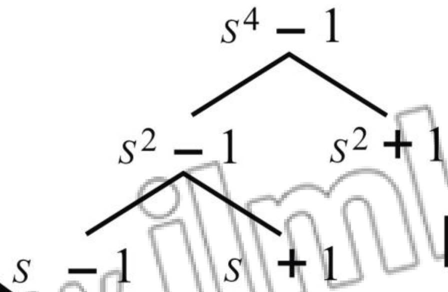

UNIT 5
FACTORIZATION

Factorization (U.B)

If a polynomial $p(x)$ can be expressed as $p(x)=g(x)h(x)$, then each of the polynomial, $g(x)$ and $h(x)$ is called a factor of $p(x)$.

For example:

$ab+ac=a(b+c)$, then a and $(b+c)$ are factors of $(ab+ac)$.

Note (K.B)

When a polynomial has been written as a product consisting only of prime factors, then it is said to be factored completely.

Important role of Factorization in Mathematics

Factorization plays an important role in mathematics as it helps to reduce the study of a complicated expression to the study of simpler expressions.

(a) Factorization of the Expression of the Type $Ka+Kb+Kc$

Example # 1 (K.B)

Factorize $5a-5b+5c$

Solution: (A.B)

$$5a-5b+5c=5(a-b+c)$$

Example # 2

Factorize

$$5a-5b-15c=5(a-b-3c)$$

(b) Factorization of the Expression of the Type $ac+ad+bc+bd$

Example (K.B)

Factorize: $ac+ad+bc+bd$

Solution:

$$ac+ad+bc+bd$$
$$=(ac+ad)+(bc+db)$$
$$=a(c+d)+b(c+d)$$
$$=(c+d)(a+b)$$

Example 2 (Page # 99)

Factorize: $pqr+qr^2-pr^2-r^3$

Solution: (A.B)

$$pqr+qr^2-pr^2-r^3$$
$$=r\left(pq+qr-pr-r^2\right)$$
$$=r\left[q(p+r)-r(p+r)\right]$$
$$=r(p+r)(q-r)$$

(c) Factorization of the Expression of Type $a^2\pm 2ab+b^2$ (K.B)

We know that

(i) $a^2+2ab+b^2=(a+b)^2=(a+b)(a+b)$

(ii) $a^2-2ab+b^2=(a-b)^2=(a-b)(a-b)$

Example # 1

Factorize: $25x^2+40x+16$

Solution: (A.B)

$$25x^2+40x+16$$
$$=(5x)^2+2(5x)(4)+(4)^2$$
$$\because a^2+2ab+b^2=(a+b)^2$$
$$=(5x+4)^2$$
$$=(5x+4)(5x+4)$$

Example # 2 **(A.B)**

Factorize: $12x^2-36x+27$

Solution:

$$12x^2-36x+27=3\left[4x^2-12x+9\right]$$
$$=3\left[(2x)^2-2(2x)(3)+(3)^2\right]$$
$$\because\ a^2-2ab+b^2=(a-b)^2$$
$$=3[2x-3]^2$$
$$=3(2x-3)(2x-3)$$

(d) Factorization of the Expression of the Type a^2-b^2

Example **(K.B)**

Factorize: (i) $4x^2-(2y-z)^2$

Solution:

(i) $4x^2-(2y-z)^2=(2x)^2-(2y-z)^2$
$$\because\ a^2-b^2=(a+b)(a-b)$$
$$=\left[2x-(2y-z)\right]\left[2x+(2y-z)\right]$$
$$=(2x-2y+z)(2x+2y-z)$$

(ii) Factorize: $6x^4-96$

Solution:

$$6x^4-96=6\left(x^4-16\right)$$
$$=6\left[\left(x^2\right)^2-4^2\right]$$
$$\because\ a^2-b^2=(a+b)(a-b)$$
$$=6\left[x^2-4\right]\left[x^2+4\right]$$
$$=6\left[x^2-2^2\right]\left[x^2+4\right]$$
$$\because\ a^2-b^2=(a+b)(a-b)$$
$$=6(x-2)(x+2)\left(x^2+4\right)$$

(e) Factorization of the Expression of the Types $a^2\pm2ab+b^2-c^2$

We know that **(K.B)**

$$a^2\pm2ab+b^2-c^2$$
$$=(a\pm b)^2-c^2=(a\pm b-c)(a\pm b+c)$$

Example **(RWP 2016)** **(A.B)**

Factorize:

(i) $x^2+6x+9-4y^2$

(ii) $1+2ab-a^2-b^2$

Solution:

(i) $x^2+6x+9-4y^2=x^2+2(x)(3)+3^2-4y^2$
$$=(x+3)^2-(2y)^2$$
$$\because\ a^2+2ab+b^2=(a+b)^2$$
$$\because\ a^2-b^2=(a+b)(a-b)$$
$$=[x+3-2y][x+3+2y]$$

(ii) $1+2ab-a^2-b^2=1-\left(a^2-2ab+b^2\right)$

(GRW 2016, RWP 2017)

$$=(1)^2-(a-b)^2$$
$$\because\ a^2-b^2=(a+b)(a-b)$$
$$=\left[1-(a-b)\right]\left[1+(a-b)\right]$$
$$=[1-a+b][1+a-b]$$

Exercise 5.1

Q.1 Factorize: **(K.B)**

(i) $2abc-4abx+2abd$

Solution:

$$2abc-4abx+2abd$$
$$=2ab(c-2x+d)$$

(ii) $9xy-12x^2y+18y^2$

Solution:

$$9xy-12x^2y+18y^2$$
$$=3y\left(3x-4x^2+6y\right)$$

(iii) $-3x^2y-3x+9xy^2$

Solution:

$$-3x^2y-3x+9xy^2$$
$$=-3x\left(xy+1-3y^2\right)$$

(iv) $5ab^2c^3-10a^2b^3c-20a^3bc^2$

Solution:

$$5ab^2c^3-10a^2b^3c-20a^3bc^2$$
$$=5abc\left(bc^2-2ab^2-4a^2c\right)$$

(v) $3x^3y(x-3y)-7x^2y^2(x-3y)$

Solution:

$$3x^3y(x-3y)-7x^2y^2(x-3y)$$
$$=(x-3y)(3x^3y-7x^2y^2)$$
$$=(x-3y)\ x^2y(3x-7y)$$
$$=x^2y(x-3y)(3x-7y)$$

(vi) $2xy^3(x^2+5)+8xy^2(x^2+5)$

Solution:

$$2xy^3(x^2+5)+8xy^2(x^2+5)$$
$$=(x^2+5)(2xy^3+8xy^2)$$
$$=(x^2+5)2xy^2(y+4)$$
$$=2xy^2(x^2+5)(y+4)$$

Q.2 Factorize **(K.B)**

(i) $5ax-3ay-5bx+3by$

Solution:

$$5ax-3ay-5bx+3by$$
$$=5ax-5bx-3ay+3by$$
$$=5x(a-b)-3y(a-b)$$
$$=(a-b)(5x-3y)$$

(ii) $3xy+2y-12x-8$

Solution:

$$3xy+2y-12x-8$$
$$=3xy-12x+2y-8$$
$$=3x(y-4)+2(y-4)$$
$$=(y-4)(3x+2)$$

(iii) $x^3+3xy^2-2x^2y-6y^3$

Solution:

$$x^3+3xy^2-2x^2y-6y^3$$
$$=x^3-2x^2y+3xy^2-6y^3$$
$$=x^2(x-2y)+3y^2(x-2y)$$
$$=(x-2y)(x^2+3y^2)$$

(iv) $(x^2-y^2)z+(y^2-z^2)x$

Solution:

$$(x^2-y^2)z+(y^2-z^2)x$$
$$=x^2z-y^2z+xy^2-xz^2$$
$$=x^2z+xy^2-xz^2-y^2z$$
$$=x^2z+xy^2-y^2z-xz^2$$
$$=x(xz+y^2)-z(xz+y^2)$$
$$=(xz+y^2)(x-z)$$

Q.3 Factorize

(i) $144a^2+24a+1$

Solution: **(K.B)**

$$144a^2+24a+1$$
$$=(12a)^2+2(12a)(1)+(1)^2$$
$$\because a^2+2ab+b^2=(a+b)^2$$
$$=(12a+1)^2$$

(ii) $\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$

(FSD 2014, MTN 2016, D.G.K 2017)

Solution:

$$\frac{a^2}{b^2}-2+\frac{b^2}{a^2}$$
$$=\left(\frac{a}{b}\right)^2-2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right)+\left(\frac{b}{a}\right)^2$$
$$\because a^2-2ab+b^2=(a-b)^2$$
$$=\left(\frac{a}{b}-\frac{b}{a}\right)^2$$

(iii) $(x+y)^2-14z(x+y)+49z^2$

Solution:

$$(x+y)^2-14z(x+y)+49z^2$$
$$\because a^2-2ab+b^2=(a-b)^2$$
$$=(x+y)^2-2(x+y)(7z)+(7z)^2$$
$$=(x+y-7z)^2$$

(iv) $12x^2-36x+27$

(SWL 2017, BWP 2016, FSD 2016)

Solution:

$$12x^2-36x+27$$
$$=3(4x^2-12x+9)$$

$\because a^2-2ab+b^2=(a-b)^2$

$=3\left[(2x)^2-2(2x)(3)+(3)^2\right]$

$=3(2x-3)^2$

Q.4 Factorize

(i) $3x^2-75y^2$ **(K.B+U.B)**

(LHR 2017, GRW 2014, BWP 2014)

