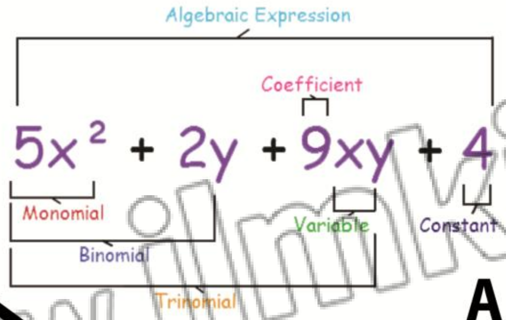


# UNIT 4
# ALGEBRAIC EXPRESSIONS AND ALGEBRAIC FORMULAS



## Algebraic Expression (LHR 2016) (K.B)

An expression consists of constants and variables connected with arithmetic operators $(+,-,\times,\div \text{ etc.})$ is called algebraic expression.

Or

When operations of addition and subtraction are applied to algebraic terms, we obtain an algebraic expression.

**For example:**

$5x^2-3x+\frac{2}{\sqrt{x}}$ and $3xy+\frac{3}{x}(x\neq 0)$ etc.

## Types of Algebraic Expressions (K.B)

There are three types of algebraic expressions:

**(i)** Polynomial Expression

**(ii)** Rational Expression

**(iii)** Irrational Expression

## Polynomials (K.B)

An expression consists of one or more terms in each of which exponent of variable is either 0 or positive integer is called polynomial expression.

A polynomial in the variable $x$ is an algebraic expression of the form

$p(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+.....a_1x+a_0,\ a_n\neq 0\rightarrow(i)$

**For example:**

$2x+3,\ x^3-27,\ xy^2z,\ 0,\ \frac{5}{4}$ etc.

## Degree of the Polynomial (K.B)

When n the highest power of variable, is a non-negative integer called the degree of the polynomial.

**For example:**

Degree of expressions $2x^3+5x^2+8x+1$ and $2x^4y^3+x^2y^2+8x$ is 3 and 7 respectively.

## Leading Coefficient (K.B)

The coefficient $a_n$ of the highest power of $x$ is called the leading coefficient of the polynomial.

**For example:**

In the polynomial $2x^3+5x^2+8x+1$ leading coefficient is 2.

## Note (U.B+K.B)

From similar properties of integers and polynomials w.r.t. addition and multiplication, we may say that polynomials behave line integers.

## Rational Expression (K.B)

The quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$ where $q(x)$ is non-zero polynomial, is called a rational expression.

**For example**:

$\frac{2x+1}{3x+8}, 3x+8\neq 0$

**Irrational Expression** **(K.B)**

An expression which cannot be expressed as the quotient $\frac{p(x)}{q(x)}$ of two polynomials $p(x)$ and $q(x)$, where $q(x) \neq 0$ is called an irrational expression.

**For example**:

$2\sqrt{x}+3, \sqrt{x}+\frac{1}{\sqrt{x}}$ etc.

**Rational Expressions behave like Rational numbers** **(SGD 2017)** **(K.B)**

Let $a$ and $b$ be two integers, then $\frac{a}{b}$ is not necessarily an integer. Therefore, number system is extended and $\frac{a}{b}$ is defined as a rational number where $a, b \in Z$ and $b \neq 0$.

Similarly if $P(x)$ and $q(x)$ are two polynomial, then $\frac{P(x)}{q(x)}$ is not necessarily a polynomial where $q(x) \neq 0$.

**Numerator** **(K.B)**

In the rational expression $\frac{P(x)}{q(x)}, P(x)$ is called the numerator.

**Denominator** **(K.B)**

In the rational expression $\frac{P(x)}{q(x)}, q(x)$ is known as the denominator of the rational expression.

**Note** **(K.B+U.B)**

Every polynomial $P(x)$ can be regarded as a rational expression, Since We can write $P(x)$ as $\frac{P(x)}{1}$. Thus every polynomial is a rational expression but every rational expression need not be a polynomial.

**Working Rule to Reduce a Rational Expression to its Lowest Terms** **(K.B)**

Let the given rational expression be $\frac{p(x)}{q(x)}$.

**Step I:** Factorize each of the two polynomials $P(x)$ and $q(x)$

**Step II:** Find H.C.F. of $P(x)$ and $q(x)$.

**Step III:** Divide the numerator $P(x)$ and the denominator $q(x)$ by the H.C.F. of $P(x)$ and $q(x)$. The rational expression so obtained, is in its lowest terms.

In other words, an algebraic fraction can be reduced to its lowest form by first factorizing both the polynomials in the numerator and the denominator and then cancelling the common factors between them.

**Example** **(A.B)**

$\frac{3x^2+18x+27}{5(x^2-9)} = \frac{3(x^2+6x+9)}{5(x^2-9)}$ (common)

$= \frac{3(x+3)(x+3)}{5(x+3)(x-3)}$ (Factorizing)

$= \frac{3(x+3)}{5(x-3)}$ (Cancelling common factors)

is in the lowest form.

**Sum, Difference and Product of Rational Expressions**

**Example # 2** **(A.B)**

**Simplify**

$\frac{1}{x-y}-\frac{1}{x+y}+\frac{2x}{x^2-y^2}$

**Solution:**

$\frac{1}{x-y}-\frac{1}{x+y}+\frac{2x}{x^2-y^2}$

$=\frac{1}{x-y}-\frac{1}{x+y}+\frac{2x}{(x-y)(x+y)}$

$$= \frac{x+y-(x-y)+2x}{(x+y)(x-y)}$$ (L.C.M of denominators)

$$= \frac{x+y-x+y+2x}{(x+y)(x-y)}$$

$$= \frac{2x+2y}{(x+y)(x-y)}$$ (Simplifying)

$$= \frac{2(x+y)}{(x+y)(x-y)}$$

$$= \frac{2}{x-y}$$ (Cancelling common factors)

**Example # 2** **(A.B)**

Find the product $\frac{x+2}{2x-3y}\cdot\frac{4x^2-9y^2}{xy+2y}$ in simplified factor.

**Solution:**

$$\frac{x+2}{2x-3y}\cdot\frac{4x^2-9y^2}{xy+2y}$$

$$= \frac{x+2}{2x-3y}\cdot\frac{(2x)^2-(3y)^2}{y(x+2)}$$

$$= \frac{(x+2)}{(2x-3y)}\times\frac{(2x-3y)(2x+3y)}{y(x+2)}$$ (Factorizing)

$$= \frac{2x+3y}{y}$$ (Reduced to the lowest form)

## Dividing a Rational Expression with Another Rational Expression

**Example** **(A.B)**

**Simplify:**

$$\frac{7xy}{x^2-4x+4}\div\frac{14y}{x^2-4}$$

**Solution:**

$$\frac{7xy}{x^2-4x+4}\div\frac{14y}{x^2-4}$$

$$= \frac{7xy}{x^2-4x+4}\times\frac{x^2-4}{14y}$$ (changing division into multiplication)

$$= \frac{7xy}{(x-2)(x-2)}\cdot\frac{(x+2)(x-2)}{14y}$$ ...(factorizing)

$$= \frac{x(x+2)}{2(x-2)}$$ ...(reduced to lowest form)

## Evaluation of Algebraic Expression for some Particular Real Number

### Value of the expression **(K.B)**

If specific numbers are substituted for the variables in an algebraic expression, the resulting number is called the value of the expression.

**Example** **(A.B)**

**Evaluate:** $\frac{3x^2\sqrt{y}+6}{5(x+y)}$ if $x=-4$ and $y=9$

**Solution:**

$$\frac{3x^2\sqrt{y}+6}{5(x+y)}$$

Putting $x=-4$ and $y=9$, we get

$$= \frac{3(-4)^2\sqrt{9}+6}{5(-4+9)}$$

$$= \frac{150}{25} = 6$$

## Exercise 4.1

**Q.1 Identify whether the following algebraic expressions are polynomials (Yes or No).**

**(i)** $3x^2+\frac{1}{x}-5$

No (Because of $\frac{1}{x}$) Ans.

**(ii)** $3x^3-4x^2-x\sqrt{x}+3$

No (Because $\sqrt{x}$ or $(x)^{\frac{1}{2}}$) Ans.

**(iii)** $x^2-3x+\sqrt{2}$

Yes (Because no variable has power in fraction). **Ans**

**(iv)** $\frac{3x}{2x-1}+8$

No (Because of $\frac{1}{2x-1}$) **Ans**

**Q.2 State whether each of the following expressions is a rational expression or not.** **(A.B+K.B)**

(i) $\frac{3\sqrt{x}}{3\sqrt{x}+5}$

Irrational **Ans**

(ii) $\frac{x^3-2x^3+\sqrt{3}}{2+3x-x^2}$

Rational

(iii) $\frac{x^2+6x+9}{x^2-9}$

Rational

(iv) $\frac{2\sqrt{x}+3}{2\sqrt{x}-3}$

Irrational

**Q.3 Reduce the following expression to the lowest form.** **(A.B+K.B)**

(i) $\frac{120x^2y^3z^5}{30x^3yz^2}$ **(SWL 2017)**

**Solution:** $\frac{{}^{4}\,\cancel{120}x^2y^3z^5}{\cancel{30}x^3yz^2}$

$=\frac{120x^2y^3z^5}{30x^3yz^2}$

$=4x^{2-3}y^{3-1}z^{5-2}$

$=4x^{-1}y^2z^3$

$=\frac{4y^2z^3}{x}$

(ii) $\frac{8a(x+1)}{2(x^2-1)}$

**(LHR 2017, FSD 2015, 16, SWL 2015, BWP 2017, MTN 2017, D.G.K 2017)**

**Solution:** $\frac{8a(x+1)}{2(x^2-1)}$

$=\frac{{}^{4}\,\cancel{8}a(x+1)}{\cancel{2}(x^2-1)}$

$=\frac{4a\cancel{(x+1)}}{(x-1)\cancel{(x+1)}}$

$=\frac{4a}{x-1}$

(iii) $\frac{(x+y)^2-4xy}{(x-y)^2}$

**Solution:** $\frac{(x+y)^2-4xy}{(x-y)^2}$

$\therefore (x+y)^2=x^2+y^2+2xy$

$\therefore (x-y)^2=x^2+y^2-2xy$

$=\frac{x^2+y^2+2xy-4xy}{x^2+y^2-2xy}$

$=\frac{x^2+y^2-2xy}{x^2+y^2-2xy}$

$=\frac{\cancel{(x-y)^2}}{\cancel{(x-y)^2}}$

$=1$

(iv) $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

**Solution:** $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$

$(a^3+b^3)=(a-b)(a^2+ab+b^2)$

$=\frac{\cancel{(x^3-y^3)}(x^2-2xy+y^2)}{\cancel{(x^3-y^3)}}$

$=x^2-2xy+y^2$

$\therefore (x-y)^2=x^2-2xy+y^2$

$=(x-y)^2$

(v) $\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

**(GRW 2018, MTN 2013, 15, RWP 2018)**

**Solution:** $\frac{(x+2)(x^2-1)}{(x+1)(x^2-4)}$

$=\frac{(x+2)\left[(x)^2-(1)^2\right]}{(x+1)\left[(x)^2-(2)^2\right]}$

$$=\frac{\cancel{(x+2)}(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x-2)\cancel{(x+2)}}$$

