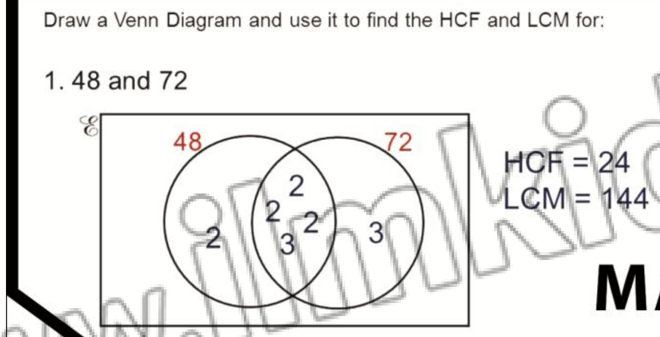

UNIT 6

ALGEBRAIC MANIPULATION

Highest Common Factor (H.C.F.)

(LHR 2014, GRW 2014, 17, FSD 2016, 17, MTN 2014, 15, SWL 2016, 17, RWP 2017)

If two or more algebraic expressions are given, then their common factor of highest power is called the H.C.F of the expressions. **(K.B)**

For example:

H.C.F of $39x^7y^3z$ **and** $91x^5y^6z^7$ **is** $13x^5y^27$

Least Common Multiple (L.C.M.)

If an algebraic expression $p(x)$ is exactly divisible by two or more expressions, then $p(x)$ is called the Common Multiple of the given expressions. The Least Common Multiple (L.C.M.) is the product of common factors together with non-common factors of the given expressions.

For example: **(K.B)**

L.C.M of $39x^7y^3z$ **and** $91x^5y^6z^7$ **is**

$273x^7y^6z^7$.

The Method used to find the H.C.F. of Expressions

Following are two methods:

(i) By factorization

(ii) By Division

Example

Find H.C.F of the following Polynomials $x^2-4, x^2+4x+4, 2x^2+x-6$ **(U.B)**

(i) **By Factorization** **(A.B)**

Solution:

$$x^2-4=x^2-2^2=(x-2)(x+2)$$

$$x^2+4x+4=(x)^2+2(x)(2)+(2)^2$$

$$=(x+2)^2$$

$$2x^2+x-6=2x^2+4x-3x-6$$

$$=2x(x+2)-3(x+2)$$

$$=(x+2)(2x-3)$$

Hence, H.C.F. $=x+2$

(ii) H.C.F. by Division

Use division method to find the H.C.F. of the polynomials **(A.B)**
$p(x)=x^3-7x^2+14x-8$ and $q(x)=x^3-7x+6$

Solution:

$$\begin{array}{r l}
 & \qquad\qquad 1 \\ \cline{2-2}
x^3-7x+6\big) & \cancel{x^3}-7x^2+14x-8 \\
 & \underline{\pm\cancel{x^3} \qquad\quad \mp 7x \pm 6} \\
 & -7x^2+21x-14
\end{array}$$

Here the remainder can be factorized as $-7x^2+21x-14=-7\left(x^2-3x+2\right)$

We ignore –7 because it is not common to both the given polynomials and consider x^2-3x+2

$$\begin{array}{r l}
 & \qquad x+3 \\ \cline{2-2}
x^2-3x+2\big) & \cancel{x^3}+0x^2-7x+6 \\
 & \underline{\pm\cancel{x^3} \mp 3x^2 \pm 2x} \\
 & 3x^2-\cancel{9x}+\cancel{6} \\
 & \underline{\pm\cancel{3x^2} \mp \cancel{9x} \pm \cancel{6}} \\
 & \qquad 0
\end{array}$$

Hence H.C.F. of $p(x)$ and $q(x)$ is x^2-3x+2.

Note

(i) In finding H.C.F. by division, if required, any expression can be multiplied by a suitable integer to avoid fraction. **(K.B)**

(ii) In case we are given three polynomials, then as a first step we find H.C.F. of any two of them and then find the H.C.F. of this H.C.F and the third polynomial.

Working Rule to Find L.C.M. of given Algebraic Expressions

(i) Factorize the given expressions completely i.e., to simplest form. **(K.B)**

(ii) Then the L.C.M. is obtained by taking the product of each factor appearing in any of the given expressions, raised to the highest power with which that factor appears.

Example

Find the L.C.M. of $p(x)=12\left(x^3-y^3\right)$ and $q(x)=8\left(x^3-xy^2\right)$ **(A.B)**

Solution:

By prime factorization of the given expressions, we have

$p(x)=12\left(x^3-y^3\right)=2^2\times 3(x-y)\left(x^2+xy+y^2\right)$ And

$q(x)=8\left(x^3-xy^2\right)=8x\left(x^2-y^2\right)$

$=2^3x(x-y)(x+y)$

Common factors $=2^2(x-y)$

Non common factors = $2\times 3\times x(x+y)\left(x^2+xy+y^2\right)$

Hence L.C.M. = Common factors × Non common factors

$$= 2^3 \times 3 \times x(x+y)(x-y)(x^2+xy+y^2)$$

$$= 24x(x+y)(x^3-y^3)$$

Note

Product of $p(x)$ and $q(x)$ is equal to product of H.C.F and L.C.M

(i) $p(x) \times q(x) = L.C.M \times H.C.F$ **(K.B+U.B)**

(ii) L.C.M. = $\dfrac{p(x)\times q(x)}{\text{H.C.F.}}$ **(iii)** H.C.F. = $\dfrac{p(x)\times q(x)}{\text{L.C.M.}}$ **(K.B+U.B)**

(iv) $p(x) = \dfrac{\text{L.C.M.}\times\text{H.C.F.}}{q(x)}$ **(v)** $q(x) = \dfrac{\text{L.C.M.}\times\text{H.C.F.}}{p(x)}$ **(K.B+U.B)**

Important Note

L.C.M. and H.C.F. are unique except for a factor of (-1) **(K.B)**

Example # 1

Find H.C.F. of the polynomials.

$p(x) = 20(x^3+3x^2-2x)$, $q(x) = 9(5x^4+40x)$ **(A.B)**

Solution:

We have

$$p(x) = 20\left[2x^3+3x^2-2x\right] = 20x\left[2x^2+3x-2\right]$$

$$= 20x\left[2x^2+4x-x-2\right] = 20x\left[2x(x+2)-1(x+2)\right]$$

$$= 20x(2x-1)(x+2) = 2^2\times 5\times x(x+2)(2x-1)$$

$$q(x) = 9(5x^4+40x) = 9\times 5x(x^3+8)$$

$$= 45x\left[x^3+2^3\right] = 45x(x+2)(x^2-2x+4)$$

$$= 3^2\times 5\times x(x+2)(x^2-2x+4)$$

Thus H.C.F $= 5x(x+2)$

Now using the formula, L.C.M. $= \dfrac{p(x)\times q(x)}{\text{H.C.F.}}$

Putting the values

$$\text{L.C.M.} = \frac{2^2\times 5\times x(x+2)(2x-1)\times 5\times 3^2\times x(x+2)(x^2-2x+4)}{5x(x+2)}$$

$$= 4\times 5\times 9\times x(x+2)(2x-1)(x^2-2x+4)$$

$$= 180x(x+2)(2x-1)(x^2-2x+4)$$

Application of H.C.F. and L.C.M.

Example **(A.B)**

The sum of two numbers is 120 and H.C.F. is 12. Find the numbers.

Solution:

Let the number be $12x$ and $12y$, where x, y are numbers prime to each other. then

Then $12x+12y=120$

i.e., $x+y=10$

Thus we have to find two numbers whose sum is 10. The possible such pairs of numbers are $(1,9),(2,8),(3,7),(4,6),(5,5)$

The pairs of numbers which are prime to each other are $(1,9)$ and $(3,7)$ Thus the required numbers are

$1\times12, 9\times12, 3\times12, 7\times12$

i.e. 12, 108, and 36, 84.

Exercise 6.1

Q.1 Find the H.C.F of the following expressions.

(LHR 2017, GRW 2017, SGD 2013, 17, SWL 2013, 15, RWP 2016, MTN 2017)

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$ **(A.B)**

Solution:

$$39x^7y^3z = 3\times13\times x.x.x.x.x.x.x.y.y.y.z$$

$$91x^5y^6z^7 = 7\times13\times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$$

H.C.F $=13\times x.x.x.x.x.y.y.y.z$

H.C.F = $13x^5y^3z$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$

Solution:

$$102xy^2z = 2\times3\times17\times x.y.y.z$$

$$85x^2yz = 5\times17\times x.x.y.z$$

$$187xyz^2 = 11\times17\times x.y.z.z$$

H.C.F $=17xyz$

Q.2 Find the H.C.F of the following expression by factorization.

