UNIT 5

LINEAR INEQUALITIES AND LINEAR PROGRAMMING

Linear programming:

The method of solving the linear inequalities is called linear programming.

Linear Inequality:

Inequalities are expressed by the following four symbols;
>(greater then); <(less than); ≥(greater than or equal to); ≤(less than or equal to)
The inequalities $ax+b<c$, $ax+b\le c$, $ax+b>c$, $ax+b\ge c$ are the linear inequalities in one variable and the inequalities $ax+by<c$, $ax+by\le c$, $ax+by>c$, $ax+by\ge c$ are the linear inequalities in two variables x and y .

The following operations will not effect the order of inequality while changing it to simpler equivalent form.

(i) Adding or subtracting a constant to each side of it.

(ii) Multiplying or dividing each side of it by a positive constant.

Note that order of an inequality is changed by multiplying or dividing its each side by a negative constant.

Graphing of a linear inequality in two variables:

Generally a linear inequality in two variables x and y can be one of the following forms:

$ax+by<c$; $ax+by>c$; $ax+by\le c$; $ax+by\ge c$

Where a,b and c are constants and a,b are not both zero.

We know that a graph of a linear equation of the form $ax+by=c$ is a line which divides the plane into two disjoint regions as stated below:

(i) The set of ordered pairs (x,y) such that $ax+by<c$

(ii) The set of ordered pairs (x,y) such that $ax+by>c$

The regions (i) and (ii) are called half planes and the line $ax+by=c$ is called the **boundary of each half plane.**

A vertical line divides the plane into **left and right half planes** while a non-vertical line divides the plane into **upper and lower half planes.**

A solution of a linear inequality in x and y is an ordered pair of numbers which satisfies the inequality.

Note that linear equation $ax+by=c$ is called "associated or corresponding equation" of each above mentioned inequalities.

Procedure for Graphing a linear Inequality in two Variables:

(i) The corresponding equation of the inequality is first graphed by using 'dashes' if the inequality involves the symbols > or < and a solid line is drawn of the inequality involves the symbols ≥ or ≤.

(ii) A test point (not on the graph of the corresponding equation) is chosen which determines that the half plane is on which side of the boundary line.

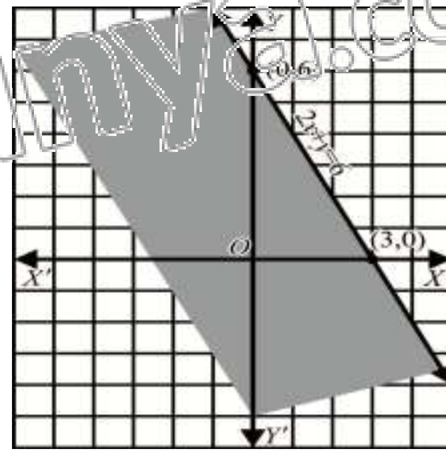
Corner points:

A point of solution region where two of its boundary lines intersect is called a **corner point** or **vertex** of the solution region.

EXERCISE 5.1

Q.1 Graph the solution set of each of the following linear inequality in *xy*–plane:

(i) $2x+y\le 6$



Solution:

$2x+y\le 6$

The associated equation of the above inequality is:

$2x+y=6\ldots$(i)

When $x=0$, then (i) becomes $y=6$

$\therefore$ $(0,6)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=3$

$\therefore$ $(3,0)$ is another point on the line (i).

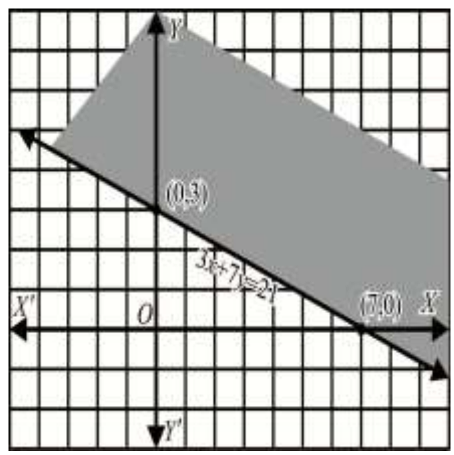
Take (0,0) as a test point. Put it in $2x+y\le 6$

$2(0)+0\le 6$

$0\le 6$ (True)

Because test point (0,0) satisfies the given inequality, so its solution is towards the origin, which is the closed half plane, as shown by shaded region in the figure.

(ii) $3x+7y\ge 21$



Solution:

$3x+7y\ge 21$

The associated equation of the above inequality is:

$3x+7y=21\ldots$(i)

When $x=0$, then (i) becomes $y=3$

$\therefore$ $(0,3)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=7$

$\therefore$ $(7,0)$ is another point on the line (i).

Now we sketch the given inequality.

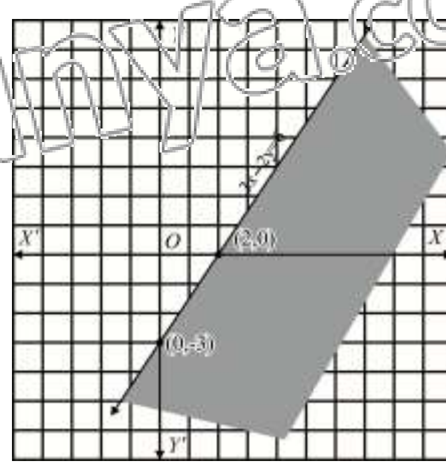
Take (0,0) as a test point. Put it in $3x+7y\ge 21$

$3(0)+7(0)\ge 21$

$0\ge 21$ (False)

Because test point (0,0) does not satisfy the given inequality, so its solution is away from the origin, which is the closed half plane, as shown by shaded region in the figure.

(iii) $3x-2y\ge 6$



Solution:

$3x-2y\ge 6$

The associated equation of the above inequality is:

$3x-2y=6\ldots$(i)

When $x=0$, then (i) becomes $y=-3$

$\therefore$ $(0,-3)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=2$

$\therefore$ $(2,0)$ is another point on the line (i).

Take (0,0) as a test point. Put it in $3x-2y\ge 6$

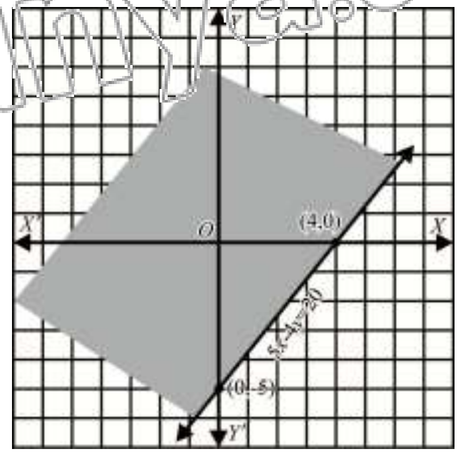
$3(0)-2(0)\ge 6$

$0 \geq 6$ (False)

Because test point (0,0) does not satisfy the given inequality, so its solution is away from the origin, which is the closed half plane, as shown by shaded region in the figure.

