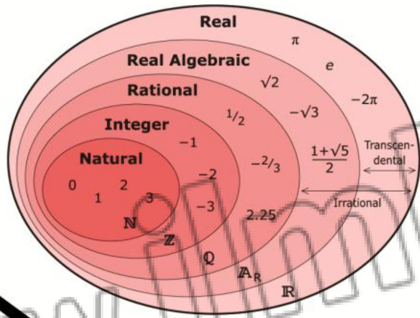


UNIT 2
REAL AND COMPLEX NUMBERS

Some Important Sets

Natural Numbers (D.G.K 2018) (K.B)

The numbers $1,2,3,...$ which we use for counting certain objects are called natural numbers or positive integers.
It is denoted by N.
i.e. $N=\{1,2,3,...\}$

Whole Numbers (RWP 2019) (K.B)

If we include 0 in the set of natural number, the resulting set is called set of Whole Numbers.
It is denoted by W.
i.e., $W=\{0,1,2,3,...\}$

Integers (RWP 2019) (K.B)

The set of integers consist of positive counting numbers, 0 and negative counting numbers.
It is denoted by Z.
i.e. $Z=\{...,-3,-2,-1,0,1,2,3,...\}$
or $Z=\{0,\pm1,\pm2,\pm3,...\}$

Rational Numbers (K.B)

All numbers of the form $\frac{p}{q}$ where p,q are integers and q is not zero are called rational numsbers. **For example**, $\frac{2}{3},-\frac{5}{4}$ etc.
It is denoted by Q.
i.e. $Q=\left\{\frac{p}{q} \mid p,q\in Z\wedge q\neq 0\right\}$

Irrational Numbers (K.B)

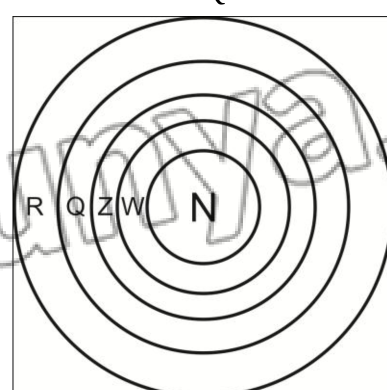
The numbers which cannot be expressed as $\frac{p}{q}$, where p and q are integers are called irrational numbers. **For example**, $\pi,\sqrt{3}$ etc.
It is denoted by Q'.
i.e., $Q'=\left\{x \mid x\neq\frac{p}{q}, p,q\in Z\wedge q\neq 0\right\}$

Real Numbers (K.B)

The union of the set of rational numbers and irrational numbers is known as the set of real numbers.
It is denoted by R,
i.e., $R=Q\cup Q'$

Note (U.B)

(i) $N\subset W\subset Z\subset Q$



i.e. $N\subset W\subset Z\subset Q\subset R$

a. are disjoint sets

(ii) For each prime number p, $\sqrt{p}$ is an irrational number

(iii) Square roots of all positive non-square integers are irrational

Types of Decimal Fraction (K.B)

There are three types of decimal fractions:

(i) Terminating decimal fractions

(ii) Recurring and non-terminating decimal fractions

(iii) Non-terminating and non- recurring decimal fraction

Types of Rational Numbers (K.B)

There are two types of rational numbers:

(i) Terminating decimal fractions

(ii) Recurring and non-terminating decimal fractions

Terminating Decimal Fractions (K.B)

The decimal fraction in which there are finite number of digits in its decimal part is called a **terminating decimal** fraction.

For example: $\frac{2}{5}=0.4$, $\frac{3}{8}=0.375$ etc.

Recurring and Non-terminating Decimal Fractions (K.B)

The decimal fraction (non-terminating) in which some digits are repeated again and again in the same order in its decimal part is called **recurring and non-terminating** decimal fraction.

For example:

$\frac{2}{9}=0.2222...,\ \frac{4}{11}=0.363636...$ etc.

Non-Recurring and Non-terminating Decimal Fractions (K.B)

The decimal fraction (non-terminating) in which some digits are not repeated again and again in the same order in its decimal part is called **non-recurring and non-terminating** decimal fraction.

These numbers are also called **irrational numbers**.

For example:

$\sqrt{2}=1.414213...,\ \pi=3.141592...$ etc.

Representation of Real Numbers on Number Line (K.B)

The real number are represented geometrically by points on a number line ℓ Such that each real number '*a*' corresponds to one and only one point on number line ℓ and to each point *p* on number line ℓ there corresponding precisely one real number.

Exercise 2.1

Q.1 Identity which of the following are rational and irrational numbers? (U.B)

Part #	Number	Type
(i)	$\sqrt{3}$	Irrational number
(ii)	$\frac{1}{6}$	Rational number
(iii)	π	Irrational number
(iv)	$\frac{15}{2}$	Rational number
(v)	7.25	Rational number
(vi)	$\sqrt{29}$	Irrational number

Q.2 Convert the following fractions into decimal fractions.

(i) $\frac{17}{25}$ **(U.B)**

Solution: $\frac{17}{25}$

$$\begin{array}{r|l} & 0.68 \\ 25 & 170 \\ & -150 \\ \hline & 200 \\ & -160 \\ \hline & 40 \end{array}$$

$\Rightarrow \frac{17}{25}=0.68$ **Ans**

(ii) $\frac{19}{4}$ **(A.B)**

Solution: $\frac{19}{4}$

$$\begin{array}{r} 4.75 \\ 4\overline{)19.000} \\ 16 \\ \hline 30 \\ 28 \\ \hline 20 \\ 20 \\ \hline 0 \end{array}$$

$\Rightarrow \frac{19}{4} = 4.75$ **Ans**

(iii) $\frac{57}{8}$ **(A.B)**

Solution: $\frac{57}{8}$

$$\begin{array}{r} 7.125 \\ 8\overline{)\quad 57} \\ -56 \\ \hline 10 \\ 8 \\ \hline 20 \\ -16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

$\Rightarrow \frac{57}{8} = 7.125$ **Ans**

(iv) $\frac{205}{18}$ **(A.B)**

Solution: $\frac{205}{18}$

$$\begin{array}{r} 11.388 \\ 18\overline{)205.000} \\ \hline 25 \\ 18 \\ \hline 70 \\ -54 \\ \hline 160 \\ -144 \\ \hline 160 \\ -144 \\ \hline 16 \\ 208 \\ \hline 18 \end{array}$$

$\Rightarrow \frac{205}{18} = 11.3889$ **Ans**

(v) $\frac{5}{8}$ **(A.B)**

Solution: $\frac{5}{8}$

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ 48 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$\Rightarrow \frac{5}{8} = 0.625$ **Ans**

(vi) $\frac{25}{38}$ **(A.B)**

Solution: $\frac{25}{38}$

$$\begin{array}{r} 0.65789... \\ 38\overline{)\quad 250} \\ -228 \\ \hline 220 \\ -190 \\ \hline 300 \\ -266 \\ \hline 340 \\ -304 \\ \hline 360 \\ -342 \\ \hline 18 \end{array}$$

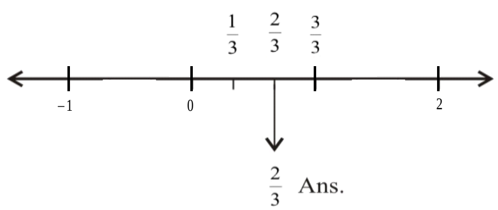
$\Rightarrow \frac{25}{38} = 0.65789$ **Ans**

Q.3 Which of the following statements are true and which are false? **(U.B)**

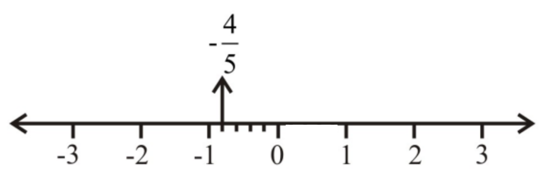
Part	Statement	T/F
(i)	$\frac{2}{3}$ is an irrational number	False
(ii)	π is an irrational number	True
(iii)	$\frac{1}{9}$ is a terminating fraction	False
(iv)	$\frac{3}{4}$ is a terminating fraction	True
(v)	$\frac{4}{5}$ is a recurring fraction	False

Q.4 Represent the following numbers on the number line.

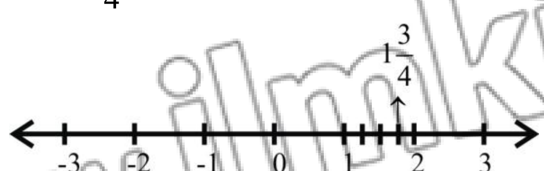
(i) $\frac{2}{3}$ **(A.B)**



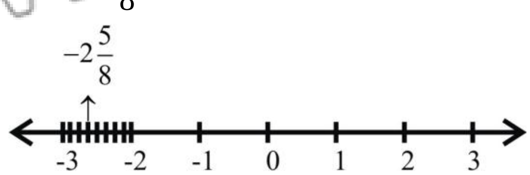
(ii) $-\frac{4}{5}$ **(A.B)**



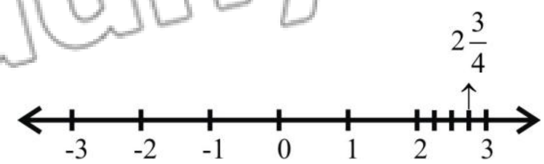
(iii) $1\frac{3}{4}$ **(A.B)**



(iv) $-2\frac{5}{8}$ **(A.B)**



(v) $2\frac{3}{4}$ **(SWL 2019)** **(A.B)**

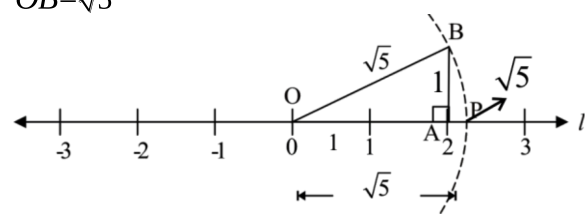


(vi) $\sqrt{5}$ **(A.B)**

$=\sqrt{4+1}=\sqrt{2^2+1^2}$

$(\text{Hypoteneus})^2=(\text{Base})^2+(\text{Perpencicular})^2$

$\overline{OB}=\sqrt{5}$



Q.5 Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$ **(A.B)**

(LHR 2019, SGD 2017)

Solution:

Rational number between

$\frac{3}{4}$ and $\frac{5}{9}$

$=\left[\frac{3}{4}+\frac{5}{9}\right]\div 2$

$=\left[\frac{27+20}{36}\right]\div 2$

$=\frac{47}{36}\times\frac{1}{2}$

$=\frac{47}{72}$

Q.6 Express the following recurring decimals as the rational number $\frac{p}{q}$ where p,q are integer and $q\neq 0$.

