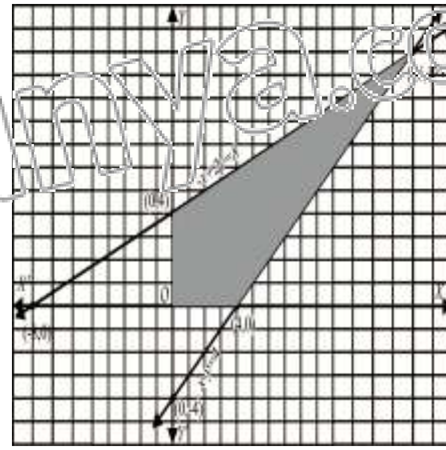
EXERCISE 5.3

Q.1 **Maximize** $f(x,y)=2x+5y$

Subject to the constraints

$2y-x\le 8;\ x-y\le 4;\ x\ge 0;\ y\ge 0.$



Solution:

The associated equations of the above inequalities are:

$2y-x=8$...(i)

When $x=0$, then (i) becomes $y=4$

$\therefore$ $(0,4)$ is a point on the line (i).

When $y=0$, then (i) becomes $x=-8$

$\therefore$ $(-8,0)$ is a point on the line (i).

Take (0,0) as a test point. Put it in

$2y-x\le 8$

$2(0)-0\le 8$

$0\le 8$ (True)

$x-y=4$...(ii)

When $x=0$, then (ii) become $y=-4$

$\therefore$ $(0,-4)$ is a point on the line (ii).

When $y=0$, then (ii) becomes $x=4$

$\therefore$ $(4,0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in

$x-y\le 4$

$0-0\le 4$

$0\le 4$ (True)

Because (0,0) satisfies all the given inequalities, so their solution is towards origin. The common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\ge 0,\ y\ge 0$, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $2y-x=8$ and $x-y=4$ we get $(16,12)$.

The corner points of the feasible region for the given system of linear inequalities are $(0,0),(4,0),(0,4)$ and $(16,12)$.

Now we evaluate the objective function $f(x,y)=2x+5y$ at these corner points.

Corner point	$f(x,y)=2x+5y$
$(0,0)$	$f(0,0)=2(0)+5(0)=0$
$(4,0)$	$f(4,0)=2(4)+5(0)=8$
$(0,4)$	$f(0,4)=2(0)+5(4)=20$
$(16,12)$	$f(16,12)=2(16)+5(12)=92$

Hence f is maximum at the corner point (16,12).

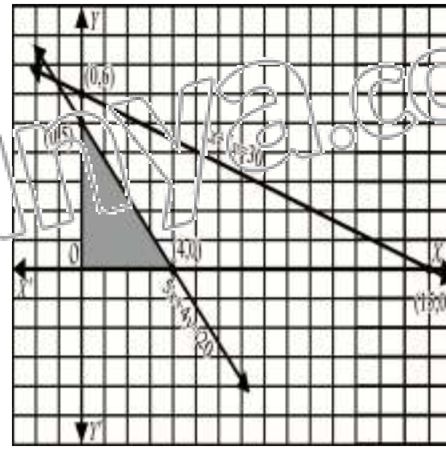
Q.2 **Maximize** $f(x, y) = x + 3y$

Subject to the constraints

$2x + 5y \le 30;\ 5x + 4y \le 20;\ x \ge 0;\ y \ge 0.$

Solution:

The associated equations of the above inequalities are:



$2x + 5y = 30$...(i)

When $x = 0$, then (i) becomes $y = 6$

$\therefore$ $(0, 6)$ is a point on the line (i).

When $y = 0$, then (i) becomes

$x = 15$

$\therefore$ $(15, 0)$ is a point on the line (i).

Take (0,0) as a test point. Put it in

$2x + 5y \le 30$

$2(0) + 5(0) \le 30$

$0 \le 30$ (True)

$5x + 4y = 20$...(ii)

When $x = 0$, then (ii) become $y = 5$

$\therefore$ $(0, 5)$ is a point on the line (ii).