(iv) $5x - 4y \leq 20$

Solution:



$5x - 4y \leq 20$

The associated equation of the above inequality is:

$5x - 4y = 20$ ...(i)

When $x = 0$, then (i) becomes $y = -5$

$\therefore$ $(0,-5)$ is a point on the line (i).

When $y = 0$, then (i) becomes $x = 4$

$\therefore$ $(4,0)$ is another point on the line (i).

Take (0,0) as a test point. Put it in $5x - 4y \leq 20$

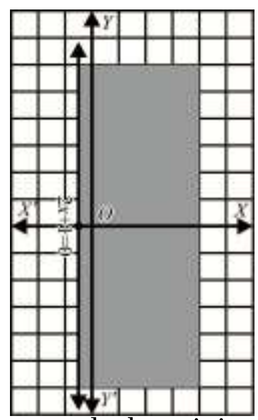
$5(0) - 4(0) \leq 20$

$0 \leq 20$ (True)

Because test point (0,0) satisfies the given inequality, so its solution is towards the origin, which is the closed half plane, as shown by shaded region in the figure.

(v) $2x + 1 \geq 0$

Solution:



$2x + 1 \geq 0$

The associated equation of the above inequality is:

$2x + 1 = 0$ ...(i)

From (i) $x = \frac{-1}{2}$

Take (0,0) as a test point. Put it in $2x + 1 \geq 0$

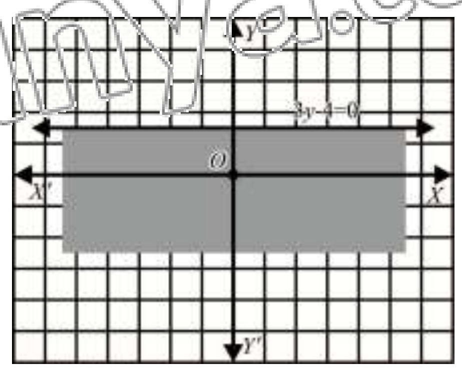
$2(0) + 1 \geq 0$

$1 \geq 0$ (True)

Because test point (0,0) satisfies the given inequality, so its solution is towards the origin, which is the closed half plane, as shown by shaded region in the figure.

(vi) $3y - 4 \leq 0$

Solution:



$3y - 4 \leq 0$

The associated equation of the above inequality is:

$3y - 4 = 0$ ...(i)

From (i) $y = \frac{4}{3}$

Take (0,0) as a test point. Put it in $3y - 4 \leq 0$

$3(0) - 4 \leq 0$

$-4 \leq 0$ (True)

Because test point (0,0) satisfies the given inequality, so its solution is towards the origin, which is the closed half plane, as shown by shaded region in the figure.

Q.2 Indicate the solution set of the following systems of linear inequalities by shading.

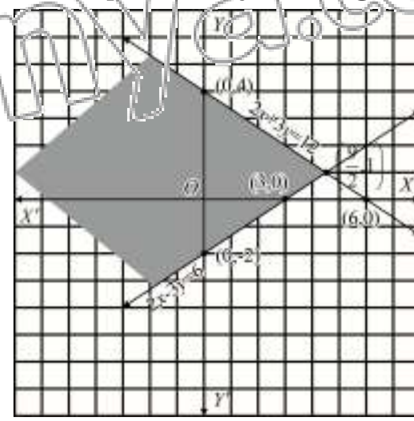
(i) $2x-3y\le 6$

$2x+3y\le 12$

Solution:

$2x-3y\le 6$, $2x+3y\le 12$

The associated equations of the above inequalities are:



$2x-3y=6$...(i)

When $x=0$, then (i) becomes $y=-2$

$\therefore$ $(0,-2)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=3$

$\therefore$ $(3,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $2x-3y\le 6$

$2(0)-3(0)\le 6$

$0\le 6$ (True)

$2x+3y=12$...(ii)

When $x=0$, then (ii) becomes $y=4$

$\therefore$ $(0,4)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=6$

$\therefore$ $(6,0)$ is a point on the line (ii).

Take (0,0) as a test point.

Put it in $2x+3y\le 12$

$2(0)+3(0)\le 12$

$0\le 12$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, the common solution for both the inequalities is shown by the shaded region as shown in the figure.

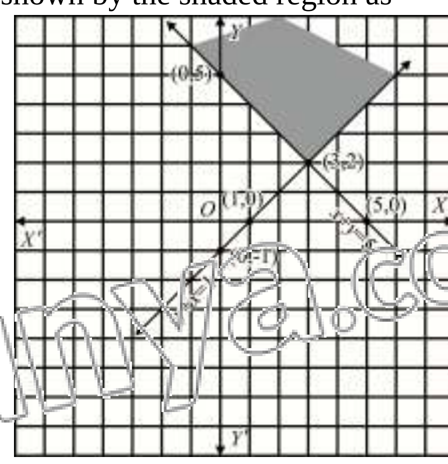
(ii) $x+y\ge 5$

$-y+x\le 1$

Solution:

$x+y\ge 5$, $-y+x\le 1$

The associated equations of the above inequalities are:



$x+y=5$...(i)

When $x=0$, then (i) becomes $y=5$

$\therefore$ $(0,5)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=5$

$\therefore$ $(5,0)$ is a point on the line (i).

$-y+x=1$...(ii)

When $x=0$, then (ii) becomes $y=-1$

$\therefore$ $(0,-1)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=1$

$\therefore$ $(1,0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in

$x+y\geq 5$

$0+0\geq 5$

$0\geq 5$ (False)

Take (0,0) as a test point. Put it in

$-y+x\leq 1$

$-0+0\leq 1$

$0\leq 1$ (True)

Because (0,0) does not satisfy the inequality (i), so its solution is away from the origin. Also, (0,0) satisfies the inequality (ii), so its solution is towards origin. The common solution for both the inequalities is shown by the shaded region as shown in the figure.

