UNIT 10

Graphs of Functions

Functions and their Graphs

A function can be expressed in various forms, including an equation, a graph, a numerical table or a verbal description. For example, the area of a circle depends on its radius.

In such cases, one variable y depends on another variable x. This relationship is expressed as:

$y = f(x)$

Here, f denotes the function, x is the independent variable (input) and y is the dependent variable (output) determined by the value of x.

Graph of Linear Functions

A linear function is a mathematical expression that represents a straight-line relationship between two variables. Its general form is $f(x) = mx + c$, where "m" is the slope or gradient of the line, indicating how steep it is and "c" is the y-intercept (the point where the line crosses the y-axis). It can also be written as $y = mx + c$.

Example 1: Sketch the graph of $y = 2x - 1$.

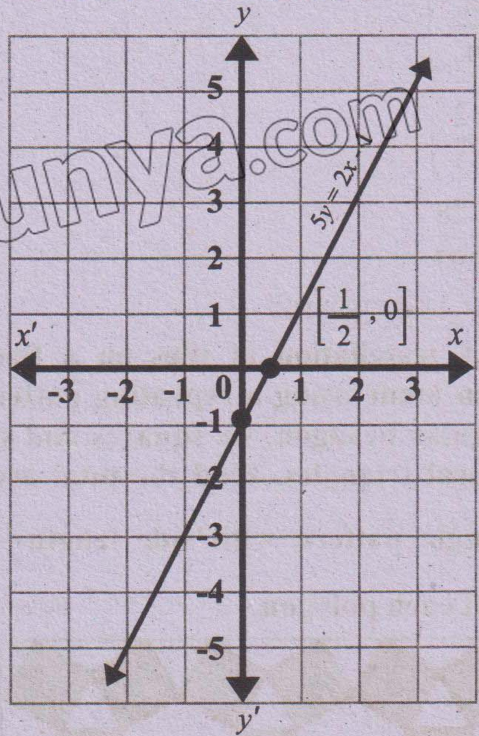
09310001

Solution: To sketch the graph of a linear function, we can find its x and y intercepts.

Put $x = 0$, we get $y = -1$. So $(0, -1)$ is the y-intercept.

Put $y = 0$, we get $x = \frac{1}{2}$. So $\left(\frac{1}{2}, 0\right)$ is the x-intercept.

The graph is a straight line that rises to the right because slope is positive.



Graph of Quadratic Functions

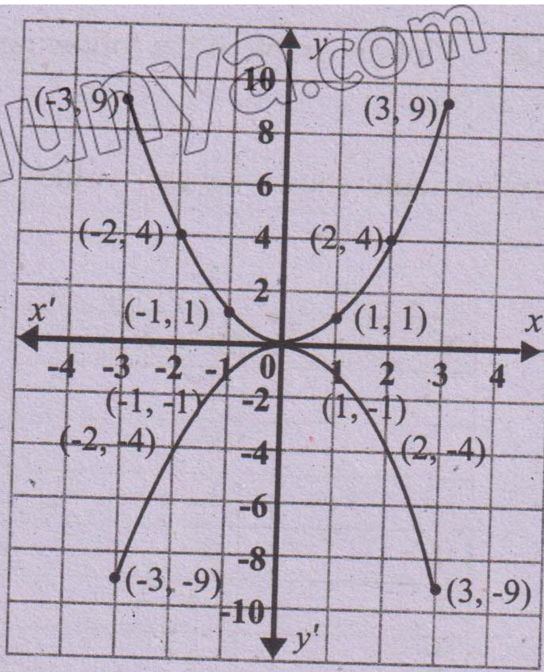
A quadratic function is a type of polynomial function that involves x^2 term. Its general form is: $y = ax^2 + bx + c$

Where a, b, c are constants and $a \neq 0$.

Example 2: Plot the graphs of $y = x^2$ and $y = -x^2$ on the same diagram. 09310002

Solution: The following table shows several values of x and the given functions are evaluated at those values:

x	$y = x^2$	$y = -x^2$
-3	$(-3)^2 = 9$	-9
-2	$(-2)^2 = 4$	-4
-1	$(-1)^2 = 1$	-1
0	$(0)^2 = 0$	0
1	$(1)^2 = 1$	-1
2	$(2)^2 = 4$	-4
3	$(3)^2 = 9$	-9



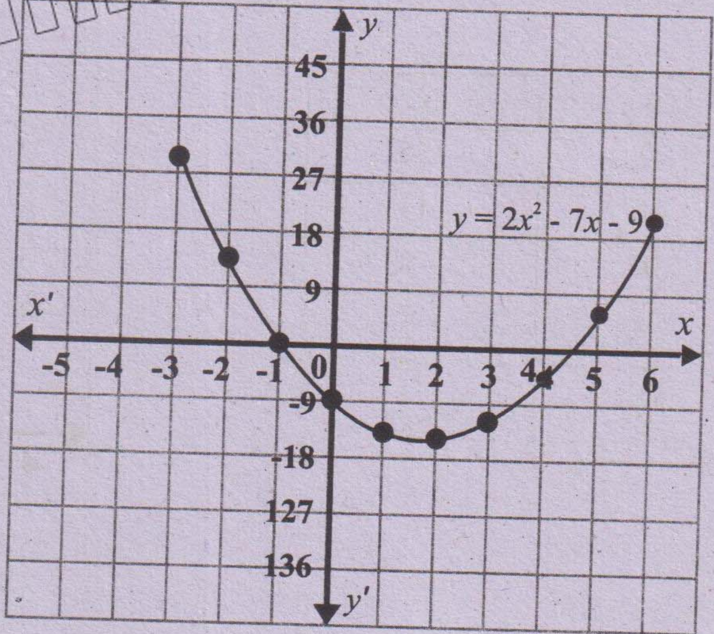
(i) Graph of $y = x^2$ represents parabola, passing through origin and opens upward.

(ii) Graph of $y = -x^2$ represents parabola, passing through origin and opens downward.

Example 3: Sketch the graph of $y = 2x^2 - 7x - 9$ for $-3 \leq x \leq 6$. 09310003

Solution: The values of x and y are given in the table and sketched in figure below:

x	y
-3	30
-2	13
-1	0
0	-9
1	-14
2	-15
3	-12
4	-5
5	6
6	21



Graph of Cubic Functions

A cubic function is a type of polynomial function of degree 3. Its standard form is:

$y = ax^3+bx^2+cx+d$ Where a, b, c are constants and $a \neq 0$.

Remember!

- The graph of a cubic function is a curve that can have at most two turning points.
- It has a general "S-shaped" appearance and depending on the coefficients, the shape may vary.
- Such functions are much more complicated and show more varied behaviour than linear and quadratic ones.

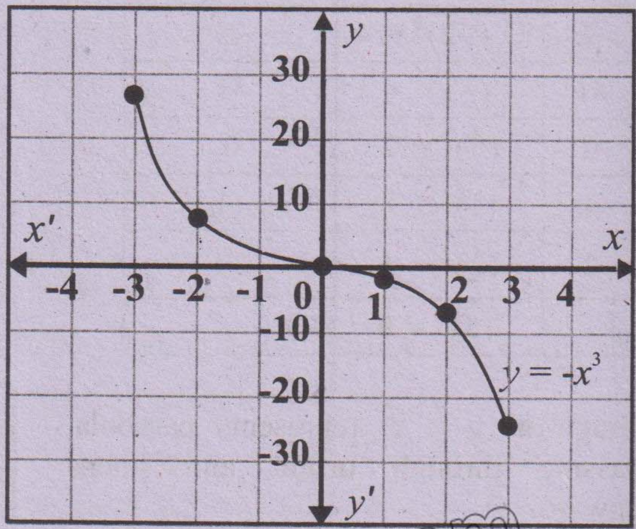
Example 4: Plot the graph of the following cubic function for $-3 \le x \le 3$: $y = -x^3$

09310004

Solution:

The following table shows several values of x and the given function is evaluated at those values:

x	$y = -x^3$
–3	27
–2	8
–1	1
0	0
1	–1
2	–8
3	–27



The curve passes through the origin.

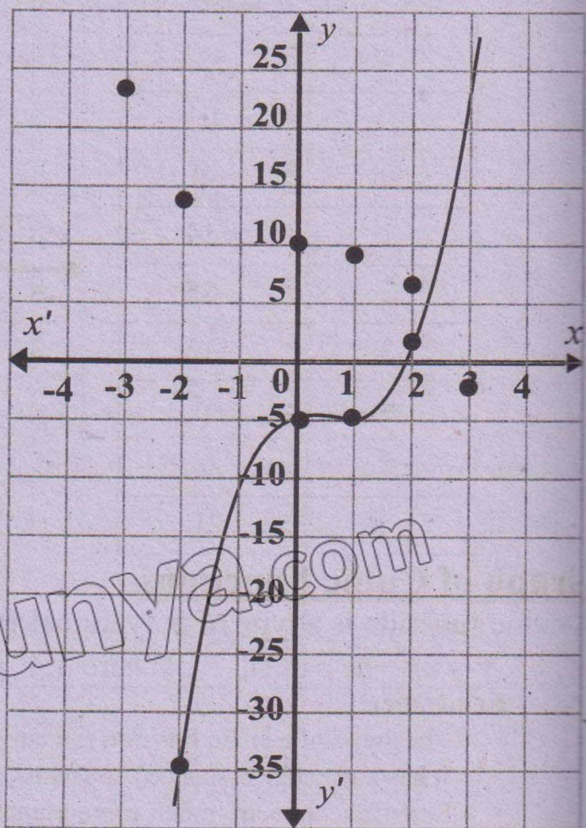
Example 5: Plot the graph of $y = 2x^3 - 3x^2 + x - 5$ for $-2 \le x \le 3$.

09310005

Solution:

The following table shows several values of x and the given function is evaluated at those values:

x	y
–2	–35
–1	–11
0	–5
1	–5
2	1
3	25



The graph tells us that when $x = 0$, the function's value is -5.

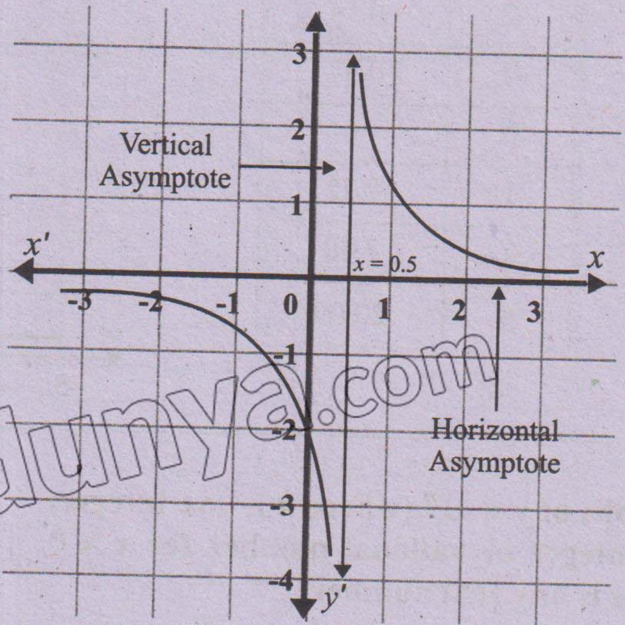
Graph of Reciprocal Functions

A reciprocal function is a function of the form: $y = \frac{a}{x}$ Where a is any real number and $x \neq 0$.

Example 6: Sketch the graph of the following reciprocal function: $y = \frac{1}{x - 0.5}, x \neq 0.5$

Solution: The following table shows several values of x and the given function is evaluated at those values:

x	y
–1	–0.67
–0.5	–1
–0.2	–1.43
0	–2
0.2	–3.3
0.5	undefined
1	2
1.2	1.43
1.5	1
2	0.67
2.2	0.59
2.5	0.5
3	0.4



Graph of Exponential Functions ($y = ka^x$ where x is real number, $a > 1$)

An exponential function is a mathematical function of the form:

$$y = ka^x$$

Where a, k are constants, x is variable and $a > 1$.

Example 7: Plot the graph of the exponential function $y = 2^x$ for $-6 \leq x \leq 6$.

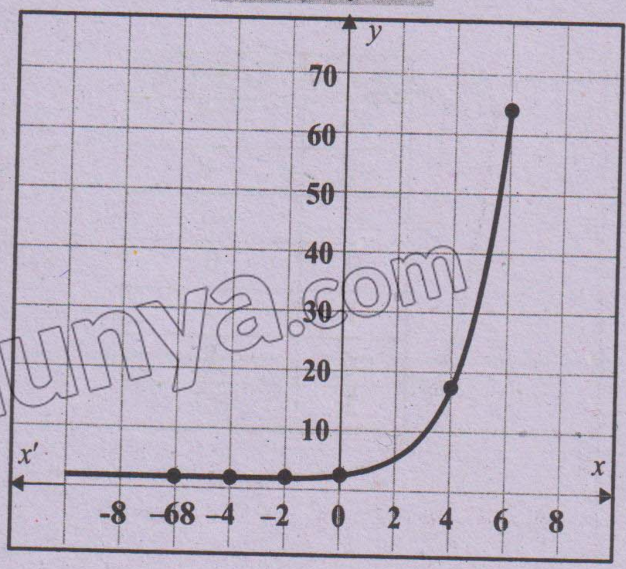
Solution: The function $y = 2^x$ has base 2 and variable exponent x. Values of (x, y) are given in the table below:

09310007

x	–6	–4	–2	0	2	4	6
$y = 2^x$	0.02	0.06	0.25	1	4	16	64

Then graph these points as in figure below.

