EXERCISE 6.7

Q.1 Find equations of the tangent and normal to each of the following at the indicated point

(i) $y^2 = 4ax$ **at** $(at^2, 2at)$

Solution:

$y^2 = 4ax$...(i)

Replace $x = \frac{x+x_1}{2}, y^2 = yy_1$ in (i)

$yy_1 = 4a\left(\frac{x+x_1}{2}\right)$

$yy_1 = 2a(x+x_1)$

Put $x_1 = at^2, y_1 = 2at,$ we get;

$y(2at) = 2a(x+at^2)$

$yt = x + at^2$

$\boxed{y = \frac{1}{t}x + at}$ is equation of tangent

Slope of tangent line is

$m_1 = \frac{1}{t}$

Slope of normal is

$m = -\frac{1}{m_1}$

$m = -t$

Equation of normal passing through $P(at^2, 2at)$ with slope $m = -t$ is given by

$y - y_1 = m(x - x_1)$

$y - 2at = -t(x - at^2)$

$\boxed{y = -tx + at^3 + 2at}$

(ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\cos\theta, b\sin\theta)$

Solution:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$...(i)

Replace x^2 by xx_1 and y^2 by yy_1 in (i)

$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

Put $x_1 = a\cos\theta, y_1 = b\sin\theta,$ we get;

$\frac{(a\cos\theta)x}{a^2} + \frac{(b\sin\theta)y}{b^2} = 1$

$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

$\boxed{x(b\cos\theta) + y(a\sin\theta) - ab = 0}$

is equation of tangent

Slope of tangent is $m_1 = \frac{-b\cos\theta}{a\sin\theta}$

$m_1 = -\frac{-b}{a}\cot\theta$

Slope of normal is

$m = -\frac{1}{m_1} = -\frac{1}{\frac{-b}{a}\cot\theta} = \frac{a}{b}\tan\theta$

Equation of normal passing through $(a\cos\theta, b\sin\theta)$ with slope $m = \frac{a}{b}\tan\theta$ is given by

$y - y_1 = m(x - x_1)$

$y - b\sin\theta = \frac{a}{b}\tan\theta(x - a\cos\theta)$

$y - b\sin\theta = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$

$b\cos\theta(y - b\sin\theta) = a\sin\theta(x - a\cos\theta)$

$(b\cos\theta)y - b^2\cos\theta\sin\theta = (a\sin\theta)x - a^2\cos\theta\sin\theta$

$(a\sin\theta)x - (b\cos\theta)y - a^2\cos\theta\sin\theta + b^2\cos\theta\sin\theta = 0$

Dividing by $\cos\theta\sin\theta$ we get;

$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} - a^2 + b^2 = 0$

$\boxed{ax\sec\theta - by\ \mathrm{cosec}\,\theta = a^2 - b^2}$

(iii) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ **at** $(a\sec\theta, b\tan\theta)$

Solution:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$...(i)

Replace x^2 by xx_1 and y^2 by yy_1

$$\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$$

Put $x_1=a\sec\theta, y_1=b\tan\theta$

$$\frac{x(a\sec\theta)}{a^2}-\frac{y(b\tan\theta)}{b^2}=1$$

$$\boxed{\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1}$$

is equation of tangent

Slope of tangent line is

$$m_1=\frac{-\frac{\sec\theta}{a}}{-\frac{\tan\theta}{b}}$$

$$m_1=\frac{\sec\theta}{a}\times\frac{b}{\tan\theta} \quad =\frac{b}{a}\sec\theta\times\cot\theta$$

$$m_1=\frac{b}{a}\frac{1}{\cos\theta}.\frac{\cos\theta}{\sin\theta}=\frac{b}{a}.\text{cosec}\,\theta$$

