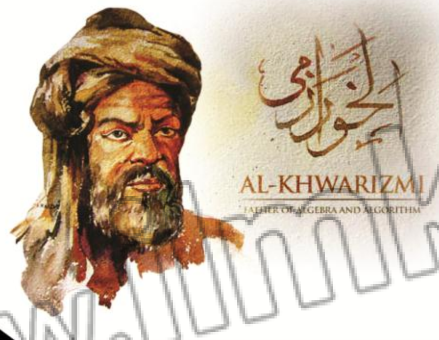

UNIT 3
LOGARITHMS

Need of Scientific Notation **(K.B)**

There are so many numbers that we use in science and technical work that are either very small or large.
While writing these numbers in ordinary notation (Standard notation) there is always chance of making an error by omitting a zero or writing more than actual number of zeros. To overcome this problem, scientists have developed a concise, precise and convenient method to write very small or very large numbers, that is called scientific notation of expressing an ordinary number.

Scientific Notation **(K.B)**

(LHR 2018, SGD 2017, RWP 2017)

A number written in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer, is called the scientific notation.

For example:
The distance from the Earth to the Sun is 150,000,000 Km approximately.
In scientific notation 150,000,000 *km* = 1.5×10^8 km.

Example # 1 **(A.B)**

Write each of the following ordinary numbers in scientific notation

(i) 30600 **(ii)** 0.000058

Solution:

(i) $30600 = 3.06 \times 10^4$
(move decimal point four places to the left)

(ii) $0.000058 = 5.8 \times 10^{-5}$
(move decimal point five places to the right)

Note **(U.B+K.B)**

Steps to change an ordinary number into scientific notation:

(i) Place the decimal point after the first non-zero digit of given number.
(ii) We multiply the number obtained in step (i), by 10^n if we shifted the decimal points n places to the left
(iii) We multiply the number obtained in step (i) by 10^{-n} if we shifted the decimal points n places to the right.

On the other hand, if we want to change a number from scientific notation to ordinary (standard) notation, we simply reverse the process.

Example # 2 **(A.B)**

Change each of the following numbers from scientific notation to ordinary notation.

(i) 6.35×10^6 **(ii)** 7.61×10^{-4}

Solution

(i) $6.35 \times 10^6 = 6350000$
(Move the decimal point six places to the right)

(ii) $7.61 \times 10^{-4} = 0.000761$
(GRW 2018, SGD 2019, D.G.K 2017)
(Move the decimal point four places to the left).

Exercise 3.1

Q.1 **Express each of the following numbers in scientific notations.**

(i) 5700 **(MTN 2017, FSD 2018)** **(A.B)**
$= 5.7 \times 10^3$

(ii) 49,800,000 **(A.B)**
$= 4.98 \times 10^7$

(iii) 96000000 **(MTN 2017)** **(A.B)**
$= 9.6 \times 10^7$

(iv) 416.9 **(A.B)**

$= 4.169\times10^{2}$

(v) 83000 **(A.B)**

$= 8.3\times10^{4}$

(vi) 0.00643 **(A.B)**

(LHR 2017, FSD 2017, BWP 2017, D.G.K 2017)

$= 6.43\times10^{-3}$

(vii) 0.0074 **(A.B)**

$= 7.4\times10^{-3}$

(viii) 60,000,000 **(A.B)**

$= 6\times10^{7}$

(ix) 0.00000000395 **(A.B)**

(SWL 2019, D.G.K 2013)

$= 3.95\times10^{-9}$

(x) $\dfrac{275000}{0.0025}$ **(LHR 2013)** **(A.B)**

$= \dfrac{2.75\times10^{5}}{2.5\times10^{-3}}$

Q.2 Express the following number in ordinary notation.

(LHR 2017, GRW 2017, 19, 21, SWL 2019, FSD 2017, BWP 2019)

(i) 6×10^{-4} **(A.B)**

$= 0.0006$

(ii) 5.06×10^{10} **(A.B)**

$= 50600000000$

(iii) 9.018×10^{-6} **(A.B)**

$= 0.000009018$

(iv) 7.865×10^{8} **(A.B)**

$= 786500000$

Logarithm **(K.B)**

(LHR 2018, RWP 2014, D.G.K 2017)

Logarithm are useful tools for accurate and rapid computations. Logarithms base 10 are known as common logarithms and those with base *e* are known as natural logarithm.

Logarithm of a Real Number **(K.B+U.B)**

If $a^x = y$ they x is called the logarithm of y to the base 'a' and is written as $\log_a y = x$, where $a > 0, a \neq 1$ and $y > 0$

The relation $a^x = y$ and $\log_a y = x$ are equivalent.

Thus

$$a^x = y \Leftrightarrow \log_a y = x$$

Note **(U.B+K.B)**

- $a^n = y$ and $\log_a y = x$ are respectively exponential and logarithmic forms of the same solution.
  $3^2 = 9$ is equivalent to $\log_3 9 = 2$ and
  $2^{-1} = \dfrac{1}{2}$ is equivalent to $\log_2\left[\dfrac{1}{2}\right] = -1$
- Logarithm of a negative number is not defined at this stage.
- Idea of logarithm was given by a Muslim mathematician Abu Muhammad Musa Al Khwarizmi.
- Logarithmic table with base *e* was prepared was John Napier.
- Logarithmic table with base 10 was prepared was Professor Henry Briggs.
- Anti-logarithm table was prepared by Jobst Burgi in 1620 A.D.
- Napier's logarithm is also known as **Natural logarithm**.
- *e* is an irrational number whose approx value is 2.718.

Example # 3 **(A.B)**

Find $\log_4 2$, i.e., find log of 2 to the base 4

Solution:

Let $\log 4^2 = x$

Then its exponential form is $4^x = 2$

i.e. $2^{2x} = 2^1$

$2x = 1 \quad \because$ bases are same

$\Rightarrow x = \dfrac{1}{2}$

$\therefore \log_4 2 = \dfrac{1}{2}$

Deductions from Definition of logarithm **(K.B)**

(i) Since $a^0 = 1 \Rightarrow \log_a 1 = 0$

Logarithm of unity to any base is 0.

(ii) Since $a^1 = a \Rightarrow \log_a a = 1$

Logarithm of a number with itself as base is 1.

Common Logarithm (K.B)

In daily life we use decimal system (means system of base 10) so in numerical calculations, the base of logarithm is taken as 10. This logarithm is called common logarithm or Briggesian logarithm in honour of Henry Briggs, an English mathematician and astronomer, who developed them.

Characteristic (K.B)

Integral part of the logarithm of a number is called characteristic. It may be positive or negative.

For example:

Characteristic of 99.6 is 1.

How to find Characteristic (U.B)

It is always one less than the number of digits in the integral part of the number.

Or

When a number b is written in the scientific notation, i.e. in the form $b = a \times 10^n$ where $1 \le a < 10$, the power of 10 i.e. n will give the characteristic of log b.