Graph of $y = 2^x$



Example 8:

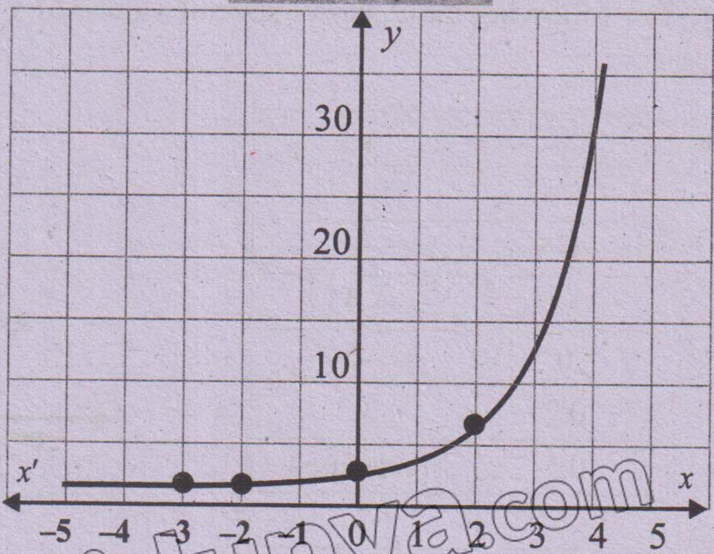
Plot the graph of the exponential function, $y = e^x$. **09310008**

Solution: The function $y = e^x$ has base e and variable power x. We know $e = 2.7182818$, correct to two decimal places $e = 2.72$.

Table of x and y values is given below:

x	$y = e^x$
-3	0.05
-2	0.14
-1	0.37
0	1
1	2.72
2	7.40
3	20.09

Graph of $y = e^x$



Graphs of $y = ax^n$ (where n is +ve integer, -ve integer or rational number for $x > 0$ and a is any real number)

The graph of the function $y = ax^n$, where n is a positive integer, negative integer or rational number for $x > 0$ and a is any real number, exhibits distinct behaviours depending on the value of n. Following are the examples of these cases:

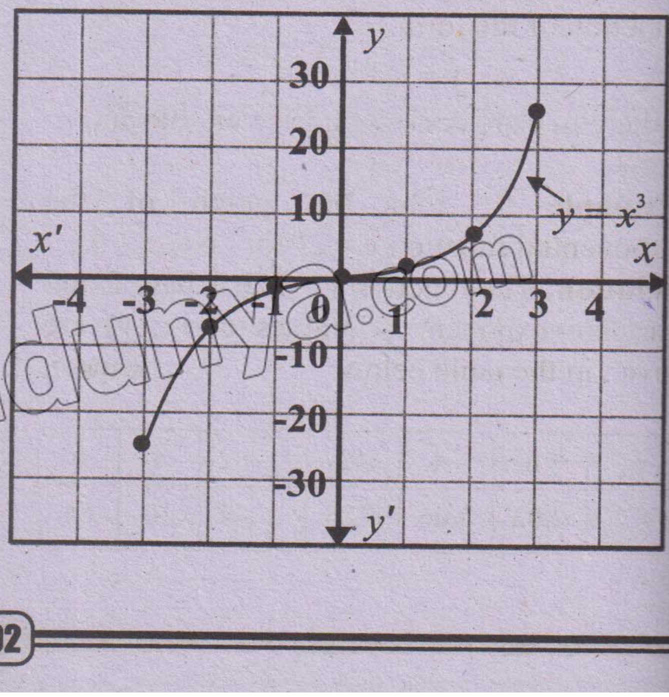
(i) When n is positive integer ($n = 3$)

Example 9: Plot the graph of $y = x^3$ for $-3 \leq x \leq 3$. **09310009**

Solution: The table shows several values of x and the given function is evaluated at those values:

The curve passes through the origin.

x	$y = x^3$
–3	–27
–2	–8
–1	–1
0	0
1	1
2	8
3	27



(ii) When n is negative integer ($n = -1$)

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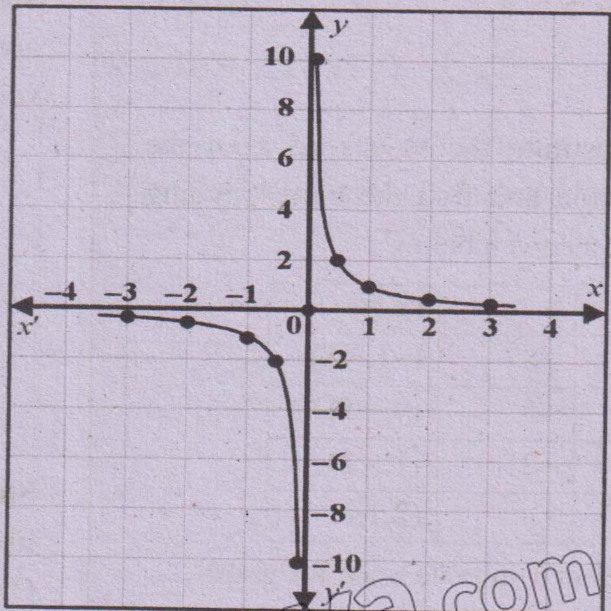
Example 10: Plot the graph of $y = x^{-1}$

Solution: $y = x^{-1} = \dfrac{1}{x}$

The following table shows several values of x and the given function

X	$y = \dfrac{1}{x}$
–3	–0.3
–2	–0.5
–1	–1
–0.5	–2
–0.1	–10
0.1	10
0.5	2
1	1
2	0.5
3	0.3



The above graph consists of two branches, one in the first quadrant and the other in the third quadrant. Both branches approach but never touch the x-axis or the y-axis.

(iii) When n is rational number $\left(n = \dfrac{1}{5}\right)$

09310011

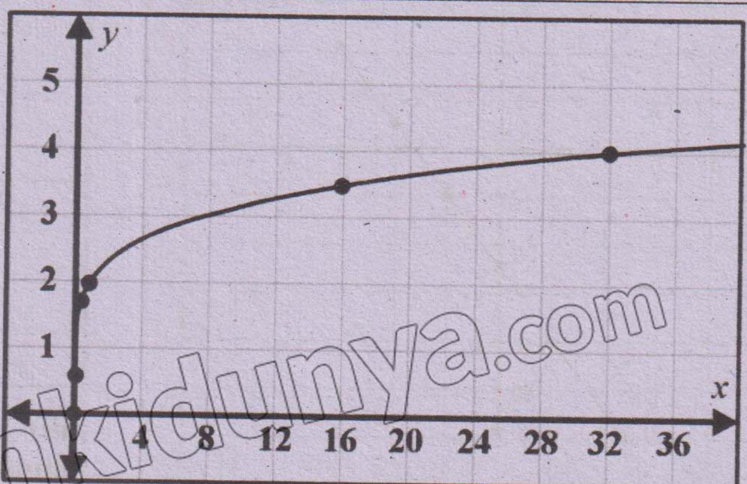
Example 11: Plot the graph of $y = 2x^{\frac{1}{5}}$.

09310012

Solution: $y = 2x^{\frac{1}{5}}$

The following table shows several values of x and the given function is evaluated at those values.

x	y
0	0
0.01	0.64
0.5	1.74
1	2
16	3.48
32	4



Exercise 10.1

Q.1 Sketch the graph of the following linear functions:

(i) $y = 3x - 5$ 09310013

Solution:

$y = 3x - 5$

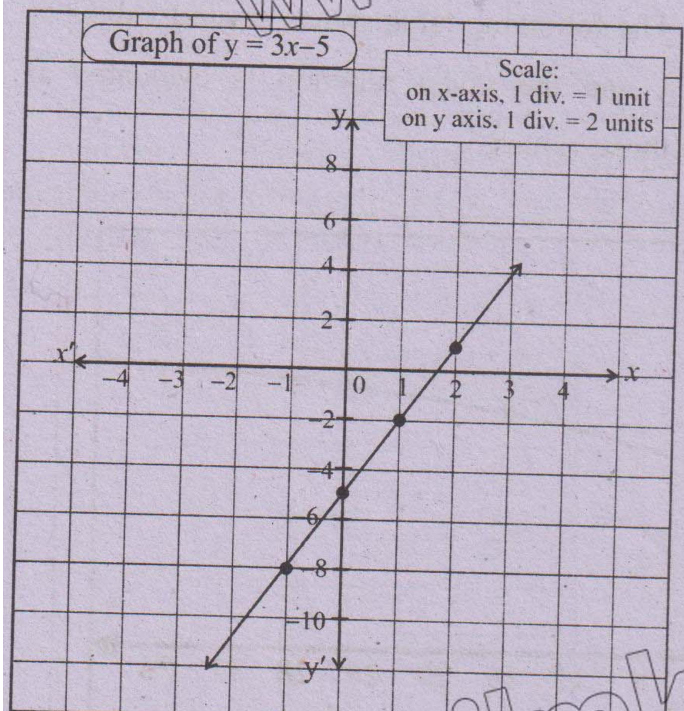
Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below

x	y	(x, y)
−1	−8	(−1, −8)
0	−5	(0, −5)
1	−2	(1, −2)
2	1	(2,1)

Step-II: Select a suitable scale for graph like on x – axis, 1 division = 1 unit

on y– axis, 1 division = 2 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(ii) $y = -2x + 8$ 09310014

Solution:

$y = -2x + 8$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below

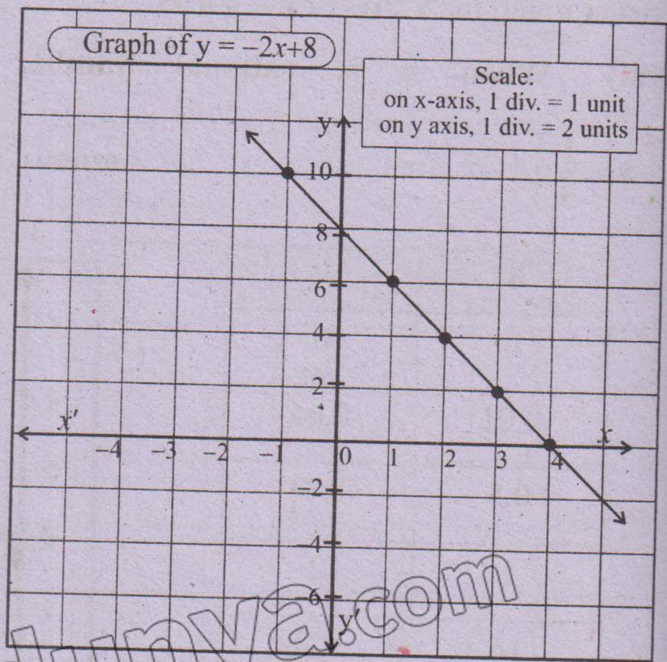
x	y	(x, y)
−1	10	(−1, 10)
0	8	(0, 8)
1	6	(1, 6)
2	4	(2, 4)
4	0	(4,0)

Step-II: Select a suitable scale for graph like

On x, axis, 1 division = 1 unit

On y, axis, 1 division = 2 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(iii) $y = 0.5x - 1$ 09310015

Solution

$y = 0.5x - 1$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

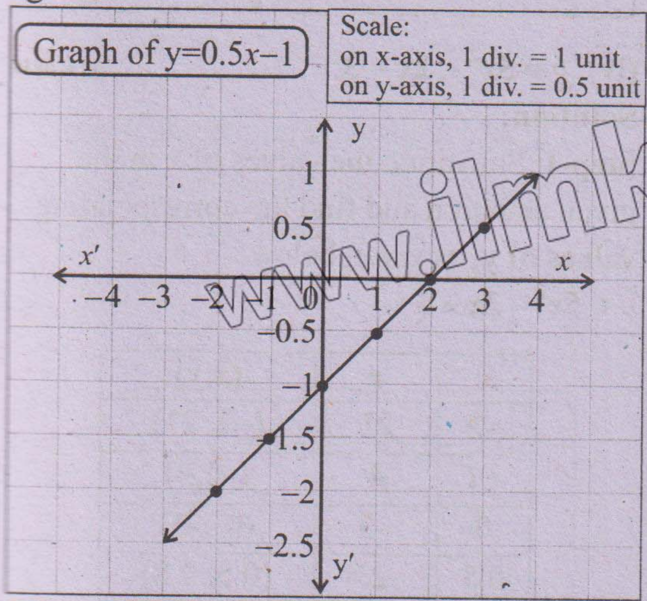
x	y	(x, y)
−2	−2	(−2, −2)
−1	−1.5	(−1,−1.5)
0	−1	(0, −1)
1	−0.5	(1, −0.5)
2	0	(2, 0)
3	0.5	(3, 0.5)

Step-II: Select a suitable scale for graph like

On x − axis, 1 division = 1 unit

On y− axis, 1 division = 0.5 unit

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Q.2 Sketch the graph of the following quadratic and cubic functions:

(i) $y = x^3 + 2x^2 - 5x - 6; -3.5 \le x \le 2.5$

09310016

Solution:

$y = x^3 + 2x^2 - 5x - 6; -3.5 \le x \le 2.5$

Step-I: Substitute the values of x from −3.5 to 2.5 in the given equation and find the corresponding values of y, as given below.