(i) $0.\overline{5}$ **(A.B)**

Solution:

Suppose

$x=0.\overline{5}$

$x=0.555...$

Multiplying both sides by 10

$10\times x=10\times 0.555...$

$10x = 5.555...$
$10x = 5 + 0.555...$
$10x = 5 + x$
$10x - x = 5$
$9x = 5$
$x = \frac{5}{9}$
$\therefore \quad 0.\overline{5} = \frac{5}{9}$

(ii) $0.\overline{13}$ **(RWP 2019, D.G.K 2017)** **(A.B)**

Solutions:

Suppose
$x = 0.\overline{13}$
$x = 0.131313...$
Multiplying both sides by 100
$100x = 100 \times 1.131313...$
$100x = 13.1313...$
$100x = 13 + 0.1313...$
$100x = 13 + x$
$100x - x = 13$
$99x = 13$
$x = \frac{13}{99}$
$\therefore \quad 0.\overline{13} = \frac{13}{99}$

(iii) $0.\overline{67}$ **(A.B)**

Solutions:

Suppose
$x = 0.\overline{67}$
$x = 0.676767...$
Multiplying both sides by 100
$100 \times x = 100 \times 0.676767...$
$100x = 67.6767...$
$100x = 67 + 0.6767...$
$100x = 67 + x$
$100x - x = 67$
$99x = 67$
$x = \frac{67}{99}$
$\therefore \quad 0.\overline{67} = \frac{67}{99}$

Properties of Real Numbers under Addition

(i) Closure property **(K.B)**

$\forall\ a,b \in R$
$a + b \in R$

For example:
If –3 and $5 \in R$
Then $-3 + 5 = 2 \in R$

(ii) Commutative Property **(K.B)**

$\forall\ a,b \in R$
$a + b = b + a$

For example:
If 2 and $3 \in R$
Then $2 + 3 = 3 + 2$
or $5 = 5$

(iii) Associative Property **(K.B)**

$\forall\ a,b,c \in R$
$a + b + c = a + (b + c)$

For example:
If $5,7,3 \in R$
Then $(5+7)+3 = 5+(7+3)$
Or $12 + 3 = 5 + 10$
$15 = 15$

(iv) Additive Identity **(K.B)**

The exists a unique real number 0, called additive identity such that
$a + 0 = a = 0 + a,\ \forall a \in R$

For example:
If $5, 0 \in R$
Then $5 + 0 = 0 + 5 = 5$

(v) Additive Inverse **(K.B)**

For ever $a \in R$ there exists a unique real number –a, called additive inverse of a such that
$a + (-a) = 0 = (-a) + a$

For example:
Additive inverse of 3 is –3
Since
$3 + (-3) = 0 = (-3) + 3$

Properties of Real Numbers under Multiplication

(i) Closure property **(K.B)**

$\forall\ a,b \in R$

$ab \in R$

For example:

If -3 and $5 \in R$

Then $(-3)(5) = -15 \in R$

(ii) Commutative Property **(K.B)**

$\forall\ a,b \in R$

$ab = ba$

For example:

If $\frac{1}{3}, \frac{3}{2} \in R$

Then $\left(\frac{1}{3}\right)\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)\left(\frac{1}{3}\right)$

Or $\frac{1}{2} = \frac{1}{2}$

(iii) Associative Property **(K.B)**

$\forall\ a,b,c \in R$

$(ab)c = a(bc)$

For example:

If $2,3,5 \in R$

Then $(2\times 3)\times 5 = 2\times(3\times 5)$

Or $6\times 5 = 2\times 15$

Or $30 = 30$

(iv) Multiplicative Identity **(K.B)**

The exists a unique real number 1, called the multiplicative identity such that

$a.1 = a = 1,\ \forall a \in R$

For example:

If $5,1 \in R$

Then $5\times 1 = 1\times 5 = 5$

(v) Multiplicative Inverse **(K.B)**

For every non zero real number, there exists a unique real number a^{-1} or $\frac{1}{a}$, called multiplicative inverse of a, such that

$aa^{-1} = 1 = a^{-1}a$

$a\times\frac{1}{a} = 1 = \frac{1}{a}\times a$

For example:

If $5, \frac{1}{5} \in R$

Then $5\times\frac{1}{5} = 1 = \frac{1}{5}\times 5$

So 5 and $\frac{1}{5}$ are multiplicative inverse of each other.

Multiplication is Distributive over Addition and Subtraction **(K.B)**

$\forall\ a,b,c \in R$

$a(b+c) = ab + ac$ (Left distributive law)

$(a+b)c = ac + bc$ (Right distributive law)

For example:

If $2,3,5 \in R$ then

$2(3+5) = 2\times 3 + 2\times 5$

Or $2(8) = 6 + 10$

Or $16 = 16$

And for all $a,b,c \in R$

$a(b-c) = ab - ac$ (Left distributive law)

$(a-b)c = ac - bc$ (Right distributive law)

For example:

If $2,5,3 \in \mathrm{R}$ then

$2(5-3) = 2\times 5 - 2\times 3$

Or $2\times 2 = 10 - 6$

Or $4 = 4$

Note **(K.B +U.B)**

(i) The symbol $\forall$ means "for all"

(ii) a is the multiplicative inverse of a^{-1} i.e. $a = \left(a^{-1}\right)^{-1}$

(iii) If a, b are real number their sum is written as *a+b* and product as *ab* or $a\times b$ or $a.b$ or (a)(b).

Properties of Equality of Real Number

(i) Reflexive Property (K.B)

$a = a \quad \forall a \in R$

For example:

2=2

(ii) Symmetric Property (K.B)

$\forall a, b \in R$

If $a = b$, then $b = a$

For example:

If $2 = x$, then $x = 2$

(iii) Transitive Property (K.B)

$\forall a, b, c \in R$

If $a = b$ and $b = c$ then $a = c$

For example:

If $x = 2$ and $y = 2$ then $x = y$

(iv) Additive Property (K.B)

$\forall a, b, c \in R$

If $a = b$, then $a + c = b + c$

For example:

If $2 = 2$, then $2 + 3 = 2 + 3$

(v) Multiplicative Property (K.B)

$\forall a, b, c \in R$

If $a = b$, then $ac = bc$

For example:

If $2 = 2$, then $2 \times 3 = 2 \times 3$

(vi) Cancellation Property for Addition (K.B)

$\forall a, b, c \in R$

If $a + c = b + c$, then $a = b$

For example:

If $x + 2 = y + 2$, then $x = y$

(vii) Cancellation Property of Multiplication (K.B)

$\forall a, b, c \in R$

If $ac = bc, c \neq 0$ then $a = b$

For example:

If $2x = 8$, then $x = 4$

Properties of Inequalities of Real Numbers

(i) Trichotomy Property (K.B)

$\forall a, b \in R$

$a < b$ or $a = b$ or $a > b$

For example:

$x < 0$ or $x = 0$ or $x > 0$

Any one statement is true, not all.

(ii) Transitive Property (K.B)

$\forall a, b, c \in R$

(a) $a < b$ and $b < c \Rightarrow a < c$

(b) $a > b$ and $b > c \Rightarrow a > c$

For example:

$5 < 6$ and $6 < 8 \Rightarrow 5 < 8$

Or $8 > 6$ and $6 > 5 \Rightarrow 8 > 5$

(iii) Additive Property (K.B)

$\forall a, b, c \in R$

(a) $a < b \Rightarrow a + c < b + c$ and
$a < b \Rightarrow c + a < c + b$

(b) $a > b \Rightarrow a + c > b + c$
$a > b \Rightarrow c + a > c + b$

For example:

$5 < 6 \Rightarrow 5 + 10 < 6 + 10$

Or $20 > 10 \Rightarrow 20 + 5 > 10 + 5$

(iv) Multiplicative Property (K.B)

$\forall a, b, c \in R$

<u>**Case: 1**</u> $c > 0$

(a) $a > b \Rightarrow ca > cb$
$a > b \Rightarrow ac > cb$

(b) $a < b \Rightarrow ac < bc$
$a < b \Rightarrow ca < cb$

For example:

$5 > 2 \Rightarrow 5 \times 4 > 2 \times 4$

<u>**Case: 2**</u> $c < 0$

(a) $a > b \Rightarrow ac < bc$
$a > b \Rightarrow ca < bc$

(b) $a < b \Rightarrow ac > bc$
$a < b \Rightarrow ca > cb$

For example:

$5 > 2 \Rightarrow -4 \times 5 < -4 \times 2$

i.e $-20 < -8$

(v) Multiplicative Inverse Property (K.B)

$\forall a, b \in R$ and $a \neq 0, b \neq 0$

(a) $a < b \Leftrightarrow \frac{1}{a} > \frac{1}{b}$

(b) $a > b \Leftrightarrow \frac{1}{a} < \frac{1}{b}$

For example:

$1 < 5 \Leftrightarrow \frac{1}{1} > \frac{1}{5}$

i.e. $1 > 0.2$

Exercise 2.2

Q.1 Identify the property used in the following: (A.B+K.B+U.B)

Solution:

Part #	Statement	Property
(i)	$a+b=b+a$	Commutative Property *w.r.t* +
(ii)	$(ab)c=a(bc)$	Associative Property *w.r.t* ×
(iii)	$7\times1=7$	Multiplicative Identity
(iv)	$x>y$ or $x=y$ or $x<y$	Trichotomy
(v)	$ab=ba$	Commutative *w.r.t* ×
(vi)	$a+c=b+c \Rightarrow a=b$	Cancellation Property of +
(vii)	$5+(-5)=0$	Additive Inverse
(viii)	$7\times\frac{1}{7}=1$	Multiplicative Inverse
(ix)	$a>b \Rightarrow ac>bc\,(c>0)$	Multiplicative

Q.2 Fill in the following blanks by stating the properties of real numbers used. (K.B+U.B)

Solution:

$3x+3(y-x)$

$=3x+3y-3x$ **Distributive property**

$=3x-3x+3y$ **Commutative *w.r.t* +**

$=0+3y$ **Additive Inverse**

$=3y$ **Additive identity**

Q.3 Give the name of property used in the following:

Solution:

(i) $\sqrt{24}+0=\sqrt{24}$ **(A.B)**

Ans. Additive Identity

(ii) $-\frac{2}{3}\left[5+\frac{7}{2}\right]=\left[-\frac{2}{3}\right](5)+\left[-\frac{2}{3}\right]\left[\frac{7}{2}\right]$ **(A.B)**

Ans. Distributive Property

(iii) $\pi+(-\pi)=0$ **(A.B)**

Ans. Additive Inverse

(iv) $\sqrt{3}.\sqrt{3}$ is a real number. **(A.B)**

Ans. Closure property w.r.t ×.

(v) $\left[-\frac{5}{8}\right]\left[-\frac{8}{5}\right]=1$ **(A.B)**

Ans. Multiplicative Inverse

RADICAL AND RADICANDS

Concept of Radicals and Radicands

(K.B)

If n is a positive integer greater than 1 and a is a real number, then any real number x such that $x^n=a$ is called the nth root of a, and in symbols is written as:

$x=\sqrt[n]{a}$ or $x=(a)^{1/n}$.

In the radical $\sqrt[n]{a}$, the symbol $\sqrt{\ }$ is called the **radical sign**, n is called the **index** of the radical and the real number a under the radical sign is called the **radicand** or **base**.