(i) $x^2+5x+6, x^2-4x-12$ **(LHR 2013, GRW 2015)**

Solution:

Factorizing x^2+5x+6

$$= x^2+3x+2x+6$$

$$= x(x+3)+2(x+3)$$

$$= (x+3)(x+2)$$

Factorizing $x^2-4x-12$

$=x^2-6x+2x-12$

$=x(x-6)+2(x-6)$

$=(x-6)(x+2)$

So,

H.C.F = $(x+2)$

(ii) $x^3-27, x^2+6x-27, 2x^2-18$ **(A.B)**

Solution:

Factorizing x^3-27

$=(x)^3-(3)^3$

$=(x-3)\left[(x)^2+(x)(3)+(3)^2\right]$ $\therefore\ a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(x-3)(x^2+3x+9)$

Factorizing $x^2+6x-27$

$=x^2+9x-3x-27$

$=x(x+9)-3(x+9)$

$=(x+9)(x-3)$

Factorizing $2x^2-18$

$=2(x^2-9)$

$=2\left[(x)^2-(3)^2\right]$

$=2(x-3)(x+3)$ $\therefore\ a^2-b^2=(a+b)(a-b)$

So,

H.C.F = $(x-3)$

(iii) $x^3-2x^2+x, x^2+2x-3, x^2+3x-4$

Factorizing x^3-2x^2+x

$=x(x^2-2x+1)$

$=x(x^2-x-x+1)$

$=x\left[x(x-1)-1(x-1)\right]$

$=x(x-1)(x-1)$

Factorizing x^2+2x-3

$=x^2+3x-x-3$

$=x(x+3)-1(x+3)$

$=(x+3)(x-1)$

Factorizing x^2+3x-4

$=x^2+4x-x-4$

$=x(x+4)-1(x+4)$

$=(x+4)(x-1)$

So, H.C.F $=(x-1)$

(iv) $18(x^3-9x^2+8x), 24(x^2-3x+2)$ **(A.B)**

Solution:

$18(x^3-9x^2+8x)$

$=2\times3^2\times x(x^2-9x+8)$

$=2\times3^2\times x(x^2-8x-x+8)$

$=2\times3^2\times x[x(x-8)-1(x-8)]$

$=2\times3^2\times x(x-8)(x-1)$

$24(x^2-3x+2)$

$=2^3\times3(x^2-3x+2)$

$=2^3\times3(x^2-2x-x+2)$

$=2^3\times3[x(x-2)-1(x-2)]$

$=2^3\times3(x-2)(x-1)$

So,

H.C.F = $2\times3(x-1)$

H.C.F = $6(x-1)$

(v) $36(3x^4+5x^3-2x^2), 54(27x^4-x)$

Factorizing $36(3x^4+5x^3-2x^2)$

$=3\times3\times2\times2\times x^2(3x^2+5x-2)$

$=3\times3\times2\times2\times x^2(3x^2+6x-x-2)$

$=3\times3\times2\times2\times x^2[3x(x+2)-1(x+2)]$

$=3\times3\times2\times2\times x^2(x+2)(3x-1)$

Factorizing $54(27x^4-x)$

$=3\times3\times3\times2\times x(27x^3-1)$

$=3\times3\times3\times2\times x\left[(3x)^3-(1)^3\right]$

$=3\times3\times3\times2\times x(3x-1)\left[(3x)^2+(3x)(1)+(1)^2\right]$ $\quad\therefore a^3-b^3=(a-b)(a^2+ab+b^2)$

$=3\times3\times3\times2\times x(3x-1)(9x^2+3x+1)$

So,

H.C.F = $3\times3\times2\times x(3x-1)$

$=18x(3x-1)$

Q.3 Find the H.C.F. of the following by division method.

(i) $x^3+3x^2-16x+12,\ x^3+x^2-10x+8$ **(A.B)**

Solution: $x^3+3x^2-16x+12,\ x^3+x^2-10x+8$

$$1$$

$$x^3+x^2-10x+8\overline{)\ \cancel{x^3}+3x^2-16x+12}$$

$$\underline{\pm\cancel{x^3}\pm x^2\mp 10x\pm 8}$$

$$2x^2-6x+4$$

$$2\left(x^2-3x+2\right)$$

$$x+4$$

$$x^2-3x+2\overline{)\ \cancel{x^3}+x^2-10x+8}$$

$$\underline{\cancel{-x^3}\mp 3x^2\pm 2x}$$

$$4x^2-12x+8$$

$$\underline{\pm 4x^2\mp 12x\pm 8}$$

$$\times$$

H.C.F = $\left(x^2-3x+2\right)$

(ii) $x^4+x^3-2x^2+x-3,\ 5x^3+3x^2-17x+6$

Solution: $x^4+x^3-2x^2+x-3,\ 5x^3+3x^2-17x+6$

$$x+2$$

$$5x^3+3x^2-17x+6\overline{)x^4+x^3-2x^2+x-3}$$

$$\underline{\times 5}$$

$$5x^4+5x^3-10x^2+5x-15$$

$$\underline{\pm 5x^4\pm 3x^3\mp 17x^2\pm 6x}$$

$$2x^3+7x^2-x-15$$

$$\underline{\times 5}$$

$$10x^3+35x^2-5x-75$$

$$\underline{\pm 10x^3\pm 6x^2\mp 34x\pm 12}$$

$$29x^2+29x-87$$

$$29(x^2+x-3)$$

$$5x-2$$

$$x^2+x-3\overline{)5x^3+3x^2-17x+6}$$

$$\underline{\pm 5x^3\pm 5x^2\mp 15x}$$

$$-2x^2-2x+6$$

$$\underline{\mp 2x^2\mp 2x\pm 6}$$

$$\times$$

H.C.F $=\left(x^2+x-3\right)$

(iii) $2x^5-4x^4-6x, x^5+x^4-3x^3-3x^2$ **(A.B)**

$$\begin{array}{r|l} & \quad 1 \\ 2x^5-4x^4-6x & \overline{x^5+x^4-3x^3-3x^2} \end{array}$$

$$\underline{\times 2}$$

$$2x^5+2x^4-6x^3-6x^2$$

$$\underline{-2x^5 \mp 4x^4 \qquad\qquad \mp 6x}$$

$$6x^4-6x^3-6x^2+6x$$

$$6\left(x^4-x^3-x^2+x\right)$$

$$\begin{array}{r|l} & \quad 2x+2 \\ x^4-x^3-x^2+x & \overline{2x^5-4x^2-6x} \end{array}$$

$$\underline{\pm 25x^5 \pm 2x^2 \qquad \mp 2x^4 \mp 2x^3}$$

$$2x^4+2x^3-6x^2-6x$$

$$\underline{\pm 24x^3 \mp 2x^2 \mp 2x^2 \mp 2x}$$

$$4x^3-4x^2-8x$$

$$\left(x^3-x^2-2x\right)$$

$$\begin{array}{r|l} & \quad x \\ x^3-x^2-2x & \overline{\cancel{x^4}-\cancel{x^3}-x^2+x} \end{array}$$

$$\underline{\cancel{\mp x^4}\ \cancel{\mp x^3} \mp 2x^3}$$

$$x^2+x$$

$$\begin{array}{r|l} & \quad x-2 \\ x^2+x & \overline{\cancel{x^3}-x^2-2x} \end{array}$$

$$\underline{\cancel{\pm x^3} \pm x^2}$$

$$-2x^2-2x$$

$$\underline{\mp 2x^2 \mp 2x}$$

$$\times$$

H.C.F $=x^2+x=x\left(x+1\right)$

Q.4 Find the L.C.M. of the following expressions.

(i) $39x^7y^3z$ **and** $91x^5y^6z^7$ **(A.B)**

Solution:

$$39x^7y^3z=3\times 13\times x.x.x.x.x.x.x.y.y.y.z$$

$$91x^5y^6z^7=7\times 13\times x.x.x.x.x.y.y.y.y.y.y.z.z.z.z.z.z.z$$

Common=$13x^5y^3z$

Uncommon=$3\times 7\times x^2y^3z^6$

$$=21x^2y^3z^6$$

L.C.M= common factors × uncommon factors

$$=13x^5y^3z\times 21x^2y^3z^6$$

$$=273x^7y^6z^7$$

(ii) $102xy^2z, 85x^2yz$ **and** $187xyz^2$ **(LHR 2015)**

Solution:

$102xy^2z = 2\times3\times17.x.y.y.z$

$85x^2yz = 5\times17\times x.x.y.z$

$187xyz^2 = 11\times17.x.y.z.z$

Common $= 17xyz$

Uncommon $= 2\times3\times5\times11.xyz$

$= 330\,xyz$

L.C.M = common × uncommon

$= 17xyz\times330xyz$

$= 5610x^2y^2z^2$

Q.5 Find the L.C.M of the following by factorizing. **(A.B)**

(i) $x^2-25x+100$ **and** x^2-x-20

Solution:

Factorizing $x^2-25x+100$

$= x^2-20x-5x+100$

$= x(x-20)-5(x-20)$

$= (x-20)(x-5)$

Factorizing x^2-x-20

$= x^2-5x+4x-20$

$= x(x-5)+4(x-5)$

$= (x-5)(x+4)$

Common factors $= (x-5)$

Non common factors $= (x-20)(x+4)$

So, L.C.M = common factors × non common factors

L.C.M = $(x-5)(x+4)(x-20)$

(ii) $x^2+4x+4, x^2-4, 2x^2+x-6$

Solution:

Factorizing x^2+4x+4

$= x^2+2x+2x+4$

$= x(x+2)+2(x+2)$

$= (x+2)(x+2)$

Factorizing x^2-4

$= (x)^2-(2)^2$

$= (x-2)(x+2) \quad \therefore\ a^2-b^2=(a+b)(a-b)$

Factorizing $2x^2+x-6$

$$=2x^2+4x-3x-6$$
$$=2x(x+2)-3(x+2)$$
$$=(x+2)(2x-3)$$

Common factors $=(x+2)$

Non common factors = $(x-2)(2x-3)$

So, L.C.M = common factors × non common factors

$$\text{L.C.M} =(x+2)(x+2)(x-2)(2x-3)$$
$$=(x+2)^2(x-2)(2x-3)$$

(iii) $2(x^4-y^4),3(x^3+2x^2y-xy^2-2y^3)$

Factorizing $2(x^4-y^4)$

$$=2\left[(x^2)^2-(y^2)^2\right]$$
$$=2(x^2+y^2)(x^2-y^2) \qquad \therefore\ a^2-b^2=(a+b)(a-b)$$
$$=2(x^2+y^2)(x+y)(x-y)$$

Factorizing $3(x^3+2x^2y-xy^2-2y^3)$

$$=3\left[x^2(x+2y)-y^2(x+2y)\right]$$
$$=3(x+2y)(x^2-y^2) \qquad \therefore\ a^2-b^2=(a+b)(a-b)$$
$$=3(x+2y)(x+y)(x-y)$$

Common factors $=(x+y)(x-y)$

Non common factors = $2(x^2-y^2)(x+2y)$

So, L.C.M = common factors × non common factors

$$\text{L.C.M} = (x+y)(x-y)(x^2+y^2)(x+2y)\times2\times3$$
$$=6(x^2-y^2)(x^2+y^2)(x+2y)$$
$$=6(x+2y)(x^4-y^4)$$

(iv) $4(x^4-1),6(x^3-x^2-x+1)$ **(A.B)**

Solution:

Factorizing $4(x^4-1)$

$$=2\times2\left[(x^2)^2-(1)^2\right] \qquad \therefore\ a^2-b^2=(a+b)(a-b)$$
$$=2\times2(x^2+1)(x^2-1)$$
$$=2\times2(x^2+1)(x+1)(x-1)$$

Factorizing $6(x^3 - x^2 - x + 1)$

$$= 2\times 3\left[x^2(x-1) - 1(x-1)\right]$$

$$= 2\times 3\left[(x-1)(x^2\text{-}1)\right]$$

$$= 2\times 3(x-1)(x-1)(x+1)$$

Common factors = $2(x+1)(x-1)$

Non common factors = $2\times 3(x^2+1)(x-1)$

So, L.C.M = common factors × non common factors

$$L.C.M = 2\times 2\times 3(x-1)(x+1)(x-1)(x^2+1)$$

$$= 12(x-1)(x^4-1)$$

Q.6 For what value of k is $(x+4)$ the H.C.F of $x^2+x-(2k+2)$ and $2x^2+kx-12$?

