## EXERCISE 6.8

**Q.1 Find an equation of each of the following with respect to new parallel axes obtained by shifting the origin to the indicated point.**

**(i)** $x^2+16y-16=0, \quad O'(0,1)$

**Solution:**

Equations of transformation are

$x=X+h$ and $y=Y+k$

Here $h=0, k=1$, so $x=X$ and $y=Y+1$

Put these values in given equation, we get

$X^2+16(Y+1)-16=0$

$X^2+16Y+16-16=0$

$\boxed{X^2+16y=0}$

**(ii)** $4x^2+y^2+16x-10y+37=0, \quad O'(-2,5)$

**Solution:**

Equations of transformation are

$x=X+h$ and $y=Y+k$

Here $h=-2, k=5$, so $x=X-2$ and $y=Y+5$

Put these values in given equation, we get;

$4(X-2)^2+(Y+5)^2+16(X-2)-10(Y+5)+37=0$

$4(X^2+4-4X)+(Y^2+25+10Y)+16X-32-10Y-50+37=0$

$4X^2+16-16X+Y^2+25+10Y+16X-32-10Y-50+37=0$

$\boxed{4X^2+Y^2-4=0}$

**(iii)** $9x^2+4y^2+18x-16y-11=0, \ O'(-1,2)$

**Solution:**

Equations of transformation are

$x=X+h, y=Y+k$

Here $h=-1, k=2$, so $x=X-1, y=Y+2$

Put these values in given equation we get;

$9(X-1)^2+4(Y+2)^2+18(X-1)-16(Y+2)-11=0$

$9(X^2+1-2X)+4(Y^2+4+4Y)+18(X-1)-16(Y+2)-11=0$

$9X^2+9-18X+4Y^2+16+16Y+18X-18-16Y-32-11=0$

$\boxed{9X^2+4Y^2-36=0}$

**(iv)** $x^2-y^2+4x+8y-11=0, \ O'(-2,4)$

**Solution:**

Equations of transformation are

$x=X+h$ and $y=Y+k$

Here $h=-2, k=4$, so $x=X-2$ and $y=Y+4$

Put these values in given equation , we get;

$$(X-2)^2-(Y+4)^2+4(X-2)+8(Y+4)-11=0$$

$$X^2+4-4X-(Y^2+16+8Y)+4(X-2)+8(Y+4)-11=0$$

$$X^2+4-4X-Y^2-16-8Y+4X-8+8Y+32-11=0$$

$$\boxed{X^2-Y^2+1=0}$$

**(v)** $9x^2-4y^2+36x+8y-4=0$; $O'(-2,1)$

**Solution:**

Equations of transformation are

$x=X+h$ and $h=Y+k$

Here $h=-2, k=1$, so $x=X-2$ and $y=Y+1$,

Put these values in given equation, we get;

$$9(X-2)^2-4(Y+1)^2+36(X-2)+8(Y+1)-4=0$$

$$9(X^2+4-4X)-4(Y^2+2Y+1)+36(X-2)+8(Y+1)-4=0$$

$$9X^2+36-36X-4Y^2-4-8Y+36X-72+8Y+8-4=0$$

$$\boxed{9X^2-4Y^2-36=0}$$

**Q.2 Find coordinates of the new origin (axes remaining parallel) so that first degree terms are removed from the transformed equation of each of the following. Also find the transformed equation.**

**(i)** $3x^2-2y^2+24x+12y+24=0$

**Solution:**

$3x^2-2y^2+24x+12y+24=0$ …(i)

Let the coordinates of the new origin be $(h,k)$, so equations of transformation are

$x=X+h, y=Y+k$

Substituting these values of '$x$' and '$y$' in given equation we get;

$$3(X+h)^2-2(Y+k)^2+24(X+h)+12(Y+k)+24=0$$

$$3(X^2+h^2+2Xh)-2(Y^2+k^2+2Yk)+24X+24h+12Y+12k+24=0$$

$$3X^2+3h^2+6Xh-2Y^2-2k^2-4kY+24X+24h+12Y+12k+24=0$$

$$(3X^2-Y^2)+X(6h+24)+Y(-4k+12)+24h+12k+3h^2-2k^2+24=0 \quad \text{…(ii)}$$

In order to remove first degree terms, we put coefficients of $X$ and $Y$ equal to zero

