EXERCISE 5.2

Q.1 Graph the feasible region of the following system of linear inequalities and find the corner points in each case.

(i) $2x-3y\le 6$

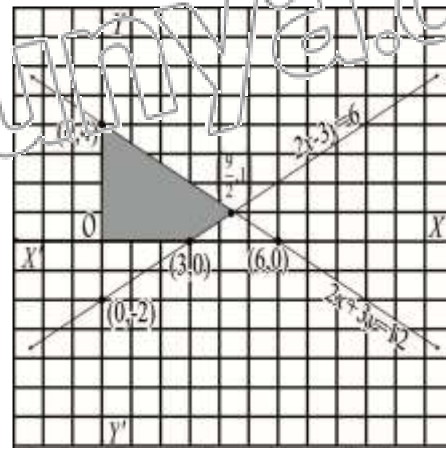
$2x+3y\le 12$

$x\ge 0,\ y\ge 0$

Solution:

$2x-3y\le 6,\ 2x+3y\le 12,\ x\ge 0,\ y\ge 0$

The associated equations of the above inequalities are:



$2x-3y=6$...(i)	$2x+3y=12$...(ii)
When $x=0$, then (i) becomes $y=-2$	When $x=0$, then (ii) becomes $y=4$
$\therefore$ $(0,-2)$ is a point on the line (i).	$\therefore$ $(0,4)$ is a point on the line (ii).
When $y=0$, then (i) becomes $x=3$	When $y=0$, then (ii) becomes $x=6$
$\therefore$ $(3,0)$ is a point on the line (i).	$\therefore$ $(6,0)$ is a point on the line (ii).
Take (0,0) as a test point.	Take (0,0) as a test point.
Put it in $2x-3y\le 6$	Put it in $2x+3y\le 12$
$2(0)-3(0)\le 6$	$2(0)+3(0)\le 12$
$0\le 6$ (True)	$0\le 12$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\ge 0,\ y\ge 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $2x-3y=6$ and $2x+3y=12$ we get $\left(\frac{9}{2},1\right)$.

The corner points of the feasible region of the given system of linear inequalities are

$(0,0)$, $(3,0)$, $\left(\frac{9}{2},1\right)$ and $(0,4)$.

(ii) $x+y\le 5$

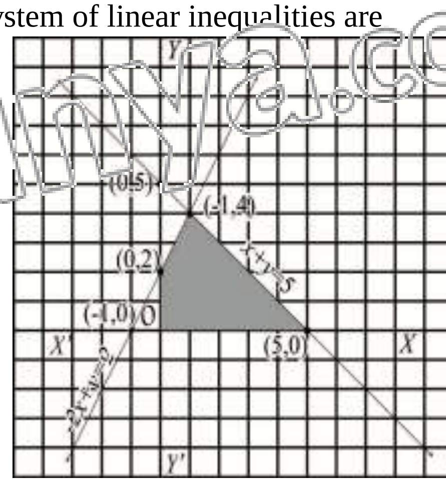
$-2x+y\le 2$

$x\ge 0,\ y\ge 0$

Solution:

$x+y\le 5,\ -2x+y\le 2,\ x\ge 0,\ y\ge 0$

The associated equations of the above inequalities are:



$x+y=5$...(i)

When $x=0$, then (i) becomes $y=5$

$\therefore$ $(0,5)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=5$

$\therefore$ $(5,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $x+y\le 5$

$0+0\le 5$

$0\le 5$ (True)

$-2x+y=2$...(ii)

When $x=0$, then (ii) becomes $y=2$

$\therefore$ $(0,2)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=-1$

$\therefore$ $(-1,0)$ is a point on the line (ii).

Take (0,0) as a test point.

