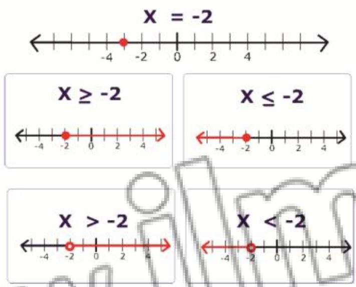

UNIT 7
LINEAR EQUATIONS AND INEQUALITIES

Linear Equation (U.B)

(LHR 2014, GRW 2014, FSD 2014, 15)

A polynomial equation of degree one is called linear equation.

A linear equation in one unknown variable x is an equation of the form $ax + b = 0$, where $a, b \in R$ and $a \neq 0$.

Technique for Solving Linear Equations (K.B)

The procedure for solving linear equations in one variable is summarized in the following

- If fractions are present, we multiply each side by the L.C.M of the denominators to eliminate them.
- To remove parentheses we use the distributive property.
- Combine alike terms, if any on both sides.
- Use the addition property of equality (add or subtract) to get all the variables on left side and constants on the other side.
- Use the multiplicative property of equality to isolate the variable.
- Verify the answer by replacing the variable in the original equation.

Example 1 (Page # 131) **(A.B)**

$$\frac{3x}{2}-\frac{x-2}{3}=\frac{25}{6}$$

(GRW 2017, SWL 2017, MTN 2016, D.G.K 2015)

Solution:

$$\frac{3x}{2}-\frac{x-2}{3}=\frac{25}{6}$$

$$\frac{3(3x)-2(x-2)}{6}=\frac{25}{6}$$

Multiplying both sides by 6

$$9x-2(x-2)=25$$
$$9x-2x+4=25$$
$$7x=25-4$$
$$7x=21$$
$$x=\frac{21}{7}$$
$$x=3$$

Check

Substituting $x=3$ in original equation,

$$\frac{3}{2}(3)-\frac{3-2}{3}=\frac{25}{6}$$
$$\frac{9}{2}-\frac{1}{3}=\frac{25}{6}$$
$$\frac{27-2}{6}=\frac{25}{6}$$
$$\frac{25}{6}=\frac{25}{6}, \text{ which is true}$$

$\therefore$ Solution Set $=\{3\}$

Note

Some fractional equation may have no solution.

Example 2 (Page # 132) **(A.B)**

Solve: $\frac{3}{y-1}-2=\frac{3y}{y-1}$, $y \neq 1$

(LHR 2013, GRW 2016, SWL 2014, 16, BWP 2016, D.G.K 2017)

Solution:

$$\frac{3}{y-1}-2=\frac{3y}{y-1}$$

To clear fractions we multiply both sides by $\text{L.C.M} = y-1$

$3-2(y-1)=3y$

$3-2y+2=3y$

$-5y=-5$

$\Rightarrow y=1$

Check

Substituting $y=1$ in the given equation, we have

$\frac{3}{1-1}-2=\frac{3(1)}{1-1}$

$\frac{3}{0}-2=\frac{3}{0}$

But $\frac{3}{0}$ is undefined, so $y=1$ cannot be a solution.

Thus, Solution Set $=\{\ \}$

Example 3 (Page # 132) **(A.B)**

Solve: $\frac{3x-1}{3}-\frac{2x}{x-1}=x \quad , \quad x\neq 1$

Solution:

$\frac{3x-1}{3}-\frac{2x}{x-1}=x$

To clear fractions we multiply each side by $3(x-1)$, we get

$(x-1)(3x-1)-6x=3x(x-1)$

$3x^2-4x+1-6x=3x^2-3x$

$-10x+1=-3x$

$-10x+3x=-1$

$-7x=-1$

$7x=1$

$\Rightarrow x=\frac{1}{7}$

Check

On substituting $x=\frac{1}{7}$ the original equation is verified a true statement. That means the restriction $x\neq 1$ has no effect on the solution because $\frac{1}{7}\neq 1$.

$\therefore$ Solution Set $=\left\{\frac{1}{7}\right\}$

Equation Involving Radicals but Reducible to Linear form

Radical Equation (K.B)

When the variable in an equation occurs under a radical sign, the equation is called a radical equation.

For example:

$\sqrt{2x-3}-7=0$

Extraneous Solutions (K.B)

When raising each sides of the radical equation to a certain power may produce a non equivalent equation that has more solutions than the original equation. These additional solutions are called extraneous solutions.

Note (U.B)

- We must check out answer(s) for such solutions when working with radical equations.
- An important point to be noted is that raising each side to an odd power will always give an equivalent equation whereas raising each side to an even power might not do so.

Example 1 (Page # 133) **(A.B)**

(a)

(b) Solve the equation $\sqrt{2x-3}-7=0$

Solution:

$\sqrt{2x-3}-7=0 \rightarrow (i)$

$\sqrt{2x-3}=7$

Squaring both sides

$2x-3=49$

$2x=52$

$\Rightarrow x=26$

Check

Put $x=26$ in the equation (i)

$$\sqrt{2(26)-3}-7=0$$
$$\sqrt{52-3}-7=0$$
$$\sqrt{49}-7=0$$
$$7-7=0$$
$$0=0$$

Hence, Solution Set $=\{26\}$.

(c)

$$\sqrt[3]{3x+5}=\sqrt[3]{x-1}\rightarrow(i)$$

Taking cube of each side

$$3x+5=x-1$$
$$2x=-6$$
$$x=-3$$

Check

Put $x=-3$ in equation (i)

$$\sqrt[3]{3(-3)+5}=\sqrt[3]{-3-1}$$
$$\Rightarrow\sqrt[3]{-4}=\sqrt[3]{-4}$$

Thus $x=-3$ satisfies the originals equation

Thus, the solution set $=\{-3\}$

Example # 2

Solve and check $\sqrt{5x-7}-\sqrt{x+10}=0$

Solution:

$$\sqrt{5x-7}-\sqrt{x+10}=0\rightarrow(i)$$
$$\sqrt{5x-7}=\sqrt{x+10}$$

Squaring both sides

$$5x-7=x+10$$
$$5x-x=10+7$$
$$4x=17$$
$$\Rightarrow x=\frac{17}{4}$$

Check

Substituting $x=\frac{17}{4}$ in equation (i)

$$\sqrt{5x-7}-\sqrt{x+10}=0$$
$$\sqrt{5\left(\frac{17}{4}\right)-7}-\sqrt{\frac{17}{4}+10}=0$$
$$\sqrt{\frac{85-28}{4}}-\sqrt{\frac{17+40}{4}}=0$$
$$\sqrt{\frac{57}{4}}-\sqrt{\frac{57}{4}}=0$$
$$0=0$$

i.e., $x=\frac{17}{4}$ makes the given equation true

Thus, Solution Set $=\left\{\frac{17}{4}\right\}$

Example # 3 (Page # 135) **(A.B)**

Solve $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

Solution:

$$\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$$

Squaring both sides, we get

$$\left(\sqrt{x+7}+\sqrt{x+2}\right)^2=\left(\sqrt{6x+13}\right)^2$$
$$\left(\sqrt{x+7}\right)^2+\left(\sqrt{x+2}\right)^2+2\sqrt{x+7}.\sqrt{x+2}=6x+13$$
$$x+7+x+2+2\sqrt{(x+7)(x+2)}=6x+13$$
$$2x+9+2\sqrt{x^2+2x+7x+14}=6x+13$$
$$2\sqrt{x^2+9x+14}=6x-2x+13-9$$
$$2\sqrt{x^2+9x+14}=4x+4$$
$$2\sqrt{x^2+9x+14}=2(2x+2)$$
$$\Rightarrow\sqrt{x^2+9x+14}=2x+2$$

Again squaring both sides

$$\left(\sqrt{x^2+9x+14}\right)^2=(2x+2)^2$$
$$x^2+9x+14=4x^2+8x+4$$
$$0=4x^2-x^2+8x-9x+4-14$$

Or $3x^2-x-10=0$

$$\Rightarrow 3x^2-6x+5x-10=0$$
$$3x(x-2)+5(x-2)=0$$
$$(x-2)(3x+5)=0$$

Either $x-2=0$ or $3x+5=0$

$$\Rightarrow x=2 \text{ or } x=-\frac{5}{3}$$

On checking we see that $x=2$ satisfies the equation, but $x=-\frac{5}{3}$ does not satisfy the equation.