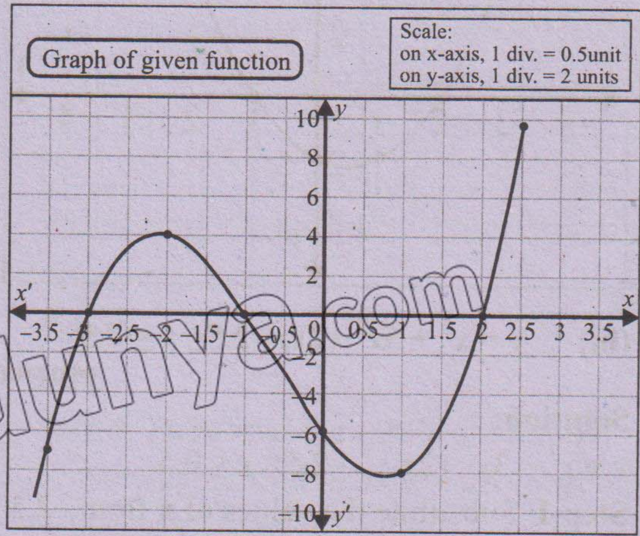
x	y	(x, y)
2.5	9.6	(2.5, 9.6)
2	0	(2, 0)
1	−8	(1, −8)
0	−6	(0, −6)
−1	0	(−1, 0)
−2	4	(−2, 4)
−3	0	(−3, 0)
−3.5	−6.8	(−3.5, −6.8)

Step-II Select a suitable scale for graph like

On x - axis, 1 division = 0.5unit

On y- axis, 1 division = 2 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(ii) $y = x^2 + x - 2$ **09310017**

Solution:

$y = x^2 + x - 2$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below

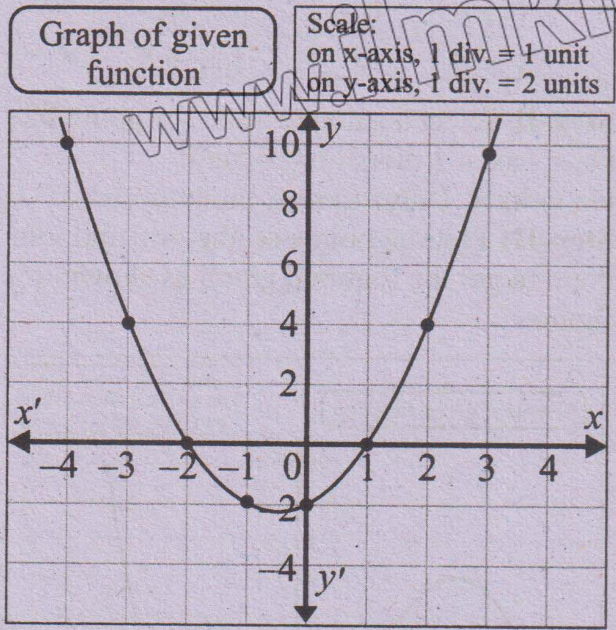
x	y	(x, y)
−4	10	(−4, 10)
−3	4	(−3, 4)
−2	0	(−2, 0)
−1	−2	(−1, −2)
0	−2	(0, −2)
1	0	(1, 0)
2	4	(2, 4)
3	10	(3, 10)

Step-II: Select a suitable scale for graph like

on −x− axis, 1 division = 1 unit

on −y− axis, 1 division = 2 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(iii) $y = x^3 + 3x^2 + 2x;\ -2.5 \le x \le 0.5$

09310018

Solution:

$y = x^3 + 3x^2 + 2x;\quad -2.5 \le x \le 0.5$

Step-I: Substitute the values of x from −2.5 to 0.5 in the given equation and find the corresponding values of y, as given below.

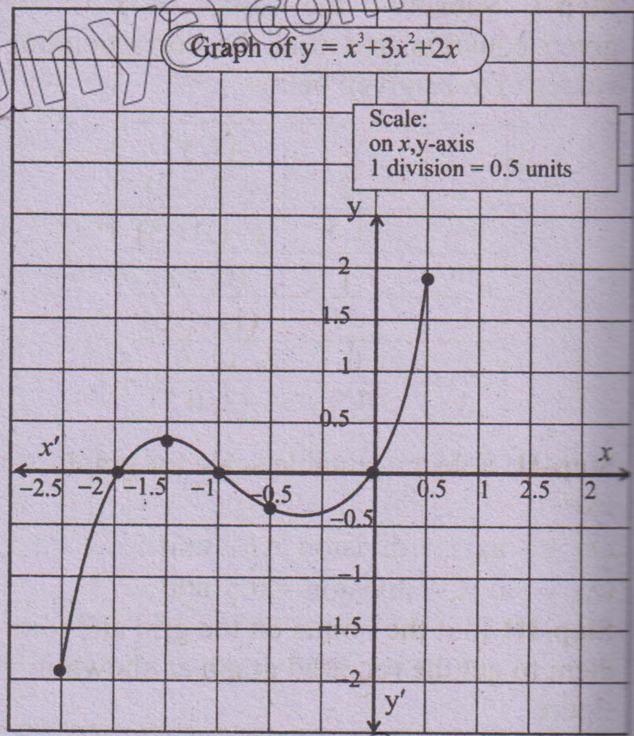
x	y	(x, y)
−2.5	−1.8	(−2.5, −1.9)
−2	0	(−2,0)
−1.5	0.4	(−1.5,0.4)
−1	0	(−1,0)
−0.5	−0.4	(−0.5,−0.4)
0	0	(0,0)
0.5	1.8	(0.5,1.8)

Step-II Select a suitable scale for graph like

On x - axis, 1 division = 0.5unit

On y- axis, 1 division = 0.5 unit

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(iv) $y = 5x^2 - 2x - 3$

09310019

Solution:

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

$y = 5x^2 - 2x - 3$

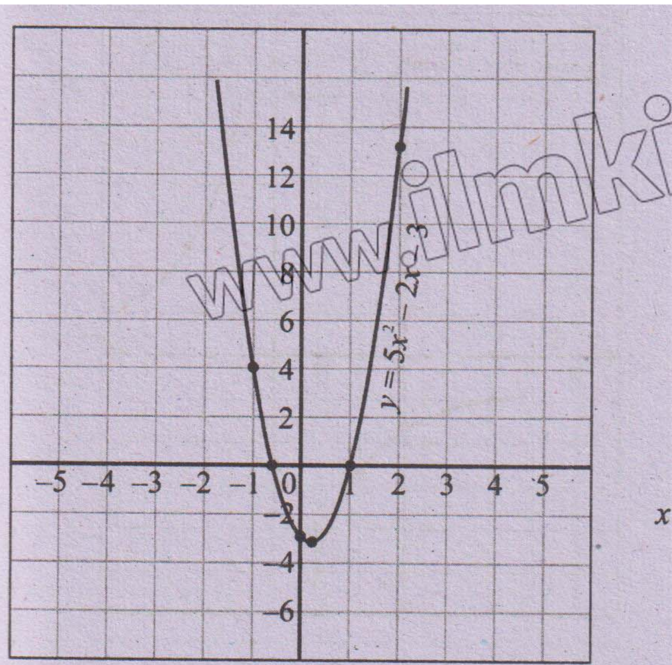
x	*y*	(x,y)
−2	21	(−2, 21)
−1	4	(−1,4)
0	−3	(0, −3)
0.5	−2.8	(0.5, 2.8)
1	0	(1,0)
2	13	(2,13)

Step-II Select a suitable scale for graph like

On x - axis, 1 division = 1 unit

On y- axis, 1 division = 2 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Q.3 Sketch the graph of the following functions:

(i) $y = 4^x$ **09310020**

Solution: $y = 4^x$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

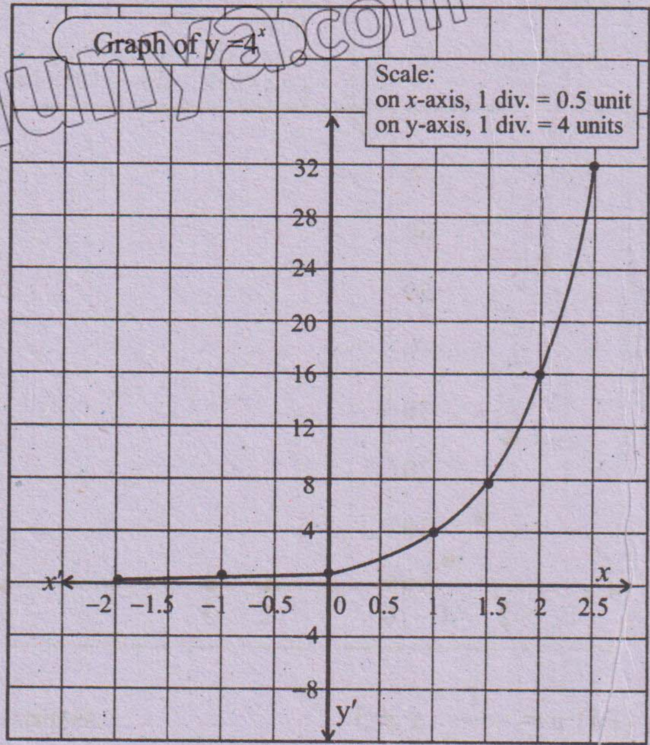
x	$y = 4^x$	(x, y)
−2	0.06	(−2,0.06)
−1	0.25	(−1,0.25)
0	1	(0,1)
0.5	2	(0.5, 2)
1	4	(1,4)
1.5	8	(1.5, 8)
2	16	(2,16)
2.5	32	(2.5, 32)

Step-II Select a suitable scale for graph like

On $x-$ axis, 1 division = 0.5 unit

On y− axis, 1 division = 4 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(ii) $y = 5^{-x}$ **09310021**

Solution:

$y = 5^{-x}$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

x	$y = 5^{-x}$	(x, y)
−2.5	56	(−2.5,56)
−2	25	(−2, 25)
−1	5	(−1, 5)
0	−1	(0, 1)
1	0.2	(1, 0.2)
2	0.04	(2, 0.04)
3	0.008	(3, 0.008)

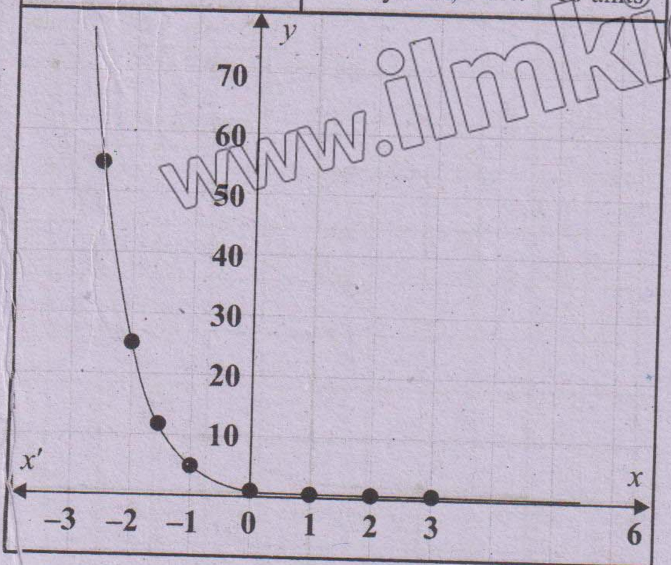
Step-II Select a suitable scale for graph like

On $x-$ axis, 1 division = 1 unit

On y− axis, 1 division = 10 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.

