UNIT 7

VECTORS

<u>Scalar Quantity</u>:

All those quantities which requires only magnitude for complete description are called **scalar quantities** e.g. time, density, temperature and length etc.

<u>Vector Quantity</u>:

All those quantities which requires magnitude as well as direction for their complete description are called **vector quantities** e.g. weight, force, momentum, displacement, velocity etc.

<u>Geometric Interpretation of Vector</u>:

Geometrically, a vector is represented by a directed line segment $\overrightarrow{AB}$ with A its initial point and B its terminal point.

<u>Magnitude of a Vector</u>:

The magnitude or length or norm of vector $\overrightarrow{AB}$ is its absolute value and is written as $\left|\overrightarrow{AB}\right|$.

<u>Unit Vector</u>:

A unit vector $\hat{\underline{v}}$ (read as $\underline{v}$ hat) of a given vector $\underline{v}$ is a vector with magnitude one and direction same as vector $\underline{v}$. Mathematically

$$\text{Unit Vector} = \frac{\text{Vector}}{\text{Magnitude of Vector}}$$

i.e. $\hat{\underline{v}} = \dfrac{\underline{v}}{|\underline{v}|}$

<u>Null Vector</u>:

A vector whose terminal point coincides with its initial point is called **null or zero vector.**

<u>Negative of a Vector</u>:

Two vectors $\underline{u}$ and $\underline{v}$ are called negative of each other, if they have same magnitude but opposite direction.

<u>Multiplication of a Vector by a scalar (Number)</u>:

Multiplication of a vector $\underline{v}$ by a scalar 'n' is a vector whose magnitude is n times that of '$\underline{v}$' i.e. $n\underline{v}$.

(i) If n is positive, then $\underline{v}$ and $n\underline{v}$ are in the same direction.

(ii) If n is negative, then $\underline{v}$ and $n\underline{v}$ are in opposite directions.

Equal Vectors:

Two vectors $\underline{u}$ and $\underline{v}$ are called equal vectors, if they have same magnitude and same direction.

Parallel Vectors:

Two vectors $\underline{u}$ and $\underline{v}$ are parallel if and only if they are non-zero scalar multiple of each other i.e.

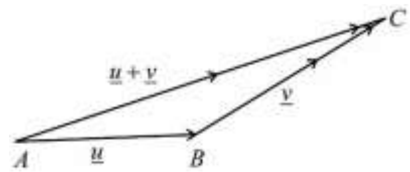
$$\underline{u} = \lambda\underline{v},\ \lambda \neq 0$$

Triangle Law of Addition:

If two vectors $\underline{u}$ and $\underline{v}$ are represented by the two sides AB and BC of a triangle such that the terminal point of $\underline{u}$ coincide with the initial point of $\underline{v}$, then the third side AC of the triangle gives vector sum $\underline{u}+\underline{v}$, that is

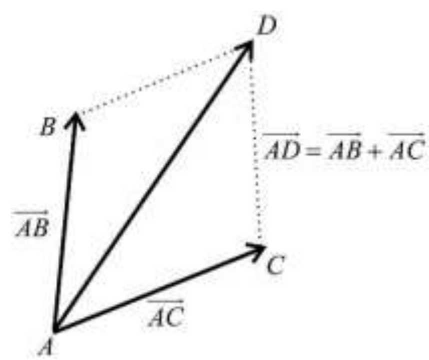
$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$$

$$\Rightarrow \quad \underline{u}+\underline{v}=\overrightarrow{AC}$$



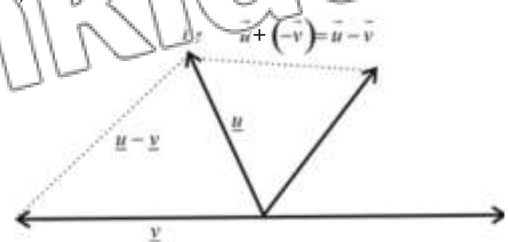
Parallelogram Law of Addition:

If two vector $\underline{u}$ and $\underline{v}$ are represented by two adjacent sides ***AB*** and ***AC*** of a parallelogram as shown in the figure, then diagonal ***AD*** give the sum or resultant of $\overrightarrow{AB}$ and $\overrightarrow{AC}$, that is $\overrightarrow{AD}=\overrightarrow{AB}+\overrightarrow{AC}=\underline{u}+\underline{v}$



Subtraction of two vectors:

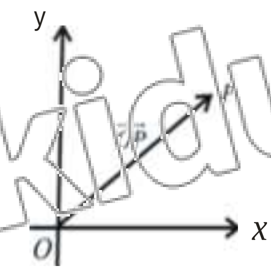
Let $\underline{u}$ and $\underline{v}$ are non-zero vector then subtraction of $\underline{v}$ and $\underline{u}$ is defined as the addition of $\underline{u}$ and $-\underline{v}$ i.e. $\vec{u}+\left(-\vec{v}\right)=\vec{u}-\vec{v}$



Position Vector:

A vector which describes the location of a point w.r.t origin is called position vector.

The vector, whose initial point is the origin **O** and whose terminal point **P** is called the position vector of the point **P** and is written as $\overrightarrow{OP}$



Vector in a Plane:

Let R be a set of real numbers. The Cartesian plane is defined to be the $R^2 = \{(x, y) : x, y \in R\}$.

The Unit Vectors $\underline{i}$, $\underline{j}$:

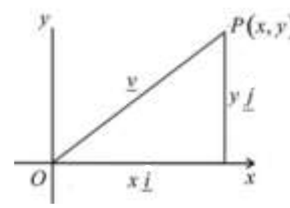
$\underline{i}$ and $\underline{j}$ are called unit vectors along x-axis and y-axis respectively.

They are written as

$$\underline{i} = [1,0],\ \underline{j} = [0,1]$$

$$|\ \underline{i}\ | = \sqrt{(1)^2 + (0)^2} = 1$$

$$|\ \underline{j}\ | = \sqrt{(0)^2 + (1)^2} = 1$$



A vector $\vec{v}$ can be written as

$$\underline{v} = [x, y] = [x,0] + [0, y] = x[1,0] + y[0,1] = x\underline{i} + y\underline{j}$$

Similarly, sum of two vectors $\underline{u}$ and $\underline{v}$ can be written as

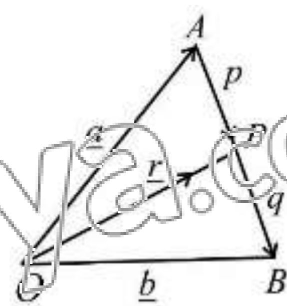
$$\underline{u} = [x_1, y_1],\quad \underline{v} = [x_2, y_2]$$

$$\underline{u} + \underline{v} = [x_1 + x_2\ , y_1 + y_2] = (x_1 + x_2)\underline{i} + (y_1 + y_2)\underline{j}$$

The Ratio Formula:

Let A and B are two points whose position vectors are $\underline{a}$ and $\underline{b}$ respectively. If a point P divides AB in the ratio $p:q$, then the position vector of P is given by

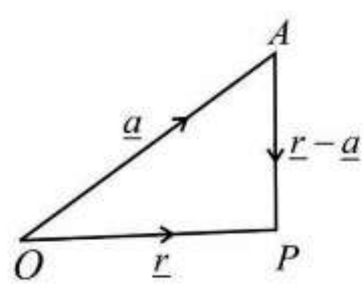
$$\underline{r} = \frac{q\underline{a} + p\underline{b}}{q + p}$$



Proof:

Given $\underline{a}$ and $\underline{b}$ are position vectors of the points A and B respectively. Let $\underline{r}$ be the position vector of the point P which divides the line segment $\overrightarrow{AB}$ in the ratio $p:q$. That is

$$m\overline{AP} : m\overline{PB} = p:q$$

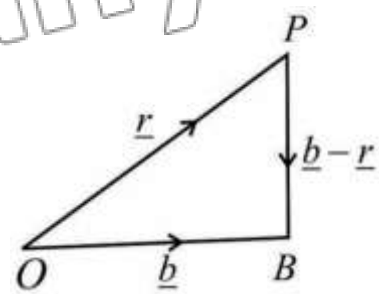


so

$$\frac{m\overline{AP}}{m\overline{PB}} = \frac{p}{q}$$

$\Rightarrow \quad q\left(m\overline{AP}\right) = p\left(m\overline{PB}\right)$

Thus $\quad q\left(\overline{AP}\right) = p\left(\overline{PB}\right)$



$\Rightarrow \quad q(\underline{r} - \underline{a}) = p(\underline{b} - \underline{r})$

$\Rightarrow \quad q\underline{r} - q\underline{a} = p\underline{b} - p\underline{r}$

$\Rightarrow \quad p\underline{r} + q\underline{r} = p\underline{b} + q\underline{a}$

$\Rightarrow \quad \underline{r}(p+q) = q\underline{a} + p\underline{b}$

$\Rightarrow \quad \underline{r} = \dfrac{q\underline{a} + p\underline{b}}{q+p}$

Corollary:

If P is the midpoint of AB, then $p:q = 1:1$

$\because$ Position vector of $P = \underline{r} = \dfrac{\underline{a}+\underline{b}}{2}$

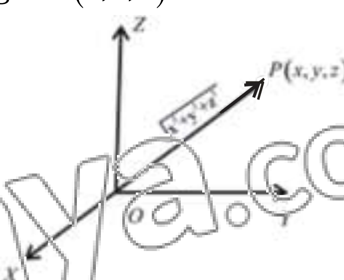
Vectors in Space:

The set $R^3 = \left\{(x,y,z) : x, y.z \in R\right\}$ is called the 3-dimensional space.

(i) Position Vector

The position vector of a point $P(x,y,z)$ in space, from the origin $O(0,0,0)$ is

$\overrightarrow{OP} = x\underline{i} + y\underline{j} + z\underline{k}$



The magnitude of $\overrightarrow{OP}$ is the distance of point P from the origin, i.e.

$\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2 + z^2}$

(ii) The unit vectors $\underline{i}, \underline{j}, \underline{k}$

$\underline{i}, \underline{j}$ and $\underline{k}$ are called unit vectors along X, Y, Z axes respectively. They are written as:

$\underline{i} = [1,0,0] \quad \underline{j} = [0,1,0] \quad \underline{k} = [0,0,1]$

$|\,\underline{i}\,| = \sqrt{(1)^2 + (0)^2 + (0)^2} = 1$

$|\,\underline{j}\,| = \sqrt{(0)^2 + (1)^2 + (0)^2} = 1$

$|\,\underline{k}\,| = \sqrt{(0)^2 + (0)^2 + (1)^2} = 1$

A vector $\overline{v}$ can be written as:

$$\underline{v} = [x, y, z] = [x, 0, 0] + [0, y, 0] + [0, 0, z]$$
$$= x[1, 0, 0] + y[0, 1, 0] + z[0, 0, 1]$$
$$= x\underline{i} + y\underline{j} + z\underline{k}$$

Similarly, sum of two vectors $\underline{u}$ and $\underline{v}$ can be written as:

$$\underline{u} = [x_1, y_1, z_1],\ \underline{v} = [x_2, y_2, z_2]$$
$$\underline{u} + \underline{v} = [x_1 + x_2, y_1 + y_2, z_1 + z_2]$$
$$= (x_1 + x_2)\underline{i} + (y_1 + y_2)\underline{j} + (z_1 + z_2)\underline{k}$$

