## EXERCISE 5.2

Q. 1 Graph the feasible region of the following system of linear inequalities and find the corner points in each case.
$2 x-3 y \leq 6$
(i) $\quad 2 x+3 y \leq 12$
$x \geq 0$,
Solution:
$2 x-3 y \leq 5.2 x+3 y \leq 12 x \geq 0, y \geq 0$
The isscciated equatoms of the above
inccilatides are:
$2 x-3 y=6$...(i)
When $x=0$, then (i) becomes $y=-2$
$\therefore(0,-2)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=3$



$$
2 x+3 y=12 \ldots(\text { ii) }
$$

When $x=0$, then (ii) becomes $y=4$
$\therefore(0,4)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=6$
$\therefore(6,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $2 x+3 y \leq 12$
$2(0)+3(0) \leq 12$
$0 \leq 12$ (True)

$$
\because
$$

$0 \leq 6$ (True)
Because $(0,0)$ satisfies both the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0, y \geq 0$ so, solution region will be in first quadrant
Now we find the corner points of the feasible region.
Solving $2 x-3 y=6$ and $2 x+3 y=12$ we get $\left(\frac{9}{2}, 1\right)$.
The corner points of the feasible region of the given system of linear inequalities are
$(0,0),(3,0),\left(\frac{9}{2}, 1\right) \operatorname{and}(0,4)$.
$x+y \leq 5$
(ii)
$-2 x+y \leq 2$
$x \geq 0$, Solution:

$$
x+y \leq \leq-x-2, x \geq 0, y \geq 0
$$

17 he issociated equations of the above inequalities are:

$x+y=5 \ldots$ (i)
$-2 x+y=2 \ldots$ (ii)
When $x=0$, then (i) becomes $y=5$
$\therefore(0,5)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=5$
$\therefore(5,0)$ is a point on the line $(\dot{)} . \square-1,0)$ is a docint on the line (ii).
Take (0,0) as a test pbiat.
Put it in $x+y \leq 5$
When $x=0$, then (ii) becones $y=2$ $\therefore(0,2)$ is a point on the iife ( 11 \%.

When $y=5$, then (in be omes $x=-1$

Take $(0,0)$ as a test point.

P- $0 \leq 5$
$0 \leq 5$ (True)
Put it in $-2 x+y \leq 2$
$-2(0)+0 \leq 2$

Because $(0,0)$ satisfies both the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0, y \geq 0$ so, solution region will be in first quadrant
Now we find the corner points of the feasible region.
Solving $x+y=5$ and $-2 x+y=2$ we get $(1,4)$.
The corner points of the feasible region of the given system of linear inequalities are $(0,0),(5,0),(1,4) \operatorname{and}(0,2)$.
$x+y \leq 5$
(iii) $-2 x+y \geq 2$
$x \geq 0, y \geq 0$

## Solution:

$x+y \leq 5,-2 x+y \geq 2, x \geq 0, y \geq 0$
The associated equations of the above inequalities are:
$x+y=5 \ldots$..(i)
When $x=0$, then (i) becomes $y=5$
$\therefore(0,5)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=5$
$\therefore(5,0)$ is a point on the line (i).

$-2 x+y=2 \ldots$..ii)
When $y=0$, then (ii) becres $x=-1$
$\therefore(-1,0)$ is 1 ront on che ine (iii)
Wher $y=0$, then (ii) becomes $y=2$
0,2 is a/puint on the line (ii).
Take (0, as as a testoon.
Take $(0,0)$ as a test point.
Put it in $-2 x+y \geq 2$
$-2(0)+0 \geq 2$
$0 \leq 5 \quad$ (True)
$0 \geq 2$ (False)

Because $(0,0)$ satisfies the inequality (i), so its solution is towards origin, while $(0,0)$ does not satisfy the inequality (ii), so its solution is away from the origin. So, the comp solution of the given system of inequalities is shown by the shared reg. on as show in the figure. As $x \geq 0, y \geq 0$ so, solution region will be nair t quadrant
Now we find the corner po lints of the feasi\#er reich.
Solving $-x-y=5$ and $-2 x+y=2$ use ret $(1,4)$.
The contr print of he feasible -cion of the given system of linear inequalities are ( 0,2 ,,$(1,4) \cdot \operatorname{ng}(0.5)$.
$3 x+79 \leq 21$

$$
\begin{aligned}
& x-y \leq 3 \\
& x \geq 0, y \geq 0
\end{aligned}
$$

## Solution:

$3 x+7 y \leq 21, x-y \leq 3, x \geq 0, y \geq 0$
The associated equations of the above inequalities are:

$3 x+7 y=21 \ldots$ (i)

$$
x-y=3 \ldots(\mathrm{ii})
$$

When $x=0$, then (i) becomes $y=3$
When $x=0$, then (ii) becomes $y=-3$
$\therefore(0,3)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=7$
$\therefore(7,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $3 x+7 y \leq 21$
$\therefore(0,-3)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=3$
$\therefore(3,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point.
Put it in $x-y \leq 3$
$3(0)+7(0) \leq 21$

$$
0-0 \leq 3
$$

$0 \leq 21$ (True)
Because $(0,0)$ satisfies bott the given inequalities, so the ir solution is towards origin. So, the common solution of the given sj ste n of nequantied is shown by the shaded region as shown in the figure $A_{S} x=P, \quad A \geq 0$ so, oration region will be in first quadrant
Solving $3 x-7 y=3.1$ and $x-y=3$ we get $\left(\frac{21}{5}, \frac{6}{5}\right)$.
The corner points of the feasible region of the given system of linear inequalities are $(0,0),(3,0),\left(\frac{21}{5}, \frac{6}{5}\right) \operatorname{and}(0,3)$.
$3 x+2 y \geq 6$
(v) $x+y \leq 4$
$x \geq 0, y \geq 0$

## Solution:

$3 x+2 y \geq 6, x+y \leq 4, x \geq 9, v \geq 0$
The asseciatid equatigno of the above inequaitie ate:

$3 x+2 y=6 \ldots(i)$
When $x=0$, then (i) becomes $y=3$
$\therefore(0,3)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=2$

$$
x+y=4 . . .(\mathrm{ii})
$$

When $x=0$, then (ii) becomes $y=4$
$\therefore(0,4)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=4$
$\therefore(2,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $3 x+2 y \geq 6$
Take $(0,0)$ as a test point.
$3(0)+2(0) \geq 6$
$0+0 \leq 4$
$0 \geq 6$ (False)
$0 \leq 4 \quad$ (True)
Because $(0,0)$ satisfies the inequality (ii), so its solution is towards origin, while $(0,0)$ does not satisfy the inequality (i), so its solution is away from the origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0, y \geq 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.
The corner points of the feasible region of the given system of linear inequalities are $(2,0),(4,0),(0,4)$ and $(0,3)$.
$5 x+7 y \leq 35$
(vi)
$x-2 y \leq 4$
$x \geq 0, y \geq 0$
Solution:
$5 x+7 y \leq 35, x-2 y \leq 4, x=0, y \geq 0$
Theinsocia ted ewations of the above
Ir ealaties are

$5 x+7 y=35 \ldots$. (i)
When $x=0$, then (i) becomes $y=5$
$\therefore(0,5)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=$ (7)
$\therefore(7,0)$ is a point on the line i$)$.
Take $(0,0)$ as a test point.
Put it in $5 x+7 x \leq 35$
$5(0)+7(0)=5 \leq$
(1) $\leq 55$ (True)

$$
x-2 y=4 \ldots(\text { ii) }
$$

When $x=0$, then (ii) becomes $\square$
$(0,-2)$ is a point on he line ii).
When $y=0$ then (ii becomes $x=4$
$(4, \%)$ is a point on be line (ii).
Take $(0,0)$ as a test point.
Put it in $x-2 y \leq 4$

$$
0-2(0) \leq 4
$$

$$
0 \leq 4 \quad \text { (True) }
$$

Because $(0,0)$ satisfies both the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0, y \geq 0$ so, solution region will be in first quadrant
Now we find the corner points of the feasible region.
Solving $5 x+7 y=35$ and $x-2 y=4$ we get $\left(\frac{98}{17}, \frac{15}{17}\right)$.
The corner points of the feasible region of the given system of linear inequalities are $(0,0),(4,0),\left(\frac{98}{17}, \frac{15}{17}\right)$ and $(0,5)$.