Graph of $y = 5^{-x}$	Scale: on x-axis, 1 div. = 1 unit; on y-axis, 1 div. = 10 units



(iii) $y = \dfrac{1}{x-3}\ \ x \neq 3$ **09310022**

Solution:

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

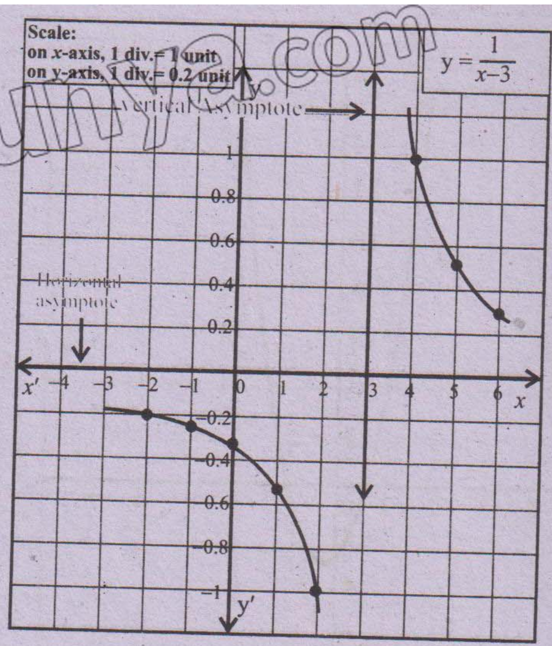
$$y = \frac{1}{x-3}\ ,\ (x \neq 3)$$

x	y	(x, y)
−3	−0.16	(−3,−0.16)
−2	−0.2	(−2, −0.2)
−1	−0.25	(−1, −0.25)
0	−0.3	(0, −0.3)
1	−0.5	(1, 0.5)
2	−1	(2, −1)
3	∞	(3, ∞)
4	1	(4, 1)
5	0.5	(5, 0.5)
6	0.3	(6,0.3)

Step-II Select a suitable scale for graph like

On x – Axis, 1 division = 1 units

On y –axis, 1 division = 0.1 units



(iv) $y = \dfrac{2}{x} + 3\ ,\ (x \neq 0)$ **09310023**

Solution: $y = \dfrac{2}{x} + 3\ ,\ (x \neq 0)$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

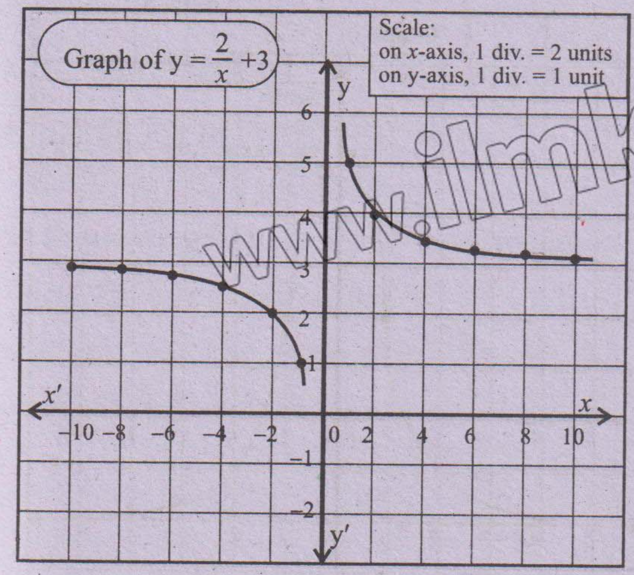
x	y	(x, y)
−10	2.8	(−10, 2.8)
−8	2.75	(−8, 2.75)
−6	2.66	(−6, 2.66)
−4	2.5	(−4, 2.5)
−2	2	(−2, 2)
−1	1	(−1, 1)
0	∞	(0, ∞)
1	5	(1, 5)
2	4	(2, 4)
4	3.5	(4, 3.5)
6	3.3	(6, 3.3)
8	3.25	(8, 3.25)
10	3.2	(10, 3.2)

Step-II Select a suitable scale for graph like

On x – Axis, 1 division = 2 units

On y –axis, 1 division = 1 unit

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(v) $y = x^{\frac{1}{2}}$ **09310024**

Solution: $y = x^{\frac{1}{2}}$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

y = $y = x^{\frac{1}{2}} = \sqrt{x}$

Square is not defined for negative values,

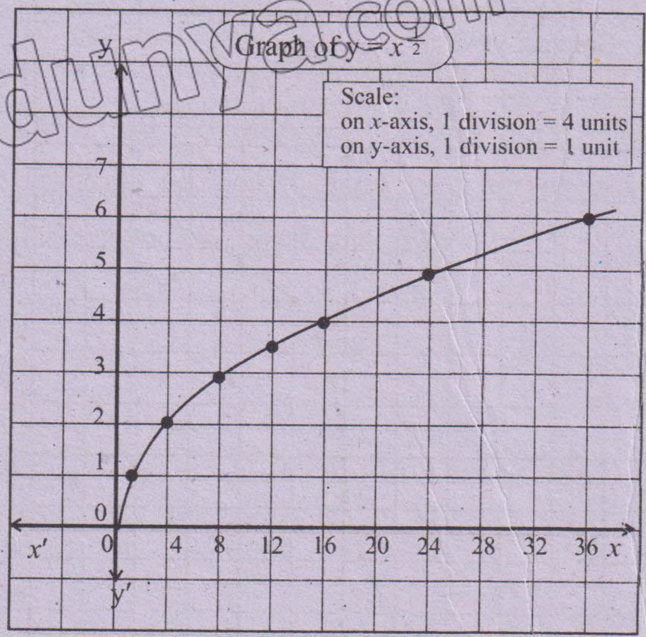
x	$y = x^{\frac{1}{2}} = \sqrt{x}$	(x, y)
0	$y = \sqrt{0} = 0$	(0, 0)
1	$y = \sqrt{1} = 1$	(1, 1)
4	$y = \sqrt{4} = 2$	(4, 2)
9	$y = \sqrt{9} = 3$	(9, 3)
16	$y = \sqrt{16} = 4$	(16, 4)
25	$y = \sqrt{25} = 5$	(25, 5)
36	$\sqrt{36} = 6$	(36, 6)

Step-II Select a suitable scale for graph like

On x – Axis, 1 division = 4 units

On y –axis, 1 division = 1 unit

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(vi) $y = 3x^{\frac{1}{3}}$ **09310025**

Solution:

$y = 3x^{\frac{1}{3}} = 3\sqrt[3]{x}$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

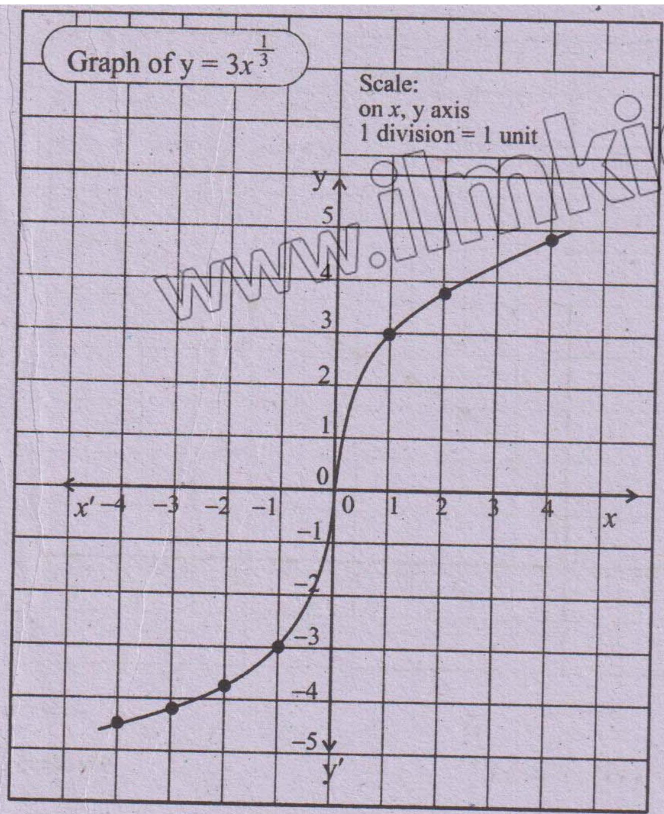
x	**y**
−4	−4.8
−3	−4.3
−2	−3.8
−1	−3
0	0
1	3
2	3.8
3	4.3
4	4.8

Step-II Select a suitable scale for graph like

On x-Axis, 1 division = 1 unit

On y-Axis, 1 division = 1 unit

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(vii) $y = 2x^{-2}$ **09310026**

Solution:

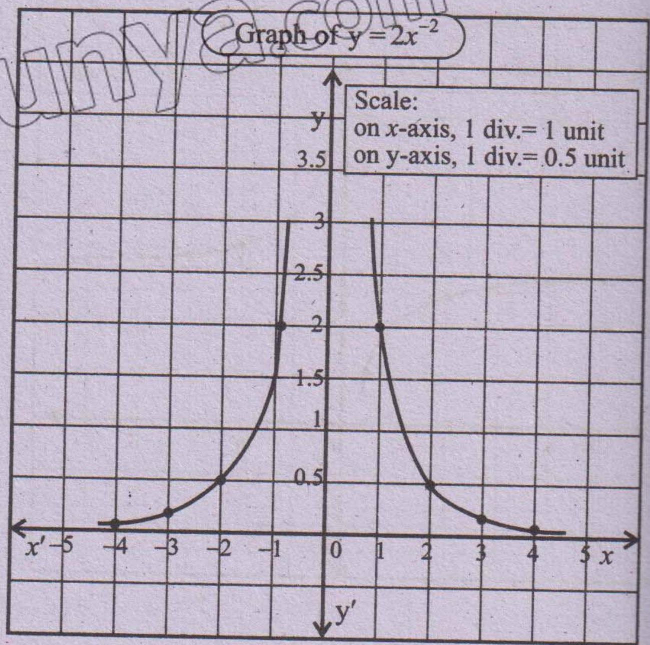
$y = 2x^{-2} = \dfrac{2}{x^2}$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

x	y	(x, y)
–4	0.13	(–4, 0.13)
–3	0.22	(4, 0.22)
–2	0.5	(–,0.5)
–1	2	(–1, 2)
0	∞	(0, ∞)
1	2	(1, 2)
2	0.5	(2, 0.5)
3	0.22	0.22

Step-II Select a suitable scale for graph like
On x-Axis, 1 division = 1 unit
On y-Axis, 1 division = 0.5 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Exponential Growth/ Decay of a Practical Phenomenon through its Graph:

Exponential Growth:

In exponential growth, such as population expansion, compound interest in finance or the spread of infectious diseases, the graph starts slowly but accelerates rapidly as time progresses. The curve increases steeply, showcasing how growth becomes more pronounced with time due to constant proportional changes.

Exponential Decay:

Conversely, in exponential decay, observed in cooling of objects or depreciation of assets, the graph starts high and decreases sharply before levelling off, indicating a gradual reduction over time. These graphs are essential for interpreting trends, making predictions and informing decision-making in diverse fields.

Example 12: The population of a village was 753 in 2010. If the population grows according to the equation $p = 753e^{0.03t}$, where p is the number of persons in the population at time t,

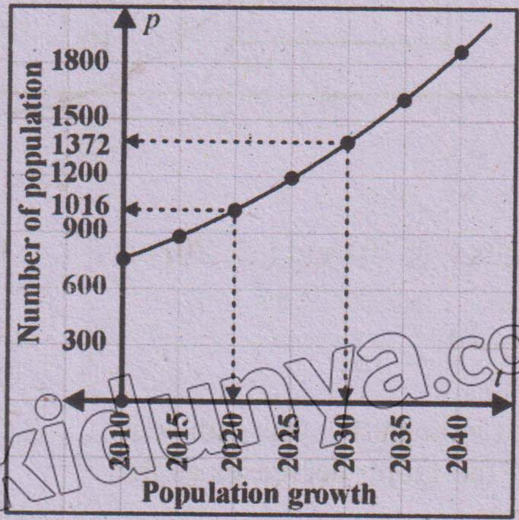
(a) Graph the population equation for $t = 0$ (in 2010) to $t = 30$ (in 2040).

(b) From the graph, estimate the population (i) in 2020 and (ii) in 2030.

Solution: (a) The general shape of the exponential is known; however, since the graph is being used for estimations, an accurate graph over the required interval, $t = 0$ to $t = 30$, is required.

Calculate a table of values for different time periods and sketched in below figure:

t	p
0	753
5	874.9
10	1016.4
15	1180.9
20	1372.1
25	1594.1
30	1852.1



(b) From graph,

(i) In 2020 ($t = 10$) the population is 1016 persons.

(ii) In 2030 ($t = 20$) the population is 1372 persons.

Gradients of Curves by Drawing Tangents

The gradient or slope of a graph at any point is equal to the gradient of the tangent to the curve at that point. Remember that a tangent is a line that just touches a curve only at one point (and doesn't cross it).