Examples:

Number	Scientific Notation	Characteristic of the logarithm
1.02	1.02×10^o	0
1662.4	1.6624×10^3	3
0.872	8.72 × 10-1	–1
0.00345	3.45 ×10-3	–3

Note (K.B+U.B)

- Characteristic of the logarithm of a number less then 1, is always negative and one more then the number of zeros immediately after the decimal paint of the number.
- Instead of –3 or –1 characteristic is written as $\bar{3}, \bar{2}$ or $\bar{1}$ ($\bar{3}$ is read as bar 3) to avoid the mantissa becoming negative. i.e. $\bar{2}.3748$ does not mean -2.3748. In $\bar{2}.3748$,2 is negative but .3748 is positive where as in -2.3748 both 2 and .3748 are negative.

Mantissa (K.B)

Fractional part of the logarithm of a number is called mantissa. It may be positive or negative.

For example:

If $\log x = 2.3451$, then mantissa is 0.3451

Finding the Mantissa of the Logarithm of a Number (K.B)

Mantissa is found by making use of logarithm tables. These tables have been constructed to obtain the logarithms up to 7 decimal places. A four-figure logarithmic table provides sufficient accuracy.

A logarithmic table is divided into 3 parts.

(a) The first part of the table is the extreme left column headed b7 blank square. This column Contain numbers from 10 to 99 corresponding to the first two digits of the number whose logarithm is required.

(b) The second part of the table consists of 10 columns headed by 0,1,2,…..,9; These headings correspond to third from the left of the number. The numbers under these columns record mantissa of the logarithms with decimal point omitted for simplicity.

(c) The third part of the table further consists of small columns known as mean differences columns heard by 1,2,3…..9 these heading correspond to the fourth digit from the left of the number. The readings of these columns are added to the mantissa recorded in second part (b) above.

Example # 1 **(A.B)**

Find the mantissa of the logarithms of 43.254.

Solutions:

Rounding off 43.254 we consider only the four significant digits 4325.

(i) We first locate the row corresponding to 43 in the log tables and

(ii) Proceed horizontally tell we reach the column corresponding to 2. The number at the intersection is 6355.

(iii) Again proceeding horizontally till the mean difference column corresponding to 5 intersects this row, we get the number 5 at the intersection.

(iv) Adding the two numbers 6355 and 5 we get. 0.6360 as the mantissa of the logarithm of 43.25.

Example # 2 **(A.B)**

Find the mantissa of the logarithm of 0.002347

Solution:

Here, we consider only the four significant digits 2347

We first locate the row corresponding to 23 in the logarithm tables and along the same row to its intersection with the column corresponding to 4 the resulting number is 3692. The number at the intersection of this row and the mean difference column corresponding to 7 is 13. Hence the sum of 3692 and 13 gives the mantissa of the logarithm of 0.002347 as 0.3705.

Note **(K.B+U.B)**

The logarithms of number having the same sequence of significant digits have the same mantissa e.g., the mantissa of log of number 0.002347 and 0.2347 is 0.3705.

Example # 3 **(A.B)**

Find (i) log 278.23 (ii) log 0.07058

Solution:

(i) 278.23 can be round off as 278.2
The characteristic is 2
The mantissa using log tables, is 0.4443
Thus, Log 278.23 = 2.4443

(ii) The characteristic of log 0.07058 is $\bar{2}$
Using log tables the mantissa is 0.8487
So that
log 0.07058 = $\bar{2}.8487$

Antilogarithm **(K.B)**

(GRW 2018, MTN 2015)

The number whose logarithm is given is called antilogarithm.

If $\log_a y = x$, then y is the antilogarithm of x, or y = antilog x.

Example **(A.B)**

Find the number whose logarithms are (i) 1.3247 (ii) $\bar{2}.1324$

Solution:

(i) **1.32147**

Reading along the row corresponding to .32 (as mantissa = 0.3247), we get 2109 at the intersection of this row with the column correspond to 4. The number at the intersection of this row and the mean difference column corresponding to 7 is 3 adding 2109 and 3 we get 2112.

Since the characteristic is 1 (it is increased by 1), there for the decimal point is fixed after two digits from left to right in 2112.

Hence

antilog of 1.3247 = 21.12

(ii) **$\bar{2}$.1324**

Proceeding as in (i) the significant figures corresponding to the mantissa 0.1324 are 1356. Since the characteristics is $\bar{2}$, its numerical value 2 is decreased by 1. Hence there will be one zero after the decimal point

Hence

antilog of $\bar{2}.1324$ = 0.01356

Exercise 3.2

Q.1 Find the common logarithms of each of the following numbers.

(i) $\log 232.92$ **(K.B)**

Solution:

$Ch = 2$

Mantissa = 0.3672

$\because \log 232.92 = 2.3672$

(ii) $\log 29.326$ **(K.B)**

Solution:

$Ch = 1$

Mantissa = 0.4672

$\Rightarrow \log 29.326 = 1.4672$

(iii) $\log 0.00032$ **(K.B)**

Solution:

$Ch = \bar{4}$

Mantissa = 0.5051

$\Rightarrow \log 0.00032 = \bar{4}.5051$

(iv) $\log 0.3206$ **(K.B)**

Solution:

$Ch = \bar{1}$

Mantissa = 0.5059

$\Rightarrow \log 0.3206 = \bar{1}.5059$

Q.2 If log $31.09 = 1.4926$, find the value of the following. **(K.B)**

(i) $\log 3.109$

(ii) $\log 310.9$

(iii) $\log 0.003109$

(iv) $\log 0.3109$

Solution:

If log $31.09 = 1.4926$

Then, mantissa = 0.4926

(i) $\log 3.109$

Characteristics = 0

Mantissa = 0.4926

$\log 3.109 = 0.4926$

(ii) $\log 310.9$

Characteristics = 2

Mantissa =0.4926

$\log 310.9 = 2.4926$

(iii) $\log 0.003109$

Characteristics = $\bar{3}$

Mantissa = 0.4926

$\log 0.003109 = \bar{3}.4926$

(iv) $\log 0.3109$

Characteristics = $\bar{1}$

Mantissa = 0.4926

$\log 0.3109 = \bar{1}.4926$

Q.3 Find the numbers whose common logarithms are **(K.B)**

(i) 3.5621

Solution:

Let required number = x

Then, $\log x = 3.5621$

Taking antilog on both sides

antilog $\log x =$ antilog 3.5621

$\Rightarrow x = 3649.0$

Hence, required number = 3649

(ii) $\bar{1}.7427$

Solution:

Let required number = x

Then, $\log x = \bar{1}.7427$

Taking antilog on both sides

antilog $\log x =$ antilog $\bar{1}.7427$

$\Rightarrow x = 0.5530$

Hence, required number = 0.5530

Q.4 What replacement for the unknown in each of the following will make the true statements? **(U.B+K.B+A.B)**

(i) $\log_3 81 = L$

Solution:

$\log_3 81 = L$

Writing in exponential form.

