

Linear programniig:
The mether of oing the ines rincqualities is called linear programming. Linear Ineguatity:
red alities dre expressed by the following four symbols; , gevater then); <(less than); $\geq$ (greater than or equal to); $\leq$ (less than or equal to)
The inequalities $a x+b<c, a x+b \leq c, a x+b>c, a x+b \geq c$ are the linear inequalities in one variable and the inequalities $a x+b y<c, a x+b y \leq c, a x+b y>c, a x+b y \geq c$ are the linear inequalities in two variables $x$ and $y$.
The following operations will not effect the order of inequality while changing it to simpler equivalent form.
(i) Adding or subtracting a constant to each side of it.
(ii) Multiplying or dividing each side of it by a positive constant.

Note that order of an inequality is changed by multiplying or dividing its each side by a negative constant.
Graphing of a linear inequality in two variables:
Generally a linear inequality in two variables $x$ and $y$ can be one of the following forms:
$a x+b y<c ; a x+b y>c ; a x+b y \leq c ; a x+b y \geq c$
Where $a, b$ and $c$ are constants and $a, b$ are not both zero.
We know that a graph of a linear equation of the form $a x+b y=c$ is a line which divides the plane into two disjoint regions as stated below:
(i) The set of ordered pairs $(x, y)$ such that $a x+b y<c$
(ii) The set of ordered pairs $(x, y)$ such that $a x+b y>c$

The regions (i) and (ii) are called half planes and the line $a x+b y=c$ is called the boundary of each half plane.
A vertical line divides the plane into left and right half planes while a non-vertical line divides the plane into upper and lower half planes.
A solution of a linear inequality in $x$ and $y$ is an ordered pair of numbers which the inequality.
Note that linear equation $a x+b y=-c$ is called "aspociatt d or corresponding equation" of each above mentioned ine ua lities nor fraphing a linear In equar wariales:
(i) The corres pondingequation of the ir equality is first graphed by using 'dashes' if the inequal ty inp. ves he s.mbols >and a solid line is drawn of the inequality involves the ormpors $\geq \mathrm{cr}^{2}$.
(ii) A wost noint (nct on the graph of the corresponding equation) is chosen which determines it at he half plane is on which side of the boundary line.
oner points:
A point of solution region where two of its boundary lines intersect is called a corner point or vertex of the solution region.

## EXERCISE 5.1

Q. 1 Graph the solution set of each of the following linear inequality in $x y$-nlane:
(i) $2 x+y \leq 6$

## Solution:

$$
2 x+y \leq 6
$$

The associated equation of the abse nequality
$2 x+y=6 \ldots$ (i)
When $x=0$, then (i) becones $p=6$
$\therefore(0,6$ is a pont on the live ( $(1)$.
Nae, $y=0$ then (i) becomes $x=3$
. $(u, 0)$ is another point on the line (i).
Take $(0,0)$ as a test point. Put it in $2 x+y \leq 6$
$2(0)+0 \leq 6$
$0 \leq 6$ (True)


Because test point $(0,0)$ satisfies the given inequality, so its solution is towards the origin, which is the closed half plane, as shown by shaded region in the figure.
(ii) $3 x+7 y \geq 21$

## Solution:

$3 x+7 y \geq 21$
The associated equation of the above inequality is:
$3 x+7 y=21 \ldots$..(i)
When $x=0$, then (i) becomes $y=3$
$\therefore(0,3)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=7$
$\therefore(7,0)$ is another point on the line (i).
Now we sketch the given inequality.
Take $(0,0)$ as a test point. Put it in $3 x+7 y \geq 21$
$3(0)+7(0) \geq 21$

$0 \geq 21$ (False)
Because test point $(0,0)$ does not satisfy the given inequality, so its solution is away from the origin, which is the closed half plane, as shown by shaded region in the figure.
(iii) $\quad 3 x-2 y \geq 6$

## Solution:

$3 x-2 y \geq 6$
The associated equation or the above nequalt. is: $3 x-2 y=5 .$. (i)
When $x=0$, then $(i)$ beomes $~ R=-3$
$\therefore(0,-3)$ is a point on the line (i).
$\sqrt{7 h e} y=1$ then (i) becomes $x=2$
$(2,0)$ is another point on the line (i).
Take $(0,0)$ as a test point. Put it in $3 x-2 y \geq 6$
$3(0)-2(0) \geq 6$