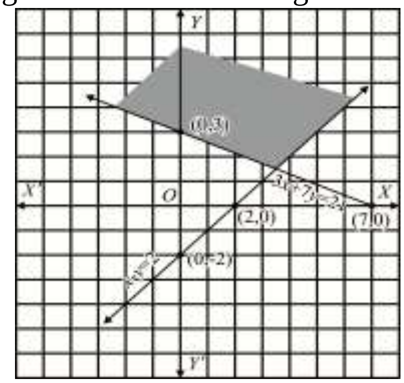
(iii) $3x+7y\geq 21$

$x-y\leq 2$

Solution:

$3x+7y\geq 21$, $x-y\leq 2$

The associated equations of the above inequalities are:



$3x+7y=21$...(i)

When $x=0$, then (i) becomes $y=3$

$\therefore$ $(0,3)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=7$

$\therefore$ $(7,0)$ is a point on the line (i).

Take (0, 0) as a test point.

Put it in $3x+7y\geq 21$

$3(0)+7(0)\geq 21$

$0\geq 21$(False)

$x-y=2$...(ii)

When $x=0$, then (ii) becomes $y=-2$

$\therefore$ $(0,-2)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=2$

$\therefore$ $(2,0)$ is a point on the line (ii).

Take (0, 0) as a test point.

Put it in $x-y\leq 2$

$0-0\leq 2$

$0\leq 2$ (True)

Because (0,0) does not satisfy the inequality (i), so its solution is away from the origin. Also, (0,0) satisfies the inequality (ii), so its solution is towards origin. The common solution for both the inequalities is shown by the shaded region as shown in the figure.

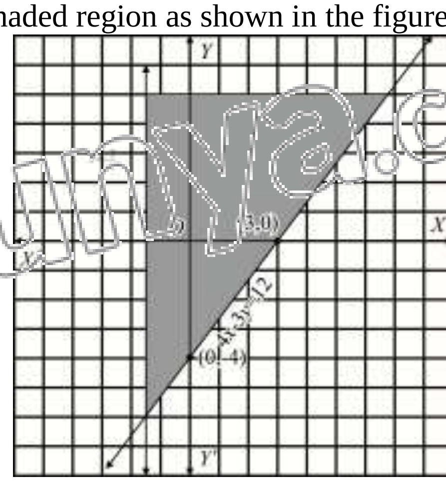
(iv) $4x-3y\leq 12$

$x\geq -\frac{3}{2}$

Solution:

$4x-3y\leq 12$, $x\geq -\frac{3}{2}$

The associated equations of the above inequalities are:



$4x-3y=12$...(i)

$x=-\frac{3}{2}$ ...(ii)

When $x=0$, then (i) becomes $y=-4$

$\therefore$ $(0,-4)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=3$

$\therefore$ $(3,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $4x-3y\le 12$

$4(0)-3(0)\le 12$

$0\le 12$ (True)

Here $x=-\frac{3}{2}$ is a vertical line.

Take (0,0) as a test point.

Put it in $x\ge -\frac{3}{2}$

$0\ge -\frac{3}{2}$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, the common solution for both the inequalities is shown by the shaded region as shown in the figure.

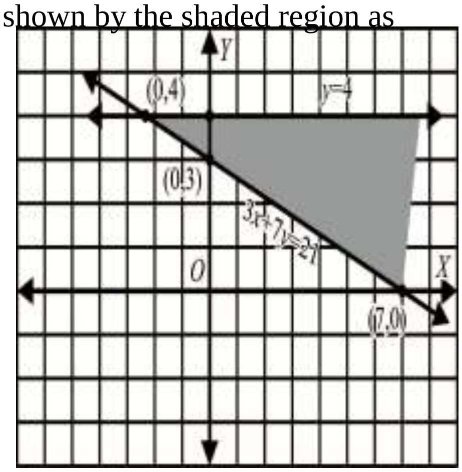
(v) $3x+7y\ge 21$

$y\le 4$

Solution:

$3x+7y\ge 21$, $y\le 4$

The associated equations of the above inequalities are:



$3x+7y=21\ldots$(i)

When $x=0$, then (i) becomes $y=3$

$\therefore$ $(0,3)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=7$

$\therefore$ $(7,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $3x+7y\ge 21$

$3(0)+7(0)\ge 21$

$0\ge 21$ (False)

$y=4\ldots$(ii)

Here $y=4$ is a horizontal line.

Take (0,0) as a test point.

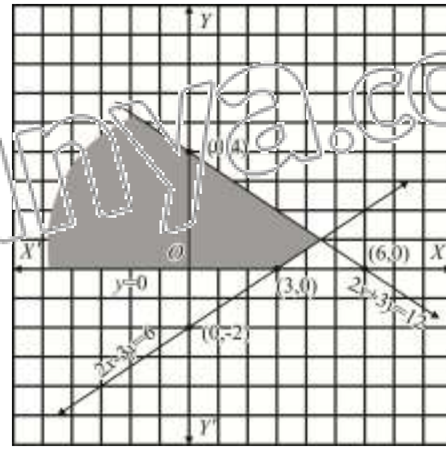
Put it in $y\le 4$

$0\le 4$ (True)

Because (0,0) does not satisfy the inequality (i), so its solution is away from the origin. Also, (0,0) satisfies the inequality (ii), so its solution is towards origin. The common solution for both the inequalities is shown by the shaded region as shown in the figure.

Q.3 Indicate the solution region of the following systems of linear inequalities by shading:

(i)
$2x-3y\le 6$
$2x+3y\le 12$
$y\ge 0$



Solution:

$2x-3y\le 6,\ 2x+3y\le 12,\ y\ge 0$

The associated equations of the above inequalities are:

$2x-3y=6\ \ldots$(i)

When $x=0$, then (i) becomes $y=-2$

$\therefore$ $(0,-2)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=3$

$\therefore$ $(3,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $2x-3y\le 6$

$2(0)-3(0)\le 6$

$0\le 6$ (True)

$2x+3y=12\ \ldots$(ii)

When $x=0$, then (ii) becomes $y=4$

$\therefore$ $(0,4)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=6$

$\therefore$ $(6,0)$ is a point on the line (ii).

Take (0,0) as a test point.

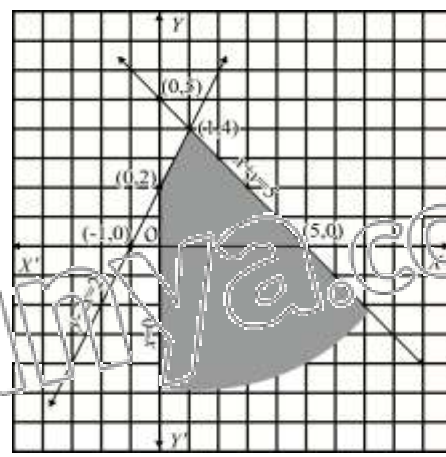
Put it in $2x+3y\le 12$

$2(0)+3(0)\le 12$

$0\le 12$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, $y\ge 0$ indicates that the solution will be above x-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

(ii)
$x+y\le 5$
$y-2x\le 2$
$x\ge 0$



Solution:

$x+y\le 5,\ y-2x\le 2,\ x\ge 0$

The associated equations of the above inequalities are:

$x+y=5\ \ldots$(i)

When $x=0$, then (i) becomes $y=5$

$\therefore$ $(0,5)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=5$

$y-2x=2\ \ldots$(ii)

When $x=0$, then (ii) becomes $y=2$

$\therefore$ $(0,2)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=-1$

$\therefore$ $(5,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $x+y\le 5$

$0+0\le 5$

$0\le 5$ (True)

$\therefore$ $(-1,0)$ is a point on the line (ii).

