Chapter 15

ELECTROMAGNETIC INDUCTION

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Recall that a changing magnetic flux through a circuit causes an emf to be induced in the circuit.
- Know that the induced emf lasts so long as the magnetic flux keeps changing.
- Determine Motional emf.
- Use Faraday's law of electromagnetic induction to determine the magnitude of induced emf.
- Apply Lenz's law to determine the direction of induced emf.
- Recognize self and mutual induction.
- Define mutual inductance, self-inductance and its unit henry.
- Know and use the formula $E = \frac{1}{2} LI^2$.
- Calculation the energy stored in an inductance.
- Describe the principle, construction and operation of an AC and DC generator.
- Describe the principle, construction and operation of DC motor.
- Recognize back emf in motors and back motor effect in generators.
- Use $\frac{N_s}{N_p} = \frac{V_s}{V_p}$ and $V_pI_p = V_sI_s$ for an ideal transformer.
- Apply transformer equation to solve the problems.

Q.1 Define electromagnetic induction.

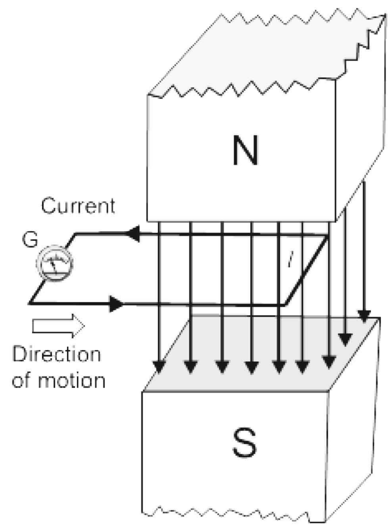
Ans. As soon as Oersted discovered that electric currents produce magnetic fields, many scientists began to look for the reverse effect, that is, to cause an electric current by means of a magnetic field. In 1831 Michael Faraday in England and at the same time Joseph Henry in USA observed that an emf is set up in a conductor when it moves across a magnetic field. **If the moving conductor was connected to a**

sensitive galvanometer, it would show an electric current flowing through the circuit as long as the conductor is kept moving in the magnetic field. The emf produced in the conductor is called induced emf, and the current generated is called the induced current. This phenomenon is known as electromagnetic induction.

Q.2 What is induced emf and induced current? Explain with different experiment.

Ans. INDUCED emf AND INDUCED CURRENT

There are many ways to produce induced emf. Figure illustrates one of them. Consider a straight piece of wire of length l placed in the magnetic field of a permanent magnet. The wire is connected to a sensitive galvanometer. This forms a closed path or loop without any battery. In the beginning when the loop is at rest in the magnetic field, no current is shown by the galvanometer. If we move the loop from left to right, the length l of the wire is dragged across the magnetic field and a current flows through the loop. On reversing the direction of motion of the loop, current also reverses its direction. This is indicated by the deflection of the galvanometer in opposite direction.



The induced current depends upon the speed with which conductor moves and upon the resistance of the loop. If we change the resistance of the loop by inserting different resistors in the loop and move it in the magnetic field with the same speed every time, we find that the product of induced current I and the resistance R of the loop remains constant, i.e.,

$$I \times R = \text{Constant}$$

This constant is the induced emf. The induced emf leads to an induced current when the circuit is closed. The current can be increased by

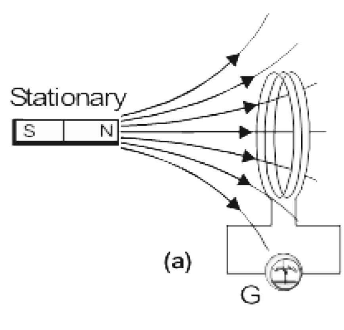
(i) Using a stronger magnetic field.

(ii) Moving the loop faster.

(iii) Replacing the loop by a coil of many turns.

If we perform the above experiment in the other way, i.e., instead of moving the loop across the magnetic field, we hold the loop stationary and move the magnet, then it can be easily observed that the results are the same. Thus it can be concluded that it is the relative motion of the loop and the magnet that causes the induced emf.

In fact, this relative motion changes the magnetic flux through the loop, therefore, we can say that an induced emf is produced in a loop if the magnetic flux through it changes. The greater the rate of change of flux, the larger is the induced emf.

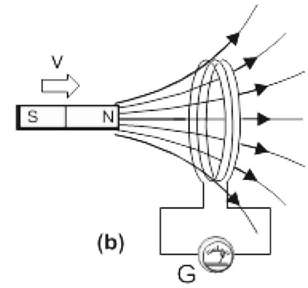


(a)

There are some other methods described below in which an emf is induced in a loop by producing a change of magnetic flux through it.

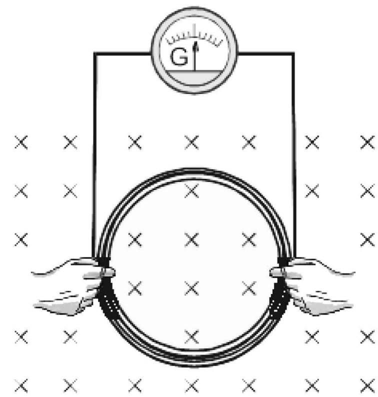
(1) Figure (a) shows a bar magnet and a coil of wire to which a galvanometer is connected. When there is no relative motion between the magnet and the coil, the galvanometer indicates no current in the circuit. As soon as the bar magnet is moved towards

the coil, a current appears in it figure (b). As the magnet is moved, the magnetic flux through the coil changes, and this changing flux produces the induced current in the coil. When the magnet moves away from the coil, a current is again induced but now in opposite direction. The current would also be induced if the magnets were held stationary and the coil is moved.

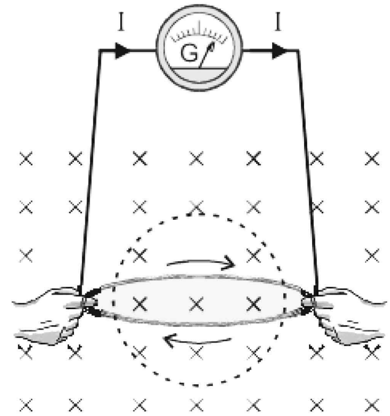


(b)

(2) There is another method in which the current is induced in a coil by changing the area of the coil in a constant magnetic field. Fig. 15.3a shows that no current is induced in the coil of constant area that is placed in a constant magnetic field. However, when the coil is being distorted so as to reduce its area, (Fig. 15.3b) and induced emf and hence current appears. The current vanishes

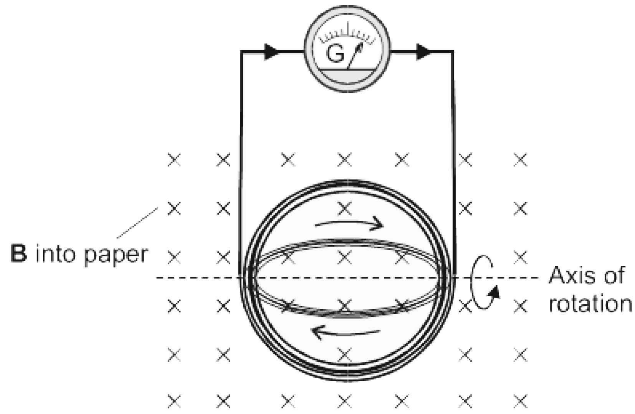


(a)

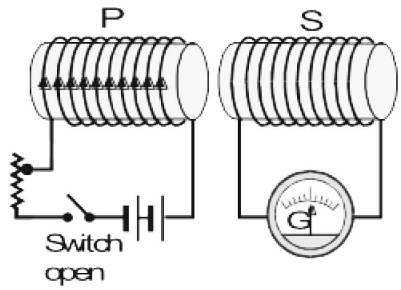


(b)

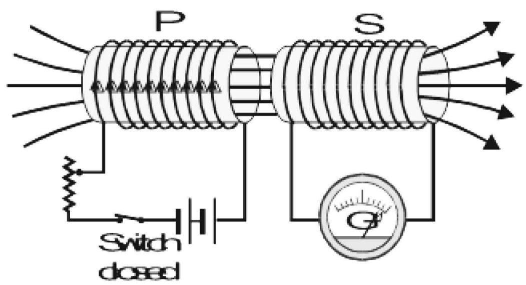
when the area is no longer changing. If the distorted coil is brought to its original circular shape thereby increasing the area, an oppositely directed current is induced which lasts as long as the area is changing.