Note **(K.B)**

$\sqrt[2]{a}$ is usually written as $\sqrt{a}$

Difference between Radical form and Exponential form **(U.B)**

In radical form radical sign is used, $x=\sqrt[n]{a}$ is a radical form.

For example: $\sqrt[3]{x}, \sqrt[5]{x^2}$ etc.

In exponential form, exponent is used in place of radicals. $x=(a)^{1/n}$ is exponential form.

For example: $x^{3/2}, (z)^{2/7}$ etc.

Properties of Radicals **(U.B)**

Let $a, b \in R$ and m, n be positive integer. Then,

(i) $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

(ii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iii) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$

(iv) $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

(v) $\sqrt[n]{a^n} = a$

Exercise 2.3

Q.1 Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify. **(A.B)**

(i) $\sqrt[3]{-64}$

$=(-64)^{\frac{1}{3}}$

(ii) $2^{\frac{3}{5}}$ **(A.B)**

$=\sqrt[5]{2^3}$

(iii) $-7^{\frac{1}{3}}$

$-\sqrt[3]{7}$

(iv) $y^{-\frac{2}{3}}$

$=\sqrt[3]{y^{-2}}$

Q.2 Tell whether the following statements are true or false?

(i) $5^{\frac{1}{5}} = \sqrt{5}$ **False**

(ii) $2^{\frac{2}{3}} = \sqrt[3]{4}$ **True**

(iii) $\sqrt{49} = \sqrt{7}$ **False**

(iv) $\sqrt[3]{x^{27}} = x^3$ **False**

Q.3 Simplify the following radical expression.

(i) $\sqrt[3]{-125}$

Solution: **(A.B)**

$=\sqrt[3]{-125}$

$=\sqrt[3]{-5\times -5\times -5}$

$=\sqrt[3]{(-5)^3} \quad \because a^m \times a^n = a^{mn}$

$=-5 \quad \because \sqrt[m]{(a)^m} = a$

$\Rightarrow \sqrt[3]{-125} = -5$

(ii) $\sqrt[4]{32}$ **(LHR 2018)** **(A.B)**

Solutions:

$\sqrt[4]{32}$

$=\sqrt[4]{2\times 2\times 2\times 2\times 2}$

$=\sqrt[4]{2^4\times 2}$

$=\sqrt[4]{2^4}\times\sqrt[4]{2} \quad \because \sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

$=2\sqrt[4]{2} \quad \because \sqrt[m]{(a)^m} = a$

$\Rightarrow \sqrt[4]{32} = 2\sqrt[4]{2}$

(iii) $\sqrt[5]{\frac{3}{32}}$ **(A.B)**

(LHR 2017, 21, GRW 2019, SWL 2018, 19, RWP 2019)

Solution:

$\sqrt[5]{\frac{3}{32}}$

$=\frac{\sqrt[5]{3}}{\sqrt[5]{32}} \quad \because \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$=\frac{\sqrt[5]{3}}{\sqrt[5]{2\times 2\times 2\times 2\times 2}}$

$=\frac{\sqrt[5]{3}}{\sqrt[5]{(2)^5}}$

$=\frac{\sqrt[5]{3}}{2} \quad \because \sqrt[n]{a^n} = a$

$\Rightarrow \sqrt[5]{\frac{3}{32}} = \frac{\sqrt[5]{3}}{2}$

(iv) $\sqrt[3]{-\frac{8}{27}}$ **(A.B)**

Solution:

$$\sqrt[3]{-\frac{8}{27}}$$
$$=\sqrt[3]{\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)}$$
$$=\sqrt[3]{\left(-\frac{2}{3}\right)^3}$$
$$=-\frac{2}{3} \qquad \because \sqrt[n]{a^n}=a$$
$$\Rightarrow \sqrt[3]{-\frac{8}{27}}=-\frac{2}{3}$$

LAW OF EXPONENTS / INDICES

Base and Exponent (K.B+U.B)

In the exponential notation a^n (read as a to the nth power) we call 'a' as the base and 'n' as the exponent or the power to which the base is raised.

Laws of Exponents (K.B+U.B)

If $a,b\in R$ and m, n are positive integers, then

(i) $a^m.a^n=a^{m+n}$

(ii) $\left(a^m\right)^n=a^{mn}$

(iii) $(ab)^n=a^nb^n$

(iv) $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}, b\neq 0$

(v) $\frac{a^m}{a^n}=a^{m-n}, a\neq 0$

(vi) $a^0=1$ *where* $a\neq 0$

(vii) $a^{-n}=\frac{1}{a^n}$ *where* $a\neq 0$

Example # 2 **(A.B)**

(ii) Simplify: $\frac{4(3)^n}{3^{n+1}-3^n}$

Solution:

$$\frac{4(3)^n}{3^{n+1}-3^n}$$
$$=\frac{4(3)^n}{3^n\times 3-3^n} \qquad \because a^m.a^n=a^{m+n}$$
$$=\frac{4\cancel{(3)^n}}{\cancel{3^n}(3-1)} \qquad \because \frac{a^n}{a^n}=1$$
$$=\frac{4}{2}=2$$
$$\Rightarrow \frac{4(3)^n}{3^{n+1}-3^n}=2$$

Exercise 2.4

Q.1 Use laws of exponents to simplify.

(i) $\frac{(243)^{-\frac{2}{3}}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$ **(A.B)**

Solution:

$$\frac{(243)^{-\frac{2}{3}}(32)^{-1/5}}{\sqrt{(196)^{-1}}}$$
$$=\frac{\left(3^5\right)^{-\frac{2}{3}}\times\left(2^5\right)^{-1/5}}{\sqrt{\left[(14)^2\right]^{-1}}} \qquad \text{(Factorization)}$$
$$=\frac{(3)^{-\frac{10}{3}}\times 2^{-1}}{\sqrt{\left[(14)^{-1}\right]^2}} \qquad \because \left(a^m\right)^n=a^{mn}$$
$$=\frac{(3)^{-\frac{10}{3}}\times 2^{-1}}{(14)^{-1}} \qquad \because \sqrt[n]{a^n}=a$$
$$=\frac{7\times\cancel{2}}{(3)^{\frac{10}{3}}\times\cancel{2}} \qquad \because a^{-n}=\frac{1}{a^n},\ \frac{a^n}{a^n}=1$$
$$=\frac{7}{3^{\frac{10}{3}}}$$
$$=\frac{7}{\sqrt[3]{3^{10}}} \qquad \because \sqrt[n]{a}=(a)^{1/n}$$

$$= \frac{7}{\sqrt[3]{\overline{3\times3\times3}\times\overline{3\times3\times3}\times\overline{3\times3\times3}\times3}}$$

$$= \frac{7}{\sqrt[3]{3^3\times3^3\times3^3\times3}}$$

$$= \frac{7}{\sqrt[3]{3^3}\times\sqrt[3]{3^3}\times\sqrt[3]{3^3}\times\sqrt[3]{3}}$$

$$\because \sqrt[n]{ab} = \sqrt[n]{a}\ \sqrt[n]{b}$$

$$= \frac{7}{3\times3\times3\times\sqrt[3]{3}} \quad \because \sqrt[n]{a^n} = a$$

$$= \frac{7}{27\sqrt[3]{3}}$$

$$\Rightarrow \frac{(243)^{-\frac{2}{3}}(32)^{-1/5}}{\sqrt{(196)^{-1}}} = \frac{7}{27\sqrt[3]{3}}$$

(ii) $\left(2x^5y^{-4}\right)\left(-8x^{-3}y^2\right)$ **(A.B)**

Solution:

$$\left(2x^5y^{-4}\right)\left(-8x^{-3}y^2\right)$$

$$= 2(-8)x^5.x^{-3}.y^{-4}.y^2$$

$$= -16x^{5-3}y^{-4+2} \quad \because a^m.a^n = a^{m+n}$$

$$= -16x^2y^{-2}$$

$$= \frac{-16x^2}{y^2} \quad \because a^{-n} = \frac{1}{a^n}$$

$$\Rightarrow \left(2x^5y^{-4}\right)\left(-8x^{-3}y^2\right) = \frac{-16x^2}{y^2}$$

(iii) $\left[\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^o}\right]^{-3}$ **(A.B)**

Solution:

$$\left[\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^o}\right]^{-3}$$

$$= \left[x^{-2-4}y^{-1+3}z^{-4-0}\right]^{-3} \quad \because \frac{a^m}{a^n} = a^{m-n}$$

$$= \left(x^{-6}y^{+2}z^{-4}\right)^{-3}$$

$$= \left(x^{-6}\right)^{-3}\left(y^2\right)^{-3}\left(y^{-4}\right)^{-3} \because (ab)^n = a^nb^n$$

$$= x^{18}y^{-6}z^{12} \quad \because \frac{a^m}{a^n} = a^{m-n}$$

$$= \frac{x^{18}z^{12}}{y^6} \quad \because a^{-n} = \frac{1}{a^n}$$

$$\Rightarrow \left[\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^o}\right]^{-3} = \frac{x^{18}z^{12}}{y^6}$$

(iv) $\frac{(81)^n.3^5-(3)^{4n-1}(243)}{\left(9^{2n}\right)\left(3^3\right)}$ **(A.B)**

Solution:

$$\frac{(81)^n.3^5-(3)^{4n-1}(243)}{\left(9^{2n}\right)\left(3^3\right)}$$

$$= \frac{\left(3^4\right)^n.3^5-3^{4n}.3^{-1}.3^5}{\left(3^2\right)^{2n}.3^3} \text{ (factorization)}$$

$$= \frac{3^{4n}.3^5-3^{4n}.3^{-1+5}}{3^{4n}.3^3} \quad \because \left(a^m\right)^n = a^{mn}$$

$$= \frac{3^{4n}.3^5-3^{4n}.3^4}{3^{4n}.3^3} \quad \because a^m.a^n = a^{m+n}$$

$$= \frac{3^{4n}.3^4(3-1)}{3^{4n}.3^3} \text{ (taking common)}$$

$$= 3^{4n-4n}.3^{4-3}.(2) \quad \because \frac{a^m}{a^n} = a^{m-n}$$

$$= 3^0.3^1.2$$
$$= 1\times3\times2$$
$$= 6$$

Q.2 Show that

$$\left[\frac{x^a}{x^b}\right]^{a+b}\times\left[\frac{x^b}{x^c}\right]^{b+c}\times\left[\frac{x^c}{x^a}\right]^{c+a} = 1$$

(K.B+A.B+U.B)

(LHR 2018, 19, SGD 2017, SWL 2017)