Slope of normal is

$$m=-\frac{1}{m_1}=\frac{-a}{b\,\text{cosec}\,\theta}=\frac{-a\sin\theta}{b}$$

Equation of the normal passing through $(a\sec\theta, b\tan\theta)$ with

$m=-\frac{a}{b}\sin\theta$ is given by

$$y-y_1=m(x-x_1)$$

$$y-b\tan\theta=\frac{-a\sin\theta}{b}(x-a\sec\theta)$$

$$by-b^2\tan\theta=-a\sin\theta x+a^2\sin\theta\sec\theta$$

$$(a\sin\theta)x+by=\frac{a^2\sin\theta}{\cos\theta}+b^2\tan\theta$$

$$(a\sin\theta)x+by=(a^2+b^2)\tan\theta$$

Dividing by $\tan\theta$ on both sides we get;

$$\left(\frac{a\sin\theta}{\tan\theta}\right)x+\frac{by}{\tan\theta}=a^2+b^2$$

$$\boxed{(a\cos\theta)x+(b\cot\theta)y=a^2+b^2}$$

Q.2 Write equation of the tangent to the given conic at the indicated point

(i) $3x^2=-16y$ **at the points whose ordinate is** -3

Solution:

$$3x^2=-16y\ \ldots\text{(i)}$$

Put ordinate, $y=-3$ in (i)

$$3x^2=-16(-3)$$

$$3x^2=48$$

$$x^2=16$$

$$x=\pm4$$

Hence points of contact are $A(4,-3)$ and $B(-4,-3)$

Now for equation of tangent, we replace x^2 by xx_1, y by $\frac{y+y_1}{2}$

$$3xx_1=-\left(\frac{y+y_1}{2}\right)$$

$$3xx_1=-8(y+y_1)$$

Hence equation of tangent at $A(4,-3)$ is

$$3x(4)=-8(y-3)$$

$$3x=-2(y-3)$$

$$\boxed{3x+2y-6=0}$$

Equation of tangent at $B(-4,-3)$ is

$$3x(-4)=-8(y-3)$$

$$3x=2(y-3)$$

$$\boxed{3x-2y+6=0}$$

(ii) $3x^2-7y^2=20$ **at the points where** $y=-1$

Solution:

$$3x^2-7y^2=20\ \ldots\text{(i)}$$

Put $y=-1$ in (1), we get;

$$3x^2-7(-1)^2=20$$

$$3x^2-7=20$$

$$3x^2=27$$

$x^2 = 9$

$\Rightarrow x = \pm 3$

Hence points of contact are

$A(3,-1), B(-3,-1)$

For equation of tangent, put

$x^2 = xx_1, y^2 = yy_1$ in (i)

$3xx_1 - 7yy_1 = 20$

So equation of tangent at $A(3,-1)$ is

$3x(3) - 7y(-1) = 20$

$\boxed{9x + 7y = 20}$

Equation of tangent at $B(-3,-1)$ is

$3(-3)x - 7y(-1) = 20$

$-9x + 7y = 20$

$\boxed{9x - 7y + 20 = 0}$

(iii) $3x^2 - 7y^2 + 2x - y - 48 = 0$ **at the points where** $x = 4$

Solution:

$3x^2 - 7y^2 + 2x - y - 48 = 0 \ldots$(i)

Put $x = 4$ in (i)

$3(4)^2 - 7y^2 + 2(4) - y - 48 = 0$

$48 - 7y^2 + 8 - y - 48 = 0$

$-7y^2 - y + 8 = 0$

$7y^2 + y - 8 = 0$

$7y^2 + 8y - 7y - 8 = 0$

$y(7y + 8) - 1(7y + 8) = 0$

$(7y + 8)(y - 1) = 0$

$7y + 8 = 0, y - 1 = 0$

$y = \frac{-8}{7}, y = 1$

Hence points of contact are

$A\left(4, \frac{-8}{7}\right), B(4,1)$

For equation of tangent, replace

x^2 by xx_1, y^2 by yy_1, x by $\frac{x + x_1}{2}$

and y by $\frac{y + y_1}{2}$ in given equation

$3xx_1 - 7yy_1 + 2\left(\frac{x + x_1}{2}\right) - \left(\frac{y + y_1}{2}\right) - 48 = 0$