Put it in $-2x+y\le 2$

$-2(0)+0\le 2$

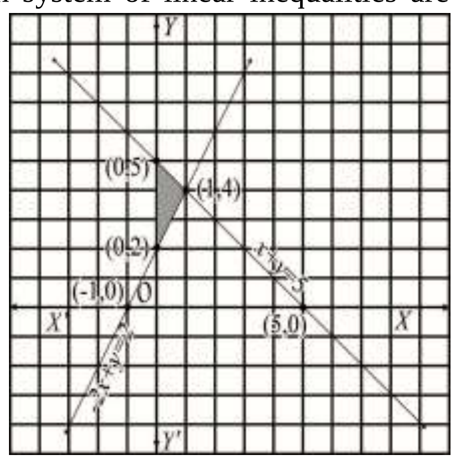
$0\le 2$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\ge 0,\ y\ge 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $x+y=5$ and $-2x+y=2$ we get $(1,4)$.

The corner points of the feasible region of the given system of linear inequalities are $(0,0)$, $(5,0)$, $(1,4)$ and $(0,2)$.



(iii) $x+y\le 5$

$-2x+y\ge 2$

$x\ge 0,\ y\ge 0$

Solution:

$x+y\le 5, -2x+y\ge 2, x\ge 0, y\ge 0$

The associated equations of the above inequalities are:

$x+y=5$...(i)

When $x=0$, then (i) becomes $y=5$

$\therefore$ $(0,5)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=5$

$\therefore$ $(5,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $x+y\le 5$

$0+0\le 5$

$0\le 5$ (True)

$-2x+y=2$...(ii)

When $y=0$, then (ii) becomes $x=-1$

$\therefore$ $(-1,0)$ is a point on the line (ii).

When $x=0$, then (ii) becomes $y=2$

$\therefore$ $(0,2)$ is a point on the line (ii).

Take (0,0) as a test point.

Put it in $-2x+y\ge 2$

$-2(0)+0\ge 2$

$0\ge 2$ (False)

Because (0,0) satisfies the inequality (i), so its solution is towards origin, while (0,0) does not satisfy the inequality (ii), so its solution is away from the origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0,\ y \geq 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $x+y=5$ and $-2x+y=2$ we get $(1,4)$.

The corner points of the feasible region of the given system of linear inequalities are $(0,2)$, $(1,4)$ and $(0,5)$.

(iv) $3x+7y \leq 21$

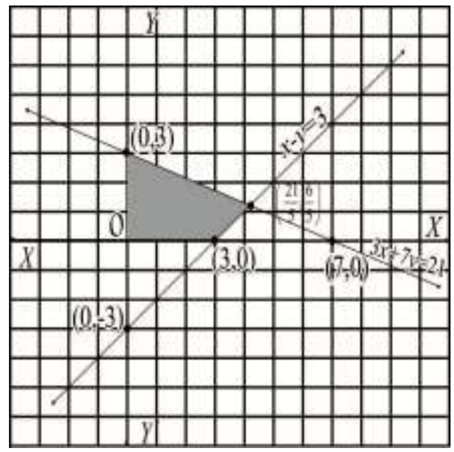
$x-y \leq 3$

$x \geq 0,\ y \geq 0$

Solution:

$3x+7y \leq 21,\ x-y \leq 3,\ x \geq 0,\ y \geq 0$

The associated equations of the above inequalities are:



$3x+7y=21$...(i)

When $x=0$, then (i) becomes $y=3$

$\therefore$ $(0,3)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=7$

$\therefore$ $(7,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $3x+7y \leq 21$

$3(0)+7(0) \leq 21$

$0 \leq 21$ (True)

$x-y=3$...(ii)

When $x=0$, then (ii) becomes $y=-3$

$\therefore$ $(0,-3)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=3$

$\therefore$ $(3,0)$ is a point on the line (ii).

Take (0,0) as a test point.