$\therefore$ **Solution Set** $=\{2\}$ and $x=-\frac{5}{3}$ is an extraneous root.

Inconsistent Equation **(U.B)**

Equations having no solution are called inconsistent equation.

For example

$2x+3=2x-5$

Conditional Equation **(U.B)**

An equation which is true for some specific value(s) of the variable is called conditional equation.

For example

$2x+3=0$

Equivalent Equations **(U.B)**

Equations have same solutions are called equivalent equations.

For example

$x+2y=5 \quad ; \quad 2x+4y=10$

Exercise 7.1

Q.1 Solve the following equations

(i) $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$ **(LHR 2013)**

Solution:

$$\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$$
$$\frac{4x-3x}{6}=\frac{6x+1}{6}$$
$$x=6\frac{(6x+1)}{6}$$
$$x=6x+1$$
$$-6x+x=1$$
$$-5x=1$$
$$x=\frac{1}{-5}$$
$$x=-\frac{1}{5}$$

Check

Substitution $x=-\frac{1}{5}$

$$\frac{2}{3}\times\frac{-1}{5}-\frac{1}{2}\times\frac{-1}{5}=\frac{-1}{5}+\frac{1}{6}$$
$$\frac{-2}{15}+\frac{1}{10}=\frac{-6+5}{30}$$
$$\frac{-2\times2+1\times3}{30}=\frac{-1}{30}$$
$$\frac{-4+3}{30}=\frac{-1}{30}$$
$$\frac{-1}{30}=\frac{-1}{30}$$

Solution Set $=\left\{-\frac{1}{5}\right\}$

(ii) $\frac{x-3}{3}-\frac{x-2}{2}=-1$

(FSD 2017, RWP 2016, D.G.K 2016)

Solution

$$\frac{x-3}{3}-\frac{x-2}{2}=-1$$

By taking L.C.M

$$\frac{2(x-3)-3(x-2)}{6}=-1$$
$$2x-6-3x+6=-6$$
$$-x=-6$$
$$\Rightarrow x=6$$

Check

$$\frac{x-3}{3}-\frac{x-2}{2}=-1$$

When $x=6$

$$\frac{6-3}{3}-\frac{6-2}{2}=-1$$
$$\frac{3}{3}-\frac{4}{2}=-1$$
$$\frac{6-12}{6}=-1$$
$$\frac{-6}{6}=-1$$
$$-1=-1$$

Solution Set $=\{6\}$

(iii) $\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$

Solution:

$$\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$$

Taking L.C.M of brackets

$$\frac{1}{2}\left(\frac{6x-1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1-6x}{2}\right)$$
$$\frac{6x-1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1-6x}{6}$$
$$\frac{6x-1+8}{12}=\frac{5+1-6x}{6}$$
$$\frac{\cancel{6}(6x+7)}{\cancel{12}^{2}}=6-6x$$
$$\frac{6x+7}{2}=6-6x$$
$$6x+7=2(6-6x)$$
$$6x+7=12-12x$$
$$6x+12x=12-7$$
$$18x=5$$
$$x=\frac{5}{18}$$

Check **(A.B)**

$$\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$$

When $x=\frac{5}{18}$

$$\frac{1}{2}\left[\frac{15}{18}-\frac{1}{6}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{1}{2}-3\left(\frac{5}{18}\right)\right]$$
$$\frac{1}{2}\left[\frac{5-3}{18}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{1}{2}-\frac{5}{6}\right]$$
$$\frac{1}{2}\left[\frac{\cancel{2}}{18}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{3-5}{6}\right]$$
$$\frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{18}\right]+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{-\cancel{2}^{1}}{\cancel{6}^{3}}\right]$$
$$\frac{1}{18}+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left[\frac{-1}{3}\right]$$
$$\frac{1+12}{18}=\frac{5}{6}-\frac{1}{9}$$
$$\frac{13}{18}=\frac{15-2}{18}$$
$$\frac{13}{18}=\frac{13}{18}$$

Solution Set $=\left\{\frac{5}{18}\right\}$

(iv) $x+\frac{1}{3}=2\left[x-\frac{2}{3}\right]-6x$

Solution:

$$x+\frac{1}{3}=2\left[x-\frac{2}{3}\right]-6x$$
$$\frac{3x+1}{3}=2\left[\frac{3x-2}{3}\right]-6x$$
$$\frac{3x+1}{3}=\frac{6x-4}{3}-6x$$

Taking L.C.M of right side

$$\frac{3x+1}{3}=\frac{6x-4-18x}{3}$$
$$\frac{3x+1}{\cancel{3}}=\frac{(-12x-4)}{\cancel{3}}$$
$$3x+1=-12x-4$$
$$3x+12x=-4-1$$
$$15x=-5$$
$$x=\frac{-5}{15}$$
$$x=\frac{-1}{3}$$

Check **(A.B)**

$$x+\frac{1}{3}=2\left[x-\frac{2}{3}\right]-6x$$

When $x=\frac{-1}{3}$

$$\frac{-\cancel{1}}{\cancel{3}}+\frac{\cancel{1}}{\cancel{3}}=2\left[\frac{-1}{3}-\frac{2}{3}\right]-6\left(\frac{-1}{3}\right)$$
$$0=2\left[\frac{-1-2}{3}\right]+\frac{\cancel{6}^{2}}{\cancel{3}}$$
$$0=2\left[\frac{-\cancel{3}}{\cancel{3}}\right]+2$$
$$0=2(-1)+2$$
$$0=-\cancel{2}+\cancel{2}$$
$$0=0$$

Solution Set $=\left\{\frac{-1}{3}\right\}$

(v) $\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

Solution

$\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

$\frac{5x-15-6x}{6}=\frac{9-x}{9}$

$\frac{-15-x}{6}=\frac{9-x}{9}$

$9(-15-x)=6(9-x)$

$-135-9x=54-6x$

$-135-54=-6x+9x$

$-189=3x$

$\frac{-189}{3}=x$

$x=-63$

Check

$\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

When $x=-63$

$\frac{5(-63-3)}{6}-(-63)=1-\frac{(-63^{7})}{9}$

$\frac{5(-66^{11})}{6}+63=1+7$

$-55+63=8$

$8=8$

Solution Set $=\{-63\}$

(vi) $\frac{x}{3x-6}=2-\frac{2x}{x-2}, x\neq 2$ **(A.B)**

Solution:

$\frac{x}{3x-6}=2-\frac{2x}{x-2}, x\neq 2$

$\frac{x}{3(x-2)}=\frac{2(x-2)-2x}{x-2}$

$\frac{x}{3(x-2)}=\frac{2x-4-2x}{x-2}$

$\frac{x}{3(x-2)}=\frac{-4}{x-2}$

$x(x-2)=-4\times 3(x-2)$

$x(x-2)=-12(x-2)$

$x(x-2)+12(x-2)=0$

$(x-2)(x+12)=0$

$x-2=0,\text{or } x+12=0$

$x=2,\text{or } x=-12$

$x=2$ (Rejected because $x\neq 2$)

