UNIT 11

Loci and Construction

Introduction

A locus plural loci is a set of points that follow a given rule. Loci are also useful for understanding and predicting patterns. For instance, consider two people walking around a room, each maintaining a fixed distance from the other. The possible locations are where each person form a specific path.

Construction of Triangles

A triangles is a closed figure having three sides and three angles. We construct triangle in the following cases:

(a) When measure of all three sides are given.

(b) When measure of two sides are given.

(c) When measure of one side and measure of two angles are given.

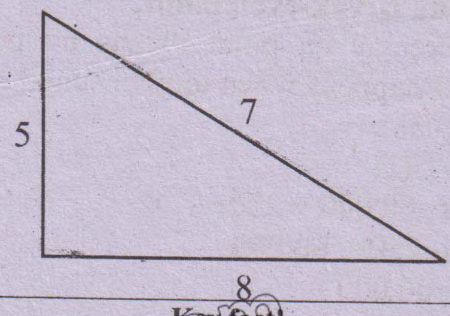
(d) When measure of two sides and an angle opposite to one of them is given.

> **Remember!**
> There are types of triangles w.r.t. sides:
> **Scalene triangles:**
> All sides are different.
> **Equilateral triangle:**
> All sides of equal length.
> There are three types of triangles w.r.t. angles:
> **Acute angled triangle:**
> All angles are of measure less than 90°.
> **Obtuse angled triangle:**
> One angle is of measure greater than 90°.
> **Right angled triangle:**
> One angle is of measure equal to 90°.

Triangle Inequality Theorem

The sum of the measure of any two sides of a triangle is always greater than the measure of the third side. For example, we can see in the figure adding any two lengths then this will be greater than the third side i.e.,

$5+7 > 8, 5+8 > 7$ and $7+8 > 5$.

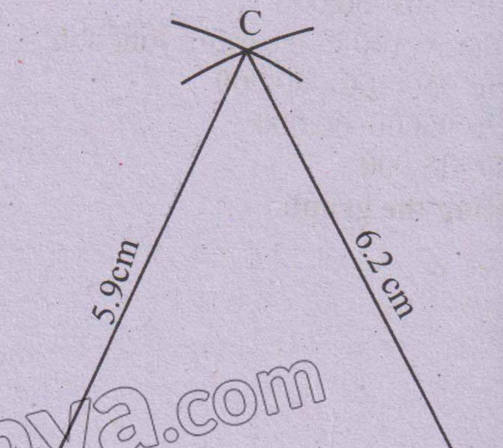


> **Key fact!**
> - An equilateral triangle is acute angled triangle.
> - A right angled triangle cannot be equilateral.

(a) Construction of a triangle when measure of three sides is given

Example 1: Construct a triangle of sides 5.3cm, 5.9cm and 6.2cm. **09311001**

Solution:



Steps of construction

(i) Draw a line segment AB of length 5.3 cm long.

(ii) Using a pair of compasses, draw two arcs with centres at points A and B of radii 5.9 cm and 6.2cm respectively.

(iii) These two arcs interest each other at point C.

(iv) Join A and B with C.

Hence, ΔABC is the required triangle.

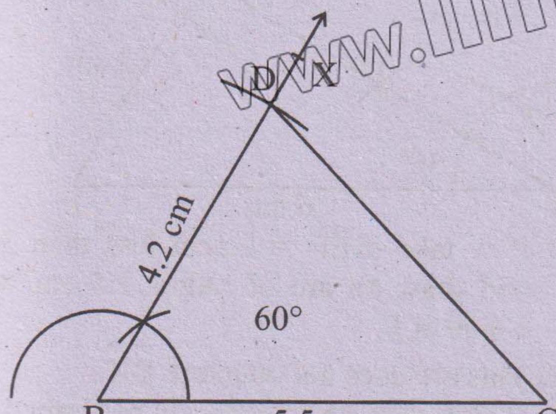
Note: The angles 30°, 45°, 60°, 75°, 90°, 105° 120°, 135° and 150° are constructed with the help a pair of compasses. Other angles are drawn using protractor.

Do you know?
When three sides are given, we can draw any length first.

b) **Construction of a triangle when the measure of two sides and their included angle are given**

Example 2: Construct a triangle BCD in which measures of two sides are 5.5 cm and 4.2 cm and measure of their included angle is 60°. **09311002**

Solution:



Step of Construction

(i) Draw a line segment BC of length 5.5cm.

(ii) Draw an angle 60° at point B using a pair of compasses and draw a ray $\overrightarrow{BX}$ through this angle.

(iii) Draw an arc of radius 4.2 cm with centre at point B intersecting $\overrightarrow{BX}$ at point D.

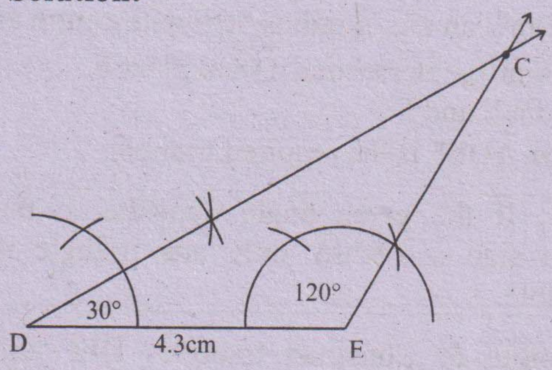
(i) Join C and D.

Hence ΔBCD is the required triangle.

(c) Construction of a triangle when measure of one side and two angles are given

Example 3: Draw a triangle CDE when $m\overline{DE}$ = 4.3cm, m∠D=30° and m∠E=120°. **09311003**

Solution:



Steps of construction

(i) Draw $m\overline{DE}$ = 4.3cm.

(ii) Draw angles 30° and 120° at points D and E respectively using a pair of compasses and draw two rays through these angles from D and E.

(iii) These two rays intersect each other at point C.

Hence, ΔCDE is the required triangle.

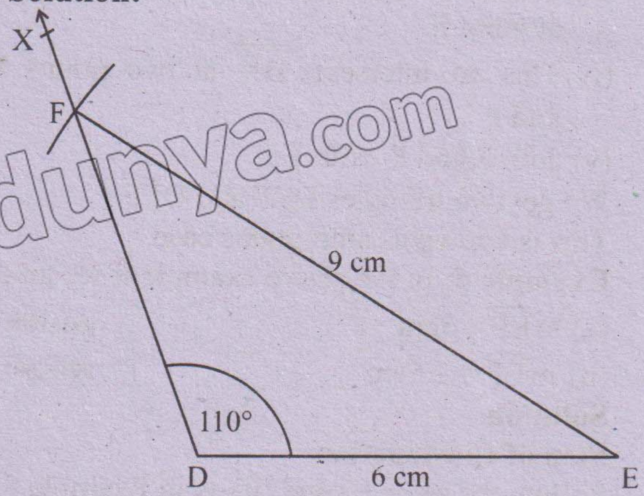
(d) Construction of a triangle when measure of two sides and angel opposite to one of the given two cases.

(i) If measure of one angle is greater than or equal to 90°.

(ii) If the measure of angle is less than 90°.

Example 4: Construct a triangle DEF when $m\overline{DE}$ = 6cm, m∠D = 110° and when $m\overline{EF}$ = 9cm. **09311004**

Solution:



(i) Draw $m\overline{DE} = 4.3$ cm,

(ii) Construct $m\angle D = 110°$ using protector and draw $\overrightarrow{DX}$ through this angle.

(iii) Draw an arc of radius 9cm with centre at point E, intersecting $\overrightarrow{DX}$ at point F.

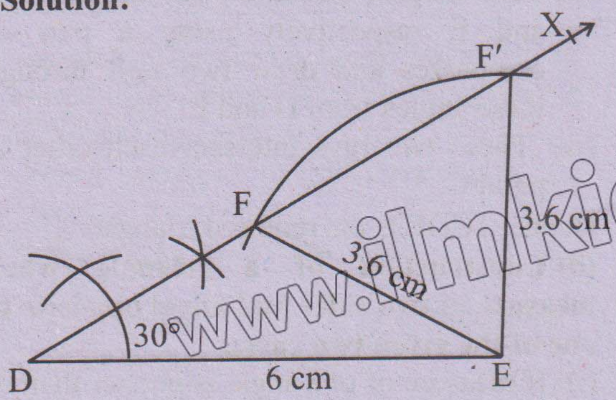
(iv) Join E and F.

Hence, ΔDEF is the required triangle.

Note: If the given angle opposite to the given side is obtuse, only one triangle is possible.

Example 5: Construct triangles DEF and DEF′ when $m\overline{DE}$ = 6cm, $m\angle D = 30°$ and $m\overline{EF}$ = 3.6cm. **09311005**

Solution:



Step of construction:

(i) Draw $m\overline{DE}$ = 6cm.

(ii) Construct an angle 30° at point D using a pair of compasses and draw $\overrightarrow{DX}$ through this angle

(iii) Draw an arc of radius 3.6 cm with centre at point E.

(iv) This arc intersects $\overrightarrow{DX}$ at two points F and F′.

(v) Join F and F′ with E.

We get two triangles DEF and DEF′.

This is known as ambiguous case.

Example 6: In the above example if we take:

(a) $m\overline{EF}$ = 3cm **09311006**

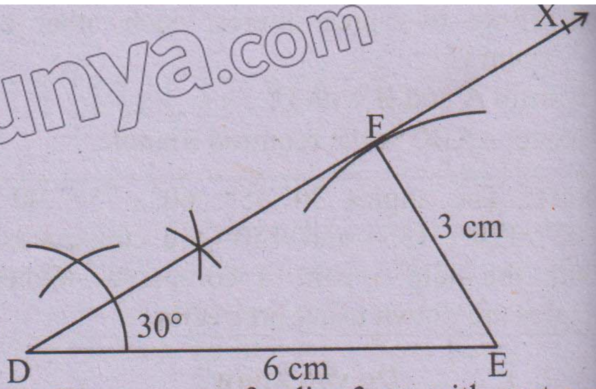
(b) $m\overline{EF}$ = 2.5cm **09311007**

Solution

Step of construction

Follow the same (i) and (ii) as in Example 5.

Case (a)

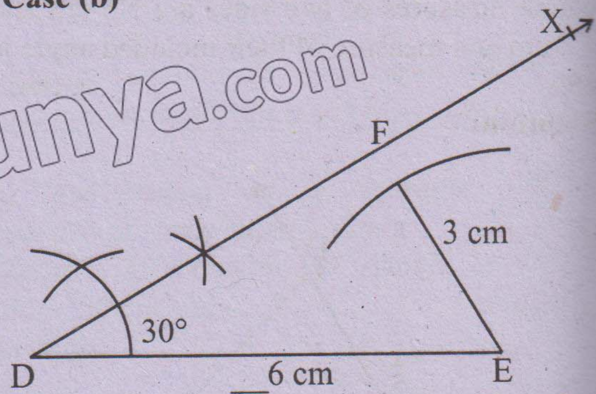


(i) Draw an arc of radius 3 cm with centre at point E which touches $\overrightarrow{DX}$ at point F.