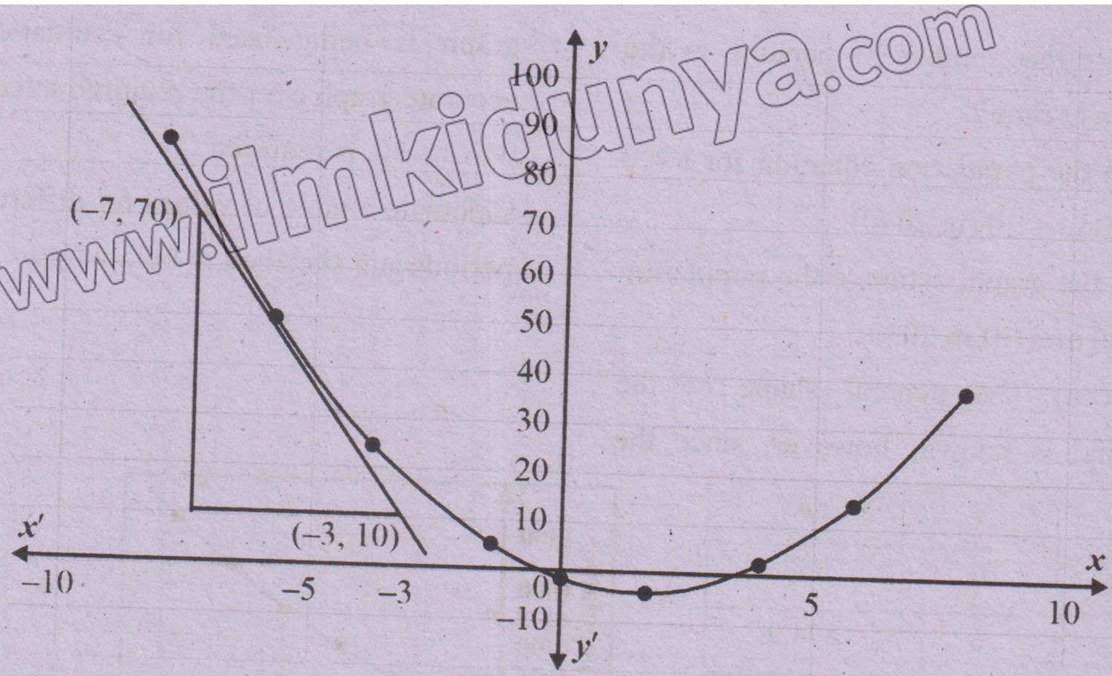
The gradient between two points is defined as:

$$\text{Gradient} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 13: Sketch the graph of $y = x^2 - 3x - 2$ for values of x from -8 to 8, draw a tangent line at $x = -6$ and determine the gradient. **09310027**

Solution: Calculate the y-values for given values of x. The results are given in the table and sketched in below figure:

x	–8	–6	–4	–2	0	2	4	6	8
y	86	52	26	8	–2	–4	2	16	38



Consider two points (–3, 10) and (–7, 70) on the tangent line.

So, gradient $= \dfrac{70-10}{-7+3} = -15$.

Since the gradient is negative, this indicates that the height of the graph decreases as the value of x increases.

10.2 Applications of Graph in Real-Life

In tax payment scenarios, graphing concepts help identify optimal income levels, tax brackets, and liability. In income and salary problems, graphing facilitates analysis of compensation packages and income growth. By sketching salary against experience, patterns or anomalies in compensation structures become apparent. In cost and profit analysis, graphing enables businesses to visualize cost-profit relationships, determine break-even points, and optimize production levels.

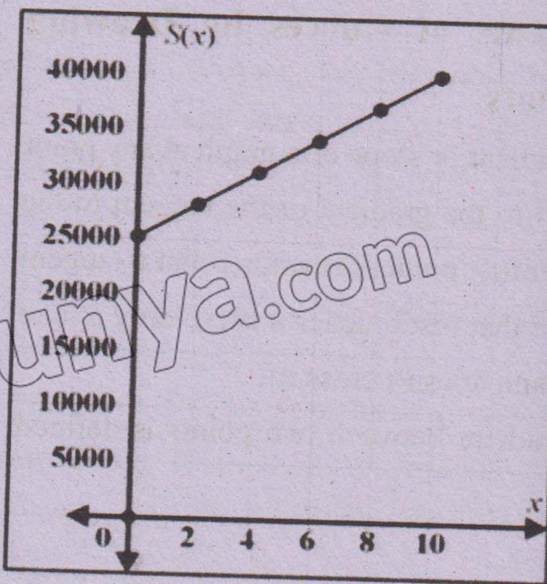
Example 14: Majid's salary S(x) in rupees is based on the following formula:

$$S(x) = 25000+1500x,$$

where x is the number of years he worked. Sketch and interpret the graph of salary function for $0 \le x \le 10$. 09310028

Solution: Table values and graph are given below:

x	$S(x)$
0	25000
2	28000
4	31000
6	34000
8	37000
10	40000



Majid's salary increases linearly with years of service and rises by Rs. 1500 for every year.

Example 15: A company manufactures footballs. The cost of manufacturing x footballs is $C(x) = 90,000 + 600x$. The revenue from selling x footballs is $R(x) = 1,800x$. Find the break-even point and determine the profit or loss when 150 footballs are sold. Draw the graphs of both the functions and identify the break-even point. **09310029**

Solution: Given that

Cost function: $C(x) = 90,000 + 600x$

Revenue function: $R(x) = 1,800x$

The break-even point occurs when

$R(x) = C(x)$

$1800x = 90000 + 600x$

$1800x - 600x = 9000$

$1200x = 90000$

$\Rightarrow x = 75$

So, at the break-even point, 75 footballs are produced or sold.

Next, we find the profit for 150 footballs

When $x = 150$, revenue:

$R(150) = 1,800(150)$

$= \text{Rs. } 270,000$

and $C(150) = 90,000 + 600(150)$

$= \text{Rs. } 180,000$

Now profit: $P(x) = R(x) - C(x)$

Substitute $x = 150$

$P(150) = R(150) - C(150)$

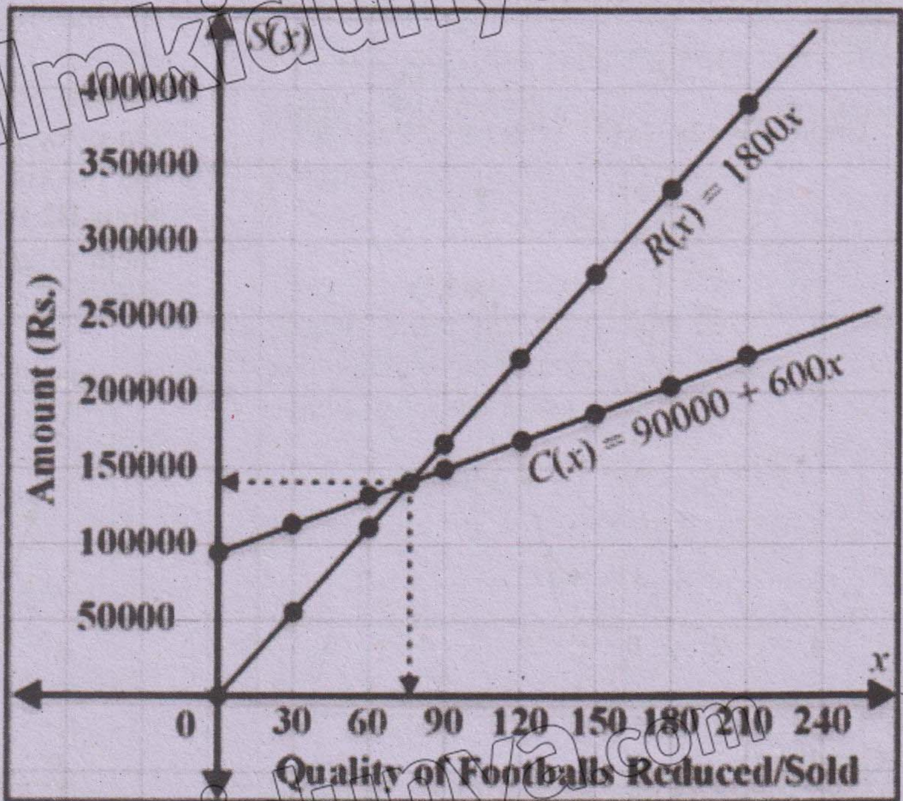
$= \text{Rs. } 270,000 - \text{Rs. } 180,000$

$= \text{Rs. } 90,000$

Thus, a company earns a profit of Rs. 90,000 when selling 150 footballs.

Table values of x, $C(x)$ and $R(x)$ are given below:

x	$C(x)$	$R(x)$
0	90000	0
30	108,000	54,000
60	126,000	108,000
90	144,000	162000
120	162,000	216,000
150	180,000	270,000
180	198,000	324,000
210	216,000	378,000



Exercise 10.2

Q.1 Plot the graph of $y = 2x^2 - 4x + 3$ from −1 to 3. Draw tangent at (2,3) and find gradient. **09310030**

Solution

$y = 2x^2 - 4x + 3$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

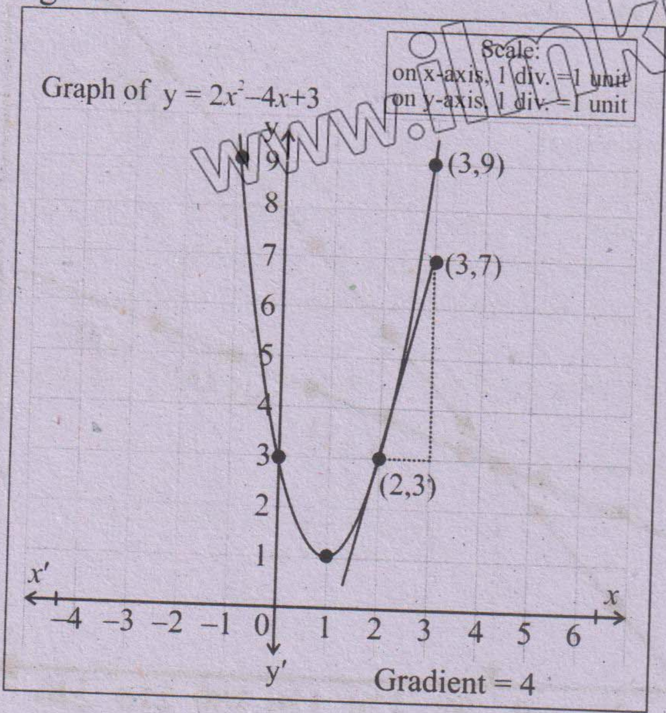
x	y	(x, y)
−1	9	(−1, 9)
0	3	(0,3)
1	1	(1, 1)
2	3	(2, 3)
3	9	(3, 9)

Step-II Select a suitable scale for graph like

On x-Axis, 1 division = 1 unit

On y-Axis, 1 division = 1 unit

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Gradient:

Draw a tangent line at point (2,3) . Take any two points on the tangent line. Let two points on the tangent line are: A(2,3) and B(3,7)

We know that

$$\text{Gradient } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7-3}{3-2} = \frac{4}{1} = 4$$

Thus gradient of tangent line is 4.

Q.2 Plot the graph of $y = 3x^2 + x + 1$ and draw tangent at (1, 5). Also find gradient of the tangent line at this point. **09310030**

Solution:

$y = 3x^2 + x + 1$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below.

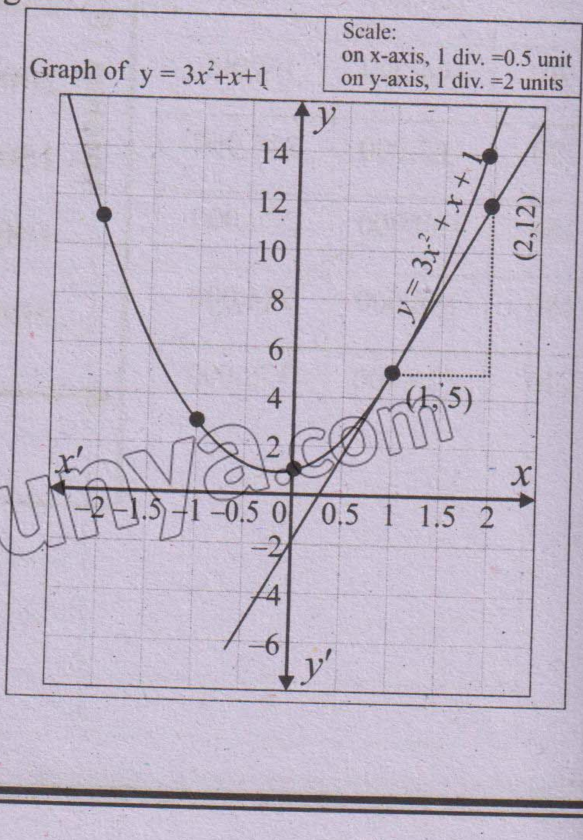
x	y	(x,y)
−2	11	(−2, 11)
−1	3	(−1, 3)
0	1	(0, 1)
1	5	(1,5)
2	15	(2, 15)

Step-II Select a suitable scale for graph like

On x-Axis, 1 division = 0.5 unit

On y-Axis, 1 division = 2 unit

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Gradient:

Draw a tangent line at point (1,5). Take any two points on the tangent line. Let two points on the tangent line are: A(1,5) and B(2,12)

We know that

$$\text{Gradient } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{12-5}{2-1} = \frac{7}{1} = 7$$

Thus gradient of tangent line is 7.