Put it in $x-y \leq 3$

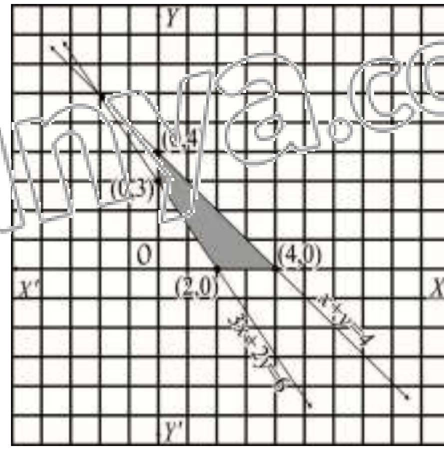
$0-0 \leq 3$

$0 \leq 3$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0,\ y \geq 0$ so, solution region will be in first quadrant

Solving $3x+7y=21$ and $x-y=3$ we get $\left(\frac{21}{5},\frac{6}{5}\right)$.

The corner points of the feasible region of the given system of linear inequalities are $(0,0)$, $(3,0)$, $\left(\frac{21}{5},\frac{6}{5}\right)$ and $(0,3)$.



(v)
$3x+2y\geq 6$
$x+y\leq 4$
$x\geq 0,\ y\geq 0$

Solution:

$3x+2y\geq 6,\ x+y\leq 4,\ x\geq 0,\ y\geq 0$

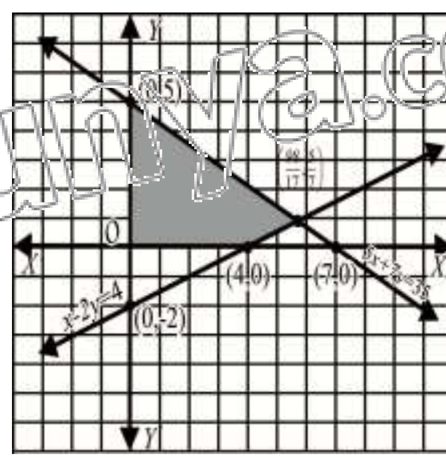
The associated equations of the above inequalities are:

$3x+2y=6$...(i)	$x+y=4$...(ii)
When $x=0$, then (i) becomes $y=3$	When $x=0$, then (ii) becomes $y=4$
$\therefore$ $(0,3)$ is a point on the line (i).	$\therefore$ $(0,4)$ is a point on the line (ii).
When $y=0$, then (i) becomes $x=2$	When $y=0$, then (ii) becomes $x=4$
$\therefore$ $(2,0)$ is a point on the line (i).	$\therefore$ $(4,0)$ is a point on the line (ii).
Take (0,0) as a test point.	Take (0,0) as a test point.
Put it in $3x+2y\geq 6$	Put it in $x+y\leq 4$
$3(0)+2(0)\geq 6$	$0+0\leq 4$
$0\geq 6$ (False)	$0\leq 4$ (True)

Because (0,0) satisfies the inequality (ii), so its solution is towards origin, while (0,0) does not satisfy the inequality (i), so its solution is away from the origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\geq 0,\ y\geq 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

The corner points of the feasible region of the given system of linear inequalities are $(2,0),\ (4,0),(0,4)$ and $(0,3)$.



(vi)
$5x+7y\leq 35$
$x-2y\leq 4$
$x\geq 0,\ y\geq 0$

Solution:

$5x+7y\leq 35,\ x-2y\leq 4,\ x\geq 0,\ y\geq 0$

The associated equations of the above inequalities are

$5x+7y=35$...(i)

When $x=0$, then (i) becomes $y=5$

$\therefore$ $(0,5)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=7$

$\therefore$ $(7,0)$ is a point on the line (i).

Take (0,0) as a test point.

Put it in $5x+7y\le 35$

$5(0)+7(0)\le 35$

$0\le 35$ (True)

$x-2y=4$...(ii)

When $x=0$, then (ii) becomes $y=-2$

$\therefore$ $(0,-2)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=4$

$\therefore$ $(4,0)$ is a point on the line (ii).

Take (0,0) as a test point.

