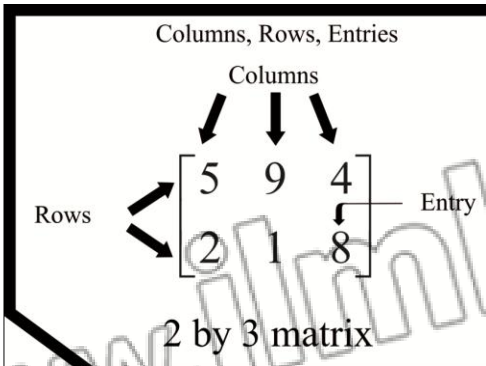


UNIT 1
MATRICES AND DETERMINANTS

Applications of Matrices (K.B)

The matrices and determinants are used in the field of mathematics, physics, statistics, Electronics and other branches of science. The matrices have played a very important role in this age of computer science.

The Idea of Matrices (K.B)

The idea of matrices was given by Arthur Cayley, an English mathematician of nineteen century who first developed, "Theory of Matrices" in 1858.

Matrix (K.B)

(D.G.K 2017, GRW 2017, FSD 2018, SGD 2018)

A rectangular array or a formation of a collection of real number say 0, 1, 2, 3 and 4 and 7 such as $\begin{matrix} 1 & 3 & 4 \\ 7 & 2 & 0 \end{matrix}$ and then enclosed by brackets '[ ]' is said to form a matrix.

The matrices are denoted conventionally by the capital letters A, B, C,…..,M, N etc of the English alphabets.

For example:

$$A=\begin{bmatrix} 1 & 3 & 4 \\ 7 & 2 & 0 \end{bmatrix}, B=\begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix} \text{ etc.}$$

Rows and Columns of a Matrix (K.B)

It is important to understand an entity of a matrix with the following formation.

Rows of a Matrix (K.B)

(BWP 2015, 16, SWL 2018)

In matrix, the entries presented in horizontal way are called rows.

$$A=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 8 \\ 7 & 1 & 5 \end{bmatrix}\begin{matrix} \to R_1 \\ \to R_2 \\ \to R_3 \end{matrix}.$$

Columns of a Matrix (K.B)

(SGD 2016, 18)

In matrix, all the entries presented in vertical way are called columns of matrix.

Matrix B has three columns as shown by C_1 C_2 and C_3.

$$B=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 8 \\ 7 & 1 & 5 \end{bmatrix}$$
$$\begin{matrix} \downarrow & \downarrow & \downarrow \\ C_1 & C_2 & C_3 \end{matrix}$$

Entries or Elements of a Matrix (K.B)

The real numbers used in the formation of a matrix are termed as entries or elements of a matrix.

Order of a Matrix (K.B)

The number of rows and columns in a matrix specifies its order. If a matrix M has m rows and n columns then M is said to be of order, $m-by-n$.

For example

Order of matrix $\begin{bmatrix} 2 & 0 & 5 \\ 4 & 1 & 3 \end{bmatrix}$ is $2-by-3$

Equal Matrices (K.B)

Let A and B be two matrices. Then A is said to be equal to B, and denoted by $A=B,$ if and only if;

(i) the order of A= the order of B

(ii) their corresponding entries are equal.

For example:

$A=\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 2+1 \\ -4 & 4-2 \end{bmatrix}$ are equal matrices.

Exercise 1.1

Q.1 Find the order of the following matrices.

$A=\begin{bmatrix}2 & 3\\ -5 & 6\end{bmatrix}$ **(A.B)**

It has 2 rows & 2 columns that's why its order is 2- by-2.

$B=\begin{bmatrix}2 & 0\\ 3 & 5\end{bmatrix}$

It has 2 rows & 2 columns. So, its order is

2- by -2.

$C=\begin{bmatrix}2 & 4\end{bmatrix}$

It has 1 row and 2 columns. So, its order is 1-by-2.

$D=\begin{bmatrix}4\\ 0\\ 6\end{bmatrix}$

It has 3 rows and 1 column. So, its order is 3 – by -1.

$E=\begin{bmatrix}a & d\\ b & e\\ c & f\end{bmatrix}$

It has 3 rows and 2 columns. So, its order is 3 – by –2.

$F=\begin{bmatrix}2\end{bmatrix}$

It has 1 row & 1 column. So, its order is 1- by -1.

$G=\begin{bmatrix}2 & 3 & 0\\ 1 & 2 & 3\\ 2 & 4 & 5\end{bmatrix}$

It has 3 rows and 3 columns. So, its order is 3 –by -3

$H=\begin{bmatrix}2 & 3 & 4\\ 1 & 0 & 6\end{bmatrix}$

It has 2 rows & 3 columns. So, its order is 2- by -3

Q.2 Which one of the following matrices are equal?

1) $A=\begin{bmatrix}3\end{bmatrix}$, **2)** $B=\begin{bmatrix}3 & 5\end{bmatrix}$,

3) $C=\begin{bmatrix}5-2\end{bmatrix}$ **4)** $D=\begin{bmatrix}5 & 3\end{bmatrix}$

5) $E=\begin{bmatrix}4 & 0\\ 6 & 2\end{bmatrix}$ **6)** $F=\begin{bmatrix}2\\ 6\end{bmatrix}$

7) $G=\begin{bmatrix}3-1\\ 3+3\end{bmatrix}$ **8)** $H=\begin{bmatrix}4 & 0\\ 6 & 2\end{bmatrix}$

9) $I=\begin{bmatrix}3 & 3+2\end{bmatrix}$ **10)** $J=\begin{bmatrix}2+2 & 2-2\\ 2+4 & 2+0\end{bmatrix}$

Solution:

Order of $A=\begin{bmatrix}3\end{bmatrix}$ is equal to Order of $C=\begin{bmatrix}5-2\end{bmatrix}=\begin{bmatrix}3\end{bmatrix}$

Order of $B=\begin{bmatrix}3 & 5\end{bmatrix}$ is equal to Order of $I=\begin{bmatrix}3 & 3+2\end{bmatrix}=\begin{bmatrix}3+5\end{bmatrix}$

$D=\begin{bmatrix}5 & 3\end{bmatrix}$ has no equal matrix

$E=\begin{bmatrix}4 & 0\\ 6 & 2\end{bmatrix}$ has equal matrices

Order of $\Rightarrow H=\begin{bmatrix}4 & 0\\ 6 & 2\end{bmatrix}$ Order of

$J=\begin{bmatrix}2+2 & 2-2\\ 2+4 & 2+0\end{bmatrix}$

Order of $F=\begin{bmatrix}2\\ 6\end{bmatrix}$ is equal to

Order of $G=\begin{bmatrix}3-1\\ 3+3\end{bmatrix}$

Q.3 Find the values of a, b, c & d which satisfy the matrix equation.

$$\begin{bmatrix}a+c & a+2b\\ c-1 & 4d-6\end{bmatrix}=\begin{bmatrix}0 & -7\\ 3 & +2d\end{bmatrix}$$ **(A.B)**

(LHR 2017)

Solution:

$$\begin{bmatrix}a+c & a+2b\\ c-1 & 4d-6\end{bmatrix}=\begin{bmatrix}0 & -7\\ 3 & +2d\end{bmatrix}$$

As Matrices are equal so their corresponding entries are same.

$a+c=0 \rightarrow (1)$

$a+2b=-7 \rightarrow (2)$

$c-1=3 \rightarrow (3)$

$4d-6=2d \rightarrow (4)$

Solving equation (3)
$c - 1 = 3$
$c = 3 + 1$
$c = 4$

Solving equation (1)
$a + c = 0$
$a + 4 = 0$
$a = -4$

Solving equation (2)
$a + 2b = -7$
$-4 + 2b = -7$
$2b = -7 + 4$
$2b = -3$
$b = \frac{-3}{2}$

Solving equation (4)
$4d - 6 = 2d$
$4d - 2d = 6$
$2d = 6$
$d = \frac{6}{2} = 3$

Result:

$a=-4,\ b=\frac{-3}{2},\ c=4,\ d=3$

Types of Matrices

(i) Row Matrix (K.B)

A matrix is called a row matrix, if it has only one row.
Example the matrix $M=\begin{bmatrix}2 & -1 & 7\end{bmatrix}$ is a row matrix of order 1–by–2.

(ii) Column Matrix (K.B)

A matrix is called a column matrix if it has only one column.

e.g., $M=\begin{bmatrix}1\\0\end{bmatrix}$ and $N=\begin{bmatrix}2\\0\\1\end{bmatrix}$ are column matrices of order 2–by–1 and 3–by–2 respectively.

(iii) Rectangular Matrix (K.B)

(GRW 2015, MTN 2015, RWP 2016, D.G.K 2018)

A matrix M is called rectangular if, the number of rows of M is not equal to the number of columns of M.

e.g., $B=\begin{bmatrix}a & b & c\\ d & e & d\end{bmatrix}$.

The order of B is 2-by-3

(iv) Square Matrix (K.B)

(FSD 2015, 17, LHR 215, SGD 2017)

A matrix is called a square matrix if its number of rows is equal to its number of columns.

e.g., $A=\begin{bmatrix}2 & -1\\ 0 & 3\end{bmatrix}$

the order of A is 2-by-2

(v) Null or Zero Matrix (K.B)

(LHR 2018, D.G.K 2015)

A matrix M is called a null or zero matrix if each of its entries is 0.

e.g., $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$ and $\begin{bmatrix}0\\0\end{bmatrix}$

are null matrix of order 2-by-2, 1-by-2, 3-by-3 and 2-by-1 respectively

Note (U.B)

Null matrix is represented by O.

(vi) Transpose of a Matrix (K.B)

A matrix obtained by changing the rows into columns or columns into rows of a matrix is called transpose of that matrix.
If A is a matrix, then its transpose is denoted by A^t.

e.g., If $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 0\\ -1 & 4 & -2\end{bmatrix}$,

then $A^t=\begin{bmatrix}1 & 2 & -1\\ 2 & 1 & 4\\ 3 & 0 & -2\end{bmatrix}$

Note **(U.B)**

If a matrix A is of order 2-by-3, then order of its transpose A^t is 3-by-2.

(vii) Negative of a Matrix **(K.B)**

Let A be matrix. Then its negative, $-A$ is obtained by changing the signs of all the entries of A,

i.e., If

$$A=\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}, \text{then} -A=\begin{bmatrix}-1 & 2\\ -3 & -4\end{bmatrix}$$

(viii) Symmetric Matrix **(K.B)**

(SGD 2015, 17, BWP 2015, FSD 2016, SWL 2016, 17, MTN 2017)

A square matrix is symmetric if it is equal to its transpose i.e., matrix A is symmetric

if $A^t = A$.

e.g.,

(i) If $M=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 4 & 0\end{bmatrix}$

is a square matrix, then

$$M^t=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 4 & 0\end{bmatrix}=M.$$

Thus M is a symmetric matrix.

(ix) Skew-Symmetric Matrix **(K.B)**

(D.G.K 2018)

A square matrix A is said to be skew-symmetric if $A^t = -A$.

e.g., $A=\begin{bmatrix}0 & 2 & 3\\ -2 & 0 & 1\\ -3 & -1 & 0\end{bmatrix}$, then

$$A^t=\begin{bmatrix}0 & -2 & -3\\ 2 & 0 & -1\\ 3 & 1 & 0\end{bmatrix}=\begin{bmatrix}0 & -2 & -3\\ -(-2) & 0 & -1\\ -(-3) & -(-1) & 0\end{bmatrix}$$

$$=-\begin{bmatrix}0 & 2 & 3\\ -2 & 0 & 1\\ -3 & -1 & 0\end{bmatrix}=-A$$

Since $A^t = -A$, therefore A is a skew symmetric matrix.

(x) Diagonal Matrix **(K.B)**

(RWP 2015, MTN 2016, BWP 2018)

A square matrix A is called a diagonal matrix if at least any one of the entries of its diagonal is not zero and non-diagonal entries are zero.

i.e., $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$.

Is diagonal matrices of order 3-by-3.

(xi) Scalar Matrix **(K.B)**

(BWP 2015, 18, MTN 2016, FSD 2016, LHR 2017, GRW 2018)

A diagonal matrix is called a scalar matrix, if all the diagonal entries are same and non-zero.

For example $\begin{bmatrix}k & 0 & 0\\ 0 & k & 0\\ 0 & 0 & k\end{bmatrix}$ where k is a constant $\neq 0,1$

For example $A=\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$

is scalar matrix of order 3-by-3 respectively.

(xii) Identity Matrix **(LHR 2018)** **(K.B)**

A diagonal matrix is called identity (unit) matrix if all diagonal entries are 1 and it is denoted by I.

e.g., $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ is a 3-by-3 identity matrix.

Note **(K.B+U.B)**

(i) The scalar matrix and identity matrix are diagonal matrices.

(ii) Every diagonal matrix is not a scalar or identity matrix.

Exercise 1.2

Q.1 Identify the following matrices.

$A=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ **(U.B)**

It's all members are 0. So, it's a null matrix.

$B=\begin{bmatrix}2 & 3 & 4\end{bmatrix}$ **(U.B)**

It has only 1 row. So, it's a row matrix.

$C=\begin{bmatrix}4\\ 0\\ 6\end{bmatrix}$ **(U.B)**

It has only 1 column. So, it's a column matrix.

$D=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ **(U.B)**

Its an identity matrix because its diagonal entries are 1 and non-diagonal enteries are zero.

$E=\begin{bmatrix}0\end{bmatrix}$ **(U.B)**

It has only 0. So, it's a null matrix.

$F=\begin{bmatrix}5\\ 6\\ 7\end{bmatrix}$ **(U.B)**

It has only 1 column. So, it's a column matrix.

Q.2 Identify the following matrices.

(i) $\begin{bmatrix}-8 & 2 & 7\\ 12 & 0 & 4\end{bmatrix}$ **(U.B)**

Its number of rows & columns are not equal. So, it's a rectangular matrix.

(ii) $\begin{bmatrix}3\\ 0\\ 1\end{bmatrix}$ **(U.B)**

It has only one column. So, it's a column matrix.

(iii) $\begin{bmatrix}6 & -4\\ 3 & -2\end{bmatrix}$ **(U.B)**

The number of rows & columns are equal. So, it's a square matrix.

$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Identity matrix – Because Diagonal entries are 1 and non-diagonal entries are 0.

(iv) $\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$ **(U.B)**

Number of rows & columns are not equal. So, it's a rectangular matrix.

(v) $\begin{bmatrix}3 & 10 & -1\end{bmatrix}$ **(U.B)**

It's a row matrix because it has only 1 row.

(vi) $\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}$ **(U.B)**

It's a column matrix because it has only one column.

(vii) $\begin{bmatrix}1 & 2 & 3\\ -1 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}$ **(U.B)**

It's a square matrix because number of rows & columns are equal.

(viii) $\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$ **(U.B)**

It's a null matrix because all elements are 0.

Q.3 Identify the matrices.

(1) $A=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$ **(U.B)**

It's a scalar-matrix because it non-diagonal entries are 0 & diagonal entries are same.

(2) $B=\begin{bmatrix}2 & 0\\ 0 & -1\end{bmatrix}$ **(U.B)**

It's a diagonal matrix because its non-diagonal entries are 0.

(3) $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(U.B)**

It's a unit matrix because diagonal-entries are 1.

(4) $D = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$ **(U.B)**

It's a diagonal matrix because non-diagonal entries are 0.

(5) $E = \begin{bmatrix} 5-3 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ **(U.B)**

It's a scalar matrix because diagonal entries are same.

Q.4 Find the negative of matrices.

(1) $A = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ **(A.B)**

$$-A = -\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(2) $B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ **(A.B)**

$$-B = -\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

$$-B = \begin{bmatrix} -3 & +1 \\ -2 & -1 \end{bmatrix}$$

(3) $C = \begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$ **(A.B)**

$$-C = -\begin{bmatrix} 2 & 6 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -6 \\ -3 & -2 \end{bmatrix}$$

(4) $D = \begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$ **(A.B)**

$$-D = -\begin{bmatrix} -3 & 2 \\ -4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} +3 & -2 \\ +4 & -5 \end{bmatrix}$$

(5) $E = \begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$ **(A.B)**

$$-E = -\begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & +5 \\ -2 & -3 \end{bmatrix}$$

Q.5 Find the transpose.

(1) $A = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$ **(A.B)**

$$A^t = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}^t$$

$$A^t = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix}$$

(2) $B = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}$ **(LHR 2019)** **(A.B)**

$$B^t = \begin{bmatrix} 5 & 1 & -6 \end{bmatrix}^t$$

$$B^t = \begin{bmatrix} 5 \\ 1 \\ -6 \end{bmatrix}$$

(3) $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$ **(FSD 2016)** **(A.B)**

$$C^t = \begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}^t$$

$$C^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \end{bmatrix}$$

(4) $D = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$ **(A.B)**

$$D^t = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}^t$$

$$D^t = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$$

(5) $E=\begin{bmatrix}2 & 3\\ -4 & 5\end{bmatrix}$ **(A.B)**

$E^t=\begin{bmatrix}2 & 3\\ -4 & 5\end{bmatrix}^t$

$E^t=\begin{bmatrix}2 & -4\\ 3 & 5\end{bmatrix}$

(6) $F=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ **(A.B)**

$F^t=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}^t$

$F^t=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$

Q.6 Verify that if $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ **and**

$B=\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix}$ **then**

(i) $(A^t)^t = A$ **(SWL 2018, SGD 2015, 17)** **(A.B)**

Verification:

$A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$

$A^t=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}^t$

$A^t=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$

$\left(A^t\right)^t=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}^t$

$\left(A^t\right)^t=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$

$\left(A^t\right)^t=A$

Hence Proved.