Q.3 The strength of students in a school was 1000 in 2016. If the strength decay according to the equation $S = 1000e^{-t}$, where S is the number of students at time t.

(a) Graph the given equation for t = 0 (in 2016) to t = 9 (in 2025). 09310031

(b) From the graph, estimate the student's strength in 2019 and in 2023. 09310032

Solution: (a)

$S = 1000\, e^{-t}$ Or $S = \frac{1000}{e^{t}}$

Step-I: Substitute the values of "t" in the given equation and find the corresponding values of S, as given below

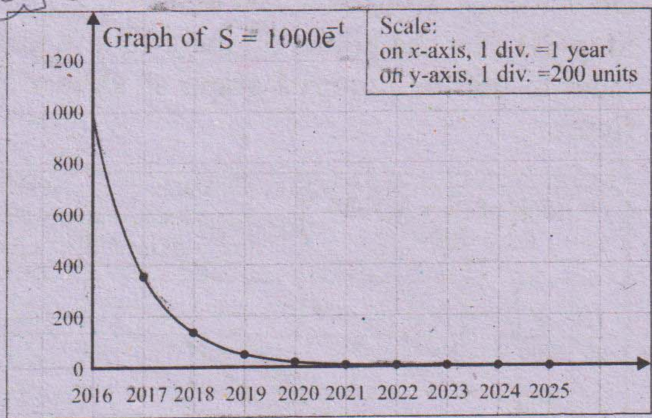
2016, (t = 0)	1000
2017, (t = 1)	367.9 ≈ 368
2018, (t = 2)	135.3≈ 135
2019, (t = 3)	49.8 ≈ 50
2020, (t = 4)	18.3≈ 18
2021, (t = 5)	6.7≈ 7
2022, (t = 6)	2.48≈ 3
2023, (t = 7)	0.91≈ 1
2024, (t = 8)	0.33≈ 0
2025, (t = 9)	0.12≈ 0

Step-II Select a suitable scale for graph like

On x-Axis, 1 division = t = 1 unit (year)

On y-Axis, 1 division = 200 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Solution: (b)

The estimated students' strength in 2019 is 50.

The estimated students' strength in 2023 is 1.

Q.4 The demand and supply functions for a product are given by the equations $P_d = 400 - 5Q$, $P_s = 3Q + 24$: Plot the graph of each function over the interval Q = 0 to Q = 300. 09310033

Solution:

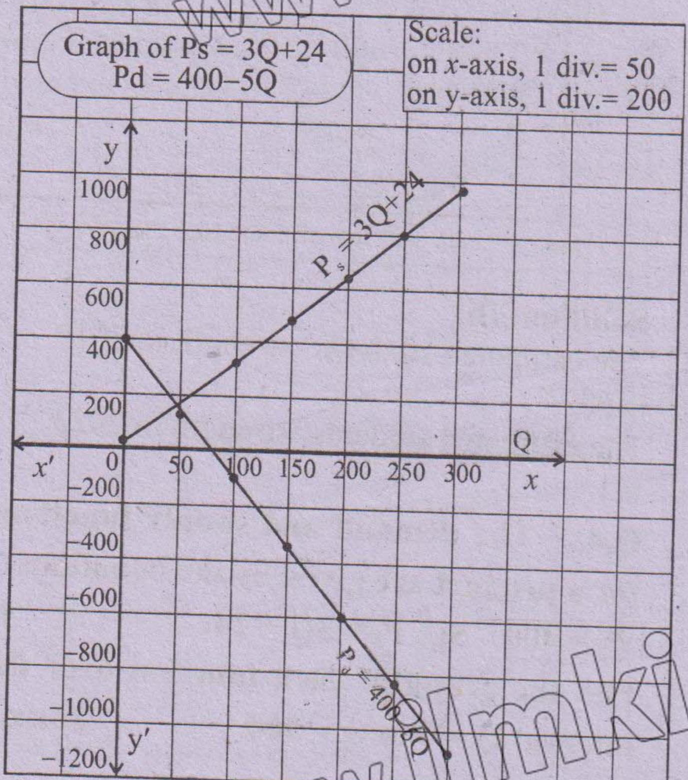
$P_d = 400 - 5Q$ and $P(s) = 3Q + 24$

The interval ($Q = 0$ to $Q = 300$)

Step-I: Substitute the values of "Q" in the given functions and find the corresponding values of P_d and p_s, as given below

Q	$P_d = 400 - 5Q$	$P_s = 3Q + 24$
0	400	24
50	150	174
100	–100	324
150	–350	474
200	–600	624
250	–850	774
300	–1100	924

Step-II Select a suitable scale for graph like
On x-Axis, 1 division = 50 units
On y-Axis, 1 division = 200 units
Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Q.5 Shahid's salary S(x) in rupees is based on the following formula:

$$S(x) = 45000 + 4500x,$$

where x is the number of years he has been with the company. Sketch and interpret the graph of salary function for $0 \le x \le 5$. 09310034

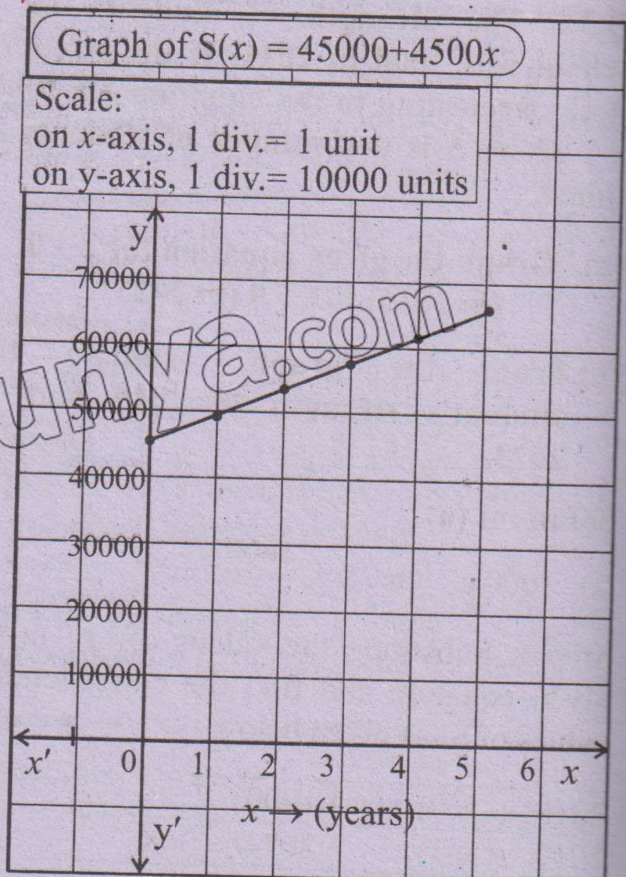
Solution:

$S(x) = 45000+4500x$ and $0 \le x \le 5$

Step-I: Substitute the values of "x" in the given function S(x) and find the corresponding values of S(x), as given below.

x	S(x)
0	45000
1	49,500
2	54,000
3	58,500
4	63,000
5	67,500

Step-II Select a suitable scale for graph like
On x-Axis, 1 division = 1 unit
On y-Axis, 1 division = 10000 units
Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Interpretation of Salary Graph:

The graph shows that Shahid's starting salary is Rs.45,000. After every 1 year Shahid's salary increases by Rs.4,500.

Q.6 A company manufactures school bags. The cost function of producing x bags is $C(x) = 1200 + 20x$ and the revenue from selling x bags is $R(x) = 50x$.

(a) Find the break-even point. 09310035

(b) Determine the profit or loss when 250 bags are sold. 09310036

(c) Plot graphs of both the functions and identify the break-even point. 09310037

Solution(a):

The break-even point is no profit or loss.

It happens when total cost and revenue are equal.

$R(x)=C(x)$

$50x=1200+20x$

$50x-20x=1200$

$30x=1200$

$x=\frac{1200}{30}=40$

The company reaches at breakeven point at the production of 40 bags.

(b) Determine the profit or loss when 250 bags are sold.

Solution:

Since break-even is at 40 bags. Surely it will be profit at 250 bags.

Profit $= R(x)-C(x)$

Profit $= 50x-(1200+20x)$

Profit $= 50x-1200-20x$

Profit $= 30x-1200$

Put $x = 250$ in profit function

Profit $= 30(250)-1200$

Profit $= 7,500-1,200$

Profit $= 6,300$.

(c) Plot graphs of both the functions and identify the break-even point.

Solution:

$R(x) = 50x$ and $C(x) = 1200 + 20x$

Step-I: Substitute the values of "x" in the given function S(x) and find the corresponding values of S(x), as given below.

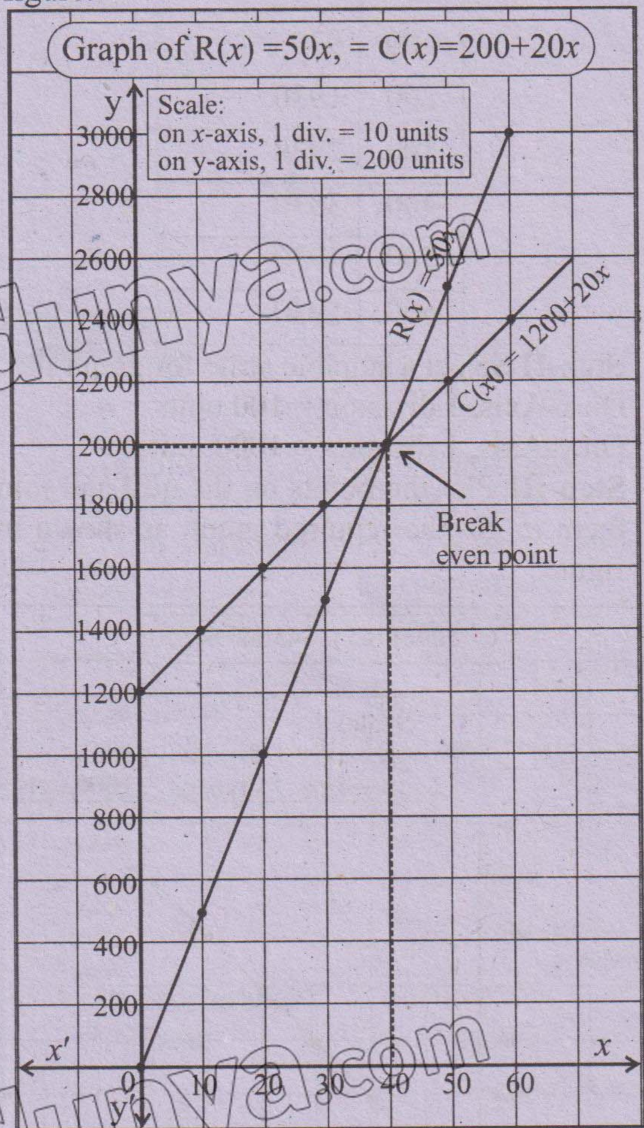
x	$R(x) = 50x$	$C(x)=1200+20x$
0	0	1200
10	500	1400
20	1000	1600
30	1500	1800
40	2000	2000
50	2500	2200
60	3000	2400

Step-II Select a suitable scale for graph like

On x-Axis, 1 division = 10 units

On y-Axis, 1 division = 200 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Q.7 A newspaper agency fixed cost of Rs.70 per edition and marginal printing and distribution costs of Rs. 40 per copy. Profit function is $p(x) = 10x - 70$, where x

is the number of newspapers. Plot the graph and find profit for 500 newspaper.

09310038

Solution:

Plotting the Graph:

$p(x) = 10x - 70$

Step-I: Substitute the values of "x" in the given function S(x) and find the corresponding values of S(x), as given below.

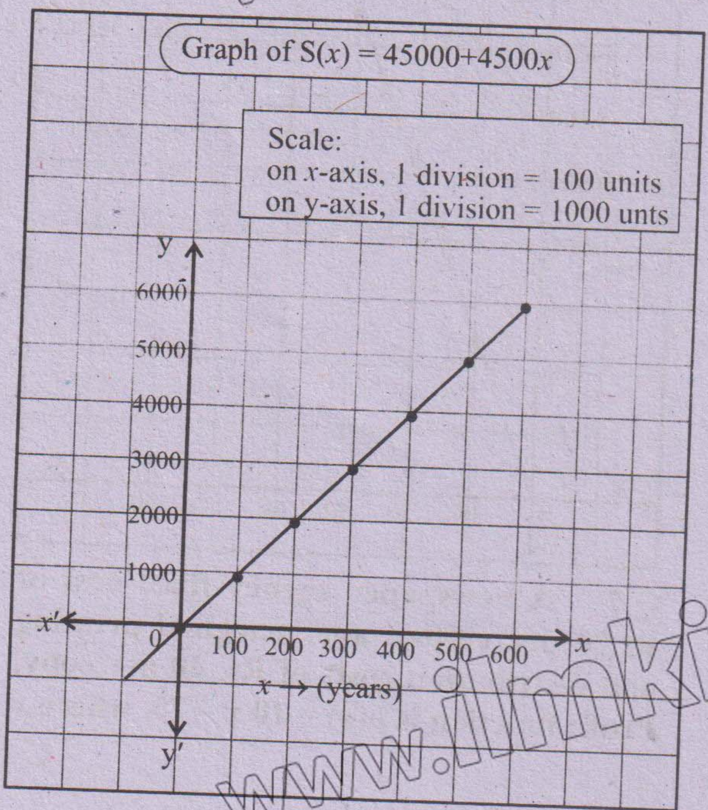
x	p(x)
0	–70
100	930
200	1930
300	2930
400	3930
500	4930
600	5930

Step-II Select a suitable scale for graph like

On x-Axis, 1 division = 100 units

On y-Axis, 1 division = 1000 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Finding the profit:

Profit function is $p(x) = 10x - 70$;

Put $x = 500$ in p(x)

$p(500) = 10(500) - 70$

$p(500) = 5000 - 70$

$p(500) = 4,930$

Thus profit for 500 news papers is Rs.4,930.