Take (0,0) as a test point.

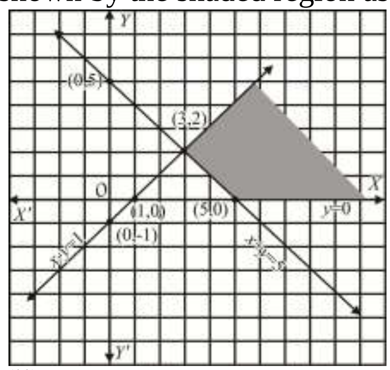
Put it in $y-2x\le 2$

$0-2(0)\le 2$

$0\le 2$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, $x\ge 0$ indicates that the solution will be on the right side of y-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

(iii)
$x+y\ge 5$
$x-y\ge 1$
$y\ge 0$



Solution:

$x+y\ge 5,\ x-y\ge 1,\ y\ge 0$

The associated equations of the above inequalities are:

$x+y=5\ldots$(i)

When $x=0$, then (i) becomes $y=5$

$\therefore$ $(0,5)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=5$

$\therefore$ $(5,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $x+y\ge 5$

$0+0\ge 5$

$0\ge 5$ (False)

$x-y=1\ldots$(ii)

When $x=0$, then (ii) becomes $y=-1$

$\therefore$ $(0,-1)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=1$

$\therefore$ $(1,0)$ is a point on the line (ii).

Take (0,0) as a test point.

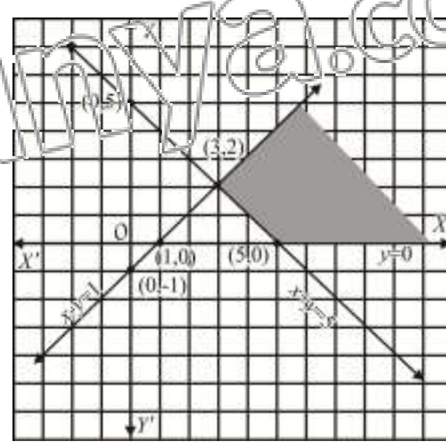
Put it in $x-y\ge 1$

$0-0\ge 1$

$0\ge 1$ (False)

Because (0,0) does not satisfies both the given inequalities, so their solution is away from the origin. Also, $y\ge 0$ indicates that the solution will be above x-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

(iv)
$3x+7y\le 21$
$x-y\le 2$
$x\ge 0$



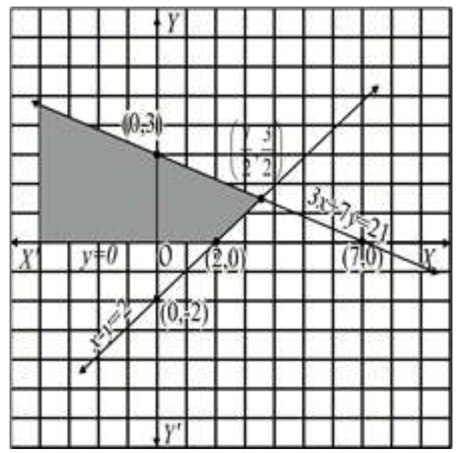
Solution:

$3x+7y\le 21,\ x-y\le 2,\ x\ge 0$

The associated equations of the above inequalities are:

$3x+7y=21 \ldots$(i)	$x-y=2 \ldots$(ii)
When $x=0$, then (i) becomes $y=3$	When $x=0$, then (ii) becomes $y=-2$
$\therefore$ $(0,3)$ is a point on the line (i).	$\therefore$ $(0,-2)$ is a point on the line (ii).
When $y=0$, then (i) becomes $x=7$	When $y=0$, then (ii) becomes $x=2$
$\therefore$ $(7,0)$ is a point on the line (i).	$\therefore$ $(2,0)$ is a point on the line (ii).
Take (0,0) as a test point.	Take (0,0) as a test point.
Put it in $3x+7y\le 21$	Put it in $x-y\le 2$
$3(0)+7(0)\le 21$	$0-0\le 2$
$0\le 21$ (True)	$0\le 2$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, $x\ge 0$ indicates that the solution will be on the right side of y-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.



(v)
$$3x+7y\le 21$$
$$x-y\le 2$$
$$y\ge 0$$

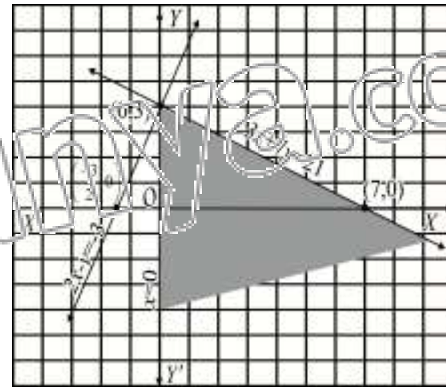
Solution:

$3x+7y\le 21,\ x-y\le 2,\ y\ge 0$

The associated equations of the above inequalities are:

$3x+7y=21 \ldots$(i)	$x-y=2 \ldots$(ii)
When $x=0$, then (i) becomes $y=3$	When $y=0$, then (ii) becomes $x=2$
$\therefore$ $(0,3)$ is a point on the line (i).	$\therefore$ $(2,0)$ is a point on the line (ii).
When $y=0$, then (i) becomes $x=7$	When $x=0$, then (ii) becomes $y=-2$
$\therefore$ $(7,0)$ is a point on the line (i).	$\therefore$ $(0,-2)$ is a point on the line (ii).
Take (0,0) as a test point.	Take (0,0) as a test point.
Put it in $3x+7y\le 21$	Put it in $x-y\le 2$
$3(0)+7(0)\le 21$	$0-0\le 2$
$0\le 21$ (True)	$0\le 2$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, $y\ge 0$ indicates that the solution will be above x-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.