(ii) Join E with F. Here, $\overline{EF}$ will be perpendicular to $\overrightarrow{DX}$.

Hence, ΔDEF is the required triangle, which is a right angled triangle.

Case (b)



(i) if w take $m\overline{EF}$ = 2.5cm less than 3cm and draw an arc of radius 2.5 cm with centre at E.

(ii) This arc does not intersect $\overrightarrow{DX}$.

So, in this case, no triangle can be formed.

We constructed three cases when acute angle is given: • If $m\overline{EF} > 3$ cm, two triangles are possible. • If $m\overline{EF} = 3$ cm, only one triangle is possible. • If $m\overline{EF} < 3$ cm, no triangle is possible.

Perpendicular Bisectors and Medians of a Triangle

Perpendicular Bisector:

A perpendicular bisector is a line that intersects a line segment at right angle and dividing it into two equal parts. In other words, it intersects the line segment at its midpoint and form right angle (90°) with it.

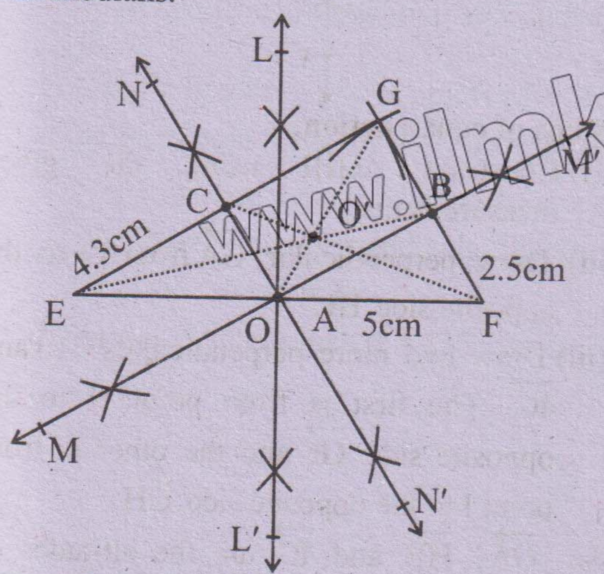
Median: A median of a triangle is a line segment that joins a vertex to the midpoint of the side that is opposite to that vertex.

Point of concurrency: A point of concurrency is the single point where three or more lines, rays or line segments intersect or meet in a geometric figure. This concept is commonly used in triangles, where several important types of points of concurrency exist.

Example 7: Draw perpendicular bisector of the triangle EFG with $m\overline{EF} = 5cm, m\overline{FG} = 2.5cm$ and $m\overline{EG} = 4.3cm$. **09311008**

Solution:

First we draw perpendicular bisectors and then medians.



Steps of construction:

(i) Draw ΔGEF as explained in the previous examples.

(ii) Draw two arcs above and below $\overline{EF}$ with more than half of $m\overline{EF}$ with centre at E.

(iii) Draw two arcs above and below $\overline{EF}$ with radius more than half of $m\overline{EF}$ with centre at F.

(iv) Draw a line through the points of intersection of the arcs in steps (ii) and (iii), we get the perpendicular bisectors LL' of the side $\overline{EF}$ at A.

(v) Draw two more perpendicular bisectors MM' and NN' of the sides $\overline{FG}$ and $\overline{EG}$ at B and C respectively.

(vi) Join the point G with opposite midpoint A so $\overline{GA}$ is the median.

(vii) Join the point F with opposite midpoint C, we get median $\overline{FC}$ and joint E with opposite midpoint B, we get median $\overline{EB}$.

Hence, we see that the perpendicular bisector $\overleftrightarrow{LL'}$, $\overleftrightarrow{MM'}$ and $\overleftrightarrow{NN'}$ are concurrent at point O or A and the medians $\overline{GA}$, $\overline{EB}$ and $\overline{FC}$ are concurrent at point O′.

Circumcentre: The point of concurrency of perpendicular bisector of the sides of a triangle is called circumscentre.

Centroid: The point concurrency of the medians of a triangle is called centroid of the triangle.

Angle Bisector of a Triangle

An angle bisector is a line or ray that divides an angle into two equal parts, creating two smaller angles that are congruent (each having half the measure of the original angle.

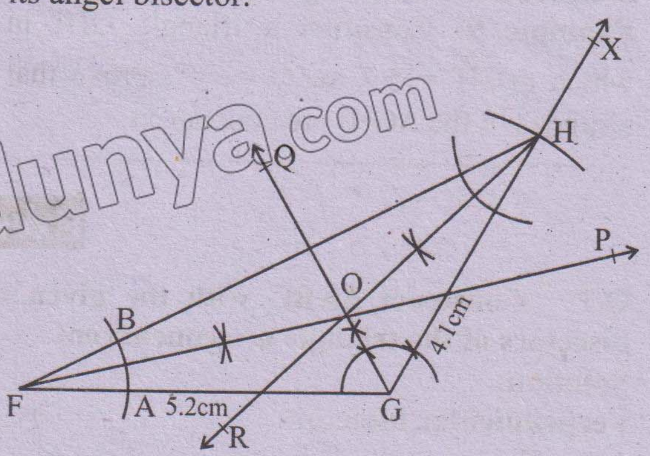
Example 8: Draw angle bisector of a triangle FGH if:

$m\overline{FG} = 5.2$ m, $m\overline{GH} = 4.1cm$ and

$m\angle FGH = 120°$ **09311009**

Solution:

We first construct triangle FGH, then draw its angel bisector.



Steps of construction:

(i) Construction ΔFGH with given lengths and angle.

(ii) Draw an arc of suitable radius with centre at point F intersecting sides $\overline{FG}$ and $\overline{FH}$ at points A and B.

(iii) Draw two arcs with centres at points A and B with suitable radius

(iv) Draw a ray from F passing through the point of intersection of the arc in step (iii).

Which is the required angle bisector $\overrightarrow{FP}$ of the angle F.

(v) Draw two more angle bisectors $\overrightarrow{GQ}$ and $\overrightarrow{HR}$ of the angles G and H. respectively.

We see that the angle bisector $\overrightarrow{FP}$, $\overrightarrow{GQ}$ and $\overrightarrow{HR}$ intersect at one point O. i.e. the angle bisectors of the triangle are concurrent.

Incentre:
The point of concurrency of the angle bisectors of a triangle is called incentre of the triangle.

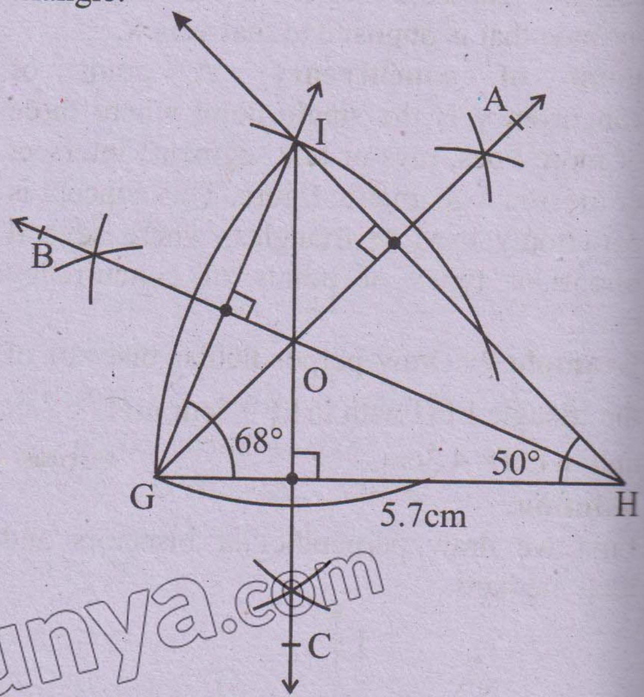
Altitudes of Triangle

Altitude is a ray drawn perpendicular from a vertex to the opposite side of the triangle. There are three altitudes of the triangle which meet at a single point i.e. the altitudes of a triangle are concurrent.

Orthocentre: The point of concurrency of the altitudes of the triangle is called orthocenter of the triangle.

Example 9: Construct a triangle GHI in which $m\overrightarrow{GH} = 5.7$, $m\angle G = 50°$. Prove that altitudes of the ΔGHI are concurrent.

Solution:
First, we construct ΔGHI using the given measurements and then draw altitudes of the triangle.



Steps of construction:

(i) Construct ΔGHI using the given measurements.

(ii) Draw perpendicular $\overrightarrow{GA}$ from G to the opposite side HI.

(iii) Draw two more perpendiculars $\overrightarrow{HB}$ and $\overrightarrow{IC}$. The first is from point H to the opposite side $\overrightarrow{GI}$ and the other is from point I to the opposite side $\overline{GH}$.

So, $\overrightarrow{GA}$, $\overrightarrow{HB}$ and $\overrightarrow{IC}$ are the altitudes of ΔGHI and they intersect at one point O. i.e., the altitudes of ΔGHI are concurrent.

Exercise 11.1

Q.1 Construct ΔABC with the given measurements and verify that the perpendicular bisectors of the triangle are concurrent.

Solution:

Perpendicular bisectors

(i) $m\overline{AB} = 5cm$, $m\overline{BC} = 6cm$, $m\overline{AC} = 7cm$

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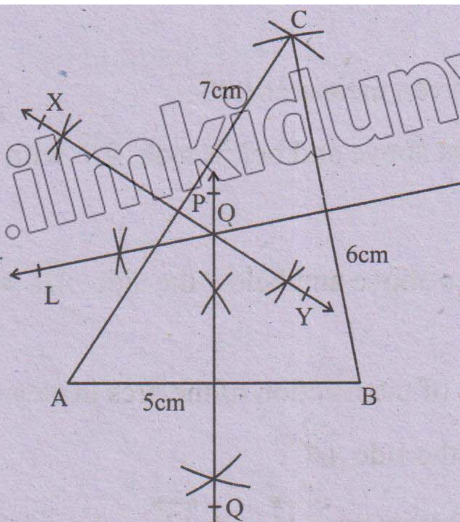


Steps of construction:

i. Construct a ΔABC using the given measurements.

ii. With centre at A, draw two arcs above and below side $\overline{AB}$ with radius more than half of $m\overline{AB}$.