Proof:

$$\text{L.H.S} = \left[\frac{x^a}{x^b}\right]^{a+b}\times\left[\frac{x^b}{x^c}\right]^{b+c}\times\left[\frac{x^c}{x^a}\right]^{c+a}$$

$$= \left(x^{a-b}\right)^{a+b}\times\left(x^{b-c}\right)^{b+c}\times\left(x^{c-a}\right)^{c+a}$$

$$\because \frac{a^m}{a^n} = a^{m-n}$$

$$= x^{(a-b)(a+b)}\times x^{(b-c)(b+c)}\times x^{(c-a)(c+a)}$$

$$\because \left(a^m\right)^n = a^{mn}$$

$$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2}$$

$$= x^{a^2-b^2+b^2-a^2+c^2-a^2} \quad \because a^m.a^n = a^{m+n}$$

$$= x^\circ \quad \because a^0 = 1$$

$= 1 =$ R.H.S

Proved

Q.3 Simplify

(i) $\dfrac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{-\frac{1}{3}} \times (9)^{\frac{1}{4}}}$ **(A.B)**

Solution:

$$\frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{-\frac{1}{3}} \times (9)^{\frac{1}{4}}}$$

$$= \frac{2^{\frac{1}{3}} \times (3^3)^{\frac{1}{3}} \times (2\times2\times3\times5)^{\frac{1}{2}}}{(2\times2\times3\times3\times5)^{\frac{1}{2}} \times (2^2)^{-\frac{1}{3}} \times (3^2)^{\frac{1}{4}}}$$

(factorization)

$$= \frac{2^{\frac{1}{3}} \times 3 \times (2^2)^{\frac{1}{2}} \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{(2^2)^{\frac{1}{2}} \times (3^2)^{\frac{1}{2}} \times (5)^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}}$$

$$\because (a^m)^n = a^{mn}, \ (ab)^n = a^n b^n$$

$$= \frac{2^{\frac{1}{3}} \times 3 \times 2 \times 3^{\frac{1}{2}} \times 5^{\frac{1}{2}}}{2 \times 3 \times 5^{\frac{1}{2}} \times 2^{-\frac{2}{3}} \times 3^{\frac{1}{2}}}$$

$$= 2^{\frac{1}{3}+1-1+\frac{2}{3}} \times 3^{1+\frac{1}{2}-1-\frac{1}{2}} \times 5^{\frac{1}{2}-\frac{1}{2}}$$

$$\because \frac{a^m}{a^n} = a^{m-n}, \because a^m.a^n = a^{m+n}$$

$$= 2^{\frac{1}{3}+\frac{2}{3}} \times 3^0 \times 5^0$$

$$= 2^{\frac{1}{3}+\frac{2}{3}} \times 1 \times 1 \quad \because a^0 = 1$$

$$= 2^{\frac{3}{3}}$$

$$= 2$$

$$\Rightarrow \frac{2^{\frac{1}{3}} \times (27)^{\frac{1}{3}} \times (60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}} \times (4)^{-\frac{1}{3}} \times (9)^{\frac{1}{4}}} = 2$$

(ii) Simplify: $\sqrt{\dfrac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$ **(A.B)**

(GRW 2019, RWP 2018, 19)

Solution:

$$\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$$

$$= \sqrt{\frac{(6^2)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{1}{2}}}} \quad \text{(factorization)}$$

$$= \sqrt{\frac{6^2 \times 5}{\left(\frac{100}{4}\right)^{\frac{1}{2}}}} \quad \because a^{-n} = \frac{1}{a^n}, (a^m)^n = a^{mn}$$

$$= \sqrt{\frac{6^2 \times 5}{(25)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{(5^2)^{\frac{1}{2}}}} \quad \because (a^m)^n = a^{mn}$$

$$= \sqrt{\frac{6^2 \times 5}{5}} \quad \because \frac{a^n}{a^n} = 1$$

$$= \sqrt{6^2}$$

$$= 6 \quad \because \sqrt[n]{a^n} = a$$

$$\Rightarrow \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}} = 6$$

(iii) $5^{2^3} \div (5^2)^3$ **(A.B)**

(LHR 2018, 21, GRW 2017, 21, SWL 2019, FSD 2021, SGD 2017, 21)

Solution:

$$5^{2^3} \div (5^2)^3$$

$$= 5^8 \div 5^6 \quad \because (a^m)^n = a^{mn}$$

$$= 5^{8-6} \quad \because \frac{a^m}{a^n} = a^{m-n}$$

$= 5^2$

$= 25$

$\Rightarrow 5^{2^3} \div (5^2)^3 = 25$

(iv) $(x^3)^2 \div x^{3^2}, x \neq 0$ **(A.B)**

(LHR 2017, FSD 2017, SWL 2017, D.G.K 2018)

Solution:

$(x^3)^2 \div x^{3^2}$

$= x^6 \div x^9 \quad \because (a^m)^n = a^{mn}$

$= x^{6-9} \quad \because \dfrac{a^m}{a^n} = a^{m-n}$

$= x^{-3}$

$= \dfrac{1}{x^3} \quad \because a^{-n} = \dfrac{1}{a^n}$

$\Rightarrow (x^3)^2 \div x^{3^2} = \dfrac{1}{x^3}$

Need of Complex Numbers **(K.B)**

Since square of a real number is non-negative. So the solution of the equation $x^2 + 1 = 0$ or $x^2 = -1$ does not exist in R. To overcome this inadequacy of real numbers, we need a number whose square is –1. Thus the mathematicians were tempted to introduce a larger set of numbers called the set of complex numbers which contains R and every number whose square is negative. They invented a new number $\sqrt{-1}$, called the imaginary unit, and denoted it by the letter *i* (iota) having the property that $i^2 = -1$.

Invention of iota (Complex Number) **(K.B)**

The swiss mathematician Leonard Euler $(1707 - 1783)$ was the first to use the symbol i for the number $\sqrt{-1}$.

Note **(U.B+K.B)**

Number like $\sqrt{-1}, \sqrt{-5}$ etc are called pure imaginary numbers

Definition of a Complex Number **(U.B)**

A number of the form $a + bi$ where a and b are real number and $i = \sqrt{-1}$, is called a complex number and is represented by *z*.

i.e. $z = a + ib$

Set of Complex Number **(U.B)**

The set of all complex number is denoted by C.

$C = \{z \mid z = a + bi, where\ a, b \in R\ and\ i = \sqrt{-1}\}$

Note **(U.B+K.B)**

(i) The numbers a and b, called the real and imaginary parts of Z are denoted as $a = \text{Re}(Z)$ and $b = Im(Z)$

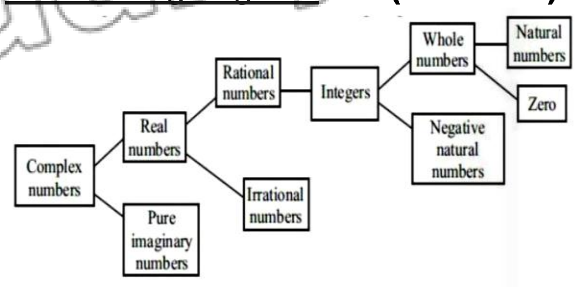
(ii) Every $a \in R$ may be identified with complex numbers of the form $a + 0i$ taking *b* = 0. Therefore, every real number is also a complex number. Thus $R \subset C$.

(iii) Every complex number is not a real number.

(iv) If *a* = 0, then *a* + *bi* reduces to a purely imaginary number *bi*. The set of purely imaginary numbers is also contained in *C*.

(v) If $a = b = 0$, then $z = 0 + i0$ is called the complex number 0.

The set of complex numbers is shown in the following diagram: **(U.B+K.B)**



Conjugate of a Complex Number **(U.B+K.B)**

If we change i to $-i$ in $z=a+bi$ we obtain another complex number $a-bi$ called the complex conjugate of z and is denoted by $\bar{z}$ (read z bar)

Thus if $z=-1-i$, then $\bar{z}=-1+i$

The number $a+bi$ and $a-bi$ are called conjugates of each other.

Note **(U.B+K.B)**

(i) $\bar{\bar{z}}=z$

(ii) The conjugate of a real number $z=a+0i$ coincides with the number itself since $\bar{z}=\overline{a+0i}=a-0i=a$.

The Equality of Complex Number

(U.B)

For all $a,b,c,d\in R$,

$a+bi=c+di$ If and only if $a=c$ and $b=d$

e.g., $2x+y^2i=4+9i$ if and only if

$2x=4$ and $y^2=9$, i.e, $x=2$ and $y=\pm 3$

Properties of Complex Number

(U.B)

Properties of real number R are also valid for the set of Complex numbers

(i) $z_1=z_1$ **(Reflexive Law)**

(ii) If $z_1=z_2$ then $z_2=z_1$ **(Symmetric Law)**

(iii) If $z_1=z_2$ and $z_2=z_3$ then $z_1=z_3$ **(Transitive Law)**

Exercise 2.5

Q.1 Evaluate

(i) i^7 **(A.B)**

Solution:

i^7

$=i^6.i$

$=\left(i^2\right)^3.i$

$=\left(-1\right)^3.i \quad \because i^2=-1$

$=-1\times i$

$=-i$

(ii) i^{50} **(A.B)**

(LHR 2021, FSD 2017, 21, SWL 2018, SGD 2018, BWP 2017, 21)

Solution:

i^{50}

$=\left(i^2\right)^{25}$

$=\left(-1\right)^{25} \quad \because i^2=-1$

$=-1$

(iii) i^{12} **(LHR 2017)** **(A.B)**

Solution:

i^{12}

$=\left(i^2\right)^6$

$=\left(-1\right)^6 \quad \because i^2=-1$

$=1$

(iv) $\left(-i\right)^8$ **(A.B)**

(RWP 2021, MTN 2021, SWL 2021, D.G.K 2017)

Solution:

$\left(-i\right)^8$

$=i^8$

$=\left(i^2\right)^4$

$=\left(-1\right)^4 \quad \because i^2=-1$

$=1$

(v) $\left(-i\right)^5$ **(FSD 2017)** **(A.B)**

Solution:

$\left(-i\right)^5$

$=-i^5$

$=-i^4.i$

$=-\left(i^2\right)^2.i$

$=-\left(-1\right)^2.i \quad \because i^2=-1$

$=-\left(1\right)\left(i\right)$

$=-i$

(vi) i^{27} **(A.B)**

Solution:

i^{27}

$=i^{26}.i$

$=\left(i^2\right)^{13}.i$

$=\left(-1\right)^{13}.i \quad \because i^2=-1$

$=-1.i$

$= -i$

Q.2 Write the conjugate of the following numbers. **(K.B)**

Part #	Complex Number	Conjugate of Number
(i)	$2+3i$	$2-3i$
(ii)	$3-5i$	$3+5i$
(iii)	$-i$	i
(iv)	$-3+4i$	$-3-4i$
(v)	$-4-i$	$-4+i$
(vi)	$i-3$	$-i-3$

Q.3 Write the real and imaginary part of the following numbers. **(K.B)**

Part #	Complex Number	Real Part	Imaginary Part
(i)	$1+i$	1	1
(ii)	$-1+2i$	-1	2
(iii)	$-2-2i$	-2	-2
(iv)	$-3i$	0	-3
(v)	$2+0i$	2	0

Q.4 Find the value of x and y if $x+iy+1=4-3i$ **(A.B)**

(GRW 2017, FSD 2016, RWP 2017, 18, MTN 2019, D.G.K 2016, BWP 2021)

Solution:

Here

$x+iy+1=4-3i$

$x+iy=4-3i-1$

$x+iy=3-3i$

By comparing real and imaginary part, we get

$x=3$ and $y=-3$

Basic Operations on Complex Numbers

(i) Addition **(K.B)**

Let $z = a+ib$ *and* $z_2 = c+id$ be two complex numbers and $a,b,c,d \in R$

Then

$z_1+z_2 = (a+bi)+(c+di)$

$= (a+c)+(b+d)i$

i.e., the sum of two complex numbers is the sum of the corresponding real and the imaginary parts.