$3^L = 81$

$3^L = 3^4$

$\because$ Bases are equal so

$L = 4$

(ii) $\log_a 6 = 0.5$ **(GRW 2017)**

Solution:

$\log_a 6 = 0.5$

In exponential form

$$a^{0.5}=6$$
$$a^{\frac{1}{2}}=6$$
$$\sqrt{a}=6$$
Taking square on both sides
$$\sqrt{(a)}^2=(6)^2$$
$$a=36$$

(iii) $\log_5 n=2$

Write in exponential form
$$5^2=n$$
$$25=n$$
Or $n=25$

(iv) $10^P=40$

Solution:
$$10^P=40$$
Changing into logarithmic form
$$P=\log_{10}40$$
$$=\log 40$$
$$\Rightarrow P=1.6021$$

Q.5 Evaluate.

(i) $\log_2\frac{1}{128}$ **(U.B)**

(LHR 2017, MTN 2017, SWL 2019)

Solution:

Suppose $\log_2\frac{1}{128}=x$

Writing in exponential form.
$$2^x=\frac{1}{128}$$
$$2^x=\frac{1}{2^7}$$
$$2^x=2^{-7}$$
$\because$ Bases are equal so
$$x=-7$$

(ii) **log 512 to the base** $2\sqrt{2}$ **(U.B)**

(GRW 2016, SWL 2013, MTN 2015, RWP 2016, D.G.K 2018)

Solution:
$$\log_{2\sqrt{2}}512=x$$
Writing in exponential form
$$\left(2\sqrt{2}\right)^x=512$$
$$\left(2^1.2^{\frac{1}{2}}\right)^x=2^9$$
$$\left(2^{\frac{3}{2}}\right)^x=2^9$$
$$2^{\frac{3}{2}x}=2^9$$
$\because$ Bases are equal so
$$\frac{3}{2}x=9$$
$$x=\frac{9\times 2}{3}$$
$$x=\frac{\cancel{18}^{6}}{\cancel{3}}$$
$$x=6$$

Q.6 Find the value of x from the following statements.

(i) $\log_2 x=5$ **(A.B)**

(GRW 2021, MTN 2018, 21, BWP 2019, SGD 2021)

Solution:
$$\log_2 x=5$$
Write in exponential form.
$$2^5=x$$
$$32=x$$
$$x=32$$

(ii) $\log_{81}9=x$ **(A.B)**

(LHR 2016, GRW 2013, FSD 2015, 17, SWL 2017, MTM 2013, D.G.K 2017)

Solution:
$$\log_{81}9=x$$
Writing in the exponential form.
$$81^x=9$$
$$\left(9^2\right)^x=9$$
$$9^{2x}=9$$
$\because$ Bases are equal so
$$2x=1$$
$$x=\frac{1}{2}$$

(iii) $\log_{64} 8 = \frac{x}{2}$ **(A.B)**

(LHR 2013, GRW 2013, 17, SWL 2016, BWP 2018, SGD 2013, 14, 21, MTN 2019, FSD 2015, RWP 2013, 16)

Solution:

$\log_{64} 8 = \frac{x}{2}$

Writing in exponential form.

$64^{\frac{x}{2}} = 8$

$\left(8^{\not{2}}\right)^{\frac{x}{\not{2}}} = 8$

$8^x = 8$

$\because$ Bases are equal so

$x = 1$

(iv) $\log_x 64 = 2$ **(A.B)**

(LHR 2018, 21, FSD 2021, RWP 2019, MTN 2018, 21, SWL 2021)

Solution:

$\log_x 64 = 2$

Writing in exponential form

$x^2 = 64$

$x^2 = 8^2$

$\because$ Exponents are equal so

$x = 8$

(v) $\log_3 x = 4$ **(A.B)**

Solution:

$\log_3 x = 4$

Writing in exponential form

$3^4 = x$

$81 = x$

Or $x = 81$

Laws of Logarithm (K.B+U.B)

(MTN 2018, RWP 2017)

(i) $\log_a (mn) = \log_a m + \log_a n$

(ii) $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

(iii) $\log_a m^n = n \log_a m$

(iv) $\log_a n = \log_b n \times \log_a b$

First Law of Logarithm (K.B+U.B)

$$\log_a (mn) = \log_b m + \log_a n$$

Proof:

Let

$\log_a m = x$ and $\log_a n = y$

Writing in exponential form

$a^x = m \rightarrow (i)$ and $a^y = n \rightarrow (ii)$

Multiplying equation (i) and (ii)

$a^x \times a^y = mn$

$a^{x+y} = mn$

In logarithmic form

$\log_a (mn) = x + y$

Putting the values of x and y

$\log_a (mn) = \log_a m + \log_a n$

Hence proved

Note **(K.B+U.B)**

(i) $\log_a (mn) \neq \log_a m \times \log_a n$

(ii) $\log_a m + \log_a n \neq \log_a (m+n)$

(iii) $\log_a (mnp...) = \log_a m + \log_a n + \log_a p + ...$

Example # 1 **(A.B)**

Evaluate: 291.3 ×42.36

Solution:

Let $x = 291.3 \times 42.36$

Taking log on both sides

$\log x = \log (291.3 \times 42.36)$

$\log x = \log 291.3 + \log 42.36$

$\because \log_a mn = \log_a m + \log_a n$

$= 2.4643 + 1.6269$

$= 4.0912$

Taking antilog on both sides

antilog x = antilog 4.0912

$x = 12340$

$\because$ 291.3 ×42.36 = 12340

Second Law of Logarithm (U.B+K.B)

$$\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$$

Proof:

Let

$\log_a m = x$ and $\log_a n = y$

Writing in exponential form

$a^x = m \rightarrow (i)$ and $a^y = n \rightarrow (ii)$

Dividing equation (i) and (ii)

$$\frac{a^x}{a^y} = \frac{m}{n}$$

$$\Rightarrow a^{x-y} = \frac{m}{n}$$

In logarithmic form

$$\log_a\left(\frac{m}{n}\right) = x - y$$

Putting the values of x and y

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

Note **(U.B+K.B)**

(i)	$\log_a\left(\frac{m}{n}\right) \neq \frac{\log_a m}{\log_a n}$
(ii)	$\log_a m - \log_a n \neq \log_a (m-n)$
(iii)	$\log_a\left(\frac{1}{n}\right) = \log_a 1 - \log_a n = -\log_a n$ $\because (\log_a 1 = 0)$

Example # 1 **(A.B)**

Evaluate: $\frac{291.3}{42.36}$

Solution:

Let $x = \frac{291.3}{42.36}$

Taking log on both sides

$$\log x = \log \frac{291.3}{42.36}$$

$$\log x = \log 291.3 - \log 42.36$$

$$\because \log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

$= 2.4643 - 1.6269$

$= 0.8374$

Taking antilog on both sides

antilog x = antilog 0.8374

$x = 6.877$

$$\Rightarrow \frac{291.3}{42.36} = 6.877$$

Third law of logarithm **(U.B+K.B)**

$$\log_a (m)^n = n \log_a m$$

Proof:

Let $\log_a m = x$

Writing in exponential form

$a^x = m$

Taking nth power on both sides

$$(a^x)^n = m^n$$

$$a^{nx} = m^n$$

In logarithmic form

$$\log_a m^n = nx$$

Putting the value of x

$$\log_a m^n = \text{n} \log_a m$$

Example # 1 **(A.B)**

Evaluate: $\sqrt[4]{(0.0163)^3}$

Solution:

Let $y = \sqrt[4]{(0.0163)^3}$

$$= \left[(0.0163)^3\right]^{1/4} = (0.0163)^{\frac{3}{4}}$$

Taking log on both sides

$$\log y = \log (0.0163)^{\frac{3}{4}}$$

$$\log y = \frac{3}{4}(\log 0.0163)$$

$$= \frac{3}{4}(\bar{2}.2122)$$

$$= \frac{3}{4}(-2 + 0.2122)$$

$$= \frac{3}{4}(-1.7878)$$

$= -1.3408$

$= -1.3408 + 2 - 2$

$= \bar{2}.6592$

Taking antilog on both sides

antilog y = antilog $\bar{2}.6592$

$y = 0.04562$

$$\Rightarrow \sqrt[4]{(0.0163)^3} = 0.04562$$

Fourth law of logarithm / Change of Base Formula **(U.B+K.B)**

$$\log_a n = \log_b n \times \log_a b \text{ or } \frac{\log_b n}{\log_b a}$$

Proof:

Let $\log_b n = x$.

Writing in exponential form

$n = b^x$

Taking log to the base a on both sides

$\log_a n = \log_a b^x$

Applying 3rd law of logarithm

$\log_a n = x \log_a b$

Putting the value of x

Thus $\log_a n = \log_b n \times \log_a b$

$$\log_a n = \log_b n \times \frac{1}{\log_b a}$$

$$\log_a n = \frac{\log_b n}{\log_b a}$$

Note **(K.B+U.B)**

- $\log_a n = \log_b n \times \log_a b$

  Putting n = a in the above result, we get

  $\log_a a = \log_b a \times \log_a b$

  $1 = \log_b a \times \log_a b$

  Or $\log_a b = \dfrac{1}{\log_b a}$
- $\log_{10} e = \log 2.718 = 0.4343$
- $\log_e 10 = \dfrac{1}{\log_{10} e} = \dfrac{1}{0.4343} = 2.3026$

Example: **(A.B)**

Calculate $\log_2 3 \times \log_3 8$

Solution:

$$\log_2 3 \times \log_2 8 = \frac{\log 3}{\log 2} \times \frac{\log 8}{\log 3}$$

$$\because \log_a n = \frac{\log_b n}{\log_b a}$$

$$= \frac{\log 8}{\log 2} = \frac{\log 2^3}{\log 2}$$

$$\because \log_a (m)^n = n \log_a m$$

$$= \frac{3 \log 2}{\log 2}$$

$$\Rightarrow \log_2 3 \times \log_3 8 = 3$$

Exercise 3.3

Q.1 Write the following into sum or difference $\log(A \times B)$ **(A.B)**

(i) $\log(A \times B)$

Solution: $\log(A \times B)$

$$\log A \times B = \log A + \log B$$

$$\because \log_a (mn) = \log_b m + \log_a n$$

(ii) $\log \dfrac{15.2}{30.5}$

Solution:

$$\log \frac{15.2}{30.5}$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \log 15.2 - \log 30.5$$

(iii) $\log \dfrac{21 \times 5}{8}$

Solution:

$$\log \frac{21 \times 5}{8}$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \log(21 \times 5) - \log 8$$

$$\because \log_a (mn) = \log_b m + \log_a n$$

$$= \log 21 + \log 5 - \log 8$$

(iv) $\log \sqrt[3]{\dfrac{7}{15}}$

Solution:

$$\log \sqrt[3]{\frac{7}{15}} = \log \left(\frac{7}{15}\right)^{\frac{1}{3}}$$

$$= \frac{1}{3} \log \left(\frac{7}{15}\right)$$

$$\because \log_a (m)^n = n \log_a m$$

$$= \frac{1}{3}(\log 7 - \log 15)$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \frac{1}{3} \log 7 - \frac{1}{3} \log 15$$

(v) $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

Solution:

$$\log \frac{(22)^{\frac{1}{3}}}{5^3} = \log 22^{\frac{1}{3}} - \log 5^3$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \frac{1}{3}\log 22 - 3\log 5$$

$$\because \log_a (m)^n = n\log_a m$$

(vi) $\log \frac{25\times 97}{29}$

Solution:

$$\log \frac{25\times 47}{29} = \log(25\times 47) - \log 29$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \log 25 + \log 47 - \log 29$$

$$\because \log_a (mn) = \log_b m + \log_a n$$

Q.2 Express

$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$ **as a single logarithm.**

Solution:

$$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$$

$$= \log x - \log x^2 + \log(x+1)^3 - \log(x^2-1)$$

$$\because \log_a (m)^n = n\log_a m$$

$$= \log\left(\frac{x}{x^2}\right) + \log\frac{(x+1)^3}{x^2-1}$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \log\left(\frac{x}{x^2}\times\frac{(x+1)^3}{x^2-1}\right)$$

$$\because \log_a (mn) = \log_b m + \log_a n$$

$$= \log\left(\frac{x(x+1)^3}{x^2(x^2-1)}\right)$$

$$= \log\frac{\cancel{x}(x+1)^2\cancel{(x+1)}}{x\times\cancel{x}(x-1)\cancel{(x+1)}}$$

$$= \log\frac{(x+1)^2}{x(x-1)}$$

Q.3 Write the following in the form of a single logarithm. **(A.B)**

(i) $\log 21 + \log 5$

Solution:

$$\log 21 + \log 5$$

$$\because \log_a (mn) = \log_b m + \log_a n$$

$$= \log(21\times 5)$$

(ii) $\log 25 - 2\log 3$

Solution: $\log 25 - 2\log 3$

$$= \log 25 - 2\log 3$$

$$= \log 25 - \log 3^2$$

$$\because \log_a (m)^n = n\log_a m$$

$$= \log\frac{25}{3^2} \quad \because \log_a \frac{m}{n} = \log_a m - \log_a n$$

(iii) $2\log x - 3\log y$

(BWP 2017, SWL 2018, 19, MTN 2019, D.G.K 2019)

Solution:

$$2\log x - 3\log y$$

$$= \log x^2 - \log y^3$$

$$= \log\frac{x^2}{y^3} \quad \because \log_a \frac{m}{n} = \log_a m - \log_a n$$

(iv) $\log 5 + \log 6 - \log 2$ **(FSD 2018)**

Solution:

$$\log 5 + \log 6 - \log 2$$

$$\because \log_a (mn) = \log_b m + \log_a n$$

$$= \log(5\times 6) - \log 2$$

$$\because \log_a \frac{m}{n} = \log_a m - \log_a n$$

$$= \log\frac{5\times 6}{2} \text{ **Ans**}$$

Q.4 Calculate the following.