Linear Inequalities and Linear Programming
$0 \geq 6$ (False)
Because test point $(0,0)$ does not satisfy the given inequality, so its solution is away from the origin, which is the closed half plane, as shown by shaded regior in mefigure
(iv) $5 x-4 y \leq 20$

## Solution:

$5 x-4 y \leq 20$
The associated equation of the above inequality is:
$5 x-4 y=20 .$. (i)
The $1 x=0$. then (i) becomes $y=-5$
$(0,-5)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=4$
$\therefore(4,0)$ is another point on the line (i).
Take $(0,0)$ as a test point. Put it in $5 x-4 y \leq 20$

$5(0)-4(0) \leq 20$
$0 \leq 20$ (True)
Because test point $(0,0)$ satisfies the given inequality, so its solution is towards the origin, which is the closed half plane, as shown by shaded region in the figure.
(v) $2 x+1 \geq 0$

Solution:
$2 x+1 \geq 0$
The associated equation of the above inequality is:
$2 x+1=0$...(i)
From (i) $x=\frac{-1}{2}$
Take $(0,0)$ as a test point. Put it in $2 x+1 \geq 0$
$2(0)+1 \geq 0$
$1 \geq 0 \quad$ (True)


Because test point $(0,0)$ satisfies the given inequality, so its solution is towards the origin, which is the closed half plane, as shown by shaded region in the figue
(vi) $3 y-4 \leq 0$

## Solution:

$3 y-4 \leq 0$
The associatid equation of he above inequality is. $3 y-4-0$.(i)
From (i) $y=\frac{4}{3}$

race (๒,0) as a test point. Put it in $3 y-4 \leq 0$
$3(0)-4 \leq 0$

$-4 \leq 0$
(True)

Because test point $(0,0)$ satisfies the given inequality, so its solution is towards the origin, which is the closed half plane, as shown by shaded region in the figure.
Q. 2 Indicate the solution set of the following systems of linear inequanities ity sharifg.
(i)
$2 x-3 y \leq 6$
$2 x+3 y \leq 12$
Solution:
$2 x-3 y=6 \quad 2 x+3 y \leq 12$
The associater eq lations of the above inecualilie are:
$2 x-3 y=6$...(i)
When $x=0$, then (i) becomes $y=-2$
$\therefore(0,-2)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=3$
$\therefore(3,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $2 x-3 y \leq 6$

$$
2(0)-3(0) \leq 6
$$

$0 \leq 6$ (True)

$2 x+3 y=12$...(ii)
When $x=0$, then (ii) becomes $y=4$
$\therefore(0,4)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=6$
$\therefore(6,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $2 x+3 y \leq 12$

$$
\begin{aligned}
& 2(0)+3(0) \leq 12 \\
& 0 \leq 12 \text { (True) }
\end{aligned}
$$

Because $(0,0)$ satisfies both the given inequalities, so their solution is towards the origin.
Also, the common solution for both the inequalities is shown by the shaded region as shown in the figure.
$x+y \geq 5$
(ii)
$-y+x \leq 1$

## Solution:

$$
x+y \geq 5 \quad, \quad-y+x \leq 1
$$

The associated equations of the above inequalities are:

. $(,, \sigma$ is a point on the line (i).
When $y=0$, then (i) becomes $x=5$
$\therefore(5,0)$ is a point on the line (i).
$-y+x=1$...(ii)
When $x=0$, then (ii) becomes $y=-1$
$\therefore(0,-1)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=1$
$\therefore(1,0)$ is a point on the line (ii).

Take $(0,0)$ as a test point. Put it in
$x+y \geq 5$
$0+0 \geq 5$
$0 \geq 5$ (False)

Take $(0,0)$ as a test point. Put it in
$-y+x \leq 1$
$-0+0 \leq 1$
$0 \leq 1 \quad$ (Tr T


Because $(0,0)$ does not salisly the ncquatity (i) so its solytion is ay vay from the origin. Also, ( $00 \%$ atisfies the inequaliky (ii), seits solntion is towards origin. The common solutione bo h the inequatios is sinovn the shaded region as shown in the figure.
$3 x+7 y \geq 21$
(iii)
$x \cdot \sqrt{2} \leq 2$
Bonution.
$3 x+7 y \geq 21$
$x-y \leq 2$
The associated equations of the above inequalities are:

$3 x+7 y=21 \ldots$ (i)
When $x=0$, then (i) becomes $y=3$
$\therefore(0,3)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=7$
$\therefore(7,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $3 x+7 y \geq 21$
$3(0)+7(0) \geq 21$
$0 \geq 21$ (False)
$x-y=2 \ldots$ (ii)
When $x=0$, then (ii) becomes $y=-2$
$\therefore(0,-2)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=2$
$\therefore(2,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $x-y \leq 2$
$0-0 \leq 2$
$0 \leq 2$ (True)

Because $(0,0)$ does not satisfy the inequality (i), so its solution is away from the origin. Also, $(0,0)$ satisfies the inequality (ii), so its solution is towards origin. The common solution for both the inequalities is shown by the shaded region as shown in the figure.
$4 x-3 y \leq 12$
(iv)

$$
x \geq-\frac{3}{2}
$$

Solution:

## $4 x-3 y \leq 12$

The assorinted equations of the abov inequal ties are:


$4 x-3 y=12 \ldots$ (i)

$$
x=-\frac{3}{2} \ldots(\mathrm{ii})
$$

When $x=0$, then (i) becomes $y=-4$
$\therefore(0,-4)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=3$
$\therefore$ (3, (1) s a print on the line (i).
Here $x=-\frac{3}{2}$ is a vertical line.
Take ( 0,0 as a tes point.
Put $\operatorname{tip} x=-\frac{3}{2}$
$0 \geq-\frac{3}{2} \quad$ (True)
$\operatorname{Tak}(0,0)$ ar a test poiilt.
Paxif i(1) $\frac{12}{}-3 y \leq 12$
$4(0)-3(0) \leq 12$
$0 \leq 12$ (True)
Because $(0,0)$ satisfies both the given inequalities, so their solution is towards the origin.
Also, the common solution for both the inequalities is shown by the shaded region as shown in the figure.
(v)

$$
3 x+7 y \geq 21
$$

$y \leq 4$

## Solution:

$$
3 x+7 y \geq 21 \quad, \quad y \leq 4
$$

The associated equations of the above inequalities are:

$3 x+7 y=21 \ldots$ (i)

$$
y=4 \ldots \text { (ii) }
$$

When $x=0$, then (i) becomes $y=3$
Here $y=4$ is a horizontal line.
$\therefore(0,3)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=7$
$\therefore(7,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $3 . x+7 y \geqslant 21$
$3(0)+(0) \geq 2:$
$5=21$ ralier,
Decause $(0,0)$ does not satisfy the inequality (i), so its solution is away from the origin. Also, $(0,0)$ satisfies the inequality (ii), so its solution is towards origin. The common solution for both the inequalities is shown by the shaded region as shown in the figure.
Q. 3 Indicate the solution region of the following systems of linear inequalities by shading:

$$
2 x-3 y \leq 6
$$

(i)

$$
\begin{aligned}
& 2 x+3 y \leq 12 \\
& y \geq 0
\end{aligned}
$$

Solution:

$$
2 x-3
$$



The assoc atec qua ions of the above inectalitie tre:

$2 x-3 y=6$
When $x=0$, then (i) becomes $y=-2$
$\therefore(0,-2)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=3$
$\therefore(3,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $2 x-3 y \leq 6$

$$
\begin{aligned}
& 2(0)-3(0) \leq 6 \\
& 0 \leq 6 \text { (True) }
\end{aligned}
$$




$$
2 x+3 y=12 \ldots \text { (ii) }
$$

When $x=0$, then (ii) becomes $y=4$
$\therefore(0,4)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=6$
$\therefore(6,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $2 x+3 y \leq 12$

$$
\begin{array}{ll} 
& 2(0)+3(0) \leq 12 \\
0 \leq 12 \quad \text { (True) }
\end{array}
$$

Because $(0,0)$ satisfies both the given inequalities, so their solution is towards the origin.
Also, $y \geq 0$ indicates that the solution will be above $x$-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

$$
x+y \leq 5
$$

(ii) $y-2 x \leq 2$
$x \geq 0$
Solution:

$$
x+y \leq 5, y-2 x \leq 2, x \geq 0
$$

The associated equations of the above inequalities are:

$y-2 x=2 \ldots$ (ii)
$\sqrt{\text { resen }} x=0$, then (i) becomes $y=5$ $(0,5)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=5$


When $x=0$, then (ii) becomes $y=2$
$\therefore(0,2)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=-1$
$\therefore(5,0)$ is a point on the line (i).
$\therefore(-1,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $x+y \leq 5$
$0+0 \leq 5$
$0 \leq 5$ (True)