(ii) $\left(B^t\right)^t=B$ **(A.B)**

(SWL 2016, D.G.K 2016, 18)

Verification:

$B=\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix}$

$B^t=\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix}^t$

$B^t=\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}$

$\left(B^t\right)^t=\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}^t$

$\left(B^t\right)^t=\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix}$

$\left(B^t\right)^t=B$

Hence proved

ADDITION AND SUBTRACTION OF MATRICES

Addition of Matrices **(K.B)**

Let A and B be any two matrices with real number entries. The matrices A and B are conformable for addition, if they have the same order.

e.g., $A=\begin{bmatrix}2 & 3 & 0\\ 1 & 0 & 6\end{bmatrix}$ and $B=\begin{bmatrix}-2 & 3 & 4\\ 1 & 2 & 3\end{bmatrix}$

are conformable for addition.

Addition of A and B, written A+B is obtained by adding the entries of the matrix A to the corresponding entries of the matrix B.

e.g., $A+B=\begin{bmatrix}2 & 3 & 0\\ 1 & 0 & 6\end{bmatrix}+\begin{bmatrix}-2 & 3 & 4\\ 1 & 2 & 3\end{bmatrix}$

$=\begin{bmatrix}2+(-2) & 3+3 & 0+4\\ 1+1 & 0+2 & 6+3\end{bmatrix}$

$=\begin{bmatrix}0 & 6 & 4\\ 2 & 2 & 9\end{bmatrix}$

Subtraction of Matrices **(K.B)**

If A and B are two matrices of same order, then subtraction of matrix B from matrix A is obtained by subtracting the entries of matrix B from the corresponding entries of matrix A and it is denoted by A – B

e.g.,

$A=\begin{bmatrix}2 & 3 & 4\\ 1 & 5 & 0\end{bmatrix}$ and $B=\begin{bmatrix}0 & 2 & 2\\ -1 & 4 & 3\end{bmatrix}$ are conformable for subtraction.

i.e.,

$$A-B=\begin{bmatrix}2 & 3 & 4\\ 1 & 5 & 0\end{bmatrix}-\begin{bmatrix}0 & 2 & 2\\ -1 & 4 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2\text{-}0 & 3\text{-}2 & 4\text{-}2\\ 1\text{-}(-1) & 5\text{-}4 & 0\text{-}3\end{bmatrix}=\begin{bmatrix}2 & 1 & 2\\ 2 & 1 & -3\end{bmatrix}$$

Note **(U.B)**

That the order of a matrix is unchanged under the operation of matrix addition and matrix subtraction.

Multiplication of a Matrix by a Real Number **(K.B)**

Let A be any matrix and the real number K be a scalar. Then the scalar multiplication of matrix A with K is obtained by multiplying each entry of matrix A with K. It is denoted by KA

$$\text{Let } A=\begin{bmatrix}1 & -1 & 4\\ 2 & -1 & 0\\ -1 & 3 & 2\end{bmatrix}$$

be a matrix of order 3–by–3 and k = –2 be a real number.

Then $kA=(-2)A$

$$=(-2)\begin{bmatrix}1 & -1 & 4\\ 2 & -1 & 0\\ -1 & 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}(-2)(1) & (-2)(-1) & (-2)(4)\\ (-2)(2) & (-2)(-1) & (-2)(0)\\ (-2)(-1) & (-2)(3) & (-2)(2)\end{bmatrix}$$

$$kA=\begin{bmatrix}-2 & 2 & -8\\ -4 & 2 & 0\\ 2 & -6 & -4\end{bmatrix}$$

Scalar multiplication of a matrix leaves the order of the matrix unchanged.

Commutative and Associative Laws of Addition of Matrices **(K.B)**

(a) Commutative law under addition **(U.B)**

If A and B are two matrices of the same order, then A+B = B+A is called commutative law under addition.

Let

$$A=\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix},\ B=\begin{bmatrix}3 & -2 & 5\\ -1 & 4 & 1\\ 4 & 2 & -4\end{bmatrix}$$

$$\text{Then. } A+B=\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}+\begin{bmatrix}3 & -2 & 5\\ -1 & 4 & 1\\ 4 & 2 & -4\end{bmatrix}$$

$$=\begin{bmatrix}2+3 & 3-2 & 0+5\\ 5-1 & 6+4 & 1+1\\ 2+4 & 1+2 & 3-4\end{bmatrix}$$

$$A+B=\begin{bmatrix}5 & 1 & 5\\ 4 & 10 & 2\\ 6 & 3 & -1\end{bmatrix}$$

$$=\begin{bmatrix}5 & 1 & 5\\ 4 & 10 & 2\\ 6 & 3 & -1\end{bmatrix}$$

Thus the commutative law of addition of matrices is verified A + B = B + A

Similarly

$$B+A=\begin{bmatrix}3 & -2 & 5\\ -1 & 4 & 1\\ 4 & 2 & -4\end{bmatrix}+\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}$$

(b) Associative Law Under Addition **(U.B)**

If A, B and C are three matrices of same order, then $(A+B)+C=A+(B+C)$ is called associative law under addition.

Let $A=\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}$, $B=\begin{bmatrix}3 & -2 & 5\\ -1 & 4 & 1\\ 4 & 2 & -4\end{bmatrix}$

and $C=\begin{bmatrix}1 & 2 & 3\\ -2 & 0 & 4\\ 1 & 2 & 0\end{bmatrix}$

$$\text{Then } (A+B)+C=\left(\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}+\begin{bmatrix}3 & -2 & 5\\ -1 & 4 & 1\\ 4 & 2 & -4\end{bmatrix}\right)+\begin{bmatrix}1 & 2 & 3\\ -2 & 0 & 4\\ 1 & 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2+3 & 3-2 & 0+5\\ 5-1 & 6+4 & 1+1\\ 2+4 & 1+2 & 3-4\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ -2 & 0 & 4\\ 1 & 2 & 0\end{bmatrix}$$

$$=\begin{bmatrix}5 & 1 & 5\\ 4 & 10 & 2\\ 6 & 3 & -1\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ -2 & 0 & 4\\ 1 & 2 & 0\end{bmatrix}=\begin{bmatrix}6 & 3 & 8\\ 2 & 10 & 6\\ 7 & 5 & -1\end{bmatrix}$$

$$A+(B+C)=\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}+\left(\begin{bmatrix}3 & -2 & 5\\ -1 & 4 & 1\\ 4 & 2 & -4\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\ -2 & 0 & 4\\ 1 & 2 & 0\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}+\begin{bmatrix}3+1 & -2+2 & 5+3\\ -1-2 & 4+0 & 1+4\\ 4+1 & 2+2 & -4+0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3 & 0\\ 5 & 6 & 1\\ 2 & 1 & 3\end{bmatrix}+\begin{bmatrix}4 & 0 & 8\\ -3 & 4 & 5\\ 5 & 4 & -4\end{bmatrix}=\begin{bmatrix}6 & 3 & 8\\ 2 & 10 & 6\\ 7 & 5 & -1\end{bmatrix}$$

Thus the associative law of addition is verified:

$(A+B)+C=A+(B+C)$

Additive Identity of Matrices (U.B)

If A and B are two matrices of same order and A+B = A = B+A

then matrix B is called additive identity of matrix A

For any matrix A and zero matrix O of same order, O is called additive identity of A as $A+0=A=0+A$

e.g., Let $A=\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}$ and $O=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

then $A+O=\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}+\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}=\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}=A$

$O+A=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}+\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}=\begin{bmatrix}1 & 2\\3 & 5\end{bmatrix}=A$

Additive Inverse of a Matrix (U.B)

If A and B are two matrices of same order such that A+B = O = B+A

Then A and B are called additive inverse of each other.

Additive inverse of any matrix A is obtained by changing to negative of the symbols (entries) of each non zero entry of A

Let $A=\begin{bmatrix}1 & 2 & 1\\0 & -1 & -2\\3 & 1 & 0\end{bmatrix}$

then

$$B=(-A)=-\begin{bmatrix}1 & 2 & 1\\0 & -1 & -2\\3 & 1 & 0\end{bmatrix}=\begin{bmatrix}-1 & -2 & -1\\0 & 1 & 2\\-3 & -1 & 0\end{bmatrix}$$

is additive inverse of A.

It can be verified as

$$A+B=\begin{bmatrix}1 & 2 & 1\\0 & -1 & -2\\3 & 1 & 0\end{bmatrix}+\begin{bmatrix}-1 & -2 & -1\\0 & 1 & 2\\-3 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(1)+(-1) & (2)+(-2) & (1)+(-1)\\0+0 & (-1)+(1) & (-2)+(2)\\(3)+(-3) & (1)+(-1) & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}=O$$

$$B+A=\begin{bmatrix}-1 & -2 & -1\\0 & 1 & 2\\-3 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & 2 & 1\\0 & -1 & -2\\3 & 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(-1)+(1) & (-2)+(2) & (-1)+(1)\\0+0 & (1)+(-1) & (2)+(-2)\\(-3)+(3) & (-1)+(1) & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}=O$$

Since A+B = 0 = B+A

Therefore, A and B are additive inverse of each other.

Exercise 1.3

Q.1 Which of the following matrices are comfortable for addition?

$A=\begin{bmatrix}2 & 1\\-1 & 3\end{bmatrix}$, $B=\begin{bmatrix}3\\1\end{bmatrix}$ **(K.B)**

$C=\begin{bmatrix}1 & 0\\2 & -1\\1 & -2\end{bmatrix}$, $D=\begin{bmatrix}2+1\\3\end{bmatrix}$ **(K.B)**

$E=\begin{bmatrix}-1 & 0\\1 & 2\end{bmatrix}$, $F=\begin{bmatrix}3 & 2\\1+1 & -4\\3+2 & 2+1\end{bmatrix}$ **(K.B)**

Solution:

In the above matrices following matrices are suitable for addition.

(i) A and E are conformable for addition because their order is same and both are square matrix.

(ii) B and D are conformable for addition because the order is same i-e they have two rows and 1columns and both are rectangular matrices and column matrix.

(iii) C and F are conformable for addition because their order is same i-e they have three 3 rows and 2 columns and they are rectangular matrices.

Q.2 Find the additive inverse of the following matrices:

(1) $A=\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$ **(FSD 2015, MTN 2016)** **(A.B)**

Solution:

Additive inverse of a matrix is negative matrix.

$A=\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$ is

$$-A=-\begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}=\begin{bmatrix} (-1)\times 2 & (-1)4 \\ (-1)(-2) & (-1)1 \end{bmatrix}$$

$$-A=\begin{bmatrix} -2 & -4 \\ 2 & -1 \end{bmatrix}$$

(2) $B=\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$ **(A.B)**

Solution: $B=\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

It's additive inverse is

$$-B=-\begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$$

$$-B=\begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$$

(3) $C=\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ **(A.B)**

Solution: $C=\begin{bmatrix} 4 \\ -2 \end{bmatrix}$

$$-C=-\begin{bmatrix} 4 \\ -2 \end{bmatrix}=\begin{bmatrix} -1\times 4 \\ -1\times -2 \end{bmatrix}$$

The additive inverse is

$$-C=\begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

(4) $D=\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$ **(A.B)**

Solution: $D=\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}$

The additive inverse is

$$-D=-\begin{bmatrix} 1 & 0 \\ -3 & -2 \\ 2 & 1 \end{bmatrix}=\begin{bmatrix} -1\times 1 & -1\times 0 \\ -1\times -3 & -1\times -2 \\ -1\times 2 & -1\times 1 \end{bmatrix}$$

$$-D=\begin{bmatrix} -1 & 0 \\ 3 & 2 \\ -2 & -1 \end{bmatrix}$$

(5) $E=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(A.B)**

Solution: $E=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The additive inverse of the given matrix is:

$$-E=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} -1\times 1 & -1\times 0 \\ -1\times 0 & -1\times 1 \end{bmatrix}$$

$$-E=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

(6) $F=\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$ **(A.B)**

Solution: $F=\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$

Its additive inverse is

$$-F=-\begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

$$=\begin{bmatrix} -1\times\sqrt{3} & -1\times 1 \\ -1\times -1 & -1\times\sqrt{2} \end{bmatrix}$$

$$-F=\begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$$

Q.3 **If** $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix},$

$C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}, D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix},$

then find. **(A.B)**

(i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Solution:

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So, $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

The order of matrix A and the given matrix order is same. So, they can be added easily.

$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$

(ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ **(SGD 2017)** **(A.B)**

Solution:

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So, $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 1+(-2) \\ -1+3 \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(iii) $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$ **(A.B)**

Solution:

As $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

So, $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+(-2) & -1+(1) & 2+3 \end{bmatrix}$

$= \begin{bmatrix} 1-2 & -1+1 & 5 \end{bmatrix}$

$= \begin{bmatrix} -1 & 0 & 5 \end{bmatrix}$

(iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ **(A.B)**

Solution:

As $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

So, $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 2+1 & 3+0 \\ -1+2 & 0+0 & 2+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 0 & 3 \end{bmatrix}$

(v) $2A$ **(RWP 2018)** **(A.B)**

Solution:

As $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

So,

$2A = 2\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2(-1) & 2\times 2 \\ 2\times 2 & 2\times 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 4 \\ 4 & 2 \end{bmatrix}$

(vi) $(-1)B$ **(A.B)**

Solution:

As $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

So,

$(-1)B = (-1)\times\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} (-1)\times\ 1 \\ (-1)\times(-1) \end{bmatrix}$

$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(vii) $(-2)C$ **(SWL 2018)** **(A.B)**

Solution:

As $C=\begin{bmatrix}1 & -1 & 2\end{bmatrix}$

So,

$$(-2)C=(-2)\times\begin{bmatrix}1 & -1 & 2\end{bmatrix}$$
$$=\begin{bmatrix}(-2)(1) & (-2)(-1) & (-2)(2)\end{bmatrix}$$
$$=\begin{bmatrix}-2 & 2 & -4\end{bmatrix}$$

(viii) $3D$ **(A.B)**

Solution:

As $D=\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{bmatrix}$

So, $3D=(3)\begin{bmatrix}1 & 2 & 3\\ -1 & 0 & 2\end{bmatrix}$

$$=\begin{bmatrix}3\times1 & 3\times2 & 3\times3\\ 3\times-1 & 3\times0 & 3\times2\end{bmatrix}$$
$$=\begin{bmatrix}3 & 6 & 9\\ -3 & 0 & 6\end{bmatrix}$$

(ix) $3C$ **(A.B)**

Solution:

As $C=\begin{bmatrix}1 & -1 & 2\end{bmatrix}$

So, $3C=(3)\times\begin{bmatrix}1 & -1 & 2\end{bmatrix}$

$$=\begin{bmatrix}3\times1 & 3\times-1 & 3\times2\end{bmatrix}$$
$$=\begin{bmatrix}3 & -3 & 6\end{bmatrix}$$

Q.4 Perform the indicated operations and simplify the following:

(i) $\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$ **(A.B)**

Solution:

$$\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1+0 & 0+2\\ 0+3 & 1+0\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1+1 & 2+1\\ 3+1 & 1+0\end{bmatrix}$$
$$=\begin{bmatrix}2 & 3\\ 4 & 1\end{bmatrix}$$

(ii) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\left(\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\right)$ **(A.B)**

Solution:

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\left(\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}-\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\right)$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0-1 & 2-1\\ 3-1 & 0-0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1-1 & 0+1\\ 0+2 & 1+0\end{bmatrix}$$
$$=\begin{bmatrix}0 & 1\\ 2 & 1\end{bmatrix}$$

(iii) $\begin{bmatrix}2 & 3 & 1\end{bmatrix}+\left(\begin{bmatrix}1 & 0 & 2\end{bmatrix}-\begin{bmatrix}2 & 2 & 2\end{bmatrix}\right)$ **(A.B)**

Solution:

$$\begin{bmatrix}2 & 3 & 1\end{bmatrix}+\left(\begin{bmatrix}1 & 0 & 2\end{bmatrix}-\begin{bmatrix}2 & 2 & 2\end{bmatrix}\right)$$
$$=\begin{bmatrix}2 & 3 & 1\end{bmatrix}+\begin{bmatrix}1-2 & 0-2 & 2-2\end{bmatrix}$$
$$=\begin{bmatrix}2 & 3 & 1\end{bmatrix}+\begin{bmatrix}-1 & -2 & 0\end{bmatrix}$$
$$=\begin{bmatrix}2-1 & 3-2 & 1-0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

(iv) $\begin{bmatrix}1 & 2 & 3\\ -1 & -1 & -1\\ 0 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 3 & 3 & 3\end{bmatrix}$ **(A.B)**

Solution:

$$\begin{bmatrix}1 & 2 & 3\\ -1 & -1 & -1\\ 0 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 1 & 1\\ 2 & 2 & 2\\ 3 & 3 & 3\end{bmatrix}$$
$$=\begin{bmatrix}1+1 & 2+1 & 3+1\\ -1+2 & -1+2 & -1+2\\ 0+3 & 1+3 & 2+3\end{bmatrix}$$
$$=\begin{bmatrix}2 & 3 & 4\\ 1 & 1 & 1\\ 3 & 4 & 5\end{bmatrix}$$

(v) $\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 0\\0 & 2 & -1\end{bmatrix}$ **(A.B)**

Solution:

$$\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\3 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0 & -2\\-2 & -1 & 0\\0 & 2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2+0 & 3-2\\2-2 & 3-1 & 1+0\\3+0 & 1+2 & 2-1\end{bmatrix}$$

$$=\begin{bmatrix}2 & 2 & 1\\0 & 2 & 1\\3 & 3 & 1\end{bmatrix}$$

(vi) $\left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ **(A.B)**

Solution:

$$\left(\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}+\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}\right)+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1+2 & 2+1\\0+1 & 1+0\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\1 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3+1 & 3+1\\1+1 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}4 & 4\\2 & 2\end{bmatrix}$$

Q.5 For the matrices $A=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$,

$B=\begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$ **and** $C=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$,

verify the following rules:

(i) $A+C=C+A$ **(K.B)**

Verification:

L.H.S.= A+C

$$=\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & 2+0 & 3+0\\2+0 & 3-2 & 1+3\\1+1 & -1+1 & 0+2\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$$

R.H.S.= C+A

$$=\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1+1 & 0+2 & 0+3\\0+2 & -2+3 & 3+1\\1+1 & 1-1 & 2-0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2 & 3\\2 & 1 & 4\\2 & 0 & 2\end{bmatrix}$$

$$A+C=C+A$$

Hence proved

(ii) $A+B=B+A$ **(K.B)**

Verification:

L.H.S.= A+B

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}1+1&2-1&+3+1\\2+2&3-2&1+2\\1+3&-1+1&0+3\end{bmatrix}$$

$$=\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}$$

R.H.S.= B+A

$$=\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}+\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}1+1&-1+2&1+3\\2+2&-2+3&2+1\\3+1&1-1&3+0\end{bmatrix}$$

$$=\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}$$

$A+B=B+A$

Hence proved

(iii) $B+C=C+B$ **(K.B)**

Verification:

L.H.S. = B+C

$$=\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}$$

$$=\begin{bmatrix}1-1&-1+0&1+0\\2+0&-2-2&2+3\\3+1&1+1&3+2\end{bmatrix}$$

$$=\begin{bmatrix}0&-1&1\\2&-4&5\\4&2&5\end{bmatrix}$$

R.H.S. = C+B

$$=\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}-1+1&0-1&0+1\\0+2&-2-2&3+2\\1+3&1+1&2+3\end{bmatrix}$$

$$=\begin{bmatrix}0&-1&1\\2&-4&5\\4&2&5\end{bmatrix}$$

L.H.S. = R.H.S.

