SHORT QUESTIONS

2.1 Define the terms (i) unit vector (ii) Position vector (iii) Components of a vector.

Ans. (i) **Unit Vector:** A vector whose magnitude is one called unit vector. It is used to find the direction of a vector. The formula for the unit vector is

$$\hat{A} = \frac{\vec{A}}{|A|}$$

(ii) **Position Vector:** It is a vector that describe the location of a particle with respect to origin.

The position vector $\vec{r}$ of point P(a, b) in x-y plane is given by

$$\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$$

where $\hat{k}$ is the unit vector along z-axis. In three dimension, the position and $\vec{r}$ from origin will

$$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along x, y and z-axis respectively.

(iii) **Component of a Vector:** The part of a vector effective in a particular direction is called the components of a vector. Usually a vector has two or more components, one along x-axis is called horizontal component and other along y-axis is called vertical component.

2.2 The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?

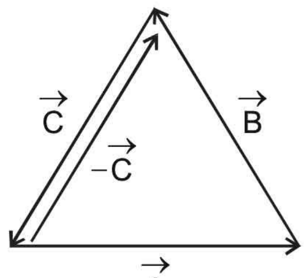
Ans. The resultant of three vectors of equal magnitudes is equal to zero if they are represented by the three adjacent sides of a triangle as shown. If we have three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. By using head to tail rule where $-\vec{C}$ is the resultant of $\vec{A}$ and $\vec{B}$.

Hence

$$-\vec{C} = \vec{A} + \vec{B}$$

$$\vec{A} + \vec{B} + \vec{C} = 0$$

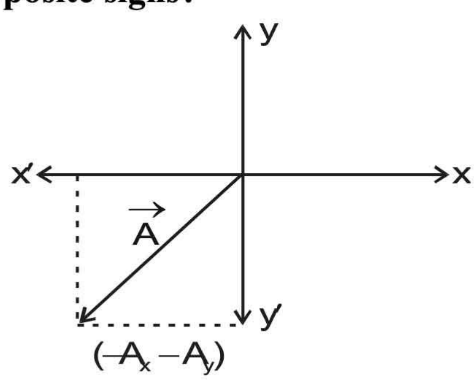
Thus the vector sum of three vectors is zero.

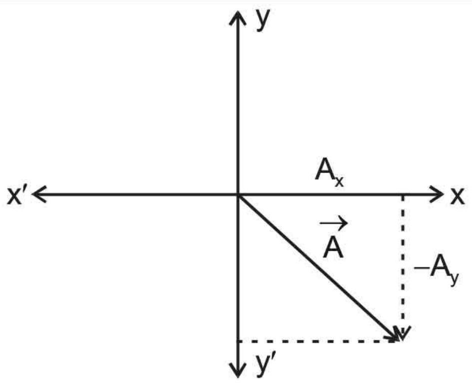


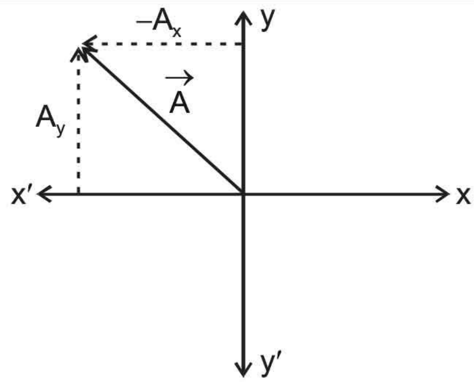
2.3 Vector A lies in the xy-plane. For what orientation will both of its rectangular components be negative. For what orientation will its components have opposite signs?

Ans. *Case-I:* If a vector $\vec{A}$ lies in third quadrant then both of its rectangular components A_x and A_y will be negative as shown.

Case-II: If a vector $\vec{A}$ lies in second and fourth quadrant then both of it, rectangular components A_x and A_y have in opposite sign as shown.