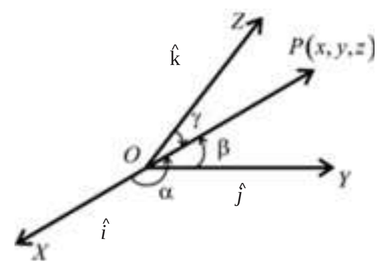
(iii) Distance between two points in space:

The distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ in space is given by:

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(iv) Direction Angles and Direction Cosines of a vector:

Let $\underline{r} = \overrightarrow{OP} = x\underline{i} + y\underline{j} + z\underline{k}$ be a non-zero vector, let α, β and γ denote the angles formed between $\underline{r}$ and the unit coordinate vector $\underline{i}, \underline{j}$ and $\underline{k}$ respectively, such that $0 \le \alpha \le \pi$, $0 \le \beta \le \pi$ and $0 \le \gamma \le \pi$.



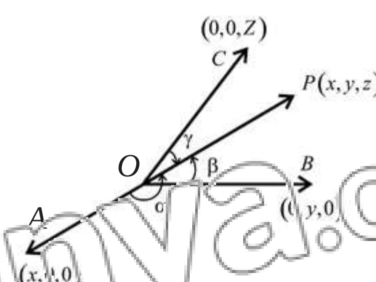
(i) The angles α, β, γ are called the direction angles.

(ii) The numbers $\cos\alpha, \cos\beta$ and $\cos\gamma$ are called direction cosines.

Important Result:

Prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$



Proof:

Let $\underline{r} = [x, y, z] = x\underline{i} + y\underline{j} + z\underline{k}$

$$\therefore\ |\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

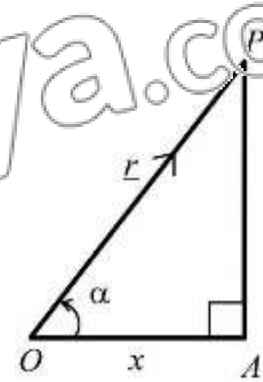
Then $\frac{\underline{r}}{|\underline{r}|} = \left[\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right]$ is the unit vector in the direction of the vector $\underline{r} = \overrightarrow{OP}$. It can be visualized that the triangle OAP is a right triangle with $\angle A = 90^o$. Therefore in right triangle OAP,

$$\cos\alpha = \frac{\overline{OA}}{\overline{OP}} = \frac{x}{r},$$

Similarly

$\cos\beta = \frac{y}{r}, \ \cos\gamma = \frac{z}{r}$

The numbers $\cos\alpha = \frac{x}{r}$, $\cos\beta = \frac{y}{r}$ and $\cos\gamma = \frac{z}{r}$ are called the **direction cosines** of $\overrightarrow{OP}$.



$\therefore \ \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2}$

$= \frac{x^2 + y^2 + z^2}{r^2} = \frac{r^2}{r^2} = 1$

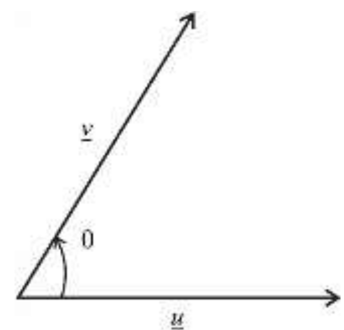
<u>The Scalar Product of two Vectors</u>:

Definition: 1

The scalar or dot product of two vectors $\underline{u}$ and $\underline{v}$ in a plane or in space is

$\underline{u}.\underline{v} = |\underline{u}|\,|\underline{v}|\cos\theta$

where θ is the angle between $\underline{u}$ and $\underline{v}$ and $0 \le \theta \le \pi$.