$3xx_1 - 7yy_1 + (x + x_1) - \left(\frac{y + y_1}{2}\right) - 48 = 0$

At $A\left(4, \frac{-8}{7}\right)$ equation of tangent is

$3x(4) - 7y\left(\frac{-8}{7}\right) + (x + 4) - \frac{1}{2}\left(y - \frac{8}{7}\right) - 48 = 0$

$12x + 8y + x + 4 - \frac{y}{2} + \frac{4}{7} - 48 = 0$

$13x + \frac{15}{2}y - \frac{304}{7} = 0$

$\boxed{182x + 105y - 608 = 0}$

At $B(4,1)$ equation of tangent is

$3x(4) - 7y(1) + (x + 4) - \frac{1}{2}(y + 1) - 48 = 0$

$12x - 7y + x + 4 - \frac{1}{2}y - \frac{1}{2} - 48 = 0$

$24x - 14y + 2x + 8 - y - 1 - 96 = 0$

$\boxed{26x - 15y - 89 = 0}$

Q.3 Find equations of the tangent to each of the following through the given point:

(i) $x^2 + y^2 = 25$ **through** $(7,-1)$

Solution:

We can check that point $(7,-1)$ does not satisfy the given equation so equation of the tangent is

$y = mx \pm a\sqrt{1 + m^2}$

Put $x = 7, y = -1$ and $a = 5$ (radius of circle), we get;

$-1 = 7m \pm 5\sqrt{1 + m^2}$

$-1 - 7m = \pm 5\sqrt{1 + m^2}$ ...(i)

Squaring on both sides, we get;

$(-1 - 7m)^2 = \left(\pm 5\sqrt{1 + m^2}\right)^2$

$1 + 49m^2 + 14m = 25(1 + m^2)$

$1 + 49m^2 + 14m = 25 + 25m^2$

$24m^2 + 14m - 24 = 0$

$12m^2 + 7m - 12 = 0$

$m = \frac{-7 \pm \sqrt{(7)^2 - 4(12)(-12)}}{2(12)}$

$m = \frac{-7 \pm \sqrt{49 + 576}}{24}$

$m = \frac{-7 \pm \sqrt{625}}{24}$

$m = \frac{-7 \pm 25}{24}$

$m = \frac{-7+25}{24}$, $m = \frac{-7-25}{24}$

$= \frac{18}{24}$, $= \frac{-32}{24}$

$m = \frac{3}{4}$, $m = \frac{-4}{3}$

$m = \frac{3}{4}, -\frac{4}{3}$

When $m = \frac{3}{4}$, equation (i)

$-1 - 7m = -5\sqrt{1+m^2}$ is satisfied, equation of tangent is

$y = mx - a\sqrt{1+m^2}$

$y = \frac{3}{4}x - 5\sqrt{1 + \frac{9}{16}}$

$y = \frac{3}{4}x - 5\sqrt{\frac{25}{16}}$

$y = \frac{3}{4}x - 5 \times \frac{5}{4}$

$y = \frac{3}{4}x - \frac{25}{4}$

$4y = 3x - 25$

$\boxed{3x - 4y - 25 = 0}$

When $m = \frac{-4}{3}$, equation (i)

$-1 - 7m = 5\sqrt{1+m^2}$ is satisfied, equation of tangent is

$y = mx + a\sqrt{1+m^2}$

$y = \frac{-4}{3}x + 5\sqrt{1 + \frac{16}{9}}$

$y = \frac{-4}{3}x + 5\sqrt{\frac{25}{9}}$

$y = \frac{-4}{3}x + 5 \times \frac{5}{3}$

$3y = -4x + 25$

$\boxed{4x + 3y - 25 = 0}$

(ii) $y^2 = 12x$ **through** (1,4)

Solution:

$y^2 = 12x$

Here $4a = 12 \Rightarrow a = 3$

Equation of tangent is

$y = mx + \frac{a}{m}$ ...(i)

Put $x = 1, y = 4, a = 3$ in (i)

$4 = m + \frac{3}{m}$

$4 = \frac{m^2 + 3}{m}$

$4m = m^2 + 3$

$m^2 - 4m + 3 = 0$

$m^2 - 3m - m + 3 = 0$

$m(m-3) - 1(m-3) = 0$

$(m-1)(m-3) = 0$

$m - 1 = 0, \mathrm{m} - 3 = 0$

$m = 1, m = 3$

When m = 1 then (i) becomes

$\boxed{y = x + 3}$

When m = 3 then (i) becomes

$y = 3x + \frac{3}{3}$

$\boxed{y = 3x + 1}$

(iii) $x^2 - 2y^2 = 2$ **through** $(1, -2)$

Solution:

$x^2 - 2y^2 = 2$

$\frac{x^2}{2} - y^2 = 1$

$\Rightarrow a^2 = 2\ , b^2 = 1$

Equations of tangents for hyperbola are given by

$y = mx \pm \sqrt{a^2m^2 - b^2}$ …(i)

Put $x = 1, y = -2, a^2 = 2, b^2 = 1$ in (i) we get;

$-2 = m \pm \sqrt{2m^2 - 1}$

$-2 - m = \pm\sqrt{2m^2 - 1}$

$-(2+m) = \pm\sqrt{2m^2 - 1}$...(ii)

Squaring on both sides we get;

$4 + m^2 + 4m = 2m^2 - 1$

$2m^2 - m^2 - 4m - 4 - 1 = 0$

$m^2 - 4m - 5 = 0$

$m^2 - 5m + m - 5 = 0$

$m(m-5) + 1(m-5) = 0$

$(m+1)(m-5) = 0$

$m + 1 = 0, m - 5 = 0$

$m = -1, 5$

When $m = -1$, equation (ii)

$-(2+m) = -\sqrt{2m^2 - 1}$ is satisfied

Equation of tangent is

$y = mx - \sqrt{2m^2 - 1}$

$y = -x - \sqrt{2(-1) - 1}$

$y = -x - \sqrt{2 - 1}$

$y = -x - 1$

$\boxed{x + y + 1 = 0}$

When $m = 5$, equation (ii)

$-(2+m) = -\sqrt{2m^2 - 1}$ is satisfied

Equation of tangent is

$y = mx - \sqrt{2m^2 - 1}$

$y = 5x - \sqrt{2(5)^2 - 1}$

$y = 5x - 7$

$\boxed{5x - y - 7 = 0}$

Q.4 Find equations of the Normals to the parabola $y^2 = 8x$ which are parallel to the line $2x + 3y - 10$

Solution:

given equation of line is

$2x + 3y - 10 = 0$ …(i)

Slope of the given line is $m_1 = -\dfrac{2}{3}$

As required normal lines are parallel to (i), so slope of normal line is

$m = m_1 = -\dfrac{2}{3}$

$y^2 = 8x$ …(ii)

Differentiate w.r.t. x

$2yy' = 8$

$y' = \dfrac{4}{y}$

At (x_1, y_1) , $y' = \dfrac{4}{y_1} =$ slope of tangent

Slope of normal at $(x_1, y_1) = -\dfrac{y_1}{4}$

But given slope of normal is $-\dfrac{2}{3}$

Hence $-\dfrac{y_1}{4} = -\dfrac{2}{3}$

$y_1 = \dfrac{8}{3}$

Since (x_1, y_1) lies on (ii) so

$y_1^2 = 8x_1$

$\left(\dfrac{8}{3}\right)^2 = 8x_1 \qquad \because y_1 = \dfrac{8}{3}$

$\dfrac{64}{9} = 8x_1$

$x_1 = \dfrac{8}{9}$

So point of intersection is $\left(\dfrac{8}{9}, \dfrac{8}{3}\right)$

Hence required equation of normal to (ii) at $\left(\frac{8}{9},\frac{8}{3}\right)$ with slope $m=-\frac{2}{3}$ is