B+C=C+B

Hence proved

(iv) $A+\left(B+A\right)=2A+B$ **(K.B)**

Verification:

L.H.S.= A+ (B+A)

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\left(\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}+\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}\right)$$

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}$$

$$=\begin{bmatrix}3&3&7\\6&4&4\\5&-1&3\end{bmatrix}$$

R.H.S. = 2A+B

$$=2\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}2&4&6\\4&6&2\\2&-2&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}3&3&7\\6&4&4\\5&-1&3\end{bmatrix}$$

L.H.S. =R.H.S.

A+ (B+A) =2A+B

Hence proved

(v) $(C-B)+A=C+(A+B)$ **(A.B+K. B)**

Verification:

L.H.S. = (C-B) + A

$$=\left(\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)+\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}-2&1&-1\\-2&0&1\\-2&0&-1\end{bmatrix}+\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}-1&3&2\\0&3&2\\-1&-1&-1\end{bmatrix}$$

R.H.S.= C+ (A-B)

$$=\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}+\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}+\begin{bmatrix}0&3&2\\0&5&-1\\-2&-2&-3\end{bmatrix}$$

$$=\begin{bmatrix}-1&3&2\\0&3&2\\-1&-1&-1\end{bmatrix}$$

L.H.S. = R.H.S.

(C-B)+A=C+ (A-B)

Hence proved

(vi) $2A+B=A+(A+B)$ **(K.B)**

Verification:

L.H.S.= 2A+B

$$=2\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}2&4&6\\4&6&2\\2&-2&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}3&3&7\\6&4&4\\5&-1&3\end{bmatrix}$$

R.H.S.= A + (A+B)

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}$$

$$=\begin{bmatrix}3&3&7\\6&4&4\\5&-1&3\end{bmatrix}$$

L.H.S. = R.H.S.

2A+B=A+ (A+B)

Hence proved

(vii) $(C-B)-A=(C-A)-B$

(A.B + K. B)

Verification:

L.H.S.= (C – B) – A

$$=\left(\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)-\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}-2&1&-1\\-2&0&1\\-2&0&-1\end{bmatrix}-\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}$$

$$=\begin{bmatrix}-3&-1&-4\\-4&-3&0\\-3&1&-1\end{bmatrix}$$

R.H.S.= (C-A) –B

$$=\left(\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}-\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}\right)-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}-2&-2&-3\\-2&-5&2\\0&2&2\end{bmatrix}-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$=\begin{bmatrix}-3 & -1 & -4\\ -4 & -3 & 0\\ -3 & 1 & -1\end{bmatrix}$$

L.H.S. = R.H.S.

(C-B)-A=(C-A)-B

Hence proved

$(A+B)+C=A+(B+C)$ **(U.B)**

Verification:

L.H.S.= (A+B) +C

$$=\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 1 & 4\\ 4 & -1 & 6\\ 5 & 1 & 5\end{bmatrix}$$

R.H.S. = A+ (B+C)

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 3 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}+\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1 & 1 & 4\\ 4 & -1 & 6\\ 5 & 1 & 5\end{bmatrix}$$

L.H.S. = R.H.S.

(A+B)+C=A+ (B+C)

Hence proved

(viii) $A+(B-C)=(A-C)+B$ **(A.B + K. B)**

Verification:

L.H.S. = A+ (B-C)

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\left(\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}2 & -1 & 1\\ 2 & 0 & -1\\ 2 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & 4\\ 4 & 3 & 0\\ 3 & -1 & 1\end{bmatrix}$$

R.H.S.= (A-C) +B

$$=\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}-\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 3\\ 1 & 1 & 2\end{bmatrix}\right)+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+1 & 2-0 & 3-0\\ 2-0 & 3+2 & 1-3\\ 1-1 & -1-1 & 0-2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 2 & 3\\ 2 & 5 & -2\\ 0 & -2 & -2\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3 & 1 & 4\\ 4 & 3 & 0\\ 3 & -1 & 1\end{bmatrix}$$

L.H.S. = R.H.S.

A+ (B-C) = (A-C) +B

Hence proved

(ix) $2A+2B=2(A+B)$ **(U.B)**

Verification:

L.H.S. = 2A + 2B

$$=2\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+2\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}2 & 4 & 6\\ 4 & 6 & 2\\ 2 & -2 & 0\end{bmatrix}+\begin{bmatrix}2 & -2 & 2\\ 4 & -4 & 4\\ 6 & 2 & 6\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & 8\\ 8 & 2 & 6\\ 8 & 0 & 6\end{bmatrix}$$

R.H.S.= 2 (A+B)

$$=2\left(\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 1 & -1 & 0\end{bmatrix}+\begin{bmatrix}1 & -1 & 1\\ 2 & -2 & 2\\ 3 & 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}1+1 & 2-1 & 3+1\\ 2+2 & 3-2 & 1+2\\ 1+3 & -1+1 & 0+3\end{bmatrix}$$

$$=2\begin{bmatrix}2 & 1 & 4\\ 4 & 1 & 3\\ 4 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & 8\\ 8 & 2 & 6\\ 8 & 0 & 6\end{bmatrix}$$

L.H.S. = R.H.S.

2A+2B=2(A+B)

Hence proved

Q.6 If $A=\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}$ **and** $B=\begin{bmatrix}0 & 7\\ -3 & 8\end{bmatrix}$ **find:**

(i) $3A-2B$ **(A.B)**

Solution:

$$3A-2B=3\begin{bmatrix}1 & -2\\ 3 & 4\end{bmatrix}-2\begin{bmatrix}0 & 7\\ -3 & 8\end{bmatrix}$$

$$=\begin{bmatrix}3 & -6\\ 9 & 12\end{bmatrix}-\begin{bmatrix}0 & 14\\ -6 & 16\end{bmatrix}$$

$$=\begin{bmatrix}3 & -20\\ 15 & -4\end{bmatrix}$$

(ii) $2A^t-3B^t$ **(A.B)**

Solution:

$$A^t=\begin{bmatrix}1 & 3\\ -2 & 4\end{bmatrix}$$

$$B^t=\begin{bmatrix}0 & -3\\ 7 & 8\end{bmatrix}$$

Now

$$2A^t-3B^t=2\begin{bmatrix}1 & 3\\ -2 & 4\end{bmatrix}-3\begin{bmatrix}0 & -3\\ 7 & 8\end{bmatrix}$$

$$=\begin{bmatrix}2 & 6\\ -4 & 8\end{bmatrix}-\begin{bmatrix}0 & -9\\ 21 & 24\end{bmatrix}$$

$$=\begin{bmatrix}2 & 15\\ -25 & -16\end{bmatrix}$$

Q.7 If $2\begin{bmatrix}2 & 4\\ -3 & a\end{bmatrix}+3\begin{bmatrix}1 & b\\ 8 & -4\end{bmatrix}=\begin{bmatrix}7 & 10\\ 18 & 1\end{bmatrix}$ **(A.B)**

(LHR 2017)

Solution:

$$2\begin{bmatrix}2 & 4\\ -3 & a\end{bmatrix}+3\begin{bmatrix}1 & b\\ 8 & -4\end{bmatrix}=\begin{bmatrix}7 & 10\\ 18 & 1\end{bmatrix}$$

$$\begin{bmatrix}4 & 8\\ -6 & 2a\end{bmatrix}+\begin{bmatrix}3 & 3b\\ 24 & -12\end{bmatrix}=\begin{bmatrix}7 & 10\\ 18 & 1\end{bmatrix}$$

$$\begin{bmatrix}7 & 8+3b\\ 18 & 2a+(-12)\end{bmatrix}=\begin{bmatrix}7 & 10\\ 18 & 1\end{bmatrix}$$

By comparing equal matrices, we get

8 + 3b = 10 _________ (i)

2a – 12 = 1 __________ (ii)

By solving equation (ii)

2a – 12 = 1

2a = 1 + 12

2a = 13

$a=\frac{13}{2}$

By solving equation (i)

8 + 3b = 10

3b = 10 – 8

3b = 2

$b=\frac{2}{3}$

Q.8 If $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ **and** $B=\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix}$

Then verify that

(i) $(A+B)^t=A^t+B^t$ **(A.B)**

Verification:

L.H.S. = $(A+B)^t$

$$=\left(\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix}\right)^t$$

$$=\begin{bmatrix}2 & 3\\ 2 & 1\end{bmatrix}^t$$

$$=\begin{bmatrix}2 & 2\\ 3 & 1\end{bmatrix}\rightarrow(i)$$

R.H.S.=A^t+B^t

$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \rightarrow (ii)$

From (*i*) and (*ii*)

L.H.S. = R.H.S.

$(A + B)^t = A^t + B^t$

Hence Proved

(ii) $(A-B)^t = A^t - B^t$ **(BWP 2017)** **(A.B)**

Verification:

L.H.S.= $(A – B)^t$

$(A - B)^t = \left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \right)^t$

$= \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix}^t$

$(A – B)^t = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$

R.H.S. = $A^t – B^t$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t - \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}^t$

$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1-1 & 0-2 \\ 2-1 & 1-0 \end{bmatrix}$

$= \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$

L.H.S. = R.H.S.

$(A-B)^t = A^t - B^t$

Hence proved

(iii) $A+A^t$ **is a symmetric** **(K.B)**

(BWP 2014)

Verification:

To show that $A+A^t$ is symmetric, we will show that $(A+A^t)^t = (A+A^t)$

$A+A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 1+1 \end{bmatrix}$

$A+A^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \rightarrow (i)$

Now

$(A+A^t)^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}^t$

$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \rightarrow (ii)$

From (*i*) and (*ii*)

$(A+A^t)^t = (A+A^t)$

Hence $A+A^t$ is symmetric Proved

(iv) $A-A^t$ **is a skew symmetric (K.B)**

Verification:

To show that A-A^t is skew symmetric we will show that $(A-A^t)^t = -(A-A^t)$

$A - A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1-1 & 2-0 \\ 0-2 & 1-1 \end{bmatrix}$

$A-A^t = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

$(A-A^t)^t = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}^t$

$= \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} \rightarrow (i)$

$-(A-A^t) = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \rightarrow (ii)$

From (i) and (ii)

$(A-A^t)^t = -(A-A^t)$

Hence $A – A^t$ **is a skew symmetric, Proved.**

(v) $B+B^t$ **is a symmetric** **(K.B)**

Verification:

The show that B + B^t is symmetric we will show that $(B+B^t)^t=(B+B^t)$

$$B+B^t=\begin{bmatrix}1&1\\2&0\end{bmatrix}+\begin{bmatrix}1&1\\2&0\end{bmatrix}^t$$

$$=\begin{bmatrix}1&1\\2&0\end{bmatrix}+\begin{bmatrix}1&2\\1&0\end{bmatrix}$$

$$=\begin{bmatrix}1+1&1+2\\2+1&0+0\end{bmatrix}$$

$$B+B^t=\begin{bmatrix}2&3\\3&0\end{bmatrix}\rightarrow(i)$$

$$\left(B+B^t\right)^t=\begin{bmatrix}2&3\\3&0\end{bmatrix}^t$$

$$=\begin{bmatrix}2&3\\3&0\end{bmatrix}\rightarrow(ii)$$

From (i) and (ii)

$$\left(B+B^t\right)^t=\left(B+B^t\right)$$

Hence $B+B^t$ **is a symmetric proved**

(vi) $B-B^t$ **is a skew symmetric (K.B)**

Verification:

To show that B – B^t is skew symmetric, we will show that $(B-B^t)^t=-(B-B^t)$

$$B-B^t=\begin{bmatrix}1&1\\2&0\end{bmatrix}-\begin{bmatrix}1&1\\2&0\end{bmatrix}^t$$

$$=\begin{bmatrix}1&1\\2&0\end{bmatrix}-\begin{bmatrix}1&2\\1&0\end{bmatrix}$$

$$=\begin{bmatrix}1-1&1-2\\2-1&0-0\end{bmatrix}$$

$$B-B^t=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$$

$$(B-B^t)^t=\begin{bmatrix}0&-1\\1&0\end{bmatrix}^t$$

$$=\begin{bmatrix}0&1\\-1&0\end{bmatrix}\rightarrow(i)$$

$$-(B-B^t)=-\begin{bmatrix}0&-1\\1&0\end{bmatrix}$$

$$=\begin{bmatrix}0&1\\-1&0\end{bmatrix}\rightarrow(ii)$$

From (i) and (ii)

$$\left(B-B^t\right)^t=-\left(B-B^t\right)$$

Hence B - B^t **is skew symmetric, proved**

Multiplication of Matrices

Two matrices A and B are conformable for multiplication giving product AB if the number if columns of A is equal to the number of rows of B.

e.g., Let $A=\begin{bmatrix}1&2\\3&0\end{bmatrix}$ and $B=\begin{bmatrix}4\\1\end{bmatrix}$.

Here number of columns of A is equal to the number of rows of B. So A and B matrices are conformable for multiplication.

Multiplication of two matrices is explained by the following examples.

(i) If $A=\begin{bmatrix}1&2\end{bmatrix}$ and $B=\begin{bmatrix}2&0\\3&1\end{bmatrix}$

Then $AB=\begin{bmatrix}1&2\end{bmatrix}\begin{bmatrix}2&0\\3&1\end{bmatrix}$

$$=\begin{bmatrix}1\times2+2\times3&1\times0+2\times1\end{bmatrix}$$

$$=\begin{bmatrix}2+6&0+2\end{bmatrix}=\begin{bmatrix}8&2\end{bmatrix}$$

is a 1-by-2 matrix

(ii) If $A=\begin{bmatrix}1&3\\2&-3\end{bmatrix}$ $B=\begin{bmatrix}-1&0\\3&2\end{bmatrix}$

Then $AB=\begin{bmatrix}1&3\\2&-3\end{bmatrix}\times\begin{bmatrix}-1&0\\3&2\end{bmatrix}$

$$=\begin{bmatrix}1\times(-1)+3\times3&1\times0+3\times2\\2(-1)+(-3)(3)&2\times0+(-3)(2)\end{bmatrix}$$

$$AB=\begin{bmatrix}-1+9&0+6\\-2-9&0-6\end{bmatrix}=\begin{bmatrix}8&6\\-11&-6\end{bmatrix}$$

Associative Law Under Multiplication

If A, B and C are three matrices conformable for multiplication then associative law under multiplication is given as

$$(AB)C=A(BC)$$ **(U.B)**

e.g., If

$A=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}$ and $C=\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$

Then

L.H.S. = (AB)C

$$(AB)C=\left(\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}\right)\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2\times0+3\times3 & 2\times1+3\times1\\ -1\times0+0\times3 & -1\times1+0\times1\end{bmatrix}\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0+9 & 2+3\\ 0+0 & -1+0\end{bmatrix}\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}9 & 5\\ 0 & -1\end{bmatrix}\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}9\times2+5\times(-1) & 9\times2+5\times0\\ 0\times2+(-1)\times(-1) & 0\times2+(-1)\times(0)\end{bmatrix}$$

$$=\begin{bmatrix}18-5 & 18+0\\ 0+1 & 0+0\end{bmatrix}=\begin{bmatrix}13 & 18\\ 1 & 0\end{bmatrix}$$

R.H.S. =

$$A(BC)=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\left(\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}0\times2+1\times(-1) & 0\times2+1\times0\\ 3\times2+1\times-1 & 3\times2+1\times0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}-1 & 0\\ 5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}2(-1)+3\times5 & 2\times0+3\times6\\ (-1)(-1)+0\times5 & -1\times0+0\times6\end{bmatrix}$$

$$=\begin{bmatrix}-2+15 & 0+18\\ 1+0 & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}13 & 18\\ 1 & 0\end{bmatrix}=(AB)C$$

The associative law under multiplication of matrices is verified.