Solution:

$P(x) = x^2 + x - (2k+2)$

Since x+4 is H.C.F of $P(x)$ and $q(x)$

$\therefore x+4$ is a factor of $P(x)$

Hence $P(-4) = 0$

$x^2 + x - (2k+2) = 0$

By putting the value of x

$(-4)^2 + (-4) - (2k+2) = 0$

$16 - 4 - 2k - 2 = 0$

$-2k + 10 = 0$

$2k = 10$

$$k = \frac{\cancel{10}^{5}}{\cancel{2}}$$

$k = 5$

Q.7 If $(x+3)(x-2)$ is the H.C.F. of $P(x) = (x+3)(2x^2-3x+k)$ and $q(x) = (x-2)$ $(3x^2+7x-l)$, find k and l. **(A.B)**

Solution:

Since (x–2)(x+3) is H.C.F of $p(x)$ and $q(x)$, then (x–2) is factor of $(2x^2-3x+k)$ and (x+3)is factor of $(3x^2+7x-l)$.

Consider

$2x^2 - 3x + k$

Put x–2 = 0 or x = 2

$= 2(2)^2 - 3(2) + k$

$= 8 - 6 + k$

$= 2 + k$

Remainder is equal to zero

$2+k=0$

$k=-2$

Now consider

$3x^2+7x-l$

Put $x+3=0$ or $x=-3$

$=3(-3)^2+7(-3)-l$

$=27-21-l$

$=6-l$

Remainder is equal to zero

$6-l=0$

$l=6$

Q.8 The L.C.M. and H.C.F. of two polynomials $P(x)$ and $q(x)$ are $2(x^4-1)$ and $(x+1)(x^2+1)$ respectively. If $P(x)=x^3+x^2+x+1$, find $q(x)$.

Solution:

We know that

$p(x)\times q(x)=\text{L.C.M.}\times\text{H.C.F.}$

$$q(x)=\frac{\text{L.C.M.}\times\text{H.C.F.}}{P(x)}$$

By putting the values

$$q(x)=\frac{2(x^4-1)(x+1)(x^2+1)}{x^3+x^2+x+1}$$

$$q(x)=\frac{2(x^4-1)(x+1)(x^2+1)}{x^2(x+1)+1(x+1)}=\frac{2(x^4-1)\cancel{(x+1)}\cancel{(x^2+1)}}{\cancel{(x+1)}\cancel{(x^2+1)}}$$

$$q(x)=2(x^4-1)$$

Q.9 Let $p(x)=10(x^2-9)(x^2-3x+2)$ and $q(x)=10x(x+3)(x-1)^2$, if the H.C.F. of $p(x),\mathrm{q}(x)$ is $10(x+3)(x-1)$. Find their L.C.M.

Solution:

We know that

$$\text{L.C.M.}=\frac{p(x)\times q(x)}{\text{H.C.F.}}$$

By putting the values

$$\text{L.C.M.}=\frac{\cancel{10}(x^2-9)(x^2-3x+2)\times 10x\cancel{(x+3)}(x-1)^{\cancel{2}}}{\cancel{10}\cancel{(x+3)}\cancel{(x-1)}}$$

$$\text{L.C.M.}=10x(x^2-9)(x^2-3x+2)(x-1)$$

Or L.C.M $=10x\left(x^2-9\right)\left(x^2-2x-x+2\right)(x-1)$

$$=10x\left(x^2-9\right)\left[x(x-2)-1(x-2)\right](x-1)$$

$$=10x\left(x^2-9\right)(x-1)^2(x-2)$$

Q.10 Let the product of L.C.M. and H.C.F. of two polynomials be $(x+3)^2(x-2)(x+5)$. If one polynomial is $(x+3)(x-2)$ and the second polynomial is $x^2+kx+15$, find the value of k.

Solution:

We know that

$$p(x)\times q(x)=\text{L.C.M.}\times\text{H.C.F.}$$

By putting the values

$$(x+3)(x-2)\left(x^2+kx+15\right)=(x+3)^2(x-2)(x+5)$$

$$x^2+kx+15=\frac{(x+3)^{\cancel{2}}\cancel{(x-2)}(x+5)}{\cancel{(x+3)}\cancel{(x-2)}}$$

$$x^2+kx+15=(x+3)(x+5)$$

$$x^2+kx+15=x^2+5x+3x+15$$

$$x^2+kx+15=x^2+8x+15$$

$$kx=\cancel{x^2}+8x+\cancel{15}-\cancel{x^2}-\cancel{15}$$

$$kx=8x$$

$$k=\frac{8\cancel{x}}{\cancel{x}}$$

$$k=8$$

Q.11 Waqas wishes to distribute 128 bananas and also 176 apples equally among a certain number of children. Find the highest number of children who can get fruit in this way.

Solution:

Finding H.C.F of 176 and 128.

$$\begin{array}{r|l} & 1 \\ 128 & 176 \\ & \underline{128} \end{array}$$

$$\begin{array}{r|l} & 2 \\ 48 & 128 \\ & \underline{-96} \end{array}$$

$$\begin{array}{r|l} & 1 \\ 32 & 48 \\ & \underline{-32} \end{array}$$

$$\begin{array}{r|l} & 2 \\ 16 & 32 \\ & \underline{-32} \\ & 0 \end{array}$$

Highest no. of children =16

Basic operations on Algebraic Fractions (K.B)

We shall carryout the operations of addition, difference product and division on algebraic fractions by giving some examples. We assume that all fractions are defined.

Example # 2 **(A.B)**

Express the product $\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$ as an algebraic expression reduced to lowest form, $x\neq 2,-2,1$

Solution:

By factorizing completely, we have

$$\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$$

$$=\frac{(x)^3-(2)^3}{x^2-2^2}\times\frac{x^2+4x+2x+8}{x^2-2(x)(1)+1}$$

$\because\ a^3-b^3=(a-b)(a^2+ab+b^2),\ a^2-b^2=(a+b)(a-b),\ a^2-2ab+b^2=(a-b)^2$

$$=\frac{(x-2)(x^2+2x+4)\times[x(x+4)+2(x+4)]}{(x-2)(x+2)\times(x-1)^2}$$

$$=\frac{(x-2)(x^2+2x+4)\times(x+2)(x+4)}{(x-2)(x+2)\times(x-1)^2}$$

$$=\frac{(x^2+2x+4)(x+4)}{(x-1)^2}$$

Exercise 6.2

Q.1 Simplify each of the following as a rational expression.

(i) $\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$

Solution:

$$\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$$

$$=\frac{x^2-3x+2x-6}{(x)^2-(3)^2}+\frac{x^2+6x-4x-24}{x^2-4x+3x-12}$$ $\therefore\ a^2-b^2=(a+b)(a-b)$ **(K.B)**

$$=\frac{x(x-3)+2(x-3)}{(x-3)(x+3)}+\frac{x(x+6)-4(x+6)}{x(x-4)+3(x-4)}$$

$$=\frac{(\cancel{x-3})(x+2)}{(\cancel{x-3})(x+3)}+\frac{(x+6)(\cancel{x-4})}{(\cancel{x-4})(x+3)}$$

$$=\frac{(x+2)}{(x+3)}+\frac{(x+6)}{(x+3)}$$

$$=\frac{x+2+x+6}{x+3}$$

$$=\frac{8+2x}{x+3}$$

$$=\frac{2(x+4)}{x+3}$$

Q.2 $\left[\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$ **(A.B)**

Solution:

$$\left[\frac{x+1}{x-1}-\frac{x-1}{x+1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{\left(x+1\right)^2-\left(x-1\right)^2}{\left(x-1\right)\left(x+1\right)}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{x^2+2x+1-(x^2+1-2x)}{(x-1)(x+1)}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$ $\therefore\ a^2-b^2=\left(a+b\right)\left(a-b\right)$ **(K.B)**

$$=\left[\frac{\cancel{x^2}+2x\cancel{+1}-\cancel{x^2}\cancel{-1}+2x}{x^2-1}-\frac{4x}{x^2+1}\right]+\left[\frac{4x}{x^4-1}\right]$$

$$=\left[\frac{4x}{x^2-1}-\frac{4x}{x^2+1}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{4x(x^2+1)-4x(x^2-1)}{(x^2-1)(x^2+1)}\right]+\frac{4x}{x^4-1}$$

$$=\left[\frac{4x^3+4x-4x^3+4x}{x^4-1}\right]+\frac{4x}{x^4-1}$$

$$=\frac{8x}{x^4-1}+\frac{4x}{x^4-1}$$

$$=\frac{8x+4x}{x^4-1}$$

$$=\frac{12x}{x^4-1}$$

Q.3 $\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$ **(A.B)**

Solution:

$$\frac{1}{x^2-8x+15}+\frac{1}{x^2-4x+3}-\frac{2}{x^2-6x+5}$$

$$=\frac{1}{x^2-3x-5x+15}+\frac{1}{x^2-3x-1x+3}-\frac{2}{x^2-5x-x+5}$$

$$= \frac{1}{x(x-3)-5(x-3)} + \frac{1}{x(x-3)-1(x-3)} - \frac{2}{x(x-5)-1(x-5)}$$

$$= \frac{1}{(x-3)(x-5)} + \frac{1}{(x-3)(x-1)} - \frac{2}{(x-5)(x-1)}$$

$$= \frac{(x-1)+(x-5)-2(x-3)}{(x-3)(x-5)(x-1)}$$

$$= \frac{\cancel{x}-\cancel{1}+\cancel{x}-\cancel{5}-\cancel{2x}+\cancel{6}}{(x-3)(x-5)(x-1)}$$

$$= \frac{0}{(x-3)(x-5)(x-1)}$$

$$= 0$$

Q.4 $\dfrac{(x+2)(x+3)}{x^2-9} + \dfrac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution:

$$\frac{(x+2)(x+3)}{x^2-9} + \frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$$

$$= \frac{(x+2)(x+3)}{(x)^2-(3)^2} + \frac{(x+2)\left[2(x^2-16)\right]}{(x-4)(x^2-3x+2x-6)}$$

$$= \frac{(x+2)(\cancel{x+3})}{(x-3)(\cancel{x+3})} + \frac{(x+2)\left[2\left\{(x)^2-(4)^2\right\}\right]}{(x-4)\left[x(x-3)+2(x-3)\right]} \qquad \therefore\ a^2-b^2=(a+b)(a-b)$$ **(K.B)**

$$= \frac{(x+2)}{(x-3)} + \frac{(\cancel{x+2})\left[2(x+4)(\cancel{x-4})\right]}{(\cancel{x-4})(x-3)(\cancel{x+2})}$$

$$= \frac{(x+2)}{(x-3)} + \frac{2(x+4)}{(x-3)}$$

$$= \frac{x+2}{x-3} + \frac{2x+8}{x-3}$$

$$= \frac{x+2+2x+8}{x-3}$$

$$= \frac{3x+10}{x-3}$$

Q.5 $\dfrac{x+3}{2x^2+9x+9} + \dfrac{1}{2(2x-3)} - \dfrac{4x}{4x^2-9}$ **(A.B)**

Solution:

$$\frac{x+3}{2x^2+9x+9} + \frac{1}{2(2x-3)} - \frac{4x}{4x^2-9}$$

$$=\frac{x+3}{2x^2+6x+3x+9}+\frac{1}{2(2x-3)}-\frac{4x}{(2x)^2-(3)^2}$$

$$=\frac{x+3}{2x(x+3)+3(x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$ $\therefore\ a^2-b^2(a+b)(a-b)$ **(A.B)**

$$=\frac{(\cancel{x+3})}{(\cancel{x+3})(2x+3)}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{1}{2x+3}+\frac{1}{2(2x-3)}-\frac{4x}{(2x-3)(2x+3)}$$

$$=\frac{2(2x-3)+(2x+3)-4x\times 2}{2(2x-3)(2x+3)}$$

$$=\frac{4x-6+2x+3-8x}{2(2x-3)(2x+3)}$$

$$=\frac{-2x-3}{2(2x-3)(2x+3)}$$

$$=\frac{-1(\cancel{2x+3})}{2(2x-3)(\cancel{2x+3})}$$

$$=\frac{-1}{2(2x-3)}$$

Q.6 $A-\frac{1}{A}$, where $A=\frac{a+1}{a-1}$

Solution:

Here $A=\frac{a+1}{a-1}$

$$A-\frac{1}{A}=\frac{a+1}{a-1}-\frac{1}{\frac{a+1}{a-1}}$$ (by putting value of A)

$$=\frac{a+1}{a-1}-\frac{a-1}{a+1}$$

$$=\frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)}$$

$$=\frac{a^2+2a+1-(a^2-2a+1)}{a^2-1}$$ $\therefore\ (a+b)^2=a^2+2ab+b^2$

$(a-b)^2=a^2-2ab+b^2$

$$=\frac{\cancel{a^2}+2a\cancel{+1}-\cancel{a^2}+2a\cancel{-1}}{a^2-1}$$

$$=\frac{4a}{a^2-1}$$

Q.7 $\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$ **(A.B)**

Solution:

$$\left[\frac{x-1}{x-2}+\frac{2}{2-x}\right]-\left[\frac{x+1}{x+2}+\frac{4}{4-x^2}\right]$$

$$=\left[\frac{x-1}{x-2}+\frac{2}{-x+2}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-x^2+4}\right]$$

$$=\left[\frac{x-1}{x-2}-\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}+\frac{4}{-\left(x^2-4\right)}\right]$$

$$=\left[\frac{x-1}{x-2}-\frac{2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{x^2-4}\right]$$

$$=\left[\frac{x-1-2}{x-2}\right]-\left[\frac{x+1}{x+2}-\frac{4}{(x+2)(x-2)}\right]$$

$$=\frac{(x-3)}{(x-2)}-\left[\frac{(x+1)(x-2)-4}{(x+2)(x-2)}\right]$$

$$=\frac{x-3}{x-2}-\left[\frac{x^2-2x+x-2-4}{(x+2)(x-2)}\right]$$

$$=\frac{x-3}{x-2}-\left[\frac{x^2-x-2-4}{(x+2)(x-2)}\right]$$

$$=\frac{x-3}{x-2}-\frac{x^2-x-6}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x^2-3x+2x-6}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{x(x-3)+2(x-3)}{(x-2)(x+2)}$$

$$=\frac{x-3}{x-2}-\frac{(x-3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$=\frac{x-\cancel{3}}{\cancel{x}-2}-\frac{x-\cancel{3}}{\cancel{x}-2}$$

$= 0$

Q.8 What rational number should be subtracted from **(A.B+K.B)**

$$\frac{2x^2+2x-7}{x^2+x-6} \text{ to get } \frac{x-1}{x-2}$$

Solution:

Let required rational number be *P*(*x*)

According to condition

$$\frac{2x^2+2x-7}{x^2+x-6}-P(x)=\frac{x-1}{x-2}$$

$$P(x)=\frac{2x^2+2x-7}{x^2+x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x^2+3x-2x-6}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{x(x+3)-2(x+3)}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7}{(x+3)(x-2)}-\frac{x-1}{x-2}$$

$$=\frac{2x^2+2x-7-(x-1)(x+3)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-\left(x^2+3x-x-3\right)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-\left(x^2+2x-3\right)}{(x+3)(x-2)}$$

$$=\frac{2x^2+2x-7-x^2-2x+3}{(x+3)(x-2)}$$

$$=\frac{x^2-4}{(x+3)(x-2)}$$

$$=\frac{x^2-2^2}{(x+3)(x-2)} \qquad \therefore\ a^2-b^2=(a+b)(a-b)$$ **(K.B)**

$$=\frac{(x+2)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$$

$$=\frac{x+2}{x+3}$$

Q.9 $\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$

Solution:

$$\frac{x^2+x-6}{x^2-x-6}\times\frac{x^2-4}{x^2-9}$$

$$=\frac{x^2+3x-2x-6}{x^2-3x+2x-6}\times\frac{x^2-2^2}{x^2-3^2}$$

$$=\frac{x(x+3)-2(x+3)}{x(x-3)+2(x-3)}\times\frac{(x-2)(x+2)}{(x-3)(x+3)} \qquad \therefore\ a^2-b^2=(a+b)(a-b)\ \textbf{(A.B+K.B)}$$

$$=\frac{(\cancel{x+3})(x-2)}{(x-3)(\cancel{x+2})}\times\frac{(x-2)(\cancel{x+2})}{(x-3)(\cancel{x+3})}$$

$$=\frac{(x-2)^2}{(x-3)^2}$$

Q.10 $\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$

Solution:

$$\frac{x^3-8}{x^2-4}\times\frac{x^2+6x+8}{x^2-2x+1}$$

$$=\frac{(x)^3-(2)^3}{(x^2)-(2)^2}\times\frac{x^2+4x+2x+8}{x^2-x-x+1}$$

$$=\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}\times\frac{x(x+4)+2(x+4)}{x(x-1)-1(x-1)} \qquad \therefore\ a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$a^2-b^2=(a+b)(a-b)$$

$$=\frac{x^2+2x+4}{(\cancel{x+2})}\times\frac{(x+4)(\cancel{x+2})}{(x-1)(x-1)} \qquad \textbf{(A.B+K.B)}$$

$$=\frac{(x^2+2x+4)(x+4)}{(x-1)^2}$$

Q.11 $\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$

Solution:

$$\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$$

$$=\frac{x(x^3-8)}{2x^2+6x-x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}$$

$$=\frac{x\left[(x)^3-(2)^3\right]}{2x(x+3)-1(x+3)}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x(x-2)}\quad\therefore\ a^3-b^3=(a-b)\left(a^2+ab+b^2\right)\textbf{(K.B)}$$

$$=\frac{\cancel{x}\left(\cancel{x-2}\right)\left(\cancel{x^2+2x+4}\right)}{\left(\cancel{2x-1}\right)\left(\cancel{x+3}\right)}\times\frac{\cancel{2x-1}}{\cancel{x^2+2x+4}}\times\frac{\cancel{x+3}}{\cancel{x}\left(\cancel{x-2}\right)}$$

$$=1$$

Q.12 $\dfrac{2y^2+7y-4}{3y^2-13y+4}\div\dfrac{4y^2-1}{6y^2+y-1}$

Solution:

$$\frac{2y^2+7y-4}{3y^2-13y+4}\div\frac{4y^2-1}{6y^2+y-1}$$

$$=\frac{2y^2+8y-1y-4}{3y^2-12y-y+4}\div\frac{(2y)^2-(1)^2}{6y^2+3y-2y-1}\qquad\therefore\ a^2-b^2=(a+b)(a-b)\textbf{(K.B)}$$

$$=\frac{2y(y+4)-1(y+4)}{3y(y-4)-1(y-4)}\div\frac{(2y-1)(2y+1)}{3y(2y+1)-1(2y+1)}$$

$$=\frac{(y+4)(2y-1)}{(3y-1)(y-4)}\div\frac{(2y-1)\left(\cancel{2y+1}\right)}{(3y-1)\left(\cancel{2y+1}\right)}$$

$$=\frac{(y+4)\left(\cancel{2y-1}\right)}{\left(\cancel{3y-1}\right)(y-4)}\times\frac{\left(\cancel{3y-1}\right)}{\left(\cancel{2y-1}\right)}$$

$$=\frac{y+4}{y-4}$$

Q.13 $\left[\dfrac{x^2+y^2}{x^2-y^2}-\dfrac{x^2-y^2}{x^2+y^2}\right]\div\left[\dfrac{x+y}{x-y}-\dfrac{x-y}{x+y}\right]$