Solution:

$3x^2-75y^2$

$=3(x^2-25y^2)$

$\because a^2-b^2=(a+b)(a-b)$

$=3\left[(x)^2-(5y)^2\right]$

$=3(x+5y)(x-5y)$

(ii) $x(x-1)-y(y-1)$ **(SGD 2015)**

Solution:

$x(x-1)-y(y-1)$

$=x^2-x-y^2+y$

$=x^2-y^2-x+y$

$=(x^2-y^2)-(x-y)$

$\because a^2-b^2=(a+b)(a-b)$

$=\left[(x+y)(x-y)\right]-(x-y)$

$=(x-y)(x+y-1)$

(iii) $128am^2-242an^2$

(MTN 2017, BWP 2014, D.G.K 2014)

Solution:

$128am^2-242an^2$

$=2a(64m^2-121n^2)$

$=2a\left[(8m)^2-(11n)^2\right]$

$\because a^2-b^2=(a+b)(a-b)$

$=2a(8m+11n)(8m-11n)$

(iv) $3x-243x^3$ **(MTN 2017, FSD 2017)**

Solution:

$3x-243x^3$

$=3x(1-81x^2)$

$=3x\left[(1)^2-(9x)^2\right]$

$\because a^2-b^2=(a+b)(a-b)$

$=3x(1+9x)(1-9x)$

Q.5 Factorize

(i) x^2-y^2-6y-9

Solution:

x^2-y^2-6y-9

$=x^2-\left[y^2+6y+9\right]$

$=x^2-[(y)^2+2(y)(3)+(3)^2]$

$\because (a+b)^2=a^2+2ab+b^2$

$=x^2-(y+3)^2$

$\because a^2-b^2=(a+b)(a-b)$

$=(x+y+3)[x-(y+3)]$

$=(x+y+3)(x-y-3)$

(ii) x^2-a^2+2a-1 **(GRW 2016)**

Solution:

x^2-a^2+2a-1

$=x^2-\left[a^2-2a+1\right]$

$\because a^2-2ab+b^2=(a-b)^2$

$=x^2-(a-1)^2$

$\because a^2-b^2=(a+b)(a-b)$

$=\left[x+(a-1)\right]\left[x-(a-1)\right]$

$=(x+a-1)(x-a+1)$

(iii) $4x^2-y^2-2y-1$

Solution:

$4x^2-y^2-2y-1$

$=4x^2-(y^2+2y+1)$

$=4x^2-\left[(y)^2+2(y)(1)+(1)^2\right]$

$\because (a+b)^2=a^2+2ab+b^2$

$= 4x^2 - (y+1)^2$

$= (2x)^2 - (y+1)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= [2x + (y+1)][2x - (y+1)]$

$= (2x+y+1)(2x-y-1)$

(iv) $x^2 - y^2 - 4x - 2y + 3$ **(LHR 2016)**

Solution:

$x^2 - y^2 - 4x - 2y + 3$

$= x^2 - 4x + 4 - y^2 - 2y - 1$

$= (x^2 - 4x + 4) - (y^2 + 2y + 1)$

$= [(x)^2 - 2(x)(2) + (2)^2]$

$- [(y)^2 + 2(y)(1) + (1)^2]$

$\because a^2 - 2ab + b^2 = (a-b)^2$

$\because a^2 + 2ab + b^2 = (a+b)^2$

$= (x-2)^2 - (y+1)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= (x - 2 + y + 1)[x - 2 - (y+1)]$

$= (x - 2 + y + 1)(x - 2 - y - 1)$

$= (x + y - 2 + 1)(x - y - 2 - 1)$

$= (x + y - 1)(x - y - 3)$

(v) $25x^2 - 10x + 1 - 36z^2$ **(GRW 2016)**

Solution:

$25x^2 - 10x + 1 - 36z^2$

$= (5x)^2 - 2(5x)(1) + (1)^2] - 36z^2$

$\because a^2 - 2ab + b^2 = (a-b)^2$

$= (5x-1)^2 - (6z)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= [(5x-1) + 6Z][(5x-1) - 6Z]$

$= (5x - 1 + 6z)(5x - 1 - 6z)$

(vi) $x^2 - y^2 - 4xz + 4z^2$

Solution:

$x^2 - y^2 - 4xz + 4z^2$

$= x^2 - 4xz + 4z^2 - y^2$

$= [(x)^2 - 2(x)(2z) + (2z)^2] - y^2$

$\because a^2 - 2ab + b^2 = (a-b)^2$

$= (x-2z)^2 - (y)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= (x - 2z + y)(x - 2z - y)$

$= (x + y - 2z)(x - y - 2z)$

Factorization of the Expression of the Types

$a^4 + a^2b^2 + b^4$ or $a^4 + 4b^4$ **(K.B)**

Example # 1 **(A.B)**

Factorize: $81x^4 + 36x^2y^2 + 16y^4$

Solution:

$81x^4 + 36x^2y^2 + 16y^4$

$= (9x^2)^2 + 72x^2y^2 + (4y^2)^2 - 36x^2y^2$

$\because (a+b)^2 = a^2 + 2ab + b^2$

$= (9x^2 + 4y^2)^2 - (6xy)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= [9x^2 + 4y^2 + 6xy][9x^2 - 6xy + 4y^2]$

Example # 2 **(A.B)**

Factorize: $9x^4 + 3y^4$

Solution:

$9x^4 + 36y^4$

$= 9x^4 + 36y^4 + 36x^2y^2 - 36x^2y^2$

$= (3x^2)^2 + 2(3x^2)(6y^2) + (6y^2)^2 - (6xy)^2$

$\because (a+b)^2 = a^2 + 2ab + b^2$

$= [3x^2 + 6y^2]^2 - (6xy)^2$

$\because a^2 - b^2 = (a+b)(a-b)$

$= [3x^2 + 6y^2 - 6xy][3x^2 + 6y^2 + 6xy]$

$= [3x^2 - 6xy + 6y^2][3x^2 + 6xy + 6y^2]$

Factorization of the Expression of the Type $x^2 + px + q$ **(K.B)**

Example

Factorize: **(A.B)**

(i) $x^2-7x+12$

(ii) $x^2+5x-36$

(i) $x^2-7x+12$

$\because (-3)+(-4)=-7$ and $(-3)(-4)=12$

Hence

$x^2-7x+12=x^2-3x-4x+12$

$=x(x-3)-4(x-3)$

$=(x-3)(x-4)$

(ii) $x^2+5x-36$

$\because 9+(-4)=5$ and $9\times(-4)=-36$

Hence

$x^2+5x-36=x^2+9x-4x-36$

$=x(x+9)-4(x+9)$

$=(x+9)(x-4)$

Factorization of the Expression of the Type $ax^2+bx+c,\ a\neq 0$:

Example

Factorize: $9x^2+21x-8$

Solution:

$\because ac=(9)(-8)=-72$ and

$24+(-3)=21$

Hence

$9x^2+21x-8$

$=9x^2+24x-3x-8$

$=3x(3x+8)-(3x+8)$

$=(3x+8)(3x-1)$

Factorization of the Expressions of the Types **(K.B)**

$(ax^2+bx+c)(ax^2+bx+d)+k$

$(x+a)(x+b)(x+c)(x+d)+k$

$(x+a)(x+b)(x+c)(x+d)+kx^2$

Example # 1 **(A.B)**

Factorize:

$(x^2-4x-5)(x^2-4x-12)-144$

Solution:

$(x^2-4x-5)(x^2-4x-12)-144$

Let $y=x^2-4x$

Then

$(x^2-4x-5)(x^2-4x-12)-144$

$=(y-5)(y-12)-144$

$=y^2-17y-84$

$=y^2-21y+4y-84$

$=y(y-21)+4(y-21)$

$=(y-21)(y+4)$

Putting the value of y

$=(x^2-4x-21)(x^2-4x+4)$

$=(x^2-7x+3x-21)(x^2-2(x)(2)+2^2)$

$\because a^2-2ab+b^2=(a-b)^2$

$=[x(x-7)+3(x-7)][x-2]^2$

$=(x-7)(x+3)(x-2)(x-2)$

Example # 2 **(A.B)**

Factorize:

$(x+1)(x+2)(x+3)(x+4)-120$

Solution:

We observe that $1+4=2+3$.