$$y-y_1=m(x-x_1)$$

$$y-\frac{8}{3}=-\frac{2}{3}\left(x-\frac{8}{9}\right)$$

$$y-\frac{8}{3}=-\frac{2}{3}\left(\frac{9x-8}{9}\right)$$

Multiply with 27 we get;

$$27y-72=-2(9x-8)$$

$$27y-72=-18x+16$$

$$\boxed{18x+27y-88=0}$$

Q.5 Find equations the tangents to the ellipse $\frac{x^2}{4}+y^2=1$ which are parallel to the line $2x-4y+5=0$

Solution:

$$\frac{x^2}{4}+y^2=1 \text{...(i)}$$

$$\Rightarrow a^2=4, b^2=1$$

Equation of tangents to (i) are given by

$$y=mx\pm\sqrt{a^2m^2+b^2}$$

Put $a^2=4, \text{b}^2=1$ we get;

$$y=mx\pm\sqrt{4m^2+1} \text{...(ii)}$$

As (ii) are parallel to $2x-4y+5=0$, so slope of tangent line is $m=\frac{1}{2}$

Put $m=\frac{1}{2}$ in (ii), we get required equations of tangent lines

$$y=\frac{1}{2}x\pm\sqrt{4\left(\frac{1}{2}\right)^2+1}$$

$$y=\frac{1}{2}x\pm\sqrt{4\left(\frac{1}{4}\right)+1}$$

$$y=\frac{1}{2}x\pm\sqrt{2}$$

$$2y=x\pm2\sqrt{2}$$

$$x-2y\pm2\sqrt{2}=0$$

Hence required equations of tangents are

$$\boxed{x-2y+2\sqrt{2}=0}$$

$$\boxed{x-2y-2\sqrt{2}=0}$$

Q.6 Find equations of the tangents to the conic $9x^2-4y^2=36$ parallel to $5x-2y+7=0$

Solution:

$$9x^2-4y^2=36$$

Dividing by 36 on both sides, we get;

$$\frac{x^2}{4}-\frac{y^2}{9}=1 \ldots\text{(i)}$$

$$\Rightarrow a^2=4, b^2=9$$

Equations of tangents to (i) are

$$y=mx\pm\sqrt{a^2m^2-b^2}$$

$$y=mx\pm\sqrt{4m^2-9} \ldots\text{(ii)}$$

As (ii) are parallel to $5x-2y+7=0$

So slope of (ii) is $m=\frac{5}{2}$

Hence equations of tangents are given by

$$y=\frac{5}{2}x\pm\sqrt{4\left(\frac{5}{2}\right)^2-9}$$

$$y=\frac{5}{2}x\pm\sqrt{4\left(\frac{25}{4}\right)-9}$$

$$y=\frac{5}{2}x\pm\sqrt{25-9}$$

$$y=\frac{5}{2}x\pm\sqrt{16}$$

$$y=\frac{5}{2}x\pm4$$

$$2y=5x\pm8$$

$$5x-2y\pm8=0$$

Hence required equations of tangents are

$$\boxed{5x-2y+8=0, 5x-2y-8=0}$$

Q.7 Find equations of the common tangents to the given conics

(i) $x^2 = 80y$ **and** $x^2 + y^2 = 81$

Solution:

$x^2 = 80y \ldots$(i)

Equation of tangent is

$y = mx - am^2$

$y = mx - 20m^2 \; (4a = 80 \Rightarrow a = 20)$

$x^2 + y^2 = 81 \ldots$(ii)

Equation of tangent is

$y = mx \pm a\sqrt{m^2+1}$

$y = mx \pm 9\sqrt{m^2+1} \; (a^2 = 81 \Rightarrow a = 9)$

For a common tangent, we must have

$9\sqrt{m^2+1} = -20m^2$

Squaring on both sides we get;