$6h+24=0 \Rightarrow h=-4$

$-4k+12=0 \Rightarrow k=3$

Hence new origin is $O(h,k)=O'(-4,3)$

Now put value of $h$ and $k$ in (ii) we get;

$$3X^2-2Y^2+X(6(-4)+24)+Y(-4(3)+12)+24(-4)+12(3)+3(-4)^2-2(3)^2+24=0$$

$$3X^2-2Y^2-96+36+48-18+24=0$$

$$\boxed{3X^2-2Y^2-6=0}$$

**(ii)** $25x^2+9y^2+50x-36y-164=0$

**Solution:**

$25x^2+9y^2+50x-36y-164=0$ ...(i)

Let the coordinates of the new origin be $(h,k)$, so equations of transformation are

$x=X+h, y=Y+k$

Substituting these values of '$x$' and '$y$' in given equation we get;

$25(X+h)^2+9(Y+k)^2+50(X+h)-36(Y+k)-164=0$

$25(X^2+h^2+2hX)+9(Y^2+k^2+2kY)+50(X+h)-36(Y+k)-164=0$

$25X^2+9Y^2+50hX+50X+18kY-36Y+50h-36k+25h^2+9k^2-164=0$

$25X^2+9Y^2+X(50h+50)+Y(18k-36)+50h-36k+25h^2+9k^2-164=0$ ... (ii)

In order to remove first degree terms, we put coefficients of $X$ and $Y$ equal to zero

$50h+50=0 \Rightarrow h=-1$

$18k-36=0 \Rightarrow k=2$

Hence new origin is $O'(h,k)=O'(-1,2)$

Now put values of '$h$' and '$k$' in (ii) we get;

$$25X^2+9Y^2+X(50(-1)+50)+Y(18(2)-36)+50(-1)-36(2)+25(-1)^2+9(2)^2-164=0$$

$\Rightarrow 25X^2+9Y^2+X(0)+Y(0)-50-72+25+36-164=0$

$\Rightarrow \boxed{25X^2+9Y^2-225=0}$

**(iii)** $x^2-y^2-6x+2y+7=0$

**Solution:**

$x^2-y^2-6x+2y+7=0$ ...(i)

Let the coordinates of the new origin be $(h,k)$, so equations of transformation are

$x=X+h, y=Y+k$

Substituting these values of '$x$' and '$y$' in given equation we get;

$(X+h)^2-(Y+k)^2-6(X+h)+2(Y+k)+7=0$

$X^2+h^2+2Xh-Y^2-k^2-2kY-6X-6h+2Y+2k+7=0$

$X^2-Y^2+2Xh-6X-2kY+2Y-6h+2k+h^2-k^2+7=0$

$X^2-Y^2+X(2h-6)+Y(-2k+2)-6h+2k+h^2-k^2+7=0$ ...(ii)

In order to remove first degree terms, we put coefficients of $x$ and $y$ equal to zero,

$2h-6=0 \Rightarrow h=3$

$-2k+2=0 \Rightarrow k=1$

Hence new origin is $O'(h,k)=O'(3,1)$

Now put values of $h$ and $k$ in (ii) we get;

$X^2-Y^2+X(2(3)-6)+Y(-2(1)+2)-6(3)+2(1)+(3)^2-(1)^2+7=0$

$X^2-Y^2+X(0)+Y(0)-18+2+9-1+7=0$

$\boxed{X^2-Y^2-1=0}$

**Q.3 In each of the following, find an equation referred to the new axes obtained by rotation of axes about the origin through the given angle:**