When $y = 0$, then (ii) become $x = 4$

$\therefore$ $(4, 0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in

$5x + 4y \le 20$

$5(0) + 4(0) \le 20$

$0 \le 20$ (True)

Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \ge 0,\ y \ge 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

The corner points of the feasible region for the given system of linear inequalities are $(0, 0), (4, 0)$ and $(0, 5)$.

Now we evaluate the objective function $f(x, y) = x + 3y$ at these corner points.

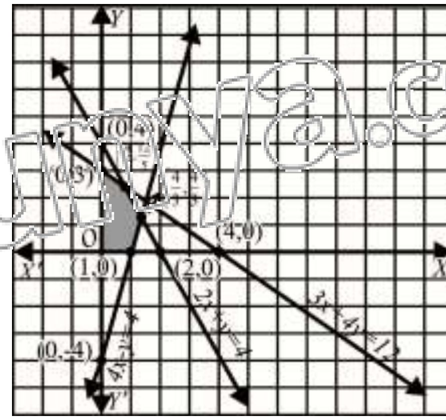
Corner point	$f(x, y) = x + 3y$
$(0, 0)$	$f(0, 0) = (0) + 3(0) = 0$
$(4, 0)$	$f(4, 0) = (4) + 3(0) = 4$
$(0, 5)$	$f(0, 5) = (0) + 3(5) = 15$

Hence f is maximum at the corner point (0,5).

Q.3 **Maximize** $z = 2x + 3y$

Subject to the constraints

$3x + 4y \le 12;\ 2x + y \le 4;\ 4x - y \le 4;\ x \ge 0;\ y \ge 0.$



Solution:

The associated equations of the above inequalities are:

$3x + 4y = 12$ (i)

When $x = 0$, then (i) becomes $y = 3$

$\therefore$ $(0,3)$ is a point on the line (i).

When $y = 0$, then (i) becomes $x = 4$

$\therefore$ $(4,0)$ is a point on the line (i).

Take (0,0) as a test point. Put it in

$3x + 4y \le 12$

$3(0) + 4(0) \le 12$

$0 \le 12$ (True)

$2x + y = 4$ (ii)

When $x = 0$, then (ii) becomes $y = 4$

$\therefore$ $(0,4)$ is a point on the line (ii).

When $y = 0$, then (ii) becomes $x = 2$

$\therefore$ $(2,0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in

$2x + y \le 4$

$2(0) + 0 \le 4$

$0 \le 4$ (True)

$4x - y = 4$ (iii)

When $x = 0$, then (iii) becomes

$y = -4$

$\therefore$ $(0,-4)$ is a point on the line (iii).

When $y = 0$, then (iii) becomes

$x = 1$

$\therefore$ $(1,0)$ is a point on the line (iii).

Take (0,0) as a test point. Put it in

$4x - y \le 4$

$4(0) - 0 \le 4$

$0 \le 4$ (True)

Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \ge 0,\ y \ge 0$ so, solution region will be in first quadrant.

Now we find the corner points of the feasible region.

Solving $2x + y = 4$ and $3x + 4y = 12$ we get $\left(\frac{4}{5}, \frac{12}{5}\right)$.

Solving $2x + y = 4$ and $4x - y = 4$ we get $\left(\frac{4}{3}, \frac{4}{3}\right)$.

The corner points of the feasible region for the given system of linear inequalities are $(0,0), (1,0), (0,3), \left(\frac{4}{3}, \frac{4}{3}\right)$ and $\left(\frac{4}{5}, \frac{12}{5}\right)$.

Now we evaluate the objective function $z = f(x, y) = 2x + 3y$ at these corner points.

Corner point	$z = f(x,y) = 2x+3y$
$(0,0)$	$z = f(0,0) = 2(0)+3(0) = 0$
$(1,0)$	$z = f(1,0) = 2(1)+3(0) = 2$
$(0,3)$	$z = f(0,3) = 2(0)+3(3) = 9$
$\left(\frac{4}{3},\frac{4}{3}\right)$	$z = f\left(\frac{4}{3},\frac{4}{3}\right) = 2\left(\frac{4}{3}\right)+3\left(\frac{4}{3}\right) = \frac{20}{3}$
$\left(\frac{4}{5},\frac{12}{5}\right)$	$z = f\left(\frac{4}{5},\frac{12}{5}\right) = 2\left(\frac{4}{5}\right)+3\left(\frac{12}{5}\right) = \frac{44}{5}$

Hence z is maximum at the corner point (0,3).