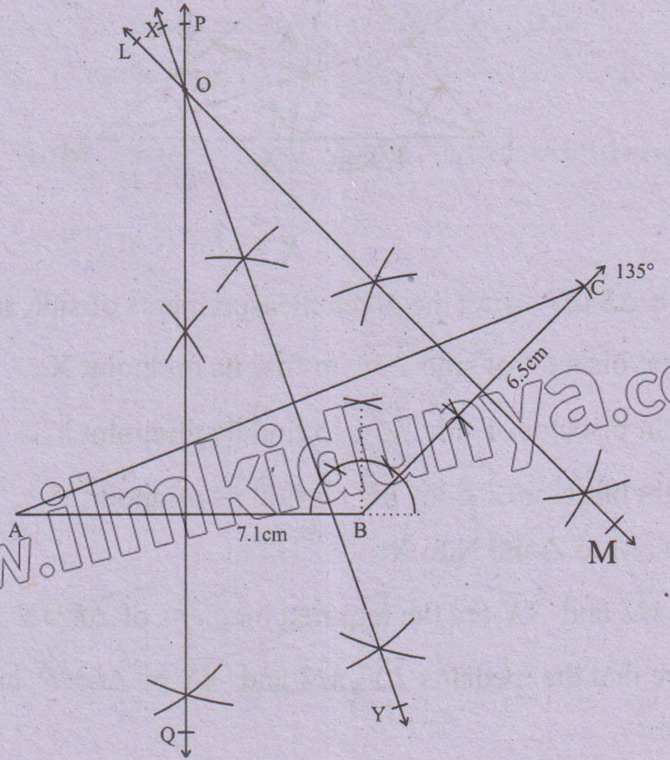
iii. With centre at B, draw two arcs above and below the side $\overline{AB}$ with the same radius as in step ii.

iv. Draw a line through the points of intersection of the arcs in step (ii) and (iii), we get the perpendicular bisector $\overleftrightarrow{PQ}$ of the side $\overline{AB}$.

v. Draw two more perpendicular bisectors $\overleftrightarrow{LM}$ and $\overleftrightarrow{XY}$ of the sides $\overline{BC}$ and $\overline{AC}$ respectively.

vi. We observe that perpendicular bisectors $\overleftrightarrow{PQ}$, $\overleftrightarrow{LM}$ and $\overleftrightarrow{XY}$ of sides of ΔABC are concurrent at O, in side the triangle.

(ii) $m\overline{AB} = 7.1cm$, $m\angle B = 135°$, $m\overline{BC} = 6.5cm$ **09311011**



Steps of construction:

i. Construct a ΔABC using the given measurements.

ii. With centre at A, draw two arcs above and below side $\overline{AB}$ with radius more than half of $m\overline{AB}$.

iii. With centre at B, draw two arcs above and below the side $\overline{AB}$ with the same radius as in step ii.

iv. Draw a line through the points of intersection of the arcs in step (ii) and (iii), we get the perpendicular bisector $\overleftrightarrow{PQ}$ of the side $\overline{AB}$.

v. Draw two more perpendicular bisectors $\overleftrightarrow{LM}$ and $\overleftrightarrow{XY}$ of the sides $\overline{BC}$ and $\overline{AC}$ respectively.

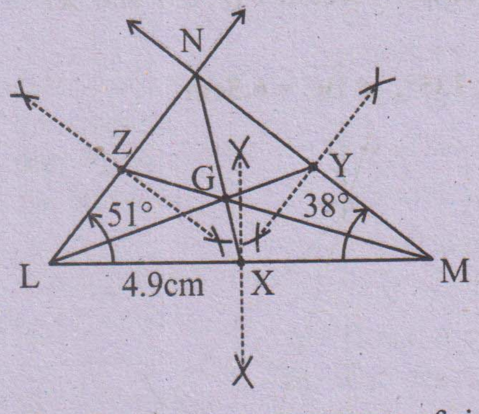
vi. We observe that perpendicular bisectors $\overleftrightarrow{PQ}$, $\overleftrightarrow{LM}$ and $\overleftrightarrow{XY}$ of sides of ΔABC are concurrent at O, out side the triangle.

Q.2 Construct ΔLMN of the following measurements and verify that the medians of the triangle are concurrent.

Medians of ΔLMN

(i) $m\overline{LM} = 4.9$ cm, $m\angle L = 51°$, $m\angle M = 38°$ **09311012**

Solution:



i. Construct a ΔLMN using the given measurements of side and angles.

ii. Draw a right bisector of side $\overline{LM}$ to find its midpoint X.

iii. Draw a right bisector of side $\overline{MN}$ to find its midpoint Y.

iv. Draw a right bisector of side $\overline{LN}$ to find its midpoint Z.

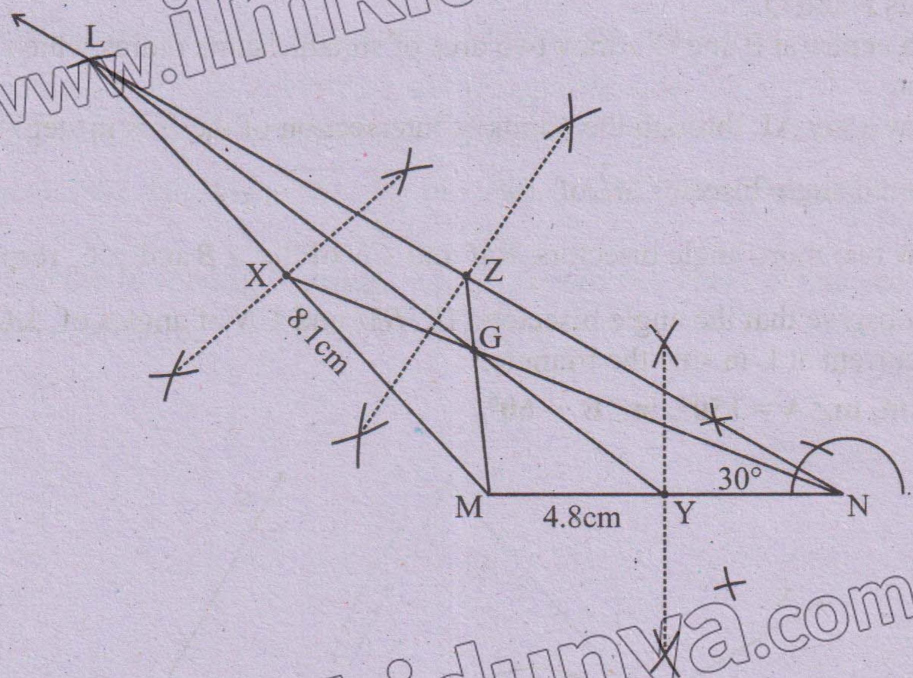
v. Join L to Y, M to Z and N to X.

vi. Thus $\overline{LY}$, $\overline{MZ}$ and $\overline{NX}$ are the required medians of ΔLMN.

We observe that the medians $\overline{LY}$, $\overline{MZ}$ and $\overline{NX}$ of ΔLMN are concurrent at G, inside the triangle.

(ii) $m\overline{MN} = 4.8cm$, $m\angle N = 30°$, $m\overline{LM} = 8.1$ cm 09311013

Solution:



Steps of construction:

i. Construct a ΔLMN using the given measurements of side and angles.

ii. Draw a right bisector of side $\overline{LM}$ to find its midpoint X.

iii. Draw a right bisector of side $\overline{MN}$ to find its midpoint Y.

iv. Draw a right bisector of side $\overline{LN}$ to find its midpoint Z.

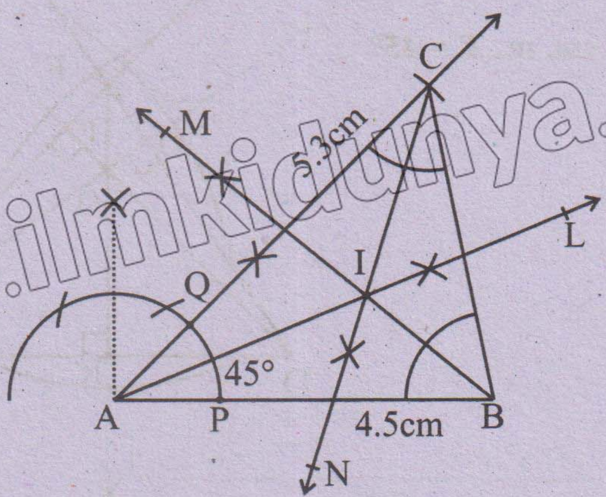
v. Join L to Y, M to Z and N to X.

vi. Thus, $\overline{LY}$, $\overline{MZ}$ and $\overline{NX}$ are the required medians of ΔLMN.
We observe that the medians $\overline{LY}$, $\overline{MZ}$ and $\overline{NX}$ of ΔLMN are concurrent at G, inside the triangle.

Q.3 Verify that the angle bisectors of ΔABC are concurrent with the following measurement:

(i) $m\overline{AB} = 4.5$ cm, $m\angle A = 45°$, m = 5.3 cm 09311014

Solution:



Steps of construction:

i. Construct a ΔABC using the given measurements.

ii. With centre at A, draw an arc of suitable radius intersecting the sides $\overline{AB}$ and $\overline{AC}$ at points P and Q.

iii. With centre at P and Q, draw two arcs of suitable same radius which intersect each other.

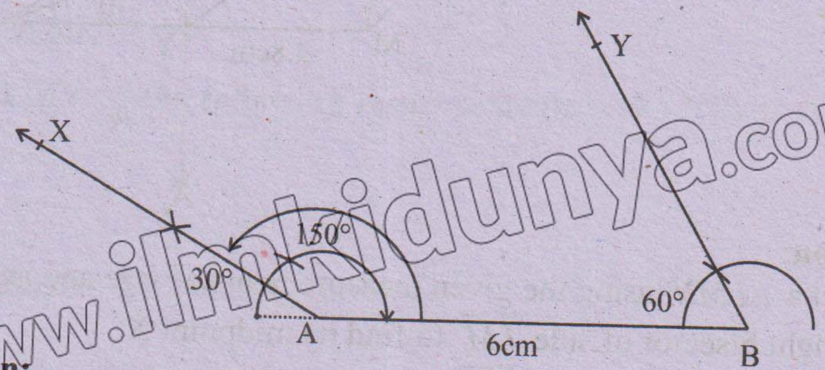
iv. Draw a ray AL through the points of intersection of the arcs in step (iii), which is the required angle bisector $\overrightarrow{AL}$ of $\angle A$.

v. Draw two more angle bisectors $\overrightarrow{BM}$ and $\overrightarrow{CN}$ of the $\angle B$ and $\angle C$ respectively.

We observe that the angle bisectors $\overrightarrow{AL}$, $\overrightarrow{BM}$ and $\overrightarrow{CN}$ of angles of ΔABC are concurrent at I, in side the triangle.

(ii) $m\overline{AB} = 6$ cm, $m\angle A = 150°$, $m\angle B = 60°$

Solution: 09311015



Steps of construction:

i. Draw $m\overline{AB} = 6cm$

ii. At vertex A, draw an angle of 150^o with help of compass

iii. At vertex B, draw an angle of 60^o with help of compass

iv. We observe that construction of required triangle according to given measurements is not possible.