$81(m^2+1) = 400m^4$

$400m^4 - 81m^2 - 81 = 0$

$m^2 = \dfrac{-(-81) \pm \sqrt{(-81)^2 - 4(400)(-81)}}{2(400)}$

$m^2 = \dfrac{81 \pm \sqrt{6561 + 129600}}{800}$

$m^2 = \dfrac{81 \pm \sqrt{136161}}{800}$

$m^2 = \dfrac{81 \pm 369}{800}$

$m^2 = \dfrac{81+369}{800}, m^2 = \dfrac{81-369}{800}$

$m^2 = \dfrac{450}{800}, \quad m^2 = -\dfrac{288}{800}$

$m^2 = \dfrac{9}{16}$, neglect $m^2 = -\dfrac{238}{800}$

because m^2 cannot be negative

$m = \pm\dfrac{3}{4}$

Hence equations of the common tangents to (i) are

$y = mx - 20m^2$

$y = \pm\dfrac{3}{4}x - 20\left(\dfrac{9}{16}\right)$

$y = \pm\dfrac{3}{4}x - \dfrac{45}{4}$

$4y = \pm 3x - 45$

$\boxed{\pm 3x - 4y - 45 = 0}$

(ii) $y^2 = 16x$ **and** $x^2 = 2y$

Solution:

$y^2 = 16x$...(i)

Equation of tangent is

$y = mx + \dfrac{a}{m}$

$y = mx + \dfrac{4}{m}$...(ii) $\because (4a = 16 \Rightarrow a = 4)$

and

$x^2 = 2y$...(iii)

Equation of tangent is

$y = mx - am^2$

$y = mx - \dfrac{1}{2}m^2$...(iv) $\because \left(4a = 2 \Rightarrow a = \dfrac{1}{2}\right)$

For a common tangent, we must have

$\dfrac{4}{m} = \dfrac{-1}{2}m^2$

$\dfrac{8}{m} = -m^2$

$m^2 + \dfrac{8}{m} = 0$

$m^3 + 8 = 0$

$(m+2)(m^2 - 4m + 4) = 0$

$m + 2 = 0, \; m^2 - 4m + 4 = 0$

$m = -2 \qquad (m^2 - 4m + 4 = 0$ gives imaginary roots)

Put $m = -2$ in (ii)

$y = -2x + \dfrac{4}{-2}$

$y = -2x - 2$

$\boxed{2x + y + 2 = 0}$

Q.8 Find the points of intersection of given conics

(i) $\frac{x^2}{18}+\frac{y^2}{8}=1$ **and** $\frac{x^2}{3}-\frac{y^2}{3}=1$

Solution: Given conics are

$\frac{x^2}{18}+\frac{y^2}{8}=1\ \ldots(i)$

$\frac{x^2}{3}-\frac{y^2}{3}=1\ \ldots(ii)$

Multiply (ii) by $\frac{1}{6}$ and then subtracting from (i) we get;

$$\frac{x^2}{18}+\frac{y^2}{8}=1$$
$$\pm\frac{x^2}{18}\mp\frac{y^2}{18}=\pm\frac{1}{6}$$
$$\frac{13}{72}y^2=\frac{5}{6}$$

$y^2=\frac{60}{13}$

$y=\pm\sqrt{\frac{60}{13}}$

Put $y=\pm\sqrt{\frac{60}{13}}$ in (i) we get;

$\frac{1}{18}x^2+\frac{1}{8}\left(\frac{60}{13}\right)=1$

$\frac{1}{18}x^2+\frac{15}{26}=1$

$\frac{1}{18}x^2=1-\frac{15}{26}$

$\frac{1}{18}x^2=\frac{11}{26}$

$x^2=\frac{99}{13}\Rightarrow x=\pm\sqrt{\frac{99}{13}}$

Hence points of intersection are

$\left(\pm\sqrt{\frac{99}{13}},\pm\sqrt{\frac{60}{13}}\right)$

(ii) $x^2+y^2=8$ **and** $x^2-y^2=1$

Solution:

Given conics are

$x^2+y^2=8\ \ldots(i)$

$x^2-y^2=1\ \ldots(ii)$

Adding (i) and (ii) we get;

$2x^2=9\Rightarrow x^2=\frac{9}{2}$

$x=\pm\frac{3}{\sqrt{2}}$

Put $x=\pm\frac{3}{\sqrt{2}}$ in (i) we get;

$\frac{9}{2}+y^2=8\Rightarrow y^2=8-\frac{9}{2}$

$y^2=\frac{7}{2}$

$y=\pm\sqrt{\frac{7}{2}}$

Hence points of intersection are

$\left(\pm\frac{3}{\sqrt{2}},\pm\sqrt{\frac{7}{2}}\right)$

(iii) $3x^2-4y^2=12$ **and** $3y^2-2x^2=7$

Solution:

$3x^2-4y^2=12\ \ldots(i)$

$-2x^2+3y^2=7\ \ldots(ii)$

Multiply equation (i) by 2 and (ii) by 3 then by adding the equations, we get;

$$6x^2-8y^2=24$$
$$-6x^2+9y^2=21$$
$$y^2=45$$

$y=\pm3\sqrt{5}$

Put $y=\pm3\sqrt{5}$ in(i) we get;

$3x^2-4(45)=12$

$3x^2=192$

$x^2=64$

$x=\pm8$

Hence points of intersection are

$(\pm8,\pm3\sqrt{5})$

(iv) $3x^2+5y^2=60$ **and** $9x^2+y^2=124$

Solution:

$3x^2+5y^2=60 \ldots$(i)

$9x^2+y^2=124 \ldots$(ii)

Multiply (i) by 3 and then subtracting (ii) from it

$$\begin{array}{r} 9x^2+15y^2=180 \\ \pm 9x^2 \pm y^2 = \pm 124 \\ \hline 14y^2=56 \end{array}$$

$y^2=4$

$y=\pm 2$

Put $y=\pm 2$ in (i) we get;

$3x^2+5(\pm 2)^2=60$

$3x^2+5(4)=60$

$3x^2=40$

$x^2=\frac{40}{3}$

$x=\pm 2\sqrt{\frac{10}{3}}$

Hence points of intersections are

$\left(\pm\sqrt{\frac{40}{3}},\pm 2\right)$

(v) $4x^2+y^2=16$ **and**

$x^2+y^2+2y-8=0$

Solution: Given conics are

$4x^2+y^2=16 \ldots$(i)

$x^2+y^2+2y-8=0 \ldots$(ii)

Subtracting (ii) from (i) we get;

$$\begin{array}{r} 4x^2 + y^2 = 16 \\ -x^2 \pm y^2 \pm 2y = \mp 8 \\ \hline 3x^2 - 2y = 8 \end{array}$$

$3x^2=\frac{2y+8}{3} \ldots$ (iii)

Put in (i) we get;

$4\left(\frac{2y+8}{3}\right)+y^2=16$

$8y+32+3y^2=48$

$3y^2+8y-16=0$

$\Rightarrow y=\frac{-8\pm\sqrt{(8)^2-4(3)(-16)}}{2(3)}$

$y=\frac{8\pm\sqrt{256}}{6}$

$\Rightarrow \boxed{y=-4,\frac{4}{3}}$

Put $y=-4$ in (iii)

$x^2=0 \Rightarrow x=0$

Put $y=\frac{4}{3}$ in (iii)

$x^2=\frac{32}{9}$

$\Rightarrow x=\pm\frac{4\sqrt{2}}{3}$

Hence the points of intersection are

$(0,-4)$ and $\left(\frac{\pm 4\sqrt{2}}{3},\frac{4}{3}\right)$