For example:

$(3-8i)+(5+2i)$

$= (3+5)+(-8+2)i$

$= 8 - 6i$

(ii) Multiplication **(K.B)**

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers and $a, b, c, d \in R$.
Then

(a) Multiplication of a complex number with a scalar

If $k \in R$, then $kz_1 = k(a+bi) = ka + kbi$

For example:

If $z = 23 - 2i$

Then $5z = 5(3-2i)$

$= 15 - 10i$

(b) Multiplication of two complex numbers

$z_1 z_2 = (a+bi)(c+di)$

$= (ac - bd) + (ad + bc)i$

The multiplication of any two complex numbers $(a+bi)$ and $(c+di)$ is explained as

$z_1 z_2 = (a+bi)(c+di) = a(c+di) + bi(c+di)$

$= ac + adi + bci + bdi^2$

$= ac + adi + bci + bd(-1) \because i^2 = -1$

$= (ac - bd) + (ad + bc)i \quad$ (combining like terms)

For example:

$(2-3i)(4+5i)$

$= 8 + 10i - 12i - 15i^2$

$= 23 - 2i \qquad \because i^2 = -1$

(iii) Subtraction **(K.B)**

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers

$z_1 - z_2 = (a+bi) - (c+di)$

$= (a-c) + (b-d)i$

i.e., the difference of two complex number is the difference of the corresponding real and imaginary parts.

For example:

$(-2+3i) - (2+i)$

$= (-2-2) + (3-1)i$

$= -4 + 2i$

(iv) Division **(K.B+A.B)**

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers such that $z_2 \neq 0$
Then

$$\frac{z_1}{z_2}=\frac{a+bi}{c+di}$$

(Multiplying the numerator and denominator by $c-di,$ the complex conjugate of $c+di$)

$$=\frac{a+bi}{c+di}\times\frac{c-di}{c-di}$$

$$=\frac{ac+bci-adi-bdi^2}{c^2-(di)^2}$$

$$=\frac{ac+bci-adi+bd}{c^2+d^2},\quad \because i^2=-1$$

$$=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

$$=\frac{ac+bd}{c^2+d^2}+\left(\frac{bc-ad}{c^2+d^2}\right)i$$

Example # 4

Solve $(3-4i)(x+yi)=1+0.i$ for real numbers x and $y,$ where $i=\sqrt{-1}$ **(A.B)**

Solution:

Here

$$(3-4i)(x+yi)=1+0.i$$

$$(3-4i)(x+yi)=1+0=1$$

$$x+yi=\frac{1}{3-4i}$$

$$x+yi=\frac{1}{3-4i}\times\frac{3+4i}{3+4i}$$

$$=\frac{3-4i}{(3)^2-(4i)^2}$$

$$=\frac{3-4i}{9-16(-1)}\quad \because i^2=-1$$

$$=\frac{3-4i}{9+16}$$

$$=\frac{3-4i}{25}$$

$$=\frac{3}{25}-\frac{4}{25}i$$

Equating the real and imaginary part, we obtain

$$x=\frac{3}{25}\quad\text{and}\quad y=\frac{4}{25}$$

$\because$ Solution Set $=\left\{\left(\dfrac{3}{25},\dfrac{4}{25}\right)\right\}$

Exercise 2.6

Q.1 Identify the following statement as true or false.

(i) $\sqrt{-3}\sqrt{-3}=3$ **False** **(K.B)**

(ii) $i^{73}=-i$ **False** **(K.B)**

(iii) $i^{10}=-1$ **True** **(K.B)**

(iv) **Complex conjugate of** $\left(-6i+i^2\right)$ **is** $\left(-1+6i\right)$ **True** **(K.B)**

(v) **Difference of a complex number** $z=a+bi$ **and its conjugate is a real number. False** **(K.B)**

(vi) **If** $(a-1)-(b+3)i=5+8i,$ **then** $a=6$ **and** $b=-11$. **True** **(K.B)**

(vii) **Product of a complex number and its conjugate is always a non-negative real number. True** **(K.B)**

Q.2 Express the each complex number in the standard form $a+bi$**, where** a **and** b **are real number.**

(i) $(2+3i)+(7-2i)$ **(K.B)**

Solution:

$$(2+3i)+(7-2i)$$
$$=2+3i+7-2i$$
$$=2+7+3i-2i$$
$$=9+i$$
$$\Rightarrow (2+3i)+(7-2i)=9+i$$

(ii) $2(5+4i)-3(7+4i)$ **(A.B)**

Solution:

$$2(5+4i)-3(7+4i)$$
$$=10+8i-21-12i$$
$$=10-21+8i-12i$$
$$=-11-4i$$
$$\Rightarrow 2(5+4i)-3(7+4i)=-11-4i$$

(iii) $(-3+5i)-(4+9i)$ **(A.B)**

Solution:

$$(-3+5i)-(4+9i)$$
$$=+3-5i-4-9i$$
$$=3-4-5i-9i$$
$$=-1-14i$$
$$\Rightarrow (-3+5i)-(4+9i)=-1-14i$$

(iv) $2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$ **(A.B)**

Solution:

$$2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}$$
$$=2(-1)+6i^2-i+3\left(i^2\right)^8-6\left(i^2\right)^9.i+4\left(i^2\right)^{12}.i$$
$$\because i^2=-1$$
$$=-2+6(-1)i+3(-1)^8-6(-1).i+4(-1)^{12}.i$$
$$=-2-6i+3-6(-1)i+4(+1)i$$
$$=1-\cancel{6i}+\cancel{6i}+4i$$
$$=1+4i$$
$$\Rightarrow 2i^2+6i^3+3i^{16}-6i^{19}+4i^{25}=1+4i$$

Q.3 Simplify and write your answer in the form $a+bi$

(i) $(-7+3i)(-3+2i)$ **(A.B)**

(GRW 2017, FSD 2019, SWL 2017, BWP 2019, D.G.K 2017)

Solution:

$$(-7+3i)(-3+2i)$$
$$=-7(-3+2i)+3i(-3+2i)$$
$$=21-14i-9i+6i^2$$
$$=21-23i+6(-1)$$
$$=21-23i-6$$
$$=21-6-23i$$
$$=15-23i$$
$$\Rightarrow (-7+3i)(-3+2i)=15-23i$$

(ii) $\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$ **(A.B)**

Solution:

$$\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)$$
$$=\left(2-\sqrt{4\times-1}\right)\left(3-\sqrt{4\times-1}\right)$$
$$=\left(2-\sqrt{4i^2}\right)\left(3-\sqrt{4i^2}\right)$$
$$=\left(2-\sqrt{2^2i^2}\right)\left(3-\sqrt{2^2i^2}\right)$$
$$=(2-2i)(3-2i)$$
$$=2(3-2i)-2i(3-2i)$$
$$=6-4i-6i+4i^2$$

$=6-10i+4(-1)$
$=6-10i-4$
$=2-10i$

$\Rightarrow\left(2-\sqrt{-4}\right)\left(3-\sqrt{-4}\right)=2-10i$

(iii) $\left(\sqrt{5}-3i\right)^2$ **(A.B)**

Solution:

$\left(\sqrt{5}-3i\right)^2$

$=\left(\sqrt{5}\right)^2+(3i)^2-2\left(\sqrt{5}\right)(3i)$

$=5+9i^2-6\sqrt{5}i$

$=5+9(-1)-6\sqrt{5}i$

$=5-9-6\sqrt{5}i$

$=-4-6\sqrt{5}i$

$\Rightarrow\left(\sqrt{5}-3i\right)^2=-4-6\sqrt{5}i$

(iv) $(2-3i)\left(\overline{3-2i}\right)$ **(A.B)**

Solution:

$(2-3i)\left(\overline{3-2i}\right)$

$=(2-3i)(3+2i)$

$=2(3+2i)-3i(3+2i)$

$=6+4i-9i-6i^2$

$=6-5i-6(-1)$

$=6-5i+6$

$=6+6-5i$

$=12-5i$

$\Rightarrow(2-3i)\left(\overline{3-2i}\right)=12-5i$

Q.4 Simplify and write your answer in the form $a+bi$.

(i) $\dfrac{-2}{1+i}$ **(FSD 2019, D.G.K 2017)** **(A.B)**

Solution:

$\dfrac{-2}{1+i}$

$=\dfrac{-2}{1+i}\times\dfrac{1-i}{1-i}$

$=\dfrac{-2(1-i)}{(1)^2-(i)^2}$

$=\dfrac{-2+2i}{1-i^2}$

$=\dfrac{-2+2i}{1-(-1)}$

$=\dfrac{-2+2i}{1+1}$

$=\dfrac{-2+2i}{2}$

$=-\dfrac{2}{2}+\dfrac{2i}{2}$

$=-1+i$

$\Rightarrow\dfrac{-2}{1+i}=-1+i$

(ii) $\dfrac{2+3i}{4-i}$ **(A.B)**

Solution:

$\dfrac{2+3i}{4-i}$

$=\dfrac{2+3i}{4-i}\times\dfrac{4+i}{4+i}$

$=\dfrac{(2+3i)(4+i)}{(4)^2-(i)^2}$

$=\dfrac{2(4+i)+3i(4+i)}{16-(-1)}$

$=\dfrac{8+2i+12i+3i^2}{16+1}$

$=\dfrac{8+4i+3(-1)}{17}$

$=\dfrac{8+14i-3}{17}$

$=\dfrac{8-3+14i}{17}$

$=\dfrac{5+14i}{17}$

$= \frac{5}{17} + \frac{14}{17}i$

$\Rightarrow \frac{2+3i}{4-i} = \frac{5}{17} + \frac{14}{17}i$

(iii) $\frac{9-7i}{3+i}$ **(GRW 2021, MTN 2018) (A.B)**

Solution:

$\frac{9-7i}{3+i}$

$= \frac{9-7i}{3+i} \times \frac{3-i}{3-i}$

$= \frac{(9-7i)(3-i)}{(3)^2-(i)^2}$

$= \frac{9(3-i)-7i(3-i)}{9-(-1)}$

$= \frac{27-9i-21i+7i^2}{9+1}$

$\frac{27-30i+7(-1)}{10}$

$= \frac{27-30i-7}{10}$

$= \frac{27-7-30i}{10}$

$= \frac{20-30i}{10}$

$= \frac{20}{10} - \frac{30i}{10}$

$= 2-3i$

$\Rightarrow \frac{9-7i}{3+i} = 2-3i$

(iv) $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$ **(A.B)**

Solution:

$\frac{2-6i}{3+i} - \frac{4+i}{3+i}$

$= \frac{2-6i-(4+i)}{3+i}$

$= \frac{2\text{-}6\text{i-}4\text{-}i}{3+i}$

$= \frac{2-4-6i-i}{3+i}$

$= \frac{-2-7i}{3+i}$

$= \frac{-2-7i}{3+i} \times \frac{3-i}{3-i}$

$= \frac{-2-7i}{3+i} \times \frac{3-i}{3-i}$

$= \frac{-2(3-i)-7i(3-i)}{(3)^2-(i)^2}$

$= \frac{-6+2i-21i+7i^2}{9-(-1)}$

$= \frac{-6-19i+7(-1)}{9+1}$

$= \frac{-6-19i-7}{10}$

$= \frac{-6-7-19i}{10}$

$= \frac{-13-19i}{10}$

$= \frac{-13}{10} - \frac{19i}{10}$

$\Rightarrow \frac{2-6i}{3+i} - \frac{4+i}{3+i} = \frac{-13}{10} - \frac{19i}{10}$

(v) $\left[\frac{1+i}{1-i}\right]^2$ **(A.B)**

Solution:

$\left[\frac{1+i}{1-i}\right]^2$

$= \frac{(1+i)^2}{(1-i)^2}$

$= \frac{(1)^2+(i)^2+2ab}{(1)^2+(i)^2-2ab}$

$$=\frac{(1)^2+(i)^2+2(1)(i)}{(1)^2+(i)^2-2(1)(i)}$$

$$=\frac{1+(-1)+2i}{+1+(-1)-2i}$$

$$=\frac{\cancel{1}-\cancel{1}+2i}{\cancel{1}-\cancel{1}-2i}$$

$$=\frac{\cancel{2i}}{-\cancel{2i}}$$

$$=-1$$

$$\Rightarrow\left[\frac{1+i}{1-i}\right]^2=-1+0i$$

(vi) $\dfrac{1}{(2+3i)(1-i)}$ **(A.B)**

Solution:

$$\frac{1}{(2+3i)(1-i)}$$

$$=\frac{1}{2(1-i)+3i(1-i)}$$

$$=\frac{1}{2-2i+3i-3i^2}$$

$$=\frac{1}{2+i-3(-1)}$$

$$=\frac{1}{2+i+3}$$

$$=\frac{1}{2+3+i}$$

$$=\frac{1}{5+i}$$

$$=\frac{1}{5+i}\times\frac{5-i}{5-i}$$

$$=\frac{1(5-i)}{(5)^2-(i)^2}$$

$$=\frac{5-i}{25-(-1)}$$

$$=\frac{5-i}{25+1}$$

$$=\frac{5-i}{26}$$

$$=\frac{5}{26}-\frac{1i}{26}$$

$$\Rightarrow\frac{1}{(2+3i)(1-i)}=\frac{5}{26}-\frac{1i}{26}$$

Q.5 Calculate $(a)\,\bar{z}\ (b)\,z+\bar{z}\ (c)\,z-\bar{z}\ (d)\,z\bar{z}$ **for each of the following.**

(i) $z=-i$ **(A.B)**

Solution: $z=-i$

(a) $\bar{z}=i$

(b) $z+\bar{z}=-i+i$
$=0$

(c) $z-\bar{z}=(-i)-(i)$
$=-2i$

(d) $z\bar{z}=(-i)(i)$
$=-i^2$
$=-(-1)$
$=1$

(ii) $z=2+i$ **(A.B)**

Solution:

$z=2+i$

(a) $\bar{z}=2-i$

(b) $z+\bar{z}=(2+i)+(2-i)$
$=2+\cancel{i}+2-\cancel{i}$
$=2+2$
$=4$

(c) $z-\bar{z}=(2+i)-(2-i)$
$=\cancel{2}+i-\cancel{2}+i$
$=i+i$
$=2i$

(d) $z\bar{z}=(2+i)(2-i)$
$=(2)^2-(i)^2$
$=4-i^2$
$=4-(-1)$

$= 4+1$
$= 5$

(iii) $z = \frac{1+i}{1-i}$ **(A.B)**

Solution:

$z = \frac{1+i}{1-i}$

$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{1(1+i)+i(1+i)}{(1-i)(1+i)}$

$= \frac{1+i+i+(-1)}{(1)^2-(i)^2}$

$= \frac{1+2i+(-1)}{1-(-1)}$

$= \frac{1+2i-1}{1+1}$

$= \frac{2i}{2}$

$\Rightarrow z = i$

(a) $\bar{z} = -i$

(b) $z+\bar{z} = i+(-i)$

$= i - i$

$= 0$

(c) $z-\bar{z} = i-(-i)$

$= i+i$

$= 2i$

(d) $z\bar{z} = (i)(-i)$

$= -i^2$

$= -(-1)$

$= 1$

(iv) $z = \frac{4-3i}{2+4i}$ **(A.B)**

Solution:

$z = \frac{4-3i}{2+4i}$

$z = \frac{4-3i}{2+4i} \times \frac{2-4i}{2-4i}$

$= \frac{4(2-4i)-3i(2-4i)}{(2+4i)(2-4i)}$

$= \frac{8-16i-6i+12i^2}{(2)^2-(4i)^2}$

$= \frac{8-22i+12(-1)}{4-16i^2}$

$= \frac{8-22i-12}{4-16(-1)}$

$= \frac{8-12-22i}{4+16}$

$= \frac{-4-22i}{20}$

$= \frac{-4}{20} - \frac{22}{20}i$

$\Rightarrow z = -\frac{1}{5} - \frac{11}{10}i$

(a) $\bar{z} = \frac{-1}{5} + \frac{11}{10}i$ **(A.B)**

(b) $z+\bar{z} = \left(-\frac{1}{5}-\frac{11}{10}i\right)+\left(-\frac{1}{5}+\frac{11}{10}i\right)$ **(A.B)**

$= -\frac{1}{5} - \frac{11}{10}i - \frac{1}{5} + \frac{11}{10}i$

$= -\frac{1}{5} - \frac{1}{5}$

$= \frac{-1-1}{5}$

$= -\frac{2}{5}$

(c) $z-\bar{z} = \left(-\frac{1}{5}-\frac{11}{10}i\right)-\left(-\frac{1}{5}+\frac{11}{10}i\right)$ **(A.B)**

$= -\frac{1}{5} - \frac{11}{10}i + \frac{1}{5} - \frac{11}{10}i$

$= -\frac{11}{10}i - \frac{11}{10}i = \frac{-11i-11i}{10}$

$= -\frac{22^{11}i}{10^{5}}$

$= -\frac{11}{5}i$

(d) $z\bar{z} = \left(-\frac{1}{5} - \frac{11}{10}i\right)\left(-\frac{1}{5} + \frac{11}{10}i\right)$ **(A.B)**

$= \left(-\frac{1}{5}\right)^2 - \left(\frac{11}{10}i\right)^2$

$= \frac{1}{25} - \frac{121}{100}i^2$

$= \frac{1}{25} - \frac{121}{100}(-1)$

$= \frac{1}{25} + \frac{121}{100}$

$= \frac{4+121}{100}$

$= \frac{125}{100}$

$= \frac{5}{4}$

Q.6 If $z = 2+3i$ and show that. (A.B)

(i) $\overline{z+w} = \bar{z} + \bar{w}$

Proof: L.H.S = $\overline{z+w}$

$z + w = 2+3i+5-4i$

$= 2+5+3i-4i$

$= 7-i$

$\Rightarrow \overline{z+w} = \overline{7-i}$

$= 7+i$ … (i)

R. H. S $= \bar{z} + \bar{w}$

$= \left(\overline{2+3i}\right) + \left(\overline{5-4i}\right)$

$= 2-3i+5+4i$

$= 2+5-3i+4i$

$= 7+i$ … (ii)

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{z+w} = \bar{z} + \bar{w}$

Hence proved

(ii) $\overline{z-w} = \bar{z} - \bar{w}$ **(A.B)**

Proof: L.H.S.= $\overline{z-w}$

$z - w = (2+3i) - (5-4i)$

$= 2+3i-5+4i$

$= 2-5+3i+4i$

$= -3+7i$

$\Rightarrow \overline{z-w} = \overline{-3+7i}$

$= -3-7i$ …(i)

R.H.S = $\bar{z} - \bar{w}$

$= \left(\overline{2+3i}\right) - \left(\overline{5-4i}\right)$

$= 2+3i-(5+4i)$

$= 2-3i-5-4i$

$= -3-7i$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{z-w} = \overline{z-w}$

Hence proved

(iii) $\overline{zw} = \bar{z}\,\bar{w}$ **(A.B)**

Proof: L.H.S = $\overline{zw}$

$zw = (2+3i)(5+4i)$

$= 2(5-4i) + 3i(5-4i)$

$= 10-8i+15i-12i^2$

$= 10+7i-12(-1)$

$= 10+7i+12$

$= 22+7i$

$\Rightarrow \overline{zw} = \overline{22+7i}$

$= 22-7i$

R.H.S = $\overline{zw}$

$= \left(\overline{2+3i}\right)\left(\overline{5-4i}\right)$

$= (2-3i)(5+4i)$

$= 2(5+4i)-3i(5+4i)$

$= 10+8i-15i-12i^2$

$= 10-7i-12(-1)$

$= 10-7i+12$

$= 22-7i$

From (i) and (ii) we get

L.H.S=R.H.S

$\overline{zw} = \overline{z}\,\overline{w}$

Hence proved

(iv) $\overline{\left[\dfrac{z}{w}\right]} = \dfrac{\overline{z}}{\overline{w}}$, where $\neq 0$ **(A.B)**

Proof: L.H.S.= $\overline{\left[\dfrac{z}{w}\right]}$

$$\frac{z}{w} = \frac{2+3i}{5-4i}\times\frac{5+4i}{5+4i}$$

$$= \frac{2(5+4i)+3i(5+4i)}{(5-4i)(5+4i)}$$

$$= \frac{10+8i+15i+12i^2}{(5)^2-(4i)^2}$$

$$= \frac{10+23i+12(-1)}{25-16i^2}$$

$$= \frac{10+23i-12}{25-(6(-1))}$$

$$= \frac{10+23i-12}{25+16}$$

$$= \frac{-2+23i}{41}$$

Now

$\overline{\left[\dfrac{z}{w}\right]}$

$$= \left(\overline{\frac{-2+23i}{41}}\right)$$

$$= \frac{-2}{41}-\frac{23}{41}i \quad \rightarrow (i)$$

R.H.S = $\dfrac{\overline{z}}{\overline{w}}$

$$= \frac{\left(\overline{2+3i}\right)}{\overline{(5-4i)}}$$

$$= \frac{2-3i}{5+4i}$$

$$= \frac{2-3i}{5+4i}\times\frac{5-4i}{5-4i}$$

$$= \frac{2(5-4i)-3i(5-4i)}{(5+4i)(5-4i)}$$

$$= \frac{10-8i-15i+12i^2}{(5)^2-(4i)^2}$$

$$= \frac{10-23i+12(-1)}{25-16i^2}$$

$$= \frac{10-23i+12(-1)}{25-16(-1)}$$

$$= \frac{10-23i-12}{25+16}$$

$$= \frac{-2-23i}{41}$$

$= \frac{-2}{41} - \frac{23}{41} i \quad \rightarrow (ii)$

From (i) and (ii) we get

L.H.S=R.H.S

Hence Proved

$\frac{1}{2}(z+\bar{z})$ is the real part of z.