$3x+7y\le 21$

(vi) $2x-y\ge -3$

$x\ge 0$

Solution:

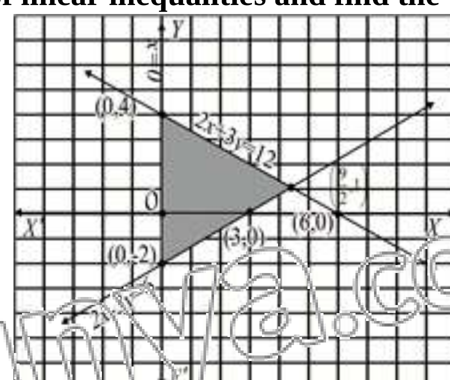
$3x+7y\le 21,\ 2x-y\ge -3,\ x\ge 0$

The associated equations of the above inequalities are:

$3x+7y=21\ldots$(i)	$2x-y=-3\ldots$(ii)
When $x=0$, then (i) becomes $y=3$	When $x=0$, then (ii) becomes $y=3$
$\therefore$ $(0,3)$ is a point on the line (i).	$\therefore$ $(0,3)$ is a point on the line (ii).
When $y=0$, then (i) becomes $x=7$	When $y=0$, then (ii) becomes $x=\frac{-3}{2}$
$\therefore$ $(7,0)$ is a point on the line (i).	$\therefore$ $\left(\frac{-3}{2},0\right)$ is a point on the line (ii).
Take (0,0) as a test point.	Take (0,0) as a test point.
Put it in $3x+7y\le 21$	Put it in $2x-y\ge -3$
$3(0)+7(0)\le 21$	$2(0)-0\ge -3$
$0\le 21$ (True)	$0\ge -3$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, $x\ge 0$ indicates that the solution will be on the right side of y-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

Q.4 Graph the solution region of the following system of linear inequalities and find the corner points in each case:



$2x-3y\le 6$

(i) $2x+3y\le 12$

$x\ge 0$

Solution:

$2x-3y\le 6,\ 2x+3y\le 12,\ x\ge 0$

The associated equations of the above inequalities are:

$2x-3y=6\ldots$(i)	$2x+3y=12\ldots$(ii)
When $x=0$, then (i) becomes $y=-2$	When $x=0$, then (ii) becomes $y=4$
$\therefore$ $(0,-2)$ is a point on the line (i).	$\therefore$ $(0,4)$ is a point on the line (ii).
When $y=0$, then (i) becomes $x=3$	When $y=0$, then (ii) becomes $x=6$
$\therefore$ $(3,0)$ is a point on the line (i).	$\therefore$ $(6,0)$ is a point on the line (ii).
Take (0,0) as a test point.	Take (0,0) as a test point.
Put it in $2x-3y\le 6$	Put it in $2x+3y\le 12$
$2(0)-3(0)\le 6$	$2(0)+3(0)\le 12$

$0 \le 6$ (True) $0 \le 12$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, $x \ge 0$ indicates that the solution will be on the right side of y-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

Now we find the corner points of the solution region

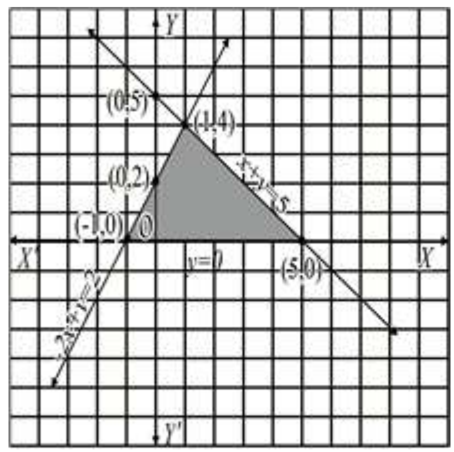
Solving $2x-3y=6$ and $2x+3y=12$, we get $\left(\frac{9}{2},1\right)$.

Hence corner points of the solution region of the given system of linear inequalities are $(0,4),(0,-2)$ and $\left(\frac{9}{2},1\right)$.

(ii) $x+y \le 5$

$-2x+y \le 2$

$y \ge 0$



Solution:

$x+y \le 5,\ -2x+y \le 2,\ y \ge 0$

The associated equations of the above inequalities are:

$x+y=5 \ldots$(i)

When $x=0$, then (i) becomes $y=5$

$\therefore$ $(0,5)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=5$

$\therefore$ $(5,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $x+y \le 5$

$0+0 \le 5$

$0 \le 5$ (True)

$-2x+y=2 \ldots$(ii)

When $x=0$, then (ii) becomes $y=2$

$\therefore$ $(0,2)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=-1$

$\therefore$ $(-1,0)$ is a point on the line (ii).

Take (0,0) as a test point.

Put it in $-2x+y \le 2$

$-2(0)+0 \le 2$

$0 \le 2$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, $y \ge 0$ indicates that the solution will be above *x*-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

Now we find the corner points of the solution region.

Solving $x+y=5$ and $-2x+y=2$ we get $(1,4)$.

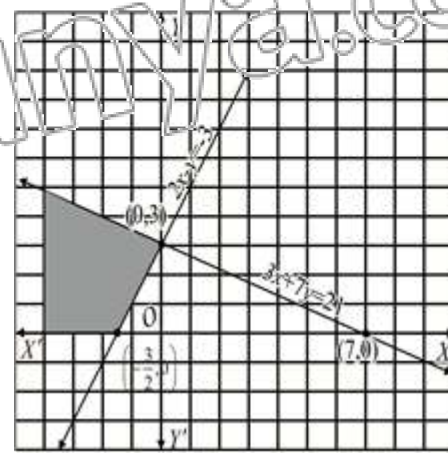
Hence corner points of the solution region of the given system of linear inequalities are $(1,4),(-1,0)$ and $(5,0)$.

(iii) $3x+7y\le 21$
$2x-y\le -3$
$y\ge 0$

Solution:

$3x+7y\le 21,\ 2x-y\le -3,\ y\ge 0$

The associated equations of the above inequalities are:



$3x+7y=21\ldots$(i)

When $x=0$, then (i) becomes $y=3$

$\therefore\ (0,3)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=7$

$\therefore\ (7,0)$ is a point on the line (i).

Take (0,0) as a test point.
Put it in $3x+7y\le 21$

$3(0)+7(0)\le 21$

$0\le 21$ (True)

$2x-y=-3\ldots$(ii)

When $x=0$, then (ii) becomes $y=3$

$\therefore\ (0,3)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=-\dfrac{3}{2}$

$\therefore\ \left(-\dfrac{3}{2},0\right)$ is a point on the line (ii).