Put it in $x-2y\le 4$

$0-2(0)\le 4$

$0\le 4$ (True)

Because (0,0) satisfies both the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\ge 0,\ y\ge 0$ so, solution region will be in first quadrant

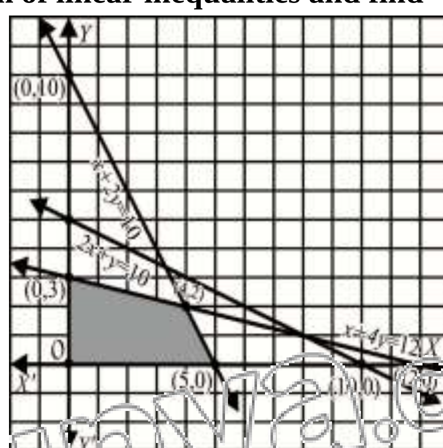
Now we find the corner points of the feasible region.

Solving $5x+7y=35$ and $x-2y=4$ we get $\left(\frac{98}{17},\frac{15}{17}\right)$.

The corner points of the feasible region of the given system of linear inequalities are $(0,0),(4,0),\left(\frac{98}{17},\frac{15}{17}\right)$ and $(0,5)$.

Q.2 Graph the feasible region of the following system of linear inequalities and find the corner points in each case.

(i)
$2x+y\le 10$
$x+4y\le 12$
$x+2y\le 10$
$x\ge 0,\ y\ge 0$



Solution:

$2x+y\le 10,\ x+4y\le 12,\ x+2y\le 10,\ x\ge 0,\ y\ge 0$

The associated equations of the above inequalities are:

$2x+y=10$...(i)

When $x=0$, then (i) becomes $y=10$

$\therefore$ $(0,10)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=5$

$\therefore$ $(5,0)$ is a point on the

$x+4y=12$...(ii)

When $x=0$, then (ii) becomes $y=3$

$\therefore$ $(0,3)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=12$

$\therefore$ $(12,0)$ is a point on

$x+2y=10$...(iii)

When $x=0$, then (iii) becomes $y=5$

$\therefore$ $(0,5)$ is a point on the line (iii).

When $y=0$, then (iii) becomes $x=10$

$\therefore$ $(10,0)$ is a point on

line (i).
Take (0,0) as a test point. Put it in
$2x+y\le 10$
$2(0)+0\le 10$
$0\le 10$ (True)

the line (ii).
Take (0,0) as a test point. Put it in
$x+4y\le 12$
$0+4(0)\le 12$
$0\le 12$ (True)

the line (iii).
Take (0,0) as a test point. Put it in
$x+2y\le 10$
$0+2(0)\le 10$
$0\le 10$ (True)

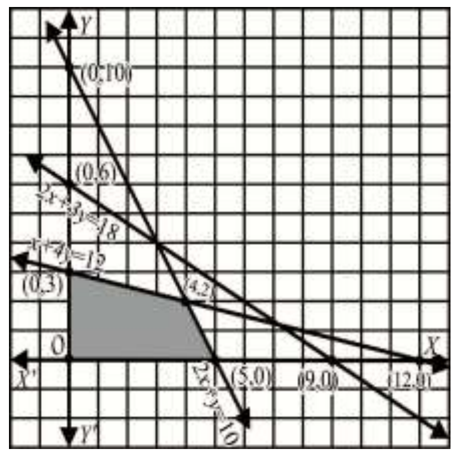
Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\ge 0,\ y\ge 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $2x+y=10$ and $x+4y=12$ we get $(4,2)$.

The corner points of the feasible region of the given system of linear inequalities are $(0,0),(5,0),(4,2)$ and $(0,3)$.