(i) $\log_3 2\times\log_2 81$

(LHR 2019, FSD 2017, 19)

Solution:

$\log_3 2\times\log_2 81$

$=\frac{\log 2}{\log 3}\times\frac{\log 81}{\log 2}$

$=\frac{\log 81}{\log 3}$

$=\frac{\log 3^4}{\log 3}$

$\because \log_a(m)^n = n\log_a m$

$=\frac{4\log 3}{\log 3}$

$=4$ **Ans**

(ii) $\log_5 3\times\log_3 25$

Solution:

$\log_5 3\times\log_3 25$

Applying 4th law of logarithm

$=\frac{\log 3}{\log 5}\times\frac{\log 25}{\log 3}$

$=\frac{\log 25}{\log 5}$

$=\frac{\log 5^2}{\log 5}$

$\because \log_a(m)^n = n\log_a m$

$=\frac{2\log 5}{\log 5}$

$=2$ **Ans**

Q.5 If $\log 2=0.3010, \log 3=0.4771$ **and** $\log 5=0.6990$, **then find the values of the following.** **(A.B)**

(i) $\log 32$

$=\log 2^5$

$\because$ using 3^{rd} law of logarithm

$=5\log 2$

By putting the value of $\log 2$

$=5(0.3010)$

$=1.5050$ **Ans**

(ii) $\log 24$

Solution:

$\log 24$

$=\log(2^3\times 3)$

$=\log 2^3+\log 3$

$\because \log_a(m)^n = n\log_a m$

$=3\log 2+\log 3$

By putting the value

$=3(0.3010)+0.4771$

$=0.9030+0.4771$

$=1.3801$

(iii) $\log\sqrt{3\frac{1}{3}}$

Solution:

$\log\sqrt{3\frac{1}{3}}$

$=\log\left(\frac{10}{3}\right)^{\frac{1}{2}}$

$\because \log_a(m)^n = n\log_a m$

$=\frac{1}{2}\log\left[\frac{2\times 5}{3}\right]$

Using 1st and 2nd laws of logarithm

$=\frac{1}{2}(\log 2+\log 5-\log 3)$

By putting the values

$=\frac{1}{2}(0.3010+0.69900-0.4771)$

$=\frac{1}{2}(1-0.4771)$

$=\frac{1}{2}(0.5229)$

$=0.26145$

(iv) $\log\frac{8}{3}$

Solution:

$\log\frac{8}{3}$

$=\log\frac{2^3}{3}$

$\because \log_a\frac{m}{n}=\log_a m-\log_a n$

$=\log 2^3-\log 3$

$\because \log_a(m)^n=n\log_a m$

$=3\log 2-\log 3$

By putting the values

$=3(0.3010)-0.4771$

$=0.9030-0.4771$

$=0.4259$ **Ans**

(v) $\log 30$

Solution:

$\log 30$

$=\log(5\times 2\times 3)$

$\because$ using first law of logarithm

$=\log 5+\log 2+\log 3$

By putting the values

$=(0.6990)+(0.3010)+(0.4771)$

$=1.4771$ **Ans**

Application of Laws of Logarithm in Numerical Calculations

So far we have applied laws of logarithm to simple type of products, quotients, powers or roots of numbers. We now extend their application to more difficult examples to verify their effectiveness in simplification.

Example 1: **(A.B+K.B+U.B)**

Show that $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}=\log 2$

Solution:

L.H.S. = $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}$

$\because \log_a\frac{m}{n}=\log_a m-\log_a n$

$= 7[\log 16-\log 15]+5[\log 25-\log 24]+3[\log 81-\log 80]$

$= 7\left[\log 2^4-\log(3\times 5)\right]+5\left[\log 5^2-\log\left(2^3\times 3\right)\right]+3\left[\log 3^4-\log\left(2^4\times 5\right)\right]$

$\because \log_a(mn)=\log_b m+\log_a n$

$= 7\left[4\log 2-(\log 3+\log 5)\right]+5\left[2\log 5-(3\log 2+\log 3)\right]+3\left[4\log 3-[4\log 2+\log 5]\right]$

$= 28\log 2-15\log 2-12\log 2-7\log 3-5\log 3+12\log 3-7\log 5+10\log 5-3\log 5$

$= (28-15-12)\log 2+(-7-5+12)\log 3+(-7+10-3)\log 5$

$= (1)\log 2+(0)\log 3+(0)\log 5$

$= \log 2+0+0$

$= \log 2=\text{R.H.S}$

Example # 2 **(A.B)**

Evaluate: $\sqrt[3]{\dfrac{0.0792\times(18.99)^2}{(5.79)^4\times 0.9474}}$

Solution:

Let y $=\sqrt[3]{\dfrac{0.07921\times(18.99)^2}{(5.79)^4\times(0.9474)}}$

$$=\left[\frac{0.07921\times(18.99)^2}{(5.79)^4\times 0.9474}\right]^{\frac{1}{3}}$$

Taking log on both sides

$$\log y=\log\left[\frac{0.07921\times(18.99)^2}{(5.79)^4\times 0.9474}\right]^{\frac{1}{3}}$$

$$\because\ \log_a(m)^n=n\log_a m$$

$$=\frac{1}{3}\log\left[\frac{0.07921\times(18.99)^2}{(5.79)^4\times(0.9474)}\right]$$

$$\because\ \log_a\frac{m}{n}=\log_a m-\log_a n$$

$$=\frac{1}{3}\left[\log 0.07921\times(18.99)^2\right]-\log\left[(5.79)^4\times 0.9474\right]$$

$$\because \log_a(mn)=\log_b m+\log_a n$$

$$=\frac{1}{3}\left[(\log 0.07921+2\log 18.99)-(4\log 5.79+\log 0.9474)\right]$$

$$=\frac{1}{3}\left[\log 0.07921+2\log 18.99-4\log 5.79-\log 0.9474\right]$$

$$=\frac{1}{3}\left[\bar{2}.8988+2(1.2786)-4(0.7627)-\bar{1}.9765\right]$$

$$=\frac{1}{3}\left[\bar{2}.8988+2.5572-3.0508-\bar{1}.9765\right]$$

$$=\frac{1}{3}\left[1.4660-3.0273\right]=\frac{1}{3}\left(\bar{2}.4287\right)$$

$$=\frac{1}{3}\left[\bar{3}+1.4287\right]$$

$$=\bar{1}+0.4762=\bar{1}.4762$$

Or y = antilog $\bar{1}.4762$

y = 0.2993

Example # 3 **(A.B)**

Given $A = Aoe^{-kd}$ **If k = 2 what be the value of d to make** $A = \frac{Ao}{2}$?