Take (0,0) as a test point.
Put it in $2x-y\le -3$

$2(0)-0\le -3$

$0\le -3$ (False)

Because (0,0) satisfies the inequality (i), so its solution is towards the origin, and (0,0) does not satisfy the inequality (ii), so its solution is away from the origin. Also, $y\ge 0$ indicates that the solution will be above *x*-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.
The corner points of the solution region of the given system of linear inequalities are

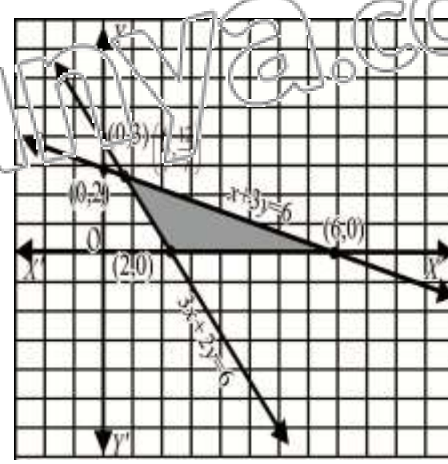
$(0,3)$ and $\left(-\dfrac{3}{2},0\right)$.

(iv) $x+3y\le 6$
$3x+2y\ge 6$
$y\ge 0$

Solution:

$x+3y\le 6,\ 3x+2y\ge 6,\ y\ge 0$

The associated equations of the above inequalities are:



$x+3y=6\ldots$(i)

When $x=0$, then (i) becomes $y=2$

$\therefore$ $(0,2)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=6$

$\therefore$ $(6,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $x+3y\le 6$

$0+3(0)\le 6$

$0\le 6$ (True)

$3x+2y=6\ldots$(ii)

When $x=0$, then (ii) becomes $y=3$

$\therefore$ $(0,3)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=2$

$\therefore$ $(2,0)$ is a point on the line (ii).

Take (0,0) as a test point.

Put it in $3x+2y\ge 6$

$3(0)+2(0)\ge 6$

$0\ge 6$ (False)

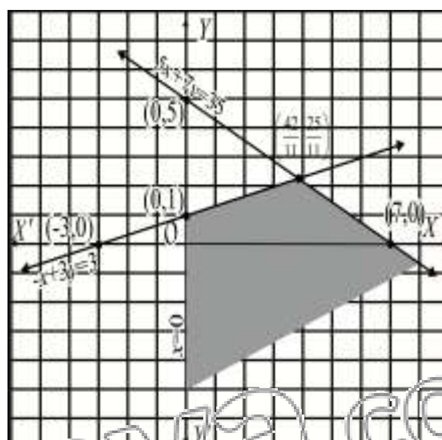
Because (0,0) satisfy the inequality (i), so its solution is towards the origin, and (0,0) does not satisfies the inequality (ii), so its solution is away from origin. Also, $y\ge 0$ indicates that the solution will be above *x*-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

Now we find the corner points of the solution region.

Solving $3x+2y=6$ and $x+3y=6$ we get $\left(\frac{6}{7},\frac{12}{7}\right)$.

The corner points of the solution region of the given system of linear inequalities are $(2,0),(6,0)$ and $\left(\frac{6}{7},\frac{12}{7}\right)$.

(v)
$$5x+7y\le 35$$
$$-x+3y\le 3$$
$$x\ge 0$$



Solution:

$5x+7y\le 35,\ -x+3y\le 3,\ x\ge 0$

The associated equations of the above inequalities are:

$5x+7y=35\ldots$(i)

When $x=0$, then (i) becomes $y=5$

$\therefore$ $(0,5)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=7$

$\therefore$ $(7,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $5x+7y\le 35$

$5(0)+7(0)\le 35$

$0\le 35$ (True)

$-x+3y=3\ldots$(ii)

When $x=0$, then (ii) becomes $y=1$

$\therefore$ $(0,1)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=-3$

$\therefore$ $(-3,0)$ is a point on the line (ii).

Take (0,0) as a test point.

Put it in $-x+3y\le 3$

$-0+3(0)\le 3$

$0\le 3$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, $x \geq 0$ indicates that the solution will be on the right side of x-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

Now we find the corner points of the solution region.

Solving $5x+7y=35$ and $-x+3y=3$ we get $\left(\frac{42}{11}, \frac{25}{11}\right)$.

The corner points of the solution region of the given system of linear inequalities are $(0,1)$ and $\left(\frac{42}{11}, \frac{25}{11}\right)$.

(vi) $5x+7y \leq 35$

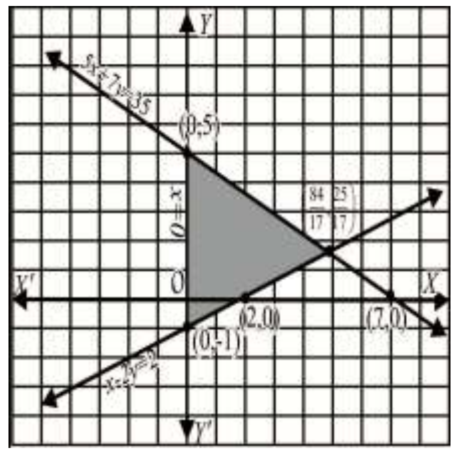
$x-2y \leq 2$

$x \geq 0$

Solution:

$5x+7y \leq 35,\ x-2y \leq 2,\ x \geq 0$

The associated equations of the above inequalities are:



$5x+7y=35 \ldots$(i)

When $x=0$, then (i) becomes $y=5$

$\therefore$ $(0,5)$ is a point on the line (i).

When $x=0$, then (ii) becomes $y=-1$

$\therefore$ $(0,-1)$ is a point on the line (ii).

Take (0,0) as a test point.

Put it in $5x+7y \leq 35$

$5(0)+7(0) \leq 35$

$0 \leq 35$ (True)

$x-2y=2 \ldots$(ii)

When $y=0$, then (i) becomes $x=7$

$\therefore$ $(7,0)$ is a point on the line (i).

When $y=0$, then (ii) becomes $x=2$

$\therefore$ $(2,0)$ is a point on the line (ii).

Take (0,0) as a test point.

Put it in $x-2y \leq 2$

$0-2(0) \leq 2$

$0 \leq 2$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards the origin. Also, $x \geq 0$ indicates that the solution will be on the right side of y-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

Now we find the corner points of the solution region.

Solving $5x+7y=35$ and $x-2y=2$ we get $\left(\frac{84}{17}, \frac{25}{17}\right)$.

The corner points of the solution region of the given system of linear inequalities are

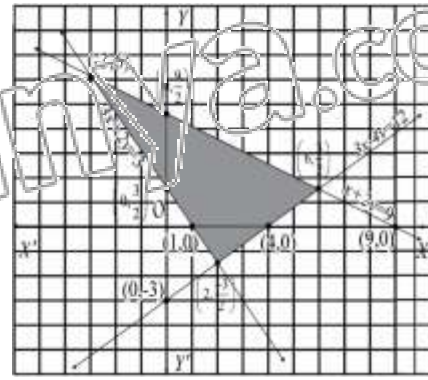
$(0,5), (0,-1)$ and $\left(\frac{84}{17}, \frac{25}{17}\right)$.