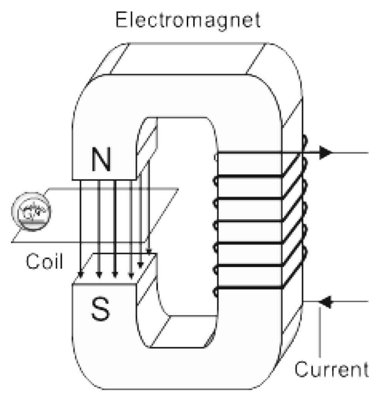
(3) An induced current can also be generated when a coil of constant area is rotated in a constant magnetic field. Here, also, the magnetic flux through the coil changes. This is the basic principle used in electric generators.



(4) A very interesting method to induce current in a coil involves by producing a change of magnetic flux in a nearby coil. Fig. 15.5 shows two coils placed side by side. The coil P is connected in series with a battery, a rheostat and a switch, while the other coil S is connected to a galvanometer only. Since there is no battery in the coil S, one might expect that the current through it will always be zero. Now, if the switch of the coil P is suddenly closed, a momentary current is induced in coil S. This is indicated by the galvanometer, which suddenly deflects and then returns to zero. When the current charges from zero in coil P, the magnetic flux due to this current also changes in coil P. This changing flux is also linked with the coil S that causes the induced current in it. Current in coil P can also be changed with the help of rheostat.



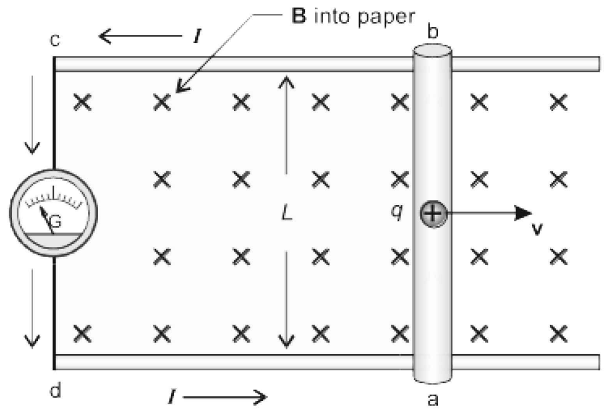
(5) It is also possible to link the changing magnetic flux with a coil by using an electromagnet instead of a permanent magnet. The coil is placed in the magnetic field of an electromagnet. Both the coil and the electromagnet are stationary. The magnetic flux through the coil is changed by changing the current of the electromagnet, thus producing the induced current in the coil.



Q.3 Define motional emf and derive a relation for it.

Ans. MOTION emf

The emf induced by the motion of a conductor across a magnetic field is called motional emf.



Consider a conducting rod of length L placed on two parallel metal rails separated by a distance L. A galvanometer is connected between the ends c and d of the rails. This forms a complete conducting loop abcda. A uniform magnetic field $\vec{B}$ is applied directed into the page. Initially when the rod is stationary, galvanometer indicates no current in the loop. If the rod is pulled to the right with constant velocity **v**, the galvanometer indicates a current flowing through the loop. Obviously, the current is induced due to the motion of the conducting rod across the magnetic field. The moving rod is acting as a source of emf $\varepsilon = V_b - V_a = \Delta V$.

When the rod moves, a charge q within the rod also moves with the same velocity $\vec{v}$ in the magnetic field $\vec{B}$ and experiences a force given by

$$\vec{F} = q\vec{v} \times \vec{B}$$

The magnitude of the force is

$$F = qvB \sin\theta$$

Since angle θ between v and B is 90°, so

$$F = qvB$$

Applying the right hand rule, we see that F is directed from a to b in the rod. As a result the charge migrates to the top end of the conductor. As more and more of the charges migrate, concentration of the charge is produced at the top b and a deficiency of charges at the bottom end a. This redistribution of charge sets up an electrostatic field E directed from b to a. The electrostatic force on the charge is $F_o = q\mathbf{E}$ directed from b to a. The system quickly reaches an equilibrium state in which these two forces on the charge are balanced. If E_o is the electric intensity in this state then

$$qE_o = qvB$$

$$E_o = vB \quad \ldots\ldots \text{(i)}$$

The motional emf ε will be equal to the potential difference $\Delta V = V_b - V_a$ between the two ends of the moving conductor in this equilibrium state. The gradient of potential will be given by $\Delta V/L$. As the electric intensity is given by the negative of the gradient therefore, i.e., $E_o = -\frac{\Delta V}{L}$ ...... (ii)

Using eq. (i)

or $\Delta V = -LE_o$

$= -(LvB)$

The motional emf

$\varepsilon = \Delta V = -LvB$

This is the magnitude of motional emf. However, if the angle between v and B is θ, then

$\varepsilon = -vBL \sin\theta$

Due to induced emf positive charges would flow along the path abcda, therefore the induced current is anticlockwise in the diagram. As the current flows the quantity of the charge at the top decreases so the electric intensity decreases but the magnetic force remains the same. Hence the equilibrium is disturbed in favour of magnetic force. Thus as the charges reach the end a of the conductor due to current flow, they are carried to the top end b of the conductor by the unbalanced magnetic force and the current continues to flow.

Interesting Information

This heater operates on the principle of electromagnetic induction. The water in the metal pot is boiling whereas that in the glass pot is not. Even the glass top of the heater is cool to touch. The coil just beneath the top carries ac current that produces changing magnetic flux. Flux linking with pots induce emf in them. Current is generated in the metal pot that heats up the water, but no current flows through the glass pan, why?

Ans. **In metal pot induced emf is produced due to rate of change of flux and this will produced heating effect. The main cause of this heating effect is eddy current but in glass pot there is no induced emf is produced.**

Q.4 State and explain Faraday's law of electromagnetic induction. Also calculate induced emf.

Ans. FARADAY'S LAW AND INDUCED emf

This law states that "the average emf induced in a conducting coil of N loops is equal to the negative of the rate at which the magnetic flux through the coil is changing with time".

Explanation

Consider a conducting rod L moves from position 1 to position 2 in a small interval of time Δt. The distance travelled by the rod in time Δt is

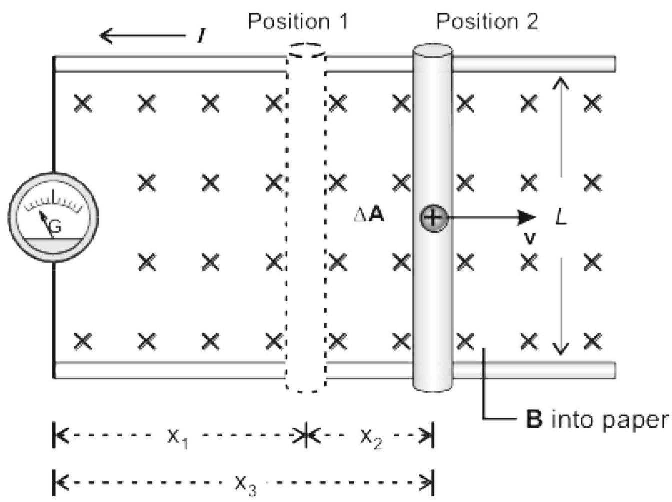
$\Delta x = x_2 - x_1$

Since the rod is moving with constant velocity v, therefore

$v = \frac{\Delta x}{\Delta t}$

As the motional emf is

$\varepsilon = -vBL$



Putting value of 'v'

$$\varepsilon = -\frac{\Delta x}{\Delta t} \times BL$$

$$\varepsilon = -\frac{\Delta x BL}{\Delta t} \qquad \ldots\ldots \text{(i)}$$

As the rod moves through the distance Δx, the increase in the area of loop is given by

$$\Delta A = \Delta x \,.\, L$$

Thus increase in the flux through the loop is given by

$$\Delta\Phi = B\Delta A$$

or

$$\Delta\Phi = \Delta x \,.\, L \,.\, B$$

Putting this in eq. (i)