$$A = A_o e^{-kd}$$

$$\frac{A_o}{2} = A_o e^{-kd}$$

$$\frac{A_o}{2A_o} = e^{-kd}$$

$$\frac{1}{2} = e^{-kd}$$

Substituting k = 2 and where e = 2.718

$$\frac{1}{2} = (2.718)^{-2d}$$

Taking Common log on both sides

$$\log 10 \frac{1}{2} = \log 10 (2.718)^{-2d}$$

$$\log_{10} 1 - \log_{10} 2 = -2d \log_{10}(2.718)$$

$$0 - 0.3010 = -2d(0.4343)$$

$$-0.3010 = -0.8686(d)$$

$$\frac{-0.3010}{-0.8686} = d$$

$$d = 0.3465$$

Exercise 3.4

Q.1 Use log tables to find the value of

(i) 0.8176×13.64 **(A.B)**

Solution:

Suppose

$x = 0.8176 \times 13.64$

Taking log on both sides

$\log x = \log(0.8176 \times 13.64)$

According to first law of logarithm

$\log x = \log 0.8176 + \log 13.64$

$= \bar{1}.9125 + 1.1348$

$\log x = -1 + 0.9125 + 1.1348$

$\log x = 1.0473$

Taking antilog on both sides

$x = \text{antilog} 1.0473$

$x = 11.15$

(ii) $(789.5)^{\frac{1}{8}}$

Solution:

Let $x = (789.5)^{\frac{1}{8}}$

Taking log on both sides

$\log x = \log(789.5)^{\frac{1}{8}}$

According to third law

$\log x = \frac{1}{8}\log(789.5)$

$\log x = \frac{1}{8}(2.8974)$

$= \frac{2.8974}{8}$

$\log x = 0.3622$

Taking antilog on both sides

x = antilog 0.3622

$x = 2.302$

(iii) $\frac{0.678\times 9.01}{0.0234}$

Solution:

Suppose

$x=\frac{0.678\times 9.01}{0.0234}$

Taking log on both sides

$\log x=\log\frac{0.678\times 9.01}{0.0234}$

According to 1st and 2nd law of log

$\log x=\log 0.678+\log 9.01-\log 0.0234$

$\log x=\bar{1}.8312+0.9547-\bar{2}.3692$

$=-1+0.8312+0.9547-(-2+0.3692)$

$=2.4167$

Taking antilog on both sides

x = antilog 2.4167

$x=261.0$

(iv) $\sqrt[5]{2.709}\times\sqrt[7]{1.239}$

Solution:

Let $x=\sqrt[5]{2.709}\times\sqrt[7]{1.239}$

$=(2.709)^{\frac{1}{5}}\times(1.239)^{\frac{1}{7}}$

Taking log on both sides

$\log x=\log\left[(2.709)^{\frac{1}{5}}\times(1.239)^{\frac{1}{7}}\right]$

According to law of logarithm

$\log x=\log(2.709)^{\frac{1}{5}}+\log(1.239)^{\frac{1}{7}}$

According to third law of logarithm

$\log x=\frac{1}{5}\log(2.709)+\frac{1}{7}\log(1.239)$

$\log x=\frac{1}{5}\log(2.709)+\frac{1}{7}\log(1.239)$

$=\frac{1}{5}0.4328+\frac{1}{7}0.0931$

$=0.0866+0.0133$

$=0.0999$

Taking antilog on both sides

x = antilog 0.999

$x=1.259$

(v) $\frac{1.23\times 0.6975}{0.0075\times 1278}$

Solution:

Suppose

$x=\frac{1.23\times 0.6975}{0.0075\times 1278}$

$\log x=\log\frac{1.23\times 0.6975}{0.0075\times 1278}$

$=\log(1.23\times 0.6975)-\log(0.0075\times 1278)$

$=\log 1.23+\log 0.6975-(\log 0.0075+\log 1278)$

$=\log 1.23+\log 0.6975-\log 0.0075-\log 1278$

$=0.0899+\bar{1}.8435-\bar{3}.8751-3.1065$

$=0.8999+(-1+0.8435)-(-3+0.8751)-3.1065$

$=-1.0482$

$\log x=-2+2-1.0482$

$\log x=0.9518-2+0.9518$

$\log x=\bar{2}.9518$

Taking antilog on both sides

x = antilog $\bar{2}.9518$

$=0.08950$

(vi) $\sqrt[3]{\frac{0.7214\times 20.37}{60.8}}$

Solution:

Let $x=\sqrt[3]{\frac{0.7214\times 20.37}{60.8}}$

$x=\left[\frac{0.7214\times 20.37}{60.8}\right]^{\frac{1}{3}}$

Taking log on both sides

$\log x=\log\left(\frac{0.7214\times 20.37}{60.8}\right)^{\frac{1}{3}}$

3rd of logarithm

$\log x=\frac{1}{3}\log\left[\frac{0.7214\times 20.37}{60.8}\right]$

According to first and 2nd law

$\log x=\frac{1}{3}[\log 0.7214+\log 37-\log 60.8]$

$\log x=\frac{1}{3}[\bar{1}.8582+1.3089-1.7839]$

$=\frac{1}{3}[-1+0.8582+1.3089-1.7839]$

$=\frac{1}{3}(-0.6168)$
$=-0.2056$
$\log x$ is in negative, so
$\log x=-1+1-0.2056$
$=-1+0.79144$
$=\bar{1}.7944$
Taking antilog on both sides
x = antilog $\bar{1}.7944$
= 0.6229

(vii)

Solution: $\frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Suppose: $x=\frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

$x=\frac{83\times(92)^{\frac{1}{3}}}{127\times(246)^{\frac{1}{5}}}$

Taking on both sides

$\log x=\log\frac{83\times(92)^{\frac{1}{3}}}{127\times(246)^{\frac{1}{5}}}$

According to 1st and 2nd law of log

$\log x=\log 83+\log(92)^{\frac{1}{3}}-\log 127-\log(246)^{\frac{1}{5}}$

According to third law of log

$\log x=\log 83+\frac{1}{3}\log 92-\log 27-\frac{1}{5}\log 246$

$\log x=(1.9191)+\frac{1}{3}(1.9638)-(2.1038)$

$-\frac{1}{5}(2.3909)$

= 1.9191 + 0.65460 – 2.1038 – 0.47818
$=1.9191+0.6546-2.1038-0.47818$
$=-0.0083$
$\log x$ is in negative, so
$\log x=-1+1-0.0083$
$=-1+0.9917$
$=\bar{1}.9917$
Taking antilog log on both sides
$x=$ antilog $\bar{1}.9917$
$x=0.9811$

(viii) $\frac{(438)^3\sqrt{0.056}}{(388)^4}$

Solution:

Suppose: $x=\frac{(438)^3\sqrt{0.056}}{(388)^4}$

$x=\frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}$

$x=\frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}$

Taking log on both sides

$\log x=\log\left(\frac{(438)^3(0.056)^{\frac{1}{2}}}{(388)^4}\right)$

According to 1st and 2nd law

$\log x=\log(438)^3+\log(0.056)^{\frac{1}{2}}-\log(388)^4$

According to third law

$\log x=3\log(438)+\frac{1}{2}\log(0.056)-4\log(38)$

$\log x=3(2.6415)+\frac{1}{2}(\bar{2}.7482)-4(2.5888)$

$=7.9245+\frac{1}{2}(-2+0.7482)-10.3552$

$=7.9245+\frac{1}{2}(-1.2518)-10.3552$

$=7.9245-0.6259-10.3552$
$=-3.0566$
log is in negative, so
$\log x=-4+4-3.0566$
$=-4+0.9434=\bar{4}.9434$
Taking antilog on both sides
$x=$antilog $\bar{4}.9434$
$=0.0008778$

Q.2 A gas is expanding according to the law $pv^n = C$.
Find C when p = 80, v = 3.1 and $n=\frac{5}{4}$.