Solution:

$$\left[\frac{x^2+y^2}{x^2-y^2}-\frac{x^2-y^2}{x^2+y^2}\right]\div\left[\frac{x+y}{x-y}-\frac{x-y}{x+y}\right]$$

$$=\left[\frac{\left(x^2+y^2\right)^2-\left(x^2-y^2\right)^2}{\left(x^2-y^2\right)\left(x^2+y^2\right)}\right]\div\left[\frac{(x+y)^2-(x-y)^2}{(x-y)(x+y)}\right]$$

$$=\left[\frac{\left(x^4+2x^2y^2+y^4\right)-\left(x^4-2x^2y^2+y^4\right)}{\left(x^2-y^2\right)\left(x^2+y^2\right)}\right]\div\left[\frac{\left(x^2+2xy+y^2\right)-\left(x^2-2xy+y^2\right)}{x^2-y^2}\right]$$

$$\because\ (a+b)^2=a^2+2ab+b^2,\quad (a-b)^2=a^2-2ab+b^2$$

$$=\left[\frac{\cancel{x^4}+2x^2y^2+\cancel{y^4}-\cancel{x^4}+2x^2y^2-\cancel{y^4}}{\left(x^2-y^2\right)\left(x^2+y^2\right)}\right]\div\left[\frac{\cancel{x^2}+2xy+\cancel{y^2}-\cancel{x^2}+2xy-\cancel{y^2}}{x^2-y^2}\right]$$

$$=\left[\frac{4x^2y^2}{\left(x^2-y^2\right)\left(x^2+y^2\right)}\right]\div\left[\frac{4xy}{x^2-y^2}\right]$$

$$=\frac{4x^2y^2}{\left(x^2-y^2\right)\left(x^2+y^2\right)}\times\frac{x^2-y^2}{4xy}$$

$$=\frac{\cancel{4xy}.xy}{\left(\cancel{x^2-y^2}\right)\left(x^2+y^2\right)}\times\frac{\cancel{x^2-y^2}}{\cancel{4xy}}$$

$$=\frac{xy}{x^2+y^2}$$

SQUARE ROOT OF ALGEBRAIC EXPRESSION (U.B)

Definition

As with number we define the square root of a given expression $p(x)$ as another expression $q(x)$ such that $q(x).q(x)=p(x)$.

As $5\times 5=25$ so square root of 25 is 5.

It means we can find square root of the expression $p(x)$ if it can be expressed as a perfect square.

Method of find Square Root (U.B)

There are two methods of find square root are

(i) By factorization **(ii)** By division

(i) By Factorization

Example # 1 **(A.B+K.B)**

Use factorization to find square root of the expression

$4x^2-12x+9$

Solution:

We have, $4x^2-12x+9$

$4x^2-6x-6x+9=2x(2x-3)-3(2x-3)$

$=(2x-3)(2x-3)=(2x-3)^2$

Hence $\sqrt{4x^2-12x+9}=\sqrt{(2x-3)^2}$ $a^2-2ab+b^2=(a-b)^2$

$=\pm(2x-3)$

Example # 2

Find the square root of $x^2+\frac{1}{x^2}+12\left(x+\frac{1}{x}\right)+38, \qquad x\neq 0$

Solution:

We have $x^2+\frac{1}{x^2}+12\left(x+\frac{1}{x}\right)+38$

$$=x^2+\frac{1}{x^2}+2+12\left(x+\frac{1}{x}\right)+36,$$

$$=\left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)(6)+(6)^2$$

$$=\left[\left(x+\frac{1}{x}\right)+6\right]^2 \qquad \text{Since } a^{2`}+2ab+b^2=(a+b)^2$$

Take square root on both sides

$$\sqrt{x^2+\frac{1}{x^2}+12\left(x+\frac{1}{x}\right)+38}=\sqrt{\left[\pm\left(x+\frac{1}{x}+6\right)\right]^2}$$

Hence the required square root is $\pm\left[x+\frac{1}{x}+6\right]$

(ii) By Division:

When it is difficult to convert the given expression into a perfect square by factorization, we use the method of actual division to find its square root. The method is similar to the division method of finding square root of numbers.

Note

We first write the given expression in descending order of powers of x.

Example # 1

Find the square root of $4x^4+12x^3+x^2-12x+4$

Solution:

We note that the given expression is already in descending order. Now the square root of the first term i.e., $\sqrt{4x^2}=2x$. So the first term of the divisor and quotient will be $2x$ in the first step. At each successive step, the remaining terms will be brought down.

$$\begin{array}{r|l} & 2x^2+3x-2 \\ 2x^2 & 4x^2+12x^3+x^2-12x+4 \\ & \pm 4x^4 \\ 4x^2+3x & 12x^3+x^2-12x+4 \\ & \pm 12x^3 \pm 9x^2 \\ 4x^2+6x-2 & -8x^2-12x+6 \\ & \mp 8x^2 \mp 12x \pm 4 \\ & 0 \end{array}$$

Thus square root of given expression is $\pm\ \left(2x^2+3x-2\right)$

Exercise 6.3

Q.1 Use factorization to find the square root of the following expression. **(A.B+U.B)**

(LHR 2014, 15, GRW 2013, FSD 2017, SGD 2016)

(i) $4x^2 - 12xy + 9y^2$

Solution:

$$\begin{aligned} 4x^2 - 12xy + 9y^2 &= 4x^2 - 6xy - 6xy + 9y^2 \\ &= 2x(2x-3y) - 3y(3x-3y) \\ &= (2x-3y)(2x-3y) \end{aligned}$$

$$4x^2 - 12y + 9y^2 = (2x-3y)^2$$

Taking square root on both sides

$$\begin{aligned} \sqrt{4x^2 - 12xy + 9y^2} &= \sqrt{[2x-3y]^2} \\ &= \pm(2x-3y) \end{aligned}$$

(ii) $x^2 - 1 + \dfrac{1}{4x^2}$

Solution:

$$x^2 - 1 + \frac{1}{4x^2}$$

$$= (x)^2 - 2(x)\left[\frac{1}{2x}\right] + \left[\frac{1}{2x}\right]^2 \qquad \because a^2 - 2ab + b^2 = (a-b)^2$$

$$= \left[x - \frac{1}{2x}\right]^2$$

Taking square root

$$\sqrt{x^2 - 1 + \frac{1}{4x^2}} = \sqrt{\left[x - \frac{1}{2x}\right]^2}$$

$$\sqrt{x^2 - 1 + \frac{1}{4x^2}} = \pm\left(x - \frac{1}{2x}\right)$$

(iii) $\dfrac{1}{16}x^2 - \dfrac{1}{12}xy + \dfrac{1}{36}y^2$ **(A.B)**

Solution:

$$\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2$$

$$= \left(\frac{1}{4}x\right)^2 - 2\left(\frac{1}{4}x\right)\left(\frac{1}{6}y\right) + \left(\frac{1}{6}y\right)^2$$

$$= \left(\frac{x}{4} - \frac{y}{6}\right)^2 \qquad \because a^2 - 2ab + b^2 = (a-b)^2$$ **(K.B)**

Taking the square root

$$\sqrt{\frac{1}{16}x^2-\frac{1}{12}xy+\frac{1}{36}y^2}=\sqrt{\left(\frac{1}{4}x-\frac{1}{6}y\right)^2}$$

$$=\pm\left(\frac{1}{4}x-\frac{1}{6}y\right)$$

$$=\pm\left(\frac{x}{4}-\frac{y}{6}\right)$$

(iv) $4(a+b)^2-12(a^2+b^2)+9(a-b)^2$

Solution:

$$4(a+b)^2-12(a2+b^2)+9(a-b)^2$$

$$=\left[2(a+b)^2\right]-2\left[2(a+b)\right]\left[3(a-b)\right]+\left[3(a-b)\right]^2$$

$$=\left[2(a+b)-3(a-b)\right]^2 \qquad \because\ a^2-2ab+b^2=(a-b)^2$$ **(K.B)**

Taking square root

$$\sqrt{4(a+b)^2-12(a^2+b^2)+9(a-b)^2}=\sqrt{\left[2(a+b)-3(a-b)\right]^2}$$

$$=\pm\left[2a+2b-3a+3b\right]$$

$$=\pm(5b-a)$$

(v) $\dfrac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

Solution:

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$$

$$=\frac{(2x^3)^2-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2+2(3x^2)(4y^2)+(4y^2)^2}$$

$$=\frac{\left[2x^3-3y^3\right]^2}{\left[3x^2+4y^2\right]^2} \qquad \because a^2\mp 2ab+b^2=(a\mp b)^2$$ **(K.B)**