Distributive Laws of Multiplication over Addition and Subtraction

(a) Let A, B and C be three matrices. Then distributive laws of multiplication over addition are given below.

(i) $A(B+C)=AB+AC$ (Left distributive law)

(U.B)

(ii) $(A+B)C=AC+BC$ (Right distributive law)

Let $A=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}$ and $C=\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$,

Then

L.H.S. = A (B+C)… (i)

$$=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\left(\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}+\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}0+2 & 1+2\\ 3-1 & 1+0\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}2 & 3\\ 2 & 1\end{bmatrix}=\begin{bmatrix}2\times2+3\times2 & 2\times3+3\times1\\ -1\times2+0\times2 & -1\times3+0\times1\end{bmatrix}$$

$$=\begin{bmatrix}4+6 & 6+3\\ -2+0 & -3+0\end{bmatrix}=\begin{bmatrix}10 & 9\\ -2 & -3\end{bmatrix}$$

R.H.S. = AB + AC

$$=\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 3 & 1\end{bmatrix}+\begin{bmatrix}2 & 3\\ -1 & 0\end{bmatrix}\begin{bmatrix}2 & 2\\ -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2\times0+3\times3 & 2\times1+3\times1\\ -1\times0+0\times3 & -1\times1+0\times1\end{bmatrix}+\begin{bmatrix}2\times2+3(-1) & 2\times2+3\times0\\ -1\times2+0\times(-1) & -1\times2+0\times0\end{bmatrix}$$

$$=\begin{bmatrix}9 & 5\\ 0 & -1\end{bmatrix}+\begin{bmatrix}1 & 4\\ -2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}9+1 & 5+4\\ 0-2 & -1-2\end{bmatrix}=\begin{bmatrix}10 & 9\\ -2 & -3\end{bmatrix}=\text{L.H.S.}$$

Which shows that

$A(B+C)=AB+AC$ similarly we can verify…(ii)

(b) Similarly the distributive laws of multiplication over subtraction are as follow.

(i) $A(B-C)=AB-AC$

(ii) $(A-B)C=AC-BC$

Let

$A=\begin{bmatrix}2 & 3\\ 1 & 0\end{bmatrix}$, $B=\begin{bmatrix}-1 & 1\\ 1 & 0\end{bmatrix}$ and $C=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$, then in… (i)

L.H.S. = A (B–C)… (i)

$$=\begin{bmatrix}2&3\\0&1\end{bmatrix}\left(\begin{bmatrix}-1&1\\1&0\end{bmatrix}-\begin{bmatrix}2&1\\1&2\end{bmatrix}\right)$$

$$=\begin{bmatrix}2&3\\0&1\end{bmatrix}\left(\begin{bmatrix}-1-2&1-1\\1-1&0-2\end{bmatrix}\right)=\begin{bmatrix}2&3\\0&1\end{bmatrix}\begin{bmatrix}-3&0\\0&-2\end{bmatrix}$$

$$=\begin{bmatrix}(2)(-3)+(3)(0)&2\times0+3(-2)\\(0)(-3)+1\times0&0\times0+(1)(-2)\end{bmatrix}$$

$$=\begin{bmatrix}-6+0&0-6\\0+0&0-2\end{bmatrix}=\begin{bmatrix}-6&-6\\0&-2\end{bmatrix}$$

R.H.S.$=AB-AC$

$$=\begin{bmatrix}2&3\\0&1\end{bmatrix}\begin{bmatrix}-1&1\\1&0\end{bmatrix}-\begin{bmatrix}2&3\\0&1\end{bmatrix}\begin{bmatrix}2&1\\1&2\end{bmatrix}$$

$$=\begin{bmatrix}(2)(-1)+(3)(1)&2(1)+3(0)\\(0)(-1)+1(1)&0(1)+1(0)\end{bmatrix}-\begin{bmatrix}2\times2+3\times1&2\times1+3\times2\\0\times2+1\times1&0\times1+1\times2\end{bmatrix}$$

$$=\begin{bmatrix}1&2\\1&0\end{bmatrix}-\begin{bmatrix}7&8\\1&2\end{bmatrix}=\begin{bmatrix}1-7&2-8\\1-1&0-2\end{bmatrix}=\begin{bmatrix}-6&-6\\0&-2\end{bmatrix}$$

Which shows that

$A(B-C)=AB-AC$

Commutative Law of Multiplication of Matrices (U.B)

Consider the matrices

$A=\begin{bmatrix}0&1\\2&3\end{bmatrix}$ and $B=\begin{bmatrix}1&0\\0&-2\end{bmatrix}$ then

$$AB=\begin{bmatrix}0&1\\2&3\end{bmatrix}\begin{bmatrix}1&0\\0&-2\end{bmatrix}$$

$$=\begin{bmatrix}0\times1+1\times0&0\times0+1\times(-2)\\2\times1+3\times0&2\times0+3(-2)\end{bmatrix}$$

$$=\begin{bmatrix}0&-2\\2&-6\end{bmatrix}$$

And

$$BA=\begin{bmatrix}1&0\\0&-2\end{bmatrix}\begin{bmatrix}0&1\\2&3\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+0\times2&1\times1+0\times3\\0\times0+(-2)\times2&0\times1+3(-2)\end{bmatrix}$$

$$=\begin{bmatrix}0&1\\-4&-6\end{bmatrix}$$

Which shows that, $AB\neq BA$

Commutative law under multiplication in matrices does not hold in general i.e., if A and B are two matrices, then $AB\neq BA$.

Commutative law under multiplication holds in particular case.

e.g., If $A=\begin{bmatrix}2&0\\0&1\end{bmatrix}$ and $B=\begin{bmatrix}-3&0\\0&4\end{bmatrix}$ then,

$$AB=\begin{bmatrix}2&0\\0&1\end{bmatrix}\begin{bmatrix}-3&0\\0&4\end{bmatrix}$$

$$=\begin{bmatrix}2\times(-3)+0\times0&2\times0+0\times4\\0\times(-3)+1\times0&0\times0+1\times4\end{bmatrix}$$

$$=\begin{bmatrix}-6&0\\0&4\end{bmatrix}$$

And

$$BA=\begin{bmatrix}-3&0\\0&4\end{bmatrix}\begin{bmatrix}2&0\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}-3\times2+0\times0&-3\times0+0\times1\\0\times2+4\times0&0\times0+4\times1\end{bmatrix}$$

$$=\begin{bmatrix}-6&0\\0&4\end{bmatrix}$$

which shows that $AB = BA$.

Multiplicative Identity of Matrices (U.B)

Let A be a matrix, another matrix B is called the identity matrix of A under multiplication if $AB = A = BA$

If $A=\begin{bmatrix}1&0\\0&-3\end{bmatrix}$, $B=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, then

$$AB=\begin{bmatrix}1&2\\0&-3\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+2\times0&1\times0+2\times1\\0\times1+(-3)\times0&0\times0+(-3)(1)\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\0 & -3\end{bmatrix}$$

$$BA=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & -3\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+0\times0 & 1\times2+0\times(-3)\\0\times1+1\times0 & 0\times2+1\times(-3)\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\\0 & -3\end{bmatrix}$$

which shows that $AB = A = BA$.

Verification of $(AB)^t = B^tA^t$:

Let, $A=\begin{bmatrix}2 & 1\\0 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\-2 & 0\end{bmatrix}$

L.H.S. = $(AB)^t$

$$=\left(\begin{bmatrix}2 & 1\\0 & -1\end{bmatrix}\begin{bmatrix}1 & 3\\-2 & 0\end{bmatrix}\right)^t$$

$$=\begin{bmatrix}2\times1+1\times(-2) & 2\times3+1\times0\\0\times1+(-1)\times(-2) & 0\times3+(-1)\times0\end{bmatrix}^t$$

$$=\begin{bmatrix}2-2 & 6+0\\0+2 & 0+0\end{bmatrix}^t=\begin{bmatrix}0 & 6\\2 & 0\end{bmatrix}^t$$

$$=\begin{bmatrix}0 & 2\\6 & 0\end{bmatrix}$$

R.H.S. = B^tA^t

$$(A)^t=\begin{bmatrix}2 & 1\\0 & -1\end{bmatrix}^t=\begin{bmatrix}2 & 0\\1 & -1\end{bmatrix} \text{ and}$$

$$(B)^t=\begin{bmatrix}1 & 3\\-2 & 0\end{bmatrix}^t=\begin{bmatrix}1 & -2\\3 & 0\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}1 & -2\\3 & 0\end{bmatrix}\begin{bmatrix}2 & 0\\1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}1\times2+(-2)\times1 & 1\times0+(-2)(-1)\\3\times2+0\times1 & 3\times0+0\times(-1)\end{bmatrix}$$

$$=\begin{bmatrix}2-2 & 0+2\\6+0 & 0+0\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}0 & 2\\6 & 0\end{bmatrix}=\text{L.H.S.}.$$

Thus $(AB)^t=B^tA^t$

Exercise 1.4

Q.1 Which of the following product of matrices is conformable for multiplication?

(i) $\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}\begin{bmatrix}-2\\3\end{bmatrix}$ **(K.B)**

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(ii) $\begin{bmatrix}1 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}-2 & -1\\1 & 3\end{bmatrix}$ **(K.B)**

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(iii) $\begin{bmatrix}1\\-1\end{bmatrix}\begin{bmatrix}0 & 1\\-1 & 2\end{bmatrix}$ **(K.B)**

No, these matrices cannot be multiplied because number of columns of 1st matrix is not equal to the number of rows of 2nd matrix.

(iv) $\begin{bmatrix}1 & 2\\0 & -1\\-1 & -2\end{bmatrix}\begin{bmatrix}1 & 0 & -1\\0 & 1 & 2\end{bmatrix}$ **(K.B)**

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

(v) $\begin{bmatrix}3 & 2 & 1\\0 & 1 & -1\end{bmatrix}\begin{bmatrix}1 & -1\\0 & 2\\-2 & 3\end{bmatrix}$ **(K.B)**

Yes, these matrices can be multiplied because number of columns of 1st matrix is equal to number of rows of 2nd matrix.

Q.2 If $A=\begin{bmatrix}3 & 0\\-1 & 2\end{bmatrix}$, $B=\begin{bmatrix}6\\5\end{bmatrix}$ **find**

(i) AB **(GRW 2018, MTN 2017, 18)** **(A.B)**

Solution:

$$AB=\begin{bmatrix}3 & 0\\-1 & 2\end{bmatrix}\begin{bmatrix}6\\5\end{bmatrix}$$

$$= \begin{bmatrix} (3\times6)+(0\times5) \\ (-1\times6)+(2\times5) \end{bmatrix}$$

$$= \begin{bmatrix} 18+0 \\ -6+10 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

(ii) *BA* **(if possible)** **(K.B)**

Solution:

BA is not possible because number of columns of B are not equal to number of rows of *A*.

Q.3 Find the following products

(i) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **(A.B)**

(GRW 2019, SGD 2016, MTN 2017)

Solution:

$$\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} (1\times4)+(2\times0) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \end{bmatrix}$$

(ii) $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix}$ **(A.B)**

(GRW 2017, FSD 2017, 18)

Solution:

$$\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \end{bmatrix} = \begin{bmatrix} (1\times5)+(2\times-4) \end{bmatrix}$$

$$= \begin{bmatrix} 5+(-8) \end{bmatrix}$$

$$= \begin{bmatrix} 5-8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \end{bmatrix}$$

(iii) $\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ **(LHR 2017)** **(A.B)**

Solution:

$$\begin{bmatrix} -3 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} (-3\times4)+(0\times0) \end{bmatrix}$$

$$= \begin{bmatrix} -12+0 \end{bmatrix}$$

$$= \begin{bmatrix} -12 \end{bmatrix}$$

(iv) $\begin{bmatrix} 6 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ -0 \end{bmatrix}$ **(A.B)**

Solution:

$$\begin{bmatrix} 6 & +0 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 6\times4+(-0)(0) \end{bmatrix}$$

$$= \begin{bmatrix} 24-0 \end{bmatrix}$$

$$= \begin{bmatrix} 24 \end{bmatrix}$$

(v) $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$ **(A.B)**

Solution:

$$\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times4+2\times0 & 1\times5+2\times(-4) \\ -3\times4+0\times0 & -3(5)+0\times(-4) \\ 6(4)+(-1)(0) & 6(5)+(-1)(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & 5-8 \\ -12+0 & -15-0 \\ 24-0 & 30+4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ -12 & -15 \\ 24 & 34 \end{bmatrix}$$

Q.4 Multiply the following matrices.

(a) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$ **(LHR 2019)** **(A.B)**

Solution:

$$\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+(3\times3) & (2\times-1)+(3\times0) \\ (1\times2)+(1\times3) & (1\times-1)+(1\times0) \\ (0\times2)+(-2\times3) & (0\times-1)+(-2\times0) \end{bmatrix}$$

$$=\begin{bmatrix}4+9 & -2+0\\ 2+3 & -1+0\\ 0-6 & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}13 & -2\\ 5 & -1\\ -6 & 0\end{bmatrix}$$

(b) $\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\\ -1 & 1\end{bmatrix}$ **(A.B)**

(LHR 2015, RWP 2015, MTN 2016)

Solution: $\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\\ -1 & 1\end{bmatrix}$

$$=\begin{bmatrix}(1\times1)+(2\times3)+(3\times-1) & (1\times2)+(2\times4)+(3\times1)\\ (4\times1)+(5\times3)+(6\times-1) & (4\times2)+(5\times4)+(6\times1)\end{bmatrix}$$

$$=\begin{bmatrix}1+6+(-3) & 2+8+3\\ 4+15+(-6) & 8+20+6\end{bmatrix}$$

$$=\begin{bmatrix}7-3 & 13\\ 19-6 & 34\end{bmatrix}$$

$$=\begin{bmatrix}4 & 13\\ 13 & 34\end{bmatrix}$$

(c) $\begin{bmatrix}1 & 2\\ 3 & 4\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}$ **(A.B)**

(RWP 2015, GRW 2016)

Solution:

$$\begin{bmatrix}1 & 2\\ 3 & 4\\ -1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}(1\times1)+(2\times4) & (1\times2)+(2\times5) & (1\times3)+(2\times6)\\ (3\times1)+(4\times4) & (3\times2)+(4\times5) & (3\times3)+(4\times6)\\ (-1\times1)+(1\times4) & (-1\times2)+(1\times5) & (-1\times3)+(1\times6)\end{bmatrix}$$

$$=\begin{bmatrix}1+8 & 2+10 & 3+12\\ 3+16 & 6+20 & 9+24\\ -1+4 & -2+5 & -3+6\end{bmatrix}$$

$$=\begin{bmatrix}9 & 12 & 15\\ 19 & 26 & 33\\ 3 & 3 & 3\end{bmatrix}$$

(d) $\begin{bmatrix}8 & 5\\ 6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\ -4 & 4\end{bmatrix}$ **(A.B)**

(LHR 2015, SGD 2018)

Solution:

$$\begin{bmatrix}8 & 5\\ 6 & 4\end{bmatrix}\begin{bmatrix}2 & -\frac{5}{2}\\ -4 & 4\end{bmatrix}$$

$$=\begin{bmatrix}(8\times2)+(5\times-4) & \left(8\times-\frac{5}{2}\right)+(5\times4)\\ (6\times2)+(4\times-4) & \left(6\times-\frac{5}{2}\right)+(4\times4)\end{bmatrix}$$

$$=\begin{bmatrix}16+(-20) & \frac{-40}{2}+20\\ 12+(-16) & \frac{-30}{2}+16\end{bmatrix}$$

$$=\begin{bmatrix}16-20 & -20+20\\ 12-16 & -15+16\end{bmatrix}$$

$$=\begin{bmatrix}-4 & 0\\ -4 & 1\end{bmatrix}$$

(e) $\begin{bmatrix}-1 & 2\\ 1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ **(A.B)**

Solution:

$$\begin{bmatrix}-1 & 2\\ 1 & 3\end{bmatrix}\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times0)+(2\times0) & (-1\times0)+(2\times0)\\ (1\times0)+(3\times0) & (1\times0)+(3\times0)\end{bmatrix}$$

$$=\begin{bmatrix}0+0 & 0+0\\ 0+0 & 0+0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

Q.5 **Let** $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$ **verify whether**

(FSD 2015, MTN 2015, D.G.K 2017)

(i) $AB=BA$ **(K.B) (A.B) (U.B)**

Verification:

L.H.S. =AB

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0-3) & (2\times2)+(0-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

R.H.S. = BA $=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$

$$=\begin{bmatrix}1\times(-1)+2\times2 & 1\times3+2\times0\\ -3\times(-1)+(-5)2 & -3\times3+(-5)(0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+4 & 3+0\\ 3-10 & -9-0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 3\\ -7 & -9\end{bmatrix}$$

Since L.H.S $\neq$ R.H.S

$AB\neq BA$

(ii) $A(BC)=(AB)C$ **(A.B)**

Verification:

L.H.S.=A(BC)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}2+2 & 1+6\\ -6+(-5) & -3+(-15)\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}4 & 7\\ -6-5 & -3-15\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}4 & 7\\ -11 & -18\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times4)+(3\times-11) & (-1\times7)+(3\times-18)\\ (2\times4)+(0\times-11) & (2\times7)+(0\times-18)\end{bmatrix}$$

$$=\begin{bmatrix}-4+(-33) & -7+(-54)\\ 8+0 & 14+0\end{bmatrix}$$

$$=\begin{bmatrix}-4-33 & -7-54\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

R.H.S.=(AB)C

$$=\left(\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\right)\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(-3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-10\times2)+(-17\times1) & (-10\times1)+(-17\times3)\\ (2\times2)+(4\times1) & (2\times1)(4\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-20+(-17) & -10+(-51)\\ 4+4 & 2+12\end{bmatrix}$$

$$=\begin{bmatrix}-20-17 & -10-51\\ 8 & 14\end{bmatrix}$$

$$=\begin{bmatrix}-37 & -61\\ 8 & 14\end{bmatrix}$$

Since

L.H.S. = R.H.S.