$\therefore (x+1)(x+2)(x+3)(x+4)-120$

$=[(x+1)(x+4)][(x+2)(x+3)]-120$

$=(x^2+5x+4)(x^2+5x+6)-120$

Put $x^2+5x=y$

$=(y+4)(y+6)-120$

$=y^2+10y+24-120$

$=y^2+10y-96$

$=y^2+16y-6y-96$

$=y(y+16)-6(y+16)$

$=(y+16)(y-6)$

Putting the value of y

$=(x^2+5x+16)(x^2+5x-6)$

$=(x^2+5x+16)(x^2+6x-x-6)$

$=(x^2+5x+16)(x(x+6)-1(x-6))$

$=(x^2+5x+16)(x+6)(x-1)$

Example # 3 **(A.B)**

Factorize: $\left(x^2-5x+6\right)\left(x^2+5x+6\right)-2x^2$

Solution:

$\left(x^2-5x+6\right)\left(x^2+5x+6\right)-2x^2$

$=\left[x^2-3x-2x+6\right]\left[x^2+3x+2x+6\right]-2x^2$

$=\left[x(x-3)-2(x-3)\right]\left[x(x+3)+2(x+3)\right]-2x^2$

$=(x-3)(x-2)(x+3)(x+2)-2x^2$

$=(x-3)(x+3)(x-2)(x+2)-2x^2$

$=\left(x^2-9\right)\left(x^2-4\right)-2x^2$

$=x^4-13x^2+36-2x^2$

$=x^4-15x^2+36$

$=x^4-12x^2-3x^2+36$

$=x^2\left(x^2-12\right)-3\left(x^2-12\right)$

$=\left(x^2-12\right)\left(x^2-3\right)$

$=\left[(x)^2-2\left(\sqrt{3}\right)^2\right]\left[(x)^2-\left(\sqrt{3}\right)^2\right]$

$\because a^2-b^2=(a+b)(a-b)$

$=\left[x-2\sqrt{3}\right]\left[x+2\sqrt{3}\right]\left[x-\sqrt{3}\right]\left[x+\sqrt{3}\right]$

Factorization of Expressions of the Types $a^3+3a^2b+3ab^2+b^3$ **(K.B)**

$a^3-3a^2b+3ab^2-b^3$

Example

Factorize: $x^3-8y^3-6x^2y+12xy^2$

Solution: ***(A.B)***

$x^3-8y^3-6x^2y+12xy^2$

$=x^3-(2y)^3-3(x)^2(2y)+3(x)\left(2y^2\right)$

$=x^3-3(x)^2(2y)+3(x)(2y)^2-(2y)^3$

$\because a^3-3a^2b+3ab^2-b^3=(a-b)^3$

$=(x-2y)^3$

$=(x-2y)(x-2y)(x-2y)$

Factorization of Expressions of the Types $a^3\pm b^3$

We recall the formulas.

$a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$

$a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$

Example # 1

Factorize: $27x^3+64y^3$

Solution:

$27x^3+64y^3=(3x)^3+(14y)^3$

$\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$

$=[3x+4y]\left[(3x)^2-(3x)(4y)+(4y)^2\right]$

$=[3x+4y]\left[9x^2-12xy+16y^2\right]$

Exercise 5.2

Q.1 Factorize

(i) $x^4+\dfrac{1}{x^4}-3$

Solution: **(A.B)**

$x^4+\dfrac{1}{x^4}-3$

$=\left(x^2\right)^2+\left(\dfrac{1}{x^2}\right)^2-3$

$=\left[(x^2)^2+\left(\dfrac{1}{x^2}\right)^2-2\right]-1$

$\because a^2-2ab+b^2=(a-b)^2$

$=\left(x^2-\dfrac{1}{x^2}\right)^2-(1)^2$

$\because a^2-b^2=(a+b)(a-b)$

$=\left(x^2-\dfrac{1}{x^2}+1\right)\left(x^2-\dfrac{1}{x^2}-1\right)$

(ii) $3x^4+12y^4$

Solution:

$3x^4+12y^4$

$=3\left(x^4+4y^4\right)$

By adding and subtracting by $2(x^2)(2y^2)$

$=3\left[\left(x^2\right)^2+\left(2y^2\right)^2+2\left(x^2\right)\left(2y^2\right)-2\left(x^2\right)\left(2y^2\right)\right]$

$\because a^2+2ab+b^2=(a+b)^2$

$= 3\left[\left(x^2+2y^2\right)^2 - 4x^2y^2\right]$

$= 3\left[\left(x^2+2y^2\right)^2 - (2xy)^2\right]$

$\because a^2 - b^2 = (a+b)(a-b)$

$= 3\left[\left(x^2+2y^2+2xy\right)\left(x^2+2y^2-2xy\right)\right]$

$= 3\left[\left(x^2+2xy+2y^2\right)\left(x^2-2xy+2y^2\right)\right]$

(iii) $a^4+3a^2b^2+4b^4$

Solution:

$a^4+3a^2b^2+4b^4$

$=\left(a^4+4b^4\right)+3a^2b^2$

$=(a^2)^2+(2b^2)^2+3a^2b^2$

By adding and subtracting by $2\left(a^2\right)\left(2b^2\right)$

$=\left(a^2\right)^2+\left(2b^2\right)^2+2\left(a^2\right)\left(2b^2\right)-2\left(a^2\right)\left(2b^2\right)+3a^2b^2$

$=\left[\left(a^2\right)^2+\left(2b^2\right)^2+2\left(a^2\right)\left(2b^2\right)\right]-2\left(a^2\right)\left(2b^2\right)+3a^2b^2$

$\because a^2+2ab+b^2=\left(a+b\right)^2$

$=(a^2+2b^2)^2-a^2b^2$

$=(a^2+2b^2)^2-(ab)^2$

$\because a^2-b^2=(a+b)(a-b)$

$=(a^2+2b^2+ab)(a^2+2b^2-ab)$

(iv) $4x^4+81$

Solution: **(A.B)**

$4x^4+81$

$=\left(2x^2\right)^2+\left(9\right)^2$

By adding and subtracting by $2\left(2x^2\right)\left(9\right)$

$=\left[\left(2x^2\right)^2+\left(9\right)^2+2\left(2x^2\right)\left(9\right)-2\left(2x^2\right)\left(9\right)\right]$

$=\left[\left(2x^2\right)^2+\left(9\right)^2+2\left(2x^2\right)\left(9\right)\right]-2\left(2x^2\right)\left(9\right)$

$\because a^2+2ab+b^2=\left(a+b\right)^2$

$=\left(2x^2+9\right)^2-36x^2$

$=\left(2x^2+9\right)^2-\left(6x\right)^2$

$\because a^2-b^2=(a+b)(a-b)$

$=\left(2x^2+9+6x\right)\left(2x^2+9-6x\right)$

$=\left(2x^2+6x+9\right)\left(2x^2-6x+9\right)$

(v) x^4+x^2+25 **(MTN 2016)**

Solution:

x^4+x^2+25

$=\left(x^4+25\right)+x^2$

$=\left[\left(x^2\right)^2+\left(5\right)^2\right]+x^2$

By adding and subtracting by $2\left(x^2\right)\left(5\right)$

$=\left[\left(x^2\right)^2+\left(5\right)^2+2\left(x^2\right)\left(5\right)-2\left(x^2\right)\left(5\right)\right]+x^2$

$=\left[\left(x^2\right)^2+\left(5\right)^2+2\left(x^2\right)\left(5\right)\right]-2\left(x^2\right)\left(5\right)+x^2$

$\because a^2+2ab+b^2=\left(a+b\right)^2$

$=\left(x^2+5\right)^2-10x^2+x^2$

$=(x^2+5)^2-9x^2$

$=(x^2+5)^2-(3x)^2$

$\because a^2-b^2=(a+b)(a-b)$

$=\left(x^2+5+3x\right)\left(x^2+5-3x\right)$

$=(x^2+3x+5)(x^2-3x+5)$

(vi) x^4+4x^2+16

Solution:

x^4+4x^2+16

$=(x^2)^2+16+4x^2$

$=(x^2)^2+(4)^2+4x^2$

By adding and subtracting by $2\left(x^2\right)\left(4\right)$

$=\left(x^2\right)^2+\left(4\right)^2+2\left(x^2\right)\left(4\right)-2\left(x^2\right)\left(4\right)+4x^2$

$=(x^2)^2+(4)^2+2(x^2)(4)-2(x^2)(4)+4x^2$

$\because a^2+2ab+b^2=\left(a+b\right)^2$

$=\left(x^2+4\right)^2-8x^2+4x^2$

$=(x^2+4)^2-4x^2$

$=(x^2+4)^2-(2x)^2$

$\because a^2-b^2=(a+b)(a-b)$

$=\left(x^2+4+2x\right)\left(x^2+4-2x\right)$

$=\left(x^2+2x+4\right)\left(x^2-2x+4\right)$

Q.2

(i) $x^2+14x+48$

Solution:

$x^2+14x+48$
$= x^2+8x+6x+48$
$= x(x+8)+6(x+8)$
$= (x+8)(x+6)$

(ii) $x^2-21x+108$

Solution:

$x^2-21x+108$
$= x^2-12x-9x+108$
$= x(x-12)-9(x-12)$
$= (x-9)(x-12)$

(iii) $x^2-11x-42$

Solution:

$x^2-11x-42$
$= x^2-14x+3x-42$
$= x(x-14)+3(x-14)$
$= (x+3)(x-14)$

(iv) $x^2+x-132$

Solution:

$x^2+x-132$
$= x^2+12x-11x-132$
$= x(x+12)-11(x+12)$
$= (x-11)(x+12)$

Q.3

(i) $4x^2+12x+5$

Solution: **(A.B)**

$4x^2+12x+5$
$= 4x^2+2x+10x+5$
$= 2x(2x+1)+5(2x+1)$
$= (2x+5)(2x+1)$

(ii) $30x^2+7x-15$ **(LHR 2014)**

Solution:

$30x^2+7x-15$
$= 30x^2+25x-18x-15$
$= 5x(6x+5)-3(6x+5)$
$= (5x-3)(6x+5)$

(iii) $24x^2-65x+21$

Solution:

$24x^2-65x+21$
$= 24x^2-56x-9x+21$
$= 8x(3x-7)-3(3x-7)$
$= (8x-3)(3x-7)$

(iv) $5x^2-16x-21$

Solution:

$5x^2-16x-21$
$= 5x^2+5x-21x-21$
$= 5x(x+1)-21(x+1)$
$= (5x-21)(x+1)$

(v) $4x^2-17xy+4y^2$

Solution:

$4x^2-17xy+4y^2$
$= 4x^2-16xy-xy+4y^2$
$= 4x(x-4y)-y(x-4y)$
$= (4x-y)(x-4y)$

(vi) $3x^2-38xy-13y^2$

Solution:

$3x^2-38xy-13y^2$
$= 3x^2-39xy+xy-13y^2$
$= 3x(x-13y)+y(x-13y)$
$= (3x+y)(x-13y)$

(vii) $5x^2+33xy-14y^2$

Solution:

$5x^2+33xy-14y^2$
$= 5x^2+35xy-2xy-14y^2$
$= 5x(x+7y)-2y(x+7y)$
$= (5x-2y)(x+7y)$

(viii) $\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4, x\neq 0$

Solution:

$$\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4$$

$$=\left(5x-\frac{1}{x}\right)^2+2\left(5x-\frac{1}{x}\right)(2)+(2)^2$$

$\because a^2+2ab+b^2=(a+b)^2$

$=\left(5x-\frac{1}{x}+2\right)^2$

$=\left(5x-\frac{1}{x}+2\right)\left(5x-\frac{1}{x}+2\right)$

Q.4

(i) $(x^2+5x+4)(x^2+5x+6)-3$

Solution: $(x^2+5x+4)(x^2+5x+6)-3$

Suppose that

$x^2+5x=y$

So,

$(x^2+5x+4)(x^2+5x+6)-3$

$=(y+4)(y+6)-3$

$=[y(y+6)+4(y+6)-3$

$=(y^2+6y+4y+24)-3$

$=(y^2+10y+24)-3$

$=y^2+10y+24-3$

$=y^2+10y+21$

$=y^2+7y+3y+21$

$=y(y+7)+3(y+7)$

$=(y+3)(y+7)$

We know that $y=x^2+5x$

$=(x^2+5x+3)(x^2+5x+7)$

(ii) $(x^2-4x)(x^2-4x-1)-20$

Solution:

$(x^2-4x)(x^2-4x-1)-20$

Suppose that

$x^2-4x=y$

So, $(x^2-4x)(x^2-4x-1)-20$

$=(y)(y-1)-20$

$=(y^2-y)-20$

$=y^2-y-20$

$=y^2-5y+4y-20$

$=y(y-5)+4(y-5)$

$=(y+4)(y-5)$

Putting the value of y

$=(x^2-4x+4)(x^2-4x-5)$

$=\left[(x)^2-2(x)(2)+(2)^2\right]\left[x^2-5x+x-5\right]$

$=(x-2)^2\left[x(x-5)+1(x-5)\right]$

$=(x-2)^2(x-5)(x+1)$

$=(x-5)(x+1)(x-2)^2$

(iii) $(x+2)(x+3)(x+4)(x+5)-15$

Solution:

$(x+2)(x+3)(x+4)(x+5)-15$

$=[(x+2)(x+5)][(x+3)(x+4)]-15$

$=[x(x+5)+2(x+5)][x(x+4)+3(x+4)]-15$

$=[x^2+5x+2x+10][x^2+4x+3x+12]-15$

$=(x^2+7x+10)(x^2+7x+12)-15$

Put $x^2+7x=y$

So,

$(x^2+7x+10)(x^2+7x+12)-15$

$=(y+10)(y+12)-15$

$=[y(y+12)+10(y+12)]-15$

$=(y^2+12y+10y+120)-15$

$=(y^2+22y+120)-15$

$=y^2+22y+120-15$

$=y^2+22y+105$

$=y^2+15y+7y+105$

$=y(y+15)+7(y+15)$

$=y(y+15)+7(y+15)$

$=(y+7)(y+15)$

Putting the value of y

$=(x^2+7x+7)(x^2+7x+15)$

(iv) $(x+4)(x-5)(x+6)(x-7)-504$

Solution:

$(x+4)(x-5)(x+6)(x-7)-504$

$=[(x+4)(x-5)][(x+6)(x-7)]-504$

$=[x(x-5)+4(x-5)][x(x-7)+6(x-7)]-504$

$=(x^2-5x+4x-20)(x^2-7x+6x-42)-504$

$=(x^2-x-20)(x^2-x-42)-504$

Suppose that

$x^2-x=y$

So,

$= (y-20)(y-42)-504$
$= [y(y-42)-20(y-42)]-504$
$= (y^2-42y-20y+840)-504$
$= y^2-62y+840-504$
$= y^2-62y+336$
$= y^2-56y-6y+336$
$= y(y-56)-6(y-56)$
$= (y-6)(y-56)$

We know that $a = x^2 - x$

$= (x^2-x-6)(x^2-x-56)$
$= (x^2-3x+2x-6)(x^2-8x+7x-56)$
$= [x(x-3)+2(x-3)][x(x-8)+7(x-8)]$
$= (x+2)(x-3)(x+7)(x-8)$

(v) $(x+1)(x+2)(x+3)(x+6)-3x^2$

Solution:

$(x+1)(x+2)(x+3)(x+6)-3x^2$
$= [(x+1)(x+6)][(x+2)(x+3)]-3x^2$
$= [x(x+6)+1(x+6)][x(x+3)+2(x+3)]-3x^2$
$= (x^2+6x+x+6)(x^2+3x+2x+6)-3x^2$
$= (x^2+6+7x)(x^2+6+5x)-3x^2$

Suppose that

$x^2+6 = y$

So,

$= (y+7x)(y+5x)-3x^2$
$= [y(y+5x)+7x(y+5x)]-3x^2$
$= (y^2+5xy+7xy+35x^2-3x^2)$
$= y^2+12xy+32x^2$
$= y^2+8xy+4xy+32x^2$
$= y(y+8x)+4x(y+8x)$
$= (y+4x)(y+8)$

We know that $y = x^2+6$

$= (x^2+6+4x)(x^2+6+8x)$
$= (x^2+4x+6)(x^2+8x+6)$

Q.5

(i) $x^3+48x-12x^2-64$

Solution:

$x^3+48x-12x^2-64$
$= x^3-12x^2+48x-64$
$\because a^3-3a^2b+3ab^2-b^3 = (a-b)^3$
$= (x)^3-3(x)^2(4)+3(x)(4)^2-(4)^3$
$= (x-4)^3$

(ii) $8x^3+60x^2+150x+125$

Solution:

$8x^3+60x^2+150x+125$
$= (2x)^3+3(2x)^2(5)+3(2x)(5)^2+(5)^3$
$\because a^3+3a^2b+3ab^2+b^3 = (a+b)^3$
$= (2x+5)^3$

(iii) $x^3-18x^2+108x-216$

Solution:

$x^3-18x^2+108x-216$
$= (x)^3-3(x)^2(6)+3(x)(6)^2-(6)^3$
$\because a^3-3a^2b+3ab^2-b^3 = (a-b)^3$
$= (x-6)^3$

(iv) $8x^3-125y^3-60x^2y+150xy^2$

Solution:

$8x^3-125y^3-60x^2y+150xy^2$
$= 8x^3-60x^2y+150xy^2-125y^3$
$= (2x)^3-3(2x)^2(5y)+3(2x)(5y)^2-(5y)^3$
$\because a^3-3a^2b+3ab^2-b^3 = (a-b)^3$
$= (2x-5y)^3$

Q.6

(i) $27+8x^3$

(GRW 2017, SWL 2014, 15, MTN 2015, SGD 2013)

Solution:

$27+8x^3$
$= (3)^3+(2x)^3$
$\because a^3+b^3 = (a+b)(a^2-ab+b^2)$
$= (3+2x)[(3)^2-(3)(2x)+(2x)^2]$
$= (3+2x)(9-6x+4x^2)$

(ii) $125x^3-216y^3$ **(SWL2013, D.GK 2017)**

Solution:

$125x^3-216y^3$
$= (5x)^3-(6y)^3$

$\because a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(5x-6y)[(5x)^2+(5x)(6y)+(6y)^2]$

$=(5x-6y)(25x^2+30xy+36y^2)$

(iii) $64x^3+27y^3$

Solution:

$64x^3+27y^3$

$=(4x)^3+(3y)^3$

$\because a^3+b^3=(a+b)(a^2-ab+b^2)$

$=(4x+3y)\left[(4x)^2-(4x)(3y)+(3y)^2\right]$

$=(4x+3y)(16x^2-12xy+9y^2)$

(iv) $8x^3+125y^3$

Solution:

$8x^3+125y^3$

$=(2x)^3+(5y)^3$

$\because a^3+b^3=(a+b)(a^2-ab+b^2)$

$=(2x+5y)\left[(2x)^2-(2x)(5y)+(5y)^2\right]$

$=(2x+5y)(4x^2-10xy+25y^2)$

REMAINDER THEORME AND FACTOR THEOREM

Remainder Theorem (K.B+U.B)

(LHR 2015, BWP 2017)

If a polynomial $p(x)$ is divided by a linear divisor $(x-a)$ until a constant remainder is obtained, then this remainder is equal to $p(a)$.

i.e. $R = P(a)$

Proof:

Let $q(x)$ be the quotient obtained after dividing $p(x)$ by $(x-a)$. As the divisor $(x-a)$ is linear, so the remainder must be a constant say R.

By division Algorithm we may write $p(x)=(x-a)q(x)+R$

This is an identity in x and so is true for all real numbers x. In particular it is true for $x=a$. Therefore,

$p(a)=(a-a)q(a)+R=0+R$

i.e., $p(a)=R$.