Because ( 0,0 ) satisf es betl the given inequalities, so their solution is towards the origin.
Al: $0 x=0$ ndicates that the solution will be on the right side of $y$-axis. Hence, the comenon solution for the given system of inequalities is shown by the shaded region as shown in the figure.
$x+y \geq 5$
(iii)
$x-y \geq 1$
$y \geq 0$

## Solution:

$x+y \geq 5, x-y \geq 1, y \geq 0$
The associated equations of the above inequalities are:
$x+y=5$
When $x=0$, then (i) becomes $y=5$
$\therefore(0,5)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=5$
$\therefore(5,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $x+y \geq 5$

$x-y=1 \ldots$ (ii)
When $x=0$, then (ii) becomes $y=-1$
$\therefore(0,-1)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=1$
$\therefore(1,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $x-y \geq 1$
$0+0 \geq 5$
$0-0 \geq 1$
$0 \geq 5$ (False)
$0 \geq 1$ (False)
Because $(0,0)$ does not satisfies both the given inequalities, so their solution is away from the origin. Also, $y \geq 0$ indicates that the solution will be above x -axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.
$3 x+7 y \leq 21$
(iv) $x-y \leq 2$
$x \geq 0$
Solution:
$3 x+7$ 721
The associated cq a io as of the above incrialities res

$3 x+7 y=21 \ldots$ (i)
$x-y=2 \ldots$ (ii)

When $x=0$, then (i) becomes $y=3$
When $x=0$, then (ii) becomes $y=$

$\therefore(0,3)$ is a point on the line (i).
$(0,-2)$ is apo it on the line (ii).
When $y=0$, then (i) becomes $x=7$
V hen,$=0$, hen ii) be comus $x=2$
$\therefore(7,0)$ is a point on the line (i).
$(2,0)$ is appoint on the line (ii).
Take (0.0) as attest point.
Take $(0,0)$ as a test point.
Put it in $3 . c+x \leq 21$
Put it in $x-y \leq 2$
$3(a)+7(0) \leq 21$

$$
\begin{aligned}
& 0-0 \leq 2 \\
& 0 \leq 2 \quad \text { (True) }
\end{aligned}
$$

$\mathrm{v} \leq 21$ (True)
Because $(0,0)$ satisfies both the given inequalities, so their solution is towards the origin.
Also, $x \geq 0$ indicates that the solution will be on the right side of $y$-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.
$3 x+7 y \leq 21$
(v)
$x-y \leq 2$
$y \geq 0$
Solution:
$3 x+7 y \leq 21, x-y \leq 2, y \geq 0$
The associated equations of the above inequalities are:

$3 x+7 y=21 \ldots$ (i)
When $x=0$, then (i) becomes $y=3$
$\therefore(0,3)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=7$
$\therefore(7,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $3 x+7 y \leq 21$
$3(0)+7(0) \leq 2 \pi$
$0 \leq 21$ ( $\operatorname{Tr} \mathrm{Le}$ )
$0 \leq 2$ (True)
Fer mere ( 0 ) satisfies both the given inequalities, so their solution is towards the origin. Also, $y \geq 0$ indicates that the solution will be above $x$-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.
$3 x+7 y \leq 21$
(vi)
$2 x-y \geq-3$
$x \geq 0$
Solution:
$3 x+7 y \leq 21,2 x-y \geq-3, \quad=0$
The associated equations of the ato ve inequa(iti) are.
$3 x+7 y=21 \ldots$ (i)
When $x=0$, then (i) becomes $y=3$
$\therefore(0,3)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=7$
$\therefore(7,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $3 x+7 y \leq 21$

$2 x-y=-3$
When $x=0$, then (ii) becomes $y=3$
$\therefore(0,3)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=\frac{-3}{2}$
$\therefore\left(\frac{-3}{2}, 0\right)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $2 x-y \geq-3$
$3(0)+7(0) \leq 21$
$2(0)-0 \geq-3$
$0 \leq 21$ (True)
$0 \geq-3$ (True)
Because $(0,0)$ satisfies both the given inequalities, so their solution is towards the origin. Also, $x \geq 0$ indicates that the solution will be on the right side of $y$-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.
Q. 4 Graph the solution region of the following system of linear inequalities and find the corner points in each case:
(i) $2 x+3 y \leq 12$
$x \geq 0$
Solution:
$2 x-3 y \leq 6,2 x+3 y \leq 12, x \geq 0$
The associated equations of the above inequalities are:
$2 x-3 y=6$..(i)
When $x=0$, then (i) becon es $y=-2$
$\therefore(0,-2)$ i; a poigt on the fine ( 1, ,
When $y=0$, then (i) becomes $x=3$

- $\sqrt{3}, 8$ is point on the line (i).