A (BC) = (AB) C

Hence proved

(iii) $A(B+C)=AB+AC$ **(A.B)**

Verification:

L.H.S.=A (B+C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1+2 & 2+1\\ -3+1 & -5+3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}3 & 3\\ -2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times3)+(3\times-2) & (-1\times3)+(3\times-2)\\ (2\times3)+(0\times-2) & (2\times3)+(0\times-2)\end{bmatrix}$$

$$=\begin{bmatrix}-3+(-6) & -3+(-6)\\ 6+0 & 6+0\end{bmatrix}$$

$$=\begin{bmatrix}-3-6 & -3-6\\ 6 & 6\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

R.H.S.=AB+AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}+\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (2\times1)+(0\times-3) & (2\times+2)+(0\times-5)\end{bmatrix}$$

$$+\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-3) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}+\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}+\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10+1 & -17+8\\ 2+4 & 4+2\end{bmatrix}$$

$$=\begin{bmatrix}-9 & -9\\ 6 & 6\end{bmatrix}$$

Since L.H.S.=R.H.S.

A (B+C) = AB+AC

Hence proved

(iv) A(B–C) = AB–AC **(A.B)**

Verification:

L.H.S.=A(B–C)

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\left(\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}\right)$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1-2 & 2-1\\ -3-1 & -5-3\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}-1 & 1\\ -4 & -8\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times-1)+(3\times-4) & (-1\times1)+(3\times-8)\\ (2\times-1)+(0l-4) & (2\times1)+(0\times-8)\end{bmatrix}$$

$$=\begin{bmatrix}+1+(-12) & -1+(-24)\\ -2+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}1-12 & -1-24\\ -2 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

R.H.S.=AB–AC

$$=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}-\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+3\times-5)\\ (2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$-\begin{bmatrix}(-1\times2)+(3\times1) & (-1\times1)+(3\times3)\\ (2\times2)+(0\times1) & (2\times1)+(0\times3)\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2+0 & 4+0\end{bmatrix}-\begin{bmatrix}-2+3 & -1+9\\ 4+0 & 2+0\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}-\begin{bmatrix}1 & 8\\ 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}-10-1 & -17-8\\ 2-4 & 4-2\end{bmatrix}$$

$$=\begin{bmatrix}-11 & -25\\ -2 & 2\end{bmatrix}$$

Since L.H.S. = R.H.S.

A (B-C) =AB-AC

Hence proved

Q.6 For the matrices $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$,

$B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$ **and** $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$

Verify that

(i) $(AB)^t = B^t A^t$ **(A.B)**

Verification:

L.H.S. = $(AB)^t$

$$(AB)=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-1\times1)+(3\times-3) & (-1\times2)+(3\times-5)\\ (+2\times1)+(0\times-3) & (2\times2)+(0\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & -2+(-15)\\ 2+0 & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & -2-15\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & -17\\ 2 & 4\end{bmatrix}^t$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}^t$$

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$$

$$A^t=\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

R.H.S. = B^tA^t

$$=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}\times\begin{bmatrix}-1 & 2\\ 3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-1)+(-3\times3) & (1\times2)+(-3\times0)\\ (2\times-1)+(-5\times3) & (2\times2)+(-5\times0)\end{bmatrix}$$

$$=\begin{bmatrix}-1+(-9) & 2+0\\ -2+(-15) & 4+0\end{bmatrix}$$

$$=\begin{bmatrix}-1-9 & 2\\ -2-15 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-10 & 2\\ -17 & 4\end{bmatrix}$$

Since L.H.S. = R.H.S.

$(AB)^t = B^tA^t$

Hence proved

L.H.S. = R.H.S.

(ii) $(BC)^t = C^tB^t$ **(A.B)**

Verification:

L.H.S. = $(BC)^t$

$$BC=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}\times\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$$

$$=\begin{bmatrix}(1\times-2)+(2\times3) & (1\times6)+(2\,x\times-9)\\ (-3\times-2)+(-5\times3) & (-3\times6)+(-5\times-9)\end{bmatrix}$$

$$=\begin{bmatrix}-2+6 & 6+(-18)\\ 6+(-15) & -18+45\end{bmatrix}$$

$$=\begin{bmatrix}4 & 6-18\\ 6-15 & 27\end{bmatrix}$$

$$=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}$$

Now

$$(BC)^t=\begin{bmatrix}4 & -12\\ -9 & 27\end{bmatrix}^t$$

$$=\begin{bmatrix}4 & -9\\ -12 & 27\end{bmatrix}$$

R.H.S. = C^tB^t

$$C^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}$$

$$B^t=\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$\text{R.H.S.} = C^tB^t=\begin{bmatrix}-2 & 3\\ 6 & -9\end{bmatrix}\times\begin{bmatrix}1 & -3\\ 2 & -5\end{bmatrix}$$

$$=\begin{bmatrix}(-2\times1)+(3\times2) & (-2\times-3)+(3\times-5)\\ (6\times1)+(-9\times2) & (6\times-3)+(-9\times-5)\end{bmatrix}$$

$$=\begin{bmatrix}-2+6 & 6+(-15)\\ 6+(-18) & -18+45\end{bmatrix}$$

$$=\begin{bmatrix} 4 & 6-15 \\ 6-18 & 27 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix}$$

Hence proved

L.H.S.=R.H.S.

Multiplicative Inverse of a Matrix

Determinant of a 2-by-2 Matrix (K.B)

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2-by-2 square matrix.

The determinant of A, denoted by A or $|A|$ is defined as

$$|A| = \det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = \lambda \in R$$

e.g., Let $B=\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$.

Then

$$|B| = \det = B\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} = 1\times 3-(-2)(1) = 3+2 = 5$$

If $M=\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$, then

$$\det M = \begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix} = 2\times 3 - 1\times 6 = 0$$

Singular Matrix (K.B)

(GRW 2017, SWL 2016, MTN 2016, 17 FSD 2018)

A square matrix A is called singular if the determinant of A is equal to zero.

For example, $A=\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ is a singular matrix, since $\det A = 1\times 0 - 0\times 2 = 0$.

Non-Singular Matrix (K.B)

(GRW 2017, SWL 2016, MTN 2016, 17 FSD 2018)

A square matrix A is called non-singular if the determinant of A is not equal to zero.

For example $A=\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ is non-singular,

Since $\det A = 1\times 2 - 0\times 1 = 2 \neq 0$.

Adjoint of a Matrix (K.B)

Adjoint of a square matrix $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the sign of other entires. Adjoint of matrix A is denoted as Adj A.

i.e., $\text{Adj A} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Multiplicative Inverse of Non Singular Matrix (K.B)

Let A and B be two non-singular square matrices of same order. Then A and B are said to be multiplicative inverse of each other if

$AB = BA = I$

The inverse of A is denoted by A^{-1}, thus

$AA^{-1} = A^{-1}A = I$

Inverse of a matrix is possible only if matrix is non singular.

Note

Inverse of identity matrix is identity matrix

Inverse of a Matrix using Adjoint (K.B)

Let $M=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix. To find the inverse of M, i.e, M^{-1} first we find the determinant as inverse is possible only of a non-singular matrix.

$$|M| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \neq 0$$

and $\text{Adj}M = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, then $M^{-1} = \dfrac{Adj\,M}{|M|}$

$$M^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{ad-bc}$$

e.g., Let $A=\begin{bmatrix} 2 & 1 \\ -1 & -3 \end{bmatrix}$, then

$$|A| = \begin{vmatrix} 2 & 1 \\ -1 & -3 \end{vmatrix} = -6-(-1) = -6+1 = -5 \neq 0$$

thus $A^{-1} = \frac{Adj\ A}{|A|} = \frac{\begin{bmatrix}-3 & -1\\ 1 & 2\end{bmatrix}}{-5}$

$= \frac{-1}{5}\begin{bmatrix}-3 & -1\\ 1 & 2\end{bmatrix} = \begin{bmatrix}\frac{3}{5} & \frac{1}{5}\\ -\frac{1}{5} & -\frac{2}{5}\end{bmatrix}$

and $AA^{-1} = \begin{bmatrix}-2 & 1\\ -1 & -3\end{bmatrix}\begin{bmatrix}\frac{3}{5} & \frac{1}{5}\\ -\frac{1}{5} & -\frac{2}{5}\end{bmatrix}$

$= \begin{bmatrix}\frac{6}{5}-\frac{1}{5} & \frac{2}{5}-\frac{2}{5}\\ \frac{-3}{5}+\frac{3}{5} & \frac{-1}{5}+\frac{6}{5}\end{bmatrix}$

$AA^{-1} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I = A^{-1}A$

Verification of $(AB)^{-1} = B^{-1}A^{-1}$: (U.B)

Let $A = \begin{bmatrix}3 & 1\\ -1 & 0\end{bmatrix}$ and $B = \begin{bmatrix}0 & -1\\ 3 & 2\end{bmatrix}$

Then det $A = 3\times 0-(-1)\times 1 = 1 \neq 0$

And det. $B = 0\times 2 - 3(-1) = 3 \neq 0$

Therefore A and B are invertible i.e. their inverses exist. Then, to verify the law of inverse of the product, take

$AB = \begin{bmatrix}3 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}0 & -1\\ 3 & 2\end{bmatrix}$

$AB = \begin{bmatrix}3\times 0+1\times 3 & 3\times(-1)+1\times 2\\ -1\times 0+0\times 3 & -1\times(-1)+0\times 2\end{bmatrix}$

$AB = \begin{bmatrix}3 & -1\\ 0 & 1\end{bmatrix}$

$\Rightarrow \det(AB) = \begin{vmatrix}3 & -1\\ 0 & 1\end{vmatrix} = 3 \neq 0$

and L.H.S. $= (AB)^{-1} = \frac{1}{3}\begin{bmatrix}1 & 1\\ 0 & 3\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & \frac{1}{3}\\ 0 & 1\end{bmatrix}$

R.H.S. $= B^{-1}A^{-1}$ where $B^{-1} = \frac{1}{3}\begin{bmatrix}2 & 1\\ -3 & 0\end{bmatrix}$,

$A^{-1} = \frac{1}{1}\begin{bmatrix}0 & -1\\ 1 & 3\end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix}2 & 1\\ -3 & 0\end{bmatrix} \cdot \frac{1}{1}\begin{bmatrix}0 & -1\\ 1 & 3\end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix}2\times 0+1\times 1 & 2\times(-1)+1\times 3\\ -3\times 0+0\times 1 & -3\times(-1)+0\times 3\end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix}0+1 & -2+3\\ 0 & 3\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1\\ 0 & 3\end{bmatrix}$

$= \begin{bmatrix}\frac{1}{3} & \frac{1}{3}\\ 0 & 1\end{bmatrix} = (AB)^{-1}$

Thus the law $(AB)^{-1} = B^{-1}A^{-1}$ is verified

Exercise 1.5

Q.1 Find the determinant of following matrices.

(i) $A = \begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$ **(A.B)**

Solution:

$A = \begin{bmatrix}-1 & 1\\ 2 & 0\end{bmatrix}$

Then,

$|A| = \begin{vmatrix}-1 & 1\\ 2 & 0\end{vmatrix}$

= (–1)(0) – (2) (1)
= 0 – 2
= –2

(ii) $B = \begin{bmatrix}1 & 3\\ 2 & -2\end{bmatrix}$ **(A.B)**

(BWP 2018, D.G.K 2018)

Solution:

$B = \begin{bmatrix}1 & 3\\ 2 & -2\end{bmatrix}$

Then,

$|B| = \begin{vmatrix}1 & 3\\ 2 & -2\end{vmatrix}$

= (1) (–2) – (2) (3)
= –2–6
= –8

(iii) $C=\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}$ **(BWP 2014, 16)** **(A.B)**

Solution:

$$C=\begin{bmatrix}3 & 2\\ 3 & 2\end{bmatrix}$$

Then,

$$|C|=\begin{vmatrix}3 & 2\\ 3 & 2\end{vmatrix}$$

$= (3)(2) - (3)(2)$

$= 6 - 6$

$= 0$

(iv) $D=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$ **(D.G.K 2018)** **(A.B)**

Solution:

$$D=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$$

Then,

$$|D|=\begin{vmatrix}3 & 2\\ 1 & 4\end{vmatrix}$$

$= (3)(4) - (2)(1)$

$= 12 - 2$

$= 10$

Q.2 Find which of the following matrices are singular or non singular?

(i) $A=\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix}$ **(A.B)**

Solution:

$$A=\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix}$$

Then,

$$|A|=\begin{vmatrix}3 & 6\\ 2 & 4\end{vmatrix}$$

$|A| = (3)(4) - (2)(6)$

$|A| = 12 - 12$

$|A| = 0$

It is a singular matrix.

(ii) $B=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$ **(A.B)**

Solution:

$$B=\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}$$

Then,

$$|B|=\begin{vmatrix}4 & 1\\ 3 & 2\end{vmatrix}$$

$|B| = (4)(2) - (3)(1)$

$|B| = 8 - 3$

$|B| = 5$

It is non singular matrix.

(iii) $C=\begin{bmatrix}7 & -9\\ 3 & 5\end{bmatrix}$ **(SGD 2018)** **(A.B)**

Solution:

$$C=\begin{bmatrix}7 & -9\\ 3 & 5\end{bmatrix}$$

Then,

$$|C|=\begin{vmatrix}7 & -9\\ 3 & 5\end{vmatrix}$$

$|C| = (7)(5) - (3)(-9)$

$|C| = 35 + 27$

$|C| = 62$

It is not equal to zero so
It is non singular matrix.

(iv) $D=\begin{bmatrix}5 & -10\\ -2 & 4\end{bmatrix}$ **(A.B)**

Solution:

$$D=\begin{bmatrix}5 & -10\\ -2 & 4\end{bmatrix}$$

Then,

$$|D|=\begin{vmatrix}5 & -10\\ -2 & 4\end{vmatrix}$$

$|D| = (5)(4) - (-2)(-10)$

$|D| = 20\text{-}20$

$|D| = 0$

It is singular matrix.

Q.3 Find the multiplicative inverse of each

(i) $A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$ **(GRW 2018)** **(A.B)**

Solution:

$A=\begin{bmatrix}-1 & 3\\ 2 & 0\end{bmatrix}$

Then,

$|A|=\begin{vmatrix}-1 & 3\\ 2 & 0\end{vmatrix}$

$|A|=(-1)(0)-(2)(3)$

$|A|=0-6$

$|A|=\ -6\neq 0$ (Non Singular)

A^{-1}exists

$AdjA=\begin{bmatrix}0 & -3\\ -2 & -1\end{bmatrix}$

$A^{-1}=\frac{1}{|A|}\times AdjA$

Putting the values

$$A^{-1}=\frac{1}{-6}\times\begin{bmatrix}0 & -3\\ -2 & -1\end{bmatrix}=\begin{bmatrix}0\times\frac{1}{-6} & -\overset{1}{\cancel{3}}\times\frac{1}{-\underset{2}{\cancel{6}}}\\ -\overset{1}{\cancel{2}}\times\frac{1}{-\underset{3}{\cancel{6}}} & -1\times\frac{1}{-\underset{2}{\cancel{6}}}\end{bmatrix}$$

$$=\begin{bmatrix}0 & \frac{1}{2}\\ \frac{1}{3} & \frac{1}{6}\end{bmatrix}$$

(ii) $B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$ **(A.B)**

(GRW 2018, FSD 2018, SGD 2018)

Solution:

$B=\begin{bmatrix}1 & 2\\ -3 & -5\end{bmatrix}$

Then,

$|B|=\begin{vmatrix}1 & 2\\ -3 & -5\end{vmatrix}$

$|B|=(-1)(-5)-(-3)(2)$

$|B|=-5+6$

$|B|=1\neq 0$ (Non Singular)

B^{-1} exists

$AdjB=\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}$

$B^{-1}=\frac{1}{|B|}\times AdjB$

Putting the values

$B^{-1}=\frac{1}{1}\times\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}$

$=\begin{bmatrix}\frac{-5}{1} & \frac{-2}{1}\\ \frac{3}{1} & \frac{1}{1}\end{bmatrix}$

$=\begin{bmatrix}-5 & -2\\ 3 & 1\end{bmatrix}$

(iii) $C=\begin{bmatrix}-2 & 6\\ 3 & -9\end{bmatrix}$ **(A.B)**

Solution:

To write in determinant form

$|C|=\begin{vmatrix}-2 & 6\\ 3 & -9\end{vmatrix}$

$|C|=(-2)(-9)-(3)(6)$

$|C|=18-18$

$|C|=0$ Singular

C^{-1} does not exists.