Q.5 Graph the solution region of the following system of linear inequalities by shading:

(i)
$3x-4y\le 12$
$3x+2y\ge 3$
$x+2y\le 9$

Solution:

$3x-4y\le 12,\ 3x+2y\ge 3,\ x+2y\le 9$



The associated equations of the above inequalities are:

$3x-4y=12$ ...(i)
When $x=0$, then (i) becomes $y=-3$
$\therefore$ $(0,-3)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=4$
$\therefore$ $(4,0)$ is a point on the line (i).
Take (0,0) as a test point. Put it in
$3x-4y\le 12$
$3(0)-4(0)\le 12$
$0\le 12$ (True)

$3x+2y=3$...(ii)
When $x=0$ then (ii) becomes $y=\frac{3}{2}$
$\therefore$ $\left(0,\frac{3}{2}\right)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=1$
$\therefore$ $(1,0)$ is a point on the line (ii).
Take (0,0) as a test point. Put it in
$3x+2y\ge 3$
$3(0)+2(0)\ge 3$
$0\ge 3$ (False)

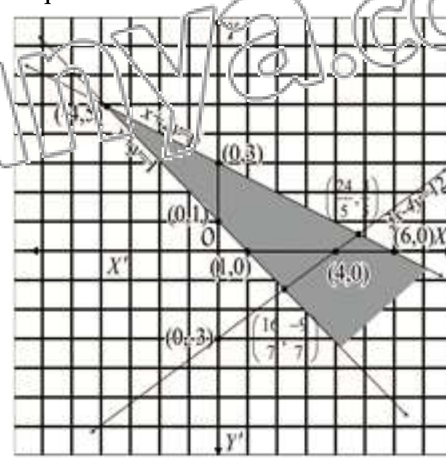
$x+2y=9$...(iii)
When $x=0$, then (iii) becomes $y=\frac{9}{2}$
$\therefore$ $\left(0,\frac{9}{2}\right)$ is a point on the line (iii).
When $y=0$, then (iii) becomes $x=9$
$\therefore$ $(9,0)$ is a point on the line (iii).
Take (0,0) as a test point. Put it in
$x+2y\le 9$
$0+2(0)\le 9$
$0\le 9$ (True)

Because (0,0) satisfies the inequalities (i) and (iii), so their solution is towards the origin, and (0,0) does not satisfy the inequality (ii), so its solution is away from the origin. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

(ii)
$3x-4y\le 12$
$x+2y\le 6$
$x+y\ge 1$

Solution:

$3x-4y\le 12,\ x+2y\le 6,\ x+y\ge 1$



The associated equations of the above inequalities are:

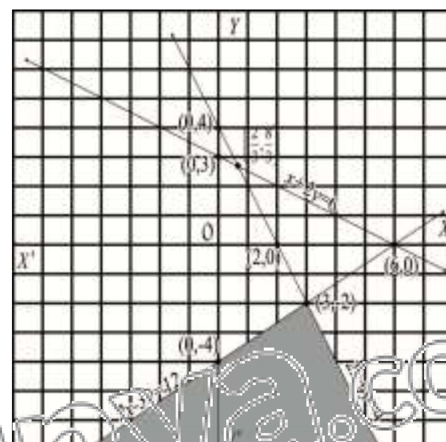
$3x-4y=12$...(i)
When $x=0$, then (i) becomes $y=-3$
$\therefore$ $(0,-3)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=4$
$\therefore$ $(4,0)$ is a point on the line (i).
Take (0,0) as a test point. Put it in
$3x-4y\le 12$
$3(0)-4(0)\le 12$
$0\le 12$ (True)

$x+2y=6$...(ii)
When $x=0$, then (ii) becomes $y=3$
$\therefore$ $(0,3)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=6$
$\therefore$ $(6,0)$ is a point on the line (ii).
Take (0,0) as a test point. Put it in
$x+2y\le 6$
$0+2(0)\le 6$
$0\le 6$ (True)

$x+y=1$...(iii)
When $x=0$, then (iii) becomes $y=1$
$\therefore$ $(0,1)$ is a point on the line (iii).
When $y=0$, then (iii) becomes $x=1$
$\therefore$ $(1,0)$ is a point on the line (iii).
Take (0,0) as a test point. Put it in
$x+y\ge 1$
$0+0\ge 1$
$0\ge 1$ (False)

Because (0,0) satisfies the inequalities (i) and (ii), so their solution is towards the origin, and (0,0) does not satisfy the inequality (iii), so its solution is away from the origin. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

(iii) $2x+y\le 4$
$2x-3y\ge 12$
$x+2y\le 6$



Solution:

$2x+y\le 4,\ 2x-3y\ge 12,\ x+2y\le 6$

The associated equations of the above inequalities are:

$2x+y=4$...(i)
When $x=0$, then (i) becomes $y=4$
$\therefore$ $(0,4)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=2$

$2x-3y=12$...(ii)
When $x=0$, then (ii) becomes $y=-4$
$\therefore$ $(0,-4)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=6$

$x+2y=6$...(iii)
When $x=0$, then (iii) becomes $y=3$
$\therefore$ $(0,3)$ is a point on the line (iii).
When $y=0$, then (iii) becomes $x=6$

$\therefore$ $(2,0)$ is a point on the line (i).
Take (0,0) as a test point. Put it in

$2x+y\le 4$

$2(0)+(0)\le 4$

$0\le 4$ (True)

$\therefore$ $(6,0)$ is a point on the line (ii).
Take (0,0) as a test point. Put it in

$2x-3y\ge 12$

$2(0)-3(0)\ge 12$

$0\ge 12$ (False)

$\therefore$ $(6,0)$ is a point on the line (iii).
Take (0,0) as a test point. Put it in

$x+2y\le 6$

$0+2(0)\le 6$

$0\le 6$ (True)

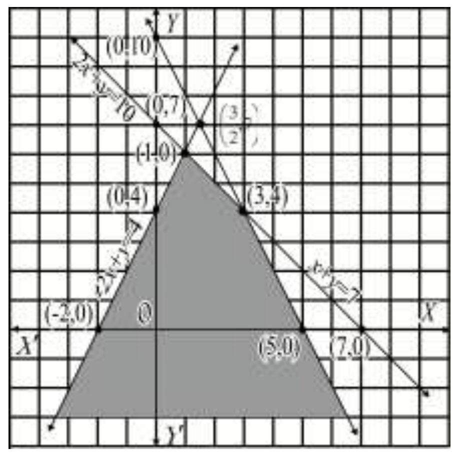
Because (0,0) satisfies the inequalities (i) and (iii), so their solution is towards the origin, and (0,0) does not satisfy the inequality (ii), so its solution is away from the origin. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