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t}$$

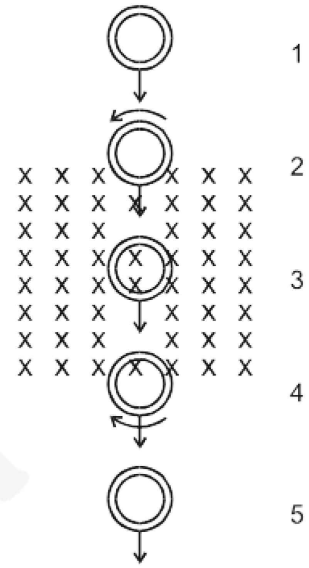
If there is a coil of N loops instead of a single loop, then the induced emf will become N times, i.e.,

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

The negative sign indicates that the direction of the induced emf is such that it opposes the change in flux.

Note: Although the above expression is derived on the basis of motional emf, but it is true in general. This conclusion was first arrived at by Faraday, so this is known as **Faraday's law of electromagnetic induction**.

Point to Ponder



A cooper ring passes through a rectangle region where a constant magnetic field is directed into the page. What do you guess about the current in the ring at the positions 2, 3 and 4?

Ans. **In position 2 current is flowing in anticlockwise direction because according to Lenz's law the direction of induced current is always such as to oppose the cause which produces it when ring enters magnetic flux increase and upper portion of ring becomes north pole, at position 2 there is no change in flux therefore $\Delta\phi = 0$ so no induced emf will produced. While at point 4 flux decreases therefore upper portion of ring will become south pole. Therefore current will flow in clockwise direction.**

Q.5 State and explain Lenz's law and direction of induced emf.

Ans. LENZ'S LAW AND DIRECTION OF INDUCED emf

The law states that, "the direction of the induced current is always so as to oppose the change which causes current".

Explanation

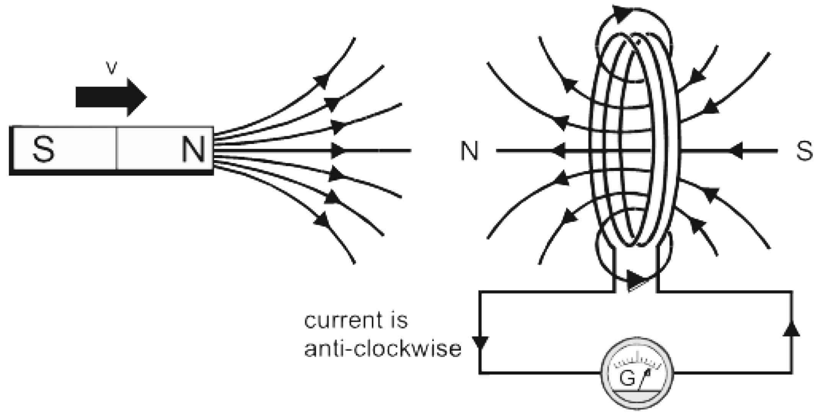
According to Faraday's law of electromagnetic induction

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

The negative sign indicates the direction of the induced emf. To determine the direction we use a method based on the discovery made by the Russian Physicist Heinrich Lenz in 1834. He found that the polarity of an induced emf always leads to an induced current that opposes, through the magnetic field of the induced current, the change inducing the emf.

The Lenz's law refers to induced currents and not to induced emf, which means that we can apply it directly to closed conducting loops or coils. However, if the loop is not closed we can imagine as if it were closed, and then from the direction of induced current, we can find the direction of the induced emf.

Let us apply the Lenz's law to the coil in which current is induced by the movement of a bar magnet. We know that a current carrying coil produces a magnetic field similar to that of a bar magnet. One face of the coil acts as the north pole while the other one as the south pole. If the coil is to oppose the motion of the bar magnet, the face of the coil towards the magnet must become a north pole. The two north poles will then repel each other. The right hand rule applied to the coil suggests that the induced current must be anticlockwise as seen from the side of the bar magnet.

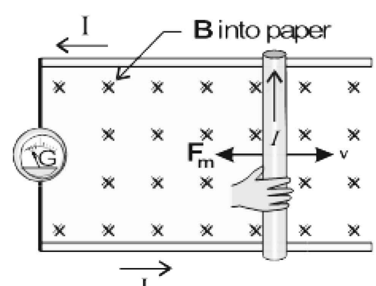


According to Lenz's law the "push" of the magnet is the "change" that produces the induced current, and the current acts to oppose the push. On the other hand if we pull the magnet away from the coil the induced current will oppose the "pull" by creating a south pole on the face of coil towards the bar magnet.

The Lenz's law is also a statement of law of conservation of energy that can be conveniently applied to the circuits involving induced currents. To understand this, consider once again the experiment in figure. When the rod moves towards right, emf is induced in it and an induced current flows through the loop in the anti-clockwise direction. Since the current carrying rod is moving in the magnetic field, it experiences a magnetic force F_m having the magnitude

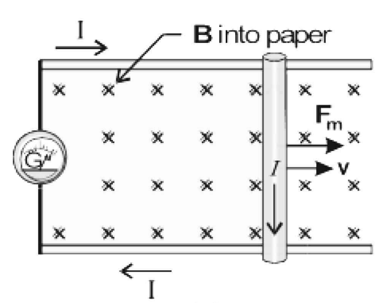
$$F_m = ILB \sin 90°$$

By right hand rule the direction of F_m is opposite to that of v, so it tends to stop the rod. An external dragging force equal to F_m in magnitude but opposite in direction must be applied to keep the rod moving with constant velocity. This dragging force provides the energy for the induced current to flow. This energy is the source of induced current, thus electromagnetic induction is exactly according to law of conservation of energy.



(a)

The Lenz's law forbids the induced current directed clockwise in this case, because the force F_m would be, then, in the direction of v that would accelerate the rod towards right as shown. This in turn would induce a stronger current, the magnetic field due to it also increases and the magnetic force increases further. Thus the motion of the wire is more accelerated and so on. Commencing with a minute quantity of energy, we obtain an every increasing kinetic energy of motion apparently from nowhere. Thus the process becomes self-perpetuating which is against the law of conservation of energy.



(b)

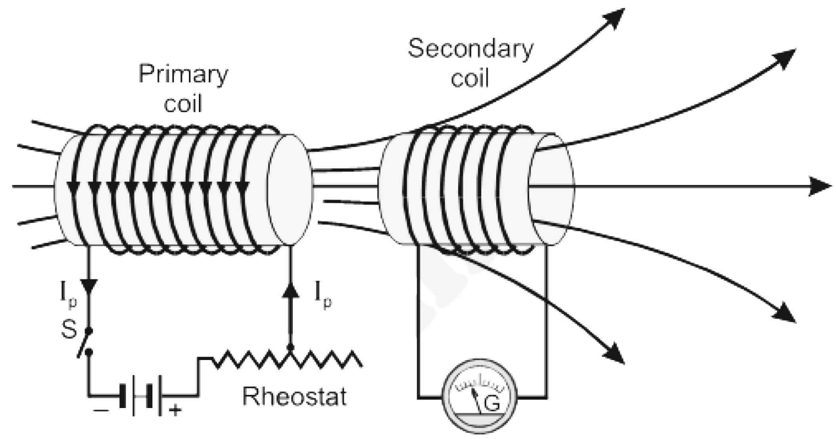
Q.6 Explain the phenomenon mutual induction.

Ans. MUTUAL INDUCTION

The phenomenon in which a changing current in one coil induces an emf in another coil is called the mutual induction.