Solution:

Given $pv^n = C$
Taking log on both sides
$\log\left(pv^n\right) = \log C$
Applying laws of logarithm
log C =log P + n log v
Putting P=80, v=3.1 and $n = \frac{5}{4}$
log C = log 80+$\frac{5}{4}$ log 3.1
=1.9031+$\frac{5}{4}$(0.4914)
=1.9031+0.6143
log C = 2.5174
Taking antilog both sides
C=Antilog (2.5174)
C=329.2

Q.3 The formula p= 90 $(5)^{-q/10}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs 18.00? (A.B)

Solution:

Given that $p = 90(5)^{\frac{-q}{10}}$
Taking log on both sides
$\log\ p = \log 90(5)^{\frac{-q}{10}}$
Applying laws of logarithm
log P = log 90 $-\frac{q}{10}$ log 5
Putting the value of P
log 18= log 90-$\frac{q}{10}$ log 5
1.2553=1.9542- $\frac{q}{10}$ × 0.6990
1.2553-1.9542 = $-\frac{q}{10}$ × 0.6990
-0.6989 × 10 = - q × 0.6990
-6.989=-q × 0.6996
6.989 = q × 0.6996
$\frac{6.989}{0.6990} = q$
q =10 approximately
Hence 10 units will be demanded

Q.4 If $A = \pi r^2$, find A, when $\pi = \frac{22}{7}$ and $r = 15$. (A.B)

Solution:

Given that $A = \pi r^2$
Taking log on both sides
log A=log πr^2
Putting $\pi = \frac{22}{7}$ and r =15
$\log A = \log \frac{22}{7}(15)^2$
Applying laws of logarithm
log A = log 22-log 7+2 log 15
= 1.3424-0.8451+2(1.1761)
= 0.4973+2.3522
log A = 2.8495
Taking antilog on both sides
A = antilog 2.8495
A = 707.1

Q.5 If $V = \frac{1}{3}\pi r^2 h$, find V, when $\pi = \frac{22}{7}$, $r = 2.5$ and h=4.2. (A.B)

Solution:

Given that $V = \frac{1}{3}\pi r^2 h$
Taking log on both sides
$\log V = \log \frac{1}{3}\pi r^2 h$
Putting the values
$\log V = \log \frac{1}{3} \times \frac{22}{7}(2.5)^2(4.2)$
Applying laws of logarithm
$= \log 1 - \log 3 + \log 22 - \log 7 + 2\log 2.5 + \log 4.2$
$= -0.4771 + 1.3424 - 0.8450 + 2(0.3979) + 0.6232$
$= -0.4771 + 1.3424 - 0.8450 + 0.7959 + 0.6232$
log V=1.4394
Taking antilog on both sides
V=antilog 1.4394
V=27.50

Review Exercise 3

Q.1 Multiple choice Questions. Choose of the correct answer. **(K.B)**

(i) If $a^x = n$, then…

(a) $a = \log_x n$ **(b)** $x = \log_n a$
(c) $x = \log_a n$ **(d)** $a = \log_n x$

(ii) The relation $y = \log_z x$ implies…

(LHR 2017, GRW 2014, FSD 2015, 16, MTN 2014, 15, 17, BWP 2017)

(a) $x^y = z$ **(b)** $z^y = x$
(c) $x^z = y$ **(d)** $y^z = x$

(iii) The logarithm of unity to any base is…

(a) 1 **(b)** 10
(c) e **(d)** 0

(iv) The logarithm of any number to itself as base is…

(LHR 2016, FSD 2014, 15, D.G.K 2014)

(a) 1 **(b)** 0
(c) e **(d)** 10

(v) log e=…,where e $\approx$ 2.718

(a) 0 **(b)** 0.4343
(c) ∞ **(d)** 1

(vi) The value of $\log\left(\dfrac{p}{q}\right)$ is…

(a) $\log p - \log q$ **(b)** $\dfrac{\log p}{\log q}$
(c) $\log p + \log q$ **(d)** $\log q - \log p$

(vii) log *p*-log *q* is same as …

(a) $\log\left(\dfrac{q}{p}\right)$ **(b)** $\log(p-q)$
(c) $\dfrac{\log p}{\log q}$ **(d)** $\log q - \log p$

(viii) $\log(m^n)$ can be written as… **(K.B+U.B)**

(LHR 2013, 14, SWL 2013, 15, BWP 2013, 14, FSD 2013, 17, RWP 2016)

(a) $(\log m)^n$ **(b)** $m \log n$
(c) $n \log m$ **(d)** $\log(mn)$

(ix) $\log_b a \times \log_c b$ can be written as… **(K.B+U.B)**

(a) $\log_a c$ **(b)** $\log_c a$
(c) $\log_a b$ **(d)** $\log_b c$

(x) $\log_y x$ will be equal to… **(U.B)**

(a) $\dfrac{\log_z x}{\log_y z}$ **(b)** $\dfrac{\log_x z}{\log_y z}$
(c) $\dfrac{\log_z x}{\log_z y}$ **(d)** $\dfrac{\log_z y}{\log_z x}$

ANSWER KEY

i	ii	iii	iv	v	vi	Vii	viii	ix	x
c	b	d	a	b	a	D	c	b	c

Q.2 Complete the following: (K.B)

(i) For common logarithm, the base is…

(ii) The integral part of the common logarithm of a number is called the …

(iii) The decimal part of the common logarithm of a number is called the …

(iv) If x = log y, then y is called the… of x.

(v) If the characteristic of the logarithm of a number have...zero(s) immediately after the decimal point.

(vi) If the characteristic of the logarithm of a number is 1, that number will have digits in its integral part.

ANSWER KEY

i	ii	iii	iv	v	vi
10	Characteristic	Mantissa	Antilogarithm	One	2

Q.3 Find the value of x in the following. (A.B)

(i) $\log_3 x = 5$

Solution:

$\log_3 x = 5$

Write in exponential form.