Proof:

$$\frac{1}{2}(z+\bar{z})$$

$$= \frac{1}{2}\left[(2+3i)+\left(\overline{2+3i}\right)\right]$$

$$= \frac{1}{2}\left[(2+3i)+(2-3i)\right]$$

$$= \frac{1}{2}\left[2+\cancel{3i}+2-\cancel{3i}\right]$$

$$= \frac{1}{2}[2+2]$$

$$= \frac{1}{\cancel{2}}\left[\cancel{4}^{2}\right]$$

$$= 2 = \text{Re}(z)$$

Hence,

$\frac{1}{2}(z+\bar{z})$ is the real part of z.

Proved

(v) $\frac{1}{2}(z-\bar{z})$ **is the imaginary part of** z. **(A.B)**

Proof:

$$\frac{1}{2}(z-\bar{z})$$

$$= \frac{1}{2}\left[(2+3i)-\left(\overline{2+3i}\right)\right]$$

$$= \frac{1}{2}\left[(2+3i)-(2-3i)\right]$$

$$= \frac{1}{2}\left[\cancel{2}+3i-\cancel{2}+3i\right]$$

$$= \frac{1}{\cancel{2}}\left[{}^{3}\cancel{6}i\right]$$

$$= 3i$$

$$= \text{Imaginary}(z)$$

Hence,

$\frac{1}{2}(z-\bar{z})$ is the imaginary part of z.

Proved

Q.7 Solve the following equations for real x and y.

(i) $(2-3i)(x+yi) = 4+i$ **(A.B)**

Solution: $(2-3i)(x+yi) = 4+i$

$$x+yi = \frac{4+i}{2-3i}$$

$$x+yi = \frac{4+i}{2-3i} \times \frac{2+3i}{2+3i}$$

$$= \frac{4(2+3i)+i(2+3i)}{(2-3i)(2+3i)}$$

$$= \frac{8+12i+2i+3i^2}{(2)^2-(3i)^2}$$

$$= \frac{8+14i+3(-1)}{4-9i^2}$$

$$= \frac{8+14i-3}{4-9(-1)}$$

$$= \frac{8-3+14i}{4+9}$$

$$= \frac{5+14i}{13}$$

$x+yi=\frac{5}{13}+\frac{14}{13}i$

By comparing real and imaginary parts, we get

$\Rightarrow x=\frac{5}{13},\ y=\frac{14}{13}$

$\therefore$ Solution Set $=\left\{\left(\frac{5}{13},\frac{14}{13}\right)\right\}$

(ii) $(3-2i)(x+yi)=2(x-2yi)+2i-1$ **(A.B)**

Solution:

$(3-2i)(x+yi)=2(x-2yi)+2i-1$

$3(x+yi)-2i(x+yi)=2x-4yi+2i-1$

$3x+3yi-2xi-2yi^2=(2x-1)+i(2-4y)$

$3x+(3y-2x)i-2y(-1)=(2x-1)+i(2-4y)$

$3x+(3y-2x)i+2y=(2x-1)+i(2-4)$

$(3x+2y)+(3y-2x)i=(2x-1)+(2-4y)i$

By comparing the real and imaginary parts.

$3x+2y=2x-1 \quad , \quad 3y-2x=2-4y$

$3x-2x+2y=-1 \quad , \quad 3y-2x=2-4y$

$x+2y=-1\rightarrow(i) \quad , \quad -2x+3y+4y=2$

$-2x+7y=2\rightarrow(ii)$

Multiply equation (i) with 2

$2(x+2y)=-1\times 2$

$2x+4y=-2\rightarrow(iii)$

By adding equation (ii) and (iii)

$\cancel{2x}+4y=\cancel{-2}$

$\underline{\cancel{-2x}+7y=\cancel{2}}$

$11y=0$

$y=\frac{0}{11}$

$y=0$

Putting $y=0$ in equation (i)

$x+2y=-1$

$x+2(0)=-1$

$x+0=-1$

$x=-1$

$\therefore$ Solution Set $=\{(0,-1)\}$

(iii) $(3+4i)^2-2(x-yi)=x+yi$ **(A.B)**

Solution: $(3+4i)^2-2(x-yi)=x+yi$

$(3+4i)^2-2(x-yi)=x+yi$

$9+24i+16i^2-2x+2yi=x+yi$

$9+24i+16(-1)-2x+2yi=x+yi$

$9+24i-16-2x+2yi=x+yi$

$9+24i-16-2x=x+2yi-yi=0$

$9+24i-16-3x+yi=0$

$-3x+yi=-9-24i+16$

$-3x+yi=16-9-24i$

$-3x+yi=7-24i$

By comparing the real and imaginary parts, we get

$-3x = 7$ and $y = -24$

$\Rightarrow x = \frac{-7}{3}$ $y = -24$

$\therefore \text{Solution Set} = \left\{\left(\frac{-7}{3}, -24\right)\right\}$

Review Exercise 2

Q.1 Multiple choice questions. Choose the correct answer.

(i) $\left(27x^{-1}\right)^{-\frac{2}{3}}$ __________ **(U.B)**

(a) $\frac{\sqrt[3]{x^2}}{9}$ **(b)** $\frac{\sqrt{x^3}}{9}$

(c) $\frac{\sqrt[3]{x^2}}{8}$ **(d)** $\frac{\sqrt{x^3}}{8}$

(ii) **Write $\sqrt[7]{x}$ in the exponential form __________** **(U.B)**

(a) x **(b)** x^7

(c) $x^{\frac{1}{7}}$ **(d)** $x^{\frac{7}{2}}$

(iii) **Write $4^{\frac{2}{3}}$ with radical sing __________** **(U.B)**

(a) $\sqrt[3]{4^2}$ **(b)** $\sqrt[2]{4^3}$

(c) $\sqrt[2]{4^3}$ **(d)** $\sqrt{4^6}$

(iv) **In $\sqrt[3]{35}$ the radicand is;** **(K.B)**

(a) 3 **(b)** $\frac{1}{3}$

(c) 35 **(d)** None

(v) $\left(\frac{25}{16}\right)^{-\frac{1}{2}} =$ __________ **(K.B)**

(a) $\frac{5}{4}$ **(b)** $\frac{4}{5}$

(c) $-\frac{5}{4}$ **(d)** $-\frac{4}{5}$

(vi) **The conjugate of $5+4i$ is __________** **(K.B)**

(a) $-5+4i$ **(b)** $-5-4i$

(c) $5-4i$ **(d)** $5+4i$

(vii) **The value of i^9 is;** **(U.B)**

(a) 1 **(b)** -1

(c) i **(d)** $-i$

(viii) **Every real number is __________** **(K.B)**

(a) Positive integer **(b)** A rational number

(c) A negative integer **(d)** A complex number

(ix) **Real point of $2ab\left(i+i^2\right)$ is __________** **(A.B)**

(a) $2ab$ **(b)** $-2ab$
(c) $2abi$ **(d)** $-2abi$

(x) **Imaginary part of $-i(3i+2)$ is __________** **(A.B)**
(a) -2 **(b)** 2
(c) 3 **(d)** -3

(xi) **Which of the following sets have the closure property w.r.t addition?** **(K.B)**
(a) $\{0\}$ **(b)** $\{0,1\}$
(c) $\{0,1\}$ **(d)** $\left\{1,\sqrt{2},\frac{1}{2}\right\}$

(xii) **Name the property of real number used in $\left[-\frac{\sqrt{5}}{2}\right]\times 1=-\frac{\sqrt{5}}{2}$ __________** **(K.B)**
(a) Additive identity **(b)** Additive inverse
(c) Multiplicative identity **(d)** Multiplicative inverse

(xiii) **If $x,y,z\in R, z<0,$ then $x<y\Rightarrow ...$** **(K.B)**
(a) $xz<yz$ **(b)** $xz>yz$
(c) $xz=yz$ **(d)** None of these

(xiv) **IF $a,b\in \in R,$ only one of $a=b$ or $a<b$** or **$a>b$ hold is called __________** **(K.B)**
(a) Trichotomy property **(b)** Transitive property
(c) Additive property **(d)** Multiplicative property

(xv) **A non-terminating, non-recurring decimal represents …** **(K.B)**
(a) A natural number **(b)** A rational number
(c) An irrational number **(d)** A prime number

ANSWER KEY

i	a	vi	c	xi	a
ii	c	vii	c	xii	c
iii	a	viii	d	xiii	b
iv	c	ix	b	xiv	a
v	b	x	a	xv	c

Q.2 **True or False? Identity**

(i)	**Division is not an associative operation.**	True	**(K.B)**
(ii)	**Every whale number is a natural number.**	False	**(K.B)**
(iii)	**Multiplicative inverse of 0.02 is 50.**	True	**(K.B)**
(iv)	**π is rational number.**	False	**(K.B)**
(v)	**Every integer is a rational number.**	True	**(K.B)**
(vi)	**Subtraction is a commutative operation.**	False	**(K.B)**
(vii)	**Every real number is a rational number.**	False	**(K.B)**

(viii) **Decimal representation of a rational number is either terminating or recurring.** **True** **(K.B)**

(ix) $1.\bar{8} = 1 + \frac{8}{9}$ **True** **(K.B)**

Q.3 Simplify the following

(i) $\sqrt[4]{81y^{-12}x^{-8}}$ **(A.B)**

Solution:

$$= \left(3^4 y^{12} x^{-8}\right)^{\frac{1}{4}}$$

$$= 3^{4\times\frac{1}{4}} y^{-12^3\times\frac{1}{4}} x^{-8^2\times\frac{1}{4}} \quad \because (ab)^n = a^n b^n$$

$$= 3y^{-3}x^{-2}$$

$$= \frac{3}{y^3x^2} \qquad \because a^{-n} = \frac{1}{a^n}$$

$$\Rightarrow \sqrt[4]{81y^{-12}x^{-8}} = \frac{3}{y^3x^2}$$

(ii) $\sqrt{25x^{10n}y^{8m}}$ **(A.B)**

Solution:

(BWP 2019, SWL 2015, D.G.K 2014, FSD 2021)

$$= \sqrt{25x^{10n}y^{8m}}$$

$$= \left(5^2 x^{10n} y^{8m}\right)^{\frac{1}{2}}$$

$$= 5^{2\times\frac{1}{2}}.x^{10n^5\times\frac{1}{2}}.y^{8m^4\times\frac{1}{2}} \quad \because (ab)^n = a^n b^n$$

$$= 5x^{5n}.y^{4m} \qquad \because \left(a^m\right)^n = a^{mn}$$

$$\Rightarrow \sqrt{25x^{10}y^{8m}} = 5x^{5n}.y^{4m}$$

Method II

$$\sqrt{25x^{10n}y^{8m}} = \sqrt{5^2\left(x^{5n}\right)^2\left(y^{4m}\right)^2}$$

$$= \sqrt{\left(5x^{5n}y^{4m}\right)^2} \quad \because (ab)^n = a^n b^n$$

$$= 5x^{5n}y^{4m} \qquad \because \sqrt[n]{a^n} = a$$

$$\Rightarrow \sqrt{25x^{10}y^{8m}} = 5x^{5n}.y^{4m}$$

(iii) $\left[\frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right]^{\frac{1}{5}}$ **(A.B)**

(BWP 2017, RWP 2014, MTN 2014, SGD 2018)

Solution:

$$= \left(x^{3+2}.y^{4+1}.z^{5+5}\right)^{\frac{1}{5}} \quad \because \frac{a^m}{a^n} = a^{m-n}$$

$$= \left(x^5y^5z^{10}\right)^{\frac{1}{5}}$$

$$= x^{5\times\frac{1}{5}} \times y^{5\times\frac{1}{5}} \times z^{10^2\times\frac{1}{5}} \quad \because (ab)^n = a^n b^n$$

$$= x.y.z^2$$

$$\Rightarrow \left[\frac{x^3y^4z^5}{x^{-2}y^{-1}z^{-5}}\right]^{\frac{1}{5}} = x.y.z^2$$

(iv) $\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}}$ **(A.B)**

Solution:

$$= \left(\frac{2^5x^{-4}y^{-4}z}{5^4x^4yz^{-4}}\right)^{\frac{2}{5}} \qquad \textbf{(Factorization)}$$

$$= \left[\frac{2^5z^{1+4}}{5^4x^{4+6}\times y^{1+4}}\right]^{\frac{2}{5}} \quad \because \frac{a^m}{a^n} = a^{m-n}$$

$$= \left[\frac{2^5z^5}{5^4x^{10}y^5}\right]^{\frac{2}{5}}$$

$$= \frac{2^{5\times\frac{2}{5}} \times z^{5\times\frac{2}{5}}}{5^{4\times\frac{2}{5}} \times x^{10^2\times\frac{2}{5}} \times y^{5\times\frac{2}{5}}} \quad \because \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$= \frac{2^2\times z^2}{5^{\frac{8}{5}}\times x^4\times y^2} \qquad \because \left(a^m\right)^n = a^{mn}$$

$$= \frac{4z^2}{5^{\frac{5}{5}+\frac{3}{5}}\times x^4y^2}$$

$$= \frac{4z^2}{5^{1+\frac{3}{5}}\times x^4y^2}$$

$$=\frac{4z^2}{5\times5^{\frac{3}{5}}x^4y^2} \quad \because \frac{a^m}{a^n}=a^{m-n}$$

$$\Rightarrow\left(\frac{32x^{-6}y^{-4}z}{625x^4yz^{-4}}\right)^{\frac{2}{5}}=\frac{4z^2}{5\times5^{\frac{3}{5}}x^4y^2}$$

Q.4 **Simplify** $\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$ **(A.B)**

Solution: $\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$

$$=\sqrt{\frac{(6^3)^{\frac{2}{3}}\times(5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{3}{2}}}} \quad \text{(factorization)}$$

$$=\sqrt{\frac{6^2\times5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}} \quad \because a^{-n}=\left(\frac{1}{a}\right)^n$$

$$=\sqrt{\frac{6^2\times5}{\left(\frac{2^2\times5^2}{2^2}\right)^{\frac{3}{2}}}}$$

$$=\sqrt{\frac{6^2\times5}{(5^2)^{\frac{3}{2}}}} \quad \because \frac{a^m}{a^m}=1$$

$$=\sqrt{\frac{6^2\times5}{(5)^3}} \quad \because (a^m)^n=a^{mn}$$

$$=\sqrt{\frac{6^2}{5^{3-1}}} \quad \because \frac{a^m}{a^n}=\frac{1}{a^{n-m}}$$

$$=\sqrt{\frac{6^2}{5^2}}$$

$$=\sqrt{\left(\frac{6}{5}\right)^2}$$

$$=\frac{6}{5} \quad \because \sqrt[n]{a^n}=a$$

Q.5 $\left(\frac{a^p}{a^q}\right)^{p+q}\times\left(\frac{a^q}{a^r}\right)^{q+r}\div5\left(a^q\cdot a^r\right)^{p-r}$

(A.B)

Solution:

$$=\frac{\left(a^{p-q}\right)^{p+q}\left(a^{q-r}\right)^{q+r}}{5\left(a^{p+r}\right)^{p-r}} \quad \because \frac{a^m}{a^n}=a^{m-n}$$

$$=\frac{a^{(p-q)(p+q)}\times a^{(q-r)(q+r)}}{5a^{(p+r)(p-r)}} \quad \because (a^m)^n=a^{mn}$$

$$=\frac{a^{p^2-q^2}\times a^{q^2-r^2}}{5a^{p^2-r^2}} \quad \because (a+b)(a-b)=a^2-b^2$$

$$=\frac{a^{p^2-q^2+q^2-r^2}}{5a^{p^2-r^2}} \quad \because a^m\times a^n=a^{m+n}$$

$$=\frac{a^{p^2-r^2-p^2+r^2}}{5} \quad \because \frac{a^m}{a^n}=a^{m-n}$$

$$=\frac{a^0}{5}$$

$$=\frac{1}{5}$$

$$\Rightarrow\left(\frac{a^p}{a^q}\right)^{p+q}\times\left(\frac{a^q}{a^r}\right)^{q+r}\div5\left(a^q\cdot a^r\right)^{p-r}$$

$$=\frac{1}{5}$$

Q.6 **Simplify:** $\left(\frac{a^{2l}}{a^{l+m}}\right)\left(\frac{a^{2m}}{a^{m+n}}\right)\left(\frac{a^{2n}}{a^{n+l}}\right)$

(A.B)

Solution:

$$\left(\frac{a^{2l}}{a^{l+m}}\right)\left(\frac{a^{2m}}{a^{m+n}}\right)\left(\frac{a^{2n}}{a^{n+l}}\right)$$

$$=a^{2l-l-m}\times a^{2m-m-n}\times a^{2n-n-l} \quad \because \frac{a^m}{a^n}=a^{m-n}$$

$= a^{l-m} \times a^{m-n} \times a^{n-l}$

$= a^{l-m+m-n+n-l} \qquad \because a^m \times a^n = a^{m+n}$

$= a^0$

$= 1$

$\Rightarrow \left(\frac{a^{2l}}{a^{l+m}}\right)\left(\frac{a^{2m}}{a^{m+n}}\right)\left(\frac{a^{2n}}{a^{n+2}}\right) = 1$

Q.7 Simplify $\sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^l}}$ **(A.B)**

Solution:

$= \sqrt[3]{a^{l-m}}\sqrt[3]{a^{m-n}}\sqrt[3]{a^{n-l}} \quad \because \frac{a^m}{a^n} = a^{m-n}$

$= \left(a^{l-m}\right)^{\frac{1}{3}} \times \left(a^{m-n}\right)^{\frac{1}{3}} \times \left(a^{n-l}\right)^{\frac{1}{3}} \quad \because \sqrt[n]{a} = a^{1/n}$

$= a^{\frac{l-m}{3}} \times a^{\frac{m-n}{3}} \times a^{\frac{n-l}{3}}$

$= a^{\frac{l-m}{3}+\frac{m-n}{3}+\frac{n-l}{3}} \quad \because a^m \times a^n = a^{m+n}$

$= a^{\frac{l-m+m-n+n-l}{3}}$

$= a^{\frac{0}{3}}$

$= a^0$

$= 1$

$\Rightarrow \sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^r}} = 1$

Method II

$\sqrt[3]{\frac{a^l}{a^m}} \times \sqrt[3]{\frac{a^m}{a^n}} \times \sqrt[3]{\frac{a^n}{a^l}}$

$= \sqrt[3]{\frac{a^l}{a^m} \times \frac{a^m}{a^n} \times \frac{a^n}{a^l}} \quad \because \sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

$= \sqrt[3]{a^{l-m} \times a^{m-n} \times a^{n-l}} \quad \because \frac{a^m}{a^n} = a^{m-n}$

$= \sqrt[3]{a^{l-m+m-n+n-l}} \quad \because a^m \times a^n = a^{m+n} = \sqrt[3]{a^0}$

$= 1 \quad \because a^0 = 1$

CUT HERE

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 **All numbers of the form** $\frac{p}{q}$ **are integers when** p,q **are integers and** q **is not zero are called ____ number**

(A) Rational **(B)** Irrational
(C) Whole number **(D)** None of these

2 $x > z$ **or** $x = z$ **or** $x < z$ **is------------property**

(A) Commutative **(B)** Trichotomy
(C) Trichotomy **(D)** None of these

3 **Imaginary part** $-i(4i+7)$ is

(A) -7 **(B)** 7
(C) ± 7 **(D)** None of these

4 $\left(\frac{25}{16}\right)^{-\frac{1}{2}}$ =____________

(A) $\frac{5}{4}$ **(B)** $\frac{4}{5}$
(C) $-\frac{5}{4}$ **(D)** $-\frac{4}{5}$

5 **The value of** $(-i)^5$ **is;**

(A) 1 **(B)** -1
(C) i **(D)** $-i$

6 $\left(27x^{-1}\right)^{\frac{-2}{3}}$ =_________:

(A) $\sqrt[3]{\frac{x^2}{9}}$ **(B)** $\sqrt{\frac{x^3}{9}}$
(C) $\sqrt[3]{\frac{x^2}{8}}$ **(D)** $\sqrt{\frac{x^3}{8}}$

7 **Which of the following sets here the closure property w.r.t. Addition**

(A) $\{0\}$ **(B)** $\{0, -1\}$
(C) $\{0, 1\}$ **(D)** $\left\{1, \sqrt{2}, \frac{1}{2}\right\}$

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Express the $0.\overline{59}$ recurring decimals as the rational number $\frac{p}{q}$.

Where p, q are integers and $q \neq 0$.

(ii) Represent on number line: $-1\frac{3}{5}$

(iii) Use law of exponents to simplify: $\left(\frac{x^9 y^{-6} z^{-8}}{x^6 y^{-10} z^{-12}}\right)^6$

(iv) Simplify: $\frac{4(3)^n}{3^{n+1} - 3^n}$

(v) Separate real and imaginary parts of $(-1+\sqrt{-2})^2$

Q.3 Answer the following Questions. **(4+4=8)**

(a) Simplify that $\frac{(81)^n \times 3^5 - (3)^{4n-1}(243)}{(9^{2n})(3^3)}$.

(b) Solve the equation $(3+4i)^2 - 2(x - yi) = x + yi$ for real x and y.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.