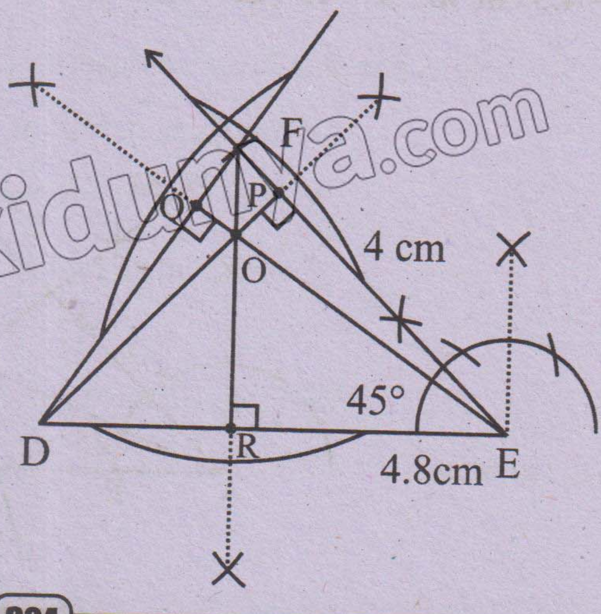
$\because 150^o + 60^o = 210^o > 180^o$

Q.4 Given the measurements of ΔDEF: $m\overline{DE} = 4.8$cm, $m\overline{EF} = 4$cm and $m\angle E = 45°$, draw altitudes of ΔDEF and find orthocenter.

Solution: 09311016

Altitudes: ΔDEF

$m\overline{DE} = 4.8$ cm, $m\overline{EF} = 4$ cm, $m\angle E = 45°$



Steps of construction:

i. Construct a ΔDEF using the given measurements.

ii. Draw perpendicular $\overline{DP}$ from vertex D to the opposite side $\overline{EF}$.

iii. Draw perpendicular $\overline{EQ}$ from vertex E to the opposite side $\overline{DF}$.

iv. Draw perpendicular $\overline{FR}$ from vertex F to the opposite side $\overline{DE}$.

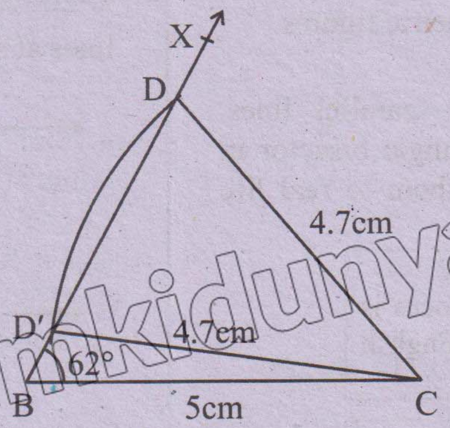
v. Thus $\overline{DP}$, $\overline{EQ}$ and $\overline{FR}$ are three required altitudes of ΔDEF.

We observe that three altitudes $\overline{DP}$, $\overline{EQ}$ and $\overline{FR}$ of ΔDEF are concurrent at O, inside the triangle.

Q.5 Construct the following triangles and find whether there exists any ambiguous.

(i) ΔBCD, $m\overline{BC} = 5cm$, $m\angle B = 62°$, $m\overline{CD} = 4.7cm$ **09311017**

Solution:



Steps of construction:

i. Draw $m\overline{BC} = 5cm$

ii. Construct $m\angle B = 62^o$ with the help of protractor and ruler and draw $m\overrightarrow{BX}$.

iii. With center C, draw an arc of radius 4.7 cm intersecting $\overrightarrow{BX}$ at D and D'.

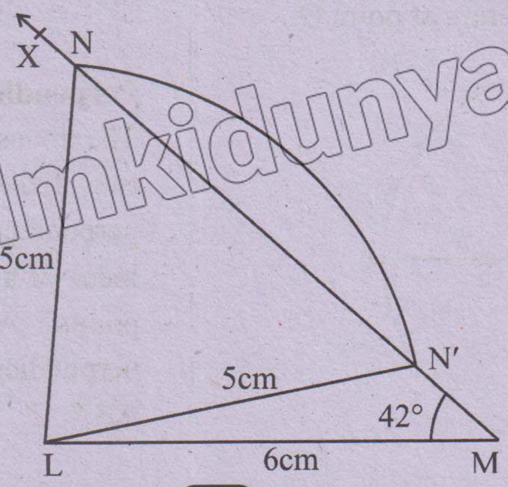
iv. Join C to D and C to D'..

Thus two triangles ΔBCD and $\Delta BCD'$ are constructed according to the given measurements.

(ii) ΔKLM; (Correction ΔLMN) **09311018**

$m\overline{LM} = 6cm$, $m\angle M = 42°$, $m\angle \overline{LM} = 5cm$

Solution:



Steps of construction:

i. Draw $m\overline{LM} = 6cm$

ii. Construct $m\angle M = 42^\circ$ with the help of protractor and ruler and draw $\overrightarrow{MX}$.

iii. With center L, draw an arc of radius 5 cm intersecting $\overrightarrow{MX}$ at N and N'.

iv. Join L to N and L to N'..

Thus two triangles ΔLMN and $\Delta LMN'$ are constructed according to the given information.

Loci and Construction

A locus

(plural loci) is a set of points that follow a given rule. In geometry, loci are often used to define the positions of points relative to one another or to other geometric figures.

Loci in two dimension

We study the loci, circle, parallel lines, perpendicular bisector and angle bisector in two dimensions and apply them to real life situations.

Do you know?

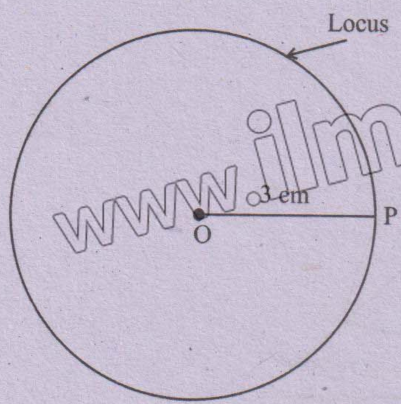
In Latin, the word locus is defined by the English term, location.

Remember!

Equidistant: Let A be a fixed point and B be a set of points. If A is at equal distance from all points of B, then A is said to be equidistant from B.

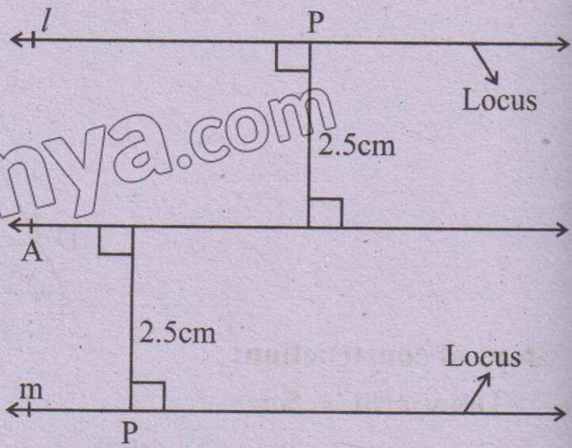
Circle

The locus of a point whose distance is constant from a fixed point is called a circle. For example, the locus of a point P whose distance is 3cm from a fixed –point O is a circle of radius 3cm and centre at point O.

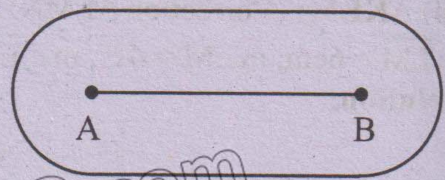


Parallel Lines

The locus of a point whose distance from a fixed line is constant are parallel lines, ℓ and m e.g. the locus of a point P whose distance is 2.5 cm from a fixed line AB are parallel lines at a distance of 2.5 cm from line AB.

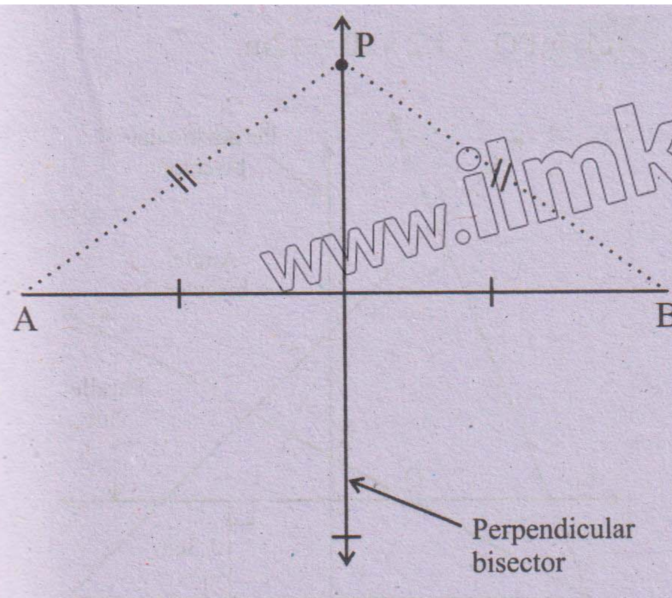


For example, a locus of points equidistant from a line segment creates a sausage shape. We can think of this type of locus as a track surrounding a line segment.



Perpendicular Bisector:

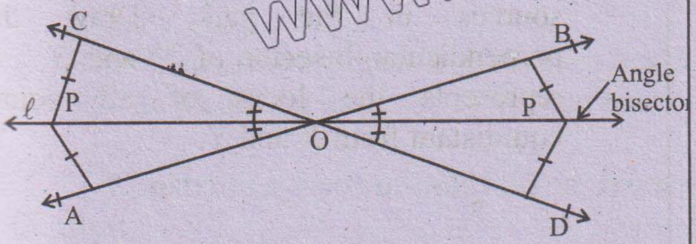
The locus of a point whose distance from two fixed points is constant is called a perpendicular bisector. For example, the locus of a point P whose distance from fixed points A and B is constant is the perpendicular bisector of the line segment AB.



Angle Bisector

The Locus of a point whose distance is constant from two intersecting lines is called an angle bisector.

For example, the locus of a point P whose distance is constant from two lines AB and CD intersecting at O is the angle bisector (ℓ) of ∠AOC and ∠BOD.



Intersection of Loci

If two or more loci intersect at a pint P, then P satisfies all given conditions of the loci. This will be explained in the following examples:

Example 10: Construct a rectangle ABCD with mAB = 5 cm and mBC = 3.2 cm. Draw the locus of all points which are:

(i) at a distance of 3.1 cm from point A. 09311019

(ii) equidistant from A and B. 09311020

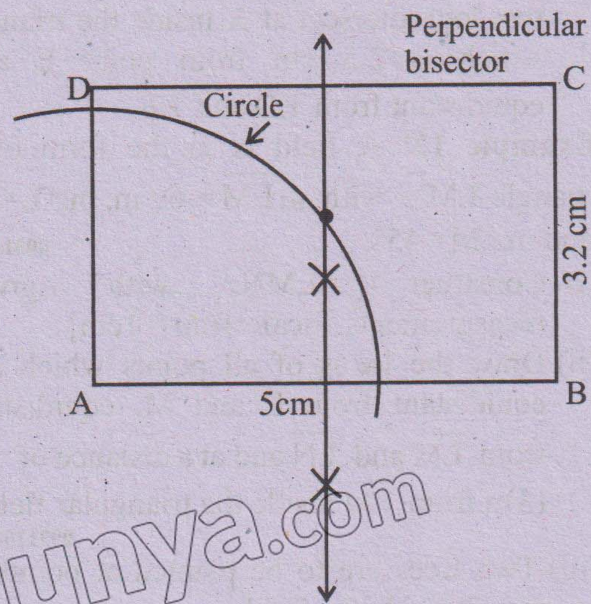
Label the point P inside the rectangle which is 3.1cm from point A and equidistant from A and B.