Q.8 Ali manufacturers expensive shirts for sale to a school. Its cost (in rupees) for x shirts is $C(x) = 1500+10x+0.2x^2$, $0 \le x \le 150$. Plot the graph and find the cost of 200 shirts.

09310039

Solution:

Plotting the Graph:

Step-I: Substitute the values of "x" in the given function C(x) and find the corresponding values of C(x), as given below.

$C(x) = 1500+10x + 0.2x^2$ and $0 \le x \le 1500$

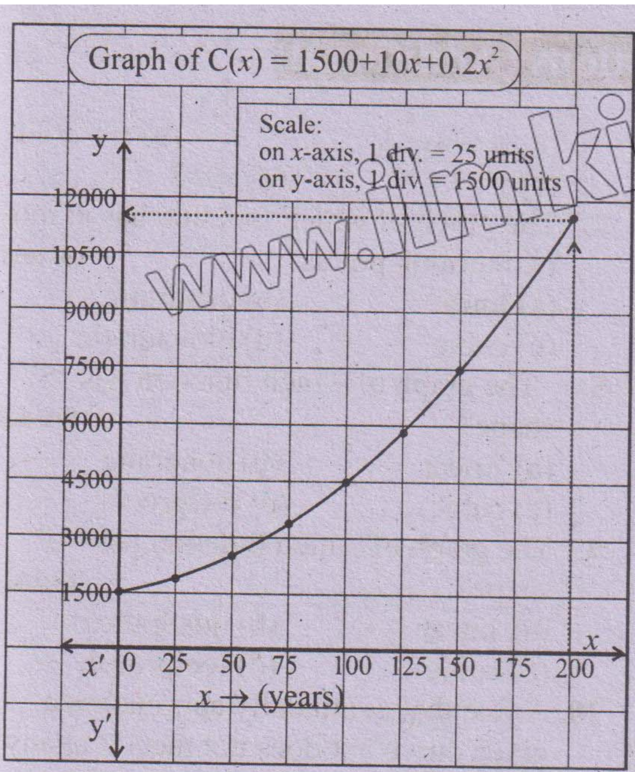
x	C(x)
0	1,500
25	1,875
50	2500
75	3375
100	4,500
125	5875
150	7,500

Step-II Select a suitable scale for graph like

On x-Axis, 1 division = 25 units

On y-Axis, 1 division = 1500 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Finding the Cost:

Cost function is $C(x) = 1500+10x + 0.2x^2$

Put $x = 200$ in $C(x)$

$C(200) = 1500+10(200)+0.2(200)^2$

$C(200) = 1500+2000+8000$

$C(200) = 11,500$

Thus cost for 200 shirts is Rs.11,500.

Review Exercise 10

Q.1 Choose the correct option.

i. $x = 5$ represents: **09310040**
(a) x-axis (b) y-axis
(c) line || to x-axis
(d) line || to y-axis

ii. Slope of the line $y = 5x + 3$ is: **09310041**
(a) 3 (b) –3
(c)5 (d)–5

iii. The y- intercepts of $y = -2x - 1$ is: **09310042**
(a) –2 (b) 2
(c) –1 (d) 1

iv. The graph of $y = x^3$, cuts the x-axis at: **09310043**
(a) $x = 0$ (b) $x = 1$
(c) $x = -1$ (d) $x = 2$

v. The graph of 3^x represents: **09310044**
(a) growth (b) decay
(c) both (a)and(b) (d) a line

vi. The graph of $y = -x^2 + 5$ opens: **09310045**
(a) upward (b) downward
(c) left side (d) right side

vii. The graph of $y = x^2 - 9$ opens: **09310046**
(a) upward (b) downward
(c) left side (d) right side

viii. $y = 5^x$ is ____________ function. **09310047**
(a) linear (b) quadratic
(c) cubic (d) exponential

ix. Reciprocal function is: **09310048**
(a) $y = 7^x$ (b) $y = \frac{2}{x}$
(c) $y = 2x^2$ (d) $y = 5x^3$

x. $y = -3x^3 + 7$ is__________ function. **09310049**
(a) exponential (b) cubic
(c) linear (d) reciprocal

Answers Key

i	d	ii	c	iii	c	iv	a	v	a
vi	b	vii	a	viii	d	ix	b	x	b

Multiple Choice Questions (Additional)

Functions and their Graphs

1. The graph of which equation is a straight line? 09310050
   (a) $y = 2x$ (b) $y = x^2$
   (c) $y = x^3$ (d) $xy=1$
2. The graph of which equation is a parabola? 09310051
   (a) $y = 2x$ (b) $y = x^2$
   (c) $y = x^3$ (d) $xy=1$
3. The graph of a quadratic function is always: 09310052
   (a) straight line (b) curves line
   (c) parabola (d) hyperbola
4. In $y=ax^2+bx+c$ if $a>0$ then parabola opens: 09310053
   (a) upward (b) downward
   (c) right ward (d) left ward
5. In $y=ax^2+bx+c$ if $a<0$ then parabola opens: 09310054
   (a) upward (b) downward
   (c) right ward (d) left ward
6. The graphs of which equation pass through the origin? 09310055
   (a) $y = 4x+2$ (b) $y = x^2+1$
   (c) $y =3x^3$ (d) $xy=8$
7. The graph of which function has at most two turning points? 09310056
   (a) linear (b) quadratic
   (c) cubic (d) biquadratic
8. The graph of which function has "S" shape? 09310057
   (a) linear (b) quadratic
   (c) cubic (d) reciprocal
9. The graph of which function has "U" shape? 09310058
   (a) linear (b) quadratic
   (c) cubic (d) reciprocal
10. A line that continually approaches a given curve but does not meet it at any finite distance is called: 09310059
    (a) horizontal line
    (b) vertical line
    (c) Tangent line
    (d) asymptotes

Answer Key

1	a	2	b	3	c	4	a	5	b	6	c	7	c	8	c	9	b	10	d

Q.2 Plot the graph of the following functions:

(i) $y = 3^{-x}$ for x from –2 to 4 09310060

Solution:

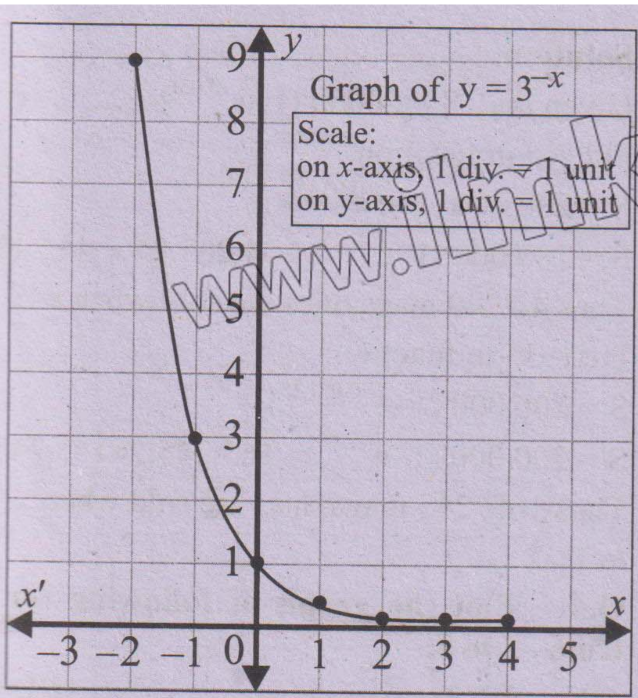
$y = 3^{-x} = \frac{1}{3^x}$.

Step-I: Substitute the values of "x" in the given function and find the corresponding values of y, as given below.

x	$y = 3^{-x}$	(x, y)
–2	9	(–2,9)
–1	3	(–1,3)
0	1	(0,1)
1	$\frac{1}{3} = 0.33$	(1,0.33)
2	$\frac{1}{9} = 0.11$	(2,0.11)
3	$\frac{1}{27} = 0.03$	(3,0.03)
4	$\frac{1}{81} = 0.01$	(4,0.01)

Step-II Select a suitable scale for graph like
On x-Axis, 1 division = 1 unit
On y-Axis, 1 division = 1 unit

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(ii) $y = \frac{2}{x}, x \neq -0$ **09310061**

Solution:

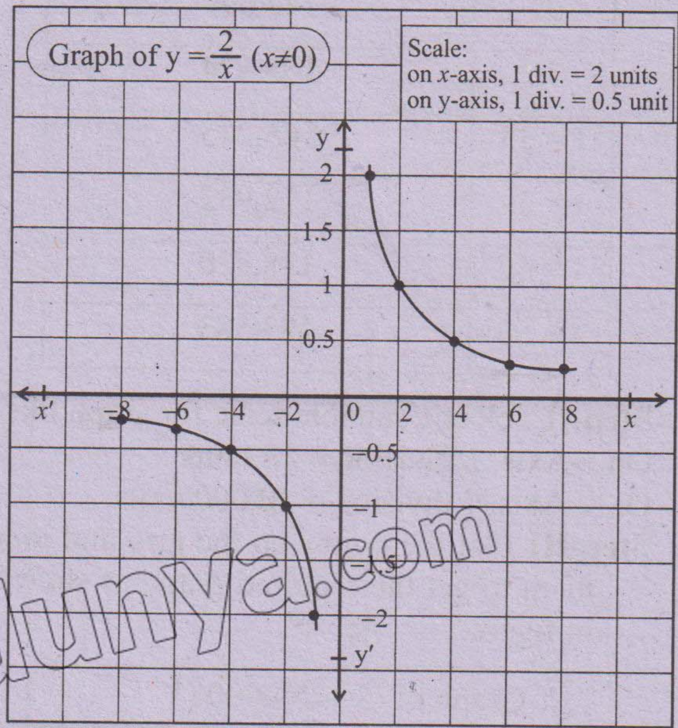
$y = \frac{2}{x}, x \neq -7$

Step-I: Substitute the values of "x" in the given function and find the corresponding values of y, as given below.

x	$y = \frac{2}{x}$
–8	–0.25
–6	–0.33
–4	–0.5
–2	–1
–1	–2
0	undefined
1	2
2	1
4	0.5
6	0.33
8	0.25

Step-II Select a suitable scale for graph like
On x-Axis, 1 division = 2 units
On y-Axis, 1 division = 0.5 unit

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Q.3 Sales for a new magazine are expected to grow according to the equation: $S = 200000\ (1 - e^{-0.05t})$, where t is given in weeks.

(a) Plot graph of sales for the first 50 weeks. **09310062**

(b) Calculate the number of magazines sold, when $t = 5$ and $t = 35$.

Solution (a): **09310063**

Graph for First 50 weeks:

Step-I: Substitute the values of "t" in the given function and find the corresponding values of S, as given below.

$S = 200000\ (1 - e^{-0.05t})$ (t is time in weeks)

We know that $e \approx 2.718 \approx 2.72$

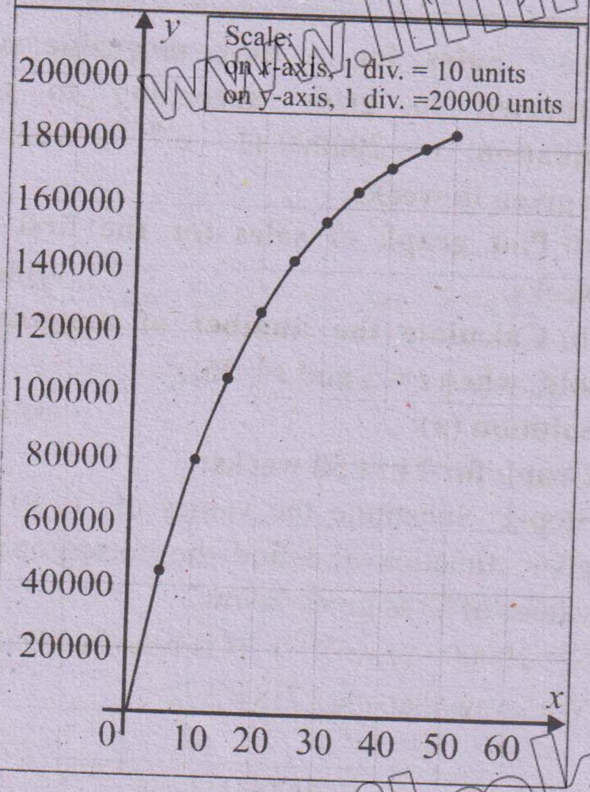
t	$S = 200000\ (1 - e^{-0.05t})$
0	0

5	44,239
10	78,694
15	105,527
20	126,424
25	142,699
30	155,374
t = 35	165,245
t = 40	172,932
t = 45	178,920
t = 50	183,583

Step-II Select a suitable scale for graph like
On x-Axis, 1 division = 10 units
On y-Axis, 1 division = 20,000 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.