$$=\frac{(x-1)}{(x-2)}$$ **Ans**

**(vi)** $\frac{x^2-4x+4}{2x^2-8}$ **(A.B+U.B)**

**(LHR 2013, FSD 2014, 16, SWL 2016, SGD 2017, BWP 2013, 17, D.G.K 2015)**

**Solution:** $\frac{x^2-4x+4}{2x^2-8}$

$$\therefore (a-b)^2 = a^2-2ab+b^2$$

$$\therefore a^2-b^2=(a+b)(a-b)$$

$$=\frac{(x)^2-2(x)(2)+(2)^2}{2(x^2-4)}$$

$$=\frac{(x-2)^2}{2\left[(x)^2-(2)^2\right]}$$

$$=\frac{(x-2)^2}{2(x+2)(x-2)}$$

$$=\frac{(x-2)\cancel{(x-2)}}{2(x+2)\cancel{(x-2)}}$$

$$=\frac{x-2}{2(x+2)}$$ **Ans**

**(vii)** $\frac{64x^5-64x}{(8x^2+8)(2x+2)}$

**(SGD 2015, SWL 2014, MTN 2014)**

**Solution:** $\frac{64x^5-64x}{(8x^2+8)(2x+2)}$

$$=\frac{64x(x^4-1)}{8(x^2+1).2(x+1)}$$

$$=\frac{64\left[(x^2)^2-(1)^2\right]}{16(x^2+1)(x+1)}$$

$$=\frac{{}^{4}\cancel{64}(x^2-1)\cancel{(x^2+1)}}{\cancel{16}\cancel{(x^2+1)}(x+1)}$$

$$=\frac{4x(x-1)\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$=4x(x-1)$$ **Ans**

**(viii)** $\frac{9x^2-(x^2-4)^2}{4+3x-x^2}$

**Solution:** $\frac{9x^2-(x^2-4)^2}{4+3x-x^2}$

$$=\frac{(3x)^2-(x^2-4)^2}{4+3x-x^2}$$

$$=\frac{(3x+x^2-4)(3x-x^2+4)}{4+3x-x^2}$$

$$=\frac{(x^2+3x-4)\cancel{(-x^2+3x+4)}}{\cancel{(-x^2+3x+4)}}$$

$$=x^2+3x-4$$ **Ans**

**Q.4 Evaluate** **(K.B+A.B+U.B)**

**(a)** $\frac{x^3y-2z}{xz}$ **for**

**(i)** $x=3, y=-1, z=-2$

**(ii)** $x=-1, y=-9, z=4$

**(b)** $\frac{x^2y^2-5z^4}{xyz}$ **for** $x=4, y=-2$ **and** $z=-1$

**Solution for 1st part**

When $x=3,\ y=-1,\ z=-2$

$$\frac{x^3y-2z}{xz}=$$

$$=\frac{(3)^3(-1)-2(-2)}{(3)(-2)}$$

$$=\frac{27(-1)+4}{-6}$$

$$=\frac{-27+4}{-6}$$

$$=\frac{-23}{-6}$$

$$=\frac{23}{6}$$ **Ans**

**Solution for 2nd Part.**

When $x=-1,\ y=-9,\ z=4$

$$\frac{x^3y-2z}{xz}=$$
$$=\frac{(-1)^3(-9)-2(4)}{(-1)(4)}$$
$$=\frac{-1(-9)-8}{-4}$$
$$=\frac{9-8}{-4}$$
$$=\frac{1}{-4}$$
$$=-\frac{1}{4}\ \textbf{Ans}$$

**(a)** $\dfrac{x^2y^3-5z^4}{xyz}$ **for** $x=4, y=-2, z=-1$

**(LHR 2016, GRW 2015, D.G.K 2016)**

**Solution:** $\dfrac{x^2y^3-5z^4}{xyz}$ **(A.B)**

$$=\frac{(4)^2(-2)^3-5(-1)^4}{(4)(-2)(-1)}$$
$$=\frac{16(-8)-5(1)}{8}$$
$$=\frac{16(-8)-5(1)}{8}$$
$$=\frac{-128-5}{8}$$
$$=-\frac{133}{8}$$
$$=-16\frac{5}{8}\ \textbf{Ans}$$

**Q.5 Perform the indicated operation and simplify.** **(A.B+U.B)**

**(i)** $\dfrac{15}{2x-3y}-\dfrac{4}{3y-2x}$

**Solution:** $\dfrac{15}{2x-3y}-\dfrac{4}{3y-2x}$

$$=\frac{15}{2x-3y}-\frac{4}{-2x+3y}$$
$$=\frac{15}{2x-3y}-\frac{4}{-(2x-3y)}$$
$$=\frac{15}{2x-3y}+\frac{4}{2x-3y}$$
$$=\frac{19}{2x-3y}\ \textbf{Ans}$$

**(ii)** $\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}$ **(A.B)**

**Solution:** $\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}$

$$=\frac{(1+2x)^2-(1-2x)^2}{(1-2x)(1+2x)}$$
$$=\frac{(1)^2+(2x)^2+2(2x)(1)-\left[(1)^2+(2x)^2-2(2x)(1)\right]}{(1)^2-(2x)^2}$$
$$=\frac{1+4x^2+4x-\left[1+4x^2-4x\right]}{1-4x^2}$$
$$=\frac{1+4x^2+4x-1-4x^2+4x}{1-4x^2}$$
$$=\frac{4x+4x}{1-4x^2}$$
$$=\frac{8x}{1-4x^2}\ \textbf{Ans}$$

**(iii)** $\dfrac{x^2-25}{x^2-36}-\dfrac{x+5}{x+6}$

**Solution:** $\dfrac{x^2-25}{x^2-36}-\dfrac{x+5}{x+6}$

$$=\frac{(x)^2-(5)^2}{(x)^2-(6)^2}-\frac{x+5}{x+6}$$
$$=\frac{(x+5)(x-5)}{(x+6)(x-6)}-\frac{x+5}{x+6}$$
$$=\frac{(x+5)(x-5)-(x-6)(x+5)}{(x+6)(x-6)}$$
$$=\frac{(x+5)\left[(x-5)-(x-6)\right]}{x^2-6^2}$$

$=\dfrac{(x+5)(x-5-x+6)}{x^2-36}$

$=\dfrac{(x+5)(1)}{x^2-36}$

$=\dfrac{x+5}{x^2-36}$ **Ans**

**(iv)** $\dfrac{x}{x-y}-\dfrac{y}{x+y}-\dfrac{2xy}{x^2-y^2}$ **(A.B)**

**Solution:** $\dfrac{x}{x-y}-\dfrac{y}{x+y}-\dfrac{2xy}{x^2-y^2}$

$=\dfrac{x(x+y)-y(x-y)}{(x-y)(x+y)}-\dfrac{2xy}{x^2-y^2}$

$=\dfrac{x^2+\cancel{xy}-\cancel{xy}+y^2}{(x)^2-(y)^2}-\dfrac{2xy}{x^2-y^2}$

$=\dfrac{x^2+y^2}{x^2-y^2}-\dfrac{2xy}{x^2-y^2}$

$=\dfrac{x^2+y^2-2xy}{x^2-y^2}$

$=\dfrac{(x-y)^2}{x^2-y^2}$

$=\dfrac{(x-y)\cancel{(x-y)}}{(x+y)\cancel{(x-y)}}$

$=\dfrac{x-y}{x+y}$ **Ans**

**(v)** $\dfrac{x-2}{x^2+6x+9}-\dfrac{x+2}{2x^2-18}$ **(A.B)**

**Solution:** $\dfrac{x-2}{x^2+6x+9}-\dfrac{x+2}{2x^2-18}$

$=\dfrac{x-2}{(x)^2+2(3)(x)+3^2}-\dfrac{x+2}{2(x^2-9)}$

$=\dfrac{x-2}{(x+3)^2}-\dfrac{x+2}{2\left[(x)^2-(3)^2\right]}$

$=\dfrac{x-2}{(x+3)^2}-\dfrac{x+2}{2(x-3)(x+3)}$

$=\dfrac{x-2}{(x+3)(x+3)}-\dfrac{x+2}{2(x+3)(x-3)}$

$=\dfrac{2(x-3)(x-2)-(x+3)(x+2)}{2(x+3)(x+3)(x-3)}$

$=\dfrac{2(x^2-2x-3x+6)-(x^2+2x+3x+6)}{2(x+3)(x+3)(x-3)}$

$=\dfrac{2(x^2-5x+6)-(x^2+5x+6)}{2(x+3)(x+3)(x-3)}$

$=\dfrac{2x^2-10x+12-x^2-5x-6}{2(x+3)^2(x-3)}$

$=\dfrac{x^2-15x+6}{2(x+3)^2(x-3)}$ **Ans**

**(vi)** $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4-1}$ **(A.B)**

**Solution:** $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4-1}$

$=\dfrac{(x+1)-(x-1)}{(x-1)(x+1)}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4-1}$

$=\dfrac{\cancel{x}+1-\cancel{x}+1}{x^2-1}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4-1}$

$=\dfrac{2}{x^2-1}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4-1}$

$=\dfrac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)}-\dfrac{4}{x^4-1}$

$=\dfrac{\cancel{2x^2}+2-\cancel{2x^2}+2}{(x^2)^2-(1)^2}-\dfrac{4}{x^4-1}$

$=\dfrac{4}{x^4-1}-\dfrac{4}{x^4-1}$

$=\dfrac{4-4}{x^4-1}$

$=\dfrac{0}{x^4-1}$

$=0$ **Ans**

**Q.6 Perform the indicated operation and simplify.**

**(i)** $\left(x^2-49\right).\frac{5x+2}{x+7}$ **(A.B)**

**Solution:** $\left(x^2-49\right).\frac{5x+2}{x+7}$

$=\left[(x)^2-(7)^2\right].\frac{5x+2}{x+7}$

$=(x+7)(x-7)\frac{(5x+2)}{(x+7)}$

$=(x-7)(5x+2)$ **Ans**

**(ii)** $\frac{4x-12}{x^2-9}\div\frac{18-2x^2}{x^2+6x+9}$ **(A.B)**

**Solution:** $\frac{4x-12}{x^2-9}\div\frac{18-2x^2}{x^2+2(x)(3)+(3)^2}$

$=\frac{4(x-3)}{(x^2)-(3)^2}\div\frac{2(9-x^2)}{(x+3)^2}$

$=\frac{4(x-3)}{(x-3)(x+3)}\times\frac{(x+3)^2}{2(9-x^2)}$

$=\frac{4}{x+3}\times\frac{(x+3)^2}{2(3+x)(3-x)}$

$=\frac{{}^2\cancel{4}x(x+3)^2}{\cancel{2}(x+3)^2(3-x)}$

$=\frac{2}{3-x}$ **Ans**

**(iii)** $\frac{x^6-y^6}{x^2-y^2}\div\left(x^4+x^2y^2+y^4\right)$ **(A.B)**

**Solution:** $\frac{x^6-y^6}{x^2-y^2}\div\left(x^4+x^2y^2+y^4\right)$

$=\frac{\left(x^2\right)^3-\left(y^2\right)^3}{x^2-y^2}\div\left(x^4+x^2y^2+y^4\right)$