Solution

Construct rectangle ABCD with given lengths.

(i) Draw a circle of radius 3.1 cm with cenre at A.

(ii) Draw perpendicular bisector of AB. The two loci intersect at p inside the rectangle which is 3.1 cm from point A and equidistant from A and B.

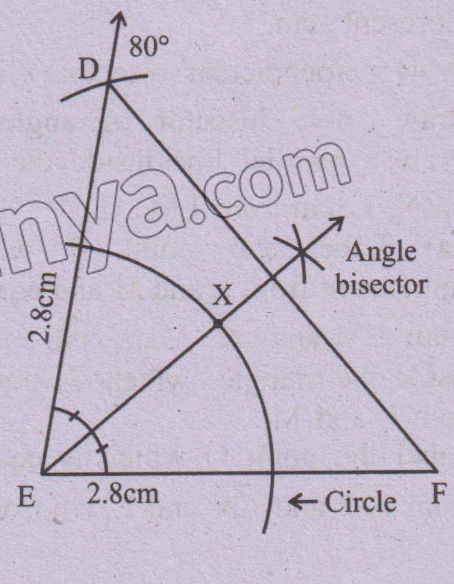


Example 11: Construct an isosceles triangle DEF with vertical angel 80° at E and $m\overline{EF}$ and $m\overline{DE}$ = 4.8 cm. Draw the locus of all points which are:

(i) at a distance of 2.8 cm from point E, 09311021

(ii) Equidistant from $\overline{DE}$ and $\overline{EF}$. 09311022

Label the point X inside the triangle which is 2.8cm from point E and equidistant from $\overline{ED}$ and $\overline{EF}$.



Solution:
Construct triangle DEF with given measurements.

(i) Draw a circle of radius 2.8cm with centre at E.

(ii) Draw angle bisector of angle $\angle E$. The two loci intersect at X inside the triangle which is 2.8 cm from point E and equidistant from $\overline{ED}$ and $\overline{EF}$.

Example 12: A field is in the form of a triangle LMN with $m\overline{LM} = 69$ m, $m\angle L = 60°$ and $m\angle M = 45°$. 09311023

(i) Construct ΔLMN with given measurements. [scale:10m = 1cm]

(ii) Draw the locus of all points which are equidistant from L and M, equidistant from $\overline{LM}$ and $\overline{LN}$ and at a distance of 13 m from $\overline{LM}$ inside the triangular field. 09311024

(iii) Two trees are to be planted at points P and Q inside the field. 09311025

(a) Mark the position of point P which is equidistant from L and M and equidistant from $\overline{LM}$ and $\overline{LN}$. 09311026

(b) Mark the position of point Q which is equidistant from $\overline{LM}$ and $\overline{LN}$ and 13m from $\overline{LM}$. 09311027

(c) Find the distance $m\overline{PQ}$. 09311028

Solution:

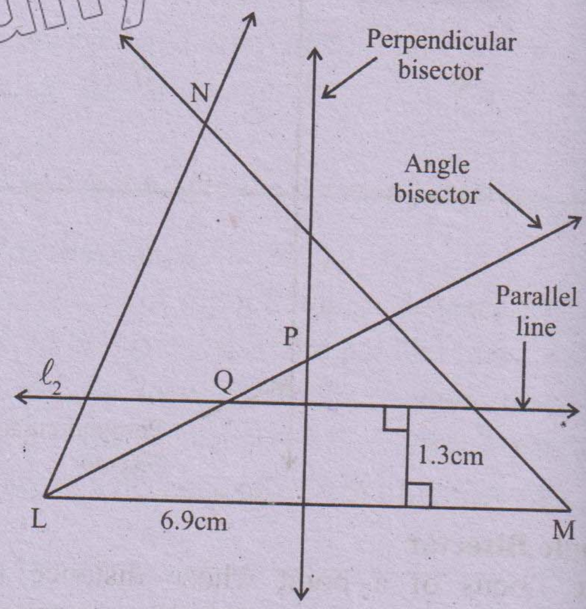
(i) Construct triangle LMN with given measurements using a scale of 10m to represent 1cm.

(ii) Draw perpendicular bisector ℓ_1 of $\overline{LM}$. Draw angle bisector of angle MLN. Draw a parallel line inside the triangle LMN, 1.3cm from $\overline{LM}$.

(iii) **(a)** Label the point P which is equidistant from L and M and equidistant from $\overline{LM}$ and $\overline{LN}$. Mark the point P inside the triangle which is equidistant from L and M.

(b) Label the point Q which is equidistant from $\overline{LM}$ and $\overline{LN}$ and 1.3cm from $\overline{LM}$.

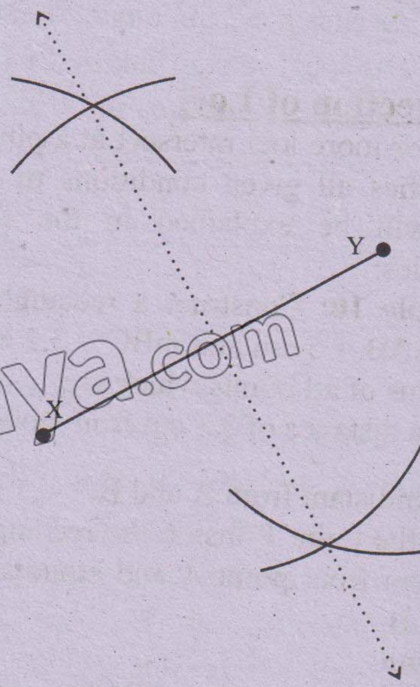
(c) $m\overline{PQ} = 1.2 \times 10 = 12$m.



Real Life Application Loci

(i) A park has two water sources at two different points. A fire hydrant needs to be placed so it is equally accessible to both sources.

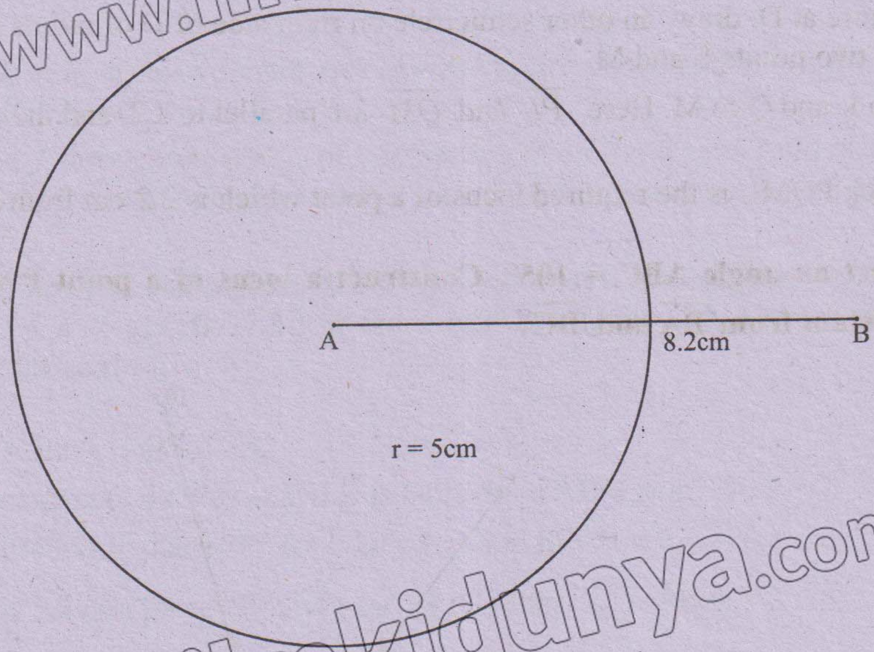
Let X and Y represent the two water sources in the park. Draw the perpendicular bisector of X and Y and represents the locus of all points equidistant from X and Y.



Exercise 11.2

Q.1 Two points A and B are 8.2cm apart. Construct the locus of points 5 cm from point A. **09311029**

Solution:



Steps of construction:

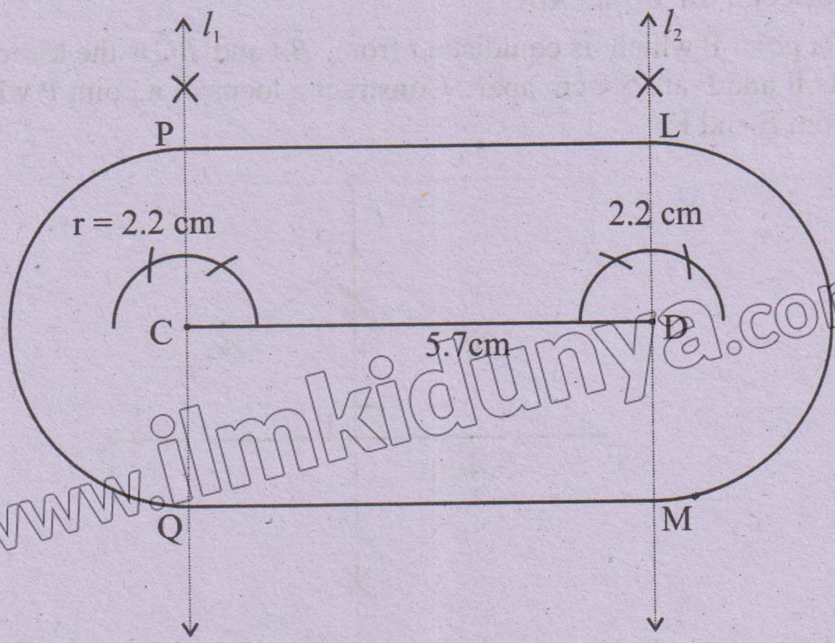
i. Draw $m\overline{AB} = 8.2cm$

ii. With centre at A, draw a circle of radius 5 cm.

Thus the locus of a point 5cm from the point A is the circle of radius 5 cm with centre at A.

Q.2 Construct a locus of point 2.2cm from line segment CD of measure 5.7cm **09311030**

Solution:



Steps of construction:

i. Using ruler draw $m\overline{CD} = 5.7cm$.

ii. Draw two perpendicular dotted lines l_1 and l_2 at points C and D respectively.

iii. With centre at C, draw a semicircle on left side of C of radius 2.2 cm cutting the line l_1 at two points P and Q.

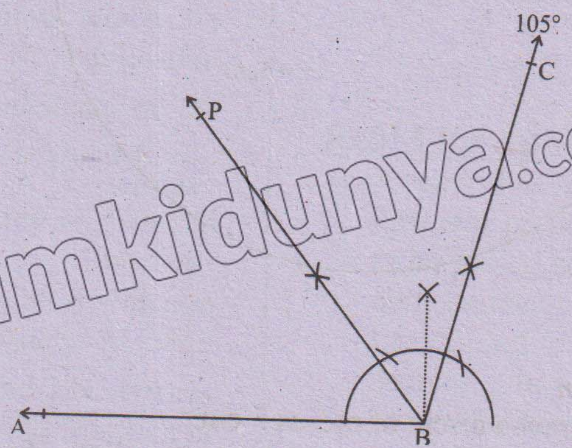
iv. With centre at D, draw an other semicircle on right side of D of radius 2.2 cm cutting the line l_2 at two points L and M.

v. Join P to L and Q to M. Here, $\overline{PL}$ and $\overline{QM}$ are parallel to $\overline{CD}$ and they are at 2.2 cm from it.