Explanation

Consider two coils placed closed to each other as shown in figure. One coil connected with a battery through a switch and a rheostat is called the primary and the other one connected with galvanometer is called the secondary. If the current in the primary is changed by varying the resistance of the rheostat, the magnetic flux in surrounding region changes. Since the secondary coil is in the magnetic field of the primary, the changing flux also links with the secondary. This causes an induced emf in the secondary.



Let the flux passing through each loop of secondary is equal to ϕ_S. Net flux passing through N_s loops = $N_s \phi_S$ where N_s is the number of turns in the secondary coil.

As $\phi = BA$

$\therefore \phi \propto B$ ...... (i)

Also $B \propto I_P$ ...... (ii)

From (i) and (ii)

$$\phi \propto I_P$$

$$\therefore N_s\phi_S \propto I_P$$

$$N_s\phi_S = MI_P$$

where $$M = \frac{N_s\phi_S}{I_P}$$

is the constant of proportionality called the mutual inductance of the two coils. It depends upon the number of turns of the coil, their area of cross-section, their closeness together and the nature of the core upon which the two coil are wound.

According to Faraday's law

$$\varepsilon_S = -N_s \frac{\Delta\phi_S}{\Delta t}$$

$$\varepsilon_S = -\Delta\left(\frac{N_s\phi_S}{\Delta t}\right)$$

Putting the value of $N_s\phi_s = MI_P$

$$\varepsilon_s = -\frac{\Delta MI_P}{\Delta t}$$

$$\therefore \quad \varepsilon_s = -\frac{M\Delta I_P}{\Delta t}$$

$$\varepsilon_s \propto -\frac{\Delta I_P}{\Delta t} \quad \text{...... (iii)}$$

This shows that emf induced in the secondary is proportional to time rate of change of current in primary.

The negative sign in eq. (iii) indicates the fact that the induced emf is in such a direction as to oppose the change of current in the primary.

Mutual Inductance

As $$M = \frac{\varepsilon_s}{\Delta I_P/\Delta t}$$

The ratio of average emf induced in the secondary to the time rate of change of current in primary is called the mutual inductance.

Unit of Mutual Inductance

The SI unit of mutual inductance is $VA^{-1}s$ which is called Henry (H).

Henry

One henry is the mutual inductance of the pair of the coils in which a rate of change of current of one ampere per second in the primary caused an induced emf of one volt in the secondary.

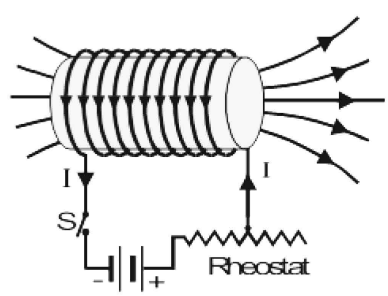
Q.7 Explain the phenomenon self induction.

Ans. SELF INDUCTION

The phenomenon in which a changing current induces an emf in itself is called self induction.

Explanation

Consider a circuit as shown in figure. A coil is connected in series with a battery and a rheostat. Magnetic flux produced through the coil due to current in it. If the current is changed by varying the rheostat quickly magnetic flux through the coil changes that causes an induced emf in the coil. Such an emf is called as self induced emf.



Let flux through one loop of the coil is equal to ϕ.

Total flux through the coil of N turns = $N\phi$

As $$\phi = BA \quad \text{...... (A)}$$

$$\phi \propto B \quad \text{...... (i)}$$

$$B \propto I \quad \text{...... (ii)}$$

From (i) and (ii)

$$\phi \propto I$$

$$\therefore \quad N\phi \propto I$$

$$N\phi = LI$$

$$L = \frac{N\phi}{I}$$

Where L is the constant of proportionality called the self inductance of the coil.

It depends upon the number of turns of the coil, its area of cross-section and the core material. By winding the coil around a ferromagnetic (iron) core, the magnetic flux and hence the inductance can be increased relative to that for an air core.

By Faraday's law

$$\varepsilon_L = -N\frac{\Delta\phi}{\Delta t}$$

$$\varepsilon_L = -\Delta\frac{(N\phi)}{\Delta t}$$

Putting the value of $N\phi = LI$

$$\therefore \quad \varepsilon_L = -\Delta\frac{(LI)}{\Delta t} \quad \text{...... (iii)}$$

$$\varepsilon_L = -L\frac{\Delta I}{\Delta t}$$

$$\varepsilon_L \propto -\frac{\Delta I}{\Delta t}$$

This shows that self induced emf in a coil is directly proportional to the time rate of change of current in the coil.

Self Inductance

As $$L = \frac{\varepsilon_L}{\frac{\Delta I}{\Delta t}}$$

The ratio of average emf to the rate of change of current in the coil is called self inductance.

Unit of Self Inductance

SI unit of self induction is also Henry (H).

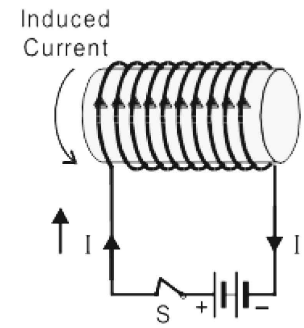
The negative sign in eq. (iii) indicates that the self induced emf must oppose the change that produced it. That is why self induced emf is sometimes called back emf. This is exactly in accordance with Lenz's law. If current is increased the induced emf will be opposite to that of battery and if current is decreased the induced emf will aid the battery. Because of their self inductance, coils of wire are known as inductors. In A.C. inductors behave like resistors.

Q.8 What type of energy is stored in an inductor? Find the relations for the energy and energy density in an inductor.

Ans. ENERGY STORED IN AN INDUCTOR

As energy can be stored in the electric field between the plates of the capacitors. Similarly energy can be stored in the magnetic field of an inductor. Inductor is a device which can store energy due to magnetic field.

Consider a coil connected to a battery and a switch in series as shown in figure. When the switch is turned on voltage V is applied across the ends of the coil and current through it rises from zero to its maximum value I. Due to the change of current, an emf induced which is opposite to that of battery. Work is done by the battery to move charges against the induced emf.



From the definition of potential difference

$$V = \frac{W}{\Delta q}$$

$$W = V\Delta q$$

Work done by the battery in moving a small charge Δq is

$$W = \varepsilon_L \Delta q \qquad \ldots\ldots \text{(i)} \qquad (\because\ V = \varepsilon_L)$$

where ε_L is the magnitude (self induced emf) of induced emf given by

$$\varepsilon_L = L\frac{\Delta I}{\Delta t}$$

Putting this value of ε_L in eq. (i)

$$\therefore \quad W = L\frac{\Delta I}{\Delta t}\Delta q$$

$$W = L\frac{\Delta q}{\Delta t}\Delta I$$

Here

$$\text{Average current} = \frac{\Delta q}{\Delta t} = \frac{0 + I}{2}$$

$$\frac{\Delta q}{\Delta t} = \frac{1}{2} I$$

$$\text{Change in current} = \Delta I = I - 0$$

$$\Delta I = I$$

$$\therefore \quad W = L\left(\frac{1}{2} I\right)(I)$$

$$W = \frac{1}{2} LI^2$$

This work is stored as potential energy in the inductor. Hence the energy stored in an inductor is

$$U_m = \frac{1}{2} LI^2 \qquad \ldots\ldots \text{(ii)}$$

Energy Stored in Terms of Magnetic Field

As for solenoid

$$B = \mu_o n I$$

As $\phi = BA$

Putting value of B

$$\therefore \quad \phi = \mu_o n A I$$

As $N\phi = LI$

$$L = \frac{N\phi}{I}$$

Putting value of ϕ

$$\therefore \quad L = \frac{N}{I}(\mu_o n A I)$$

$$L = N\mu_o n A$$

As $n = \frac{N}{l}$

$$N = nl$$

$$L = nl\mu_o nA$$

$$L = \mu_o n^2 (Al)$$

Putting this value in (ii)