Graph of $S = 200000\,(1-e^{-0.05t})$



(b) Calculate the number of magazines sold, when $t = 5$ and $t = 35$. 09310064

Solution:

Given that $S = 20000\,(1-e^{-0.05t})$ ....(i)

Put t=5 in function.

$S = 200{,}000\,(1-e^{-0.05\times 5})$

$S = 200{,}000\,(1-e^{-0.25}) = 44{,}239.84 \approx 44{,}240$

Thus 44,240 magazines are sold when t=5.

Put t=35 in function.

$S = 200{,}000\,(1-e^{-0.05\times 35})$

$S = 200{,}000\,(1-e^{-1.75}) = 165{,}245.2 \approx 165{,}245$

Thus 165, 245 magazines are sold when t=35.

Q.4 Plot the graph of following for x from −5 to 5:

(i) $y = x^2 - 3$ 09310065

Solution:

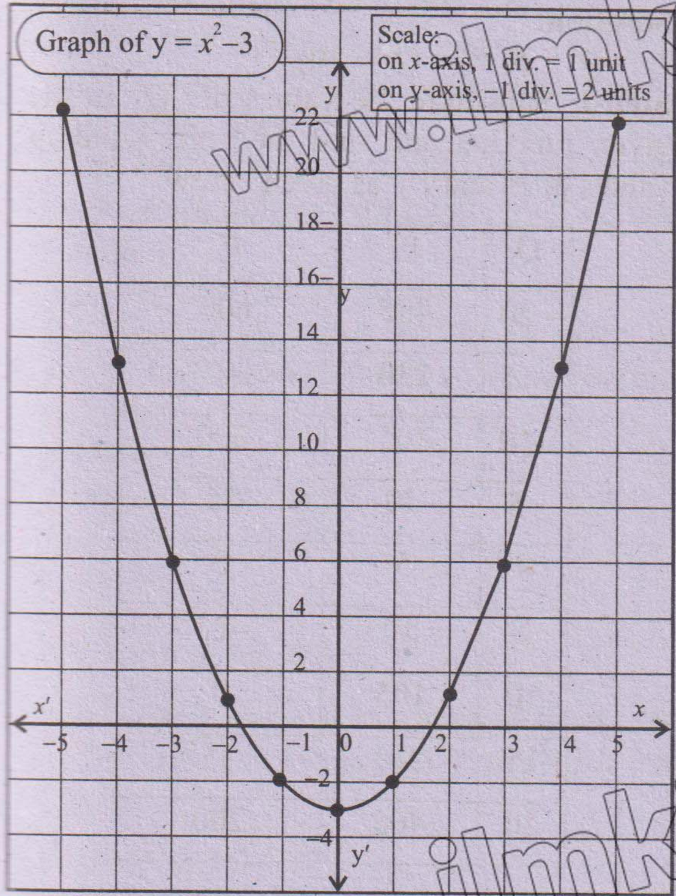
$y = x^2 - 3$

Step-I: Substitute the values of "x" in the given function and find the corresponding values of y, as given below.

x	$y = x^2 - 3$	(x, y)
−5	22	(−5,22)
−4	13	(−4,13)
−3	6	(−3,6)
−2	1	(−2,1)
−1	−2	(−1, −2)
0	−3	(0, −3)
1	−2	(1, −2)
2	1	(2,1)
3	6	(3,6)
4	13	(4,13)
5	22	(5,22)

Step-II Select a suitable scale for graph like
On x-Axis, 1 division = 1 unit
On y-Axis, 1 division = 2 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



(ii) $y = 15 - x^2$

Solution

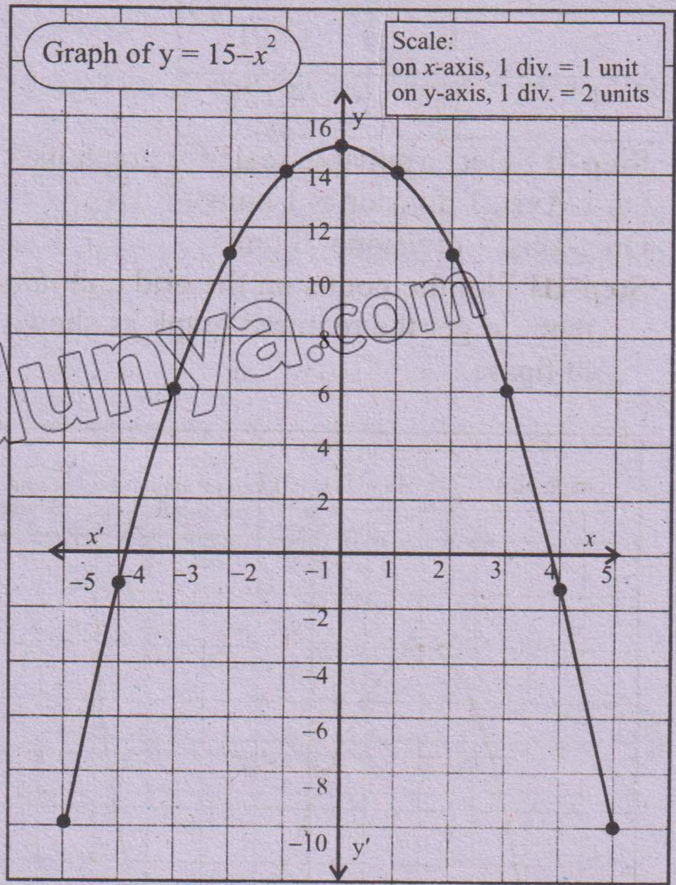
$y = 15 - x^2$ (–5 to 5)

Step-I: Substitute the values of "x" in the given function and find the corresponding values of y, as given below.

x	$y = 15 - x^2$	(x, y)
–5	–10	(–5, –10)
–4	–1	(–4, –1)
–3	6	(–3, 6)
–2	11	(–2, 11)
–1	14	(–1, 14)
0	15	(0, 15)
1	14	(–1, 14)
2	11	(2, 11)
3	6	(4, 6)
4	–1	(4, –1)
5	–10	(5, –10)

Step-II Select a suitable scale for graph like
On x-Axis, 1 division = 1 unit
On y-Axis, 1 division = 2 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Q.5 Plot the graph of $y = \frac{1}{2}(x+4)(x-1)(x-3)$ from –5 to 4. 09310066

Solution:

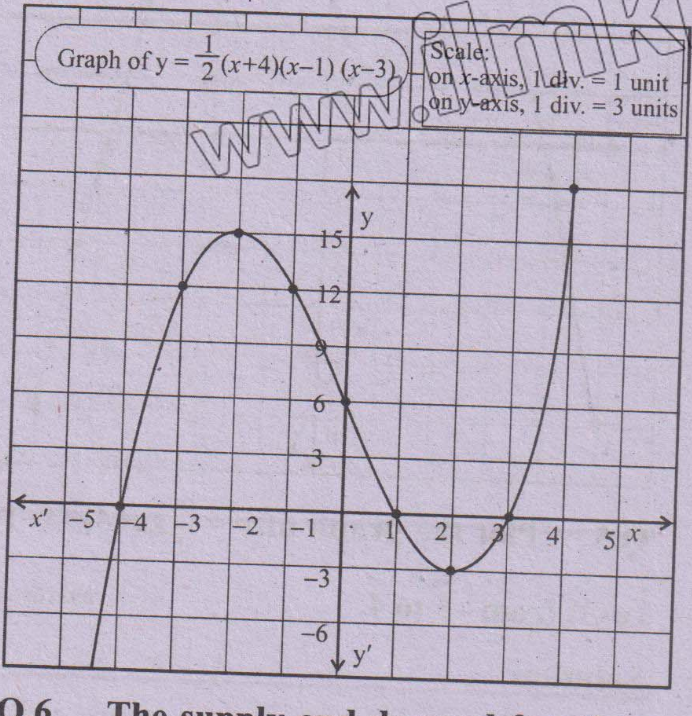
$y = \frac{1}{2}(x+4)(x-1)(x-3)$

Step-I: Substitute the values of "x" in the given function and find the corresponding values of y, as given below.

x	y	(x, y)
−4	0	(−4,0)
−3	12	(−3,12)
−2	15	(−2,15)
−1	12	(−1,12)
0	6	(0,6)
1	0	(1,0)
2	−3	(2,−3)
3	0	(3,0)
4	12	(4,12)

Step-II Select a suitable scale for graph like
On x-Axis, 1 division = 1 units
On y-axis, 1 division = 3 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Q.6 The supply and demand functions for a particular market are given by the equations:

(i) $P_s = Q^2 + 5$ and $P_d = Q^2 - 10Q$, where P represents price and Q represents quantity, 09310067

(ii) Sketch the graph of each function over the interval $Q = -20$ to $Q = 20$. 09310068

Solution:

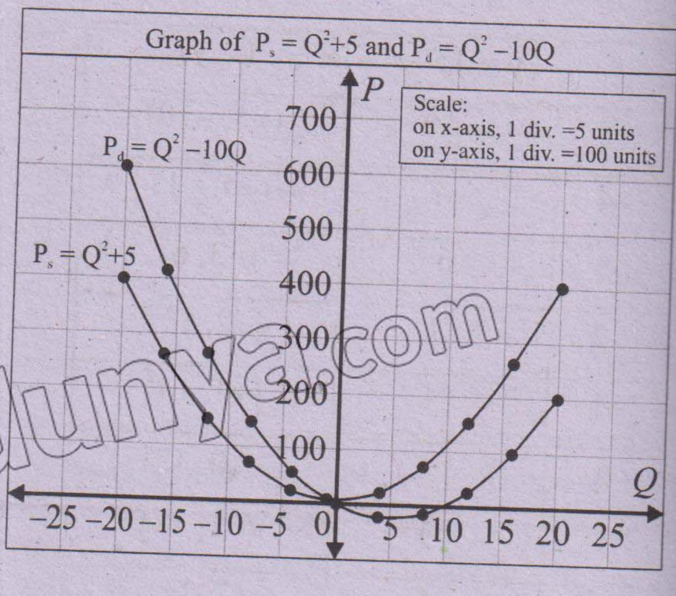
$P_s = Q^2 + 5$, $P_d = Q^2 - 10Q$

Step-I: Substitute the values of "Q" in the given function and find the corresponding values of P_s and P_d as given below.

Q	P_s	P_d
−20	405	600
−15	230	375
−10	105	200
−5	30	75
0	5	0
5	30	−25
10	105	0
15	230	75
20	405	200

Step-II Select a suitable scale for graph like
On x-Axis, 1 division = 5 units
On y-Axis, 1 division = 100 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.



Q.7 A television manufacturer company make 40 inches LEDs. The cost of manufacturing *x* LEDs is *C*(*x*) = 60,000 + 250*x* and the revenue from selling *x* LEDs is *R*(*x*) = 1200*x*. Find the break-even point and find the profit or loss when 100 LEDs are sold. Identify the break-even point graphically. 09310069

Solution:

Finding Break-even point:

The break-even point means no profit or loss. It happens when total cost and revenue are equal.

$R(x)= C(x)$

$1200x=60,000 + 250x$

$1200x-250x=60,000$

$950x=60,000$

$$x=\frac{60000}{950}=63.16\approx 63$$

Since break-even is at approximately 64 LEDs. Surely it will be profit on selling 100 LEDs.

Profit = $R(x)-C(x)$

Profit=$(1200x)-(60,000 + 250x)$

Profit=$1200x-60,000 - 250x$

Profit=$950x-60,000$

Put the x= 100 in the profit function.

Profit=950(100)–60,000

Profit=95000–60,000

Profit=35,000.

Plotting the graph:

$C(x) = 60,000 + 250x$, $R(x) = 1200x$

x	$C(x)$	$R(x)$
0	60,000	(0,0)
20	65,000	24,000
40	70,000	48,000
60	75,000	72,000
80	80,000	96,000
100	85,000	
120	90,000	

Step-II Select a suitable scale for graph like

On x-axis, 1 division = 20 units

On y-axis, 1 division = 20,000 units

Step-III Plot the points on the grid and join them to get the required graph as shown in figure.