Thus track PQML is the required locus of a point which is 2.2 cm from the line segment CD.

Q.3 Construct an angle ABC = 105°. Construct a locus of a point P which moves such that it is equidistant from $\overline{BA}$ and $\overline{BC}$.

Solution: 09311031



Steps of construction:

i. Using the ruler draw a ray BA

ii. Draw $m\angle ABC = 105°$ using the compass and ruler.

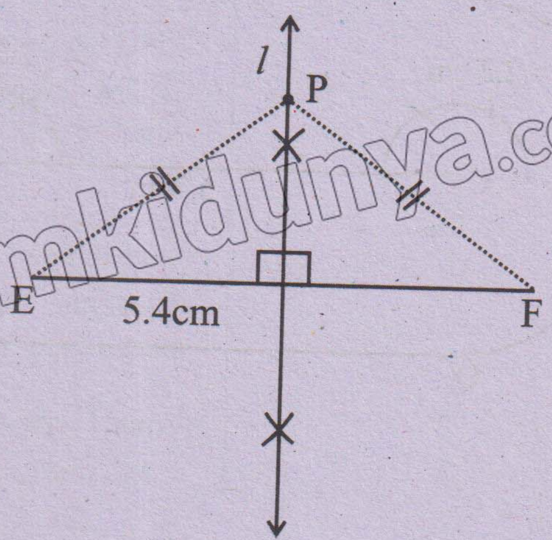
iii. Draw the bisector $\overrightarrow{BP}$ of $m\angle ABC$.

Thus locus of a point P which is equidistant from $\overrightarrow{BA}$ and $\overrightarrow{BC}$ is the bisector $\overrightarrow{BP}$ of $m\angle ABC$

Q.4 Two points E and F are 5.4 cm apart. Construct a locus of a point P which moves such that it is equidistant from E and F.

$m\overline{EF} = 5.4$ cm 09311032

Solution:



The locus of moving point which is equidistant from two given points is the "right bisector" of the line segment joining these point.

Steps of construction:

i. Draw $m\overline{EF} = 5.4cm$

ii. With centre at E draw two arcs of radius more than half of m $\overline{EF}$, above and below the $\overline{EF}$.

iii. With centre at F, draw two more arcs of same radius above and below the $\overline{EF}$ cutting the previous arcs.

iv. Draw a line l through the points of intersection of arcs and get a right bisector of $\overline{EF}$.

Thus the locus of a moving point P equidistant from the two given points E and F is the right bisector of $\overline{EF}$.

Q.5 The island has two main cities A and B 8 km apart. Kashif lives on the island exactly 6.8km from city A and exactly 7.3 km from city B. Mark with a cross the points on the island where Kashif could live. **09311033**

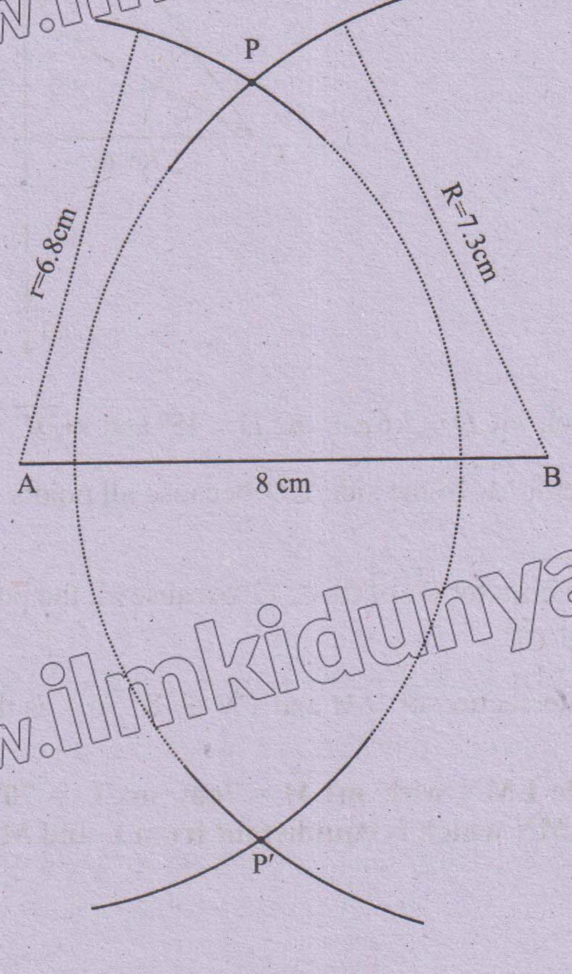
Solution:

Choose a suitable scale = 1km = 1cm

Actual distance between two cities A and B is 8km $\Rightarrow$ m $\overline{AB}$ = 8cm

Distance of Kashif's living place "P" from city A = 6.8km $\Rightarrow$ m$\overline{PA}$ = 6.8 cm

Distance of Kashif's living place "P" from city B = 7.3km $\Rightarrow$ m$\overline{PB}$ = 7.3 cm



Steps of construction:

$\Rightarrow\ m\overline{AB} = 8cm$

Consider two cities A and B as two points A and B.

P is the place where Kashif could live.

i. Draw $m\overline{AB} = 8cm$

ii. With centre at A draw two arcs of radius 6.8 cm , above and below the $\overline{AB}$.

iii. With centre at B, draw two more arcs of radius 7.3 cm above and below the $\overline{AB}$ which cross the previous arcs at two points P and P'.

The crossing points of two arcs P and P' could be the position where Kashif lives.

Q.6 Construct a triangle CDE with $m\overline{CD}$ = 7.6cm, m∠D = 45° and mDE = 5.9cm. Draw the locus of all points which are:

(a) Equidistant from C and D 09311035

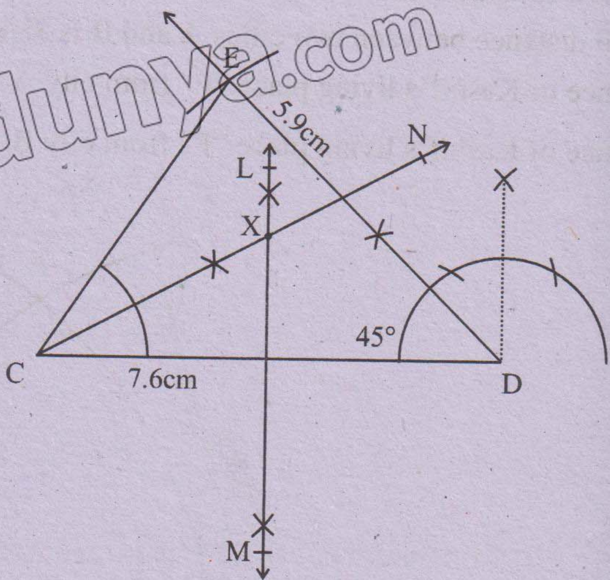
(b) Equidistant from $\overline{CD}$ and $\overline{CE}$ 09311036

Mark the pint X where the two loci intersect.

Solution:

ΔCDE,

$m\overline{CD}$ = 7.6cm, = ∠D = 45°, $m\overline{DE}$ = 5.9 cm



Steps of construction:

i. Draw ΔCDE in which $m\overline{CD} = 7.6\,cm$ $m\angle D = 45°$ and $m\overline{DE} = 5.9\,cm$.

ii. Draw the right bisector $\overleftrightarrow{LM}$ of the side $\overline{CD}$ because all points on $\overleftrightarrow{LM}$ are equidistant from points C and D.

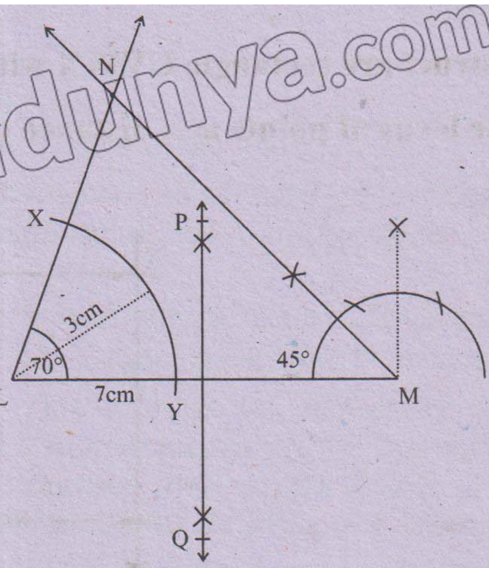
iii. Now draw the angle bisector $\overrightarrow{CN}$ of the $\angle C$ because all the points on $\overrightarrow{CN}$ are equidistant from points $\overline{CD}$ and $\overline{CE}$.

iv. Mark the point of intersection of $\overleftrightarrow{LM}$ and $\overrightarrow{CN}$ as X. So X is the point where two loci intersect.

Q.7 Construct a triangle LMN with $m\overline{LM}$ = 7cm, m∠L = 70° and m∠M = 45°. Find a point within the triangle LMN which is equidistant from L and M and 3cm from L. 09311037

Solution :

ΔLMN, m$\overline{LM}$ = 7cm, m∠L = 70°, m∠M = 45°



Steps of construction:

i. Draw ΔLMN in which $m\overline{LM} = 7\,cm$ $m\angle L = 70°$ and $m\angle M = 45°$.

ii. Draw the right bisector $\overleftrightarrow{PQ}$ of the side LM because all points on $\overleftrightarrow{PQ}$ are equidistant from points L and M.

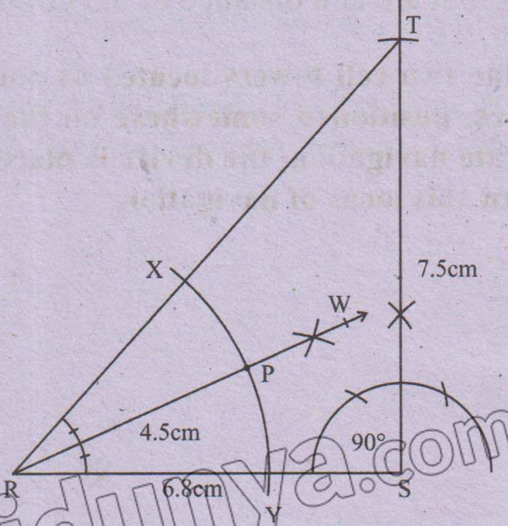
iii. With centre at L, draw an arc XY of radius 3 cm which does not cut $\overleftrightarrow{PQ}$ at any point.

iv. So the required point is not possible.

Q.8 Construct a right angled triangle RST with m$\overline{RS}$ = 6.8 cm, m∠S = 90° and m$\overline{ST}$ = 7.5cm. Find a point within the triangle RST which is equidistant from $\overline{RS}$ and $\overline{RT}$ and 4.5cm from R. **09311038**

Solution:

ΔRST; m$\overline{RS}$ = 6.8cm, m∠S = 90°, m$\overline{ST}$ = 7.5



Steps of construction:

i. Draw a right-angled ΔRST in which $m\overline{RS} = 6.8\,cm, m\angle S = 90°, m\overline{ST} = 7.5\,cm$.