$=\frac{\left(x^2-y^2\right)\left[\left(x^2\right)^2+x^2y^2+\left(y^2\right)^2\right]}{\left(x^2-y^2\right)}\div\left(x^4+x^2y^2+y^4\right)$

$=\left(x^4+x^2y^2+y^4\right)\times\frac{1}{\left(x^4+x^2y^2+y^4\right)}$

= 1 **Ans**

**(iv)** $\frac{x^2-1}{x^2+2x+1}.\frac{x+5}{1-x}$ **(A.B)**

**Solution:** $\frac{x^2-1}{x^2+2x+1}.\frac{x+5}{1-x}$

$=\frac{(x+1)(x-1)}{\left(x^2+2(x)(1)+(1)^2\right)}\times\frac{x+5}{-(x-1)}$

$=\frac{(x+1)(x-1)}{(x+1)^2}\times\frac{(x+5)}{-(x-1)}$

$=-\frac{(x+1)(x+5)}{(x+1)(x+1)}$

$=-\frac{(x+5)}{x+1}$ **Ans**

**(v)** $\frac{x^2+xy}{y(x+y)}.\frac{x^2+xy}{y(x+y)}\div\frac{x^2-x}{xy-2y}$ **(A.B)**

**Solution:** $\frac{x^2+xy}{y(x+y)}.\frac{x^2+xy}{y(x+y)}\div\frac{x^2-x}{xy-2y}$

$=\frac{x(x+y)}{y(x+y)}.\frac{x(x+y)}{y(x+y)}\div\frac{x(x-1)}{y(x-2)}$

$=\frac{x.x}{y.y}\times\frac{y(x-2)}{x(x-1)}$

$=\frac{x(x-2)}{y(x-1)}$ **Ans**

## Algebraic Formulae and their uses

**(K.B+U.B)**

**(i)** $(a+b)^2+(a-b)^2=2\left(a^2+b^2\right)$

**(ii)** $(a+b)^2-(a-b)^2=4ab$

**(iii)** $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

**(iv)** $(a+b)^3=a^3+3ab(a+b)+b^3$

**(v)** $(a-b)^3=a^3-3ab(a-b)-b^3$

**(vi)** $a^3\pm b^3=(a\pm b)\left(a^2\pm ab+b^2\right)$

**Example (Page # 83)** **(A.B)**

If a + b = 7 and a – b = 3 then find the value of **(a)** $a^2+b^2$ **(b)** $ab$

**Solution:**

We are given $a+b=7$ and $a-b=3$

**(a)** Formula

$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$

Substituting the values, we get

$$(7)^2+(3)^2=2(a^2+b^2)$$

$\Rightarrow \quad 49+9=2(a^2+b^2)$

$\Rightarrow \quad 58=2(a^2+b^2)$

$\Rightarrow \quad 29=a^2+b^2$

Or $\quad a^2+b^2=29$

**(b)** Formula

$$(a+b)^2-(a-b)^2=4ab$$

Putting the values

$\Rightarrow \quad (7)^2-(3)^2=4ab$

$\Rightarrow \quad 49-9=4ab$

$\Rightarrow \quad 40=4ab$

$\Rightarrow \quad 10=ab$

Or $\quad ab=10$

Hence $a^2+b^2=29$ and $ab=10$

**Example (Page # 84)** **(A.B)**

If $2x-3y=10$ and $xy=2$ then find the value of $8x^3-27y^3$.

**Solution:**

Here $\quad 2x-3y=10$

Taking cube on both sides

$\Rightarrow (2x-3y)^3=(10)^3$

$8x^3-27y^3-3(2x)(3y)(2x-3y)=1000$

$\because (a-b)^3=a^3-3ab(a-b)-b^3$

Putting the values

$\Rightarrow \quad 8x^3-27y^3-18(2)(10)=1000$

$\Rightarrow \quad 8x^3-27y^3-360=1000$

$\Rightarrow \quad 8x^3-27y^3=1000+360$

Hence $8x^3-27y^3=1360$

**Example 1 (Page # 86)** **(A.B)**

Factorize: $\quad 64x^3+343y^3$

**Solution:**

We have

$64x^3+343y^3$

$=(4x)^3+(7y)^3$

$\because a^3+b^3=(a+b)(a^2-ab+b^2)$

$=(4x+7y)\left[(4x)^2-(4x)(7y)+(7y)^2\right]$

$=(4x+7y)(16x^2-28xy+49y^2)$

**Example # 4** **(A.B)**

Find the product

$$\left[\frac{4x}{5}-\frac{5}{4x}\right]\left[\frac{16x^2}{25}+\frac{25}{16x^2}+1\right]$$

**Solution:**

$\left[\frac{4}{5}x-\frac{5}{4x}\right]\left[\frac{16}{25}x^2+\frac{25}{16x^2}+1\right]$

$=\left[\frac{4x}{5}-\frac{5}{4x}\right]\left[\frac{16x^2}{25}+1+\frac{25}{16x^2}\right]$

$=\left[\frac{4x}{5}-\frac{5}{4x}\right]\left[\left(\frac{4x}{5}\right)^2+\left(\frac{4x}{5}\right)\left(\frac{5}{4x}\right)+\left(\frac{5}{4x}\right)^2\right]$

$\because (a-b)(a^2+ab+b^2)=a^3-b^3$

$=\left[\frac{4x}{5}\right]^3-\left[\frac{5}{4x}\right]^3$

$=\frac{16x^3}{125}-\frac{125}{64x^3}$

## Exercise 4.2

**Q.1**

**(i) If $a+b=10$ and $a-b=6$, then find the value of $(a^2+b^2)$** **(A.B)**

**Solution:**

Formula

$2(a^2+b^2)=(a+b)^2+(a-b)^2$

Putting the values

$2(a^2+b^2)=(10)^2+(6)^2$

$2(a^2+b^2)=100+36$

$2(a^2+b^2)=136$

$(a^2+b^2)=\frac{\cancel{136}^{68}}{\cancel{2}}$

$(a^2+b^2)=68$

**(ii)** **If** $a+b=5, a-b=\sqrt{17}$, **then find the value of** $ab$. **(A.B)**

**Solution:**

$4ab=(a+b)^2-(a-b)^2$

Putting the values

$4ab=(5)^2-(\sqrt{17})^2$

$4ab=25-17$

$4ab=8$

$ab=\frac{8}{4}$

$ab=2$

**Q.2** **If** $a^2+b^2+c^2=45$ **and** $a+b+c=-1$, **then find the value of** $ab+bc+ca$. **(A.B)**

**Solution:**

We know that

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Putting the values

$(-1)^2=45+2(ab+bc+ca)$

$1=45+2(ab+bc+ca)$

$1-45=2(ab+bc+ca)$

$-44=2(ab+bc+ca)$

$\frac{-44^{22}}{2}=(ab+bc+ca)$

$(ab+bc+ca)=-22$

**Q.3** **If** $m+n+p=10$ **and** $mn+np+np=27$, **find the value of** $m^2+n^2+p^2$ **(A.B)**

**Solution:**

We know that

$(m+n+p)^2=m^2+n^2+p^2+2mn+2np+2mp$

$(10)^2=m^2+n^2+p^2+2(mn+np+mp)$

Putting the values

$100=m^2+n^2+p^2+2(27)$

$100=m^2+n^2+p^2+54$

$100-54=m^2+n^2+p^2$

$m^2+n^2+p^2=46$

**Q.4** **If** $x^2+y^2+z^2=78$ **and** $xy+yz+zx=59$, **find the value of** $x+y+z$. **(A.B)**

**Solution:**

We know that

$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

$(x+y+z)^2=78+2(xy+yz+zx)$

Putting the values

$(x+y+z)^2=78+2(59)$

$(x+y+z)^2=78+118$

$(x+y+z)^2=196$

Taking square root at both sides

$\sqrt{(x+y+z)^2}=\pm\sqrt{196}$

$x+y+z=\pm 14$

**Q.5** **If** $x+y+z=12$ **and** $x^2+y^2=64$, **find the value of** $xy+yz+zx$. **(A.B)**

**Solution:**

We know that

$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

$(12)^2=64+2(xy+yz+zx)$

$144-64=2(xy+yz+zx)$

$80=2(xy+yz+zx)$

$\frac{{}^{40}80}{2}=(xy+yz+zx)$

$40=xy+yz+zx$

$xy+yz+zx=40$ **Ans**

**Q.6** **If** $x+y=7$ **and** $xy=12$, **then find the value of** $x^3+y^3$ **(A.B)**

**Solution:**

We know that

$(x+y)^3=x^3+y^3+3xy(x+y)$

$(7)^3 = x^3 + y^3 + 3(12)(7)$

$343 = x^3 + y^3 + 252$

$343 - 252 = x^3 + y^3$

$91 = x^3 + y^3$

$x^3 + y^3 = 91$ **Ans**

**Q.7 If $3x + 4y = 11$ and $xy = 12$, then find the value of $27x^3 + 64y^3$.** **(A.B)**

**Solution:**

We know that

$(3x+4y)^3 = (3x)^3 + (4y)^3 + 3(3x)(4y)(3x+4y)$

$\because (a+b)^3 = a^3 + 3ab(a+b) + b^3$

$(3x+4y)^3 = 27x^3 + 64y^3 + 36xy(3x+4y)$

Putting the values

$(11)^3 = 27x^3 + 64y^3 + 36(12)(11)$

$1331 = 27x^3 + 64y^3 + 4752$

$1331 - 4752 = 27x^3 + 64y^3$

$-3421 = 27x^3 + 64y^3$

$27x^3 + 64y^3 = -3421$ Ans

**Q.8 If $x - y = 4$ and $xy = 21$, then find the value of $x^3 - y^3$** **(A.B)**

**Solution:**

We know that

$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(4)^3 = x^3 - y^3 - 3(21)(4)$