(iv) $2x+y\le 10$
$x+y\le 7$
$-2x+y\le 4$

Solution:

$2x+y\le 10,\ x+y\le 7,\ -2x+y\le 4$

The associated equations of the above inequalities are:



$2x+y=10$...(i)
When $x=0$, then (i) becomes $y=10$

$\therefore$ $(0,10)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=5$

$\therefore$ $(5,0)$ is a point on the line (i).
Take (0,0) as a test point. Put it in

$2x+y\le 10$

$2(0)+0\le 10$

$0\le 10$ (True)

$x+y=7$...(ii)
When $x=0$, then (ii) becomes $y=7$

$\therefore$ $(0,7)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=7$

$\therefore$ $(7,0)$ is a point on the line (ii).
Take (0,0) as a test point. Put it in

$x+y\le 7$

$0+0\le 7$

$0\le 7$ (True)

$-2x+y=4$...(iii)
When $x=0$, then (iii) becomes $y=4$

$\therefore$ $(0,4)$ is a point on the line (iii).
When $y=0$, then (iii) becomes $x=-2$

$\therefore$ $(-2,0)$ is a point on the line (iii).
Take (0,0) as a test point. Put it in

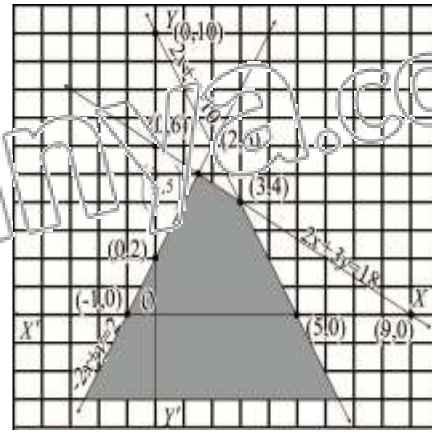
$-2x+y\le 4$

$-2(0)+0\le 4$

$0\le 4$ (True)

Because (0,0) satisfies all the inequalities (i), (ii) and (iii) so their solution is towards the origin. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

(v)
$$2x+3y\le 18$$
$$2x+y\le 10$$
$$-2x+y\le 2$$



Solution:

$2x+3y\le 18,\ 2x+y\le 10,\ -2x+y\le 2$

The associated equations of the above inequalities are:

$2x+3y=18$...(i)

When $x=0$, then (i) becomes $y=6$

$\therefore$ $(0,6)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=9$

$\therefore$ $(9,0)$ is a point on the line (i).

Take (0,0) as a test point. Put it in $2x+3y\le 18$

$2(0)+3(0)\le 18$

$0\le 18$ (True)

$2x+y=10$...(ii)

When $x=0$, then (ii) becomes $y=10$

$\therefore$ $(0,10)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=5$

$\therefore$ $(5,0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in $2x+y\le 10$

$2(0)+0\le 10$

$0\le 10$ (True)

$-2x+y=2$...(iii)

When $x=0$, then (iii) becomes $y=2$

$\therefore$ $(0,2)$ is a point on the line (iii).

When $y=0$, then (iii) becomes $x=-1$

$\therefore$ $(-1,0)$ is a point on the line (iii).

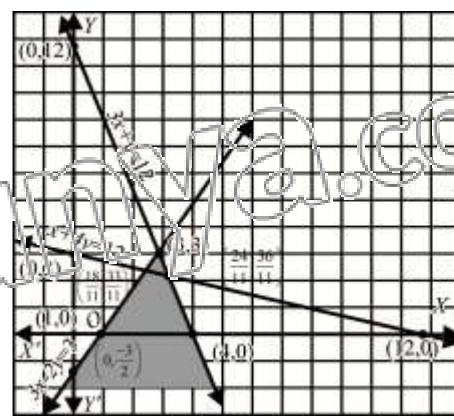
Take (0,0) as a test point. Put it in $-2x+y\le 2$

$-2(0)+0\le 2$

$0\le 2$ (True)

Because (0,0) satisfies all the inequalities (i), (ii) and (iii), so their solution is towards the origin. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

(vi)
$$3x-2y\ge 3$$
$$x+4y\le 12$$
$$3x+y\le 12$$



Solution:

$3x-2y\ge 3,\ x+4y\le 12,\ 3x+y\le 12$

The associated equations of the above inequalities are:

$3x-2y=3$...(i)

When $x=0$, then (i) becomes $y=\dfrac{-3}{2}$

$x+4y=12$...(ii)

When $x=0$, then (ii) becomes $y=3$

$\therefore$ $(0,3)$ is a point on

$3x+y=12$...(iii)

When $x=0$, then (iii) becomes $y=12$

$\therefore \left(0, \frac{-3}{2}\right)$ is a point on the line (i).
When $y = 0$, then
(i) becomes $x = 1$
$\therefore (1, 0)$ is a point on the line (i).
Take (0,0) as a test point. Put it in
$3x - 2y \geq 3$
$3(0) - 2(0) \geq 3$
$0 \geq 3$ (False)

the line (ii).
When $y = 0$, then
(ii) becomes $x = 12$
$\therefore (12, 0)$ is a point on the line (ii).
Take (0,0) as a test point. Put it in
$x + 4y \leq 12$
$0 + 4(0) \leq 12$
$0 \leq 12$ (True)

$\therefore (0,12)$ is a point on the line (iii).
When $y = 0$, then
(iii) becomes
$x = 4$
$\therefore (4, 0)$ is a point on the line (iii).
Take (0,0) as a test point. Put it in
$3x + y \leq 12$
$3(0) + 0 \leq 12$
$0 \leq 12$ (True)

Because (0,0) satisfies the inequalities (ii) and (iii), so their solution is towards the origin, and (0,0) does not satisfy the inequality (i), so its solution is away from the origin. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

Problem constraints:

The system of linear inequalities involved in the problem concerned are called problem constraints.

Non-negative constraints:

The variables used in the system of linear inequalities relating to the problem of everyday life are non- negative and are called non-negative constraints.

Decision variables:

These non-negative constraints play an important role play an important role for taking decisions. So these variables are called decision variables.

Feasible region:

A region which is restricted to the first quadrant is called feasible region.

Feasible solution:

Each point of feasible region is called feasible solution.

Feasible solution set:

A set consisting of all the feasible solutions of the system of linear inequalities is called feasible solution set.

Convex region:

If a line segment obtained by joining two points of a region lies entirely within the region is called convex region.