(ii)
$2x+3y\le 18$
$2x+y\le 10$
$x+4y\le 12$
$x\ge 0,\ y\ge 0$



Solution:
$2x+3y\le 18,\ 2x+y\le 10,\ x+4y\le 12,\ x\ge 0,\ y\ge 0$

The associated equations of the above inequalities are:

$2x+3y=18$...(i)
When $x=0$, then (i) becomes $y=6$
$\therefore$ $(0,6)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=9$
$\therefore$ $(9,0)$ is a point on the line (i).
Take (0,0) as a test point. Put it in $2x+3y\le 18$
$2(0)+3(0)\le 18$
$0\le 18$ (True)

$2x+y=10$...(ii)
When $x=0$, then (ii) becomes $y=10$
$\therefore$ $(0,10)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=5$
$\therefore$ $(5,0)$ is a point on the line (ii).
Take (0,0) as a test point. Put it in
$2x+y\le 10$
$2(0)+0\le 10$
$0\le 10$ (True)

$x+4y=12$...(iii)
When $x=0$, then (iii) becomes $y=3$
$\therefore$ $(0,3)$ is a point on the line (iii).
When $y=0$, then (iii) becomes $x=12$
$\therefore$ $(12,0)$ is a point on the line (iii).
Take (0,0) as a test point. Put it in
$x+4y\le 12$
$0+4(0)\le 12$
$0\le 12$ (True)

Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0,\ y \geq 0$ so, solution region will be in first quadrant.

Now we find the corner points of the feasible region.

Solving $2x + y = 10$ and $x + 4y = 12$ we get $(4,2)$.

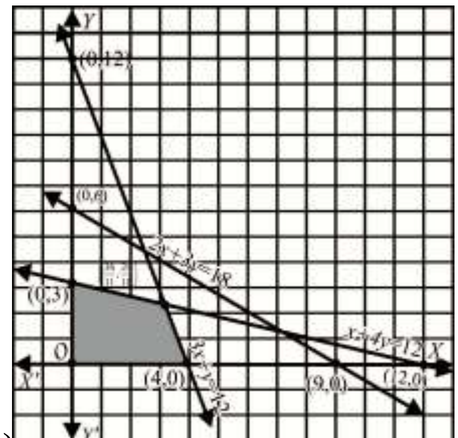
The corner points of the feasible region of the given system of linear inequalities are $(0,0), (5,0), (4,2)$ and $(0,3)$.

(iii)
$2x + 3y \leq 18$
$x + 4y \leq 12$
$3x + y \leq 12$
$x \geq 0,\ y \geq 0$

Solution:

$2x + 3y \leq 18,\ x + 4y \leq 12,\ 3x + y \leq 12,\ x \geq 0,\ y \geq 0$

The associated equations of the above inequalities are:



$2x + 3y = 18$...(i)
When $x = 0$, then (i) becomes $y = 6$
$\therefore$ $(0,6)$ is a point on the line (i).
When $y = 0$, then (i) becomes $x = 9$
$\therefore$ $(9,0)$ is a point on the line (i).
Take (0,0) as a test point. Put it in $2x + 3y \leq 18$
$2(0) + 3(0) \leq 18$
$0 \leq 18$ (True)

$x + 4y = 12$...(ii)
When $x = 0$, then (ii) becomes $y = 3$
$\therefore$ $(0,3)$ is a point on the line (ii).
When $y = 0$, then (ii) becomes $x = 12$
$\therefore$ $(12,0)$ is a point on the line (ii).
Take (0,0) as a test point. Put it in $x + 4y \leq 12$
$0 + 4(0) \leq 12$
$0 \leq 12$ (True)

$3x + y = 12$...(iii)
When $x = 0$, then (iii) becomes $y = 12$
$\therefore$ $(0,12)$ is a point on the line (iii).
When $y = 0$, then (iii) becomes $x = 4$
$\therefore$ $(4,0)$ is a point on the line (iii).
Take (0,0) as a test point. Put it in $3x + y \leq 12$
$3(0) + 0 \leq 12$
$0 \leq 12$ (True)

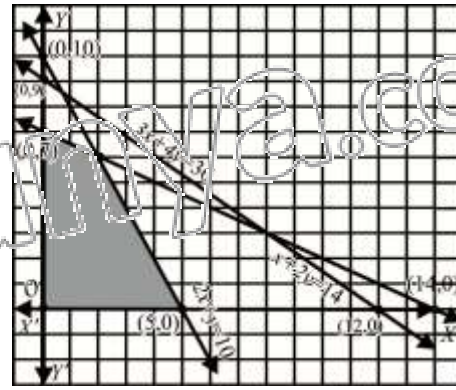
Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0,\ y \geq 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $3x + y = 12$ and $x + 4y = 12$ we get $\left(\frac{36}{11}, \frac{24}{11}\right)$.