<u>The unit vectors</u> $\underline{i}, \underline{j}, \underline{k}$:

(a) $\underline{i}\,.\,\underline{i} = |\underline{i}|\,|\underline{i}|\cos 0^o = (1)(1)(1) = 1$ as $\cos 0 = 1$

$\underline{j}\,.\,\underline{j} = |\underline{j}|\,|\underline{j}|\cos 0^o = (1)(1)(1) = 1$

$\underline{k}\,.\,\underline{k} = |\underline{k}|\,|\underline{k}|\cos 0^o = (1)(1)(1) = 1$

(b) $\underline{i}\,.\,\underline{j} = |\underline{i}|\,|\underline{j}|\cos 90^o = (1)(1)(0) = 0$ as $\cos 90^o = 0$

$\underline{j}\,.\,\underline{k} = |\underline{j}|\,|\underline{k}|\cos 90^o = (1)(1)(0) = 0$

$\underline{k}\,.\,\underline{i} = |\underline{k}|\,|\underline{i}|\cos 90^o = (1)(1)(0) = 0$

(c) $\underline{u}.\underline{v} = \underline{v}.\underline{u}$ (Dot product of two vectors is commutative)

Definition: 2

(a) If $\underline{u} = a_1\underline{i} + b_1\underline{j}$ *and* $\underline{v} = a_2\underline{i} + b_2\underline{j}$ are two vectors in a plane, then the dot product of $\underline{u}$ and $\underline{v}$ is

$\underline{u}.\underline{v} = (a_1\underline{i} + b_1\underline{j}).(a_2\underline{i} + b_2\underline{j})$

$= a_1a_2(\underline{i}.\underline{i}) + a_1b_2(\underline{i}.\underline{j}) + a_2b_1(\underline{j}.\underline{i}) + (b_1b_2)(\underline{j}.\underline{j})$

$= a_1a_2(1) + a_1b_2(0) + a_2b_1(0) + b_1b_2(1)$

$\underline{u}.\underline{v} = a_1a_2 + b_1b_2$

(b) If $\underline{u} = a_1\underline{i} + b_1\underline{j} + c_1\underline{k}$ and $\underline{v} = a_2\underline{i} + b_2\underline{j} + c_2\underline{k}$ are two vectors in space, then the dot product of $\underline{u}$ and $\underline{v}$ is

$\underline{u}.\underline{v} = \left(a_1\underline{i} + b_1\underline{j} + c_1\underline{k}\right).\left(a_2\underline{i} + b_2\underline{j} + c_2\underline{k}\right)$

$= a_1a_2\left(\underline{i}.\underline{i}\right) + a_1b_2\left(\underline{i}.\underline{j}\right) + a_1c_2\left(\underline{i}.\underline{k}\right) + a_2b_1\left(\underline{j}.\underline{i}\right)$

$+ b_1b_2\left(\underline{j}.\underline{j}\right) + b_1c_2\left(\underline{j}.\underline{k}\right) + a_2c_1\left(\underline{k}.\underline{i}\right) + b_2c_1\left(\underline{k}.\underline{j}\right) + c_1c_2\left(\underline{k}.\underline{k}\right)$

$= a_1a_2(1) + a_1b_2(0) + a_1c_2(0) + a_2b_1(0) + b_1b_2(1)$

$+ b_1c_2(0) + a_2c_1(0) + b_2c_1(0) + c_1c_2(1)$

$\underline{u}.\underline{v} = a_1a_2 + b_1b_2 + c_1c_2$

Perpendicular (Orthogonal) Vectors:

If two vectors $\underline{u}$ and $\underline{v}$ are perpendicular, then

$\underline{u}.\underline{v} = |\underline{u}|\,|\underline{v}|\cos 90^o = |\underline{u}|\,|\underline{v}|(0)$

$\underline{u}.\underline{v} = 0$

Angle between two vectors:

The angles between two vectors $\underline{u}$ and $\underline{v}$ is

(a) $\underline{u}.\underline{v} = |\underline{u}|\,|\underline{v}|\cos\theta$

$\therefore \quad \cos\theta = \dfrac{\underline{u}.\underline{v}}{|\underline{u}|\,|\underline{v}|}$