Take $(0,0)$ as a test point.
Put it in $2 x-3 y \leq 6$
$2(0)-3(0) \leq 6$
$2(0)+3(0) \leq 12$
$0 \leq 6$ (True)
$0 \leq 12 \quad$ (True)
Because $(0,0)$ satisfies both the given inequalities, so their solution is towads the pris Also, $x \geq 0$ indicates that the solution will be on the right ride of $y$-2 is. Hence the common solution for the given system of incqualites s shown by tle sthaded Regon as shown in the figure.
Now we fing the corner nornt. of the : clution lezion
Solving $2 x-x=6$ and $2 x+3 y=12$, we get $\left(\frac{9}{2}, 1\right)$.
Frinde $\mathcal{O}_{\text {ctan }}$ points of the solution region of the given system of linear inequalities are
$(0,4),(0,-2)$ and $\left(\frac{9}{2}, 1\right)$.
$x+y \leq 5$
(ii) $-2 x+y \leq 2$
$y \geq 0$

## Solution:

$x+y \leq 5,-2 x+y \leq 2, y \geq 0$
The associated equations of the above inequalities are:
$x+y=5 \ldots$ (i)
When $x=0$, then (i) becomes $y=5$
$\therefore(0,5)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=5$
$\therefore(5,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $x+y \leq 5$

$-2 x+y=2 \ldots$ (ii)
When $x=0$, then (ii) becomes $y=2$
$\therefore(0,2)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=-1$
$\therefore(-1,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $-2 x+y \leq 2$
$0+0 \leq 5$
$0 \leq 5$ (True)
Because ( $f, \mathrm{Q}$ ) satisfies $b_{0}$ th the $g / v e n$ inecualit es so their solution is towards the origin. Also, fo indicates that the plution vill be above $x$-axis. Hence, the common solution for the given s.aten of iresualities is shown by the shaded region as shown in the figure. Tory we fird dhe corner points of the solution region.
Solving $x+y=5$ and $-2 x+y=2$ we get $(1,4)$.

Hence corner points of the solution region of the given system of linear inequalities are $(1,4),(-1,0) \operatorname{and}(5,0)$.
(iii)

Solution:
$3 x+7 y \leq 21$
$2 x-y \leq-3$
$y \geq 0$
$3 x+7 y \leq 1,=\{x-, \geq-3, y \geq 0$
The associated equations of the above
inpennalithes are:
$3 x+7 y=21 \ldots$ (i)
When $x=0$, then (i) becomes $y=3$
$\therefore(0,3)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=7$
$\therefore(7,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $3 x+7 y \leq 21$
$3(0)+7(0) \leq 21$

$2 x-y=-3 \ldots$ (ii)
When $x=0$, then (ii) becomes $y=3$
$\therefore(0,3)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=-\frac{3}{2}$
$\therefore\left(-\frac{3}{2}, 0\right)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $2 x-y \leq-3$

$$
2(0)-0 \leq-3
$$

$0 \leq 21$ (True)
$0 \leq-3$
(False)

Because ( 0,0 ) satisfies the inequality (i), so its solution is towards the origin, and ( 0,0 ) does not satisfy the inequality (ii), so its solution is away from the origin. Also, $y \geq 0$ indicates that the solution will be above $x$-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.
The corner points of the solution region of the given system of linear inequalities are $(0,3)$ and $\left(-\frac{3}{2}, 0\right)$.
$x+3 y \leq 6$
(iv)

Solution:


Frae iswriated equations of the above inequalities are:
$x+3 y=6 \ldots$ (i)
When $x=0$, then (i) becomes $y=2$
$\therefore(0,2)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=6$
$\therefore(6,0)$ is a point on the 1 ne (i)
Take $(\rho, 0)$ as a test point.
Put it ir $-c+3, y \leq 5$
$0+\rho^{\prime}(0) \leq \sigma$
g-1, Qrirue)
$3 x+2 y=6 \ldots$ (ii)
When $x=0$, then (ii) becomes $y=3$
$\therefore(0,3)$ is a point on tre ine ii).
When $y=O$, onen (ii) beco nes $x=2$
(2.0) is a p puit on the line (ii).