$64 = x^3 - y^3 - 252$

$64 + 252 = x^3 - y^3$

$316 = x^3 - y^3$

$x^3 - y^3 = 316$ **Ans**

**Q.9 If $5x - 6y = 13$ and $xy = 6$, then find the value of b$125x^3 - 216y^3$** **(A.B)**

**Solution:**

We know that

$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(5x-6y)^3 = (5x)^3 - (6y)^3 - 3(5x)(6y)(5x-6y)$

$(5x-6y)^3 = 125x^3 - 216y^3 - 90xy(5x-6y)$

Putting the values

$(13)^3 = 125x^3 - 216y^3 - 90(6)(13)$

$2197 = 125x^3 - 216y^3 - 7020$

$2197 + 7020 = 125x^3 - 216y^3$

$9217 = 125x^3 - 216y^3$

$125x^3 - 216y^3 = 9217$ **Ans**

**Q.10 If $x + \frac{1}{x} = 3$ then find the value of $x^3 + \frac{1}{x^3}$** **(A.B)**

**Solution:**

We know that

$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right)$

Putting the values

$(3)^3 = x^3 + \frac{1}{x^3} + 3(3)$

$27 = x^3 + \frac{1}{x^3} + 9$

$27 - 9 = x^3 + \frac{1}{x^3}$

$18 = x^3 + \frac{1}{x^3}$

$x^3 + \frac{1}{x^3} = 18$ **Ans**

**Q.11 If $x - \frac{1}{x} = 7$, then find the value of $x^3 - \frac{1}{x^3}$** **(A.B)**

**Solution:**

We know that

$\left(x-\frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x-\frac{1}{x}\right)$

Putting the values

$(7)^3 = x^3 - \frac{1}{x^3} - 3(7)$

$$343 = x^3 - \frac{1}{x^3} - 21$$

$$343 + 21 = x^3 - \frac{1}{x^3}$$

$$364 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 364 \textbf{ Ans}$$

**Q.12** **If** $\left[3x + \frac{1}{3x}\right] = 5$, **then find the value of** $\left[27x^3 + \frac{1}{27x^3}\right]$ **(A.B)**

**Solution:**

We know that

$$\left(3x + \frac{1}{3x}\right)^3 = (3x)^3 + \left(\frac{1}{3x}\right)^3 + 3(\cancel{3x})\left(\frac{1}{\cancel{3x}}\right)\left(3x + \frac{1}{3x}\right)$$

Putting the values

$$(5)^3 = 27x^3 + \frac{1}{27x^3} + 3(5)$$

$$125 = 27x^3 + \frac{1}{27x^3} + 15$$

$$125 - 15 = 27x^3 + \frac{1}{27x^3}$$

$$110 = 27x^3 + \frac{1}{27x^3}$$

$$27x^3 + \frac{1}{27x^3} = 110$$

**Q.13** **If** $\left(5x - \frac{1}{5x}\right) = 6$, **then find the value of** $\left(125x^3 - \frac{1}{125x^3}\right)$ **(A.B)**

**Solution:**

We know that

$$\left(5x - \frac{1}{5x}\right)^3 = (5x)^3 - \left(\frac{1}{5x}\right)^3 - 3(\cancel{5x})\left(\frac{1}{\cancel{5x}}\right)\left(5x - \frac{1}{5x}\right)$$

$$(6)^3 = 125x^3 - \frac{1}{125x^3} - 3(6)$$

$$216 = 125x^3 - \frac{1}{125x^3} - 18$$

$$216 + 18 = 125x^3 - \frac{1}{125x^3}$$

$$234 = 125x^3 - \frac{1}{125x^3}$$

$$125x^3 - \frac{1}{125x^3} = 234 \textbf{ Ans}$$

**Q.14 Factorize**

**(i)** $x^3 - y^3 - x + y$ **(GRW 2015)** **(A.B)**

**Solution:** $x^3 - y^3 - x + y$

$$= (x)^3 - (y)^3 - 1(x - y)$$

$$= (x - y)(x^2 + xy + y^2) - 1(x - y)$$

$$= (x - y)(x^2 + xy + y^2 - 1) \textbf{ Ans}$$

**(ii)** $8x^3 - \frac{1}{27y^3}$ **(A.B)**

**(FSD 2015, MTN 2013, SWL 2013, BWP 2016)**

**Solution:** $8x^3 - \frac{1}{27y^3}$

$$= (2x)^3 - \left(\frac{1}{3y}\right)^3$$

$$= \left[2x - \frac{1}{3y}\right]\left[(2x)^2 + (2x)\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right]$$

$$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right) \textbf{ Ans}$$

**Q.15 Find the products, using formula.**

**(i)** $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$ **(A.B)**

**Solution:** $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

$$= (x^2 + y^2)\left[(x^2)^2 - (x^2)(y^2) + (y^2)^2\right]$$

$$\left[(x^2)^3 + (y^2)^3\right]$$

$$= x^6 + y^6 \textbf{ Ans}$$

**(ii)** $\left(x^3-y^3\right)\left(x^6+x^3y^3+y^6\right)$ **(A.B)**

**Solution:** $\left(x^3-y^3\right)\left(x^6+x^3y^3+y^6\right)$

$\left(x^3-y^3\right)\left[\left(x^3\right)^2+\left(x^3\right)\left(y^3\right)+\left(y^3\right)^2\right]$

$=\left(x^3\right)^3-\left(y^3\right)^3$

$=x^9-y^9$ **Ans**

**(iii)** $(x-y)(x+y)\left(x^2+y^2\right)\left(x^2+xy+y^2\right)$

$\left(x^2+xy+y^2\right)\left(x^4-x^2y^2+y^4\right)$

**(A.B)**

**Solution:**

$(x-y)(x+y)\left(x^2+y^2\right)\left(x^2+xy+y^2\right)$

$\left(x^2+xy+y^2\right)\left(x^4-x^2y^2+y^4\right)$

$=\left[(x-y)\left(x^2+xy+y^2\right)\right]\left[(x+y)\left(x^2-xy+y^2\right)\right]$

$\left[\left(x^2+y^2\right)\left(x^4-x^2y^2+y^4\right)\right]$

$=\left[\left(x^3-y^3\right)\left(x^3+y^3\right)\right]\left[\left(x^2\right)^3+\left(y^2\right)^3\right]$

$=\left[\left(x^3\right)^2-\left(y^3\right)^2\right]\left[\left(x^6+y^6\right)\right]$

$=\left[\left(x^6-y^6\right)\left(x^6+y^6\right)\right]$

$=\left[\left(x^6\right)^2-\left(y^6\right)^2\right]$

$=x^{12}-y^{12}$

**(iv)** $\left(2x^2-1\right)\left(2x^2+1\right)\left(4x^4+2x^2+1\right)\left(4x^4-2x^2+1\right)$

**Solution:**

$\left(2x^2-1\right)\left(2x^2+1\right)\left(4x^4+2x^2+1\right)\left(4x^4-2x^2+1\right)$

$=\left[\left(2x^2-1\right)\left(4x^4+2x^2+1\right)\right]\left[\left(2x^2+1\right)\left(4x^4-2x^2+1\right)\right]$

$=\left[\left(2x^2\right)^3-(1)^3\right]\left[\left(2x^2\right)^3+(1)^3\right]$

$=\left(8x^6-1\right)\left(8x^6+1\right)$

$=\left(8x^6\right)^2-(1)^2$

$=64x^{12}-1$ **Ans**

## SURDS AND THEIR APPLICATION

### SURD

An irrational radical with radicand is called a surd.

**For example:** **(K.B)**

$\sqrt{3},\sqrt{\frac{2}{5}},\ \sqrt[3]{7},\ \sqrt[4]{10}$

**Note** **(U.B+K.B)**

Hence the radical $\sqrt[n]{a}$ is a surd if

**(i)** $a$ is rational

**(ii)** The result $\sqrt[n]{a}$ is irrational.

### Order of a Surd **(K.B)**

If $\sqrt[n]{a}$ is an irrational number then $n$ is called index or the order of the surd and the rational number 'a' is called the radicand.

**For example:**

In $\sqrt[3]{7}$, order of order surd is 3.

Note

- $\sqrt{\pi}$ and $\sqrt{2+\sqrt{17}}$ are not surds because $\pi$ and $2+\sqrt{17}$ are not rational.
- Every surd is an irrational number but every irrational number is not a surd. e.g., the surd $\sqrt[3]{5}$ is an irrational number but the irrational number $\sqrt{\pi}$ is not a surd.

## OPERATOINS ON SURDS

**(a) Addition and Subtracting of Surd** **(U.B+K.B)**

Similar surds (i.e. surds having same irrational factors) can be added or subtracted into a single term.

**Example (Page # 89)** **(K.B)**

$\sqrt[3]{128}-\sqrt[3]{250}+\sqrt[3]{432}$

$=\sqrt[3]{64\times2}-\sqrt[3]{125\times2}+\sqrt[3]{216\times2}$

$=\sqrt[3]{4^3\times2}-\sqrt[3]{5^3\times2}+\sqrt[3]{6^3\times2}$

$=\sqrt[3]{(4)^3}\times\sqrt[3]{2}-\sqrt[3]{(5)^3}\times\sqrt[3]{2}+\sqrt[3]{(6)^3}\times\sqrt[3]{2}$

$=4\sqrt[3]{2}-5\sqrt[3]{2}+6\sqrt[3]{2}$

$=(4-5+6)\sqrt[3]{2}$

$=5\sqrt[3]{2}$

**(b) Multiplications and Division of Surds** **(K.B)**

We can multiply and divide surds of the same order by making use of the following laws of surds.

$\sqrt[n]{a}.\sqrt[n]{b}=\sqrt[n]{ab}$ and $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}}$

and the result obtained will be a surd of the same order. If surds to be multiplied or divided are not of the same order, they must be reduced to the surds of the same order.