Taking square root

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}}=\frac{\sqrt{\left(2x^3-3y^3\right)^2}}{\sqrt{\left(3x^2+4y^2\right)^2}}$$

$$=\pm\left(\frac{2x^3-3y^3}{3x^2+4y^2}\right)$$

(vi) $\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right),(x\neq 0)$

Solution:

$$\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)$$

By adding and subtracting 4

$$=x^2+\frac{1}{x^2}+2-4\left(x-\frac{1}{x}\right)$$

$$=\left(x^2+\frac{1}{x^2}-2\right)+2-4\left(x-\frac{1}{x}\right)+2$$

$$=x^2+\frac{1}{x^2}-2-4\left(x-\frac{1}{x}\right)+4$$

$$=\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right)(2)+(2)^2$$

$$=\left[\left(x-\frac{1}{x}\right)-2\right]^2 \qquad \because\ a^2-2ab+b^2=(a-b)^2$$ **(A.B+K.B)**

Taking square root

$$\sqrt{\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)}=\sqrt{\left[x-\frac{1}{x}-2\right]^2}$$

$$=\pm\left(x-\frac{1}{x}-2\right)$$

(vii) $\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$

Solution:

$$\left(x^2+\frac{1}{x^2}\right)^2-4\left(x+\frac{1}{x}\right)^2+12$$

$$=\left[x^2+\frac{1}{x^2}\right]^2-4\left[x^2+\frac{1}{x^2}+2\right]+12$$

$$=\left[x^2+\frac{1}{x^2}\right]^2-4x^2-\frac{4}{x^2}-8+12$$

$$=\left(x^2+\frac{1}{x^2}\right)^2-4\left(x^2+\frac{1}{x^2}\right)+4$$

$$=\left[x^2+\frac{1}{x^2}\right]^2-2\left[x^2+\frac{1}{x^2}\right](2)+(2)^2 \qquad \because a^2-2ab+b^2=(a-b)^2$$

$$=\left[x^2+\frac{1}{x^2}-2\right]^2$$

Taking square root

$$\sqrt{\left[x^2+\frac{1}{x^2}\right]^2-4\left[x+\frac{1}{x}\right]^2+12}=\sqrt{\left[x^2+\frac{1}{x^2}-2\right]^2}$$

$$=\pm\left(x^2+\frac{1}{x^2}-2\right)$$

(viii) $\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)$

Solution:

$$\left(x^2+3x+2\right)\left(x^2+4x+3\right)\left(x^2+5x+6\right)$$

$$=\left[x^2+2x+x+2\right]\left[x^2+3x+x+3\right]\left[x^2+3x+2x+6\right]$$

$$=\left[x(x+2)+1(x+2)\right]\left[x(x+3)+1(x+3)\right]\left[x(x+3)+2(x+3)\right]$$

$$=(x+2)(x+1)(x+3)(x+1)(x+3)(x+2)$$

$$=(x+2)^2(x+1)^2(x+3)^2$$

Taking square root

$$\sqrt{(x^2+3x+2)(x^2+4x+3)(x^2+5x+6)}=\sqrt{(x+2)^2(x+1)^2(x+3)^2}$$

$$=\pm(x+1)(x+2)(x+3)$$

(ix) $\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)$

Solution:

$$\left(x^2+8x+7\right)\left(2x^2-x-3\right)\left(2x^2+11x-21\right)$$

$$=(x^2+7x+1x+7)(2x^2-3x+2x-3)(2x^2+14x-3x-21)$$

$$=\left[(x(x+7)+1(x+7)\right]\left[x(2x-3)+1(2x-3)\right]\left[(2x(x+7)-3(x+7)\right]$$

$$=(x+7)(x+1)(2x-3)(x+1)(x+7)(2x-3)$$

$$=(x+7)^2(x+1)^2(2x-3)^2$$

Taking square root

$$\sqrt{(x^2+8x+7)(2x^2-x-3)(2x^2+11x-21)}=\sqrt{(x+7)^2(x+1)^2(2x-3)^2}$$

$$=\pm(x+1)(x+7)(2x-3)$$

Q.2 Use division method to find the square root of the following expression. **(K.B)**

(i) $4x^2+12xy+9y^2+16x+24y+16$

Solution:

$$4x^2+12xy+9y^2+16x+24y+16$$

$$\begin{array}{l} \quad 2x+3y+4 \\ 2x\,\big)\,\cancel{4x^2}+12xy+9y^2+16x+24y+16 \\ \quad \underline{\pm\cancel{4x^2}} \end{array}$$

$$\begin{array}{l} 4x+3y\,\big)\,12\cancel{xy}+\cancel{9y^2}+16x+24y+16 \\ \qquad\quad \underline{\pm12\cancel{xy}\pm\cancel{9y^2}} \\ 4x+6y+4\,\big)\,\cancel{16x}+\cancel{24y}+\cancel{16} \\ \qquad\qquad\quad \underline{\cancel{16x}\pm\cancel{24y}\pm\cancel{16}} \\ \qquad\qquad\qquad\quad 0 \end{array}$$

Square root = $\pm(2x+3y+4)$

(ii) $x^4-10x^3+37x^2-60x+36$ **(A.B+K.B)**

Solution: $x^4-10x^3+37x^2-60x+36$

$$\begin{array}{l} \quad x^2-5x+6 \\ x^2\,\big)\,\cancel{x^4}-10x^3+37x^2-60x+36 \\ \quad \underline{\pm\cancel{x^4}} \\ 2x^2-5x\,\big)\,-\cancel{10x^3}+37x^2-60x+36 \\ \qquad\quad \underline{\pm\cancel{10x^3}\pm25x^2} \\ \qquad 2x^2-10x+6\,\big)\,\cancel{12x^2}-\cancel{60x}+\cancel{36} \\ \qquad\qquad\qquad\qquad \underline{\pm\cancel{12x^2}\mp\cancel{60x}\pm\cancel{36}} \\ \qquad\qquad\qquad\qquad\qquad \times \end{array}$$

Square root $=\pm(x^2-5x+6)$

(iii) $9x^4-6x^3+7x^2-2x+1$

Solution:

$$9x^4-6x^3+7x^2-2x+1$$

$$\begin{array}{r|l}
 & 3x^2 - x + 1 \\
\hline
3x^2 & \cancel{9x^4} - 6x^3 + 7x^2 - 2x + 1 \\
 & \underline{\pm\cancel{9x^4}} \\
\hline
6x^2 - x & \cancel{-6x^3} + 7x^2 - 2x + 1 \\
 & \underline{\pm\cancel{6x^3} \pm x^2} \\
\hline
6x^2 - 2x + 1 & \cancel{6x^2} - \cancel{2x} + \cancel{1} \\
 & \underline{\pm\cancel{6x^2} \mp \cancel{2x} \pm \cancel{1}} \\
 & \quad\quad \times
\end{array}$$

Square root $\pm\left(3x^2 - x + 1\right)$

(iv) $4 + 25x^2 + 7x^2 - 2x + 1$ **(A.B)**

Solution: $4 + 25x^2 - 12x - 24x^3 + 16x^4$

$$\begin{array}{r|l}
 & 4x^2 - 3x + 2 \\
\hline
4x^2 & \cancel{16x^4} - 24x^3 + 25x^2 - 12x + 4 \\
 & \underline{\pm\cancel{16x^4}} \\
\hline
8x^2 - 3x & -\cancel{24x^3} + 25x^2 - 12x + 4 \\
 & \underline{\mp\cancel{24x^3} \pm 9x^2} \\
\hline
8x^2 - 6x + 2 & \cancel{16x^2} - \cancel{12x} + \cancel{4} \\
 & \underline{\pm\cancel{16x^2} \mp \cancel{12x} \pm \cancel{4}} \\
 & \quad\quad \times
\end{array}$$

Square root $= \pm\left(4x^2 - 3x + 2\right)$

(v) $\dfrac{x^2}{y^2} - \dfrac{10x}{y} + 27 - \dfrac{10y}{x} + \dfrac{y^2}{x^2}, (x \neq 0, y \neq 0)$

Solution: $\dfrac{x^2}{y^2} - \dfrac{10x}{y} + 27 - \dfrac{10y}{x} + \dfrac{y^2}{x^2}, (x \neq 0, y \neq 0)$

$$\frac{x}{y}-5+{}^{y}\!/\!{}_{x}$$

$$\frac{x}{y}\Big)\,\frac{\cancel{x^2}}{\cancel{y^2}}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2},$$

$$\pm\frac{\cancel{x^2}}{\cancel{y^2}}$$

$$\frac{2x}{y}-5\Big)\,-\frac{10\cancel{x}}{\cancel{y}}+27-\frac{10y}{x}+\frac{y^2}{x^2}$$

$$\mp\frac{10\cancel{x}}{\cancel{y}}\pm 25$$

$$\frac{2x}{y}-10+\frac{y}{x}\Big)\,+\cancel{2}-\frac{10\cancel{y}}{\cancel{c}}+\frac{\cancel{y^2}}{\cancel{x^2}}$$

$$\pm\cancel{2}\mp\frac{10\cancel{y}}{\cancel{x}}\pm\frac{\cancel{y^2}}{\cancel{x^2}}$$

$$\times$$

$$\textbf{Square root}=\pm\left(\frac{x}{y}-5+\frac{y}{x}\right)$$

Q.3 Find the value of k for which the following expressions will become a perfect square.