Hence Proved

Note **(K.B)**

If the divisor is $(ax-b)$, we have

$p(x)=(ax-b)q(x)+R$

Substituting $ax-b=0$ or $x=\dfrac{b}{a}$, we obtain

$p\left(\dfrac{b}{a}\right)=0.\ q\left(\dfrac{b}{a}\right)+R=0+R=R$

Thus if the divisor is linear, the above theorem provides an efficient way of finding the remainder without being involved in the process of long division.

Example # 1 **(A.B)**

Find the remainder when $9x^2-6x+2$ is divided by

(i) $(x-3)$ **(ii)** $x+3$ **(iii)** $3x+1$ **(iv)** x

Solution:

Let $p(x)=9x^2-6x+2$

(i) Put $x-3=0$ or $x=3$ in $p(x)$

$p(3)=9(3)^2-6(3)+2=65$

$\therefore R=65$

(ii) Put $x+3=0$ or $x=-3$ in $p(x)$

$p(-3)=9(-3)^2-6(-3)+2=101$

$\therefore R=101$

(iii) Put $3x+1=0$ or $x=-\dfrac{1}{3}$ in $p(x)$

$P\left(-\dfrac{1}{3}\right)=9\left(-\dfrac{1}{3}\right)^2-6\left(-\dfrac{1}{3}\right)+2=5$

$\therefore R=5$

(iv) Put $x=0$ in $p(x)$

$P(0)=9(0)^2-6(0)+2=2$

$\therefore R=2$

Example # 2 **(A.B)**

Find the value of k if the expression x^3+kx^2+3x-4 leaves a remainder of -2 when divided by $x+2$.

Solution:

$p(x)=x^3+kx^2+3x-4$

Put $x+2=0$ or $x=-2$ in $p(x)$

$p(-2)=(-2)^3+k(-2)^2+3(-2)-4$.

$=-8+4k-6-4$

$=-18+4k$

By the given condition, we have

$P(-2)=-2$

$\Rightarrow 4k-18=-2$

$4k=-2+18$

$4k=16$

$\Rightarrow \ k=4$

Zero of the Polynomial **(K.B)**

If a specific number $x=a$ is substituted for the variable x in a polynomial $p(x)$ so that value $P(a)$ is a zero then $x=a$ is called a zero of the polynomial $P(x)$.

Factor Theorem **(U.B)**

If a polynomial $P(x)$ is divided by a binomial $(x-a)$ such that remainder is zero, then $(x-a)$ is called factor of $P(x)$.

Or

The polynomial $(x-a)$ is a factor of the polynomial $P(x)$ if and only if $P(a)=0$.

Proof:

Let $q(x)$ be the quotient and R the remainder when a polynomial $P(x)$ is divided by $(x-a)$ then by division Algorithm,

$P(x)=(x-a)q(x)+R$

By the Remainder Theorem, $R=P(\text{a})$.

Hence $P(x)=(x-a)q(x)+P(a)$

(i) Now if $P(a)=0$ then $P(x)$ then $P(x)=(x-a)q(x)$

i.e., $(x-a)$ is a factor of $P(x)$

(ii) Conversely, if $(x-a)$ is a factor of $P(x)$, then the remainder upon dividing $P(x)$ by $(x-a)$ must be zero i.e. $P(a)=0$

This complete the proof.

Example # 1 **(A.B)**

Determine if $(x-2)$ is a factor of x^3-4x^2+3x+2.

Solution:

$P(x)=x^3-4x^2+3x+2$

Then the remainder for $(x-2)$ is

$P(2)=(2)^3-4(2)^2+3(2)+2$

$=8-16+6+2=0$

Since remainder is 0, $(x-2)$ is a factor of the polynomial $P(x)$.

Example # 2

Find a polynomial $p(x)$ of degree 3 that has 2, - 1 and 3 as zeros (i.e. roots).

Solution:

Since $x=2,-1,3$ are roots of $p(x)$, so by factor theorem $(x-2),(x+1)$ and $(x-3)$ are the factors of $p(x)$.

Thus,

$p(x)=a(x-2)(x+1)(x-3)$ where $a\in R$

$p(x)=a(x-2)(x^2-3x+x-3)$

$p(x)=a(x-2)(x^2-2x-3)$

$p(x)=a(x^3-2x^2-3x-2x^2+4x+6)$

$p(x)=a(x^3-4x^2+x+6)$

Is the required cubic polynomial.

Exercise 5.3

Q.1 Use the remainder theorem to find the remainder when

(i) $3x^3-10x^2+13x-6$ **is divided by** $(x-2)$.

Solution:

$P(x)=3x^3-10x^2+13x-6$

Since $P(x)$ is divided by $(x-2)$.

$\therefore P(2)=R$

$P(2)=3(2)^3-10(2)^2+13(2)-6$

$=3(2)^3-10(2)^2+13(2)-6$

$=24-40+26-6$

$R=4$

Hence 4 is the remainder

(ii) $4x^3-4x+3$ **is divided by** $(2x-1)$

Solution: **(A.B)**

$P(x)=4x^3-4x+3$

Since $P(x)$ is divided by $(2x-1)$

$\therefore R=P\left(\frac{1}{2}\right) \quad \because 2x-1=0 \Rightarrow x=\frac{1}{2}$

$P\left(\frac{1}{2}\right)=4\left[\frac{1}{2}\right]^3-4^2\times\frac{1}{2}+3$

$=4\times\frac{1}{8_2}-2+3$

$=\frac{1}{2}-2+3$

$=\frac{1-4+6}{2}=\frac{3}{2}$

$R=\frac{3}{2}$

Hence $\frac{3}{2}$ is the remainder

(iii) $6x^4+2x^3-x+2$ **is divided by** $(x+2)$ **from** $x+2=0$

Solution: Given that

$P(x)=6x^4+2x^3-x+2$

Since $P(x)$ is divided by $(x+2)$

$\therefore R=P(-2)$

$P(-2)=6(-2)^4+2(-2)^3-(-2)+2$

$=96-16+2+2$

$R=84$

Hence 84 is the remainder.

(iv) $(2x-1)^3+6(3+4x)^2-10$ **is divided by** $2x+1$.

Solution: Given that

$P(x)=(2x-1)^3+6(3+4x)^2-10$

Since $P(x)$ is divided by $2x+1$

$\therefore R=P\left(-\frac{1}{2}\right)$

$P\left(-\frac{1}{2}\right)=\left[2\left(-\frac{1}{2}\right)-1\right]^3+6\left[3+4^2\left(\frac{-1}{2}\right)\right]^2-10$

$=[-1-1]^3+6[3-2]^2-10$

$=[-2]^3+6-10=-8+6-10$

$R=-12$

Hence –12 is the remainder.

(v) $x^3-3x^2+4x-14$ **is divided by** $(x+2)$ **from** $x+2=0, x=-2$

Solution: Given that

$P(x)=x^3-3x^2+4x-14$

Since $P(x)$ is divided by $(x+2)$

$\therefore R=P(-2)$

$P(-2)=(-2)^3-3(-2)^2+4(-2)-14$

$=-8-12-8-14$

$R=-42$

Hence -42 is the remainder

Q.2

(i) **If $(x+2)$ is a factor of** $3x^2-4kx-4k^2$ **then find the values of k** $x+2=0$ $x=-2$

Solution: Given that **(A.B)**

$P(x)=3x^2-4kx-4k^2$

$P(-2)=3(-2)^2-4k(-2)-4k^2$

$P(-2)=12+8k-4k^2$

If $(x+2)$ is the factor then remainder is equal to zero

$P(-2)=0$

$12+8k-4k^2=0$

$-4(-3-2k+k^2)=0$

$k^2-2k-3=0 \quad \because -4\neq 0$

$k^2-3k+k-3=0$

$k(k-3)+1(k-3)=0$

$(k-3)(k+1)=0$

Either

$k-3=0$ or $k+1=0$

$k=3$ $\quad$ $k=-1$

(ii) If (x –1) is a factor of $x^3-kx^2+11x-6$ the find the value of k from $x-1=0$ $x=1$

Solution: Given that

$P(x)=x^3-kx^2+11x-6$

$P(1)=(1)^3-k(1)^2+11(1)-6$

$P(1)=1-k+11-6$

$P(1)=6-k$

If (x-1) is the factor then remainder is equal to zero

$P(1)=0$

$6-k=0$

$k=6$

Result:

$k=6$

Q.3 Without long division determine whether

(i) $(x-2)$ and $(x-3)$ are factor of $P(x)=x^3-12x^2+44x-48$

Solution:

Given that

$P(x)=x^3-12x^2+44x-48$

If (x-2) is the factor then remainder is equal to zero

$P(2)=(2)^3-12(2)^2+44(2)-48=8-48+88-48=0$

Hence $x-2$ is a factor of $P(x)$

For $x-3$

$R=P(3)$

$=(3)^3-12(3)^2+44(3)-48$

$=(3)^3-12(3)^2+44(3)-48$

$=27-108+132-48$

$=159-156$

$R=3\neq 0$

P(3) is not equal to zero then x – 3 is not factor of $P(x)=x3-12x^2+44x-48$

(ii) $(x-2),(x+3)$ and $(x-4)$ are factor of $q(x)=x^3+2x^2-5x-6$ from $x-2=0, x=2$

Solution:

Given that

$q(x)=x^3+2x^2-5x-6$

Put $x-2=0 \Rightarrow x=2$

$R=q(2)$

$=(2)^3+2(2)^2-5(2)-6$

$R=8+8-10-6$

$R=16-16$

$R=0$

Hence $x-2$ is factor o f $q(x)=x^3+2x^2-5x-6$

Put $x+3=0 \Rightarrow x=-3$

$R=q(-3)$

$=(-3)^3+2(-3)^2-5(-3)-6$

$=-27+18+15-6$

$R=0$

Hence $x-2$ is factor of $q(x)=x^3+2x^2-5x-6$

Put $x-4=0 \Rightarrow x=4$

$R=q(4)$

$=(4)^3+2(4)^2-5(4)-6$

$=64+32-20-6$

$R=70\neq 0$

Hence $x-4$ is not a factor of $q(x)=x^3+2x^2-5x-6$

Q.4 For what value of 'm' is the polynomial P(x) $=4x^3-7x^2+6x-3m$ exactly divisible by $x+2$?