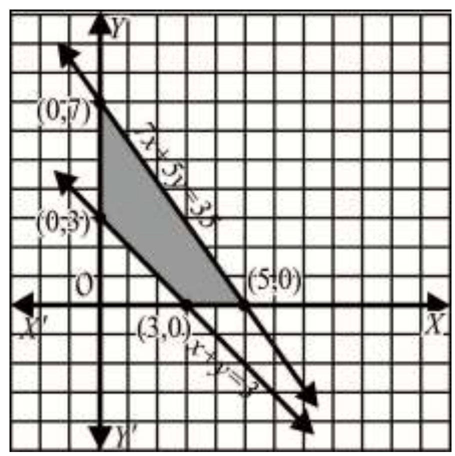
Q.4 Minimize $z = 2x + y$

Subject to the constraints

$x + y \geq 3;\ 7x+5y \leq 35;\ x \geq 0;\ y \geq 0.$

Solution:

The associated equations of the above inequalities are:



$x + y = 3$...(i)

When $x = 0$, then (i) becomes

$y = 3$

$\therefore$ $(0,3)$ is a point on the line (i).

When $y = 0$, then (i) becomes

$x = 3$

$\therefore$ $(3,0)$ is a point on the line (i).

Take (0,0) as a test point. Put it in

$x + y \geq 3$

$0 + 0 \geq 3$

$0 \geq 3$ (False)

$7x + 5y = 35$...(ii)

When $x = 0$, then (ii) becomes

$y = 7$

$\therefore$ $(0,7)$ is a point on the line (ii).

When $y = 0$, then (ii) becomes

$x = 5$

$\therefore$ $(5,0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in

$7x + 5y \leq 35$

$7(0) + 5(0) \leq 35$

$0 \leq 35$ (True)

Because (0,0) does not satisfy the inequality (i), so its solution is away from the origin, while (0,0) satisfy the inequality (ii), so its solution is towards origin. As $x \geq 0,\ y \geq 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

The corner points of the feasible region for the given system of linear inequalities are $(0,3),(3,0),(5,0)$ and $(0,7)$.

Now we evaluate the objective function $z = f(x,y) = 2x + y$ at these corner points.

Corner point	$z = f(x,y) = 2x + y$
$(0,3)$	$z = f(0,3) = 2(0) + (3) = 3$
$(3,0)$	$z = f(3,0) = 2(3) + (0) = 6$
$(5,0)$	$z = f(5,0) = 2(5) + (0) = 10$
$(0,7)$	$z = f(0,7) = 2(0) + (7) = 7$

Hence z is minimum at the corner point (0,3).

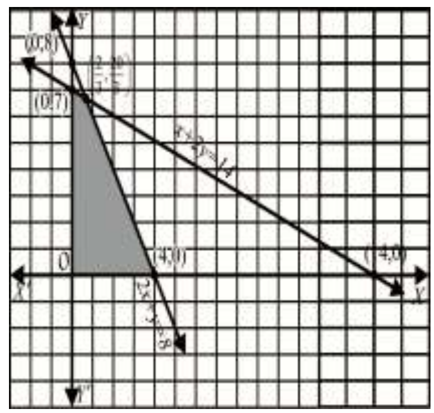
Q.5 Maximize the function defined as; $f(x,y) = 2x + 3y$

Subject to the constraints:

$2x + y \le 8;\ x + 2y \le 14;\ x \ge 0;\ y \ge 0.$

Solution:

The associated equations of the above inequalities are:



$2x + y = 8$...(i)

When $x = 0$, then (i) becomes

$y = 8$

$\therefore$ $(0,8)$ is a point on the line (i).

When $y = 0$, then (i) becomes

$x = 4$

$\therefore$ $(4,0)$ is a point on the line (i).

Take (0,0) as a test point. Put it in

$2x + y \le 8$

$2(0) + 0 \le 8$

$0 \le 8$ (True)

$x + 2y = 14$...(ii)

When $x = 0$, then (ii) becomes

$y = 7$

$\therefore$ $(0,7)$ is a point on the line (ii).