**Example (Page # 89)** **(A.B)**

**(ii)** $\dfrac{\sqrt[6]{12}}{\sqrt{3}\sqrt[3]{2}}$

We have

For $\sqrt{3}\ \sqrt[3]{2}$ the L.C.M of 2 and 3 is 6

$$\sqrt{3}=(3)^{\frac{1}{2}}=(3)^{\frac{3}{3}\times\frac{1}{2}}=(3)^{\frac{3}{6}}=\sqrt[6]{3^3}=\sqrt[6]{27}$$

$$\sqrt[3]{2}=(2)^{\frac{1}{2}}=(2)^{\frac{2}{2}\times\frac{1}{3}}=2^{\frac{2}{6}}=\sqrt[6]{2^2}=\sqrt[6]{4}$$

Hence

$$\frac{\sqrt[6]{12}}{\sqrt{3}\sqrt[3]{2}}=\frac{\sqrt[6]{12}}{\sqrt[6]{27}\times\sqrt[6]{4}}=\frac{\sqrt[6]{12}}{\sqrt[6]{27\times4}}=\sqrt[6]{\frac{12}{108}}=\sqrt[6]{\frac{1}{9}}$$

It simplest form is

$$\sqrt[6]{\left(\frac{1}{3}\right)^2}=\left[\left(\frac{1}{3}\right)^2\right]^{\frac{1}{6}}=\left[\frac{1}{3}\right]^{2\times\frac{1}{3}}=\left(\frac{1}{3}\right)^{\frac{1}{3}}=\sqrt[3]{\frac{1}{3}}$$

**Method II**

$$\frac{\sqrt[6]{12}}{\sqrt{3}\sqrt[3]{2}}$$

$$=\frac{(12)^{1/6}}{(3)^{1/2}(2)^{1/3}} \quad \text{(in exponential form)}$$

$$=\frac{(2^2\times3)^{1/6}}{(3)^{1/2}(2)^{1/3}}$$

$$=\frac{(2^2)^{1/6}\times(3)^{1/6}}{(3)^{1/2}(2)^{1/3}} \quad \because (ab)^n=a^nb^n$$

$$=\frac{2^{1/3}\times3^{1/6}}{3^{1/2}\times2^{1/3}} \quad \because (a^m)^n=a^{mn}$$

$$=\frac{3^{1/6}}{3^{1/2}} \quad \because \frac{a^n}{a^n}=1$$

$$=\frac{1}{3^{1/2-1/6}} \quad \because \frac{a^m}{a^n}=a^{m-n}$$

$$=\frac{1}{3^{2/6}}$$

$$=\frac{1}{3^{1/3}}$$

$$=\frac{1}{\sqrt[3]{3}} \quad \text{(in radical form)}$$

## Exercise 4.3

**Q.1 Express each of the following surd in the simplest form:**

**(i)** $\sqrt{180}$ **(A.B)**

**Solution:**

$$\sqrt{180}$$
$$=\sqrt{2^2\times3^2\times5}$$
$$=\sqrt{2^2}\times\sqrt{3^2}\times\sqrt{5} \quad \because \sqrt[n]{ab}=\sqrt[n]{a}\times\sqrt[n]{b}$$
$$=2\times3\times\sqrt{5}$$
$$=6\sqrt{5}$$

**(ii)** $3\sqrt{162}$ **(A.B)**

**Solution:**

$$3\sqrt{162}$$
$$=3\left(\sqrt{9^2\times2}\right)$$
$$=3\left(\sqrt{9^2}\times\sqrt{2}\right)$$
$$=3\times9\sqrt{2}$$
$$=27\sqrt{2}$$

**(iii)** $\dfrac{3}{4}\sqrt[3]{128}$ **(A.B)**

**Solution:**

$$\frac{3}{4}\sqrt[3]{128}$$
$$=\frac{3}{4}\left(\sqrt[3]{4^3\times2}\right)$$

$=\frac{3}{4}\left[\sqrt[3]{4^3}\times\sqrt[3]{2}\right]$

$=\frac{3}{4}\times 4\times\sqrt[3]{2}$

$=3\sqrt[3]{2}$

**(iv)** $\sqrt[5]{96x^6y^7z^8}$ **(A.B)**

**Solution:**

$\sqrt[5]{96x^6y^7z^8}$

$=\sqrt[5]{2^5\times 3\times x^5y^5z^5\times xy^2z^3}$

$=\sqrt[5]{2^5x^5y^5z^5}\times\sqrt[5]{3xy^2z^3}$

$=\sqrt[5]{2^5}\times\sqrt[5]{x^5}\times\sqrt[5]{y^5}\times\sqrt[5]{z^5}\times\sqrt[5]{3xy^2z^3}$

$=2xyz\sqrt[5]{3xy^2z^3}$

**Q.2 Simplify**

**(i)** $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$ **(BWP 2014)** **(A.B)**

**Solution:**

$\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

$=\sqrt{\frac{3^2\times 2}{3\times 2}}$

$\because \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}},\ \sqrt{a}\times\sqrt{b}=\sqrt{ab}$

$=\sqrt{3}$

**(ii)** $\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$ **(A.B)**

**(LHR 2017, FSD 2016, MTN 2016, SWL 2017, SGD 2017, D.G.K 2017)**

**Solution:**

$\frac{\sqrt{21}\sqrt{9}}{\sqrt{63}}$

$=\sqrt{\frac{21\times 9}{63}}\quad \because \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}},\ \sqrt{a}\times\sqrt{b}=\sqrt{ab}$

$=\sqrt{\frac{7\times 3\times 3^2}{7\times 3^2}}$

$=\sqrt{3}$

**(iii)** $=\sqrt[5]{243x^5y^{10}z^{15}}$ **(A.B)**

**Solution:**

$\sqrt[5]{243x^5y^{10}z^{15}}$

$=\sqrt[5]{3^5x^5\left(y^2\right)^5\left(z^3\right)^5}$

$=\sqrt[3]{3^5}\times\sqrt[5]{x^5}\times\sqrt[5]{\left(y^2\right)^5}\times\sqrt[3]{\left(z^3\right)^5}$

$=3\times x\times y^2\times z^3$

$=3xy^2z^3$

**(iv)** $\frac{4}{5}\sqrt[3]{125}$ **(MTN 2013, SGD 2013)** **(A.B)**

**Solution:**

$\frac{4}{5}\sqrt[3]{125}$

$=\frac{4}{5}\sqrt[3]{5\times 5\times 5}$

$=\frac{4}{5}\sqrt[3]{5^3}$

$=\frac{4}{\cancel{5}}\times\cancel{5}$

$=4$

**(v)** $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$ **(A.B)**

**Solution:**

$\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

$=\sqrt{7\times 3}\times\sqrt{7}\times\sqrt{3}$

$=\sqrt{7\times 3\times 7\times 3}$

$=\sqrt{7^2\times 3^2}$

$=\sqrt{7^2}\times\sqrt{3^2}$

$=7\times 3$

$=21$

**Q.3 Simplify by combining similar terms.**

**(i)** $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$ **(A.B)**

**(D.G.K 2017)**

**Solution:**

$\sqrt{45}-3\sqrt{20}+4\sqrt{5}$

$=\sqrt{9\times 5}-3\sqrt{5\times 4}+4\sqrt{5}$

$=\sqrt{3^2}\times\sqrt{5}-3\sqrt{2^2}\times\sqrt{5}+4\sqrt{5}$

$=3\sqrt{5}-3\times 2\sqrt{5}+4\sqrt{5}$

$=3\sqrt{5}-6\sqrt{5}+4\sqrt{5}$

$=\sqrt{5}(3-6+4)$

$=\sqrt{5}(3-2)$

$=\sqrt{5}(1)$

$=\sqrt{5}$

**(ii)** $4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$ **(A.B)**

**Solution:**

$4\sqrt{12}+5\sqrt{27}-3\sqrt{75}+\sqrt{300}$

$=4\sqrt{2^2\times 3}+5\sqrt{3^2\times 3}-3\sqrt{5^2\times 3}+\sqrt{10^2\times 3}$

$=4\times 2\sqrt{3}+5\times 3\sqrt{3}-3\times 5\sqrt{3}+10\sqrt{3}$

$=8\sqrt{3}+15\sqrt{3}-15\sqrt{3}+10\sqrt{3}$

$=8\sqrt{3}+15\sqrt{3}-15\sqrt{3}+10\sqrt{3}$

$=\sqrt{3}(8+\cancel{15}-\cancel{15}+10)$

$=\sqrt{3}(8+10)$

$=\sqrt{3}(18)$

$=18\sqrt{3}$

**(iii)** $\sqrt{3}\left(2\sqrt{3}+3\sqrt{3}\right)$ **(A.B)**

**Solution:**

$\sqrt{3}\left(2\sqrt{3}+3\sqrt{3}\right)$

$=\sqrt{3}\times\sqrt{3}(2+3)$

$=\left(\sqrt{3}\right)^2\times(5)$

$=3(5)$

$=15$

**(iv)** $2\left(6\sqrt{5}-3\sqrt{5}\right)$ **(LHR 2016)** **(A.B)**

**Solution:**

$2\left(6\sqrt{5}-3\sqrt{5}\right)$

$=2\times\sqrt{5}(6-3)$

$=2\times\sqrt{5}(3)$

$=6\sqrt{5}$

**Q.4 Simplify**

**(FSD 2016, SGD 2013, BWP 2014)**

**(i)** $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

**Solution:** **(A.B)**

$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

$\because (a+b)(a-b)=a^2-b^2$

$=(3)^2-\left(\sqrt{3}\right)^2$

$=9-3$

$=6$

**(ii)** $\left(\sqrt{5}+\sqrt{3}\right)^2$ **(A.B)**

**Solution:**

$\left(\sqrt{5}+\sqrt{3}\right)^2$

$\because (a+b)^2=a^2+2ab+b^2$

$=\left(\sqrt{5}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2$

$=5+2\sqrt{5\times 3}+3$

$=8+2\sqrt{15}$

**(iii)** $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$ **(A.B)**

**Solution:**

$\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

$\because (a+b)(a-b)=a^2-b^2$

$=\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2$

$=5-3$

$=2$

**(iv)** $\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$ **(A.B)**

**(BWP 2014)**

**Solution:**

$\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}-\frac{1}{\sqrt{3}}\right)$

$\because (a+b)(a-b)=a^2-b^2$

$=\left(\sqrt{2}\right)^2-\left(\frac{1}{\sqrt{3}}\right)^2$

$$= 2 - \frac{(1)^2}{\left(\sqrt{3}\right)^2}$$
$$= 2 - \frac{1}{3}$$
$$= \frac{6-1}{3}$$
$$= \frac{5}{3}$$

**(v)** $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x+y)\left(x^2+y^2\right)$ **(A.B)**

**Solution:**

$$\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x+y)\left(x^2+y^2\right)$$
$$\because\ (a+b)(a-b) = a^2 - b^2$$
$$= \left[\left(\sqrt{x}\right)^2 - \left(\sqrt{y}\right)^2\right](x+y)\left(x^2+y^2\right)$$
$$= (x-y)(x+y)\left(x^2+y^2\right)$$
$$= \left[(x)^2 - (y)^2\right]\left(x^2+y^2\right)$$
$$= \left(x^2-y^2\right)\left(x^2+y^2\right)$$
$$= \left[\left(x^2\right)^2 - \left(y^2\right)^2\right]$$
$$= x^4 - y^4$$

**Monomial Surd** (GRW 2017) **(K.B)**

A surd which contains a Single term is called a monomial Surd e.g., $\sqrt{2}\,\sqrt{3}$ etc.

**Binomial Surd** **(K.B)**

A surd containing two terms is called binomial surd.