Solution:

$P(x)=4x^3-7x^2+6x-3m$

From $x+2=0$, $x=-2$

$P(-2)=4(-2)^3-7(-2)^2+6(-2)-3m$

P(-2)=-32-28-12-3m=-72-3m

If (x+2) is the factor then remainder is equal to zero

$P(-2)=0$

$-72-3m=0$

$-72=3m$

$m=-\frac{72}{3}$

$m=-24$

Result:

$m = -24$

Q.5 Determine the value of k if $P(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x-3)$.

Solution:

$q(x) = x^3 - 4x + k$

Put x-3=0 or x=3

$R_1 = q(3)$

$= (3)^3 - 4(3) + k$

$= 27 - 12 + k$

$= 15 + k$

$R_1 = 15 + k$ ....(i)

$R_2 = P(3)$

$= k(3)^3 + 4(3)^2 + 3(3) - 4$

$= 27k + 36 + 9 - 4$

$R_2 = 27k + 41$ ....(ii)

Since it leaves the same remainder.

Hence $R_1 = R_2$

$15 + k = 27k + 41$

$15 - 41 = 27k - k$

$-26 = 26k$

$k = \frac{-26}{26}$

$k = -1$

Q.6 The remainder after dividing the polynomial $P(x) = x^3 + ax^2 + 7$ by $(x+1)$ is $2b$ calculate the value of a and b if this expression leaves a remainder of $(b+5)$ on being dividing by $(x-2)$

Solution:

Let

$P(x) = x^3 + ax^2 + 7$

Since $P(x)$ is divided by (x+1)

Put $x + 1 = 0$ $x = -1$

R=P(−1)

$= (-1)^3 + a(-1)^2 + 7$

$= -1 + a + 7$

$R = a + 6$

According to first condition remainder is 2b

$2b = a + 6$ …(i)

Since $P(x)$ is divided by (x-2)

Put $x - 2 = 0$ or $x = 2$

P(2)=$(2)^3$+a$(2)^2$+7

=8+4a+7

R=15+4a

According to second condition remainder is (b+5)

15+4a=b+5

4a-b=5-15

4a-b=-10 ……. (ii)

Solving equations (i) and (ii)

From equation (ii) b=10+4a

Putting the value of b in equation (i)

a+6=2(10+4a)

a=20+8a-6

-8a + a =14

-7a=14

$a = \frac{14}{-7}$

a=-2

Putting the value of a in equation (ii)

$4a - b = -10$

$4(-2) - b = -10$

$-8 - b = -10$

$-8 + 10 = b$

$2 = b$

$b = 2$

Result:

$a = -2, b = 2$

Q.7 The polynomial $x^3 + lx^2 + mx + 24$ has a factor $(x+4)$ and it leaves a remainder of 36 when divided by $(x-2)$
Find the values of l and m.

Solution: **(K.B+U.B)**

Let

$P(x) = x^3 + lx^2 + mx + 24$

From $x+4=0$ or $x=-4$

$P(-4) = (-4)^3 + l(-4)^2 + m(-4) + 24$

$P(-4) = -64 + 16l - 4m + 24$

$P(-4) = 16l - 4m - 40$

According to condition (x+4) is the factor then

$16l - 4m - 40 = 0$

$4[4l - m - 10] = 0$

$4l - m - 10 = 0$ (i)

from $x-2=0$ or $x=2$

Now $P(2) = (2)^3 + l(2)^2 + m(2) + 24$

$P(2) = 8 + 4l + 2m + 24$

$P(2) = 4l + 2m + 32$

According the condition

$4l + 2m + 32 = 36$

$4l + 2m = 36 - 32$

$4l + 2m = 4$

$4l + 2m - 4 = 0$ (ii)

Subtracting (i) from (ii)

$\cancel{4l} + 2m - 4 = 0$

$\pm\cancel{4l} \mp m \mp 10 = 0$

$3m + 6 = 0$

$3m = -6$

$m = \frac{-\cancel{6}\,2}{\cancel{3}}$

$m = -2$

Putting the value of m is equation (i)

$4l - (-2) - 10 = 0$

$4l + 2 - 10 = 0$

$4l - 8 = 0$

$4l = 8$

$l = \frac{2\cancel{8}}{\cancel{4}}$

$l = 2$

Q.8 The expression $lx^3 + mx^2 - 4$ leaves remainder of -3 and 12 when divided by $(x-1)$ and $(x+2)$ respectively. Calculate the value of l and m.

Solution:

$P(x) = lx^3 + mx^2 - 4$

from x-1=0 or x=1

$P(1) = l(1)^3 + m(1)^2 - 4$

$P(1) = l + m - 4$

According to conditions l+m-4=-3

$l + m = 4 - 3$

$l = 1 - m$ ....(i)

From $x+2=0$ or $x=-2$

$P(-2) = l(-2)^3 + m(-2)^2 - 4$

$P(-2) = -8l + 4m - 4$

According to condition

$-8l + 4m - 4 = 12$

Putting the value of l in the equation

$-8[1-m] + 4m = 16$

$-8 + 8m + 4m = 16$

$12m = 16 + 8$

$12m = 24$

$m = \frac{\cancel{24}^{2}}{\cancel{12}}$

$m = 2$

Putting the value of m is equation (i)

$l = 1 - 2$

$l = -1$

Result:

$m = 2$, $l = -1$

Q.9 The expression $ax^3 - 9x^2 + bx + 3a$ is exactly divisible by $x^2 - 5x + 6$. Find the value of a and b.

Solution:

Given that

$P(x) = ax^3 - 9x^2 + bx + 3a$

$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$

$= x[x-2] - 3[x-2]$

$= [x-2][x-3]$

Since x^2-5x+6 divides $P(x)$ completely then

$(x-2)$ and $(x-3)$ are factors of $P(x)$.

Put $x-2=0 \Rightarrow x=2$

$P(2)=a(2)^3-9(2)^2+b(2)+3a$

$P(2)=8a-36+2b+3a$

$P(2)=11a+2b-36$

According to condition (x-2) is the factor so

$11a+2b-36=0 \quad \to (i)$

From $x-3=0$ or $x=3$

$P(3)=a(3)^3-9(3)^2+b(3)+3a$

$P(3)=27a-81+3b+3a$

$P(3)=30a+3b-81$

According to condition (x-3) is the factor so

$30a+3b-81=0 \quad \to$ (ii)

$3(10a+b-27)=0$

$10a+b-27=\dfrac{0}{3}$

$b=27-10a \quad \to$ (iii)

Putting the value of b in equation (i)

$11a+2[27-10a)-36=0$

$11a+54-20a-36=0$

$-9a+18=0$

$18=9a$

$a=\dfrac{\cancel{18}\,2}{\cancel{9}}$

$a=2$

Putting the value of a in equation (iii)

$b=27-10(+2)$

$b=27-20$

$b=7$

Result:

$a=2,\ b=7$

Factorization of a Cubic Polynomial Rational Root Theorem (K.B)

Let $a_o x^n + a_1 x^{n-1} + ... + a_{n-1}x + a_n = 0$, $a_o \neq 0$ be a polynomial equation of degree n with integral coefficients. If $\frac{p}{q}$ is a rational root (expressed in lowest terms) of the equation, then p is a factor of the constant term a_n and q is factor of the leading coefficient a_o.

Example **(A.B)**

Factorize the polynomial x^3-4x^2+x+6, by using Factor Theorem

Solution:

We have $P(x)=x^3-4x^2+x+6$.

Possible factor of the constant term $P=6$ are $\pm1, \pm2, \pm3$ and ±6 and of leading coefficient $q=1$ are ±1. Thus the expected zeros (or root) of $P(x)=0$ are $\frac{p}{q}=\pm1,\pm2,,\pm3$ and ±6 if $x=a$ is a zero of $P(x)$, then $(x-a)$ will be a factor.

We use the hit and trail method to find zeros of $P(x)$ let us try $x=1$

Now $P(1)=(1)^3-4(1)^2+1+6$

$=1-4+1+6=4\neq0$

Hence $x=1$ is not a zero of $P(x)$.

Again $P(-1)=(-1)^3-4(-1)^2-1+6$

$=-1-4-1+6=0$

Hence $x=-1$ is a zero of $P(x)$ and therefore $x-(-1)=(x+1)$ a factor of $P(x)$

Now

$P(2)=(2)^3-4(2)^2+2+6$

$=8-16+2+6=0$

$\Rightarrow x=2$ is a root of $P(x)$.