2.4 If one of the rectangular components of a vector is not zero, can its magnitude be zero? Explains.

Ans. No, its magnitude cannot be zero because, the magnitude of vector contains the sum of square of its components. So if one of the components of a vector is not zero and even if they have the opposite signs then the magnitude of a vector cannot be zero. According to formula

$$A = \sqrt{A_x^2 + A_y^2}$$

If $A_y = 0$

Then $A = \sqrt{A_x^2 + 0^2}$

$$A = \sqrt{A_x^2}$$

$$A = A_x$$

$\therefore$ $A \neq 0$

So if one of the rectangular components of a vector is not zero then its magnitude cannot be zero.

2.5 Can a vector have a component greater than the vector's magnitude?

Ans. (i) No, the magnitude of a vector cannot have a component greater than its magnitude because the components of a vector is always less in magnitude of resultant vector. Only in case of equilateral triangle, they are equal.

(ii) Yes, the statement is correct if we do not take the case of rectangular component. So a vector has a component greater than vector magnitude.

2.6 Can the magnitude of a vector have a negative value?

Ans. No, the magnitude of a vector cannot have a negative value. The magnitude of a vector always has a positive value. For example, if we have a vector $-3\vec{A}$, where 3 is the magnitude of a vector and the negative sign shows its direction.

(OR)

As magnitude of $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2}$$

Hence magnitude of a vector cannot have a negative value. e.g.,

If $A_x = -5$ and $A_y = 2$

then $A = \sqrt{(-5)^2 + (2)^2}$

$$= \sqrt{25 + 4}$$

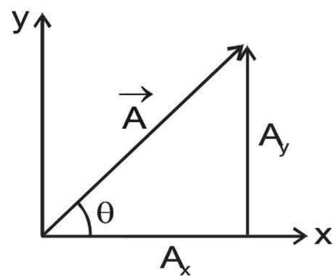
$$= \sqrt{29}$$

2.7 If $\vec{A} + \vec{B} = \vec{0}$, what can you say about the components of the two vectors?

Ans. If
$$\vec{A} + \vec{B} = \vec{0}$$
$$\vec{A} = -\vec{B}$$
In case of rectangular components
$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = -(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$
$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = -B_x\hat{i} - B_y\hat{j} - B_z\hat{k}$$
Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$
$$A_x = -B_x$$
$$A_y = -B_y$$
$$A_z = -B_z$$
So it means that if the sum of the two vectors is zero then their rectangular components will be of the same magnitude but in opposite direction.

2.8 Under what circumstances would a vector have components that are equal in magnitude?

Ans. If θ be the angle which vector $\vec{A}$ makes with horizontal line having components A_x and A_y then
$$\tan\theta = \frac{A_y}{A_x}$$
$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$
According to question



$$A_x = A_y$$
then
$$\theta = \tan^{-1}\left(\frac{A_y}{A_y}\right)$$
$$\theta = \tan^{-1}(1)$$
$$\theta = 45°$$

So, if $\vec{A}$ makes an angle of 45° with x-axis then its both components will be equal in magnitude.

2.9 Is it possible to add a vector quantity to a scalar quantity? Explain.

Ans. No, a vector quantity cannot be added to a scalar quantity because scalar has only magnitude while vector has both the magnitude and direction. So they cannot be added to each other.

2.10 Can you add zero to a null vector?

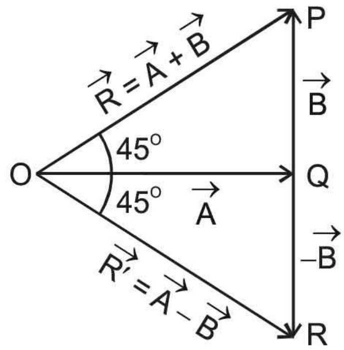
Ans. No, zero is not added to a null vector because zero is a scalar and null vector is a vector quantity.

2.11 Two vectors have unequal magnitudes. Can their sum be zero? Explain.

Ans. No, the sum of two vectors of unequal magnitude cannot be zero. It is only possible when two vectors have same magnitude and in opposite direction.

2.12 Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length.