Or

A Surd which contains sum of two monomial surds or sum of a monomial surd and a rational number is called a binomial surd.

**For example:**

$\sqrt{3}+\sqrt{7}$, $\sqrt{2}+5$ etc.

**Conjugate surds** **(K.B)**

Such pairs of binomial surds whose product is a rational number are called conjugate of each other.

Or

Two binomial surds of second order differing only in sign connecting their terms are called conjugate surds.

$\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$ are conjugate surds of each other.

$$\because\ \left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$$
$$= \left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2 = a - b$$

The conjugate of $x+\sqrt{y}$ is $x-\sqrt{y}$.

**For example:**

$$\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right) = (3)^2 - \left(\sqrt{5}\right)^2$$
$$= 9 - 5 = 4$$

which is a rational number.

**Rationalizing a Denominator** **(K.B)**

**Rationalizing Real Numbers of the types** $\frac{1}{a+b\sqrt{x}}, \frac{1}{\sqrt{x}+\sqrt{y}}$

**Example # 3** **(A.B)**

**Simplify:** $\frac{6}{2\sqrt{3}-\sqrt{6}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$

**Solution:**

First we shall rationalize the denominators and then simplify

$$\frac{6}{2\sqrt{3}-\sqrt{6}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}$$

$$=\frac{6}{2\sqrt{3}-\sqrt{6}}\times\frac{2\sqrt{3}+\sqrt{6}}{\sqrt{3}+\sqrt{6}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}\times\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}$$

$$=\frac{6\left(2\sqrt{3}+\sqrt{6}\right)}{\left(2\sqrt{3}\right)^2-\left(\sqrt{6}\right)^2}+\frac{\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}-\frac{4\sqrt{3}\left(\sqrt{6}+\sqrt{2}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{6\left(2\sqrt{3}+\sqrt{6}\right)}{12-6}+\frac{\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)}{3-2}-\frac{4\sqrt{3}\left(\sqrt{6}+\sqrt{2}\right)}{6-2}$$

$$=\frac{12\sqrt{3}+6\sqrt{6}}{12-6}+\frac{\sqrt{6}\times\sqrt{3}-\sqrt{2}\sqrt{6}}{1}-\frac{4\sqrt{3}\sqrt{6}+4\sqrt{3}\sqrt{2}}{4}$$

$$=2\sqrt{3}+\sqrt{6}+3\sqrt{2}-2\sqrt{3}-3\sqrt{2}-\sqrt{6}$$

$$=0$$

**Example # 5** **(A.B)**

If $x=3+\sqrt{8}$, then evaluate

(i) $x+\frac{1}{x}$ and (ii) $x^2+\frac{1}{x^2}$

**Solution:**

Here

$$x=3+\sqrt{8}$$

$$\frac{1}{x}=\frac{1}{3+\sqrt{8}}$$

Rationalizing the denominator

$$=\frac{1}{3+\sqrt{8}}\times\frac{3-\sqrt{8}}{3-\sqrt{8}}$$

$$=\frac{3-\sqrt{8}}{(3)^2-\left(\sqrt{8}\right)^2}$$

$$=\frac{3-\sqrt{8}}{9-8}$$

$$=3-\sqrt{8}$$

**(i)** $x+\frac{1}{x}=\left(3+\sqrt{8}\right)+\left(3-\sqrt{8}\right)$

$$=3+\sqrt{8}+3-\sqrt{8}$$

$$=6$$

**(ii)** Consider

$$x+\frac{1}{x}=6$$

Squaring both sides

$$\left(x+\frac{1}{x}\right)^2=6^2$$

$$x^2+\frac{1}{x^2}+2=36$$

$$x^2+\frac{1}{x^2}=36-2$$

$$\Rightarrow x^2+\frac{1}{x^2}=34$$

## Exercise 4.4

**Rationalize the denominator of the following**

**Q.1**

**(i)** $\frac{3}{4\sqrt{3}}$ **(A.B)**

**Solution:**

$$\frac{3}{4\sqrt{3}}$$

$$=\frac{3}{4\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{3\left(\sqrt{3}\right)}{4\left(\sqrt{3}\right)^2}$$

$$=\frac{3\sqrt{3}}{4\times3}$$

$$=\frac{\sqrt{3}}{4}$$

**(ii)** $\frac{14}{\sqrt{98}}$ **(A.B)**

**Solution:**

$$\frac{14}{\sqrt{98}}$$

$$=\frac{14}{\sqrt{98}}\times\frac{\sqrt{98}}{\sqrt{98}}$$

$$=\frac{14\left(\sqrt{98}\right)}{\left(\sqrt{98}\right)^2}$$

$$=\frac{14\left(\sqrt{7\times7\times2}\right)}{98}$$

$$=\frac{14\times7\times\sqrt{2}}{98}$$

$$=\frac{\cancel{98}\times\sqrt{2}}{\cancel{98}}$$

$$=\sqrt{2}$$

**(iii)** $\frac{6}{\sqrt{8}\sqrt{27}}$ **(A.B)**

**Solution:**

$$\frac{6}{\sqrt{8}\sqrt{27}}$$

$$=\frac{6}{\sqrt{2^2\times2}\sqrt{3^2\times3}}$$

$$=\frac{6}{2\times3\sqrt{2}.\sqrt{3}}$$

$$=\frac{1}{\sqrt{6}}$$

$$=\frac{1}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}$$

$$=\frac{\sqrt{6}}{6}$$

**(iv)** $\frac{1}{3+2\sqrt{5}}$ **(A.B)**

**Solution:**

$$\frac{1}{3+2\sqrt{5}}$$

$$=\frac{1}{3+2\sqrt{5}}\times\frac{3-2\sqrt{5}}{3-2\sqrt{5}}$$

$$=\frac{3-2\sqrt{5}}{(3)^2-\left(2\sqrt{5}\right)^2}$$

$$=\frac{3-2\sqrt{5}}{9-4(5)}$$

$$=\frac{3-2\sqrt{5}}{9-20}$$

$$=\frac{3-2\sqrt{5}}{-11}$$

$$=\frac{-3+2\sqrt{5}}{11}$$

**(v)** $\frac{15}{\sqrt{31}-4}$ **(A.B)**

**Solution:**

$$\frac{15}{\sqrt{31}-4}$$

$$=\frac{15}{\sqrt{31}-4}\times\frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{\left(\sqrt{31}\right)^2-(4)^2}$$

$$=\frac{15\left(\sqrt{31}+4\right)}{31-16}$$

$$=\frac{\cancel{15}\left(\sqrt{31}+4\right)}{\cancel{15}}$$

$$=\sqrt{31}+4$$

**(vi)** $\frac{2}{\sqrt{5}-\sqrt{3}}$ **(RWP 2016)** **(A.B)**

**Solution:**

$$\frac{2}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}\times\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$=\frac{\cancel{2}\left(\sqrt{5}+\sqrt{3}\right)}{\cancel{2}}$$

$$=\sqrt{5}+\sqrt{3}$$

**(vii)** $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ **(BWP 2017)** **(A.B)**

**Solution:**

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{\left(\sqrt{3}-1\right)^2}{3-1}$$

$$=\frac{\left(\sqrt{3}\right)^2-2\left(\sqrt{3}\right)(1)+(1)^2}{2}$$

$$=\frac{3-2\sqrt{3}+1}{2}$$

$$=\frac{4-2\sqrt{3}}{2}$$

$$=\frac{\cancel{2}\left(2-\sqrt{3}\right)}{\cancel{2}}$$

$$=2-\sqrt{3}$$

**(viii)** $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ **(A.B)**

**Solution:**

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{\left(\sqrt{5}\right)^2+2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2}{5-3}$$

$$=\frac{5+2\sqrt{15}+3}{2}$$

$$=\frac{8+2\sqrt{15}}{2}$$

$$=\frac{\cancel{2}\left(4+\sqrt{15}\right)}{\cancel{2}}$$

$$=4+\sqrt{15}$$

**Q.2 Find the conjugate of** $x+\sqrt{y}$ **(K.B)**

**(i)** $3+\sqrt{7}$

**Conjugate** $3-\sqrt{7}$

**(ii)** $4-\sqrt{5}$

**Conjugate** $4+\sqrt{5}$

**(iii)** $2+\sqrt{3}$

**Conjugate** $2-\sqrt{3}$

**(iv)** $2+\sqrt{5}$

**Conjugate** $2-\sqrt{5}$

**(v)** $5+\sqrt{7}$

**Conjugate** $5-\sqrt{7}$

**(vi)** $4-\sqrt{15}$

**Conjugate** $4+\sqrt{15}$

**(vii)** $7-\sqrt{6}$

**Conjugate** $7+\sqrt{6}$

**(viii)** $9+\sqrt{2}$

**Conjugate** $9-\sqrt{2}$

**Q.3** **(GRW 2017, SWL 2017)**

**(i)** **If** $x = 2-\sqrt{3}$, **find** $\frac{1}{x}$

**Solution:**

**Given that**

$x = 2-\sqrt{3}$

$\Rightarrow \frac{1}{x} = \frac{1}{2-\sqrt{3}}$

Rationalizing the denominator

$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$= \frac{2+\sqrt{3}}{4-3}$

$= \frac{2+\sqrt{3}}{1}$

$\frac{1}{x} = 2+\sqrt{3}$

**(ii)** **If** $x = 4-\sqrt{17}$, **find** $\frac{1}{x}$ **(LHR 2014)**

**Solution:**

Given that $x = 4-\sqrt{17}$

$\Rightarrow \frac{1}{x} = \frac{1}{4-\sqrt{17}}$

$= \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$

$= \frac{4+\sqrt{17}}{(4)^2-(\sqrt{17})^2}$

$= \frac{4+\sqrt{17}}{16-17}$

$= \frac{4+17}{-1}$

$= -1(4+\sqrt{17})$

$\Rightarrow \frac{1}{x} = -4-\sqrt{17}$

**(iii)** **If** $x = \sqrt{3}+2$, **find** $x+\frac{1}{x}$

**Solution:** Given that $x = \sqrt{3}+2$ **(A.B)**

$\frac{1}{x} = \frac{1}{\sqrt{3}+2}$

$= \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$

$= \frac{\sqrt{3}-2}{(\sqrt{3})^2-(2)^2}$

$= \frac{\sqrt{3}-2}{3-4}$

$= \frac{\sqrt{3}-2}{-1}$

$= -(\sqrt{3}-2)$

$= -\sqrt{3}+2$

Now

$x+\frac{1}{x} = (\sqrt{3}+2)+(-\sqrt{3}+2)$

$= \sqrt{3}+2-\sqrt{3}+2$

$= 2+2$

$x+\frac{1}{x} = 4$

**Q.4** **Simplify**

**(i)** $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ **(A.B)**

**Solution:**

$\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$= \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})+(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$\because (a+b)(a-b) = a^2-b^2$