(b) If $\underline{u} = a_1\underline{i} + b_1\underline{j} + c_1\underline{k}$

$\underline{v} = a_1\underline{i} + b_2\underline{j} + c_2\underline{k}$

$\underline{u}.\underline{v} = a_1a_2 + b_1b_2 + c_1c_2$

$|\underline{u}| = \sqrt{a_1^2 + b_1^2 + c_1^2} \qquad |\underline{v}| = \sqrt{a_2^2 + b_2^2 + c_2^2}$

$\because \quad \cos\theta = \dfrac{\underline{u}.\underline{v}}{|\underline{u}|\,|\underline{v}|}$

$\therefore \quad \cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}$

Projection of one vector upon another vector:

Let $\underline{u} = \overrightarrow{OA}$ and $\underline{v} = \overrightarrow{OB}$ and θ be the angle between them. Where $0 \le \theta \le \pi$.

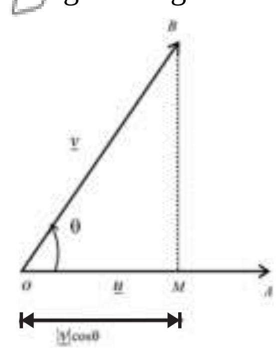
Draw $\overline{BM} \perp \overline{OA}$. Then $\overline{OM}$ is called the projection of $\underline{v}$ along $\underline{u}$. In right triangle OMB

$\cos\theta = \dfrac{\overline{OM}}{\overline{OB}}$

$\Rightarrow \quad \overline{OM} = \overline{OB}\cos\theta$

$\therefore \quad \overline{OM} = |\underline{v}|\cos\theta \qquad$ (i)

Also $\cos\theta = \dfrac{\underline{u}.\underline{v}}{|\underline{u}|\,|\underline{v}|} \qquad$ (ii)



putting (ii) in (i)

$$\overline{OM} = |\underline{v}| \frac{\underline{u}.\underline{v}}{|\underline{u}||\underline{v}|}$$

$$= \frac{\underline{u}.\underline{v}}{|\underline{u}|}$$

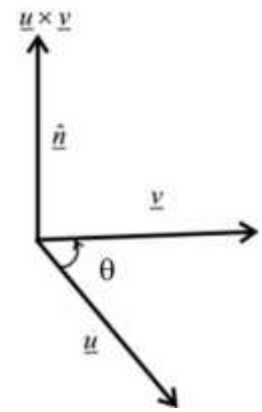
$$= \frac{|\underline{u}||\underline{v}|\cos\theta}{|\underline{u}|} = |\underline{v}|\cos\theta$$

i.e. projection of $\underline{v}$ along $\underline{u} = \frac{\underline{u}.\underline{v}}{|\underline{u}|} = |\underline{v}|\cos\theta$

Similarly.

Projection of $\underline{u}$ along $\underline{v} = \frac{\underline{u}.\underline{v}}{|\underline{v}|} = |\underline{u}|\cos\theta$.

The Cross Product or Vectors Product of Two Vector:



(i) Definition 1:

The vector or cross product of two vectors $\underline{u}$ and $\underline{v}$ in space is

$\underline{u} \times \underline{v} = |\underline{u}||\underline{v}|\sin\theta\, \underline{n}$

where θ is the angle between $\underline{u}$ *and* $\underline{v}$ and $0 \le \theta \le \pi$. $\underline{n}$ is a unit vector perpendicular to the plane of $\underline{u}$ *and* $\underline{v}$ with direction given by the "right hand rule" stated below:

"If the fingers of right hand are curled in a direction from $\underline{u}$ towards $\underline{v}$, then the thumb will point in the direction of $\underline{n}$ which is $\underline{u} \times \underline{v}$. It is important to note that $\underline{u} \times \underline{v} \ne \underline{v} \times \underline{u}$, rather $\underline{u} \times \underline{v} = -(\underline{v} \times \underline{u})$.