Hence $x=-12$

Check

$\frac{x}{3x-6}=2-\frac{2x}{x-2}$

When $x=-12$

$\frac{-12}{3(-12)-6}=2-\frac{2(-12)}{-12-2}$

$\frac{-12}{-36-6}=2+\frac{24}{-14}$

$\frac{-12}{-42}=2-\frac{24}{14}$

$\frac{12}{42}=2-\frac{12}{7}$

$\frac{2}{7}=\frac{14-12}{7}$

$\frac{2}{7}=\frac{2}{7}$

Solution Set $=\{-12\}$

(vii) $\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}$ **(A.B)**

Solution

$\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}$

$\frac{2x}{2x+5}=\frac{2(4x+10)-3\times 5}{3(4x+10)}$

$\frac{2x\times 3(4x+10)}{2x+5}=8x+20-15$

$\frac{6x\times 2(2x+5)}{(2x+5)}=8x+5$

$12x=8x+5$

$12x-8x=5$

` $4x=5$

$x=\frac{5}{4}$

Check

$$\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}$$

When $x=\frac{5}{4}$

$$\frac{2\left(\frac{5}{4}\right)}{2\left(\frac{5}{4}\right)+5}=\frac{2}{3}-\frac{5}{4\left(\frac{5}{4}\right)+10}$$

$$\frac{\frac{5}{2}}{\frac{5}{2}+5}=\frac{2}{3}-\frac{5}{5+10}$$

$$\frac{\frac{5}{2}}{\frac{5+10}{2}}=\frac{2}{3}-\frac{5}{15}$$

$$\frac{\frac{5}{2}}{\frac{15}{2}}=\frac{2}{3}-\frac{1}{3}$$

$$\frac{\cancel{5}}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{15}^{3}}=\frac{2-1}{3}$$

$$\frac{1}{3}=\frac{1}{3}$$

Solution Set $=\left\{\frac{5}{4}\right\}$

(viii) $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}\qquad x\neq 1$ **(A.B)**

Solution:

$$\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}$$

$$\frac{3\times 2x+1(x-1)}{3(x-1)}=\frac{5(x-1)+2\times 6}{6(x-1)}$$

$$\frac{6x+x-1}{3(x-1)}=\frac{5x-5+12}{6(x-1)}$$

$$\frac{7x-1}{3(x-1)}=\frac{5x-5+12}{6(x-1)}$$

$$7x-1=\frac{5x+7}{2}$$

$$2(7x-1)=5x+7$$

$$14x-2=5x+7$$

$$14x-5x=7+2$$

$$9x=9$$

$$x=\frac{9}{9}$$

$$x=1$$

No solution because $x\neq 1$.

(ix) $\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1}\qquad x\neq\pm 1$ **(A.B)**

Solution

$$\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1}$$

$$\frac{2}{(x-1)(x+1)}-\frac{1}{x+1}=\frac{1}{x+1}$$

$$\frac{2-(x-1)}{(x-1)(x+1)}=\frac{1}{x+1}$$

$$\frac{2-(x-1)}{(x-1)(x+1)}=\frac{1}{x+1}$$

$$2-x+1=\frac{(x-1)(x+1)}{(x+1)}$$

$$3-x=x-1$$

$$1+3=x+x$$

$$4=2x$$

$$\frac{4}{2}=x$$

$$x=2$$

Check

$$\frac{2}{2^2-1}-\frac{1}{2+1}=\frac{1}{2+1}$$

$$\frac{2}{4-1}-\frac{1}{3}=\frac{1}{3}$$

$$\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

$$\frac{2-1}{3}=\frac{1}{3}$$

$$\frac{1}{3}=\frac{1}{3}$$

Solution Set $=\{2\}$

(x) $\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4}$ **(A.B)**

Solution:

$$\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4}$$
$$\frac{2}{3(x+2)}=\frac{1}{6}-\frac{1}{2(x+2)}$$
$$\frac{2}{3(x+2)}=\frac{x+2-3}{6(x+2)}$$
$$\frac{2\times 6(x+2)}{3(x+2)}=x-1$$
$4=x-1$
$4+1=x$
$x=5$

Check

$$\frac{2}{3(5)+6}=\frac{1}{6}-\frac{1}{2(5)+4}$$
$$\frac{2}{15+6}=\frac{1}{6}-\frac{1}{10+4}$$
$$\frac{2}{21}=\frac{1}{6}-\frac{1}{14}$$
$$\frac{2}{21}=\frac{7-3}{42}$$
$$\frac{2}{21}=\frac{\cancel{4}^{2}}{\cancel{42}_{21}}$$
$$\frac{2}{21}=\frac{2}{21}$$

Solution Set $=\{5\}$

Q.2 Check each equation and check for extraneous solution, if any

(i) $\sqrt{3x+4}=2$

(LHR 2013, GRW 2016, SWL 2014, 16, BWP 2016, D.G.K 2017)

Solution:

$\sqrt{3x+4}=2$

Taking square on both side

$\left(\sqrt{3x+4}\right)^2=(2)^2$
$3x+4=4$
$3x=4-4$
$3x=0$
$x=\frac{0}{3}$
$x=0$

Check

$\sqrt{3x+4}=2$

When $x=0$

$\sqrt{3(0)+4}=2$
$\sqrt{4}=2$
$2=2$

L.H.S = R.H.S

Solution Set $=\{0\}$

(ii) $\sqrt[3]{2x-4}-2=0$ **(A.B)**

(LHR 2015, BWP 2017)

Solution:

$\sqrt[3]{2x-4}-2=0$
$\sqrt[3]{2x-4}=2$

Taking cube on both sides

$\left(\sqrt[3]{2x-4}\right)^3=(2)^3$
$2x-4=8$
$2x=8+4$
$2x=12$
$x=\frac{12}{2}$
$x=6$

Check

$\sqrt[3]{2x-4}-2=0$

When $x=6$

$\sqrt[3]{2x-4}-2=0$
$\sqrt[3]{2(6)-4}-2=0$
$\sqrt{12-4}-2=0$
$\sqrt[3]{8}-2=0$
$\sqrt[3]{2^3}-2=0$
$2-2=0$
$0=0$

L.H.S = R.H.S

Solution Set $=\{6\}$

(iii) $\sqrt{x-3}-7=0$ **(A.B)**
(LHR 2014, 17, FSD 2014, BWP 2013, 15, D.G.K 2014)

Solution:

$\sqrt{x-3}-7=0$

$\sqrt{x-3}=7$

Taking square on both side

$\left(\sqrt{x-3}\right)^2=(7)^2$

$x-3=49$

$x=49+3$

$x=52$

Check

$\sqrt{x-3}-7=0$

When $x=52$

$\sqrt{52-3}-7=0$

$\sqrt{49}-7=0$

$7-7=0$

$0=0$

L.H.S = R.H.S

Solution Set $=\{52\}$

(iv) $2\sqrt{t+4}=5$ **(A.B)**
(LHR 2015, MTN 2014, D.G.K 2017)

Solution:

$2\sqrt{t+4}=5$

Taking square on both side

$\left(2\sqrt{t+4}\right)^2=(5)^2$

$4(t+4)=25$

$t+4=\frac{25}{4}$

$t=\frac{25}{4}-4$

$t=\frac{25-16}{4}$

$t=\frac{9}{4}$

Check

$2\sqrt{t+4}=5$

When $t=\frac{9}{4}$

$2\sqrt{\frac{9}{4}+4}=5$

$2\sqrt{\frac{9+16}{4}}=5$

$2\sqrt{\frac{25}{4}}=5$

$2\times\frac{5}{2}=5$

$5=5$

L.H.S = R.H.S

Solution Set $=\left\{\frac{9}{4}\right\}$

(v) $\sqrt[3]{2x+3}=\sqrt[3]{x-2}$ **(A.B)**
(SWL 2015, SGD 2017)

Solution:

$\sqrt[3]{2x+3}=\sqrt[3]{x-2}$

Taking cube on both sides

$\left(\sqrt[3]{2x+3}\right)^3=\left(\sqrt[3]{x-2}\right)^3$

$2x+3=x-2$

$2x-x=-2-3$

$x=-5$

Check

$\sqrt[3]{2x+3}=\sqrt[3]{x-2}$

When $x=-5$

$\sqrt[3]{2(-5)+3}=\sqrt[3]{-5-2}$

$\sqrt[3]{-10+3}=\sqrt[3]{-7}$

$\sqrt[3]{-7}=\sqrt[3]{-7}$

L.H.S = R.H.S

Solution Set $=\{-5\}$

(vi) $\sqrt[3]{2-t}=\sqrt[3]{2t-28}$ **(A.B)**

Solution:

$\sqrt[3]{2-t}=\sqrt[3]{2t-28}$

Taking cube on both sides

$\left(\sqrt[3]{2-t}\right)^3=\left(\sqrt[3]{2t-28}\right)^3$

$2-t=2t-28$

$2+28=2t+t$

$30=3t$

$\frac{30}{3}=t$

$t=10$

Check

$\sqrt[3]{2-t}=\sqrt[3]{2t-28}$

When $t=10$

$\sqrt[3]{2-10}=\sqrt[3]{2(10)-28}$

$\sqrt[3]{-8}=\sqrt[3]{20-28}$

$\sqrt[3]{-8}=\sqrt[3]{-8}$

L.H.S = R.H.S

Solution Set $=\{10\}$

(vii) $\sqrt{2t+6}-\sqrt{2t-5}=0$ **(A.B)**

Solution:

$\sqrt{2t+6}-\sqrt{2t-5}=0$

$\sqrt{2t+6}=\sqrt{2t-5}$

Taking square on both side

$\left(\sqrt{2t+6}\right)^2=\left(\sqrt{2t-5}\right)^2$

$2t+6=2t-5$

$2t-2t=-5-6$

$0=-11$

Solution is not possible

Solution Set $=\{\}$ or ϕ

(viii) $\sqrt{\dfrac{x+1}{2x+5}}=2 \quad x\neq\dfrac{-5}{2}$ **(A.B)**

(SGD 2016, MTN 2013)

Solution:

$\sqrt{\dfrac{x+1}{2x+5}}=2$

Taking square on both side

$\left(\sqrt{\dfrac{x+1}{2x+5}}\right)^2=(2)^2$

$\dfrac{x+1}{2x+5}=4$

$x+1=4(2x+5)$

$x+1=8x+20$

$1-20=8x-x$

$-19=7x$

$-\dfrac{19}{7}=x$

Or, $x=\dfrac{-19}{7}$

Check

$\sqrt{\dfrac{x+1}{2x+5}}=2$

When $x=\dfrac{-19}{7}$

$\sqrt{\left(\dfrac{-19}{7}+1\right)\div\left[2\times\dfrac{-19}{7}+5\right]}=2$

$\sqrt{\dfrac{-19+7}{7}\div\left[\dfrac{-38}{7}+5\right]}=2$

$\sqrt{\dfrac{-12}{7}\div\left[\dfrac{-38+35}{7}\right]}=2$

$\sqrt{\dfrac{-12}{7}\div\dfrac{-3}{7}}=2$

$\sqrt{\dfrac{-12^{\,4}}{7}\times\dfrac{7}{-3}}=2$

$\sqrt{4}=2$

$2=2$

L.H.S = R.H.S

Solution Set $=\left\{\dfrac{-19}{7}\right\}$

Equation Involving Absolute Value (K.B)

Absolute Value

The absolute value of real number 'a' denoted by $|a|$ is defined as:

$$|a|=\begin{cases}a, & \text{if } a\geq 0\\ -a, & \text{if } a<0\end{cases}$$

e.g., $|6|=6,\ |0|=0$ and $|-6|=-(-6)=6$

Some Properties of Absolute Value

If $a,b\in\mathbb{R}$, then **(U.B)**

(i) $|a|\geq 0$

(ii) $|-a|=|a|$

(iii) $|ab|=|a|.|b|$

(iv) $\left|\dfrac{a}{b}\right|=\dfrac{|a|}{|b|},\ b\neq 0$

Solving Linear Equations Involving Absolute Values (K.B)

Example # 1 (Page # 136)

Solve and check: $|2x+3|=11$

Solution:

$|2x+3|=11$

By definition of absolute equation, we get

$(2x+3)=11$ or $-(2x+3)=11$

$2x+3=11$ or $2x+3=-11$

$2x=8$ or $2x=-14$

$x=4$ or $x=-7$

Check

Substituting $x=4$ in the original equation we get

$|2(4)+3|=11$

i.e., $11=11$, true

Now substituting $x=-7$, we have

$|2(-7)+3|=11$

$|-11|=11$

$11=11$, true

Hence $x=4,-7$ are the solutions to the given equation.

Thus, Solution Set $=\{-7,4\}$

Example # 2 (Page # 137)

Solve $|8x-3|=|4x+5|$

Solution:

$8x-3=\pm(4x+5)$

$8x-3=4x+5$ or $8x-3=-(4x+5)$

$4x=8$ or $12x=-2$

$x=2$ or $x=-\frac{1}{6}$

On checking, we find that $x=2,\ x=-\frac{1}{6}$ both satisfy the original equations.

Hence the Solution Set $=\left\{\frac{-1}{6},2\right\}$.

Exercise 7.2

Q.1 Identify the following statements as true or

(i)	$|x| = 0$ **has only one solution**	**True**
(ii)	**All absolute value equations have two solutions**	**False**
(iii)	**The equation** $|x| = 2$ **is equivalent to** $x=2$ *or* $x=-2$	**True**
(iv)	**The equation** $|x-4| = -4$ **has no solution**	**True**
(v)	**The equation** $|2x-3| = 5$ **is equivalent to** $2x-3=5$ *or* $2x+3=5$	**False**

Q.2

(i) Solve: $|3x-5|=4$ **(K.B)**

(LHR 2014, 17, GRW 2014, 17, FSD 2017, SWL 2016, 17, MTN 2014, 15)

Solution:

$|3x-5|=4$

$3x-5=\pm 4$

$3x-5=4$ or $3x-5=-4$

$3x=4+5$ $3x=-4+5$

$3x=9$ $3x=1$

$x=\frac{9}{3}$ $x=\frac{1}{3}$

$x=3$

Check

Put $x=3$

$|3(3)-5|=4$

$|9-5|=4$

$4=4$ True

Put $x=\frac{1}{3}$

$\left|3\times\frac{1}{3}-5\right|=4$

$|1-5|=4$

$|-4|=4$

$4=4$ True

Solution Set $=\left\{3,\frac{1}{3}\right\}$

(ii) $\frac{1}{2}|3x+2|-4=11$ **(K.B)**

(LHR 2017, SWL 2015, 16, FSD 2016, MTN 2013, 16, RWP 2016)

Solution:

$\frac{1}{2}|3x+2|-4=11$

$\frac{1}{2}|3x+2|=11+4$

$\frac{1}{2}|3x+2|=15$

$|3x+2|=2\times 15$

$|3x+2|=30$

$3x+2=\pm 30$

$3x+2=30$	$3x+2=-30$
$3x=30-2$	$3x=-30-2$
$3x=28$	$3x=-32$
$x=\frac{28}{3}$	$x=\frac{-32}{3}$

Check

$\frac{1}{2}\lvert 3x+2\rvert-4=11$	$\frac{1}{2}\left\lvert 3\times-\frac{32}{3}+2\right\rvert-4=11$
$\frac{1}{2}\lvert 3\times\frac{28}{3}+2\rvert-4=11$	$\frac{1}{2}\lvert -32+2\rvert-4=11$
$\frac{1}{2}\lvert 28+2\rvert-4=11$	$\frac{1}{2}\lvert -30\rvert-4=11$
$\frac{1}{2}\times 30-4=11$	$\frac{1}{2}(30)-4=11$
$15-4=11$	$15-4=11$
$11=11$	$11=11$

Solution Set $=\left\{\frac{28}{3},\frac{-32}{3}\right\}$

(iii) $|2x+5|=11$ **(K.B)**

(LHR 2014, 15, 16, 17, GRW 2014, 15, 16, 17, SWL 2016, 17, FSD 2014, 15, D.G.K 2014, 15, 16, 17, BWP 2017)

Solution:

$|2x+5|=11$

$2x+5=\pm 11$

$2x+5=11$	$2x+5=-11$
$2x=11-5$	$2x=-11-5$
$2x=6$	$2x=-16$
$x=\frac{6}{2}$	$x=\frac{-16}{2}$
$x=3$	$x=-8$