Hence $(x-2)$ is also a factor of $P(x)$

Similarly $P(3)=(3)^3-4(3)^2+3+6$

$=27-36+3+6=0$

$\Rightarrow x=3$ is a zero of $P(x)$.

Hence $(x-3)$ is the third factor of $P(x)$.

Thus the factorized form of

$P(x)=x^3-4x^2+x+6$

is $P(x)=(x+1)(x-2)(x-3)$

Exercise 5.4

Q.1 x^3-2x^2-x+2

Solution: Given that

$P(x)=x^3-2x^2-x+2$

P=2 and possible factors of 2 are $\pm1,\pm2$.

Here q=1 and possible factor of 1 are ±1.

So possible factor of P(x) can be $\frac{P}{q}=\pm1,\pm2$

$P(x)=x^3-2x^2-x+2$

Put x=1

$P(1)=(1)^3-2(1)^2-1+2$

$=1-2-1+2=0$

As remainder is equal to zero, (x-1) is factor.

Put x = - 1

$P(-1)=(-1)^3-2(-1)^2-(-1)+2$

$=-1-2+1+2=0$

As remainder is equal to zero, (x+1) is factor.

Put x=2

$P(2)=(2)^3-2(2)^2-(2)+2=8-8-2+2=0$

As remainder is equal to zero, (x-2) is factor.

$x^3-2x^2-x+2=(x-1)(x+1)(x-2)$

Q.2 $x^3-x^2-22x+40$

Solution:

Given that **(K.B)**

$P(x)=x^3-x^2-22x+40$

P = 40 possible factors of 40 are:

$=\pm1,\pm2,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

Here q=1 and possible factor of 1are ±1

So possible factor of P(x) will be

$\frac{P}{q}=\pm1,\pm2,\pm4,\pm5,\pm8,\pm10,\pm20,\pm40$

$P(x)=x^3-x^2-22x+40$

Put $x=2$

$P(2)=(2)^3-(2)^2-22(2)+40$

$=8-4-44+40=0$

As remainder is equal to zero, (x-2) is a factor.

Put x=4

$P(4)=(4)^3-(4)^2-22(4)+40$

$=64-16-88+40=0$

As remainder is equal to zero, (x-4) is a factor.

Put x=-5

$P(-5)=(-5)^3-(-5)^2-22(-5)+40$

$=-125-25+110+40$

$=-150+150$

$=0$

As remainder is equal to zero, (x+5) is a factor.

Hence

$x^3-x^2-22x+40=(x-2)(x-4)(x+5)$

Q.3 x^3-6x^2+3x+10

Solution:

Given that

$P(x)=x^3-6x^2+3x+10$

P=10

So possible factors of 10 are $\pm1,\pm2,\pm5,\pm10$

Here q=1 So, possible factor of 1are ±1.

So possible of factor of P(x) can be

$\frac{P}{q}=\pm1,\pm2,\pm5,\pm10$

P(x)=x^3-6x^2+3x+10

Put x=-1

$P(-1)=(-1)^3-6(-1)^2+3(-1)+10$

= -1-6-3+10 = 0

As remainder is equal to zero, (x+1) is a factor.

Put x = 2

P(2) = $(2)^3-6(2)^2+3(2)+10$

= 8 − 24+6+10 = 0

As remainder is equal to zero, (x-2) is a factor.

Put x = 5

$P(5)=(5)^3-6(5)^2+3(5)+10=125-150+15+10=0$

As remainder is equal to zero, (x-5) is a factor.

Hence

$x^3-6x^2+3x+10=(x+1)(x-2)(x-5)$

Q.4 $x^3+x^2-10x+8$

Solution:

Given that

$P(x) = x^3+x^2-10x+8$

P=8 So possible factors of 8 are $\pm1, \pm2, \pm4, \pm8$.

Here q=1 So possible factor can be

$\frac{P}{q} = \pm1, \pm2, \pm4, \pm8$

$P(x)=x^3+x^2-10x+8$

Put x=1

$P(1)=(1)^3+(1)^2-10(1)+8=1+1-10+8=0$

As remainder is equal to zero, (x-1) is a factor.

Put x=2

$P(2)=2^3+2^2-10(2)+8$

$=8+4-20+8$

$=20-20$

$=0$

As remainder is equal to zero, (x-2) is a factor.

Put $x=-4$

$P(-4) = (-4)^3+(-4)^2-10(-4)+8$

$=-64+16+40+8$

$=-64+64$

$= 0$

As remainder is equal to zero, (x+4) is a factor.

Hence

$x^3+x^2-10x+8=(x-1)(x-2)(x+4)$

Q.5 x^3-2x^2-5x+6

Solution:

Given that

$P(x)=x^3-2x^2-5x+6$

$P=6$ So factors of 6 are $\pm1, \pm2, \pm3, \pm6$

Here q=1, so factors of 1are ± 1.

So possible factors of P(x) can be

$\frac{P}{q} = \pm1, \pm2, \pm3, \pm6$

$P(x) = x^3-2x^2-5x+6$

Put x=1

$P(1) = (1)^3-2(1)^2-5(1)+6$

$=1-2-5+6$

$=-7+7$

$=0$

Remainder is equal to zero so $(x-1)$ is a factor

Put $x=-2$

$P(-2)=(-2)^3-2(-2)^2-5(-2)+6$

$=-8-8+10+6$

$=-16+16$

$=0$

Remainder is equal to zero so (x+2) is a factor

Put x=3

$P(3)=(3)^3-2(3)^2-5(3)+6$

$=27-6-15+6$

$= 27-27$

$=0$

As remainder is equal to zero, (x-3) is a factor.

Hence

$x^3-2x^2-5x+6=(x-1)(x+2)(x-3)$

Q.6 $x^3+5x^2-2x-24$

Solution: Given that

$P(x)=x^3+5x^2-2x-24$

P= –24 So possible factors of 24 are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

Here q=1. So possible factors of 1 are ± 1.

So possible factors of P(x) will be

$\frac{P}{q} = \pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

$P(x)=x^3+5x^2-2x-24$

Put x = 2

$P(2)=(2)^3+5(2)^2-2(2)-24$

$=8+20-4-24$

$=28-28$

$=0$

As remainder is equal to zero, $(x-2)$ is a factor.

Put $x=-3$

$P(-3)=(-3)^3+5(-3)^2-2(-3)-24$

$=-27+45+6-24$

$=-51+51$

$=0$

Remainder is equal to zero so (x+3) is a factor

Put $x=-4$

$$P(-4)=(-4)^3+5(-4)^2-2(-4)-24$$
$$=-64+80+8-24$$
$$=-88+88$$
$$=0$$

Remainder is equal to zero so (x+4) is a factor

Hence

$$x^3+5x^2-2x-24=(x-2)(x+3)(x+4)$$

Q.7 $3x^3-x^2-12x+4$

Solution:

Given that

$$P(x)=3x^3-x^2-12x+4$$

P=4 So possible factors of 4 are $\pm1,\pm2,\pm4$.

Here q=3 So possible factors of 3 are $\pm1,\pm3$.

So possible factors of P(x) can be

$$\frac{P}{q}=\pm1,\pm2,\pm4,\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{4}{3}$$

Put x=2

$$P(2)=3(2)^3-(2)^2-12(2)+4$$
$$=-24-4+24+4$$
$$=28-28$$
$$=0$$

As remainder is equal to zero, (x-2) is a factor.

Put $x=-2$

$$P(-2)=3(-2)^3-(-2)^2-12(-2)+4$$
$$=-24-4+24+4$$
$$=28-28$$
$$=0$$

As remainder is equal to zero, (x+2) is a factor.

Put $x=\frac{1}{3}$

$$P\left(\frac{1}{3}\right)=3\left(\frac{1}{3}\right)^3-\left(\frac{1}{3}\right)^2-\cancel{12}^4\left(\frac{1}{3}\right)+4$$
$$=\cancel{3}\left(\frac{1}{\cancel{27}_9}\right)-\frac{1}{9}-\cancel{4}+\cancel{4}$$
$$P\left(\frac{1}{3}\right)=\frac{\cancel{1}}{\cancel{9}}-\frac{\cancel{1}}{\cancel{9}}$$
$$=0$$

As remainder is equal to zero, $(3x-1)$ is a factor

Hence

$$3x^3-x^2-10x+4=(x-2)(x+2)(3x-1)$$

Q.8 $2x^3+x^2-2x-1$

Solution:

Given that

P(x)=$2x^3+x^2-2x-1$

P= 1, so possible factors of -1 are ±1.

Here q=2. So possible factors 2 are $\pm1,\pm2$.

Therefore, possible values of P(x) will be

$$\frac{P}{q}=\pm1,\pm2,\pm\frac{1}{2}$$

P(x)=$2x^3+x^2-2x-1$

Put x=1

$$P(1)=2(1)^3+(1)^2-2(1)-1$$
$$=2+1-2-1$$
$$=3-3$$
$$=0$$

As remainder is equal to zero, (x–1) is a factor.

Put $x=-1$

$$P(-1)=2(-1)^3+(-1)^2-2(-1)-1$$
$$=2+1-2-1$$
$$=3-3$$
$$=0$$

As remainder is equal to zero, (x+1) is a factor.