$3^5 = x$

$243 = x$

Or $x = 243$

(ii) $\log_4 256 = x$ **(D.G.K 2014, SGD 2017)**

Solution:

$\log_4 256 = x$

Write in exponential form

$4^x = 256$

$4^x = 4^4$

$x = 4$ $\because$ bases are same

(iii) $\log_{625} 5 = \frac{1}{4}x$ **(LHR 2014)**

Solution:

$\log_{625} 5 = \frac{1}{4}x$

Write in exponential form

$(625)^{\frac{1}{4}x} = 5$

$\left(5^4\right)^{\frac{x}{4}} = 5$

$5^x = 5^1$

$x = 1$ $\because$ bases are same

(iv) $\log_{64} x = -\frac{2}{3}$

Solution:

$\log_{64} x = -\frac{2}{3}$

Write in exponential form

$(64)^{\frac{-2}{3}} = x$

$\left(4^3\right)^{-\frac{2}{3}} = x$

$4^{-2} = x$

$\frac{1}{4^2} = x$

$\frac{1}{16} = x$

Q.4 Find the value of x in the following. **(A.B)**

(i) $\log x = 2.4543$

Solution:

$\log x = 2.4543$

Taking antilog on both sides

$x = \text{antilog}\, 2.4543$

$x = 284.6$

(ii) $\log x = 0.1821$ **(FSD 2014)**

Solution:

$\log x = 0.1821$

Taking antilog on both sides

$x = \text{antilog}\, 0.1821$

$x = 1.521$

(iii) $\log x = 0.0044$ **(LHR 2016)**

Solution:

$\log x = 0.0044$

Taking antilog on both sides

$x = \text{antilog}\ 0.0044$

$x = 1.010$

(iv) $\log x = \bar{1}.6238$

Solution:

$\log x = \bar{1}.6238$

Taking antilog on both sides

$x = \text{antilog}\, \bar{1}.6333$

$x = 0.4206$

Q.5 If $\log 2 = 0.3010$, $\log 3 = 0.4771$, **and** $\log 5 = 0.6990$ **then find the values of the following.** **(A.B)**

(i) $\log 45$

Solution:

$\log 45$

$= \log(9 \times 5)$

$= \log(3^2 \times 5)$

Applying 1st law of logarithm

$= \log 3^2 + \log 5$

Applying 3rd law of logarithm

$= 2\log 3 + \log 15$

Putting the values

$= 2(0.4771) + 0.6990$

$= 0.9542 + 0.6990$

$= 1.6532$

(ii) $\log \frac{16}{15}$

Solution:

$\log \frac{16}{15} = \log \frac{2^4}{3 \times 5}$

Applying laws of logarithm

$= \log 2^4 - \log(3 \times 5)$

$= 4\log 2 - (\log 3 + \log 5)$

$= \log 2^4 - \log 3 - \log 5$

$= 4\log 2 - \log 3 - \log 5$

Putting the values

$= 4(0.3010) - 0.4771 - 0.6990$

$= 1.2040 - 0.4771 - 0.6990$

$= 0.0279$

(iii) $\log 0.048$

Solution:

$\log 0.048 = \log \frac{48}{1000}$

$= \log \frac{2 \times 2 \times 2 \times 2 \times 3}{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= \log \frac{2^4 \times 3}{2^3 \times 5^3}$

$= \log 2^4 + \log 3 - \log 2^3 - \log 5^3$

Applying 3rd law of logarithm

$= 4\log 2 + \log 3 - 3\log 2 - 3\log 5$

$= 4(0.3010) + 0.4771 - 3(0.3010) - 3(0.6990)$

$= 1.2040 + 0.4771 - 0.9030 - 2.0970$

$= -1.3189$

$= -2 + 2 - 1.3189$

$= -2 + 0.6811$

$= \bar{2}.6811$

Q.6 Simplify the following. **(A.B)**

(i) $\sqrt[3]{25.47}$

Solution:

Let $x = \sqrt[3]{25.74} = (25.47)^{\frac{1}{3}}$

Taking log on both sides

$\log x = \log(25.47)^{\frac{1}{3}}$

Applying 3rd law of logarithm

$$= \frac{1}{3}\log 25.47$$

$$= \frac{1}{3}(1.4060)$$

$\log x = 0.4687$

Taking antilog on both sides

$x = \text{anti}\log 0.4687$

$x = 2.943$

(ii) $\sqrt[5]{342.2}$

Solution:

Let $x = \sqrt[5]{342.2}$

$$x = (242.)^{\frac{1}{5}}$$

Taking log on both sides

$$\log x = \log(342.2)^{\frac{1}{5}}$$

Applying 3rd law of logarithm

$$\log x = \frac{1}{5}\log 342.2$$

$$= \frac{1}{5}(2.5343)$$

$\log x = 0.5069$

Taking antilog on both sides

$\log x = \text{antilog}\, 0.5069$

$x = 3.213$

(iii) $\dfrac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Solution:

Let $x = \dfrac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Taking log on both sides

$$\log x = \log \frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$$

$$= \log(8.97)^3 + \log(3.95)^2 - \log(15.37)^{\frac{1}{3}}$$

$$= 3\log 8.97 + 2\log 3.95 - \frac{1}{3}\log 15.37$$

$$= 3(0.9528) + 2(0.5966) - \frac{1}{3}(1.1867)$$

$$= 2.8584 + 1.1932 - 0.3956$$

$\log x = 3.656$

Taking antilog on both sides

$x = \text{antilog}\, 3.656$

$x = 4529$

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. (7×1=7)

1 Who prepared logarithim tables with base 10.

(A) John Napier **(B)** Henry Briggs

(C) Jobst Burgi **(D)** Musa Al Khwarizmi

2 A number written in the form $a\times10^n$, where _________ and n is an integer, is called the scientific rotation

(A) $1 < a < 10$ **(B)** $0 \le a < 10$

(C) $1 \le a < 10$ **(D)** $1 \le a \le 10$

3 $\log_{\sqrt{3}}\sqrt{3} = ?$

(A) 1 **(B)** $\sqrt{3}$

(C) 3 **(D)** $\sqrt{6}$

4 $\log_y x =$

(A) $\frac{\log_z x}{\log_y z}$ **(B)** $\frac{\log_x z}{\log_y z}$

(C) $\frac{\log_z x}{\log_z y}$ **(D)** $\frac{\log_z y}{\log_z x}$

5 0.0073 **write in scientific notation**

(A) 7.3×10^3 **(B)** 7.3×10^{-3}

(C) 0.73×10^{-3} **(D)** 7.3×10^4

6 $\log e =$ ______

(A) 0 **(B)** 0.4343

(C) ∞ **(D)** 1

7 $\log p - \log q$ **is same as_________**

(A) $\log\left(\frac{q}{p}\right)$ **(B)** $\log(p-q)$

(C) $\frac{\log p}{\log q}$ **(D)** $\log\left(\frac{p}{q}\right)$

Q.2 Give Short Answers to following Questions. (5×2=10)

(i) Evaluate: $\log \frac{1}{128}$

(ii) Find the value of *x*, when $\log_{64} 8 = \frac{x}{2}$

(iii) Express in scientific notation: $\frac{275,000}{0.0025}$

(iv) Write into sum or difference of logarithms: $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

(v) Calculate: $\log_2 3 \times \log_3 8$

Q.3 Answer the following Questions. (4+4=8)

(a) If log 2 = 0.3010, log 3 = 0.4771 and log 5 = 0.6990 then find the value of log 0.048

(b) A gas is expanding according to the law $PV^n = C$ find C when P = 89, V = 3.1 and $n = \frac{5}{4}$.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.