## Q. 2 Graph the feasible region of the following system of linear inequalities and find

 the corner points in each case.$2 x+y \leq 10$
(i)

$$
x+4 y \leq 12
$$

$x+2 y \leq 10$
$x \geq 0, y \geq 0$

## Solution:

$2 x+y \leq 10, x+4 y \leq 12, x+2 y \leq 10, x \geq 0, y \geq 0$
The associated equations of the above inequalities are:
$2 x+y=10 \ldots$ (i)
When $x=0$, then (i) become:-,$=10$
$\therefore(0,10)$ is a pain on the pine (i).
When $y=0$, then (i) becomes $x=5$
$\therefore(5,0)$ is a point on the

$x+4 \mathrm{Ci}=12 \ldots \mathrm{Ci}$ When $x=0$, then (i) bodaries $y=3$ $(0,3)$ is a point on the line (ii).
When $y=0$, then
(ii) becomes $x=12$
$\therefore(12,0)$ is a point on

$\therefore(0,5)$ is a point on the line (iii).
When $y=0$, then
(iii) becomes $x=10$
$\therefore(10,0)$ is a point on
line (i).
Take $(0,0)$ as a test point. Put it in $2 x+y \leq 10$
$2(0)+0 \leq 10$
$0 \leq 10$ (True)
the line (ii).
Take $(0,0)$ as a test point. Put it in $\left[\begin{array}{l}\text { point. Put it in } \\ x+4 y \leq 12 \\ 0+48 \\ 0 \leq 12\end{array}\right]$
the line (iii).
Take ( 0,0 ) as 2 test
point Plitt
$d+z(c)=10$
$0 \leq 10$

Becaus $(0,0)$ satisfies all the given inequalities, so their solution is towards origin. So, the com mon sclittion e the given system of inequalities is shown by the shaded region as Hhoty n (i) the Ingure. As $x \geq 0, y \geq 0$ so, solution region will be in first quadrant fow we find the corner points of the feasible region.
Solving $2 x+y=10$ and $x+4 y=12$ we get $(4,2)$.
The corner points of the feasible region of the given system of linear inequalities are $(0,0),(5,0),(4,2)$ and $(0,3)$.
$2 x+3 y \leq 18$
(ii)
$2 x+y \leq 10$
$x+4 y \leq 12$
$x \geq 0, y \geq 0$

## Solution:

$2 x+3 y \leq 18,2 x+y \leq 10, x+4 y \leq 12, x \geq 0, y \geq 0$
The associated equations of the above inequalities are:

$2 x+3 y=18 \ldots($ (i)
When $x=0$, then (i) becomes $y=6$
$\therefore(0,6)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=9$
$\therefore(9,0)$ is a point on the line (i).
Take ( 0,0 ) 1 as a test nojiti.
Put it in $-3-3 y \leq 10$ $2(0)+3(0)=18$
$2 x+y=10$...(ii)
When $x=0$, then
(ii) becomes $y=10$
$\therefore(0,10)$ is a point on the line (ii).
When $y=0$, then
(ii) becrmes $x=5$
$\therefore$ (5) 0 ) is a p cint on the
line (ii.
Falke (o, on as a lest
Q.int. Put it in
$2 x+y \leq 10$
$2(0)+0 \leq 10$
$0 \leq 10$ (True)
$x+4 y=12$...(iii)
When $x=0$, then
(iii) becomes $y=3$
$\therefore(0,3)$ is a point on the line (iii).
When $y=0$, the
(iii) he onds $\in 12$
$(12,(1)$ is a point on
tteline (iii).
Take $(0,0)$ as a test
point. Put it in
$x+4 y \leq 12$
$0+4(0) \leq 12$
$0 \leq 12$ (True)