**(i)** $xy=1,\theta=45^\circ$

**Solution:**

As we know that equations of transformations are

$$x=X\cos\theta-Y\sin\theta$$
$$y=X\sin\theta+Y\cos\theta$$

Put $\theta=45^\circ$, we get

$$\begin{cases} x=X\cos 45^\circ-Y\sin 45^\circ \\ \text{and} \\ y=X\sin 45^\circ+Y\cos 45^\circ \end{cases}$$

$$\Rightarrow \begin{cases} x=\dfrac{X-Y}{\sqrt{2}} \\ \text{and} \\ y=\dfrac{X+Y}{\sqrt{2}} \end{cases}$$

Putting in given equation, we get

$$\left(\frac{X-Y}{\sqrt{2}}\right)\left(\frac{X+Y}{\sqrt{2}}\right)=1$$

$$\frac{X^2-Y^2}{2}=1$$

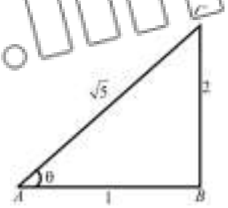
$\boxed{X^2-Y^2=2}$ is the required equation

**(ii)** $7x^2-8xy+y^2-9=0,\theta=\arctan 2$

**Solution:**

Here $\theta=\tan^{-1}2\Rightarrow\tan\theta=2$

From right angle $\Delta ABC$




$$\sin\theta=\frac{2}{\sqrt{5}},\cos\theta=\frac{1}{\sqrt{5}}$$

Using equations of transformations

$$x=X\cos\theta-Y\sin\theta$$

Put values of $\sin\theta$ and $\cos\theta$, we get

$$x=\frac{X}{\sqrt{5}}-\frac{2Y}{\sqrt{5}}$$

$$x=\frac{X-2Y}{\sqrt{5}}$$

As $y=X\sin\theta+Y\cos\theta$

Put values of $\sin\theta$ and $\cos\theta$, we get

$$y=\frac{2X}{\sqrt{5}}+\frac{Y}{\sqrt{5}}$$

$$y=\frac{2X+Y}{\sqrt{5}}$$

Substituting the values of '$x$' and '$y$' in given equation, we get

$$7\left(\frac{X-2Y}{\sqrt{5}}\right)^2-8\left(\frac{X-2Y}{\sqrt{5}}\right)\left(\frac{2X+Y}{\sqrt{5}}\right)+\left(\frac{2X+Y}{\sqrt{5}}\right)^2-9=0$$

$$7\frac{\left(X^2+4Y^2-4XY\right)}{5}-8\frac{\left(2X^2-3XY-2Y^2\right)}{5}+\frac{4X^2+Y^2+4XY}{5}-9=0$$

(Multiply by 5)

$$7\left(X^2+4Y^2-4XY\right)-8\left(2X^2-3XY-2Y^2\right)+4X^2+Y^2+4XY-45=0$$

$$\Rightarrow\left(7-16+4\right)X^2+\left(28+16+1\right)Y^2+\left(-28+24+4\right)XY-45=0$$

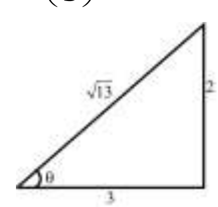
$$\Rightarrow -5X^2+45Y^2-45=0$$

$$\Rightarrow \boxed{X^2-9Y^2+9=0}$$

**(iii)** $9x^2+12xy+4y^2-x-y=0,\theta=\tan^{-1}\left(\frac{2}{3}\right)$

**Solution:**

Here $\theta=\tan^{-1}\left(\frac{2}{3}\right)\Rightarrow\tan\theta=\frac{2}{3}$



From right triangle

$$\sin\theta = \frac{2}{\sqrt{13}}, \cos\theta = \frac{3}{\sqrt{13}}$$

Using equations of transformations

$$x = X\cos\theta - Y\sin\theta$$
$$y = X\sin\theta + Y\cos\theta$$

Using values of $\sin\theta$ and $\cos\theta$, we get

$$\begin{cases} x = \dfrac{3X-2Y}{\sqrt{13}} \\ \text{and} \\ y = \dfrac{2X+3Y}{\sqrt{13}} \end{cases}$$