$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{(\sqrt{5})^2-(\sqrt{3})^2}$

$$=\frac{2\sqrt{5}-2\sqrt{6}}{5-3}$$

$$=\frac{\cancel{2}\left(\sqrt{5}-\sqrt{6}\right)}{\cancel{2}}$$

$$=\sqrt{5}-\sqrt{6}$$

**(ii)** $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$ **(A.B)**

**Solution:**

$$\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2+\sqrt{5}}$$

$$=\left(\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)+\left(\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)$$

$$+\left(\frac{1}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}\right)$$

$$=\left(\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}\right)+\left(\frac{2\times\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}\right)$$

$$+\left(\frac{2-\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2}\right)$$

$$=\left(\frac{2-\sqrt{3}}{4-3}\right)+\left(\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{5-3}\right)+\left(\frac{2-\sqrt{5}}{4-5}\right)$$

$$=\left(\frac{2-\sqrt{3}}{1}\right)+\left(\frac{2\left(\sqrt{5}+\sqrt{3}\right)}{2}\right)+\left(\frac{2-\sqrt{5}}{-1}\right)$$

$$=2-\sqrt{3}+\sqrt{5}+\sqrt{3}-2+\sqrt{5}$$

$$=\cancel{2}-\cancel{2}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}+\sqrt{5}+\sqrt{5}$$

$$=\sqrt{5}+\sqrt{5}$$

$$=2\sqrt{5}$$

**(iii)** $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$ **(A.B)**

**Solution:**

$$\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$$

$$=\left(\frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)+\left(\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$$

$$-\left(\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{7}}\right)$$

$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}\right)+\left(\frac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}\right)$$

$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2}\right)$$

$$=\left(\frac{2\left(\sqrt{5}-\sqrt{3}\right)}{2}\right)+\left(\frac{\sqrt{3}-\sqrt{2}}{1}\right)-\left(\frac{3\left(\sqrt{5}-\sqrt{2}\right)}{3}\right)$$

$$=\cancel{\sqrt{5}}-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-\cancel{\sqrt{2}}-\cancel{\sqrt{5}}+\cancel{\sqrt{2}}$$

$$=0$$

**Q.5 If** $x=2+\sqrt{3}$, **then find the value of** $x-\frac{1}{x}$ **and** $\left(x-\frac{1}{x}\right)^2$ **(A.B)**

**Solution:**

**Given that** $x=2+\sqrt{3}$

$$\Rightarrow\frac{1}{x}=\frac{1}{2+\sqrt{3}}$$

$$=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$=\frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{2-\sqrt{3}}{4-3}$$

$$=\frac{2-\sqrt{3}}{1}$$

$$=2-\sqrt{3}$$

Now

$$x-\frac{1}{x}=\left(2+\sqrt{3}\right)-\left(2-\sqrt{3}\right)$$

$$=\cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}$$

$$=\sqrt{3}+\sqrt{3}$$

$$=2\sqrt{3}$$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=\left(2\sqrt{3}\right)^2$$
$$=4\left(\sqrt{3}\right)^2$$
$$=4(3)$$
$$=12$$

**(i) If** $x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$**, find the value of** $x+\frac{1}{x}$, $x^2+\frac{1}{x^2}$ **and** $x^3+\frac{1}{x^3}$

**Solution:** **(A.B)**

**Given that** $x=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$$\Rightarrow \frac{1}{x}=\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

**Now**

$$x+\frac{1}{x}=\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}+\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$
$$=\frac{\left(\sqrt{5}-\sqrt{2}\right)^2+\left(\sqrt{5}+\sqrt{2}\right)^2}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$$

$\because \left(a+b\right)^2+\left(a-b\right)^2=2\left(a^2+b^2\right)$,

$\left(a+b\right)\left(a-b\right)=a^2-b^2$

$$=\frac{2\left[\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2\right]}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$
$$=\frac{2(5+2)}{5-2}$$
$$\Rightarrow x+\frac{1}{x}=\frac{14}{3}$$

Taking square on both sides

$$\left(x+\frac{1}{x}\right)^2=\left(\frac{14}{3}\right)^2$$

$\because \left(a+b\right)^2=a^2+b^2+2ab$

$$x^2+\frac{1}{x^2}+2\left(x\right)\left(\frac{1}{x}\right)=\frac{196}{9}$$
$$x^2+\frac{1}{x^2}=\frac{196}{9}-2$$
$$x^2+\frac{1}{x^2}=\frac{196-18}{9}$$
$$x^2+\frac{1}{x^2}=\frac{178}{9}$$

**Consider**

$$x+\frac{1}{x}=\frac{14}{3}$$

Taking cube on both sides

$$\left(x+\frac{1}{x}\right)^3=\left(\frac{14}{3}\right)^3$$

$\because \left(a+b\right)^3=a^3+b^3+3ab(a+b)$

$$x^3+\frac{1}{x^3}+3\left(x\right)\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$
$$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$
$$x^3+\frac{1}{x^3}+3\left(\frac{14}{3}\right)=\frac{2744}{27}$$
$$x^3+\frac{1}{x^3}+14=\frac{2744}{24}$$
$$x^3+\frac{1}{x^3}=\frac{2744}{27}-14$$
$$x^3+\frac{1}{x^3}=\frac{2744-378}{27}$$
$$x^3+\frac{1}{x^3}=\frac{2366}{27}$$

**Q.6 Determine the rational numbers** $a$ **and** $b$ **if** $\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$ **(A.B)**

**Solution:**

**Given that**

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

**Or** $$a+b\sqrt{3}=\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$=\frac{\left(\sqrt{3}-1\right)^2+\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)}$$

$\because\ (a+b)^2+(a-b)^2=2\left(a^2+b^2\right),$

$(a+b)(a-b)=a^2-b^2$

$$=\frac{2\left[\left(\sqrt{3}\right)^2+(1)^2\right]}{\left(\sqrt{3}\right)^2-(1)^2}$$

$$=\frac{2(3+1)}{3-1}$$

$$=\frac{2(4)}{2}$$

$a+b\sqrt{3}=4$

$a+b\sqrt{3}=4+0\sqrt{3}$

Comparing both sides

$a=4 \quad b\sqrt{3}=0\sqrt{3}$

$$b=\frac{0\sqrt{3}}{\sqrt{3}}$$

$b=0$

## Review Exercise 4

**Q.1 Multiple type questions?**

**(i) is an algebraic …** **(K.B)**

**(a)** Expression **(b)** Sentence
**(c)** Equation **(d)** In-equation

**(ii) The degree of polynomial $4x^4+3x^2y$ is** **(K.B)**

**(FSD 2014, SGD 2017, MTN 2016, BWP 2016)**

**(a)** 1 **(b)** 2
**(c)** 3 **(d)** 4

**(iii) $a^3+b^3$ is equal to** **(U.B)**

**(a)** $(a-b)\left(a^2+ab+b^2\right)$ **(b)** $(a+b)\left(a^2-ab+b^2\right)$
**(c)** $(a-b)\left(a^2-ab+b^2\right)$ **(d)** $(a-b)\left(a^2+ab+b^2\right)$

**(iv) $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$ is equal to** **(A.B)**

**(a)** 7 **(b)** -7
**(c)** -1 **(d)** 1

**(v) Conjugate of surd $a+\sqrt{b}$ is;** **(U.B)**

**(a)** $-a+\sqrt{b}$ **(b)** $a-\sqrt{b}$
**(c)** $\sqrt{a}+\sqrt{b}$ **(d)** $\sqrt{a}-\sqrt{b}$

**(vi) $\frac{1}{a-b}-\frac{1}{a+b}$ is equal to** **(A.B)**

**(LHR 2015, FSD 2015, MTN 2014, BWP 2017, D.G.K 2013, 15)**

**(a)** $\frac{2a}{a^2-b^2}$ **(b)** $\frac{2b}{a^2-b^2}$
**(c)** $\frac{-2a}{a^2-b^2}$ **(d)** $\frac{-2b}{a^2-b^2}$

(vii) $\frac{a^2-b^2}{a+b}$ is equal to (FSD 2016, SWL 2013, BWP 2014, RWP 2013, 17) (U.B)

(a) $(a-b)^2$ (b) $(a+b)^2$
(c) $a+b$ (d) $a-b$

(viii) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$ is equal to (U.B)

(a) $a^2+b^2$ (b) $a^2-b^2$
(c) $a-b$ (d) $a+b$

## ANSWER KEY

| i | ii | iii | iv | v | vi | vii | viii |
|---|---|---|---|---|---|---|---|
| a | d | b | a | b | b | d | c |

**Q.2 Fill in the blanks**

(i) The degree of polynomial $x^2y^2+3xy+y^3$ is __________ (U.B)

(ii) $x^2-4$__________ (U.B)

(iii) $x^3+\frac{1}{x^3}=\left[x+\frac{1}{x}\right]($__________$)$ (U.B)

(iv) $2\left(a^2+b^2\right)=(a+b)^2+($__________$)^2$ (U.B)

(v) $\left[x-\frac{1}{x}\right]^2=$ __________ (U.B)

(vi) Order of surd $\sqrt[3]{x}$ is __________ (U.B)

(vii) $\frac{1}{2-\sqrt{3}}=$ ______________ (U.B)