Ans. Consider two vectors $\vec{A}$ and $\vec{B}$ having equal magnitudes as shown. From head to tail rule, $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are the resultant having same magnitude because they are the hypotenuse of right angled triangle.



Since in ΔOPQ

$$\angle POQ = \angle QPO = 45°$$

and magnitude of $\vec{R}$ will be

$$|\vec{R}| = \sqrt{A^2 + B^2}$$

and in ΔOQR

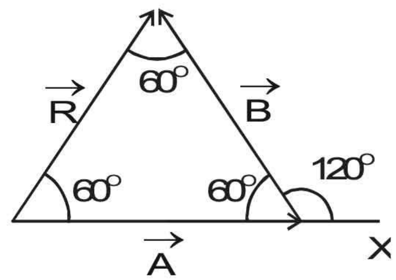
$$\angle ROQ = \angle ORQ = 45°$$

So $$\angle POQ + \angle ROQ = 45° + 45° = 90°$$

Hence the resultants $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$ are perpendicular to each other and they are in equal length.

2.13 How would the two vectors of the same magnitude have to be oriented, if they were to be combined to give a resultant equal to a vector of the same magnitude.

Ans. If two vectors $\vec{A}$ and $\vec{B}$ make an angle of 120°. Then their resultant would have the same magnitude as that of $\vec{A}$ and $\vec{B}$.



We know that:

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB + \cos\theta}$$

According to given condition A = B = R.

$$\because \quad R = \sqrt{R^2 + R^2 + 2R^2\cos\theta}$$

$$R^2 = 2R^2 + 2R^2\cos\theta$$

$$R^2 - 2R^2 = 2R^2\cos\theta$$

$$\frac{-R^2}{2R^2} = \cos\theta \quad \Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = 120°$$

2.14 The two vectors to be combined have magnitudes 60 N and 35 N. Pick the correct answer from those given below and tell why is it the only one of the three that is correct.

(i) 100N (ii) 70N (iii) 20N

Ans. When 60 N and 35 N forces are in same direction then maximum resultant is

$$= 60 + 35 = 95 \text{ N}$$

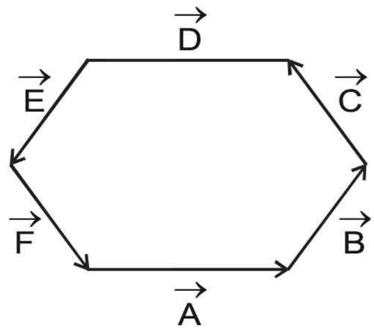
When 60 N and 35 N forces are in opposite direction then minimum resultant is

$$= 60 - 35 = 25 \text{ N}$$

Hence the range of the force is 25 N – 95 N then (ii) 70 N is correct.

2.15 Suppose the sides of a closed polygon represent vector arranged head to tail? What is the sum of these vectors?

Ans. We know that if the vectors are arranged by head to tail rule, which makes a closed polygon then its resultant is zero because there is no place to draw resultant. Consider $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}$ and $\vec{F}$ are the vectors which are arranged by head to tail rule then their resultant is zero i.e.,



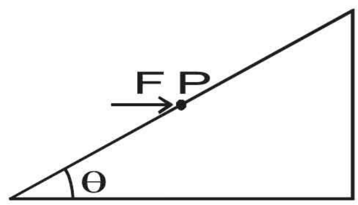
$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F} = 0$$

2.16 Identify the correct answer:

(i) Two ships X and Y are traveling in different directions at equal speeds. The actual direction of motion of X is due north but to an observer on Y, the apparent direction of motion of X is north-east. The actual direction of motion of Y as observed from the shore will be.