Check

$\lvert 2x+5\rvert=11$	$\lvert 2(-8)-8+5\rvert=11$
$\lvert 2\times 3+5\rvert=11$	$\lvert -16+5\rvert=11$
$6+5=11$	$\lvert -11\rvert=11$
$11=11$	$11=11$

Solution Set $=\{-8,3\}$

(iv) $|3+2x|=|6x-7|$ **(K.B)**

(LHR 2015, 17, FSD 2016, SWL 2013, BWP 2017)

Solution:

$|3+2x|=|6x-7|$

$3+2x=\pm(6x-7)$

$3+2x=6x-7$	$3+2x=-(6x-7)$
$3+7=6x-7$	$3+2x=-6x+7$
$10=4x$	$2x+6x=7-3$
$\frac{10}{4}=x$	$\frac{4}{8}=x$
$x=\frac{5}{2}$	$x=\frac{1}{2}$

Check

$\lvert 3+2x\rvert=\lvert 6x-7\rvert$	$\lvert 3+2x\rvert=\lvert 6x-7\rvert$
$\left\lvert 3+2\left(\frac{5}{2}\right)\right\rvert=\left\lvert \cancel{6}^3\left(\frac{5}{\cancel{2}}\right)-7\right\rvert$	$\left\lvert 3+2\times\frac{1}{2}\right\rvert=\left\lvert \cancel{6}^3\times\frac{1}{\cancel{2}}-7\right\rvert$
$\lvert 3+5\rvert=\lvert 15-7\rvert$	$\lvert 3+1\rvert=\lvert 3-7\rvert$
$\lvert 8\rvert=\lvert 8\rvert$	$\lvert 4\rvert=\lvert -4\rvert$
$8=8$	$4=4$

Solution Set $=\left\{\frac{5}{2},\frac{1}{2}\right\}$

(v) $|x+2|-3=5-|x+2|$ **(K.B)**

Solution:

$|x+2|-3=5-|x+2|$

$|x+2|+|x+2|=5+3$

$2|x+2|=8$

$|x+2|=\frac{8}{2}$

$|x+2|=4$

$x+2=\pm 4$

$x+2=4 \qquad\qquad x+2=-4$

$x=4-2 \qquad\qquad x=-4-2$

$x=2 \qquad\qquad x=-6$

Check

$|x+2|-3=5-|x+2| \qquad |x+2|-3=5-|x+2|$

$|2+2|-3=5-|2+2| \qquad |-6+2|-3=5-|-6+2|$

$14-3=5-|4| \qquad |-4|-3=5-|-4|$

$4-3=5-4 \qquad 4-3=5-4$

$1=1 \qquad 1=1$

Solution Set $=\{-6,2\}$

(vi) $\frac{1}{2}|x+3|+21=9$ **(K.B) + (U.B)**

Solution:

$$\frac{1}{2}|x+3|+21=9$$

$$\frac{1}{2}|x+3|=9-21$$

$$\frac{1}{2}|x+3|=-12$$

$$|x+3|=-12\times 2$$

$$|x+3|=-24$$

Value of absolute is never negative so solution is not possible

Solution Set $=\{\ \}$

(vii) $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$ **(A.B)**

Solution:

$$\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$$

$$\left|\frac{3-5x}{4}\right|=\frac{2}{3}+\frac{1}{3}$$

$$\left|\frac{3-5x}{4}\right|=\frac{2+1}{3}$$

$$\left|\frac{3-5x}{4}\right|=\frac{3}{3}$$

$$\left|\frac{3-5x}{4}\right|=1$$

$$\frac{3-5x}{4}=\pm 1$$

$\frac{3-5x}{4}=1$ and $\frac{3-5x}{4}=-1$

$3-5x=4 \qquad 3-5x=-4$

$-5x=4-3 \qquad -5x=-4-3$

$-5x=1 \qquad -5x=-7$

$x=\frac{1}{-5} \qquad x=\frac{-7}{-5}$

$x=-\frac{1}{5} \qquad x=\frac{7}{5}$

$\left|\frac{3-5\times\left(-\frac{1}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3} \qquad \left|\frac{3-5\times\left(+\frac{7}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$\left|\frac{3+1}{4}\right|-\frac{1}{3}=\frac{2}{3} \qquad \left|\frac{3-7}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$\left|\frac{4}{4}\right|-\frac{1}{3}=\frac{2}{3} \qquad \left|\frac{-4}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$|-1|-\frac{1}{3}=\frac{2}{3}$

$1-\frac{1}{3}=\frac{2}{3} \qquad 1-\frac{1}{3}=\frac{2}{3}$

$\frac{3-1}{3}=\frac{2}{3} \qquad \frac{3-1}{3}=\frac{2}{3}$

$\frac{2}{3}=\frac{2}{3} \qquad \frac{2}{3}=\frac{2}{3}$

Solution Set $=\left\{\frac{-1}{5},\frac{7}{5}\right\}$

(viii) $\left|\frac{x+5}{2-x}\right|=6$ **(A.B)**

Solution:

$$\left|\frac{x+5}{2-x}\right|=6$$

$$\frac{x+5}{2-x}=\pm 6$$

$$\frac{x+5}{2-x}=6$$

$$x+5=6(2-x)$$

$$x+5=12-6x$$

$x+6x=12-5$
$7x=7$
$x=\frac{7}{7}$
$x=1$

$\frac{x+5}{2-x}=-6$
$x+5=-6(2-x)$
$x+5=-12+6x$
$5+12=6x-x$
$17=5x$
$\frac{17}{5}=x$
$x=\frac{17}{5}$

Check

$\left|\frac{x+5}{2-x}\right|=6$

$\left|\frac{1+5}{2-1}\right|=6$

$\left|\frac{6}{1}\right|=6$

$6=6$

$\left|\left(\frac{17}{5}+5\right)\div\left(2-\frac{17}{5}\right)\right|=6$

$\left|\frac{17+25}{5}\div\frac{10-17}{5}\right|=6$

$\left|\frac{42}{5}\div\frac{-7}{5}\right|=6$

$|-6|=6$

$6=6$

Solution Set $=\left\{1,\frac{17}{5}\right\}$

Work of Thomas Herriot on Inequality (K.B) + (U.B)

The inequality symbols < and > were introduced by an English Mathematician Thomas Herriot (1560-1621).

Linear Inequality (K.B)+(U.B)

A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form $ax+b<0,\ a\neq 0$ where $a,b\in R$.

We may replace the symbol < by >, < or ≥ also

For example

$x<2$, $x\geq 2$ etc.

Strict (Strong) Inequality (K.B) + (U.B)

The inequality $x>y$ and $x<y$ are known as strict (or strong) inequalities.

For example

$x<2$

Non Strict (Weak) Inequality (K.B)+(U.B)

Whereas he inequalities $x\leq y$ and $y\leq x$ are called non-strict (or weak)

For example

$x\geq 2$

Properties of inequalities

(1) Law of Trichotomy (K.B)+(U.B)

For any $a,b\in\mathbb{R}$ one and only one of the following statements is true.

Either $a<b$ or $a=b$, or $a>b$

An important special case of this property is the case for $b=0$; namely,

$a<0$ or $a=0$ or $a>0$ for any $a\in\mathbb{R}$.