When $y = 0$, then (ii) becomes

$x = 14$

$\therefore$ $(14,0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in

$x + 2y \le 14$

$0 + 2(0) \le 14$

$0 \le 14$ (True)

Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \ge 0, y \ge 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

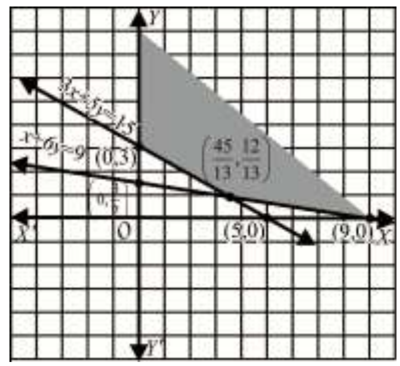
Solving $2x + y = 8$ and $x + 2y = 14$ we get $\left(\frac{2}{3}, \frac{20}{3}\right)$.

The corner points of the feasible region for the given system of linear inequalities are $(0,0),(4,0),\left(\frac{2}{3}, \frac{20}{3}\right)$ and $(0,7)$.

Now we evaluate the objective function $f(x,y)=2x+3y$ at these corner points.

Corner point	$f(x,y)=2x+3y$
$(0,0)$	$f(0,0)=2(0)+3(0)=0$
$(4,0)$	$f(4,0)=2(4)+3(0)=8$
$\left(\frac{2}{3},\frac{20}{3}\right)$	$f\left(\frac{2}{3},\frac{20}{3}\right)=2\left(\frac{2}{3}\right)+3\left(\frac{20}{3}\right)=\frac{4}{3}+\frac{60}{3}=\frac{64}{3}$
$(0,7)$	$f(0,7)=2(0)+3(7)=21$

Hence f is maximum at the corner point $\left(\frac{2}{3},\frac{20}{3}\right)$.



Q.6 Minimize $z=3x+y$;

Subject to the constraints:

$3x+5y\geq 15;\ x+6y\geq 9;\ x\geq 0;\ y\geq 0.$

Solution:

The associated equations of the above inequalities are:

$3x+5y=15$...(i)

When $x=0$, then (i) becomes

$y=3$

$\therefore$ $(0,3)$ is a point on the line (i).

When $y=0$, then (i) becomes

$x=5$

$\therefore$ $(5,0)$ is a point on the line (i).

Take (0,0) as a test point. Put it in

$3x+5y\geq 15$

$3(0)+5(0)\geq 15$

$0\geq 15$ (False)

$x+6y=9$...(ii)

When $x=0$, then (ii) becomes

$y=\frac{3}{2}$

$\therefore$ $\left(0,\frac{3}{2}\right)$ is a point on the line (ii).

When $y=0$, then (ii) becomes

$x=9$

$\therefore$ $(9,0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in

$x+6y\geq 9$

$0+6(0)\geq 9$

$0\geq 9$ (False)

Because (0,0) does not satisfies all the given inequalities, so their solution is away from origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\geq 0,\ y\geq 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $3x+5y=15$ and $x+6y=9$ we get $\left(\frac{45}{13},\frac{12}{13}\right)$

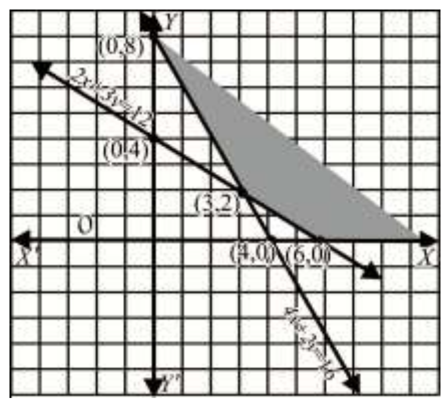
The corner points of the feasible region for the given system of linear inequalities are $(0,3),\left(\frac{45}{13},\frac{12}{13}\right)$ and $(9,0)$.

Now we evaluate the objective function $z = f(x,y) = 3x + y$ at these corner points.