The corner points of the feasible region of the given system of linear inequalities are $(0,0), (4,0), \left(\frac{36}{11}, \frac{24}{11}\right)$ and $(0,3)$.

(iv)
$x+2y\le 14$
$3x+4y\le 36$
$2x+y\le 10$
$x\ge 0,\ y\ge 0$



Solution:

$x+2y\le 14,\ 3x+4y\le 36,\ 2x+y\le 10,\ x\ge 0,\ y\ge 0$

The associated equations of the above inequalities are:

$x+2y=14$...(i)

When $x=0$, then (i) becomes $y=7$

$\therefore$ $(0,7)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=14$

$\therefore$ $(14,0)$ is a point on the line (i).

Take (0,0) as a test point. Put it in

$x+2y\le 14$

$0+2(0)\le 14$

$0\le 14$ (True)

$3x+4y=36$...(ii)

When $x=0$, then (ii) becomes $y=9$

$\therefore$ $(0,9)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=12$

$\therefore$ $(12,0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in

$3x+4y\le 36$

$3(0)+4(0)\le 36$

$0\le 36$ (True)

$2x+y=10$...(iii)

When $x=0$, then (iii) becomes $y=10$

$\therefore$ $(0,10)$ is a point on the line (iii).

When $y=0$, then (iii) becomes $x=5$

$\therefore$ $(5,0)$ is a point on the line (iii).

Take (0,0) as a test point. Put it in

$2x+y\le 10$

$2(0)+0\le 10$

$0\le 10$ (True)

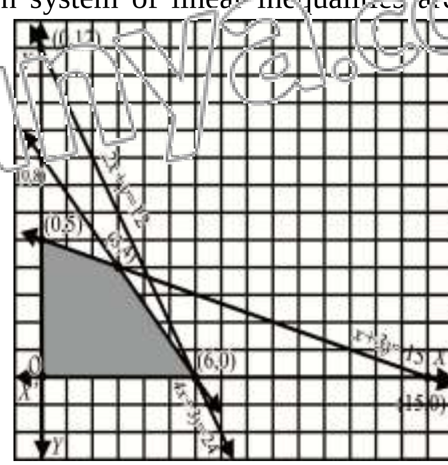
Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\ge 0,\ y\ge 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $2x+y=10$ and $x+2y=14$ we get $(2,6)$.

The corner points of the feasible region of the given system of linear inequalities are $(0,0),(5,0),(2,6)$ and $(0,7)$.

(v)
$x+3y\le 15$
$2x+y\le 12$
$4x+3y\le 24$
$x\ge 0,\ y\ge 0$



Solution:

$x+3y\le 15,\ 2x+y\le 12,\ 4x+3y\le 24,\ x\ge 0,\ y\ge 0$

The associated equations of the above inequalities are:

$x+3y=15...(i)$
When $x=0$, then (i) becomes $y=5$
$\therefore$ $(0,5)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=15$
$\therefore$ $(15,0)$ is a point on the line (i).
Take (0,0) as a test point. Put it in
$x+3y\le 15$
$0+3(0)\le 15$
$0\le 15$ (True)