$$U_m = \frac{1}{2}(\mu_o n^2 A l) I^2 \qquad \ldots\ldots \text{(iii)}$$

As $B = \mu_o n I$

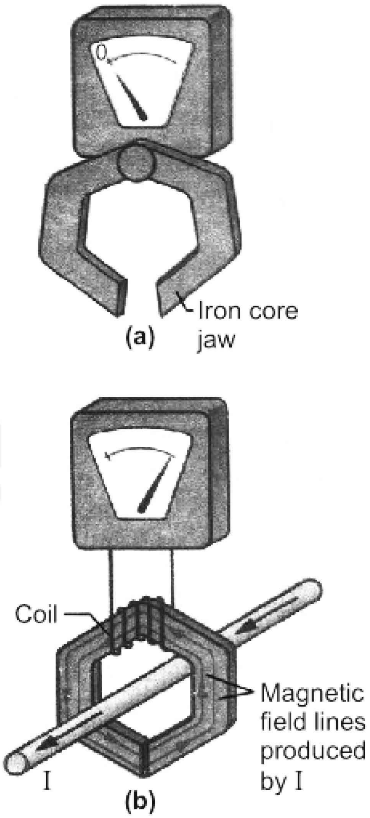
$$I = \frac{B}{\mu_o n}$$

Putting value of I in eq. (iii)

$$U_m = \frac{1}{2}\mu_o n^2 A l \left(\frac{B^2}{\mu_o^2 n^2}\right)$$

$$U_m = \frac{1}{2}\frac{B^2}{\mu_o}(Al)$$

Do You Know?



An induction ammeter with its iron-core jaw (a) open and (b) closed around a wire carrying an alternating current I. Some of the magnetic field lines that encircle the wire are routed through the coil by the iron core and lead to an induced emf. The meter detects the emf and is calibrated to display the amount of current in the wire.

Energy Density

Energy density can be defined as the energy stored per unit volume inside the solenoid.

$$E_d = \frac{E}{\text{Volume}} = \frac{U_m}{\text{Volume}}$$

$$= \frac{\frac{1}{2}\frac{B^2}{\mu_o}(Al)}{Al}$$

$$E_d = \frac{1}{2}\frac{B^2}{\mu_o}$$

Q.9 Describe the principle, construction and working of an alternating current generator.

Ans. ALTERNATING CURRENT GENERATOR

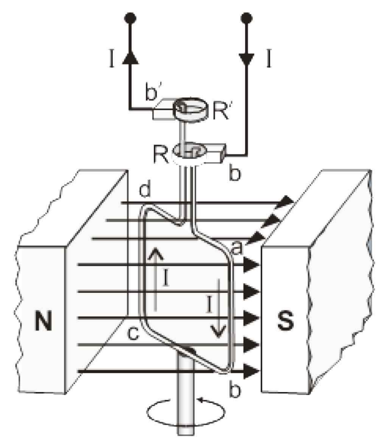
A current generator is a device that converts mechanical energy into electrical energy.

Principle

It is based on Faraday's law of electromagnetic induction i.e., when a coil is rotated in a magnetic field by some mechanical means, magnetic flux through the coil changes and hence an emf is induced in the coil.

Construction

A rectangular loop of wire of area A is placed in a uniform magnetic field $\vec{B}$, as shown in figure. The loop is rotated about z-axis through its centre at constant angular velocity ω. One end of the loop is attached to a metal ring R and the other end to the ring R′. These rings called the slip rings, are concentric with the axis of the loop and rotate with it. Rings RR′ slide against stationary carbon brushes to which external circuit is connected.



(Front view)

Working

Now we calculate the induced emf in the loop, consider its position while it is rotating anticlockwise (top view). The vertical side ab of the loop is moving with velocity $\vec{v}$ in the magnetic field $\vec{B}$. If the angle between $\vec{v}$ and $\vec{B}$ is θ, the motional emf induced in this side has the magnitude.

$$\varepsilon_{ab} = vBL \sin\theta$$

The direction of the induced current in the wire ab is the same as that of force $\vec{F}$ experienced by the +ve charges in the wire i.e., from top to bottom. The same amount of emf is induced in the side cd but the direction of the current is from bottom to the top.

The net contribution to emf by sides bc and da is zero because the force acting on the charges inside bc and da is not along the wire.

$$\therefore \quad \varepsilon_{dc} = vBL \sin\theta$$

$$\therefore \quad \varepsilon_{cd} = \varepsilon_{da} = 0$$

Since both the emf's in the sides ab and cd drives the current in the same direction around the loop, the total emf is

$$\varepsilon = \varepsilon_{ab} + \varepsilon_{cd}$$

$$= vBL \sin\theta + vBL \sin\theta$$

$$\varepsilon = 2vBL \sin\theta$$

If the loop is replaced by a coil of N turns then

$$\varepsilon = 2NvBL \sin\theta \qquad \text{...... (i)}$$

The linear speed v of the vertical wire is related to the angular speed ω as

$$v = r\omega$$

where r is the distance of the vertical wires from the centre of the coil.

∴ eq. (i) becomes

$$\varepsilon = 2Nr\omega BL \sin\theta$$

$$\varepsilon = NB(2rL)\ B \sin\theta$$

As $2rL = A$ = Area of coil

$$\therefore \quad \varepsilon = NBA\omega \sin\theta$$

Since $\theta = \omega t$

$$\therefore \quad \varepsilon = NBA\omega \sin\omega t \qquad \text{...... (ii)}$$

This equation shows that this induced emf varies sinosoidally with time. It has the maximum value when $\sin\omega t = 1$

$$\therefore \quad \varepsilon_o = NBA\omega$$

Therefore eq. (ii) becomes

$$\varepsilon = \varepsilon_o \sin\omega t \qquad \text{...... (iii)}$$

If R is the resistance of the coil, then by Ohm's law the induced current in the coil will be

$$I = \frac{\varepsilon}{R}$$

Putting value of ε from eq. (iii)

$$\therefore \quad I = \frac{\varepsilon_o \sin\omega t}{R} \qquad \text{...... (iv)}$$

Maximum induced current is

$$I_o = \frac{\varepsilon_o}{R} \qquad \text{(as max value of } \sin\omega t = 1)$$

$$\therefore \quad I = I_o \sin\omega t \qquad \text{...... (v)}$$

As angular speed ω of the coil is related to its frequency f as

$$T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{T}$$

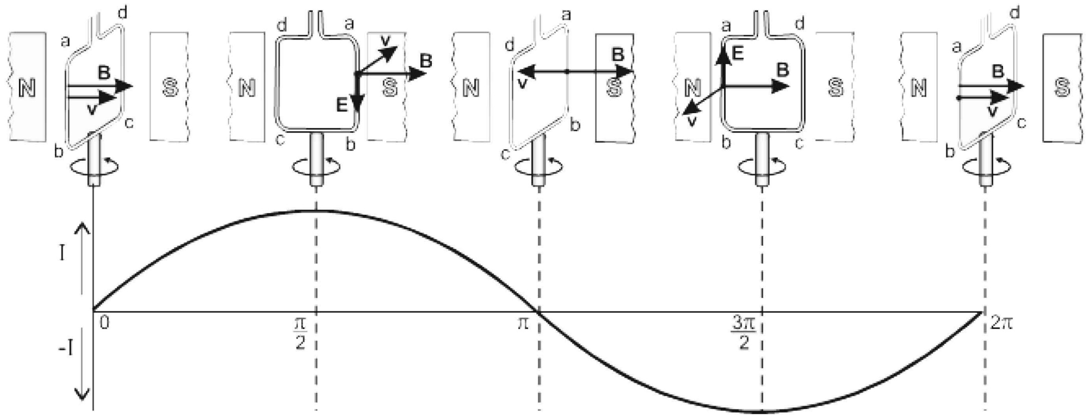
$$\omega = 2\pi f \qquad \left(\frac{1}{T} = f\right)$$

Therefore eq. (iii) and (v) becomes

$$\varepsilon = \varepsilon_o \sin 2\pi ft \qquad \text{...... (vi)}$$

$$I = I_o \sin 2\pi ft \qquad \text{...... (vii)}$$

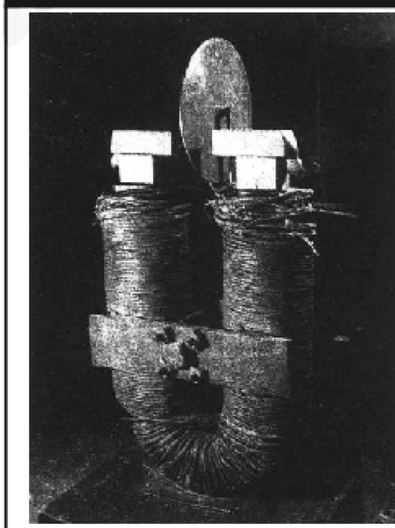
The above equation indicates that the variation of current as a function of $\theta = 2\pi ft$. Figure shows the graph for the current corresponding to different positions of one loop of the coil.