Corner point	$z = f(x,y) = 3x + y$
$(0,3)$	$z = f(0,3) = 3(0) + (3) = 3$
$\left(\frac{45}{13}, \frac{12}{13}\right)$	$z = f\left(\frac{45}{13}, \frac{12}{13}\right) = 3\left(\frac{45}{13}\right) + \frac{12}{13} = \frac{147}{13}$
$(9,0)$	$z = f(9,0) = 3(9) + (0) = 27$

Hence z is minimum at the corner point (0,3).

Q.7 Each unit of food X costs Rs. 25 and contains 2 units of protein and 4 units of iron while each unit of food Y costs Rs. 30 and contains 3 units of protein and 2 units of iron. Each animal must receive at least 12 units of protein and 16 units of iron each day. How many units of each food should be fed to each animal at the smallest possible cost?



Solution:

Let x units of food X and y units of food Y be fed to each animal. Then the cost function is $f(x,y) = 25x + 30y$

under the constraints

$2x + 3y \geq 12, 4x + 2y \geq 16, x \geq 0, y \geq 0.$

We have to minimize the cost function

$f(x,y) = 25x + 30y$.

The associated equations of the above inequalities are:

$2x + 3y = 12$...(i)

When $x = 0$, then (i) becomes

$y = 4$

$\therefore$ $(0,4)$ is a point on the line (i).

When $y = 0$, then (i) becomes

$x = 6$

$\therefore$ $(6,0)$ is a point on the line (i).

Take (0,0) as a test point. Put it in

$2x + 3y \geq 12$

$2(0) + 3(0) \geq 12$

$0 \geq 12$ (False)

$4x + 2y = 16$...(ii)

When $x = 0$, then (ii) becomes

$y = 8$

$\therefore$ $(0,8)$ is a point on the line (ii).

When $y = 0$, then (ii) becomes

$x = 4$

$\therefore$ $(4,0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in

$4x + 2y \geq 16$

$4(0) + 2(0) \geq 16$

$0 \geq 16$ (False)

Because (0,0) does not satisfies all the given inequalities, so their solution is away from origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0,\ y \geq 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $2x+3y=12$ and $4x+2y=16$ we get $(3,2)$.

The corner points of the feasible region for the given system of linear inequalities are $(0,8),(3,2)\ and\ (6,0)$.

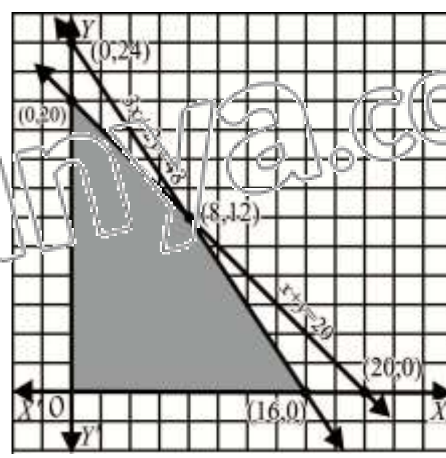
Now we evaluate the objective function $f(x,y)=25x+30y$ at these corner points.

Corner point	$f(x,y)=25x+30y$
$(0,8)$	$f(0,8)=25(0)+30(8)=240$
$(3,2)$	$f(3,2)=25(3)+30(2)=135$
$(6,0)$	$f(6,0)=25(6)+30(0)=150$

Minimum cost is at the corner point (3,2), that is, cost is minimum if 3 units of food *X* and 2 units of food *Y* are fed to each animal.

Q.8 A dealer wishes to purchase a number of fans and sewing machines. He had only Rs. 5760 to invest and has space at most for 20 items. A fan costs him Rs. 360 and a sewing machine costs Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit?

Solution:



Let *x* fans and *y* sewing machines be purchased.

Then the profit function is $f(x,y)=22x+18y$

under the constraints

$360x+240y \leq 5760, x+y \leq 20, x \geq 0, y \geq 0.$

$3x+2y \leq 48, x+y \leq 20, x \geq 0, y \geq 0.$

We have to maximize the profit function

$f(x,y)=22x+18y$.

The associated equations of the above inequalities are:

$3x+2y=48$...(i)

When $x=0$, then (i) becomes

$y=24$

$\therefore$ $(0,24)$ is a point on the line (i).

When $y=0$, then (i) becomes

$x=16$

$\therefore$ $(16,0)$ is a point on the line (i).