## ANSWER KEY

(i) 4

(ii) $(x-2)(x+2)$

(iii) $x^2-1+\frac{1}{x^2}$

(iv) $a-b$

(v) $x^2+\frac{1}{x^2}-2$

(vi) 3

(vii) $2+\sqrt{3}$

**Q.3 If $x+\frac{1}{x}=3$, find** **(A.B)**

**(i)** $x^2+\frac{1}{x^2}$ **(ii)** $x^4+\frac{1}{x^4}$

**Solution:**

**Given that** $x+\frac{1}{x}=3$

Squaring both sides

$$\left(x+\frac{1}{x}\right)^2=3^2$$

$$\because (a+b)^2=a^2+b^2+2ab$$

$$(x)^2+\left(\frac{1}{x}\right)^2+2(x)\left(\frac{1}{x}\right)=9$$

$$x^2+\frac{1}{x^2}+2=9$$

$$x^2+\frac{1}{x^2}=9-2$$

$$x^2+\frac{1}{x^2}=7$$

**(ii) For** $x^4+\frac{1}{x^4}$

**Here** $x^2+\frac{1}{x^2}=7$

Squaring both sides

$$\left(x^2+\frac{1}{x^2}\right)^2=7^2$$

$$x^4+\frac{1}{x^4}+2=49$$

$$x^4+\frac{1}{x^4}=49-2$$

$$x^4+\frac{1}{x^4}=47$$

**Q.4 If $x-\frac{1}{x}=2$ find**

(i) $x^2+\frac{1}{x^2}$ (ii) $x^4+\frac{1}{x^4}$ **(A.B)**

**Solution (i)**

Given that $x-\frac{1}{x}=2$

Squaring both sides

$$\left(x-\frac{1}{x}\right)^2=2^2$$

$$\because (a+b)^2=a^2+b^2+2ab$$

Putting the values

$$\left(x-\frac{1}{x}\right)^2=(x)^2+\left(\frac{1}{x}\right)^2-2(x)\left(\frac{1}{x}\right)$$

$$(2)^2=x^2+\frac{1}{x^2}-2$$

$$4+2=x^2+\frac{1}{x^2}$$

$$x^2+\frac{1}{x^2}=6$$

**Solution (ii)**

Here $x^2+\frac{1}{x^2}=6$

Again squaring both sides

$$\left(x^2+\frac{1}{x^2}\right)^2=6^2$$

$$x^4+\frac{1}{x^4}+2(x^2)\left(\frac{1}{x^2}\right)=36$$

$$x^4+\frac{1}{x^4}+2=36$$

$$x^4+\frac{1}{x^4}=36-2$$

$$x^4+\frac{1}{x^4}=34$$

**Q.5 Find the value of $x^3+y^3$ and $xy$ if $x+y=5$ and $x-y=3$.** **(A.B)**

**Solution:**

Formula

$$(x+y)^2-(x-y)^2=4xy$$

Putting the values

$$(5)^2-(3)^2=4xy$$

$$4xy=25-9$$

$$4xy=16$$

$xy = \frac{16^{4}}{4}$

$xy = 4$

As we know that

$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

Putting the values

$(5)^3 = x^3 + y^3 + 3\times 4\times 5$

$125 = x^3 + y^3 + 60$

$x^3 + y^3 = 125 - 60$

$x^3 + y^3 = 65$

**Q.6 If $P = 2+\sqrt{3}$ , find (A.B)**

**(i)** $P + \frac{1}{P}$

**Solution:**

Given that $P = 2+\sqrt{3}$

$\frac{1}{P} = \frac{1}{2+\sqrt{3}}$

Rationalizing the denominator

$\frac{1}{P} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$\frac{1}{P} = \frac{2-\sqrt{3}}{(2)^2 - (\sqrt{3})^2}$ $\because (a+b)(a-b) = a^2 - b^2$

$\frac{1}{P} = \frac{2-\sqrt{3}}{4-3}$

$\frac{1}{P} = \frac{2-\sqrt{3}}{1}$

$\frac{1}{P} = 2-\sqrt{3}$

$P + \frac{1}{P} = 2 + \sqrt{3} + 2 - \sqrt{3}$

$P + \frac{1}{P} = 4$

**(ii) For** $P - \frac{1}{P}$

$P - \frac{1}{P} = (2+\sqrt{3}) - (2-\sqrt{3})$

$= 2 + \sqrt{3} - 2 + \sqrt{3}$

$= 2\sqrt{3}$

**(iii) For** $P^2 + \frac{1}{P^2}$

**Here** $P + \frac{1}{P} = 4$

Squaring both sides

$\left(P + \frac{1}{P}\right)^2 = (4)^2$

$\therefore (a+b)^2 = a^2 + b^2 + 2ab$

$(P)^2 + \left(\frac{1}{P}\right)^2 + 2(P)\left(\frac{1}{P}\right) = 16$

$P^2 + \frac{1}{P^2} + 2 = 16$

$P^2 + \frac{1}{P^2} = 16 - 2$

$P^2 + \frac{1}{P^2} = 14$

**(iv) For** $P^2 - \frac{1}{P^2}$

**Formula**

$P^2 - \frac{1}{P^2} = \left(P + \frac{1}{P}\right)\left(P - \frac{1}{P}\right)$

Putting the values

$P^2 - \frac{1}{P^2} = (4)(2\sqrt{3})$

$= 8\sqrt{3}$

**Q.7 If $q = \sqrt{5}+2$ find. (A.B)**

**(i)** $q + \frac{1}{q}$ **(ii)** $q - \frac{1}{q}$

**(iii)** $q^2 + \frac{1}{q^2}$ **(iv)** $q^2 - \frac{1}{q^2}$

**Solution:**

**(i) For** $q + \frac{1}{q}$

Given that $q = \sqrt{5}+2$

$\frac{1}{q} = \frac{1}{\sqrt{5}+2}$

Rationalizing the denominator

$$\frac{1}{q}=\frac{1}{\sqrt{5}+2}\times\frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$=\frac{\sqrt{5}-2}{\left(\sqrt{5}\right)^2-(2)^2}\quad \because (a+b)(a-b)=a^2-b^2$$

$$=\frac{\sqrt{5}-2}{5-4}$$

$$=\sqrt{5}-2$$

Now

$$q+\frac{1}{q}=\left(\sqrt{5}+2\right)+\left(\sqrt{5}-2\right)$$

$$q+\frac{1}{q}=\sqrt{5}+\cancel{2}+\sqrt{5}-\cancel{2}$$

$$q+\frac{1}{q}=2\sqrt{5}$$

**(ii) For** $q-\frac{1}{q}$

$$q-\frac{1}{q}=\left(\sqrt{5}+2\right)-\left(\sqrt{5}-2\right)$$

$$=\cancel{\sqrt{5}}+2-\cancel{\sqrt{5}}+2$$

$$q-\frac{1}{q}=4$$

**(iii) For** $q^2+\frac{1}{q^2}$

$$q-\frac{1}{q}=4$$

Squaring both sides

$$\left(q-\frac{1}{q}\right)^2=(4)^2$$

$$q^2+\frac{1}{q^2}-2=16$$

$$q^2+\frac{1}{q^2}=16+2$$

$$q^2+\frac{1}{q^2}=18$$

**(iv) For** $q^2-\frac{1}{q^2}$ **(A.B)**

By using formula

$$q^2-\frac{1}{q^2}=\left(q+\frac{1}{q}\right)\left(q-\frac{1}{q}\right)$$

Putting the values

$$=\left(2\sqrt{5}\right)(4)$$

$$=8\sqrt{5}$$

**Q.8 Simplify (A.B)**

**(i)** $\dfrac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$

**Solution:**

$$\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$$

Rationalizing the denominator

$$=\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}\times\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$

$$=\frac{\left(\sqrt{a^2+2}+\sqrt{a^2-2}\right)^2}{\left(\sqrt{a^2+2}\right)^2-\left(\sqrt{a^2-2}\right)^2}$$

$$\because (a+b)^2=a^2+2ab+b^2,\ (a+b)(a-b)=a^2-b^2$$

$$=\frac{\left(\sqrt{a^2+2}\right)^2+\left(\sqrt{a^2-2}\right)^2+2\left(\sqrt{a^2+2}\right)\left(\sqrt{a^2-2}\right)}{\left(a^2+2\right)-\left(a^2-2\right)}$$

$$=\frac{a^2+\cancel{2}+a^2-\cancel{2}+2\left(\sqrt{\left(a^2+2\right)\left(a^2-2\right)}\right)}{a^2+2-a^2+2}$$

$$=\frac{2a^2+2\sqrt{a^4-4}}{4}$$

$$=\frac{\cancel{2}\left(a^2+\sqrt{a^4-4}\right)}{\cancel{4}\,2}$$

$$=\frac{a^2+\sqrt{a^4-4}}{2}$$

**(ii)** $\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}$ **(A.B)**

**Solution:**

$$\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}$$

$$=\frac{1\left(a+\sqrt{a^2-x^2}\right)-1\left(a-\sqrt{a^2-x^2}\right)}{\left(a-\sqrt{a^2-x^2}\right)\left(a+\sqrt{a^2-x^2}\right)}$$

$$\because (a+b)(a-b)=a^2-b^2$$

$$=\frac{a+\sqrt{a^2-x^2}-a+\sqrt{a^2-x^2}}{a^2-\left(\sqrt{a^2-x^2}\right)^2}$$

$$=\frac{2\sqrt{a^2-x^2}}{a^2-\left(a^2-x^2\right)}$$

$$=\frac{2\sqrt{a^2-x^2}}{a^2-a^2+x^2}$$

$$=\frac{2\sqrt{a^2-x^2}}{x^2}$$

## SELF TEST

**Time: 40 min** **Marks: 25**

**Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer.** **(7×1=7)**

**1 Polynomial means an expression with _________ terms**

**(A)** Two **(B)** Four
**(C)** Many **(D)** No term

**2 The degree of** $2x^4y^3 + x^2y^2 + 8x$ **is**

**(A)** 4 **(B)** 3
**(C)** 7 **(D)** 8

**3 The value of** $\dfrac{3x^2\sqrt{y}}{5(x+y)} + 6$**, if** $x = -4,\ y = 9$ **is:**

**(A)** 12 **(B)** $-6$
**(C)** 9 **(D)** 6

**4 In** $\sqrt[7]{5}$ **the order of surd is**

**(A)** 5 **(B)** $\dfrac{1}{2}$
**(C)** 7 **(D)** $\dfrac{1}{7}$

**5** $\dfrac{7xy}{x^2 - 4x + 4} \div \dfrac{14y}{x^2 - 4} = ?$

**(A)** $\dfrac{x(x+2)}{2(x-2)}$ **(B)** $\dfrac{2(x-2)}{x(x+2)}$
**(C)** $\dfrac{x-2}{x+2}$ **(D)** None of these

**6 If** $x + \dfrac{1}{x} = 5$ **find** $\left(x + \dfrac{1}{x}\right)^2 =$ _______

**(A)** 23 **(B)** 27
**(C)** 25 **(D)** 5

**7** $\dfrac{1}{a-b} - \dfrac{1}{a+b}$ **is equal to:**

**(A)** $\dfrac{2b}{a^2 - b^2}$ **(B)** $\dfrac{-2b}{a^2 - b^2}$
**(C)** $\dfrac{-2a}{a^2 - b^2}$ **(D)** $\dfrac{2a}{a^2 - b^2}$

**Q.2 Give Short Answers to following Questions.** **(5×2=10)**

**(i)** Simplify: $4\sqrt{3}-3\sqrt{27}+2\sqrt{75}$

**(ii)** If $x=2+\sqrt{3}$ find the value of $x-\frac{1}{x}$.

**(iii)** Simplify: $\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$

**(iv)** Simplify: $\frac{9x^2-\left(x^2-4^2\right)}{4+3x-x^2}$

**(v)** Simplify: $\sqrt[5]{243x^5y^{10}z^{15}}$

**Q.3 Answer the following Questions.** **(4+4=8)**

**(a)** If $5x-6y=13$ and $xy=6$, then find the value of $125x^3-216y^3$

**(b)** If $\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$, determine the rational numbers *a* and *b*.

**NOTE:** Parents or guardians can conduct this test in their supervision in order to check the skill of students.