(i) $4x^4-12x^3+37x^2-42x+k$ **(A.B)**

Solution: $4x^4-12x^3+37x^2-42x+k$

$$2x^2-3x+7$$

$$2x^2\Big)\,\cancel{4x^4}-12x^3+37x^2-42x+k$$

$$\pm\cancel{4x^4}$$

$$4x^2-3x\Big)\,-\cancel{12x^3}+37x^2-42x+k$$

$$-\cancel{12x^3}\pm 9x^2$$

$$4x^2-6x+7\Big)\,\cancel{28x^2}-\cancel{42x}+k$$

$$\pm\cancel{28x^2}\mp\cancel{42x}\pm 49$$

$$k-49$$

In the case of perfect square remainder is always is equal to zero so

$k-49=0$

$k=49$

(ii) $x^4-4x^3+10x^2-kx+9$

Solution:

$$x^4-4x^3+10x^2-kx+9$$

$$x^2-2x+3$$

$$=x^2\overline{)\cancel{x^4}-4x^3+10x^2-kx+9}$$

$$\underline{\pm\cancel{x^4}}$$

$$2x^2-2x\overline{)-\cancel{4x^3}+10x^2-kx+9}$$

$$\underline{\mp\cancel{4x^3}\pm4x^2}$$

$$2x^2-4x+3\overline{)6x^2-kx+9}$$

$$\underline{-6x^2\mp12x\pm9}$$

$$-kx+12x=0$$

In the case of square root remainder is always equal to zero

$k-12=0$

$k=12$

Q.4 Find the value of l and m for which the following expression will be perfect square

(i) $x^4+4x^3+16x^2+lx+m$ **(A.B)**

Solution:

$$x^4+4x^3+16x^2+lx+m$$

$$x^2+2x+6$$

$$=x^2\overline{)\cancel{x^4}+4x^3+16x^2+lx+m}$$

$$\underline{\mp\cancel{x^4}}$$

$$2x^2+2x\overline{)\cancel{4x^3}+16x^2+lx+m}$$

$$\underline{\pm\cancel{4x^3}\pm4x^2}$$

$$2x^2+4x+6\overline{)\cancel{12x^2}+lx+m}$$

$$\underline{\pm\cancel{12x^2}\pm24x\pm36}$$

$$\left(l-24\right)x=0,\quad m-36=0$$

In the case of square root remainder is always zero

$l-24=0$, $m-36=0$

$l=24$, $m=36$

(ii) $49x^4 - 70x^3 + 109x^2 + lx - m$

Solution:

$$49x^4 - 70x^3 + 109x^2 + lx - m$$

$$7x^2 - 5x + 6$$

$$7x^2 \overline{\big)\, \cancel{49x^4} - 70x^3 + 109x^2 + lx - m}$$

$$\underline{\pm \cancel{49x^4}}$$

$$14x^2 - 5x \overline{\big)\, -\cancel{70x^3} + 109x^2 + lx - m}$$

$$\underline{\mp \cancel{70x^3} \pm 25x^2}$$

$$14x^2 - 10x + 6 \overline{\big)\, \cancel{84x^2} + lx - m}$$

$$\underline{\pm \cancel{84x^2} \mp 60x \pm 36}$$

$$lx + 60x - m - 36$$

$$(l + 60)x - m - 36$$

In the case of square root remainder is always equal to zero

$l + 60 = 0 \quad -m - 36 = 0$

$l + 60 = 0\,, \quad -m = 36$

$l = -60 \quad , \quad m = -36$

Q.5 **To make the expression** $9x^4 - 12x^3 + 22x^2 - 13x + 12$ **a perfect square** **(A.B)**

Solution: $9x^4 - 12x^3 + 22x^2 - 13x + 12$

$$3x^2 - 2x + 3$$

$$= 3x^2 \overline{\big)\, \cancel{9x^4} - 12x^3 + 22x^2 - 13x + 12}$$

$$\underline{\pm \cancel{9x^4}}$$

$$6x^2 - 2x \overline{\big)\, -\cancel{12x^3} + 22x^2 - 13x + 12}$$

$$\underline{\mp \cancel{12x^3} \pm 4x^2}$$

$$6x^2 - 4x + 3 \overline{\big)\, \cancel{18x^2} - 13x + 12}$$

$$\underline{\pm \cancel{18x^2} \mp 12x \pm 9}$$

$$-x + 3$$

(i) $+x - 3$ **is to be added**

(ii) $-x + 3$ **is to be subtract from it**

(iii) $-x + 3 = 0$

$x = 3$

Review Exercise 6

Q.1 Choose the correct answer.

(i) **H.C.F of** p^3q-pq^3 **and** $p^5q^2-pq^5$ **is** ________ **(U.B)**

(a) $pq(p^2-q^2)$ **(b)** $pq(p-q)$

(c) $p^2q^2(p-q)$ **(d)** $pq(p^3-q^3)$

(ii) **H.C.F of** $5x^2y^2$ *and* $20x^3y^3$ is ________ **(K.B)**

(FSD 2014, SWL 2013, MTN 2013, D.G.K 2013)

(a) $5x^2y^2$ **(b)** $20x^3y^3$

(c) $100x^5y^5$ **(d)** $5xy$

(iii) **H.C.F of** $x-2$ **and** x^2+x-6 is ______ **(U.B)**

(a) x^2+x-6 **(b)** $x+3$

(c) $x-2$ **(d)** $x+2$

(iv) **H.C.F of** a^3+b^3 **and** a^2-ab+b^2 is ______ **(U.B)**

(a) $a+b$ **(b)** a^2-ab+b^2

(c) $(a-b)^2$ **(d)** a^2+b^2

(v) **H.C.F of** x^2-5x+6 **and** x^2-x-6 is______ **(K.B)**

(GRW 2013, FSD 2013, BWP 2013, BWP 2013, 16)

(a) $x-3$ **(b)** $x+2$

(c) x^2-4 **(d)** $x-2$

(vi) **H.C.F of** a^2-b^2 **and** a^3-b^3 is______ **(U.B)**

(a) $a-b$ **(b)** $a+b$

(c) a^2+ab+b^2 **(d)** a^2-ab+b^2

(vii) **H.C.F of** x^2+3x+2, x^2+4x+3 **and** x^2+5x+4 is______ **(K.B)**

(a) $x+1$ **(b)** $(x+1)(x+2)$

(c) $x+3$ **(d)** $(x+4)(x+1)$

(viii) **L.C.M of** $15x^2, 45xy$ **and** $30xyz$ is______ **(A.B)**

(a) $90xyz$ **(b)** $90x^2yz$

(c) $15xyz$ **(d)** $15x^2yz$

(ix) **L.C.M of** a^2+b^2 **and** a^4-b^4 is______ **(K.B)**

(a) a^2+b^2 **(b)** a^2-b^2

(c) a^4-b^4 **(d)** $a-b$

(x) **The product of two algebraic expression is equal to the ________ of their H.C.F and L.C.M** **(K.B)**

(a) Sum **(b)** Difference

(c) Product **(d)** Quotient

(xi) **Simplify** $\dfrac{a}{9a^2-b^2}+\dfrac{1}{3a-b}$ is_______ **(A.B)**

(a) $\dfrac{4a}{9a^2-b^2}$ **(b)** $\dfrac{4a-b}{9a^2-b^2}$

(c) $\dfrac{4a+b}{9a^2-b^2}$ **(d)** $\dfrac{b}{9a^2-b^2}$

(xii) **Simplify** $\frac{a^2+5a-14}{a^2-3a-18}\times\frac{a+3}{a-2}=$_______ **(A.B)**

(a) $\frac{a+7}{a-6}$ **(b)** $\frac{a+7}{a-2}$

(c) $\frac{a+3}{a-6}$ **(d)** $\frac{a-2}{a+3}$

(xiii) **Simplify the** $\frac{a^3-b^3}{a^4-b^4}\div\frac{a^2+ab+b^2}{a^2+b^2}=$_______ **(A.B)**

(a) $\frac{1}{a+b}$ **(b)** $\frac{1}{a-b}$

(c) $\frac{a-b}{a^2+b^2}$ **(d)** $\frac{a+b}{a^2+b^2}$

(xiv) **Simplify** $\left(\frac{2x+y}{x+y}-1\right)\div\left(1-\frac{x}{x+y}\right)=$ _________ **(A.B)**

(a) $\frac{x}{x+y}$ **(b)** $\frac{y}{x+y}$

(c) $\frac{y}{x}$ **(d)** $\frac{x}{y}$

(xv) **The square root of** a^2-2a+1 is ________ **(K.B)**

(LHR 2017, GRW 2017, MTN 2014, 15, 16, 17, SGD 2013)

(a) $\pm(a+1)$ **(b)** $\pm(a-1)$

(c) $a-1$ **(d)** $a+1$

(xvi) **What should be added to complete the square of** x^4+64 ?______

(a) $8x^2$ **(b)** $-8x^2$

(c) $16x^2$ **(d)** $4x^2$

(xvii) **The square root to** $x^4+\frac{1}{x^4}+2$ **is**________ **(SWL 2013, BWP 2016)** **(K.B)**

(a) $\pm\left(x+\frac{1}{x}\right)$ **(b)** $\pm\left(x^2+\frac{1}{x^2}\right)$

(c) $\pm\left(x-\frac{1}{x}\right)$ **(d)** $\pm\left(x^2-\frac{1}{x^2}\right)$

ANSWER KEYS

1	b	5	a	9	c	13	a	17	b
2	a	6	a	10	c	14	d		
3	c	7	a	11	c	15	b		
4	b	8	b	12	a	16	c		

Q.2 **Find the H.C.F of the following by factorization.**

$8x^4-128, 12x^3-96$

Solution:

$$8x^4-128 \quad =8\left(x^4-16\right) \quad =8\left[(x)^2-(4)^2\right]$$

$$=2\times2\times2\left(x^2+4\right)\left(x^2-4\right) \qquad a^2-b^2=(a+b)(a-b)$$

$$=2\times2\times2\left(x^2+4\right)(x+2)(x-2)$$

$$12x^3-96 \quad =12(x^3-8)$$

$$=\left(12(x^3-2^3\right) \qquad a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$$

$$=12(x-2)(x^2+2x+4)$$

$$2\times2\times3(x-2)\left(x^2+2x+4\right)$$

H.C.F = $2\times2(x-2)$

$=4(x-2)$

Q.3 **Find the H.C.F of the following by division method** $y^3+3y^2-3y-9,\ y^3+3y^2-8y-24$**.**

(A.B)

Solution:

$$\begin{array}{r|l} & 1 \\ \hline y^3+3y^2-8y-24 & \cancel{y^3}+\cancel{3y^2}-3y-9 \\ & \underline{\pm\cancel{y^3}\pm\cancel{3y^2}\mp8y\mp24} \\ & \qquad 5y+15 \\ & 5(y+3) \end{array}$$

$$\begin{array}{r|l} & y^2-8 \\ \hline y+3 & \cancel{y}3+3y^2-8y-24 \\ & \underline{\pm\cancel{y}3\pm3y^2} \\ & \;\;-\cancel{8y}-\cancel{24} \\ & \;\;\underline{\pm\cancel{8y}\pm\cancel{24}} \\ & \qquad\times \end{array}$$

H.C.F $=(y+3)$

Q.4 Find the L.C.M of the following by factorization.