Put $x=\frac{-1}{2}$

$$P\left(\frac{-1}{2}\right)=2\left[\frac{-1}{2}\right]^3+\left[\frac{-1}{2}\right]^2-\cancel{2}\left[\frac{-1}{\cancel{2}}\right]-1$$
$$P\left(\frac{-1}{2}\right)=2\left[\frac{-1}{\cancel{8}4}\right]+\frac{1}{4}+\cancel{1}-\cancel{1}$$
$$P\left(\frac{-1}{2}\right)=-\frac{\cancel{1}}{\cancel{4}}+\frac{\cancel{1}}{\cancel{4}}$$
$$=0$$

$$\because x=-\frac{1}{2}\Rightarrow 2x+1=0$$

As remainder is equal to zero, $(2x+1)$ is a factor.

Hence $2x^3+x^2-2x-1=(x-1)(x+1)(2x+1)$

Review Exercise 5

Q.1 Filling the blanks:

1. **The factor of** x^2-5x+6 **are __________.** **(U.B)**

(a) $x+1, x-6$ **(b)** $x-2, x-3$

(c) $x+6, x-1$ **(d)** $x+2, x+3$

2. **Factors of** $8x^3+27y^3$ **are ____________.** **(RWP 2013, SWL 2013)** **(U.B)**

(a) $(2x-3y),(4x^2+9y^2)$ **(b)** $(2x-3y),(4x^2-9y^2)$

(c) $(2x+3y),(4x^2-6xy+9y^2)$ **(d)** $(2x-3y),(4x^2+6xy+9y^2)$

3. **Factors of** $3x^2-x-2$ **are____________.** **(U.B)**

(a) $(x+1),(3x-2)$ **(b)** $(x+1),(3x+2)$

(c) $(x-1),(3x-2)$ **(d)** $(x-1),(3x+2)$

4. **Factors of** a^4-4b^4 **are___________.** **(U.B)**

(FSD 2014, 17, MTN 2015, SWL 2016, 17)

(a) $(a-b),(a+b),(a^2+4b^2)$ **(b)** $(a^2-2b^2),(a^2+2b^2)$

(c) $(a-b),(a+b)(a^2-4b^2)$ **(d)** $(a-2b),(a^2+2b^2)$

5. **What will be added to complete the square of** $9a^2-12ab$ **?.....** **(K.B)**

(a) $-16b^2$ **(b)** $16b^2$

(c) $4b^2$ **(d)** $-4b^2$

6. **Find m so that** x^2+4x+m **is a complete square** **(K.B)**

(a) 8 **(b)** –8

(c) 4 **(d)** 16

7. **Factors of** $5x^2-17xy-12y^2$ **are___________.** **(K.B)**

(a) $(x+4y),(5x+3y)$ **(b)** $(x-4y),(5x-3y)$

(c) $(x-4y),(5x+3y)$ **(d)** $(5x-4y),(x+3y)$

8. **Factors of** $27x^3-\frac{1}{x^3}$ **are** **(SWL 2014)** **(K.B)**

(a) $\left(3x-\frac{1}{x}\right),\left(9x^2+3+\frac{1}{x^2}\right)$ **(b)** $\left(3x+\frac{1}{x}\right),\left(9x^2+3+\frac{1}{x^2}\right)$

(c) $\left(3x-\frac{1}{x}\right),\left(9x^2-3+\frac{1}{x^2}\right)$ **(d)** $\left(3x+\frac{1}{x}\right),\left(9x^2-3+\frac{1}{x^2}\right)$

ANSWERS KEYS

1	2	3	4	5	6	7	8
b	c	d	b	c	c	c	a

Q.2 Completion items

(i) $x^2+5x+6=$ ____________ **(U.B)**

(ii) $4a^2-16=$ ____________ **(U.B)**

(iii) $4a^2+4ab+($_____$)$ **is a complete square.** **(U.B)**

(iv) $\frac{x^2}{y^2}-2+\frac{y^2}{x^2}=$ __________ **(K.B)**

(v) $(x+y)(x^2-xy+y^2)=$ ____________ **(K.B)**

(vi) **Factored form of** x^4-16 **is** ____________ **(K.B)**

(vii) **If** $x-2$ **is factor of** $P(x)=x^2+2kx+8$ **then** $=$ ____________ **(K.B)**

Answer Keys

(i) $(x+3)(x+2)$

(ii) $(2a+4)(2a-4)=4(a+2)(a-2)$

(iii) $(b)^2$

(iv) $\left(\frac{x}{y}-\frac{y}{x}\right)^2$

(v) x^3+y^3

(vi) $(x+2)(x-2)(x^2+4)$

(vii) -3

Q.3 Factorize the following

(i) $x^2+8x+16-4y^2$

Solution:

$$x^2+8x+16-4y^2$$
$$=\left[x^2+8x+16\right]-4y^2$$
$$=\left[(x)^2+2(x)(4)+(4)^2\right]-(2y)^2$$
$$\because a^2+2ab+b^2=(a+b)^2$$
$$=(x+4)^2-(2y)^2$$
$$\because a^2-b^2=(a+b)(a-b)$$
$$=(x+4+2y)(x+4-2y)$$
$$=(x+2y+4)(x-2y+4)$$

(ii) $4x^2-16y^2$

Solution:

$$4x^2-16y^2$$
$$=4\left[x^2-4y^2\right]$$
$$=4\left[(x)^2-(2y)^2\right]$$
$$\because a^2-b^2=(a+b)(a-b)$$
$$=4(x-2y)(x+2y)$$

(iii) $9x^2+24x+3x+8$

Solution:

$$9x^2+24x+3x+8$$
$$=3x(3x+8)+1(3x+8)$$
$$=(3x+8)(3x+1)$$

(iv) $1-64z^3$

Solution:

$1-64z^3$

$=(1)^3-(4z)^3$

$\because a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(1-4z)\left[(1)^2+(1)(4z)+(4z)^2\right]$

$=(1-4z)(1+4z+16z^2)$

(v) $8x^3-\left(\frac{1}{3y}\right)^3$

$=(2x)^3-\left(\frac{1}{3y}\right)^3$

$\because a^3-b^3=(a-b)(a^2+ab+b^2)$

$=\left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right)$

(vi) $2y^2+5y-3$

Solution:

$2y^2+5y-3$

$=2y^2+6y-y-3$

$=2y(y+3)-1(y+3)$

$=(2y-1)(y+3)$

(vii) x^3+x^2-4x-4

Solution:

x^3+x^2-4x-4

$=x^2(x+1)-4(x+1)$

$=(x+1)(x^2-4)$

$\because a^2-b^2=(a+b)(a-b)$

$=(x+1)(x-2)(x+2)$

(viii) $25m^2n^2+10mn+1$

Solution:

$25m^2n^2+10mn+1$

$=(5mn)^2+2(5mn)(1)+(1)^2$

$\because a^2+2ab+b^2=(a+b)^2$

$=(5mn+1)^2$

(ix) $1-12pq+36p^2q^2$

Solution:

$1-12pq+36p^2q^2$

$=(1)^2-2(1)(6pq)+(6pq)^2$

$\therefore a^2-2ab+b^2=(a-b)^2$

$=(1-6pq)^2$

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 What will be added to complete the square of $9a^2 - 12ab$

(A) $4b^2$ **(B)** $16a^2$

(C) $-16b^2$ **(D)** $-4b^2$

2 Factors of $3x^2 - x - 2$ **are ________**

(A) $(x + 1)(3x - 2)$ **(B)** $(x + 1)(3x + 2)$

(C) $(x - 1)(3x - 2)$ **(D)** $(x - 1)(3x + 2)$

3 Factors of $a^4 - 4b^4$ **are;**

(A) $(a-b)(a+b)(a^2-4b^2)$ **(B)** $(a-b)(a+b)(a^2+4b^2)$

(C) $(a-b)(a^2+2b^2)$ **(D)** $(a^2-2b^2)(a^2+2b^2)$

4 I f $(x-1)$ **is the factor of polynomial expression** $(x^3 - kx^2 + 11x - 6)$**, then value of** k **is:**

(A) –6 **(B)** 6

(C) –12 **(D)** 12

5 Factors $8x^3 + 27y^3$ **are:**

(A) $(2x-3y)(4x^2+6xy+9y^2)$ **(B)** $(2x-3y)(4x^2-9y^2)$

(C) $(2x+3y)$ **(D)** $(2x+3y)(4x^2-6xy+9y^2)$

6 Factorizing of $3x^2 - 75y^2$ **is:**

(A) $(3x+75y)(3x-75y)$ **(B)** $3(x+25y)(x-25y)$

(C) $3(x-25y)$ **(D)** $3(x+5y)(x-5y)$

7 If a polynomial $P(x)$ **can be expressed as** $P(x) = g(x).h(x)$**, then each of the polynomials** $g(x)$ **and** $h(x)$ **is called __________ of** $P(x)$**.**

(A) Element **(B)** Factors

(C) Member **(D)** Function

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Factorize: $128m^2 - 242an^2$.

(ii) Factorize: $8x^3 - 125y^2 - 60x^2y + 150xy^2$.

(iii) Define remainder theorem.

(iv) Factorize: $3x^4 + 12y^4$

(v) Find the value of *x* if the expression $x^3 + kx^2 + 3x - 4$ leaves a remainder of –2 when divided by $x + 2$.

Q.3 Answer the following Questions. **(4+4=8)**

(a) The polynomial $x^3 + lx^2 + mx + 24$ has a factor $(x+4)$ and it leaves a remainder of 36 when divided by $(x-2)$ find the value of l and *m*.

(b) Factorize the cubic polynomial $3x^3 - x^2 - 12x + 4$.

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.