12) \\ &= -2 + 1 + 0 = -1 \neq 0 \end{aligned}$$

So given lines are not concurrent

Homogeneous Equation:

An equation in two variables x and y is called a homogeneous equation of degree n , if the sum of exponents of x and y in each term is same and equal to n .

The equation $ax^2 + 2hxy + by^2 = 0$ is a homogenous equation of second degree which represents a pair of intersecting lines passing from origin.

Measure of the angle θ between these

$$\text{lines is given by } \tan \theta = \frac{2\sqrt{h^2 - ab}}{a + b}.$$