$12x^2-75,\ 6x^2-13x-5,\ 4x^2-20x+25$

Solution:

$$12x^2-75=3\left(4x^2-25\right)$$
$$=3\left[(2x)^2-(5)^2\right] \qquad \because\ a^2-b^2=(a+b)(a-b)$$
$$=3(2x-5)(2x+5)$$
$$6x^2-13x-5=6x^2-15x+2x-5$$
$$=3x(2x-5)+1(2x-5)$$
$$=(2x-5)(3x+1)$$
$$4x^2-20x+25=4x^2-10x-10x+25$$
$$=2x(2x-5)-5(2x-5)$$
$$=(2x-5)(2x-5)$$

Common factor = $(2x–5)$

Non common factor $=3(3x+1)(2x-5)(2x+5)$

L.C.M = common factor $\times$ non common factor

L.C.M = $(2x-5)3(3x+1)(2x+5)(2x-5)$

$$\text{L.C.M}=3(2x+5)(2x-5)^2(3x+1)$$

Q.5 If H.C.F of $x^4+3x^3+5x^2+26x+56$ and $x^4+2x^3-4x^2-x+28$ is x^2+5x+7 find their L.C.M. **(A.B+K.B)**

Solution: $p(x)=x^4+3x^3+5x^2+26x+56$ and $q(x)=x^4+2x^3-4x^2-x+28$

H.C.F. $=x^2+5x+7$, L.C.M. = ?

$$\text{L.C.M.}=\frac{P(x)\times q(x)}{\text{H.C.F.}}$$ **(K.B)**

$$\text{L.C.M}=\frac{\left(x^4+3x^2+5x^2+26x+56\right)\times\left(x^4+2x^3-4x^2-x+28\right)}{\left(x^2+5x+7\right)}$$

$$\begin{array}{r|l}
 & x^2-3x+4 \\ \hline
x^2+5x+7 & \cancel{x^4}+2x^3-4x^2-x+28 \\
 & \underline{\pm\cancel{x^4}\pm5x^3\pm7x^2} \\
 & \quad -\cancel{3x^3}-11x^2-x+28 \\
 & \quad \underline{\mp\cancel{3x^3}\mp15x^2\mp21x} \\
 & \quad +\cancel{4x^2}+\cancel{20x}+\cancel{28} \\
 & \quad \underline{\pm\cancel{4x^2}\pm\cancel{20x}\pm\cancel{28}} \\
 & \qquad 0
\end{array}$$

$$\text{L.C.M} = \frac{q(x)\,p(x)}{H.C.F}$$

$$L.C.M = \frac{(x^4+3x^3+5x^2+26x+56)\,\overset{x^2-3x+4}{\cancel{(x^4+2x^3-4x^2-x+28)}}}{\cancel{(x^2+5x+7)}}$$

$$L.C.M = (x^4+3x^3+5x^2+26x+56)(x^2-3x+4)$$

Q.6 Simplify: **(A.B)**

Solution:

(i) $\dfrac{3}{x^3+x^2+x+1}-\dfrac{3}{x^3-x^2+x-1}$

Solution: $\dfrac{3}{x^3+x^2+x+1}-\dfrac{3}{x^3-x^2+x-1}$

$$=\frac{3}{x^2(x+1)+1(x+1)}-\frac{3}{x^2(x-1)+1(x-1)}$$

$$=\frac{3}{(x^2+1)(x+1)}-\frac{3}{(x-1)(x^2+1)}$$

$$=\frac{3(x-1)-3(x+1)}{(x+1)(x-1)(x^2+1)}$$

$$=\frac{\cancel{3x}-3-\cancel{3x}-3}{(x+1)(x-1)(x^2+1)}$$

$$=\frac{-6}{(x+1)(x-1)(x^2+1)}=\frac{-6}{(x^2-1)(x^2+1)}$$

$$=\frac{-6}{(x^4-1)}$$

$$=\frac{6}{1-x^4}$$

(ii) $\frac{a+b}{a^2-b^2} \div \frac{a^2-ab}{a^2-2ab+b^2}$

Solution: $\frac{a+b}{a^2-b^2} \div \frac{a^2-ab}{a^2-2ab+b^2}$

$$=\frac{a+b}{a^2-b^2}\times\frac{a^2-2ab+b^2}{a^2-ab}$$

$a^2-2ab+b^2=(a-b)^2$

$a^2-b^2=(a+b)(a-b)$

$$=\frac{\cancel{a+b}}{(a-b)\left(\cancel{a+b}\right)}\times\frac{(a-b)^2}{a(a-b)}$$

$$=\frac{\left(\cancel{a-b}\right)^2}{a\left(\cancel{a-b}\right)^2}$$

$$=\frac{1}{a}$$

Q.7 Find the square root by using factorization. $\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27 \quad (x\neq 0)$.

(BWP 2013, 14, 16, FSD 2015,) (A.B+K.B)

Solution: $\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27$

$$=\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+25+2$$

$$=\left(x^2+\frac{1}{x^2}+2\right)+10\left(x+\frac{1}{x}\right)+25$$

$$=\left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)\times5+(5)^2$$

$$=\left[x+\frac{1}{x}+5\right]^2$$

$a^2+2ab+b^2=(a+b)^2$

Taking the square root

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27}=\sqrt{\left[x+\frac{1}{x}+5\right]^2}$$

$$=\pm\left(x+\frac{1}{x}+5\right)$$

Q.8 **Find the square roots0 by using division method.** $\frac{4x^2}{y^2}+\frac{20x}{y}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$ **(A.B)**

Solution:

$$\frac{2x}{y}+5-\frac{3y}{x}$$

$$\frac{2x}{y}\Big)\ \cancel{\frac{4x^2}{y^2}}+\frac{20x}{4}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$$

$$\pm\cancel{\frac{4x^2}{y^2}}$$

$$\frac{4x}{y}+5\Big)\ \cancel{\frac{20x}{y}}+13-\frac{30y}{x}+\frac{9y^2}{x^2}$$

$$\pm\cancel{\frac{20x}{y}}\pm 25$$

$$\frac{4x}{y}+10-\frac{3y}{x}\Big)\ -\cancel{12}-\cancel{\frac{30y}{x}}+\cancel{\frac{9y^2}{x^2}}$$

$$\mp\cancel{12}\mp\cancel{\frac{30y}{x}}\pm\cancel{\frac{9y^2}{x^2}}$$

$$\times$$

Square root $=\pm\left[\frac{2x}{y}+5-\frac{3y}{x}\right]$

CUT HERE

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 If two are more algebraic expressions are given then their common factors of highest power is called of the expressions

(A) L.C.M. **(B)** H.C.F.

(C) Multiplication **(D)** Square root

2 In $(x^2 - 4)$ and $(x^2 + 4x + 4)$, the H.C.F is

(A) $x^2 -4$ **(B)** $x + 4$

(C) $x +2$ **(D)** $(x - 2)^2$

3 L.C.M of $15x^2$, $45xy$ **and** $30xy$ **is**

(A) $90xyz$ **(B)** $90x^2yz$

(C) $15xyz$ **(D)** $15x^2yz$

4 Simplify $\frac{a^2+5a-14}{a^2-3a-18}\times\frac{a+3}{a-2}=$

(A) $\frac{a+7}{a-6}$ **(B)** $\frac{a+7}{a-2}$

(C) $\frac{a+3}{a-6}$ **(D)** $\frac{a-2}{a+3}$

5 Simplify $\frac{a}{9a^2-b^2}+\frac{1}{3a-b}=$

(A) $\frac{4a}{9a^2-b^2}$ **(B)** $\frac{4a-b}{9a^2-b^2}$

(C) $\frac{4a+b}{9a^2-b^2}$ **(D)** $\frac{b}{9a^2-b^2}$

6 The square root of $x^4+\frac{1}{x^4}+2$ **is:**

(A) $\pm\left(x+\frac{1}{x}\right)$ **(B)** $\pm\left(x^2+\frac{1}{x^2}\right)$

(C) $\pm\left(x-\frac{1}{x}\right)$ **(D)** $\pm\left(x^2-\frac{1}{x^2}\right)$

7 H.C.F of $\left(p^3q-pq^3\right)$ **and** $\left(p^5q^2-p^2q^5\right)$ **is:**

(A) $pq\left(p^2-q^2\right)$ **(B)** $pq\left(p-q\right)$

(C) $p^2q^2\left(p-q\right)$ **(D)** $pq\left(p^3-q^3\right)$

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Find the H.C.F of the following by factorization $8x^4 - 128$, $12x^3 - 96$

(ii) Find H.C.F, by division method $x^3 + 3x^2 - 16x + 12$, $x^3 + x^2 - 10x + 8$

(iii) Find L.C.M by factorization $x^2 - 25x + 100$ and $x^2 - x - 20$.

(iv) Simplify: $\frac{x^2 - x - 6}{x^2 - 9} + \frac{x^2 + 2x - 24}{x^2 - x - 12}$.

(v) Find the Square Root of the Expression: $\frac{1}{16}x^2 - \frac{1}{12}xy + \frac{1}{36}y^2$

Q.3 Answer the following Questions in detail. **(4+4=8)**

(a) To make the expression $9x^4 - 12x^3 + 22x^2 - 13x + 12$ perfect square

- **(i)** What should be subtracted from it
- **(ii)** What should be added to it
- **(iii)** What should be the value of *x*.

(b) Simplify: $\frac{x+1}{x^2 - 3x + 2} + \frac{x}{x^2 - 4x + 3} + \frac{1}{x^2 - 5x + 6}$

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.