Because $(0,0)$ satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0, y \geq 0$ so, solution region will be in first quarcan
Now we find the corner points of the feasitic region.
Solving $2 x+y=10$ and $x+4 y=1$ (2ve get ( 4,25 .
The corner points of the feasirm eqion of the given syster of livear inequalities are $(0,0),(5,0),(4,2)$ and $(0,3)$,
$2 x+3 y \leq 1 \beta$
$x+4 y \leq 12$
$3 x+y(8) 2$
$x \geq 0, y \geq 0$
Solution:
$2 x+3 y \leq 18, x+4 y \leq 12,3 x+y \leq 12, x \geq 0, y \geq 0$
The associated equations of the above inequalities are:
$2 x+3 y=18$...(i)
When $x=0$, then (i) becomes $y=6$
$\therefore(0,6)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=9$
$\therefore(9,0)$ is a point on the line (i).
Take $(0,0)$ as a test point.
Put it in $2 x+3 y \leq 18$
$2(0)+3(0) \leq 18$
$0 \leq 18$ (True)
$x+4 y=12$
When $x=0$, then
(ii) becomes $y=3$
$\therefore(0,3)$ is a point on the line (ii).
When $y=0$, then
(ii) becomes $x=12$
$\therefore(12,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point. Put it in
$x+4 y \leq 12$
$0+4(0) \leq 12$
$0 \leq 12$ (True)

(iii) becomes
$y=12$
$\therefore(0,12)$ is a point
on the line (iii).
When $y=0$, then
(iii) becomes
$x=4$
$\therefore(4,0)$ is a point on the line (iii). Take $(0,0)$ as a test point. Put it in $3 x+y \leq 12$

Because $(0,0)$ satisfies all the given inequalitios, so the ir sclution is to waras origin. So, the common solution of the given spter of inequalitie is hown by he shaded region as shown in the figure. As $x \geq 0, \sqrt{2} 0$ so, olltiyn egion will ve in first quadrant
Now we tind the comen pdits of the fela ible region.
Solving $3 x+y=12$.nct $r+4 y=12$ we get $\left(\frac{36}{11}, \frac{24}{11}\right)$.
Vhed cirner points of the feasible region of the given system of linear inequalities are $(0,0),(4,0),\left(\frac{36}{11}, \frac{24}{11}\right)$ and $(0,3)$.
(iv)
$x+2 y \leq 14$
$3 x+4 y \leq 36$
$2 x+y \leq 10$
$x \geq 0, y \geq 0$
Solution:
$x+2 y \leq 1+, 3 x+4 y \leq 20,2 x+y \leq 10$,
The assosia te t eqration of he abose inequalities are:

1) $+2 y=14 \ldots$ (i)

When $x=0$, then (i) becomes $y=7$
$\therefore(0,7)$ is a point on the line (i).
When $y=0$, then (i)
becomes $x=14$
$\therefore(14,0)$ is a point on the line (i).
Take $(0,0)$ as a test point. Put it in
$x+2 y \leq 14$
$0+2(0) \leq 14$
$0 \leq 14$ (True)
$3 x+4 y=36 \ldots$ (ii)
When $x=0$, then (ii)
becomes $y=9$
$\therefore(0,9)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=12$
$\therefore(12,0)$ is a point on the line (ii).
Take $(0,0)$ as a test point. Put it in $3 x+4 y \leq 36$
$3(0)+4(0) \leq 36$
$0 \leq 36$ (True)
$2 x+y=10 \ldots$..(iii)
When $x=0$, then (iii)
becomes $y=10$
$\therefore(0,10)$ is a point on the line (iii).
When $y=0$, then (iii) becomes $x=5$
$\therefore(5,0)$ is a point on the line (iii).
Take $(0,0)$ as a test point. Put it in $2 x+y \leq 10$ $2(0)+0 \leq 10$
$0 \leq 10$ (True)

Because $(0,0)$ satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0, y \geq 0$ so, solution region will be in first quadrant Now we find the corner points of the feasible region.
Solving $2 x+y=10$ and $x+2 y=14$ we get $(2,6)$.
The corner points of the feasible region of the given system of linear imequalities 20 $(0,0),(5,0),(2,6)$ and $(0,7)$.
$x+3 y \leq 15$
(v)
$2 x+y \leq 12$
$4 x+3 y=4$
$x \geq 0, y \geq 0$
Solution:
x+ $3 y \leq 2$. $2 x+y \leq 12,4 x+3 y \leq 24, x \geq 0, y \geq 0$
The associated equations of the above inequalities are:

$x+3 y=15 \ldots$..(i)
When $x=0$, then (i) becomes $y=5$
$\therefore(0,5)$ is a point on the line (i).
When $y=0$, then (i)
becomes $x=15$
$\therefore(15,0)$ is a poir $t$ or
the 解 (i).
Theke (O(0) asa test
D jik. Fut it in
$x+3 y \leq 15$
$0+3(0) \leq 15$
$0 \leq 15$ (True)
$2 x+y=12$...(ii) When
$x=0$, then (ii)
becomes $y=12$
$\therefore(0,12)$ is a p in on the $\therefore(0,8)$ is tpont on the line
When $y=0$, hen (ii)
becorle $x=0$
( 5,0 ) is a point on the
line (ii).
Take $(0,0)$ as a test point. Put it in $2 x+y \leq 12$
$2(0)+0 \leq 12$
$0 \leq 12$ (True)
$4 x+3 y=24$...(iii)
When $x-0$, then (iii)
becomes $=8$ lin= (iii).
When $y=0$, then (iii)
becomes $x=6$
$\therefore(6,0)$ is a point on the line (iii).
Take $(0,0)$ as a test point. Put it in
$4 x+3 y \leq 24$
$4(0)+3(0) \leq 24$
$0 \leq 24$ (True)

Because $(0,0)$ satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0, y \geq 0$ so, solution region will be in first quadrant
Now we find the corner points of the feasible region.
Solving $4 x+3 y=24$ and $x+3 y=15$ we get $(3,4)$.
The corner points of the feasible region of the given system of linear inequalities are
(vi)
$(0,0),(6,0),(3,4)$ and $(0,5)$.
$2 x+y \leq 20$
$8 x+15 y \leq 120$
$x+y \leq 11$
$x \geq 0, y \geq 0$

## Solution:

$2 x+y \leq 20,8 x+15 y \leq 120, x+y \leq 11, x \geq 0, y \geq 0$
The associated equations of the above inequalities are:
$2 x+y=20 \ldots$ (i)
When $x=0$, then (i) becomes $y=20$
$\therefore(0,20(118)$ Rpointon the lin $(\mathrm{i})$
Whan. $=$ ( ), hen (i)
1)econies $x=10$
$\therefore(10,0)$ is a point on the line (i).
$8 x+15 y=120 \ldots$ (ii)
When $x=0$, hen (ii)
kepunces $\Rightarrow \delta \cup \perp$ wecorhes $y=11$
$\therefore$ (0,8) i q puint on ne line (ii).
When $y=0$, then (ii) becomes $x=15$
$\therefore(15,0)$ is a point on the line (ii).


Take $(0,0)$ as a test point. Put it in
$2 x+y \leq 20$
$2(0)+0 \leq 20$
$0 \leq 20$ (Trge)

Because ( 0,0 ) satibf es a 1 the riven inequalities, so their solution is towards origin. So,
the emmon solution of the given system of inequalities is shown by the shaded region as
Shown in the figure. As $x \geq 0, y \geq 0$ so, solution region will be in first quadrant
Now we find the corner points of the feasible region.
Solving $8 x+15 y=120$ and $x+y=11$ we get $\left(\frac{45}{7}, \frac{32}{7}\right)$.
Solving $2 x+y=20$ and $x+y=11$ we get $(9,2)$.
The corner points of the feasible region of the given system of linear inequalities are $(0,0),(10,0),(9,2),\left(\frac{45}{7}, \frac{32}{7}\right) \operatorname{and}(0,8)$.

## Theorem of linear Programming:

The theorem of linear programming states that the maximum and minimum values of the objective function occur at corner points of the feasible region.

## Objective function:

A function which is to be maximized or minimized is called an objective function.
Optimal solution:
The feasible solution which maximizes or minimizes the objective function is called optimal solution.

## Procedure for determining optimal solution:

(i) Graph the solution set of linear inequality constraints to determine feasible region.
(ii) Find the corner points of the feasible region.

Evaluate the objective function at each confer puint ford the epima solution