Explanation from Graph

When the angle between $\vec{v}$ and $\vec{B}$ is $\theta = 0$, the plane of the loop is perpendicular to $\vec{B}$, current is zero. As θ increases, current also increases and at $\theta = 90° = \pi/2$ rad, the loop is parallel to $\vec{B}$, current is maximum, directed along abcda. On further increase in θ current becomes zero as the loop is again perpendicular to $\vec{B}$. For $180° < \theta < 270°$ current increases but reverses its direction as is clear from the figure. Current is now directed along dcbad. At $\theta = 270° = 3\pi/2$ rad, current is maximum in the reverse direction as the loop is parallel to $\vec{B}$. At $\theta = 360° = 2\pi$ rad, one rotation is completed, the loop is perpendicular to $\vec{B}$ and the current decreases to zero. After one rotation the cycle repeats itself. The current alternates in direction once in one cycle. Therefore, such a current is called the alternating current. It reverses its direction f times per second.

Do You Know?



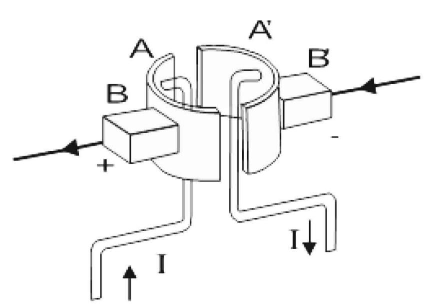
Faraday's homopolar generator with which he was able to produce a continuous induced current.

Note: In actual practice a number of coils are wound around an iron cylinder which is rotated in the magnetic field. This assembly is called an armature. The magnetic field is usually provided by an electromagnet. Armature is rotated by a fuel a engine or a turbine run by a waterfall. In some commercial generation field magnet is rotated around a stationary armature.

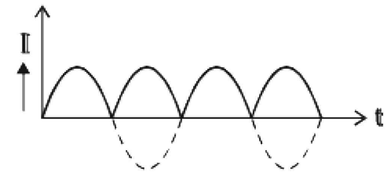
Q.10 What is D.C. generator?

Ans. D.C. GENERATOR

Alternating current generators are not suitable for many applications to run a D.C. motor. In 1834 William Sturgeon invented a simple device called a commutator that prevents the direction of current from changing. Therefore a D.C. generator is similar to the A.C. generator in construction with the difference that "slip rings" are replaced by "split rings". The "split rings" are two halves of a ring that act as a commutator figure shows the "split rings" A and A′ attached to the two ends of the coil that rotates in the magnetic field. When the current in the coil is zero and is about to change direction, the split rings also change the contacts with the carbon brushes BB′. In this way the



output from BB′ remains in the same direction, although the current is not constant in magnitude. The curve of the current is shown in figure.

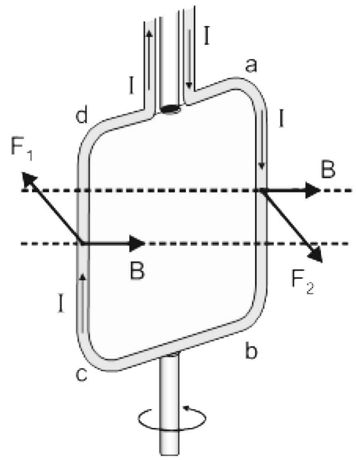


It is similar to a sine curve with the lower half inverted. The fluctuations of the output can be significantly reduced by using many coils rather than a single one. Multiple coils are wound around a cylindrical core to form the armature. Each coil is connected to a separate commutator and the output of every coil is tapped only as it reaches its peak emf. Thus the emf in the outer circuit is almost constant.

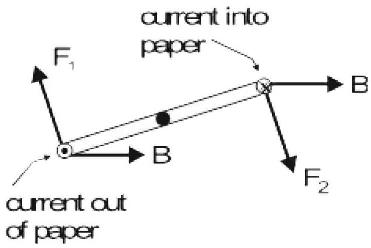
Q.11 Explain back motor effect in generators.

Ans. BACK MOTOR EFFECT IN GENERATORS

A generator is the source of electricity production. Practically, the generators are not so simple as described above. A large turbine is turned by high pressure steam or waterfall. The shaft of the turbine is attached to the coil which rotates in a magnetic field. It converts the mechanical energy of the driven turbine to electrical energy. The generator supplies current to the external circuit. The devices in the circuit that consume electrical energy are known as the "load". The greater the load the larger the current is supplied by the generator. When the circuit is open, the generator does not supply electrical energy, and a very little force is needed to rotate the coil. As soon as the circuit is closed, a current is drawn through the coil. The magnetic field exerts force on the current carrying coil. Figure shows the forces acting on the coil. Force $\vec{F_1}$ is acting on the left side of the coil whereas an equal but opposite force $\vec{F_2}$ acts on the right side of the coil. The forces are such that they produce a counter torque that opposes the rotational motion of the coil. This effect is sometimes referred to as back motor effect in the generators. The larger the current drawn, the greater is the counter torque produced. That means more mechanical energy is required to keep the coil rotating with constant angular speed. This is in agreement with the law of conservation of energy. The energy consumed by the "load" must come from the "energy source" used to drive the turbine.



Front view

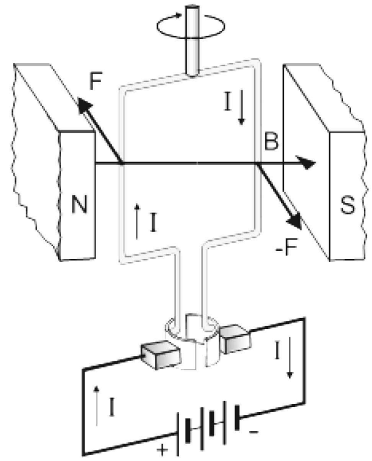


Top view

Q.12 What is D.C. motor?

Ans. D.C. MOTOR

A motor is a device which converts electrical energy into mechanical energy. We already know that a wire carrying current placed in a magnetic field experiences a force. This is the basic principle of an electric motor. In construction a D.C motor is similar to a D.C generator, having a magnetic field, a commutator and an armature. In the generator, the armature is rotated in the magnetic field and current is the output. In the D.C., magnetic field and current is the output. In the D.C. motor, the brushes are connected to a D.C supply or battery. When current flows through the armature coil, the force on the conductors produces a torque, that rotates the armature. The amount of this torque depends upon the current, the strength of the magnetic field, the area of the coil and the number of turns of the coil.



If the current in the coil were all the time in the same direction, the torque on it would be reversed after each half revolution. But at this moment, commutator reverses the direction of current that keeps the torque always in the same sense. A little problem arises due to the use of commutator. That is, the torque vanishes each time the current changes its direction. This creates jerks in the smooth running of the armature. However the problem is overcome by using more than one coils wrapped around a soft-iron core. This results in producing a more steady torque.

The magnetic field in the motor, is provided by a permanent magnet or an electromagnet. The windings of the electromagnet are usually called the field coils. The field coils may be in series or in parallel to the armature coils.