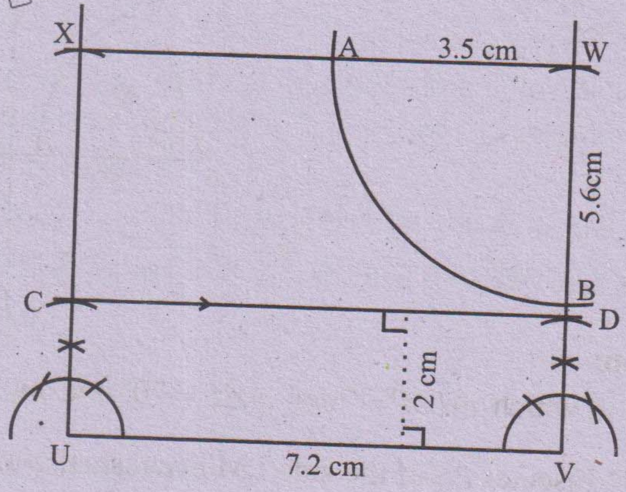
ii. Draw the angle bisector $\overrightarrow{RW}$ of the $\angle R$ because all the points on $\overrightarrow{RW}$ are equidistant from points $\overline{RS}$ and $\overline{RT}$.

iii. With centre at R, draw an arc XY of radius 4.5cm which cuts the $\overrightarrow{RW}$ at point P.

So the required point is P which is equidistant from $\overline{RS}$ and $\overline{RT}$, and 4.5cm from R.

Q.9 Construct are rectangle UVWX with measure $m\overline{UV} = 7.2$cm and $m\overline{VW} = 5.6$ cm. Draw the locus of points at a distance of 2 cm from $\overline{UV}$ and 3.5cm from W. 09311039

Solution:



Steps of construction:

i. Draw a rectangle UVWX with length $\overline{UV}cm = 7.2\ cm$ and width $m\overline{VW} = 5.6$ cm.

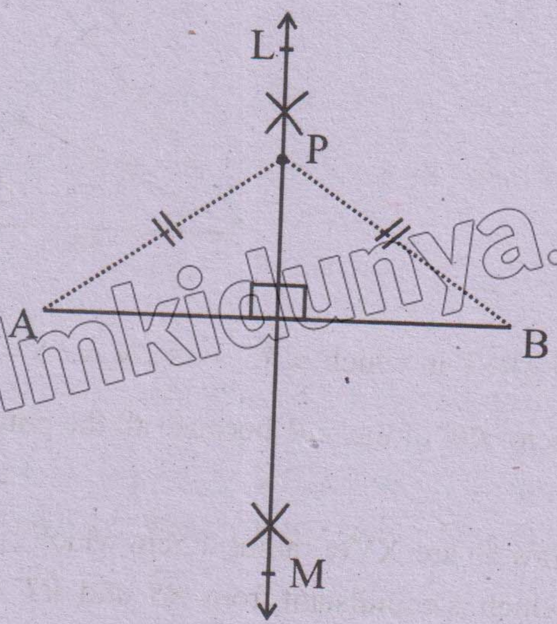
ii. With the centre at U and V draw two arcs of radius 2 cm which cut the side UX at C and side VW at D. Join C to D.

iii. Here, $\overline{CD} \parallel \overline{UV}$ and distance between them is 2 cm. So all points on $\overline{CD}$ are at distance of 2 cm from side UV.

iv. With the center at W, draw a circle of radius 3.5 cm which is a locus of all such points that are at a distance of 3.5 cm from the vertex W.

Q.10 Imagine two cell towers located at points A and B on a coordinate lane. The GPS-enabled device, positioned somewhere on the plane, receives signals from both towers. To ensure accurate navigation, the device is placed equidistant from both towers to estimate its position. Draw this locus of navigation.

Solution: 09311040



Steps of construction:

Let points A and B are showing location of cell towers.

i. Take two points A and B at suitable distance and join them to get $\overline{AB}$.

ii. With centre at A draw two arcs of radius more than half of m $\overline{AB}$, above and below the $\overline{AB}$.

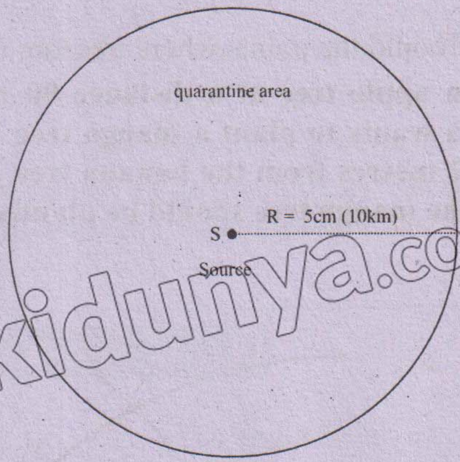
iii. With centre at B, draw two more arcs of same radius above and below the $\overline{AB}$ cutting the previous arcs.

iv. Draw a line l through the points of intersection of arcs and get a right bisector of $\overline{AB}$.

Thus every position on the right bisector l of $\overline{AB}$ is equidistant from the towers A and B.

Q.11 Epidemiologists use loci to determine infection zones, especially for contagious diseases, to predict the spread and take containment measures. In the case of a disease outbreak, authorities might determine a quarantine zone within 10km of the infection source. Draw locus of all points 10km from the source defining the quarantine area to monitor and control the disease's spread. 09311041

Solution:



Steps of construction:

i. Mark a point "S" as infection source.

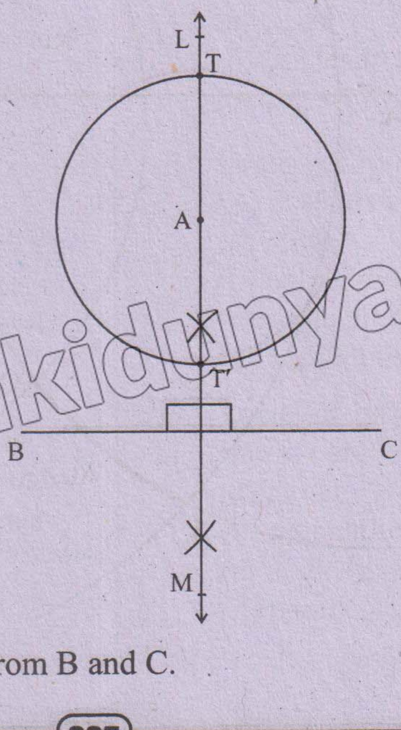
ii. Set a suitable scale i.e. 2km=1cm

iii. Using the scale 10 km = 5 cm.

iv. With the centre at S, draw a circle of radius 5cm.

v. This circle of radius 5cm represents the 10 km quarantine zone. All locations within this circle are included in the quarantine area.

Q.12 There is a treasure buried somewhere on the island. The treasure is 24 kilometres from A and equidistant from B and C. Using a scale of 1cm to represent 10km, find where the treasure could be buried. 09311042



Solution:

Steps of construction:

Treasure is 24km from A and equidistant from B and C.

i. Set a suitable scale of 10km=1cm
i.e. 24 km=2.4 cm

ii. Take any two points B and C in the plane at suitable distance and join them to get $\overline{BC}$.

iii. Draw right bisector $\overleftrightarrow{LM}$ of $\overline{BC}$.

iv. Take any point A on the right bisector $\overleftrightarrow{LM}$.

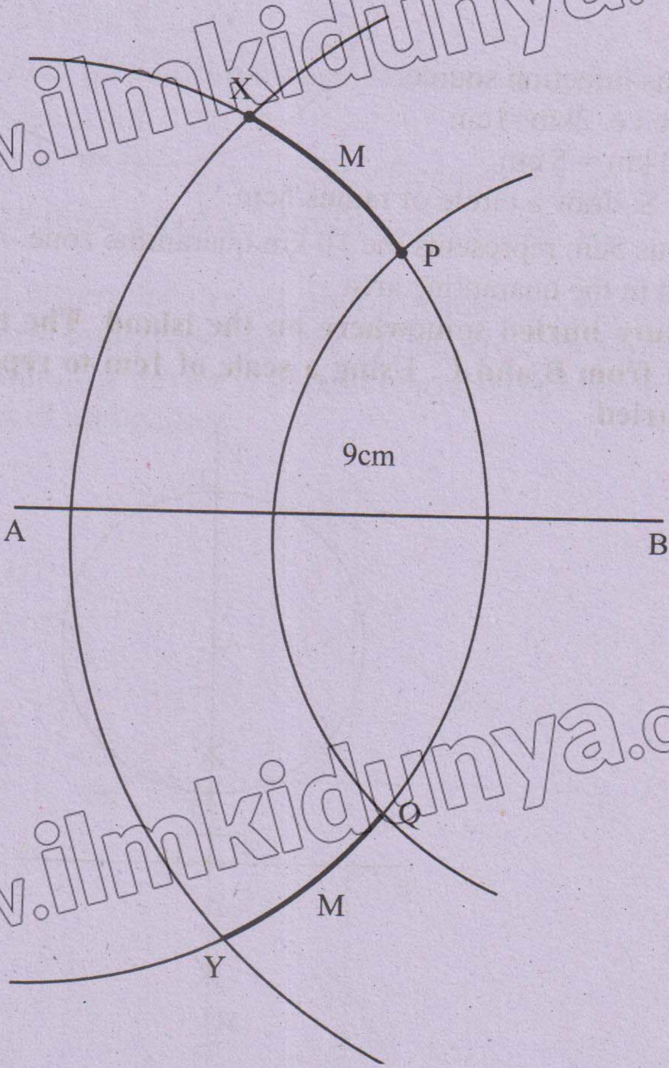
v. With centre at A, draw a circle of radius 2.4 cm such that it intersects the $\overleftrightarrow{LM}$ at points T and T'.

vi. Being points of a circle these points are at the distance of 2.4 cm from the cntre point A, and being the points of the right bisector of $\overline{BC}$ are equidistant from the endpoints B and C.

So T and T' could the points where treasure is buried.

Q.13 There is an apple tree at a distance 90 metres from banana tree in the garden of Sara's house. Sara wants to plant a mango tree M which is 64 metres from apple tree and between 54 and 82 metres from the banana tree. Using scale of 1cm to represent 10m, Find the points where the mango tree should be planted. 09311043

Solution:



Steps of construction:

Let Points A, B, and M represent the apples, bananas, and Mango trees respectively.

i. Set a suitable scale of 10m=1cm

Distance between A and B=90m i.e. $m\overline{AB} = 9cm$

Distance between A and M=64m i.e. $m\overline{AM} = 6.4cm$.