Ca. $\mathrm{e}(\mathrm{v}, 0)$ as a test point.
Put it in $3 x+2 y \geq 6$

$$
\begin{aligned}
& 3(0)+2(0) \geq 6 \\
& 0 \geq 6 \quad \text { (False) }
\end{aligned}
$$

Because $(0,0)$ satisfy the inequality (i), so its solution is towards the origin, and $(0,0)$ does not satisfies the inequality (ii), so its solution is away from origin. Also, $y \geq 0$ indicates that the solution will be above $x$-axis. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.
Now we find the corner points of the solution region.
Solving $3 x+2 y=6$ and $x+3 y=6$ we get $\left(\frac{6}{7}, \frac{12}{7}\right)$.
The corner points of the solution region of the given system of linear inequalities are
$(2,0),(6,0)$ and $\left(\frac{6}{7}, \frac{12}{7}\right)$.
$5 x+7 y \leq 35$
(v) $-x+3 y \leq 3$
$x \geq 0$
Solution:
$5 x+7 y \leq 35,-x+3 y \leq 3, x \geq 0$
The associated equations of the above inequalities are:
$5 x+7 y=35 \ldots$ (i)
$5 x+7 y=35 \ldots$ (i)
When $x=0$, then (i) beconts $y=$

$-x+3 y=3 \cdot(\mathrm{ii})$
Wher $x=0$, then (ii) becomes $y=1$
$\therefore(0,5)$ is a point on the line (i).
$\therefore(0,1)$ is a point on lie line (ii).
When (y) $=0$, then (i) becones $x=7$
$\therefore(7,0)$ is a pont on the live ( 1 ).
Take ( 0,1 ) datest point.
P) in in $5 x+7 y \leq 35$
$5(0)+7(0) \leq 35$
$0 \leq 35$ (True)
Whuen $y=0$, then (ii) becomes $x=-3$
$\therefore(-3,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $-x+3 y \leq 3$
$-0+3(0) \leq 3$
$0 \leq 3$ (True)

Because $(0,0)$ satisfies both the given inequalities, so their solution is towards the origin. Also, $x \geq 0$ indicates that the solution will be on the right side of $x$-2xis. Hence common solution for the given system of inequalities is shown by the sited resion (ac) shown in the figure.
Now we find the corner peints of the solytion dibn
Solving $5 \cdot 7 y=35$ alli $-x b 3 k=3$ ve get $\left(-\frac{1}{1} \quad 11\right)$.
The comer pciats of the solution region of the given system of linear inequalities are Ni, and $\left(\frac{-2}{11}, 11\right)$.
$5 x+7 y \leq 35$
(vi) $\quad x-2 y \leq 2$
$x \geq 0$

## Solution:

$$
5 x+7 y \leq 35, x-2 y \leq 2, x \geq 0
$$

The associated equations of the above inequalities are:


$$
x-2 y=2 \ldots(\text { ii })
$$

When $y=0$, then (i) becomes $x=7$
$\therefore(7,0)$ is a point on the line (i).
When $y=0$, then (ii) becomes $x=2$
$\therefore(2,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $x-2 y \leq 2$
$0-2(0) \leq 2$
$0 \leq 2$ (True)
$0 \leq 35$ (True)
Because $(0,0)$ satisfies both the given inequ, ities, or eir sply tion is townst orrgin. Also, $x \geq 0$ indicates that the soetion whl he on the righ sy de of $y$-axis. Hence, the common solution for the given syster of in equalities stown by the shaded region as shown int the figure.
Now werin d the orer po nts orthelsolution region.
ふiving $5 x+7 \mid,=55$ and $x-2 y=2$ we get $\left(\frac{84}{17}, \frac{25}{17}\right)$.
Ine corner points of the solution region of the given system of linear inequalities are
$(0,5),(0,-1)$ and $\left(\frac{84}{17}, \frac{25}{17}\right)$.
Q. 5 Graph the solution region of the following system of linear inequalities by shading: $3 x-4 y \leq 12$
(i) $3 x+2 y \geq 3$ $x+2 y \leq 9$

## Solution:

 $3 x-4 y \leq 12,3 x+2 y \geq 3, x+2 y \leq 9$
$x+2 y=9$...(iii)
When $x=0$, then
(iii) becomes $y=\frac{9}{2}$
$\therefore\left(0, \frac{9}{2}\right)$ is a point on
the line (iii).
When $y=0$, then
(iii) becomes
$x=9$
$\therefore(9,0)$ is a point on the line (iii).
Take $(0,0)$ as a test point. Put it in
$x+2 y \leq 9$
$0+2(0) \leq 9$
$0 \leq 9$ (True)