(2) Transitive property (K.B)+(U.B)

Let $a,b,c\in\mathbb{R}$

(i) If $a>b$ and $b>c$, then $a>c$

(ii) If $a<b$ and $b<c$, then $a<c$

(3) Additive closure property for $a,b,c\in\mathbb{R}$ (K.B)+(U.B)

(i) If $a>b$, then $a+c>b+c$

If $a<b$, then $a+c>b+c$

(ii) If $a>0$, and $b>0$ then $a+b>0$

If $a<0$, and $b<0$ then $a+b<0$

(4) Multiplicative property

(K.B)+(U.B)

Let $a,b,c,d \in \mathbb{R}$

(i) If $a>0$ and $b>0$, then $ab>0$
Whereas $a<0$ and $b>0 \Rightarrow ab>0$

(ii) If $a>b$ and $c>0$, then $ac>bc$
or if $a<b$ or $c>0$ then $ac<bc$

(iii) If $a>b$ and $c<0$, then $ac<bc$
or if $a<b$ and $c<0$ then $ac>bc$

The above property (iii) states that the sign of inequality is reversed if each side is multiplied by a negative real number

(iv) If $a>b$ and $c>d$ then $ax>bd$

Example # 4 (Page 141) **(A.B)**

Solve the inequality $4x-1\le 3\le 7+2x$, where $x\in\mathbb{R}$

Solution:

The given inequality holds if and only if both the separate inequalities $4x-1\le 3$ and $3\le 7+2x$ hold we solve each of these inequalities separately. The first inequality

$4x-1\le 3$

$4x\le 3+1$

Gives $4x\le 4$ i.e. $x\le 1 \ldots$(i)

And the second inequality $3\le 7+2x$ yields

$-4\le 2x$

i.e. $-2\le x$ which implies $x\ge -2$ …(ii)

Combing (i) and (ii) we have $-2\le x\le 1$

Thus the solution set $=\{x\,|-2\le x\le 1\}$

Solving Linear Inequalities

Example # 1 (Page # 140) **(A.B)**

Solve $9-7x>19-2x$ where $x\in\mathbb{R}$

(GRW 2013, MTN 2014, 15, 16)

Solution:

$9-7x>19-2x$

$9-5x>19$ … (adding $2x$ to each side)

$-5x>10$ …(adding -9 to each side)

$x<-2$ … (multiplying each side by $-\frac{1}{5}$)

Hence the solution set $=\{x\,|\,x<-2\}$

Example # 2 (Page # 140) **(A.B)**

Solve $\frac{1}{2}x-\frac{2}{3}\le x+\frac{1}{3}$, where $x\in\mathbb{R}$

Solution:

$\frac{1}{2}x-\frac{2}{3}\le x+\frac{1}{3}$

Multiply each side by 6

$6\times\frac{1}{2}x-6\times\frac{2}{3}\le 6x+6\times\frac{1}{3}$

Or $3x-4\le 6x+2$

$-4-2\le 6x-3x$

$-6\le 3x$

Or $-2\le x$

Or $x\ge -2$

Hence the solution set $=\{x\,|\,x\ge -2\}$

Example # 3 (Page # 140) **(A.B)**

Solve the double inequality $-2<\frac{1-2x}{3}<1$

where $x\in\mathbb{R}$

Solution:

The given inequality is a double inequality and represents two separate inequalities.

$-2<\frac{1-2x}{3}$ and $\frac{1-2x}{3}<1$

$-2<\frac{1-2x}{3}<1$

Multiplying each term by 3

$-6<1-2x<3$

Subtracting 1 from each

$-6-1<1-2x-1<3-1$

$-7<-2x<2$

Dividing by 2

$\frac{7}{2}>x>-1$

Or $-1<x<\frac{7}{2}$

$-1<x<3.5$

Thus, Solution Set is $\{x\,|1-<x<3.5\}$

Exercise 7.3

Q.1 Solve the following inequalities.

(i) $3x+1<5x-4$ **(K.B)**

(FSD 2015, SGD 2013, RWP 2016, BWP 2013, 14, D.G.K 2013)

Solution: $3x+1<5x-4$

$3x<5x-4-1$

$3x-5x<-5$

$-2x<-5$

Case-I When negative is eliminated from both sides of inequality the symbol will be change.

Case-II When negative is transferred from variable to constant side, symbol will also change.

$x>\frac{-5}{-2}$

$x>\frac{5}{2}$

Solution Set $=\left\{x\,|\,x>\frac{5}{2}\right\}$

(ii) $4x-10.3\le 21x-1.8$ **(A.B)**

(SGD 2015, D.G.K 2015)

Solution:

$4x-10.3\le 21x-1.8$

$4x-21x\le -8.5+10.3$

$-17x\le 8.5$

When negative value is shifted to other side its symbol changes.

$x\ge\frac{8.5}{-17}$

$x\ge-\frac{8.5}{17}$

$x\ge-0.5$

Solution Set $=\{x\,|\,x\ge-0.5\}$

(iii) $4-\frac{1}{2}x\ge-7+\frac{1}{4}x$ **(SWL 2013) (A.B)**

Solution:

$4-\frac{1}{2}x\ge-7+\frac{1}{4}x$

$-\frac{1}{2}x-\frac{1}{4}\ge-7-4$

$\frac{-2x-x}{4}\ge-11$

$-3x\ge-44$

When negative value is shifted the symbol changes

$x\le\frac{-44}{-3}$

$x\le\frac{44}{3}$

Solution Set $=\{x\,|\,x\le\frac{44}{3}\}$

(iv) $x-2(5-2x)\ge 6x-3\frac{1}{2}$ **(A.B)**

Solution:

$x-2(5-2x)\ge 6x-3\frac{1}{2}$

$x-10+4x\ge 6x-\frac{7}{2}$

$5x-6x\ge-\frac{7}{2}+10$

$-1x\ge\frac{-7+20}{2}$

$-x\ge-\frac{13}{2}$

When negative is shifted other side symbol changes

$x\le\frac{13}{-1\times 2}$

$x\le-\frac{13}{2}$

$x\le-6.5$

Solution Set $=\{x\,|\,x\le-6.5\}$

(v) $\frac{3x+2}{9}-\frac{2x+1}{3}>-1$ **(K.B)**

Solution:

$\frac{3x+2}{9}-\frac{2x+1}{3}>-1$

$\frac{3x+2-3(2x+1)}{9}>-1$

$3x+2-6x-3>-9$

$-3x>-9+1$

$-3x>-8$

Negative value is shifted to other side its symbols changes

$x < \frac{-8}{-3}$

$x < \frac{8}{3}$

Solution Set $= \left\{ x \mid x < \frac{8}{3} \right\}$

(vi) $3(2x+1)-2(2x+5)<5(3x-2)$ **(A.B)**

Solution:

$3(2x+1)-2(2x+5)<5(3x-2)$

$6x+3-4x-10<15x-10$

$2x-7-15x<-10$

$-13x<-10+7$

$-13x<-3$

Multiplying both sides by -1

$13x>3$

$x>\frac{3}{13}$

Solution Set $= \left\{ x \mid x > \frac{3}{13} \right\}$

(vii) $3(x-1)-(x-2)>-2(x+4)$ **(A.B)**

Solution:

$3(x-1)-(x-2)>-2(x+4)$

$3x-3-x+2>-2x-8$

$2x-1>-2x-8$

$2x+2x>-8+1$

$4x>-7$

$x>\frac{-7}{4}$

Solution Set $= \left\{ x \mid x > \frac{-7}{4} \right\}$

(viii) $2\frac{2}{3}x+\frac{2}{3}(5x-4)>-\frac{1}{3}(8x+7)$ **(A.B)**

Solution:

$2\frac{2}{3}x+\frac{2}{3}(5x-4)>-\frac{1}{3}(8x+7)$

$\frac{8}{3}x+\frac{10x-8}{3}>-\frac{(8x+7)}{3}$

$\frac{8x+10x-8}{3}>-\frac{8x+7}{3}$

Multiplying both side by 3

$\cancel{3}\times\frac{18x-8}{\cancel{3}}>-\cancel{3}\times\frac{8x+7}{\cancel{3}}$

$18x-8>-(8x+7)$

$18x-8>-8x-7$

$18x+8x>-7+8$

$26x>1$

$x>\frac{1}{26}$

Solution Set $= \left\{ x \mid x > \frac{1}{26} \right\}$

Q.2 Solve the following inequalities

(i) $-4<3x+5<8$ **(K.B)+(U.B)**
(SWL 2014, MTN 2015)

Solution:

$-4<3x+5<8$

$-4<3x+5$	and	$3x+5<8$
$-4-5<3x$		$3x<8-5$
$-9<3x$		$3x<3$
$\frac{-9}{3}<x$		$x<\frac{3}{3}$
$-3<x$		$x<1$

–3<x<1

Solution Set $= \{ x \mid -3<x<1 \}$

(ii) $-5\le\frac{4-3x}{2}<1$ **(SWL 2014)** **(A.B)**

Solution:

$-5\le\frac{4-3x}{2}<1$

$-5\le\frac{4-3x}{2}$	and	$\frac{4-3x}{2}<1$
$-10\le 4-3x$		$4-3x<2$
$3x-10\le 4$		$-3x<2-4$
$3x\le 4+10$		$-3x<-2$
$3x\le 14$		$x>\frac{-2}{-3}$
$x\le\frac{14}{3}$		$x>\frac{2}{3}$

$\frac{2}{3} < x$

$\frac{2}{3} < x \le \frac{14}{3}$

Solution Set $= \{x \mid \frac{2}{3} < x \le \frac{14}{3}\}$

(iii) $-6 < \frac{x-2}{4} < 6$ **(A.B)**

Solution:

$-6 < \frac{x-2}{4} < 6$

$-6 < \frac{x-2}{4} \rightarrow (i)$ and $\frac{x-2}{4} < 6 \rightarrow (ii)$

$(i) \Rightarrow -6 < \frac{x-2}{4}$

$-24 < x-2$

$-24+2 < x$

$-22 < x$

and

$(ii) \Rightarrow \frac{x-2}{4} < 6$

$x-2 < 24$

$x < 24+2$

$x < 26$

$-22 < x < 26$

Solution Set $= \{x \mid -22 < x < 26\}$

(iv) $3 \ge \frac{7-x}{2} \ge 1$ **(A.B)**

Solution:

$3 \ge \frac{7-x}{2} \ge 1$

$3 \ge \frac{7-x}{2} \rightarrow (i)$ and $\frac{7-x}{2} \ge 1 \rightarrow (ii)$

$(i) \Rightarrow 3 \ge \frac{7-x}{2}$

$6 \ge 7-x$

$6-7 \ge -x$

$-1 \ge -x$

Negative sign change the symbols

$1 \le x$

$(ii) \Rightarrow \frac{7-x}{2} \ge 1$

$7-x \ge 2$

$-x \ge 2-7$

$-x \ge -5$

$x \le 5$

$1 \le x \le 5$

Solution Set $= \{x \mid 1 \le x \le 5\}$

(v) $3x-10 \le 5 < x+3$ **(A.B)**

Solution:

$3x-10 \le 5 < x+3$

$3x-10 \le 5$ and $5 < x+3$

$3x \le 5+10$; $5-3 < x$

$3x \le 15$; $2 < x$

$\frac{3x}{3} \le \frac{15}{3}$

$x \le 5$

$2 < x \le 5$

Solution Set $= \{x \mid 2 < x \le 5\}$

(vi) $-3 \le \frac{x-4}{-5} < 4$ **(A.B)**

Solution:

$-3 \le \frac{x-4}{-5} < 4$

$-3 \le \frac{x-4}{-5}$ and $\frac{x-4}{-5} < 4$

$-3 \times -5 \ge x-4$; $x-4 > 4(-5)$

$15 \ge x-4$; $x > -20+4$

$15+4 \ge x$; $x > -16$

$19 \ge x$; $-16 < x$

$x \le 19$

$-16 < x \le 19$

Solution Set $= \{x \mid -16 < x \le 19\}$

(vii) $1-2x<5-x\le 25-6x$ **(A.B)**

Solution:

$1-2x<5-x\le 25-6x$

$1-2x<5-x$ and $5-x\le 25-6x$

$1-5<2x-x$ $\quad -x+6x\le 25-5$

$-4<x$ $\quad 5x\le 20$

$x\le \frac{20}{5}$

$x\le 4$

$-4<x\le 4$

$\therefore$ **Solution Set** $=\{x\,|-4<x\le 4\}$

(viii) $3x-2<2x+1<4x+17$ **(A.B)**

Solution:

$3x-2<2x+1<4x+17$

$3x-2<2x+1$ $\quad 2x+1<4x+17$

$3x-2x-2<+1$ $\quad 2x-4x<17-1$

$x<1+2$ $\quad -2x<16$

$x<3$ $\quad x>\frac{16}{-2}$

$x>-8$

$-8<x$

$-8<x<3$

$\therefore$ **Solution Set** $=\left\{x\,|-8<x<3\right\}$

Review Exercise 7

Q.1 Choose the correct answer

(i) Which of the following is the solution of the inequality $3-4x\le 11$? **(A.B)**

(a) –8 (b) –2

(c) $-\frac{14}{4}$ (d) None of these

(ii) A statement involving any of the symbols <, >, ≤or >, is called------------ **(K.B)**

(a) Equation (b) Identity

(c) Inequality (d) Linear equation

(iii) x = ------- is a solution of the inequality $-z<x>\frac{3}{2}$ **(U.B)**

(a) – 5 (b) 3

(c) 0 (d) $\frac{3}{2}$

(iv) If x is no larger than 10, then ---------- **(U.B)**

(FSD 2014, 15, SWL 2017, RWP 2014, SGD 2014, D.G.K 2013)

(a) $x\le 8$ (b) $x\ge 10$

(c) $x<10$ (d) $x>10$

(v) If the capacity of an elevator is at most 1600 pounds then ---------- **(K.B)**

(LHR 2013, GRW 2014, FSD 2014, 17, SWL 2014, 16)

(a) $c<1600$ (b) $c\ge 1600$

(c) $c\le 1600$ (d) $c>1600$

(vi) x = 0 is a solution of the inequality ----------- **(A.B)**

(a) $x>0$ (b) $3x+5<0$

(c) $x+\frac{z}{2}<0$ (d) $x-2<0$

ANSWER KEY

i	ii	iii	iv	V	vi
b	c	c	b	C	d

Q.2 Identify the following statement as true or false **(U.B)**

(i) The equation $3x-5=7-x$ is a linear equation. **(True)**

(ii) The equation $x-0.3x=0.7x$ is an identity **(True)**

(iii) The equation $-2x+3=8$ is equivalent to $-2x=11$ **(False)**

(iv) To eliminate fractions we multiply each side of an equation by the L.C.M of denominators **(True)**

(v) $4(x+3)=x+3$ is a conditional equations **(True)**

(vi) The equation $2(3x+5)=6x+12$ is an in consistent equation **(True)**

(vii) To solve $\frac{2}{3}x=12$, we should multiply each side by $\frac{2}{3}$ **(False)**

(viii) Equations having exactly the same solution are called equivalent equations. **(True)**

(ix) A solution that does not satisfy the original equation is called extra solution **(True)**

Q.3 Answer the following short question.

(i) Define a linear inequality in one variable **(K.B)**

Ans A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form $ax+b<0, a\neq 0$

(ii) State the Trichotomy and transitive properties of in equalities **(K.B)**

Ans Trichotomy Property **(K.B)**

For any $a,b\in R$ one and only one of the following statements in true. $a<b$ or $a=b,$ or $a>b$

Transitive Property

Let $a,b,c\in R$.