Substituting these values of '$x$' and '$y$' in given equation, we get

$$9\left(\frac{3X-2Y}{\sqrt{13}}\right)^2 + 12\left(\frac{3X-2Y}{\sqrt{13}}\right)\left(\frac{2X+3Y}{13}\right) + 4\left(\frac{2X+3Y}{\sqrt{13}}\right)^2 - \left(\frac{3X-2Y}{\sqrt{13}}\right) - \left(\frac{2X+3Y}{\sqrt{13}}\right) = 0$$

$$9\left(\frac{9X^2+4Y^2-12XY}{13}\right) + 12\left(\frac{6X^2+5XY-6Y^2}{13}\right) + 4\left(\frac{4X^2+9Y^2+12XY}{13}\right)$$

$$-\frac{3X}{\sqrt{13}} + \frac{2Y}{\sqrt{13}} - \frac{2X}{\sqrt{13}} - \frac{3Y}{\sqrt{13}} = 0$$

Multiply equation by 13, we get

$$\Rightarrow (81+72+16)X^2 + (36-72+36)Y^2 + (-108+60+48)XY + (-3-2)\sqrt{13}X + (2-3)\sqrt{13}Y = 0$$

$$\Rightarrow 169X^2 - 5\sqrt{13}X - \sqrt{13}Y = 0$$

Dividing by $\sqrt{13}$ we get

$$\boxed{13\sqrt{13}X^2 - 5X - Y = 0}$$

**(iv)** $x^2 - 2xy + y^2 - 2\sqrt{2}x - 2\sqrt{2}y + 2 = 0, \theta = 45^\circ$

**Solution:**

Using equations of transformations

$$x = X\cos\theta - Y\sin\theta$$
$$y = X\sin\theta + Y\cos\theta$$

Put $\theta = 45^\circ$, we get

$$\begin{cases} x = X\cos 45^\circ - Y\sin 45^\circ \\ \text{and} \\ y = X\sin 45^\circ + Y\cos 45^\circ \end{cases}$$

$$\Rightarrow \begin{cases} x = \dfrac{X-Y}{\sqrt{2}} \\ \text{and} \\ y = \dfrac{X+Y}{\sqrt{2}} \end{cases}$$

Substituting these values of '$x$' and '$y$' in given equation, we get

$$\left(\frac{X-Y}{\sqrt{2}}\right)^2 - 2\left(\frac{X-Y}{\sqrt{2}}\right)\left(\frac{X+Y}{\sqrt{2}}\right) + \left(\frac{X+Y}{\sqrt{2}}\right)^2 - 2\sqrt{2}\left(\frac{X-Y}{\sqrt{2}}\right) - 2\sqrt{2}\left(\frac{X+Y}{\sqrt{2}}\right) + 2 = 0$$

$$\left(\frac{X-Y}{2}\right)^2 - \frac{2(X^2-Y^2)}{2} + \frac{(X+Y)^2}{2} - 2(X-Y)$$

$$-2(X+Y) + 2 = 0$$

Multiply the equation by 2 we get;

$$(X-Y)^2 - 2(X^2-Y^2) + (X+Y)^2 - 4(X-Y) - 4(X+Y) + 4 = 0$$

$$\Rightarrow (X+Y)^2 + (X-Y)^2 - 2(X^2-Y^2) - 4X + 4Y - 4X - 4Y + 4 = 0$$

$$\Rightarrow 2(X^2+Y^2) - 2(X^2-Y^2) - 8X + 4 = 0$$

$$2X^2 + 2Y^2 - 2X^2 + 2Y^2 - 8X + 4 = 0$$

$$4Y^2 - 8X + 4 = 0$$

$$\boxed{Y^2 - 2X + 1 = 0}$$

**Q.4 Find measure of the angle through which the axes be rotated so that the product term XY is removed from the transformed equation. Also find the transformed equation.**