Q.13 Explain the back emf effect in motors.

Ans. BACK emf EFFECT IN MOTORS

A motor is just like a generator running in reverse. When the coil of the motor rotates across the magnetic field by the applied potential difference V, an emf ε is induced in it. The induced emf is in such a direction that opposes the emf running the motor. Due to this reason the induced emf is called back emf of the motor. The magnitude of the back emf increases with the speed of motor.

Since V and ε are opposite in polarity, the net emf in the circuit is $V - \varepsilon$. If R is the resistance of the coil and I the current drawn by the motor, then by Ohm's law

$$I = \frac{V - \varepsilon}{R} \quad \text{or} \quad V = \varepsilon + IR$$

When the motor is just started, back emf is almost zero and hence a large current passes through the coil. As the motor speeds up, the back emf increases and the current becomes smaller and smaller. However, the current is sufficient to provide torque on the coil to drive the load and to overcome losses due to friction. If the motor is overloaded, it slows down. Consequently, the back emf decreases and allows the motor to draw more current. If the motor is overloaded beyond its limits, the current could be so high that it may burn the motor out.

Q.14 Describe construction, principle and working of a transformer.

Ans. TRANSFORMER

"A transformer is an electrical device to change a given alternating emf into a larger or smaller emf".

Principle

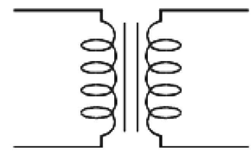
It works on the principle of mutual induction between two coils.

Construction

The transformer consists of two coils of copper, electrically insulated from each other, wound on the same iron core. The coil to which A.C power supplied is called primary and that from which power delivered to the circuit is called secondary.

Working

As there is no electrical connection between the two coils but they are magnetically linked. Suppose that an alternating emf is applied to the primary. If at some instant Δt, the flux in the primary is changing at the rate of $\frac{\Delta\phi}{\Delta t}$ then there will be back emf induced in the primary which will oppose the applied voltage.



Transformer symbol

$$\therefore \quad \text{Self induced emf} = -N_p \frac{\Delta\phi}{\Delta t}$$

If the resistance of the coil is negligible then back emf is equal and opposite to the applied voltage V_p.

$$\therefore \quad V_p = -\text{ Back emf}$$

$$V_p = -\left(-N_p \frac{\Delta\phi}{\Delta t}\right)$$

$$V_p = N_p \frac{\Delta\phi}{\Delta t} \qquad \text{...... (i)}$$

where N_P is the number of turns in the primary.

Let the flux through the primary also passes through the secondary i.e., the two coils are tightly coupled, the rate of change of flux in the secondary will also be $\frac{\Delta\phi}{\Delta t}$ and the magnitude of induced emf across the secondary is given by

$$V_s = N_s \frac{\Delta\phi}{\Delta t} \qquad \text{...... (ii)}$$

where N_s is the no. of turns in the secondary.

Divide eq. (i) by (ii)

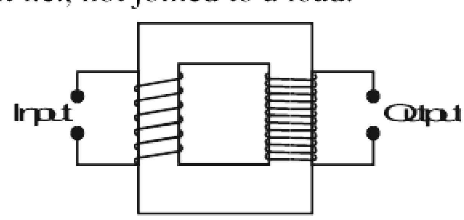
$$\frac{V_s}{V_p} = \frac{N_s \frac{\Delta\phi}{\Delta t}}{N_p \frac{\Delta\phi}{\Delta t}}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

This relation is true only when secondary coil is in a open circuit i.e., not joined to a load.

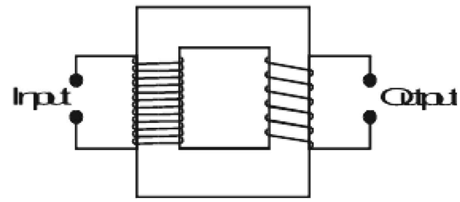
Step up Transformer

If $N_s > N_p$ then $V_s > V_p$ such a transformer in which voltage across secondary is greater than the primary voltage is called a step-up transformer.



(a) Stepup transformer

Step Down Transformer



(b) Step down transformer

If $N_s < N_p$ then $V_s < V_p$, such a transformer in which voltage across the secondary is less than the primary voltage is called step down transformer.

Electrical Power in Transformer

The electrical power in a transformer is transformed from its primary to the secondary coil by means of changing flux.

For an ideal transformer

$$P_{input} = P_{output}$$

$$V_p I_p = V_s I_s$$

$$\frac{V_p}{V_s} = \frac{I_s}{I_p}$$

I_p is the current in the primary and

I_s is the current in the secondary.

The currents are thus inversely proportional to the respective voltages.

Applications

In a step up transformer when the voltage across the secondary is raised, the value of current is reduced. This is the principle behind its use in the electric supply network where transformer increases the voltage and reduces the current so that in can be transmitter over long distance without much power loss. When current I passes through resistance R, the power loss due to heating effect is I^2R. In order to minimize the loss during transmission, it is not possible to reduce R because it requires the use of thick copper wires which become highly uneconomical. The purpose is well served by reducing I. At the **generating power station** the **voltage is stepped up** to several thousand of volts and power is transmitted at low current to long distances without much loss.

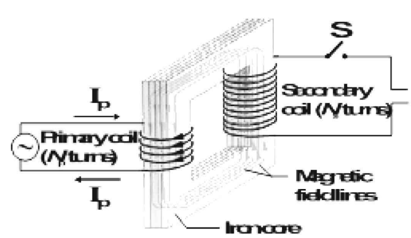
Step down transformer decreases the voltage to a safe value at the end of line where the consumer of electric power is located. Inside a house transformer may be used to step down the voltage from 250 V to 9 volts for ringing bell or operating a transistor radio. The transformers with several secondary are used in television and radio receives where several different voltages are required.

Q.15 How the power is lost in transformer?

Ans. POWER LOSSES IN TRANSFORMER

The output of a transformer is always less than input due to power losses. There are two main causes of the power loss.

(i) Eddy Current (ii) Hysteresis Loss



In order to enhance the magnetic flux, the primary and secondary coils of the transformer are wound on soft iron core. The flux generated by the coils also passes through the core. As magnetic flux changes through a solid conductor, induced currents are set up in closed paths in the body of the conductor. These induced currents are set up in a direction perpendicular to the flux and are known as eddy currents. It results in power dissipation and heating of the core material. In order to minimize the power loss due to flow of these currents, the core is laminated with insulation in between the layers of laminations which stops the flow of eddy currents (Figure).

Hysterises loss is the energy expended to magnetize and demagnetize the core material in each cycle of the A.C.

Efficiency of Transformer

Due to these power losses, a transformer is far from being an ideal. Its output power is always less than its input power. The efficiency of a transformer is defined as

$$E = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$

In order to improve the efficiency, care should be exercised, to minimize all the power losses. For example core should be assembled from the laminated sheets of a material whose hysteresis loop area is very small. The insulation between lamination sheets should be perfect so as to stop the flow of eddy currents. The resistance of the primary and secondary coils should be kept to a minimum. As power transfer from primary to secondary takes place through flux linkages, so the primary and secondary coils should be wound in such a way that flux coupling between them is maximum.

SOLVED EXAMPLES

EXAMPLE 15.1

A metal rod of length 25 cm is moving at a speed of 0.5 ms^{-1} in a direction perpendicular to a 0.25 T magnetic field. Find the emf produced in the rod.

Data

Speed of rod = v = $0.5\ ms^{-1}$

Length of rod = L = 25 cm

= 0.25 m

Magnetic flux density = B = 0.25 T

To Find

Induced emf = ε = ?

SOLUTION

Using the relation

$$\varepsilon = vBL$$

$$\varepsilon = 0.5 \times 0.25 \times 0.25$$

$$\varepsilon = 3.13 \times 10^{-2}\ JC^{-1}$$

$$\varepsilon = 3.13 \times 10^{-2}\ V$$

Result

Induced emf = ε = 3.13×10^{-2} V

EXAMPLE 15.2

A loop of wire is placed in a uniform magnetic field that is perpendicular to the plane of the loop. The strength of the magnetic field is 0.6 T. The area of the loop begins to shrink at a constant rate of $\frac{\Delta A}{\Delta t} = 0.8\ m^2s^{-1}$. What is the magnitude of emf induced in the loop while it is shrinking?