(a) East (b) West (c) South-east (d) South-west

(ii) A horizontal force F is applied to a small object P of mass m at rest on a smooth plane inclined at an angle θ to the horizontal as shown in figure. The magnitude of the resultant force acting up and along the surface of the plane, on the object is:



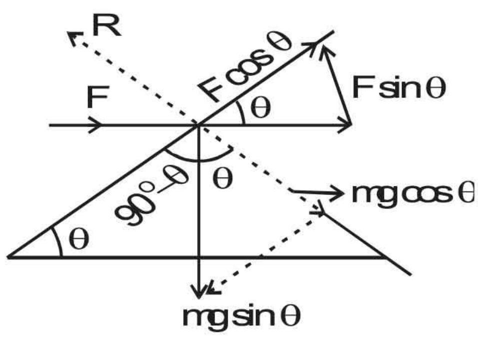
(a) $F \cos\theta - mg \sin\theta$ (b) $F \sin\theta - mg \cos\theta$

(c) $F \cos\theta + mg \cos\theta$ (d) $F \sin\theta + mg \sin\theta$

(e) $mg \tan\theta$

Ans. (i) We know that the ship x is moving towards North from shore and according to observer on ship y, the ship x is moving towards north-east direction so ship y is approaching towards the line of motion of ship x. Thus the motion of ship y is towards west so (b) is correct.

(ii) Now the horizontal force F and weight mg of the body can be resolved into its rectangular components as shown. The force acting up along the plane of the surface is



$$= F \cos\theta - mg \sin\theta$$

So (a) is correct.

2.17 If all the components of the vectors, $\vec{A}_1$ and $\vec{A}_2$ were reversed, how would this alter $\vec{A}_1 \times \vec{A}_2$?

Ans. The vectors $\vec{A}_1$ and $\vec{A}_2$ can be resolved into its rectangular components i.e.,

$$\vec{A}_1 = A_{1x}\hat{i} + A_{1y}\hat{j} + A_{1z}\hat{k}$$

$$\vec{A}_2 = A_{2x}\hat{i} + A_{2y}\hat{j} + A_{2z}\hat{k}$$

$$\vec{A}_1 \times \vec{A}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

On reversing the components

$$\vec{A}_1 = -A_{1x}\hat{i} - A_{1y}\hat{j} - A_{1z}\hat{k}$$

$$\vec{A}_2 = -A_{2x}\hat{i} - A_{2y}\hat{j} - A_{2z}\hat{k}$$

So $$\vec{A}_1 \times \vec{A}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -A_{1x} & -A_{1y} & -A_{1z} \\ -A_{2x} & -A_{2y} & -A_{2z} \end{vmatrix}$$

Taking (–1) as common from R_1 and R_2

$$\vec{A}_1 \times \vec{A}_2 = (-)(-)\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

$$\vec{A}_1 \times \vec{A}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \end{vmatrix}$$

Hence if the components of the vectors are reversed then there is no effect on $\vec{A}_1 \times \vec{A}_2$.

2.18 Name the three different conditions that could make $\vec{A}_1 \times \vec{A}_2 = 0$.

Ans. We know that

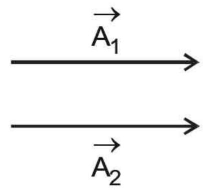
$$\vec{A}_1 \times \vec{A}_2 = A_1A_2 \sin\theta\, \hat{n}$$

Therefore the required three conditions are

(i) Either $\vec{A}_1$ or $\vec{A}_2$ is null vector i.e., $|\vec{A}_1| = |\vec{A}_2| = 0$.

$$\vec{A}_1 \times \vec{A}_2 = 0$$

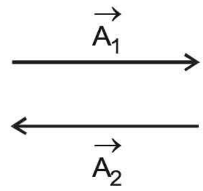
(ii) $\vec{A}_1$ and $\vec{A}_2$ are parallel i.e., $\theta = 0°$.



$$\vec{A}_1 \times \vec{A}_2 = A_1A_2 \sin 0° \, \hat{n} \qquad \because \sin 0° = 0$$

$$\vec{A}_1 \times \vec{A}_2 = 0$$

(iii) $\vec{A}_1$ and $\vec{A}_2$ are antiparallel i.e., $\theta = 180°$



$$\vec{A}_1 \times \vec{A}_2 = A_1A_2 \sin 180° \, \hat{n}$$

$$\Rightarrow \quad \vec{A}_1 \times \vec{A}_2 = 0 \qquad \because \sin 180° = 0$$

2.19 Identify true or false statements and explain the reason:

(a) A body in equilibrium implies that it is not moving nor rotating.