**(i)** $2x^2+6xy+10y^2-11=0$

**Solution:**

$2x^2+6xy+10y^2-11=0 \ldots$(i)

Comparing (i) with

$ax^2+2hxy+by^2+2gx+2fy+c=0$,

we get

$a=2, b=10, 2h=6$

As we know that angle of rotation is given by

$$\tan 2\theta=\frac{2h}{a-b}$$

$$\tan 2\theta=\frac{6}{2-10}$$

$$\tan 2\theta=\frac{6}{-8}$$

$$\tan 2\theta=-\frac{3}{4}$$

$$\frac{2\tan\theta}{1-\tan^2\theta}=-\frac{3}{4}$$

$$8\tan\theta=-3+3\tan^2\theta$$

$$3\tan^2\theta-8\tan\theta-3=0$$

$$3\tan^2\theta-9\tan\theta+\tan\theta-3=0$$

$$3\tan\theta(\tan\theta-3)+1(\tan\theta-3)=0$$

$$(\tan\theta-3)(3\tan\theta+1)=0$$

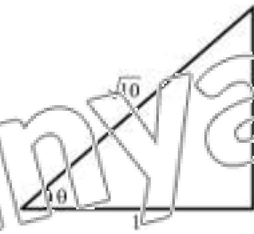
$$\tan\theta-3=0, 3\tan\theta+1=0$$

$$\tan\theta=3, \tan\theta=-\frac{1}{3}$$

$\therefore 0^\circ<\theta<90^\circ$, so $\theta$ lies in 1st quadrant and in 1st quadrant $\tan\theta$ is positive, hence

$\tan\theta=-\frac{1}{3}$ is neglected

From $\tan\theta=3$ we get;




$$\sin\theta=\frac{3}{\sqrt{10}}, \cos\theta=\frac{1}{\sqrt{10}}$$

Using equations of transformations

$$x=X\cos\theta-Y\sin\theta$$

$$y=X\sin\theta+Y\cos\theta$$

Using values of $\sin\theta$ and $\cos\theta$, we get

$$\begin{cases} x=\dfrac{X-3Y}{\sqrt{10}} \\ \text{and} \\ y=\dfrac{3X+Y}{\sqrt{10}} \end{cases}$$

Substituting values of '$x$' and '$y$' in (1), we get

$$2\left(\frac{X-3Y}{\sqrt{10}}\right)^2+6\left(\frac{X-3Y}{\sqrt{10}}\right)\left(\frac{3X+Y}{\sqrt{10}}\right)+10\left(\frac{3X+Y}{\sqrt{10}}\right)^2-11=0$$

$$2\left(\frac{X^2+9Y^2-6XY}{10}\right)+\frac{6\left(3X^2-8XY-3Y^2\right)}{10}+\frac{10\left(9X^2+Y^2+6XY\right)}{10}-11=0$$

Multiply the equation by 10, we get

$$2\left(X^2+9Y^2-6XY\right)+6\left(3X^2-8XY-3Y^2\right)+10\left(9X^2+Y^2+6XY\right)-110=0$$

$$\Rightarrow(2+18+90)X^2+(18-18+10)Y^2+(-12-48+60)XY-110=0$$

$$\Rightarrow 110X^2+10Y^2-110=0$$

Dividing by 10 on both sides

$$\boxed{11X^2+Y^2-11=0}$$

**(ii)** $xy + 4x - 3y - 10 = 0$

**Solution:**

$xy + 4x - 3y - 10 = 0 \ldots$(i)