(ii) The Unit Vectors $\underline{i}, \underline{j}, \underline{k}$ **:**

(a) $\underline{i} \times \underline{i} = |\underline{i}||\underline{i}|\sin 0^o\, \underline{n} = (1)(1)(0)\underline{n} = 0 \quad \because \sin 0^o = 0$

$\underline{j} \times \underline{j} = |\underline{j}||\underline{j}|\sin 0^o\, \underline{n} = (1)(1)(0)\underline{n} = 0$

$\underline{k} \times \underline{k} = |\underline{k}||\underline{k}|\sin 0^o\, \underline{n} = (1)(1)(0)\underline{n} = 0$

(b) $\underline{i} \times \underline{j} = |\underline{i}||\underline{j}|\sin 90^o\, \underline{k} = (1)(1)(1)\underline{k} = \underline{k} \quad \sin 90^o = 1$

$\underline{j} \times \underline{k} = |\underline{j}||\underline{k}|\sin 90^o\, \underline{i} = (1)(1)(1)\underline{i} = \underline{i}$

$\underline{k} \times \underline{i} = |\underline{k}||\underline{i}|\sin 90^o\, \underline{j} = (1)(1)(1)\underline{j} = \underline{j}$

Definition 2:

If $\underline{u} = a_1\underline{i} + b_1\underline{j} + c_1\underline{k}$ and $\underline{v} = a_2\underline{i} + b_2\underline{j} + c_2\underline{k}$ are two vectors in space, then cross product of $\underline{u}$ and $\underline{v}$ is :

$$\underline{u} \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

which is called the "Determinant formula" for $\underline{u} \times \underline{v}$.

Parallel Vectors:

If two vectors $\underline{u}$ and $\underline{v}$ are parallel, then

$$\underline{u}\times\underline{v}=|\underline{u}|\,|\underline{v}|\sin 0^{o}\ \underline{n}=|\underline{u}|\,|\underline{v}|(0)\,\underline{n}$$

$$\therefore\ \underline{u}\times\underline{v}=0$$

Angle between two Vectors:

The angle θ between two vectors $\underline{u}$ and $\underline{v}$ is

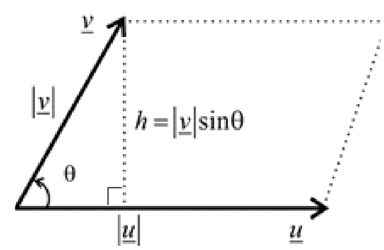
$$\underline{u}\times\underline{v}=|\underline{u}|\,|\underline{v}|\sin 0^{o}\ \underline{n}$$

$$\therefore\ \frac{\underline{u}\times\underline{v}}{\underline{n}}=|\underline{u}|\,|\underline{v}|\sin\theta$$

$$\therefore\ |\underline{u}\times\underline{v}|=|\underline{u}|\,|\underline{v}|\sin\theta$$

$$\therefore\ \sin\theta=\frac{|\underline{u}\times\underline{v}|}{|\underline{u}|\,|\underline{v}|}$$

Area of Parallelogram:

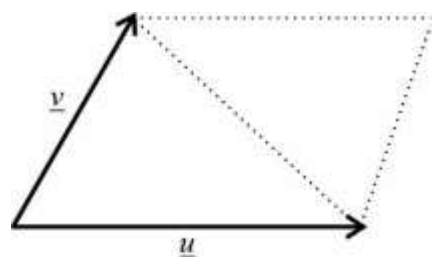


$$\text{Area of Parallelogram} = Base\times Height$$

$$=|\underline{u}|\,|\underline{v}|\sin\theta$$

$$\text{Area of Parallelogram} = |\underline{u}\times\underline{v}|$$

Area of a Triangle:



From the figure, it is clear Area of triangle

Area of Triangle = $\frac{1}{2}$ (Area of Parallelogram)

$=\frac{1}{2}|u\times v|$ where $\underline{u}$ and $\underline{v}$ are vectors along to adjacent sides of triangle.