Because $(0,0)$ satisfies the inequalities (i) and (iii), so their solution is towards the origin, and $(0,0)$ does not satisfy the inequality (ii), so its solution is away from the origin. Hence, the common solution for the given system of inequalities is shown by the shatec region as shown in the figure.
$3 x-4 y \leq 12$
(ii)
$x+2 y \leq 6$
$x+y \geq 1$

## Solution:

$3 x-4 y \leq 12, x+2 y \leq 6, x+y \geq$ The associated equations of the above irmentaitles are:

$3 x-4 y=12 \ldots(i)$
When $x=0$, then
(i) becomes
$y=-3$
$\therefore(0,-3)$ is a point on the 率e ( i ).
When $y=0$, then
(i) becorne $x=4$ (4, (O) i®apuint on the line (i).
Take $(0,0)$ as a test point. Put it in
$3 x-4 y \leq 12$
$3(0)-4(0) \leq 12$
$0 \leq 12$ (True)
$x+2 y=6$...(ii)
When $x=0$, then (ii) becomes $y=3$
$\therefore(0,3)$ is poin oh the 1 ing di.
When $y=0$, thent (ii) becomes $x=6$
$\therefore(6,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point. Put it in
$x+2 y \leq 6$
$0+2(0) \leq 6$
$0 \leq 6$ (True)
$x+y=1 \ldots$ (iii)
When $x=0$, tien (iii)
becornes
$(0,1)$ is a poimion
the line (iii).
When $y=0$, then (iii)
becomes $x=1$
$\therefore(1,0)$ is a point on the line (iii).
Take $(0,0)$ as a test
point. Put it in

$$
\begin{aligned}
& x+y \geq 1 \\
& 0+0 \geq 1 \\
& 0 \geq 1 \text { (False) }
\end{aligned}
$$

Because $(0,0)$ satisfies the inequalities (i) and (ii), so their solution is towards the origin, and $(0,0)$ does not satisfy the inequality (iii), so its solution is away from the origin. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

```
\(2 x+y \leq 4\)
(iii) \(2 x-3 y \geq 12\)
\(x+2 y \leq 6\)
```


## Solution:

$2 x+y \leq 4,2 x-3 y \geq 12, x+2 y \leq 6$
The associated equations of the above inequalities are:
$2 x+y=4 \ldots$ (i)
When $x=0$, then
(i) becemes $y=4$
$2 x=3 y=12 \ldots$.ii) When $x=0$ thek
(iii) becontes $y=-4$ $\therefore(0,-4)$ is a point on the line (ii).
When $y=0$, then
(ii) becomes $x=6$

(iii) becomes $y=3$
$\therefore(0,3)$ is a point on
the line (iii).
When $y=0$, then
(iii) becomes $x=6$
$\therefore(2,0)$ is a point on the line (i).
Take $(0,0)$ as a test point. Put it in
$2 x+y \leq 4$

$0 \leq 4$ (Tr 1 e)
$\therefore(6,0)$ is a point on the line (ii).
Take $(0,0)$ as a test ponit. Put it in $4 \sqrt{2}=3 y=$

Becalse ( 0 (C) satisfies the inequalities (i) and (iii), so their solution is towards the origin, a, 10,3$)$ does not satisfy the inequality (ii), so its solution is away from the origin. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.
$2 x+y \leq 10$
(iv)
$x+y \leq 7$
$-2 x+y \leq 4$

## Solution:

$2 x+y \leq 10, x+y \leq 7,-2 x+y \leq 4$
The associated equations of the above inequalities are:

$2 x+y=10 \ldots$ (i)
When $x=0$, then
(i) becomes $y=10$
$\therefore(0,10)$ is a point on the line (i).
When $y=0$, then
(i) becomes $x=5$
$\therefore(5,0)$ is a point on the line (i).
Take $(0,0)$ as a test point. Put it in $2 x+y-0$ $2(0)+0=10$
$x+y=7$...(ii)
When $x=0$, then
(ii) becomes $y=7$
$\therefore(0,7)$ is a point on the line (ii).
When $y=0$, then
(ii) becomes $x=7$
$\therefore(7,0)$ is a poin on