(a) If $a>b$ and $b>c,$ then $a>c$

(b) If $a<b$ and $b<c,$ then $a<c$

(iii) The formula relating degree Fahrenheit to degree Celsius is F= $\frac{9}{5}c+32$ for what value of c is F< O was **(K.B) + (A.B) + (U.B)**

Ans F= $\frac{9}{5}c + 32$

$\frac{9}{5}c + 32 = F$

Since F < 0

So $\frac{9}{5}c + 32 < 0$

$\frac{9c+160}{5}<0$

Or 9c + 160 < 0 × 5

Or 9c + 160 < 0

Or 9c< – 160

Or $c < -\frac{160}{9}$

(iv) Seven times the sum of an integer and 12 is at least 50 and at most 60. Write and solve the inequality that expresses this relationship **(U.B)**

Solution: Let the integer = y

Sum of integer and $12 = y+12$

Seven times sum of integer and $12 = 7(y+12)$

According to condition

$50\leq 7(y+12)\leq 60$

$\frac{50}{7}\leq 7\frac{(y+12)}{7}\leq\frac{60}{7}$

$\frac{50}{7}\leq y+12\leq\frac{60}{7}$

$\frac{50}{7}-12\leq y+\cancel{12}-\cancel{12}\leq\frac{60}{7}-12$

$\frac{50-84}{7}\leq y\leq\frac{60-84}{7}$

$\frac{-34}{7}\leq y\leq\frac{-24}{7}$ **Solution Set** $=\left\{y \mid \frac{-34}{7}\leq y-\frac{24}{7}\right\}$

Q.4 Solve each of the following and check for extraneous solution if any

(i) $\sqrt{2t+4}=\sqrt{t-1}$ **(A.B)**

Solution: $\sqrt{2t+4}=\sqrt{t-1}$

Taking square on both side

$\left(\sqrt{2t+4}\right)^2=\left(\sqrt{t-1}\right)^2$

$2t+4=t-1$

$2t-t=-1-4$

$t=-5$

To check

$\sqrt{2t+4}=\sqrt{t-1}$

When $t=-5$

$\sqrt{2(-5)+4}=\sqrt{t-5-1}$

$\sqrt{-10+4}=\sqrt{-6}$

$\sqrt{-6}=\sqrt{-6}$

L.H.S = R.H.S

Solution Set $=\{-5\}$

(ii) $\sqrt{3x-1}-2\sqrt{8-2x}=0$ **(A.B)**

Solution: $\sqrt{3x-1}-2\sqrt{8-2x}=0$

$\sqrt{3x-1}=2\sqrt{8-2x}$

Taking square on both side

$\left(\sqrt{3x-1}\right)^2=\left(2\sqrt{8-2x}\right)^2$

$3x-1=4(8-2x)$

$3x-1=32-8x$

$3x+8x=32+1$

$11x=33$

$x=\frac{33}{11}$

$x=3$

To check

$\sqrt{3x-1}-2\sqrt{8-2x}=0$

When $x=3$

$\sqrt{3(3)-1}-2\sqrt{8-2(3)}=0$

$\sqrt{9-1}-2\sqrt{8-6}=0$

$\sqrt{8}-2\sqrt{2}=0$

$2\sqrt{2}-2\sqrt{2}=0$

$0=0$

L.H.S = R.H.S

Solution Set $=\{3\}$

Q.5 Solve for x

(i) $|3x+14|-2=5x$ **(A.B)**

Solution: $|3x+14|-2=5x$

$|3x+14|=5x+2$

$3x+14=\pm(5x+2)$

$3x+14=5x+2$ $\quad$ $3x+14=-(5x+2)$

$14-2=5x-3x$

$12=2x$ $\quad$ $3x+14=-5x-2$

$\frac{12}{2}=x$ $\quad$ $3x+5x=-2-14$

$x=6$ $\quad$ $8x=\frac{-16}{8}$

To check $\quad$ $x=-2$

$|3x+14|-2=5x$ $\quad$ $|3x+14|-2=5x$

When $x=6$ $\quad$ when $x=-2$

$|3(6)+14|-2=5(6)$ $\quad$ $|3(-2)+14|-2=5(-2)$

$|18+14|-2=30$ $\quad$ $|-6+14|-2=-10$

32–2=30 $\quad$ 8–2 = –10

30 = 30 $\quad$ 6 = –10

Solution Set $=\{6\}$

(ii) $\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$ **(A.B)**

Solution $\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

$\frac{2}{3}|x-3|=|x+2|$

$\frac{2}{3}=\frac{|x+2|}{|x-3|}$

$\frac{x+2}{x-3}=\pm\frac{2}{3}$

$\frac{x+2}{x-3}=\frac{2}{3}$ and $\frac{x+2}{x-3}=-\frac{2}{3}$

$3(x+2)=2(x-3)$ $\quad$ $3(x+2)=-2(x-3)$

$3x+6=2x-6$ $\quad$ $3x+6=-2x+6$

$3x-2x=-6-6$ $\quad$ $3x+2x=+6-6$

$x=-12$ $\quad$ $5x=0$

To check $\quad$ $x=\frac{0}{5} \Rightarrow x=0$

$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

When $x=-12$

$\frac{1}{3}|-12-3|=\frac{1}{2}|-12+2|$

$\frac{1}{3}|-15|=\frac{1}{2}|-10|$

$\frac{1}{\cancel{3}}\left(\cancel{15}^{5}\right)=\frac{1}{\cancel{2}}\left(\cancel{10}^{5}\right)$

$5=5$

$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$

when $x=0$

$\frac{1}{3}|0-3|=\frac{1}{2}|0+2|$

$\frac{1}{3}|-3|=\frac{1}{2}|2|$

$\frac{1}{\cancel{3}}\left(\cancel{3}^{1}\right)=\frac{1}{\cancel{2}}\left(\cancel{2}^{1}\right)$

$\frac{1}{3}(3)=1$

$1=1$

Solution Set $=\{-12,0\}$

Q.6 Solve the following inequality

(iii) $-\frac{1}{3}x+5\le 1$ **(U.B)+(K.B)**

Solution $-\frac{1}{3}x+5\le 1$

$-\frac{1}{3}x\le 1-5$

$-\frac{1}{3}x\le -4$

$x\ge -4\times(-3)$

$x\ge 12$

Solution Set $=\{x\,|\,x\ge 12\}$

(i) $-3<\frac{1-2x}{5}<1$

Solution $-3<\frac{1-2x}{5}<1$

$-3<\frac{1-2x}{5}$

$-15<1-2x$

$-15-1<-2x$

$-16<-2x$

$\frac{-16}{-2}>x$

$8>x$

$x<8$

$\frac{1-2x}{5}<1$

$1-2x<5$

$-2x<5-1$

$-2x<4$

$x>\frac{4}{-2}$

$x>-2$

$-2<x$

$-2<x<8$

Solution Set $=\{x\,|\,-2<x<8\}$

CUT HERE

SELF TEST

Time: 40 min **Marks: 25**

Q.1 Four possible answers (A), (B), (C) & (D) to each question are given, mark the correct answer. **(7×1=7)**

1 If capacity " C " of an elevator is at most 1600 pounds then ___

(A) $C < 1600$ **(B)** $C \le 1600$

(C) $C \ge 1600$ **(D)** $C > 1600$

2 $x = 0$ **is a solution of the inequality ______**

(A) $x > 0$ **(B)** $3x + 5 < 0$

(C) $x + 2 < 0$ **(D)** $x - 2 < 0$

3 ____ is the member of the solution set of inequality $-2 < x < \frac{3}{2}$.

(A) –5 **(B)** 0

(C) $\frac{3}{2}$ **(D)** 3

4 The solution set of $|x - 4| = -4$ **is:**

(A) – 8 **(B)** – 16

(C) { } **(D)** 4

5 Which of the solution set of the inequality $9 - 7x > 19 - 2x$

(A) 19 **(B)** – 7

(C) 2 **(D)** – 2

6 The value of "x" from the equation $\sqrt{2x - 3} - 7 = 0$ **is:**

(A) 7 **(B)** 49

(C) 52 **(D)** 26

7 The general form of linear equation in one variable x is

(A) ax + by + c **(B)** ax^2 + bx + c

(C) ax + b = 0 **(D)** ax + by + cz = 0

Q.2 Give Short Answers to following Questions. **(5×2=10)**

(i) Find the solution set of $\frac{3}{y-1}-2=\frac{3y}{y-1}, \quad y\neq 1$

(ii) Solve the inequality: $4x-10.3\leq 21x-1.8$

(iii) Solve the inequality: $4-\frac{1}{2}x\geq -7+\frac{1}{4}x$

(iv) Solve: $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$

(v) Solve the radical equation: $\sqrt[3]{2x+3}=\sqrt[3]{x-2}$

Q.3 Answer the following Questions. **(4+4=8)**

(a) Solve the inequality: $x-2(5-2x)\geq 6x-3\frac{1}{2}$

(b) Solve the equation and check for extraneous root: $\sqrt{\frac{x+1}{2x+5}}=2, \; x\neq\frac{-5}{2}$

NOTE: Parents or guardians can conduct this test in their supervision in order to check the skill of students.