(b) If coplanar forces acting on a body form a closed polygon, then the body is said to be in equilibrium.

Ans. (a) This statement is false because the body is said to be in equilibrium if it is moving with constant velocity or rotating with constant angular velocity.

(OR)

This statement is true because when a body is not moving nor rotating then it is in static equilibrium.

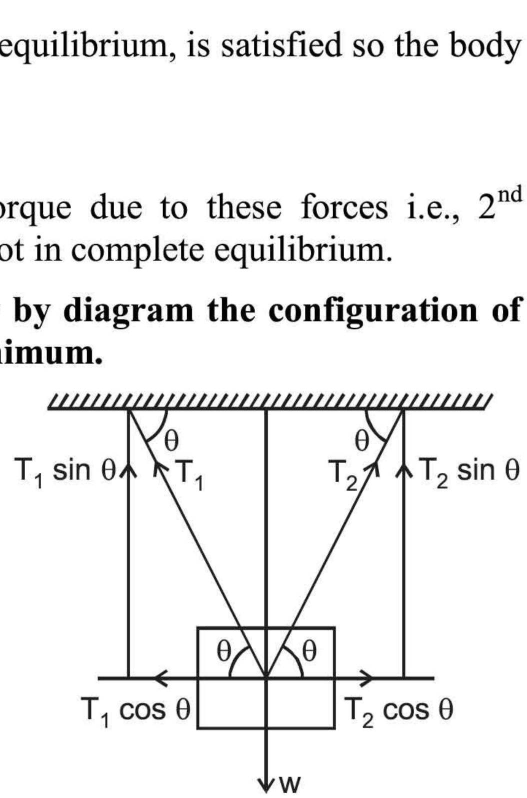
(b) This statement is true because when coplanar forces (vectors) acting on a body in the form of a closed polygon then $\sum \vec{F} = \vec{0}$ i.e., 1st condition of equilibrium, is satisfied so the body is in translational equilibrium.

(OR)

This statement is false because there may be any torque due to these forces i.e., 2nd condition of equilibrium is not satisfied so the body is not in complete equilibrium.

2.20 A picture is suspended from a wall by two strings. Show by diagram the configuration of the strings for which the tension in the settings will be minimum.

Ans. Let we suspend the picture from the wall by two strings as shown in figure.



Let T_1 and T_2 be the tension in string from figure.

Along x-axis

$$T_1 \cos\theta = T_2 \cos\theta$$

$$\Rightarrow \quad T_1 = T_2 = T$$

Along y-axis

$$T_1 \sin\theta + T_2 \sin\theta = w$$

$$T \sin\theta + T \sin\theta = w \qquad \because T_1 = T_2 = T_3$$

$$T = \frac{w}{2 \sin \theta}$$

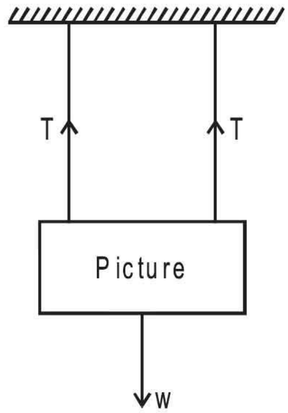
For minimum tension sin θ should have max. value and max. value of sin θ = 1.

$$\Rightarrow \quad \sin \theta = 1$$

$$\sin \theta = \sin 90°$$

$$\theta = 90°$$

So for minimum tension θ should be 90° as shown in figure.



From figure

$$2T = w$$

$$T = \frac{w}{2}$$

So only in this case, the tension in both the string will be minimum.

2.21 Can a body rotate about its centre of gravity under the action of its weight?

Ans. No, a body cannot rotate about its centre of gravity under the action of weight because the line of action of force (weight) passes through its center of gravity (pivot) i.e., movement arm r = 0.

So $\tau = rF$

$$\tau = (0) F$$

$$\tau = 0$$

So the torque is zero.