(iv) $D=\begin{bmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{bmatrix}$ **(A.B)**

Solution:

To write in determinant form

$D=\begin{bmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{bmatrix}$

$|D|=\begin{vmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{vmatrix}=\frac{1}{2}\times 2-\frac{3}{4}\times 1$

$=1-\frac{3}{4}$

$$= \frac{4-3}{4}$$

$$|D| = \frac{1}{4} \neq 0 \text{ (Non Singular)}$$

D^{-1} exists

$$AdjD = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$D^{-1} = \frac{1}{|D|} \times AdjD$$

By putting the values

$$= \frac{1}{\frac{1}{4}} \begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 1 \div \frac{1}{4} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 1 \times \frac{4}{1} \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$= 4 \begin{bmatrix} 2 & -\frac{3}{4} \\ -1 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 \times 2 & -\frac{3}{\not 4} \times \not 4 \\ -1 \times 4 & \frac{1}{\not 2_{1}} \times \not 4^{2} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

Q.4 If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ **and** $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$**, then verify that**

(i) A(AdjA)=(AdjA)A=(detA)I **(A.B)**

Verification:

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$\text{AdjA} = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$\text{detA} = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$= 1 \times 6 \quad 2 \times 4$$

$$= 6 - 8$$

$$= -2$$

$$A(AdjA) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-8 & (-2)+2 \\ 24-24 & -8+6 \end{bmatrix}$$

$$\text{A (Adj A)} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \text{ \_\_\_\_\_\_\_\_ (i)}$$

$$(AdjA)A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$(AdjA)A = \begin{bmatrix} (6)\times(1)+(-2)\times 4 & (6)\times 2+(-2)\times 6 \\ (-4)\times 1+(1)(4) & (-4)(2)+(1)(6) \end{bmatrix}$$

$$= \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix}$$

$$\text{(AdjA)A} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \text{ \_\_\_\_ (ii)}$$

$$(\det A)\text{I} = -2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \times 1 & 0 \times 2 \\ -2 \times 0 & 1 \times -2 \end{bmatrix}$$

$$\text{(detA)I} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \text{ \_\_\_\_ (iii)}$$

Hence proved

From eq (i), (ii) and (iii)

A(AdjA)=(AdjA)A=(detA)I

(ii) $BB^{-1} = I = B^{-1}B$ **(A.B)**

Solution: (U.B)

$$B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$$

To write in determinant form

$$|B| = \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix}$$

$$= -6 - (-2)$$

$= -6 + 2$

$= -4 \neq 0$ (Non singular)

B^{-1} exists.

$$AdjB = \begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} AdjB$$

$$B^{-1} = \frac{1}{-4}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

Now

$$BB^{-1} = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} \times \frac{1}{-4}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} -6+2 & 3-3 \\ -4+4 & 2-6 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{4} & 0 \\ 0 & \frac{4}{4} \end{bmatrix}$$

$$BB^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$B^{-1}B = \frac{1}{-4}\begin{bmatrix} -2 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -6+2 & 2-2 \\ -6+6 & 2-6 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-4}{-4} & 0 \\ 0 & \frac{-4}{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$B^{-1}B = I$

From (i) and (ii)

$BB^{-1} = I = B^{-1}B$

Hence proved

Q.5 Determine whether the given matrices are multiplicative inverses of each other.

(i) $\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}$ and $\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$ **(U.B)**

Solution:

$$\begin{bmatrix} 3 & 5 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 7 & -5 \\ -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 21+(-20) & -15+15 \\ 28+(-28) & -20+21 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Since, $AA^{-1} = I$, given matrices are multiplicative inverse of each other.

(ii) $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$ **(A.B + U.B)**

Solution:

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -3+4 & 2+(-2) \\ -6+6 & 4+(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Since, $AA^{-1} = I$, given matrices are multiplicative inverse of each other.

Q.6

(i) $(AB)^{-1} = B^{-1}A^{-1}$ **(A.B + U.B)**

Solution: $(AB)^{-1} = B^{-1}A^{-1}$

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

L.H.S. = $(AB)^{-1}$

$$AB = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 4\times(-4)+0(1) & 4\times(-2)+0(-1) \\ -1\times(-4)+2(1) & -1\times(-2)+2(-1) \end{bmatrix}$$

$$=\begin{bmatrix}-16+0 & -8+0\\ 4+2 & 2+(-2)\end{bmatrix}$$

$$=\begin{bmatrix}-16 & -8\\ 6 & 0\end{bmatrix}$$

To write in determinant form

$$|AB|=\begin{vmatrix}-16 & -8\\ 6 & 0\end{vmatrix}$$

$$|AB|=0-(-48)$$

$$|AB|=48$$

To write in Adj (AB)

$$\text{Adj (AB)} = \begin{bmatrix}0 & 8\\ -6 & -16\end{bmatrix}$$

$$(\text{AB})^{-1}=\frac{1}{|AB|}\times AdjAB$$

$$=\frac{1}{48}\times\begin{bmatrix}0 & 8\\ -6 & -16\end{bmatrix}$$

$$=\begin{bmatrix}\frac{0}{48} & \frac{8}{48}\\ \frac{-6}{48} & \frac{-16}{48}\end{bmatrix}$$

$$=\begin{bmatrix}0 & \frac{1}{6}\\ -\frac{1}{8} & -\frac{1}{3}\end{bmatrix}\rightarrow(i)$$

R.H. S = $\text{B}^{-1}\text{A}^{-1}$

To write in determinant form

$$|B|=\begin{vmatrix}-4 & -2\\ 1 & -1\end{vmatrix}$$

$$|B|=4-(-2)$$

$$|B|=4+2$$

$$|B|=6$$

To write in Adj B

$$AdjB=\begin{bmatrix}-1 & 2\\ -1 & -4\end{bmatrix}$$

$$B^{-1}=\frac{1}{|B|}\times AdjB$$

By putting value

$$B^{-1}=\frac{1}{6}\times\begin{bmatrix}-1 & 2\\ -1 & -4\end{bmatrix}$$

To write in determinant form

$$|A|=\begin{vmatrix}4 & 0\\ -1 & 2\end{vmatrix}$$

$$|A|=8-(-0)$$

$$|A|=8$$

To write in Adj A

$$AdjA=\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\times AdjA$$

$$=\frac{1}{8}\times\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}$$

To solve R.H.S.

$$B^{-1}A^{-1}=\frac{1}{6}\begin{bmatrix}-1 & 2\\ -1 & -4\end{bmatrix}\times\frac{1}{8}\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}$$

$$=\frac{1}{6}\times\frac{1}{8}\begin{bmatrix}-1 & 2\\ -1 & -4\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}$$

$$=\frac{1}{48}\begin{bmatrix}-2+2 & 0+8\\ -2-4 & 0-16\end{bmatrix}$$

$$=\frac{1}{48}\begin{bmatrix}0 & 8\\ -6 & -16\end{bmatrix}$$

$$=\begin{bmatrix}\frac{0}{48} & \frac{8}{48}\\ \frac{-6}{48} & \frac{-16}{48}\end{bmatrix}$$

$$=\begin{bmatrix}0 & \frac{1}{6}\\ -\frac{1}{8} & -\frac{1}{3}\end{bmatrix}\rightarrow(ii)$$

From (i) and (ii)

L.H.S. = R.H.S.

Hence proved

Solution of Simultaneous Linear Equations (K.B)

System of two linear equations in two variables in general form is given as:

$ax + by = m$
$cx + dy = n$
where a, b, c, d, m and n are real numbers.
This system is also called simultaneous linear equations.

(i) Solution of System of Linear Equation by Matrix Inversion Method **(U.B)**

Consider the following system of linear equations.
$ax + by = m$
$cx + dy = n$
Writing in matrices form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} m \\ n \end{bmatrix}$$

Let $AX = B$

where $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \end{bmatrix}$ and $B=\begin{bmatrix} m \\ n \end{bmatrix}$

or $X = A^{-1}B$

or $X=\dfrac{Adj\ A}{|A|}\times B \rightarrow (i)$

$\because\ A^{-1}=\dfrac{Adj\ A}{|A|}$ and $|A|\neq 0$

Here, $|A| = ad - bc \neq 0$

Equation $(i) \Rightarrow$

$$\begin{bmatrix} x \\ y \end{bmatrix}=\frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix}}{ad-bc}$$

$$=\begin{bmatrix} \dfrac{dm-bm}{ad-bc} \\ \dfrac{-cm+an}{ad-bc} \end{bmatrix}$$

$\Rightarrow x=\dfrac{dm-bn}{ad-bc}$ and $y=\dfrac{an-cm}{ad-bc}$

(ii) Cramer's Rule **(K.B)**

Consider the following system of linear equations.
$ax + by = m$
$cx + by = n$

We know that
$AX = B$

where $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \end{bmatrix}$ and $B=\begin{bmatrix} m \\ n \end{bmatrix}$

Or $X = A^{-1}B$ or $X=\dfrac{Adj\ A}{|A|}\times B$

Or $\begin{bmatrix} x \\ y \end{bmatrix}=\dfrac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix}}{|A|}=\dfrac{\begin{bmatrix} dm-bn \\ -cm+an \end{bmatrix}}{|A|}$

$$=\begin{bmatrix} \dfrac{dm-bn}{|A|} \\ \dfrac{-cm+an}{|A|} \end{bmatrix}$$

Or $x=\dfrac{dm-bn}{|A|}=\dfrac{|A_x|}{|A|}$

and $y=\dfrac{an-cm}{|A|}=\dfrac{|A_y|}{|A|}$

where $|A_x|=\begin{vmatrix} m & b \\ n & d \end{vmatrix}$ and $|A_y|=\begin{vmatrix} a & m \\ c & n \end{vmatrix}$

Example # 1 **(K.B)**

Solve the following system by using matrix inversion method
$4x-2y=8$
$3x+y=-4$

Solution:
$4x-2y=8$
$3x+y=-4$
Writing in matrix form

$$\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

Let $AX = B$
Or

$X = A^{-1}B$ or $X=\dfrac{Adj\ A}{|A|}\times B \rightarrow (i)$

Here

$$A=\begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$$

$A=4\times1-3(-2)=4+6=10\neq 0$

So A^{-1} is possible.

$$Adj\ A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$$

Putting the values in equation (*i*)

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ -4 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 8-8 \\ -24-16 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 0 \\ -40 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

By comparing, we get

$\Rightarrow x = 0, \quad y = -4$

$\therefore$ Solution Set $= \{(0, -4)\}$

Example # 2: **(A.B)**

Solve the following system of linear equations by using Cramer's rule

$3x - 2y = 1$

$-2x + 3y = 2$

Solution:

$3x - 2y = 1$

$-2x + 3y = 2$

Writing in matrix form

$$\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Here

$$A = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}, A_x = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}, A_y = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 \\ -2 & 3 \end{vmatrix} = 9 - 4 = 5 \neq 0$$

(A is non singular)

$$x = \frac{|A_x|}{|A|} = \frac{\begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix}}{5} = \frac{3+4}{5} = \frac{7}{5}$$

$$y = \frac{|A_y|}{|A|} - \frac{\begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}}{5} = \frac{6+2}{5} = \frac{8}{5}$$

$$\therefore \text{Solution Set} = \left\{\left(\frac{7}{5}, \frac{8}{5}\right)\right\}$$

Exercise 1.6

Q.1 Use of matrices, if possible to solve the following systems of linear equations by: **(A.B + U.B)**

(i) The matrix inversion method

(ii) The Cramer's rule

(i) $2x - 2y = 4$

$3x + 2y = 6$

(FSD 2018, SGD 2018, BWP 2018)

By matrices inversion method

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Let $AX = B$

Where $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$|A| = (2)(2) - (-2)(3)$

$|A| = 4 + 6$

$|A| = 10 \neq 0$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$$

As we know that

$AX = B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 2\times4 + 2\times6 \\ -3\times4+2\times6 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 8+12 \\ -12+12 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{20}{10} \\ \frac{0}{10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$x = 2, y = 0$

Solution: Set = {(2, 0)}

By Cramer's rule

$$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$$

$$= (2)(2) - (-2)(3)$$

$$= 4 - (-6)$$

$$= 4 + 6$$

$$= 10$$

$$|A_x| = \begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix}$$

$$= (4)(2) - (-2)(6)$$

$$= 8 + 12$$

$$= 20$$

$$|A_y| = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$$

$$= (2)(6) - (4)(3)$$

$$= 12 - 12$$

$$= 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{20}{10}$$

$$x = 2$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{0}{10}$$

$$y = 0$$

Solution: Set = {(2, 0)}

(ii) $2x + y = 3$ **(A.B)**

$6x + 5y = 1$

(GRW 2018, D.G.K 2018)

Matrices inversion method

$$\begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Let $AX = B$

where $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$$

$$= (2)(5) - (1)(6)$$

$$= 10 - 6$$

$= 4$ as $|A| \neq 0$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 5 & -1 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 5\times3+(-1\times1) \\ -6\times3+2\times1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 15+(-1) \\ -18+2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 14 \\ -16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \overset{7}{\cancel{14}} \times \frac{1}{\underset{2}{\cancel{4}}} \\ -\overset{4}{\cancel{16}} \times \frac{1}{\underset{1}{\cancel{4}}} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7}{2} \\ -4 \end{bmatrix}$$

$x = \frac{7}{2}, y = -4$

$\therefore$ **Solution Set** = $\left\{\left(\frac{7}{2}, -4\right)\right\}$

By Cramer's Rule

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 5 \end{bmatrix} B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix}$$

$= (2)(5) - (1)(6)$

$= 10 - 6$

$= 4$ as $|A| \neq 0$

Solution is possible because A is non singular matrix.

$$|A_x| = \begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix}$$

$= (3)(5) - (1)(1)$

$= 15 - 1$

$= 14$

$$|A_y| = \begin{vmatrix} 2 & 3 \\ 6 & 1 \end{vmatrix}$$

$= (2)(1) - (3)(6)$

$= 2 - 18$

$= -16$

$x = \frac{|A_x|}{|A|}$

$x = \frac{14}{4}$

$x = \frac{7}{2}$

$y = \frac{|A_y|}{|A|}$

$y = \frac{16}{4}$

$y = -4$

Solution Set $= \left\{\left(\frac{7}{2}, -4\right)\right\}$

(iii) $4x + 2y = 8$ **(FSD 2019)** **(A.B)**

$3x - y = -1$

By Matrices Inversion Method

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$= (4)(-1) - (2)(3)$

$= -4 - 6$

$= -10$ as $|A| \neq 0$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$$

As we know that

$AX = B$

$X = A^{-1}B$

$X = \frac{1}{|A|} \times AdjA \times B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -8+2 \\ -24+(-4) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-10}\begin{bmatrix} -6 \\ -28 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-6}{-10} \\ \frac{-28}{-10} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{14}{5} \end{bmatrix}$$

$$x = \frac{3}{5}, y = \frac{14}{5}$$

Solution Set $= \left\{\left(\frac{3}{5}, \frac{14}{5}\right)\right\}$

By Cramer's rule

$$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}, B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= (4)(-1) - (2)(3)$$

$$= -4 - 6$$

$$= -10 \quad \text{as} \quad |A| \neq 0$$

Solution is possible because A is non singular matrix.

$$|A_x| = \begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix}$$

$$= (8)(-1) - (2)(-1)$$

$$= -8 - (-2)$$

$$= -6$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{6}{10}$$

$$x = \frac{3}{5}$$

$$|A_y| = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$$

$$= (4)(-1) - (8)(3)$$

$$= -4 - 24$$

$$= -28$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{-28}{-10}$$

$$y = \frac{14}{5}$$

Solution Set $= \left\{\left(\frac{3}{5}, \frac{14}{5}\right)\right\}$

(iv) $3x - 2y = -6$ **(A.B)**

$5x - 2y = -10$

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

Let AX = B

Where,

$$A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$$

$$= (3)(-2) - (-2)(5)$$

$$= -6 - (-10)$$

$$= -6 + 10$$

$$= 4 \quad \text{as} \quad |A| \neq 0$$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2 & 2 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -2\times-6+2\times-10 \\ -5\times-6+3\times-10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12+(-20) \\ 30+(-30) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12-20 \\ 30-30 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -8 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-8}{4} \\ \frac{0}{4} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$x=-2,\ y=0$

Solution: Set $=\{(-2,0)\}$

By Cramer's rule

$$A=\begin{bmatrix} 3 & -2 \\ 5 & -2 \end{bmatrix}, B=\begin{bmatrix} -6 \\ -10 \end{bmatrix}$$

$$|A|=\begin{vmatrix} 3 & -2 \\ 5 & -2 \end{vmatrix}$$

$=(3)(-2)-(-2)(5)$

$=-6-(-10)$

$=-6+10$

$=4$ as $|A|\neq 0$

Solution is possible because A is non singular matrix.

$$|A_x|=\begin{vmatrix} -6 & -2 \\ -10 & -2 \end{vmatrix}$$

$=(-6)(-2)-(-2)(-10)$

$=+12-(+20)$

$=12-20$

$=-8$

$$|A_y|=\begin{vmatrix} 3 & -6 \\ 5 & -10 \end{vmatrix}$$

$=(3)(-10)-(-6)(5)$

$=-30-(-30)$

$=-30+30$

$=0$

$$x=\frac{|A_x|}{|A|}$$

$$x=\frac{-8}{4}$$

$x=-2$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{0}{4}$$

$y=0$

Solution Set $=\{(-2,0)\}$

(v) $3x-2y=4$ **(A.B)**

$-6x+4y=7$

(GRW 2015, SGD 2015, SWL 2018)

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Here $A=\begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X=\begin{bmatrix} x \\ y \end{bmatrix}, B=\begin{bmatrix} 4 \\ 7 \end{bmatrix}$

$$|A|=\begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix}$$

$=(3)(4)-(-2)(-6)$

$=12-(+12)$

$=12-12$

$=0$

Solution is not possible because A is singular matrix.