Comparing (i) with

$ax^2 + by^2 + 2gx + 2fy + 2hxy + c = 0$,

we get

$a = 0, b = 0, 2h = 1$

As we know that angle of rotation is given by

$$\tan 2\theta = \frac{2h}{a-b}$$

$$\tan 2\theta = \frac{1}{0-0}$$

$$\tan 2\theta = \infty$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

Using equations of transformations

$$x = X\cos\theta - Y\sin\theta$$

$$y = X\sin\theta + Y\cos\theta$$

Put $\theta = 45^\circ$, we get

$$\begin{cases} x = X\cos 45^\circ - Y\sin 45^\circ \\ \text{and} \\ y = X\sin 45^\circ + Y\cos 45^\circ \end{cases}$$

$$\Rightarrow \begin{cases} x = \dfrac{X-Y}{\sqrt{2}} \\ \text{and} \\ y = \dfrac{X+Y}{\sqrt{2}} \end{cases}$$

Putting values of $x$ and $y$ in (1), we get

$$\left(\frac{X-Y}{\sqrt{2}}\right)\left(\frac{X+Y}{\sqrt{2}}\right) + 4\left(\frac{X-Y}{\sqrt{2}}\right) - 3\left(\frac{X+Y}{\sqrt{2}}\right) - 10 = 0$$

$$\frac{X^2 - Y^2}{2} + \frac{4(X-Y)}{\sqrt{2}} - \frac{3(X+Y)}{\sqrt{2}} - 10 = 0$$

Multiply the equation with 2 we get;

$$X^2 - Y^2 + 4\sqrt{2}(X-Y) - 3\sqrt{2}(X+Y) - 20 = 0$$

$$X^2 - Y^2 + 4\sqrt{2}X - 4\sqrt{2}Y - 3\sqrt{2}X - 3\sqrt{2}Y - 20 = 0$$

$$\boxed{X^2 - Y^2 + \sqrt{2}X - 7\sqrt{2}Y - 20 = 0}$$

**(iii)** $5x^2 - 6xy + 5y^2 - 8 = 0$

**Solution:**

$5x^2 - 6xy + 5y^2 - 8 = 0 \ldots$(i)

Comparing (i) with

$ax^2 + by^2 + 2gx + 2fy + 2hxy + c = 0$,

we get

$a = 5, b = 5, 2h = -6$

As we know that angle of rotation is given by

$$\tan 2\theta = \frac{2h}{a-b}$$

$$\tan 2\theta = \frac{-6}{5-5}$$

$$\tan 2\theta = \frac{-6}{0} \Rightarrow \tan 2\theta = \infty$$

$$\Rightarrow 2\theta = 90^\circ \Rightarrow \theta = 45^\circ$$

Using equations of transformations

$$x = X\cos\theta - Y\sin\theta$$

$$y = X\sin\theta + Y\cos\theta$$

Put $\theta = 45^\circ$, we get

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$$\begin{cases} x = X\cos 45^\circ - Y\sin 45^\circ \\ \text{and} \\ y = X\sin 45^\circ + Y\cos 45^\circ \end{cases}$$

$$\Rightarrow \begin{cases} x = \dfrac{X-Y}{\sqrt{2}} \\ \text{and} \\ y = \dfrac{X+Y}{\sqrt{2}} \end{cases}$$

Putting values of $x$ and $y$ in (i),

we get

$$5\left(\frac{X-Y}{\sqrt{2}}\right)^2 - 6\left(\frac{X-Y}{\sqrt{2}}\right)\left(\frac{X+Y}{\sqrt{2}}\right) + 5\left(\frac{X+Y}{\sqrt{2}}\right)^2 - 8 = 0$$

$$\frac{5(X-Y)^2}{2} - \frac{6(X^2-Y^2)}{2} + \frac{5(X+Y)^2}{2} - 8 = 0$$

Multiply the equation by 2

$$5(X-Y)^2 - 6(X^2-Y^2) + 5(X+Y)^2 - 16 = 0$$

$$5(X+Y)^2 + 5(X-Y)^2 - 6(X^2-Y^2) - 16 = 0$$

$$5\left[2(X^2+Y^2)\right] - 6(X^2-Y^2) - 16 = 0$$

$$10X^2 + 10Y^2 - 6X^2 + 6Y^2 - 16 = 0$$

$$4X^2 + 16Y^2 - 16 = 0$$

$$\boxed{X^2 + 4Y^2 - 4 = 0}$$