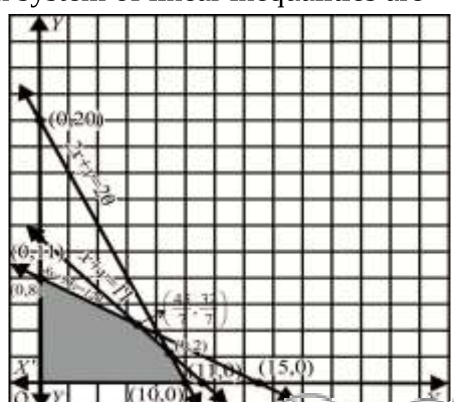
$2x+y=12...(ii)$ When $x=0$, then (ii) becomes $y=12$
$\therefore$ $(0,12)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=6$
$\therefore$ $(6,0)$ is a point on the line (ii).
Take (0,0) as a test point. Put it in
$2x+y\le 12$
$2(0)+0\le 12$
$0\le 12$ (True)

$4x+3y=24...(iii)$
When $x=0$, then (iii) becomes $y=8$
$\therefore$ $(0,8)$ is a point on the line (iii).
When $y=0$, then (iii) becomes $x=6$
$\therefore$ $(6,0)$ is a point on the line (iii).
Take (0,0) as a test point. Put it in
$4x+3y\le 24$
$4(0)+3(0)\le 24$
$0\le 24$ (True)

Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\ge 0,\ y\ge 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.
Solving $4x+3y=24$ and $x+3y=15$ we get $(3,4)$.
The corner points of the feasible region of the given system of linear inequalities are $(0,0),(6,0),(3,4)$ and $(0,5)$.

(vi)
$2x+y\le 20$
$8x+15y\le 120$
$x+y\le 11$
$x\ge 0,\ y\ge 0$



Solution:

$2x+y\le 20,\ 8x+15y\le 120,\ x+y\le 11,\ x\ge 0,\ y\ge 0$

The associated equations of the above inequalities are:

$2x+y=20...(i)$
When $x=0$, then (i) becomes $y=20$
$\therefore$ $(0,20)$ is a point on the line (i).
When $y=0$, then (i) becomes $x=10$
$\therefore$ $(10,0)$ is a point on the line (i).

$8x+15y=120...(ii)$
When $x=0$, then (ii) becomes $y=8$
$\therefore$ $(0,8)$ is a point on the line (ii).
When $y=0$, then (ii) becomes $x=15$
$\therefore$ $(15,0)$ is a point on the line (ii).

$x+y=11...(iii)$
When $x=0$, then (iii) becomes $y=11$
$\therefore$ $(0,11)$ is a point on the line (iii).
When $y=0$, then (iii) becomes $x=11$
$\therefore$ $(11,0)$ is a point on the line (iii).

Take (0,0) as a test point. Put it in	Take (0,0) as a test point. Put it in	Take (0,0) as a test point. Put it in
$2x+y\le 20$	$8x+15y\le 120$	$x+y\le 11$
$2(0)+0\le 20$	$8(0)+15(0)\le 120$	$0+0\le 11$
$0\le 20$ (True)	$0\le 120$ (True)	$0\le 11$ (True)

Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\ge 0,\ y\ge 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $8x+15y=120$ and $x+y=11$ we get $\left(\frac{45}{7},\frac{32}{7}\right)$.

Solving $2x+y=20$ and $x+y=11$ we get $(9,2)$.

The corner points of the feasible region of the given system of linear inequalities are $(0,0),(10,0),(9,2),\left(\frac{45}{7},\frac{32}{7}\right)$ and $(0,8)$.

<u>Theorem of linear Programming</u>:

The theorem of linear programming states that the maximum and minimum values of the objective function occur at corner points of the feasible region.

<u>Objective function</u>:

A function which is to be maximized or minimized is called an objective function.

<u>Optimal solution</u>:

The feasible solution which maximizes or minimizes the objective function is called optimal solution.

<u>Procedure for determining optimal solution</u>:

(i) Graph the solution set of linear inequality constraints to determine feasible region.

(ii) Find the corner points of the feasible region.

Evaluate the objective function at each corner point to find the optimal solution