$\therefore(6,0)$ is a point on
the line (iii).
Take 0,01 as a iest
polnt. Fuit in
$x+2 y \leq 6$
$0+2(0) \leq 6$
$0 \leq 6$ (True)
$2 x+3 y \leq 18$
(v) $2 x+y \leq 10$
$-2 x+y \leq 2$
Solution:
$2 x+3 y \leq 18,2 x+y \leq 10,-2 x+y$
The associated equations of thearo e inequaritios are:
$2 x+3 y=18 \ldots$ (i)
When $x=0$, then
(i) becomes $y=6$
$\therefore(0,6)$ is a point on the line (i).
When $y=0$, then (i)
becomes $x=9$
$\therefore(9,0)$ is a point on the line (i).
Take $(0,0)$ as a test point. Put it in $2 x+3 y \leq 18$
$2(0)+3(0) \leq 18$
$0 \leq 18$ (True)
$2 x+y=10 \ldots$..(ii)
When $x=0$, then
(ii) becomes $y=10$
$\therefore(0,10)$ is a point on the line (ii).
When $y=0$, then
(ii) becomes $x=5$
$\therefore(5,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point. Put it in $2 x+y \leq 10$
$2(0)+0 \leq 10$
$0 \leq 10$ (True)


Because ( 0,0 ) satisfies all the inequalities (i), (ii) and (iii), so their solution is towards the origin. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

$$
3 x-2 y \geq 3
$$

(vi)

$$
\begin{aligned}
& x+4 y \leq 12 \\
& 3 x+y \leq 12
\end{aligned}
$$

## Solution:

$3 x-2 y \geq 3, x+4 y \leq 12,3 x+y \leq 12$ The associated equations of the abpre inequalitiesore:

$$
x+4 y=12 \ldots(\mathrm{ii})
$$

When $x=0$, then
(ii) becomes $y=3$
$\therefore(0,3)$ is a point on
$-2 x+y=2 \ldots$ (iii)
When $x=0$, then
(iii) becomes $y=2$
$\therefore(0,2)$ is a point on the line (iii).
When $y=0$, then
(iii) becomes
$x=-1$
$\therefore(-1,0)$ is a point on
the line (iii).
Take $(0,0)$ as a test
point. Put it in
$-2 x+y \leq 2$
$-2(0)+0 \leq 2$
$0 \leq 2$ (True)
$A x-(0,=3 \ldots$ (i)
When $x=0$, then
(i) becomes $y=\frac{-3}{2}$

$3 x+y=12 \ldots$.(iii)
When $x=0$, then
(iii) becomes
$y=12$
$\therefore\left(0, \frac{-3}{2}\right)$ is a point on
the line (i).
When $y=0$, then
(i) becomes $x=1$
$\therefore(1,0)$ is a noint on the lie $i$ ).
Take (T, (1) as : $t \in S t$ point. Put it in
$3 \cdot v-2, \geq 30 \quad 0 \leq 12$ (True)
$3(0)-2(0) \geq 3$
$0 \geq 3$ (False)
the line (ii).
When $y=0$, then
(ii) becomes $x=12$
$\therefore(12,0)$ is a point on
the live (ii)
Take (00) a a test
point. Pit 1 ih
$x+4 y=12$
$0+4(0) \leq 12$
$\therefore(0,12)$ is a point on the line (iii).
When $y=\infty$, hen
$\left[\begin{array}{l}\text { (ii i) becones } \\ x=4 \\ 4\end{array}\right.$
$(4,0)$ ic a point on
the line (iii).
Take $(0,0)$ as a test
point. Put it in
$3 x+y \leq 12$
$3(0)+0 \leq 12$
$0 \leq 12$ (True)

Because ( 0,0 ) satisfies the inequalities (ii) and (iii), so their solution is towards the origin, and $(0,0)$ does not satisfy the inequality (i), so its solution is away from the origin. Hence, the common solution for the given system of inequalities is shown by the shaded region as shown in the figure.

## Problem constraints:

The system of linear inequalities involved in the problem concerned are called problem constraints.

## Non-negative constraints:

The variables used in the system of linear inequalities relating to the problem of everyday life are non- negative and are called non-negative constraints.

## Decision variables:

These non-negative constraints play an important role play an important role for taking decisions. So these variables are called decision variables.

## Feasible region:

A region which is restricted to the first quadrant is called feasible region.

## Feasible solution:

Each point of feasible region is called feasible solution.

## Feasible solution set:

A set consisting of all the feasible solutions of the system of linear inequalities is c feasible solution set.

## Convex region:

If a line segment obtained hy joiniggo pot in or a resy lies entre y within the region is called convex region