Scalar Triple Product:

For any three vectors $\underline{u}, \underline{v}$ *and* $\underline{w}$, the dot product of one vector with cross product of remaining two vectors is called "Scalar Triple Product" of vectors $\underline{u}, \underline{v}$ *and* $\underline{w}$. It is written as:

$$\underline{u}.(\underline{v}\times\underline{w})$$

If $\underline{u}=a_1\underline{i}+b_1\underline{j}+c_1\underline{k}$, $\underline{v}=a_2\underline{i}+b_2\underline{j}+c_2\underline{k}$ and $\underline{w}=a_3\underline{i}+b_3\underline{j}+c_3\underline{k}$ then

$$\underline{v}\times\underline{w}=\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$$

$$=\underline{i}\,(b_2c_3-c_2b_3)-\underline{j}\,(a_2c_3-c_2a_3)+\underline{k}\,(a_2b_3-b_2a_3)$$

$$\underline{u}.(\underline{v}\times\underline{w})=(a_1\underline{i}+b_1\underline{j}+c_1\underline{k}).\left[(b_2c_3-c_2b_3)\,\underline{i}-(a_2c_3-c_2a_3)\,\underline{j}+(a_2b_3-b_2a_3)\,\underline{k}\right]$$

$$=a_1(b_2c_3-c_2b_3)-b_1(a_2c_3-c_2a_3)+c_1(a_2b_3-b_2a_3)$$

$$\underline{u}.(\underline{v}\times\underline{w})=\begin{vmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$$

It is important to note that

$$\underline{u}.(\underline{v}\times\underline{w})=\underline{v}.(\underline{w}\times\underline{u})=\underline{w}.(\underline{u}\times\underline{v})$$

The Volume of the Parallelepiped:

The Scalar triple product i.e. $\underline{u}.(\underline{v}\times\underline{w})$ is **volume of a parallelepiped**. Hence it is a scalar.

The Volume of Tetrahedron:

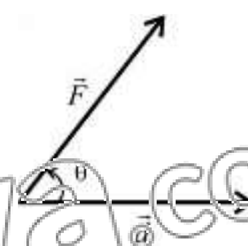
The volume of Tetrahedron $=\dfrac{1}{6}\ \underline{u}.(\underline{v}\times\underline{w})$

The properties of Scalar Triple Product:

(i) If $\underline{u},\underline{v}$ and $\underline{w}$ are coplanar then the volume of the parallelepiped is zero that is $(\underline{u}\times\underline{v}).\underline{w}=0$

(ii) If any two vector of scalar triple product are equal, then its values is zero i.e $[\underline{u}\ \underline{v}\ \underline{v}]=0$

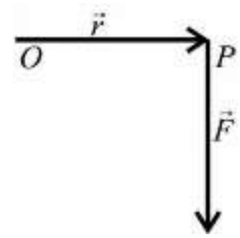
Work done by a Force:



If a constant Force $\vec{F}$ acts on a body, at an angle θ to the direction of motion, then work done by $\vec{F}$ is define to the product of the component of $\vec{F}$ in the , direction of the displacement and the distance that the body moves.

Work done $=\underline{F}.\underline{d}=(\underline{F}\cos\theta)\underline{d}=\underline{F}\underline{d}\cos\theta$

Moment of a Force (Torque):



The turning effect produced by a force is called "Torque" or "Moment" of that Force.

Moment = Perpendicular distance between point of application of force and point of rotation $\times$ Force applied.

Moment of $\vec{F}$ about $O=\overrightarrow{OP}\times\vec{F}$

$$=\vec{r}\times\vec{F}$$