(vi) $4x+y=9$ **(A.B)**

$-3x-y=-5$

By Matrices Inversion Method

$$\begin{bmatrix} 4 & 1 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

Let AX = B

Where, $A=\begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X=\begin{bmatrix} x \\ y \end{bmatrix}, B=\begin{bmatrix} 9 \\ -5 \end{bmatrix}$

$$|A|=\begin{vmatrix}4 & 1\\ -3 & -1\end{vmatrix}$$

$$=(4)(-1)-(1)(-3)$$

$$=-4+3$$

$$=-1 \qquad \text{as} \qquad |A|\neq 0$$

Solution is possible because $|A|$ is non singular

$$AdjA=\begin{vmatrix}-1 & -1\\ 3 & 4\end{vmatrix}$$

As we know that

$$X=A^{-1}B$$

$$X=\frac{1}{|A|}\times AdjA\times B$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-1 & -1\\ 3 & 4\end{bmatrix}\begin{bmatrix}9\\ -5\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-9+5\\ 27+(-20)\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-1}\begin{bmatrix}-4\\ 7\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{-4}{-1}\\ \frac{7}{-1}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ -7\end{bmatrix}$$

$$x=4, y=-7$$

Solution Set$=\{(4,-7)\}$

By Cramer's rule

$$A=\begin{bmatrix}4 & 1\\ -3 & -1\end{bmatrix}, B=\begin{bmatrix}9\\ -5\end{bmatrix}$$

$$|A|=\begin{vmatrix}4 & 1\\ -3 & -1\end{vmatrix}$$

$$=(4)(-1)-(1)(-3)$$

$$=-4-(-3)$$

$$=-4+3$$

$$=-1 \qquad \text{as} \qquad |A|\neq 0$$

Solution is possible because A is non-singular matrix

$$|A_x|=\begin{vmatrix}9 & 1\\ -5 & -1\end{vmatrix}$$

$$=(9)(-1)-(1)(-3)$$

$$=-9-(-5)$$

$$=-9+5$$

$$=-4$$

$$x=\frac{|A_x|}{|A|}$$

$$=\frac{-4}{-1}$$

$$x=4$$

$$|A_y|=\begin{vmatrix}4 & 9\\ -3 & -5\end{vmatrix}$$

$$=(4)(-5)-(9)(-3)$$

$$=-20-(-27)$$

$$=-20+27$$

$$=7$$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{7}{-1}$$

$$y=-7$$

Solution Set$=\{(4,-7)\}$

(vii) $2x-2y=4$ **(A.B)**

$-5x-2y=-10$ **(LHR 2019)**

By Matrices Inversion Method

$$\begin{bmatrix}2 & -2\\ -5 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}4\\ -10\end{bmatrix}$$

Let $A=\begin{bmatrix}3 & -2\\ 3 & 2\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}4\\ -10\end{bmatrix}$

$$|A|=\begin{vmatrix}2 & -2\\ -5 & -2\end{vmatrix}$$

$$=(2)(-2)-(-2)(-5)$$

$$=-4-(+10)$$

$$=-4-10$$

$$=-14 \qquad \text{as} \qquad |A|\neq 0$$

Solution is possible because A is non-singular matrix

$$AdjA = \begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}$$

As we know that

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \times AdjA \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -2 & 2 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -8+(-20) \\ 20+(-20) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -8-20 \\ 20-20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-14}\begin{bmatrix} -28 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-28}{-14} \\ \frac{0}{14} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$x = 2, y = 0$

Solution Set $= \{(2,0)\}$

By Cramer's rule

$$A = \begin{bmatrix} 2 & -2 \\ -5 & -2 \end{bmatrix} B = \begin{bmatrix} 4 \\ -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -2 \\ -5 & -2 \end{vmatrix}$$

$$= (2)(-2)-(-2)(-5)$$

$$= -4-(+10)$$

$$= -4-10$$

$= -14$ as $|A| \neq 0$

Solution is possible because A is non-singular matrix

$$|A_x| = \begin{bmatrix} 4 & -2 \\ -10 & -2 \end{bmatrix}$$

$$= (4)(-2)-(-2)(-10)$$

$$= -8-(+20)$$

$$= -8-20$$

$$= -28$$

$$y = \frac{|A_y|}{|A|} = ?$$

$$|A_y| = \begin{bmatrix} 2 & 4 \\ -5 & -10 \end{bmatrix}$$

$$= (2)(-10)-(4)(-5)$$

$$= -20-(-20)$$

$$= -20+20$$

$$= 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{-28}{-14}$$

$$x = 2$$

Solution Set $= \{(2,0)\}$

(viii) $3x-4y=4$ **(A.B)**

$x+2y=8$

By Matrices Inversion Method

$$\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$$

Let $A = \begin{bmatrix} 3 & -2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$$

$$= (3)(2)-(-4)(1)$$

$$= 6-(-4)$$

$$|A| = 6+4$$

$= 10$ as $|A| \neq 0$

Solution is possible because A is non singular matrix.

$$AdjA = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$$

As we know that

$AX = B$

$X = A^{-1}B$

$X = \frac{1}{|A|} \times AdjA \times B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 2\times4+4\times8 \\ -1\times+3\times8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 8+32 \\ -4+24 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 40 \\ 20 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{40}{10} \\ \frac{20}{10} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

$x = 4, y = 2$

Solution Set $= \{(4,2)\}$

By Cramer's rule

$A = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} B = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$

$= (3)(2) - (-4)(1)$

$= 6 - (-4)$

$= 6 + 4$

$= 10$ as $|A| \neq 0$

Solution is possible because a is non-singular matrix

$|A_x| = \begin{bmatrix} 4 & -4 \\ 8 & 2 \end{bmatrix}$

$|A_x| = \begin{vmatrix} 4 & -4 \\ 8 & 2 \end{vmatrix}$

$= (4)(2) - (-4)(8)$

$= 8 - (-32)$

$= 8 + 32$

$= 40$

$|A_y| = \begin{bmatrix} 3 & 4 \\ 1 & 8 \end{bmatrix}$

$|A_y| = \begin{vmatrix} 3 & 4 \\ 1 & 8 \end{vmatrix}$

$= (3)(8) - (4)(1)$

$= 24 - 4$

$= 20$

$x = \frac{|A_x|}{|A|}$

$x = \frac{40}{10}$

$x = 4$

$y = \frac{|A_y|}{|A|}$

$y = \frac{20}{10}$

$y = 2$

Solution Set $= \{(4,2)\}$

Q.2 The length of a rectangle is 4 times it width. The perimeter of the rectangle is 150cm. Find the dimensions of the rectangle. (A.B+U.B+K.B)

Solution:

Let width of rectangle = x

Length of rectangle = y

According to 1st condition

$y = 4x$

$-4x + y = 0 \quad \rightarrow ...(i)$

According to 2nd condition

2(length + Width)=Perimeter

$2(y + x) = 150$

$$y+x=\frac{\cancel{150}^{75}}{\cancel{2}}$$

$x+y=75 \qquad \rightarrow ...\text{(ii)}$

$-4x+y=0$

$x+y=75$

Changing into matrix form

$$\begin{bmatrix}-4 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 75\end{bmatrix}$$

$X=A^{-1}B$

(By matrix inversion method)

Let $A=\begin{bmatrix}-4 & 1\\ 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}0\\ 75\end{bmatrix}$

$$|A|=\begin{vmatrix}-4 & 1\\ 1 & 1\end{vmatrix}$$

$=(-4)(1)-(1)(1)$

$=-4-1$

$=-5$

$$AdjA=\begin{bmatrix}1 & -1\\ -1 & -4\end{bmatrix}$$

As we know that

$X=A^{-1}B$

$$X=\frac{1}{|A|}\times AdjA\times B$$

$$=-\frac{1}{5}\begin{bmatrix}1 & -1\\ -1 & -4\end{bmatrix}\begin{bmatrix}0\\ 75\end{bmatrix}$$

$$=-\frac{1}{5}\begin{bmatrix}0-75\\ 0-300\end{bmatrix}$$

$$=\frac{1}{-5}\begin{bmatrix}-75\\ -300\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}\frac{-75}{-5}\\ \frac{-300}{-5}\end{bmatrix}$$

$$=\begin{bmatrix}15\\ 60\end{bmatrix}$$

$x=15, y=60$

Result:

Width of rectangle $= x = 15$cm

Length of rectangle $= y = 60$cm

By Cramer's rule

$$A=\begin{bmatrix}-4 & 1\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}0\\ 75\end{bmatrix}$$

$$|A|=\begin{vmatrix}-4 & 1\\ 1 & 1\end{vmatrix}$$

$=(-4)(1)-(1)(1)$

$=-4-1$

$=-5$

$$|A_x|=\begin{vmatrix}0 & 1\\ 75 & 1\end{vmatrix}$$

$=(0)(1)-(1)(75)$

$=0-75$

$=-75$

$$|A_y|=\begin{vmatrix}-4 & 0\\ 1 & 75\end{vmatrix}$$

$=(-4)(75)-(0)(1)$

$=-300+0$

$=-300$

$$x=\frac{|A_x|}{|A|}$$

$$=\frac{-75}{-5}$$

$x=15$

$$y=\frac{|A_y|}{|A|}$$

$$=\frac{-300}{-5}$$

$y=60$

Result:

Width of rectangle = x = 15 cm

Length of rectangle = y = 60 cm

Q.3 Two sides of a rectangle differ by 3.5cm. Find the dimension of the rectangle if its perimeter is 67cm. (K.B)

Solution:

Suppose Width of rectangle = x

Length of rectangle = y

According to 1st condition

$y-x=3.5$

$-x+y=3.5 \quad \rightarrow(\text{i})$

According to 2nd condition

$2(L+B)=P$

$2(y+x)=67$

$x+y=\frac{67}{2}$

$x+y=33.5 \quad \rightarrow(\text{ii})$

Changing eq (*i*) and (*ii*) into matrix form

$$\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3.5\\ 33.5\end{bmatrix}$$

(By matrix inversion method)

Let $A=\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}3.5\\ 33.5\end{bmatrix}$

$|A|=\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}$

$=(-1)(1)-(1)(1)$

$=-1-1$

$=-2$

$AdjA=\begin{bmatrix}1 & -1\\ -1 & -1\end{bmatrix}$

As we know that

$X=A^{-1}B$

$X=\frac{1}{|A|}\times AdjA\times B$

Putting the values

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{-2}\begin{bmatrix}1 & -1\\ -1 & -1\end{bmatrix}\begin{bmatrix}3.5\\ +33.5\end{bmatrix}$$

$$=\frac{1}{-2}\begin{bmatrix}1(3.5)+(-1)(33.5)\\ -1(3.5)+(-1)(33.5)\end{bmatrix}$$

$$=\frac{1}{-2}\begin{bmatrix}3.5-33.5\\ -3.5-33.5\end{bmatrix}$$

$$=\frac{1}{-2}\begin{bmatrix}-30\\ -37\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-30}{-2}\\ \frac{-37}{-2}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}15\\ 18.5\end{bmatrix}$$

$\Rightarrow x=15,\ y=18.5$

By Cramer's rule

$A=\begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix} B=\begin{bmatrix}3.5\\ 33.5\end{bmatrix}$

$|A|=\begin{vmatrix}-1 & 1\\ 1 & 1\end{vmatrix}$

$=(-1)(1)-(1)(1)$

$=-1-1$

$=-2$

$|A_x|=\begin{vmatrix}3.5 & 1\\ 33.5 & 1\end{vmatrix}$

$=(3.5)(1)-(1)(33.5)$

$=3.5-33.5$

$=-30$

$|A_y|=\begin{vmatrix}-1 & 3.5\\ 1 & 33.5\end{vmatrix}$

$=(-1)(33.5)-(3.5)(1)$

$=-33.5-3.5$

$=-37$

$x=\frac{|A_x|}{|A|}$

$x=\frac{-30}{-2}$

$x=15$

$y=\frac{|A_y|}{|A|}$

$y=\frac{-37}{-2}$

$y=\frac{37}{2}=18.5$

Result:

Width of rectangle =x=15cm

Length of rectangle =y=18.5cm

Q.4 The third angle of an isosceles Δ is $16°$ less than the sum of two equal angles. Find three angles of the triangle. (K.B)

Solution:

Let each equal angles are x and third angle is y

According to condition $y=2x-16$

$2x-y=16$ (i)

As we know that

$x+x+y=180$

$2x+y=180$ (ii)

$2x-y=16$

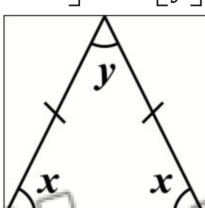
$2x+y=180$

Changing into matrix form

$$\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}16\\ 180\end{bmatrix}$$

Let $AX=B$

Where, $A=\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}16\\ 180\end{bmatrix}$



Using Matrix Inversion Method

$X=A^{-1}B$

$|A|=\begin{vmatrix}2 & -1\\ 2 & 1\end{vmatrix}$

$=2\times1-(-1)\times2$

$=2+2$

$=4\neq0$ (None singular)

A^{-1} exist

$AdjA=\begin{bmatrix}1 & 1\\ -2 & 2\end{bmatrix}$

$X=A^{-1}B$

Or $X=\frac{1}{|A|}\times AdjA\times B$

Putting the values

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{4}\begin{bmatrix}1 & 1\\ -2 & 2\end{bmatrix}\begin{bmatrix}16\\ 180\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}1\times16+1\times180\\ -2\times16+2\times180\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}16+180\\ -32+360\end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix}196\\ 328\end{bmatrix}$$

$$=\begin{bmatrix}\frac{196}{4}\\ \frac{328}{4}\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}49\\ 82\end{bmatrix}$$

$x=49$ and $y=82$

Cramer Rule

$A=\begin{bmatrix}2 & -1\\ 2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}16\\ 180\end{bmatrix}$

$|A|=\begin{vmatrix}2 & -1\\ 2 & 1\end{vmatrix}$

$=(2)(1)-(-1)(2)$

$=2-(-2)$

$=2+2$

$=4$

$|A_x|=\begin{bmatrix}16 & -1\\ 180 & 1\end{bmatrix}$

$=(16)(1)-(-1)(180)$

$=16+180$

$=196$

$$|A_y| = \begin{bmatrix} 2 & 16 \\ 2 & 180 \end{bmatrix}$$

$=(2)(180)-(16)(2)$

$=360-32$

$=328$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{196}{4}$$

$x = 49$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{328}{4}$$

$y = 82$

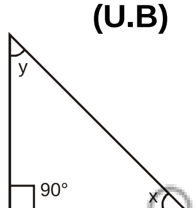
1st angle = x = 49°

2nd angle= x = 49°

3rd angle =y= 82°

Q.5 One acute angle of a right triangle is $12°$ more than twice the other acute angle. Find the acute angles of the right triangle. (U.B)

Solution:



Let one acute angle = x

And other acute angle = y

According to given condition

$x = 2y + 12$

$x - 2y = 12 \quad \rightarrow (i)$

As we know

$x + y = 90 \quad \rightarrow (ii)$

By matrices inversion method

Changing into matrix form

$$\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

Let $AX = B$

Where, $A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \end{bmatrix} B = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$

Using Matrix Inversion Method

$$|A| = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}$$

$=(1)(1)-(-2)(1)$

$=1-(-2)$

$=3$ (Non singular)

$\therefore A^{-1}$ exists

$$AdjA = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

As we know that

$X = A^{-1}B$ *or*

$$X = \frac{1}{|A|} \times AdjA \times B$$

Putting the values

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 12+180 \\ -12+90 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 192 \\ 78 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{192}{3} \\ \frac{78}{3} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 64 \\ 26 \end{bmatrix}$$

$x = 64$ and $y = 26$

Then

1st angle = x = 64°

2nd angle = y = 26°

By Cramer's rule

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} B = \begin{bmatrix} 12 \\ 90 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix}$$

$$= (1)(1) - (-2)(1)$$

$$= 1 - (-2)$$

$$= 1 + 2$$

$$= 3$$

$$|A_x| = \begin{vmatrix} 12 & -2 \\ 90 & 1 \end{vmatrix}$$

$$= (12)(1) - (-2)(90)$$

$$= 12 + 180$$

$$= 192$$

$$|A_y| = \begin{vmatrix} 1 & 12 \\ 1 & 90 \end{vmatrix}$$

$$= (90) - (12)$$

$$= 90 - 12$$

$$= 78$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{192}{3}$$

$$x = 64$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{78}{3}$$

$$y = 26$$

Result:

1st angle = x = 64°

2nd angle = y = 26°

Q.6 Two cars that are 600 km apart are moving towards each other. Their speeds differ by 6 km per hour and the cars are 123 km apart after $4\frac{1}{2}$ hours. Find the speed of each car.