Take (0,0) as a test point. Put it in

$3x+2y\le 48$

$3(0)+2(0)\le 48$

$0\le 48$ (True)

$x+y=20$...(ii)

When $x=0$, then (ii) becomes

$y=20$

$\therefore$ $(0,20)$ is a point on the line (ii).

When $y=0$, then (ii) becomes

$x=20$

$\therefore$ $(20,0)$ is a point on the line (ii).

Take (0,0) as a test point. Put it in

$x+y\le 20$

$0+0\le 20$

$0\le 20$ (True)

Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x\ge 0,\ y\ge 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $3x+2y=48$ and $x+y=20$ we get $(8,12)$.

The corner points of the feasible region for the given system of linear inequalities are $(0,0),(0,20),(8,12)$ and $(16,0)$.

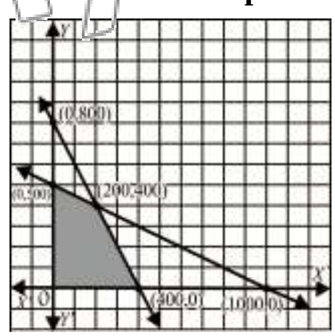
Now we evaluate the objective function $f(x,y)=22x+18y$ at these corner points.

Corner point	$f(x,y)=22x+18y$
$(0,0)$	$f(0,0)=22(0)+18(0)=0$
$(0,20)$	$f(0,20)=22(0)+18(20)=360$
$(8,12)$	$f(8,12)=22(8)+18(12)=176+216=392$
$(16,0)$	$f(16,0)=22(16)+18(0)=352$

Which shows that f is maximum at the corner point (8,12). Hence profit is maximum if the investor purchases 8 fans and 12 sewing machines.

Q.9 A machine can produce product A by using 2 units of chemical and 1 unit of a compound or can produce product B by using 1 unit of chemical and 2 units of the compound. Only 800 units of chemical and 1000 units of the compound are available. The profits per unit of A and B are Rs. 30 and Rs. 20 respectively, maximize the profit function.

Solution:



Let x be the units of product A and y be the units of product B be prepared. Then the profit function is $f(x,y)=30x+20y$ under the constraints

$2x+y\le 800,\ x+2y\le 1000,\ x\ge 0,\ y\ge 0.$

We have to maximize the profit function

$f(x,y)=30x+20y$.

The associated equations of the above inequalities are:

$2x + y = 800$...(i)

When $x = 0$, then (i) becomes

$y = 800$

$\therefore$ (0,800) is a point on the line (i).

When $y = 0$, then (i) becomes

$x = 400$

$\therefore$ (400,0) is a point on the line (i).

Take (0,0) as a test point. Put it in

$2x + y \le 800$

$2(0) + 0 \le 800$

$0 \le 800$ (True)

$x + 2y = 1000$...(ii)

When $x = 0$, then (ii) becomes

$y = 500$

$\therefore$ (0,500) is a point on the line (ii).

When $y = 0$, then (ii) becomes

$x = 1000$

$\therefore$ (1000,0) is a point on the line (ii).

Take (0,0) as a test point. Put it in

$x + 2y \le 1000$

$0 + 2(0) \le 1000$

$0 \le 1000$ (True)

Because (0,0) satisfies all the given inequalities, so their solution is towards origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \ge 0,\ y \ge 0$ so, solution region will be in first quadrant

Now we find the corner points of the feasible region.

Solving $2x + y = 800$ and $x + 2y = 1000$ we get $(200, 400)$.

The corner points of the feasible region for the given system of linear inequalities are $(0,0), (0,500), (200,400)$ and $(400,0)$.

Now we evaluate the objective function $f(x, y) = 30x + 20y$ at these corner points.

Corner point	$f(x, y) = 30x + 20y$
$(0,0)$	$f(0,0) = 30(0) + 20(0) = 0$
$(0,500)$	$f(0,500) = 30(0) + 20(500) = 10000$
$(200,400)$	$f(200,400) = 30(200) + 20(400) = 14000$
$(400,0)$	$f(400,0) = 30(400) + 20(0) = 12000$

Which shows that *f* is maximum at the corner point (200,400). Hence profit is maximum if 200 units of product *A* and 400 units of product *B* are produced.