Data

Rate of change of area = $\frac{\Delta A}{\Delta t}$ = $0.8\ m^2s^{-1}$

Magnetic flux density = B = 0.6 T

Number of turns = N = 1

To Find

Induced emf = ε = ?

SOLUTION

Applying Faraday's law, magnitude of induced emf

$$\varepsilon = N\frac{\Delta\Phi}{\Delta t}$$

$$= N\frac{\vec{B}\,.\,\Delta\vec{A}}{\Delta t}$$

$$= N\frac{B\Delta A\cos 0^\circ}{\Delta t}$$

$$= N\frac{B\Delta A}{\Delta t}$$

$$= 1 \times 0.6 \times 0.8$$

$$= 0.48\ JC^{-1}$$

$$\varepsilon = 0.48\ V$$

Result

Induced emf $= \varepsilon = 0.48$ V

EXAMPLE 15.3

An emf of 5.6 V is induced in a coil while the current in a nearby coil is decreased from 100 A to 20 A in 0.02 s. What is the mutual inductance of the two coils? If the secondary has 200 turns, find the change in flux during this interval.

Data

emf induced in the secondary $= \varepsilon_s = 5.6$ V

Change in current in primary $= \Delta I_p = 100\text{ A} - 20\text{ A} = 80$ A

Time interval for the change $= \Delta t = 0.02$ s

No. of turns in the secondary $= N_s = 200$

To Find

Mutual inductance $= M = ?$

Change in flux $= \Delta\Phi = ?$

SOLUTION

Using

$$\varepsilon_s = \frac{\Delta I_p}{\Delta t}M$$

$$5.6 = M \times \frac{80}{0.02}$$

$$M = \frac{5.6 \times 0.02}{80}$$

$$= 1.4 \times 10^{3} \text{ H}$$

By Faraday's law

$$\varepsilon_S = N_s \frac{\Delta\Phi_S}{\Delta t}$$

$$\Delta\Phi_S = \frac{\varepsilon_S \Delta t}{N_s}$$

$$= \frac{5.6 \times 0.02}{200}$$

$$= 5.6 \times 10^{-4} \text{ Wb}$$

Result

Mutual inductance = M = 1.4×10^{3} H

Change in flux = $\Delta\phi_s$ = 5.6×10^{-4} Wb

EXAMPLE 15.4

The current in a coil of 1000 turns is changed from 5 A to zero in 0.2 s. If an average emf of 50 V is induced during this interval, what is the self inductance of the coil? What is the flux through each turn of the coil when a current of 6A is flowing?

Data

Change in current = ΔI = 5A – 0

= 5A

Time interval = Δt = 0.2 s

emf induced = ε = 50 V

Steady current = I = 6 A

No. of turns of coil = N = 1000

To Find

Self inductance = L = ?

Flux through each turn = Φ = ?

SOLUTION

Using formula

$$L = \frac{\varepsilon}{\frac{\Delta I}{\Delta t}} = \frac{\varepsilon \times \Delta t}{\Delta I}$$

$$= \frac{50 \times 0.2}{5}$$

$$L = 2 \text{ H}$$

Now, using equation

$$N\Phi = LI \quad \text{or} \quad \Phi = \frac{LI}{N}$$

$$\Phi = \frac{2 \times 6}{1000}$$

$$= 1.2 \times 10^{-2}\ \text{Wb}$$

Result

Self inductance $= L = 2\ \text{H}$

Flux $= \phi = 1.2 \times 10^{-2}\ \text{Wb}$

EXAMPLE 15.5

A solenoid coil 10.0 cm long has 40 turns per cm. When the switch is closed, the current rises from zero to its maximum value of 5.0 A in 0.01 s. Find the energy stored in the magnetic field if the area of cross-section of the solenoid be 28 cm^2.

Data

Length of solenoid $= l = 10.0\ \text{cm} = 0.1\ \text{m}$

No. of turns $= n = 40$ per cm $= 4000$ per m

Area of cross section $= A = 28\ \text{cm}^2 = 2.8 \times 10^{-3}\ \text{m}^2$

Stead current $= I = 5\ \text{A}$

To Find

Energy stored $= U_m = ?$

SOLUTION

First, we calculate the inductance L using the equation

$$L = \mu_o n^2 A l$$

$$= (4\pi \times 10^{-7}) \times (4000)^2 \times 2.8 \times 10^{-3} \times 0.1$$

$$= 5.63 \times 10^{-3}\ \text{H}$$

$$\text{Energy stored} = U_m = \frac{1}{2} LI^2$$

$$= \frac{1}{2} (5.63 \times 10^{-3}) \times (5)^2$$

$$= 7.04 \times 10^{-2}\ \text{J}$$

Result

Energy stored $= U_m = 7.04 \times 10^{-2}\ \text{J}$

EXAMPLE 15.6

An alternating current generator operating at 50 Hz has a coil of 200 turns. The coil has an area of 120 cm^2. What should be the magnetic field in which the coil rotates in order to produce an emf of maximum value of 240 volts?

Data

Frequency of rotation = f = 50 Hz

No. of turns of the coil = N = 200

Area of the coil = A = 120 cm^2 = 1.2×10^{-2} m^2

Maximum emf = ε_{max} = 240 V

To Find

Magnetic flux density = B = ?

SOLUTION

Using $\omega = 2\pi f$

$$\omega = 2 \times \frac{22}{7} \times 50 = 314.3 \text{ rad s}^{-1}$$

Using $\varepsilon_o = N\omega AB$

$$B = \frac{\varepsilon_o}{N\omega A}$$

$$B = \frac{240}{200 \times 314.3 \times 1.2 \times 10^{-2}}$$

$$B = 0.32 \text{ T}$$

Result

Magnetic flux density = B = 0.32 T

EXAMPLE 15.7

A permanent magnet D.C motor is run by a battery of 24 volts. The coil of the motor has a resistance of 2 ohms. It develops a back emf of 22.5 volts when driving the load at normal speed. What is the current when motor just starts up? Also find the current when motor is running at normal speed.

Data

Operation voltage = V = 24 V

Resistance of the coil = R = 2 Ω

Back emf = ε = 22.5 V

To Find

Current = I = ?

SOLUTION

(i) When motor just starts up, the back emf $\varepsilon = 0$

Using $V = \varepsilon + IR$

$$24 = 0 + I \times 2$$

$$I = \frac{24}{2}$$

$$= 12 \text{ A}$$

(ii) When motor runs at normal speed, $\varepsilon = 22.5$ V then using

$$V = \varepsilon + IR$$

$$V = 22.5 + I \times 2$$

$$I = \frac{24 - 22.5}{2}$$

$$= 0.75 \text{ A}$$

Result

Current $= I_1 = 12$ A

$= I_2 = 0.75$ A

EXAMPLE 15.8

The turns ratio of a step up transformer is 50. A current of 20 A is passed through its primary coil at 220 volts. Obtain the value of the voltage and current in the secondary coil assuming the transformer to be ideal one.

Data

Turn ratio $= \frac{N_s}{N_p} = 50$

Current from primary $= I_p = 20$ A

Primary voltage $= V_p = 220$ V

To Find

Secondary voltage $= V_s = ?$

Secondary current $= I_s = ?$

SOLUTION

Using the equation

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}$$

But $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$$\frac{V_s}{220} = 50$$

$$V_s = 50 \times 220$$

$$= 11000 \text{ volt}$$

Since $I_s = \frac{V_p}{V_s} \times I_p$

$$= \frac{1}{50} \times 20$$

$$= 0.4 \text{ A}$$

Result

Secondary voltage = V_s = 11000 volt

Secondary current = I_s = 0.4 A