(K.B) (U.B)

Solution:

Suppose speed of 1st car = x

Suppose speed of 2nd car = y

According to 1st condition

$$x - y = 6 \quad \rightarrow (\text{i})$$

According to 2nd condition

Total distance = 600 km

Left distance = 123 km

Covered distance = 600-123 = 477 km

Total time = $4\frac{1}{2}$ hours = $\frac{9}{2}$ hours

$$\text{Total Speed} = \frac{\text{Total Distance Covered}}{\text{Total Time Taken}}$$

$$x + y = \frac{477}{\frac{9}{2}} = 477 \div \frac{9}{2} = 477 \times \frac{2}{9}$$

$$x + y = \frac{{}^{53}\cancel{477} \times 2}{\cancel{9}}$$

$$x + y = 106 \quad \rightarrow (\text{ii})$$

$x - y = 6$

$x + y = 106$

By matrices inversion method

Changing eq (*i*) and (*ii*) into matrix form

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

Let $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 6 \\ 106 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= (1)(1) - (-1)(1)$$

$$= 1 - (-1)$$

$$= 1 + 1$$

$$= 2 \quad \text{as} \quad |A| \neq 0$$

Solution is possible because a is non-singular matrix

$AdjA=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$

$X=A^{-1}B$ or

$X=\frac{1}{|A|}\times AdjA\times B$

Putting the values

$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}6\\ 106\end{bmatrix}$

$=\frac{1}{2}\begin{bmatrix}6+106.\\ -6+106\end{bmatrix}$

$=\frac{1}{2}\begin{bmatrix}112\\ 100\end{bmatrix}$

$=\begin{bmatrix}\frac{112}{2}\\ \frac{100}{2}\end{bmatrix}$

$\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}56\\ 50\end{bmatrix}$

By comparing

$x=56, y=50$

Result:

Speed of 1st car = x= 56km/h

Speed of 2nd car = y= 50km/h

By Cramer's Rule

$A=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}6\\ 106\end{bmatrix}$

$|A|=\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}$

$=(1)(1)-(-1)(1)$

$=1-(-1)$

$=1+1$

$=2$

$|A_x|=\begin{vmatrix}6 & -1\\ 106 & 1\end{vmatrix}$

$=(6)(1)-(-1)(106)$

$=6-(-106)$

$=6+106$

$=112$

$|A_y|=\begin{vmatrix}1 & 6\\ 1 & 106\end{vmatrix}$

$=(106)(1)-(6)(1)$

$=106-6$

$=100$

$x=\frac{|A_x|}{|A|}$

$x=\frac{112}{2}$

$x=56$

$y=\frac{|A_y|}{|A|}$

$y=\frac{100}{2}$

$y=50$

Result:

Speed of 1st car = x = 56km/h

Speed of 2nd car = y = 50km/h

Review Exercise 1

Q.1 Select the correct answer in each of the following.

(i) The order of matrix $\begin{bmatrix} 2 & 1 \end{bmatrix}$ is…. (SGD 2015, SWL) **(K.B)**

(a) 2-by-1 **(b)** 1-by-2
(c) 1-by-1 **(d)** 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ is called …matrix. (LHR 2014, 17, FSD 2015, 17, BWP 2015, 16, RWP 2014) **(K.B)**

(a) Zero **(b)** Unit
(c) Scalar **(d)** Singular

(iii) Which is order of a square matrix? (D.G.K 2017, RWP 2016, SWL 2016, SGD 2016) **(K.B)**

(a) 2-by-2 **(b)** 1-by-2
(c) 2-by-1 **(d)** 3-by-2

(iv) Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ is… **(K.B)**

(MTN 2016, GRW 2014, RWP 2015, SGD 2016, SWL 2017)

(a) 3-by-2 **(b)** 2-by-3
(c) 1-by-3 **(d)** 3-by-1

(v) Adjoint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ is… (LHR 2018, FSD 2018, SWL 2014 MTN 2014, SGD 2015) **(K.B)**

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ **(d)** $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) Product of $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is… (LHR 2015, GRW 2015, FSD 2015, D.G.K 2015) **(K.B)**

(a) $\begin{bmatrix} 2x+y \end{bmatrix}$ **(b)** $\begin{bmatrix} x-2y \end{bmatrix}$

(c) $\begin{bmatrix} 2x-y \end{bmatrix}$ **(d)** $\begin{bmatrix} x+2y \end{bmatrix}$

(vii) If $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix}$=0, then x is equal to… (SGD 2014, RWP 2017, MTN 2015, D.G.K 2014) **(K.B)**

(a) 9 **(b)** -6
(c) 6 **(d)** -9

(viii) If $X + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then X is equal to… **(K.B)**

(LHR 2017, GRW 2015, BWP 2014, RWP 2016, MTN 2018 SGD 2017)

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ **(b)** $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ **(d)** $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
b	c	a	b	a	c	a	d

Q.2 Complete the follwoing:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **is called ... matrix.** **(BWP 2017)**

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **is called ... matrix.** **(GRW 2015, FSD 2016)**

(iii) **Additive inverse of** $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ **is....** **(SWL 2018)**

(iv) **In matrix multiplication, in general, AB ...BA.**

(v) **Matrix A+B may be found if order of A and B is...**

(vi) **A matrix is called ... matrix if number of rows and columns are equal.**

ANSWER KEY

i	ii	iii	iv	v	vi
null	unit	$\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$	$\neq$	same	square

Q.3 If $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$, then find a and b. **(K.B+A.B)**
(LHR 2014, FSD 2017)

Solution:

$$\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$$

By comparing, we get

$a+3=-3$ $\qquad$ $b-1=2$
$a=-3-3$ $\qquad$ $b=2+1$
$a=-6$ $\qquad$ $b=3$

Q.4 If $A=\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, B=\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$, then find the following.
(SWL 2014, SWL 2015)

(i) $2A+3B$ **(K.B+A.B)**
(ii) $-3A+2B$ **(K.B+A.B)**
(iii) $-3(A+2B)$ **(K.B+A.B)**
(iv) $\frac{2}{3}(2A-3B)$ **(K.B+A.B)**

Solutoin:

(i) $2A+3B=2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}+3\begin{bmatrix} 5 & 4 \\ -2 & -1 \end{bmatrix}$

$$=\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}+\begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}$$

$$=\begin{bmatrix} 4+15 & 6-12 \\ 2-6 & 0-3 \end{bmatrix}$$

$$=\begin{bmatrix} 19 & -6 \\ -4 & -3 \end{bmatrix}$$

Solution:

(ii) $-3A+2B=-3\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}+2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$
(BWP 2016, D.G.K 2015)

$$=\begin{bmatrix} -6 & -9 \\ -3 & 0 \end{bmatrix}+\begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} -6+10 & -9-8 \\ -3-4 & 0-2 \end{bmatrix}$$

$$=\begin{bmatrix} 4 & -17 \\ -7 & -2 \end{bmatrix}$$

Solution:

(iii) $-3(A+2B)=-3\left(\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}+2\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}\right)$

$$=-3\left(\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}+\begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}\right)$$

$$=-3\begin{bmatrix} 2+10 & 3-8 \\ 1-4 & 0-2 \end{bmatrix}$$

$$=-3\begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} -36 & 15 \\ 9 & 6 \end{bmatrix}$$

Solution:

(iv) $\frac{2}{3}(2A-3B)$

$$=\frac{2}{3}\left(2\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}-3\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}\right)$$

$$=\frac{2}{3}\left(\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix}-\begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}\right)$$

$$=\frac{2}{3}\begin{bmatrix} 4-15 & 6-(-12) \\ 2+6 & 0-(-3) \end{bmatrix}$$

$$=\frac{2}{3}\begin{bmatrix} -11 & 6+12 \\ 2+6 & 0+3 \end{bmatrix}$$

$$=\frac{2}{3}\begin{bmatrix} -11 & 18 \\ 8 & 3 \end{bmatrix}$$

$$=\begin{bmatrix} -11\times\frac{2}{3} & 18\times\frac{2}{3} \\ 8\times\frac{2}{3} & 3\times\frac{2}{3} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{-22}{3} & 12 \\ \frac{16}{3} & 2 \end{bmatrix}$$

Q.5 Find the value of X, if
$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}+X=\begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$.
(K.B+A.B)
(GRW 2017, RWP 2014, D.G.K 2016)

Solution:
Given that

$$\begin{bmatrix}2 & 1\\3 & -3\end{bmatrix}+X=\begin{bmatrix}4 & -2\\-1 & -2\end{bmatrix}$$

$$X=\begin{bmatrix}4 & -2\\-1 & -2\end{bmatrix}-\begin{bmatrix}2 & 1\\3 & -3\end{bmatrix}$$

$$=\begin{bmatrix}4-2 & -2-1\\-1-3 & -2-(-3)\end{bmatrix}$$

$$=\begin{bmatrix}2 & -3\\-4 & -2+3\end{bmatrix}$$

$$X=\begin{bmatrix}2 & -3\\-4 & 1\end{bmatrix}$$ **Ans**

Q.6 If $A=\begin{bmatrix}0 & 1\\2 & -3\end{bmatrix}, B=\begin{bmatrix}-3 & 4\\5 & -2\end{bmatrix}$**, then prove that**

(i) $AB\neq BA$ **(K.B+A.B)**

Proof:

Given $A=\begin{bmatrix}0 & 1\\2 & -3\end{bmatrix}, B=\begin{bmatrix}-3 & 4\\5 & -2\end{bmatrix}$

(i) $AB\neq BA$

L.H.S.= $AB=\begin{bmatrix}0 & 1\\2 & -3\end{bmatrix}\begin{bmatrix}-3 & 4\\5 & -2\end{bmatrix}$

$$=\begin{bmatrix}0\times(-3)+1\times5 & 0\times4+1\times(-2)\\2\times(-3)+(-3)\times5 & 2\times4+(-3)\times(-2)\end{bmatrix}$$

$$=\begin{bmatrix}0+5 & 0-2\\-6-15 & 8+6\end{bmatrix}$$

$$=\begin{bmatrix}5 & -2\\-21 & 14\end{bmatrix}\rightarrow(\text{i})$$

R.H.S.= $BA=\begin{bmatrix}-3 & 4\\5 & -2\end{bmatrix}\begin{bmatrix}0 & 1\\2 & -3\end{bmatrix}$

$$=\begin{bmatrix}-3(0)+4(2) & -3(1)+4(-3)\\5(0)+(-2)(2) & 5(1)+(-2)(-3)\end{bmatrix}$$

$$=\begin{bmatrix}0+8 & -3-12\\0-4 & 5+6\end{bmatrix}$$

$$=\begin{bmatrix}8 & -15\\-4 & 11\end{bmatrix}\rightarrow(\text{ii})$$

From (i) and (ii) , we get

$L.H.S\neq R.H.S$

$AB\neq BA$

Hence proved

Q.7 If $A=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}$ **and** $B=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$**, then verify that**

(i) $(AB)^t=B^tA^t$ **(K.B+A.B)**

(ii) $(AB)^{-1}=B^{-1}A^{-1}$ **(K.B+A.B)**

verification:

Given

$A=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}$ **and** $B=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$

(i) $(AB)^t=B^tA^t$

$$AB=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$$

$$=\begin{bmatrix}3(2)+2(-3) & 3(4)+2(-5)\\1(2)+(-1)(-3) & 1(4)+(-1)(-5)\end{bmatrix}$$

$$=\begin{bmatrix}6-6 & 12-10\\2+3 & 4+5\end{bmatrix}$$

$$=\begin{bmatrix}0 & 2\\5 & 9\end{bmatrix}$$

L.H.S.= $(AB)^t=\begin{bmatrix}0 & 2\\5 & 9\end{bmatrix}^t$

$$=\begin{bmatrix}0 & 5\\2 & 9\end{bmatrix}\rightarrow(\text{i})$$

$$A^t=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}^t=\begin{bmatrix}3 & 1\\2 & -1\end{bmatrix}$$

$$B^t=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}^t=\begin{bmatrix}2 & -3\\4 & -5\end{bmatrix}$$

R.H.S.= $B^tA^t=\begin{bmatrix}2 & -3\\4 & -5\end{bmatrix}\begin{bmatrix}3 & 1\\2 & -1\end{bmatrix}$

$$=\begin{bmatrix}2\times3+(-3)\times2 & 2(1)+(-3)(-1)\\4\times3+(-5)\times2 & 4(1)+(-5)(-1)\end{bmatrix}$$

$$=\begin{bmatrix}6-6 & 2+3\\12-10 & 4+5\end{bmatrix}$$

$$=\begin{bmatrix}0 & 5\\2 & 9\end{bmatrix}\rightarrow(\text{ii})$$

From equal (i) and (ii) we get

L.H.S.=R.H.S.

$(AB)^t=B^tA^t$

Hence proved

Verification:

Given $A=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}$ **and** $B=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$

(ii) $(AB)^{-1}=B^{-1}A^{-1}$

L.H.S. = $(AB)^{-1}$

$AB=\begin{bmatrix}0 & 2\\5 & 9\end{bmatrix}$

$|AB|=\begin{vmatrix}0 & 2\\5 & 9\end{vmatrix}$

$=0\times9-2\times5$
$=0-10$
$=-10$ (Non singular)

Inverse exists

$Adj(AB)=\begin{bmatrix}9 & -2\\-5 & 0\end{bmatrix}$

L.H.S.=$(AB)^{-1}$

$=\frac{1}{|AB|}Adj(AB)$

$=\frac{1}{-10}\begin{bmatrix}9 & -2\\-5 & 0\end{bmatrix}$

$=\begin{bmatrix}\frac{9}{10} & \frac{-2}{-10}\\\frac{-5}{-10} & \frac{0}{-10}\end{bmatrix}$

$=\begin{bmatrix}\frac{-9}{10} & \frac{1}{5}\\\frac{1}{2} & 0\end{bmatrix}\quad\rightarrow(\text{i})$

R.H.S. = $B^{-1}A^{-1}$

$B=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$

$|B|=\begin{vmatrix}2 & 4\\-3 & -5\end{vmatrix}$

$=2(-5)-4\times(-3)$
$=-10+12$
$=2$ (non singular)

$\therefore B^{-1}$ exists

$AdjB=\begin{bmatrix}-5 & -4\\3 & 2\end{bmatrix}$

$B^{-1}=\frac{1}{|B|}AdjB$

$=\frac{1}{2}\begin{bmatrix}-5 & -4\\3 & 2\end{bmatrix}$

$A=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}$

$|A|=\begin{vmatrix}3 & 2\\1 & -1\end{vmatrix}$

$=3(-1)-2\times1$
$=-3-2$
$=-5$ (non singular)

$\therefore A^{-1}$ exists

$AdjA=\begin{bmatrix}-1 & -2\\-1 & 3\end{bmatrix}$

$A^{-1}=\frac{1}{|A|}\times AdjA$

$=\frac{1}{-5}\begin{bmatrix}-1 & -2\\-1 & 3\end{bmatrix}$

R.H.S.= $B^{-1}A^{-1}$

$=\left(\frac{1}{2}\begin{bmatrix}-5 & -4\\3 & 2\end{bmatrix}\right)\times\left(\frac{1}{-5}\begin{bmatrix}-1 & -2\\-1 & 3\end{bmatrix}\right)$

$=\frac{1}{2}\left(-\frac{1}{5}\right)\begin{bmatrix}-5 & -4\\3 & 2\end{bmatrix}\begin{bmatrix}-1 & -2\\-1 & 3\end{bmatrix}$

$=-\frac{1}{10}\begin{bmatrix}-5(-1)+(-4)(-1) & -5(-2)+(-4)(3)\\3(-1)+2(-1) & 3(-2)+2(3)\end{bmatrix}$

$=-\frac{1}{10}\begin{bmatrix}5+4 & 10-12\\-3-2 & -6+6\end{bmatrix}$

$=-\frac{1}{10}\begin{bmatrix}9 & -2\\-5 & 0\end{bmatrix}$

$=\begin{bmatrix}\frac{9}{-10} & \frac{-2}{-10}\\\frac{-5}{-10} & \frac{0}{-10}\end{bmatrix}$

$=\begin{bmatrix}\frac{-9}{10} & \frac{1}{5}\\\frac{1}{2} & 0\end{bmatrix}\quad\rightarrow(\text{ii})$

From equation (i) and (ii) we get

L.H.S. =R.H.S.

$(AB)^{-1}=B^{-1}A^{-1}$

Hence proved

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 **Order of transpose of** $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ **is**

(A) 3 – by – 2 **(B)** 2 – by – 3

(C) 1 – by – 3 **(D)** 3 – by – 1

2 **If** $P=\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ **and** $Q=\begin{bmatrix} 2 & 3 & 4 \\ -1 & 2 & 0 \end{bmatrix}$ **then which of the following statement is true**

(A) $P = Q$ **(B)** $P \neq Q$

(C) $P < Q$ **(D)** $P > Q$

3 **A square matrix** M **is said to be skew symmetric if** M^t

(A) M **(B)** O

(C) $-M$ **(D)** I

4 **Adjoint of** $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **is**

(A) $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ **(B)** $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$

(C) $\begin{bmatrix} -a & c \\ b & -d \end{bmatrix}$ **(D)** $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

5 **Idea of matrices was introduced by**

(A) John Napier **(B)** Al-Khwarizmi

(C) Arther Kellay **(D)** Henry Briggs

6 A (AdjA) =

(A) A^{-1} **(B)** det A

(C) A **(D)** (det A) I

7 **Usually __________ property is not possible in product of matrices**

(A) Associative **(B)** Distributive

(C) Commutative **(D)** None of these

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Find the values of a,b,c and d which satisfy the $\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$

(ii) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$ then find $\frac{2}{3}(2A - 3B)$.

(iii) Define matrix?

(iv) Define symmetric and skew symmetric matrices.

(v) Find the product of $\begin{bmatrix} 1 & 2 \\ -3 & 0 \\ 6 & -1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 0 & -5 \end{bmatrix}$.

Q.3 Answer the following Questions. **(4+4=8)**

(a) Solve with the help of matrix inverse method. $3x-2y=-6$, $5x-2y=-10$

(b) If $A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$ then prove that $(AB)^t = B^tA^t$.

Note:

Parents or guardians can conduct this test in their supervision in order to check the skill of students.