Distance between B and M=54m to 82 m i.e. $m\overline{BM} = 5.4cm$ to 8.2 cm.

ii. Draw a line segment AB of length 9 cm which means points A and B are 9cm (90m) apart from each other.

iii. Taking A as centre, draw a semicircle of radius 6.4 cm on the B side. All points on the semicircle are at a distance of 6.4 cm (64m) from point A.

iv. Taking B as centre, draw an arc of radius 5.4 cm on the A side which cuts the semicircle above and below the $\overline{AB}$ at points P and Q respectively.

v. Taking B as centre, draw another arc of radius 8.2 cm on the A side which cuts the semicircle above and below the $\overline{AB}$ at points X and Y respectively.

vi. The portion P to X on the semicircle above the $\overline{AB}$ and the portion Q to Y on the semicircle below the $\overline{AB}$ is the suitable place for M (mango tree) because this place is at a distance of 6.4cm (64m) from A (apple tree) and 5.4 cm (54m) to 8.2 cm (82m) from B (Bananas tree).

Review Exercise - 11

Q.1 Choose the correct option.

i. A triangle can be constructed if the sum of the measure of any two sides is ________ the measure of the third side. 09311044

(a) less than (b) greater than
(c) equal to
(d) greater than and equal to

ii. An equilateral triangle ____________. 09311045

(a) can be isosceles
(b) can be right angled
(c) can be obtuse angled
(d) has each angle equal to 50°

iii. If the sum of the measures of two angles is less than 90°, then the triangle is ________. 09311046

(a) equilateral (b) acute angled
(c) obtuse angled (d) right angled

iv. The line segment joining the midpoint of a side to its opposite vertex in a triangle is called ____________. 09311047

(a) median (b) perpendicular bisector
(c) angle bisector (d) circle

v. The angle bisectors of a triangle intersect at ____________. 09311049

(a) one point (b) two points
(c) three points (d) four points
(c) angle bisector (d) circle

vi. Locus of all points equidistant from a fixed point is: 09311050

(a) circle
(b) perpendicular bisector
(c) angle bisector (d) parallel bisector

vii. Locus of points equidistant from two fixed points is: 09311051

(a) circle
(b) perpendicular bisector
(c) angle bisector
(d) parallel lines

viii. Locus of points equidistant from a fixed line is / are: 09311052

(a) circle
(b) perpendicular bisector
(c) angle bisector
(d) parallel lines

ix. Locus of points equidistant from two intersecting lines is __________. 09311053

(a) circle
(b) perpendicular bisector
(c) angle bisector
(d) parallel lines

x. The set of all points which is farther than 2 km from a fixed point B is a region outside a circle of radius __________ and centre at B. 09311054

(a) 1 km (b) 1.9 km
(c) 2 km (d) 2.1 km

Answers Key

i	**b**	ii	**a**	iii	**c**	iv	**a**	v	**a**
vi	**a**	vii	**b**	viii	**d**	ix	**c**	x	**c**

Multiple Choice Questions (Additional)

Basic Concept

1. Which of the following is used to measure the angle? 09311055

(a)compass (b) protector
(c) scale (d) set square

2. Which of the following can be constructed by compass? 09311056

(a) 105^o (b) 125^o
(c) 130^o (d) 55^o

3. Which of the following cannot be constructed with compass? 09311057

(a) 15^o (b) 30^o
(c) 45^o (d) 95^o

4. Sum of interior angles of a triangle is: 09311058

(a) 60^o (b) 120^o
(c) 180^o (d) 240^o

5. In right-angled triangle one angle is right, other two angles are: 09311059

(a) right (b) obtuse
(c) acute
(d) one acute ,one obtuse

Types of triangles

6. A triangle having all sides equal is called: 09311060

(a) Isosceles (b) Scalene
(c) Equilateral (d) right

7. A triangle having all sides different is called: 09311061

(a) Isosceles (b) Scalene
(c) Equilateral (d) right

8. A triangle having two sides congruent is called: 09311062

(a) Scalene (b) Right angled
(c) Equilateral (d) Isosceles

Concepts of concurrency

9. The right bisectors of the three sides of a triangle are: 09311063

(a) Congruent (b) Collinear
(c) Concurrent (d) Parallel

10. The angle bisectors of the angles of a triangle are: 09311064

(a) Congruent (b) Collinear
(c) Concurrent (d) Parallel

11. If two medians of a triangle are congruent then the triangle will be: 09311065

(a)Isosceles (b) Equilateral
(c)Right angled (d) Acute angled

12. A perpendicular from a vertex of triangle to the opposite side is called: 09311066

(a)Altitude (b) Median
(c)Angle bisector (d) Right bisector

13. The point of concurrency of the three altitudes of a Δ is called its: 09311067

(a)Ortho centre (b) In centre
(c)Circumcentre (d) centroid

14. The bisectors of the angles of a triangle meet at a point called: 09311068

(a)In centre
(b) Ortho centre
(c) Circumcentre
(c) centroid

15. The point of concurrency of the three right bisectors of the sides of a triangle is called: 09311069
(a) Circumcentre
(b) In centre
(c) Ortho centre
(d) centroid

16. Point of concurrency of three medians of a triangle is called its: 09311070
(a) In centre (b) Ortho centre
(c) Centroid (d) Circumcentre

17. In-centre is the point of concurrency of three….. of triangle. 09311071
(a) Right bisectors
(b) Angle bisectors
(c) Altitudes
(d) Medians

18. Circumcentre is the point of concurrency of three ….. of triangle. 09311072
(a) right bisectors
(b) angle bisectors
(c) altitudes
(d) medians

19. Ortho centre is the point of concurrency of three….. of triangle. 09311073
(a) right bisectors
(b) angle bisectors
(c) altitudes (d)medians

20. Centroid is the point of concurrency of three….. of triangle. 09311074
(a)right bisectors
(b) angle bisectors
(c) altitudes
(d) medians

21. If in-center, circumcenter, orthocenter and centroid of a triangle coincide then triangle is: 09311075
(a)Isosceles
(b) Equilateral
(c)Right angled
(d) Acute angled

22. The circum center of right triangle lies on the …….of triangle. 09311076
(a) vertex (b) altitude
(c) hypotenuse (d) base

23. The ortho center of an acute triangle lies …….of triangle. 09311077
(a) Inside (b) Outside
(c) Midpoint (d) vertex of

24. The in-center of any triangle always lies …….the triangle. 09311078
(a) Outside (b) Inside
(c) Midpoint (d) on base of

25. The centroid of any triangle always lies …….the triangle. 09311079
(a) Outside (b) Inside
(c) Midpoint (d) on base of

Locus

26. Locus is a -----word. 09311080
(a) English (b) German
(c) French (d) Latin

27. A locus is a set of points that follow a given: 09311081
(a) Instructions (b) rule
(c) variable (d) value

28. To find the location equidistant from two towns, which locus do we have to draw? 09311082
(a) circle
(b) right bisector
(c) angle bisector
(d) Parallel lines

29. The garbage dumping area must be 5km away from the city. Which locus do we have to draw? 09311083
(a) circle (b) right bisector
(c) angle bisector (d) Parallel lines

30. A locus of point equidistant from a line segment creates a shape: 09311084
(a) circle (b) triangle
(c) sausage (d) rectangle

Answer Key

1	b	2	a	3	d	4	c	5	c	6	c	7	b	8	d	9	c	10	c
11	a	12	a	13	a	14	a	15	a	16	c	17	b	18	a	19	c	20	d
21	b	22	c	23	a	24	b	25	b	26	d	27	b	28	b	29	a	30	c

Q.2 Constant a right angled triangle with measures of sides 6cm, 8cm and 10cm.

Let m $\overline{AB}$ = 6cm, m $\overline{BC}$ = 8cm, m $\overline{AC}$ = 10cm

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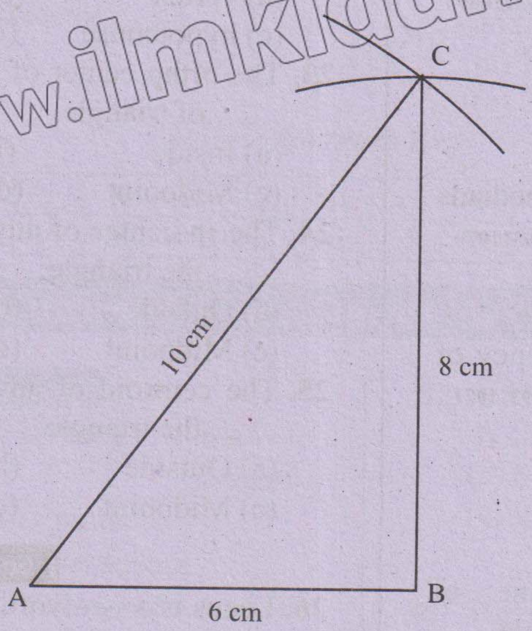

Steps of construction:

i. Let $m\overline{AB} = 6cm, m\overline{BC} = 8cm, m\overline{AC} = 10cm$

ii. Draw $m\overline{AB} = 6cm$.

iii. Taking B as a centre, draw an arc of radius 8 cm.

iv. Taking A as a centre, draw another arc of radius 10 cm which cuts the previous arc at point C.

v. Join point C to A and B.

Thus ΔABC is required triangle.

Q.3 Construct a triangle ABC with m $\overline{AB}$ = 5.3 cm, m∠A = 30° and m∠B = 120°.

Solution:

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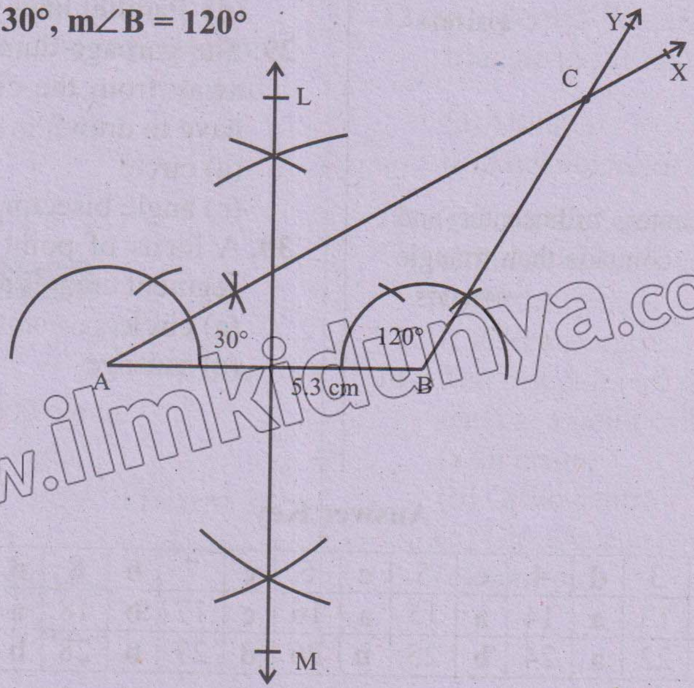
m $\overline{AB}$ = 5.3cm, m∠A = 30°, m∠B = 120°



Steps of construction:

i. Draw $m\overline{AB} = 5.3\,cm$.

ii. Using compass draw $m\angle A = 30^\circ$ and draw $\overrightarrow{AX}$.

iii. Using compass draw $m\angle B = 120^\circ$ and draw $\overrightarrow{BY}$.

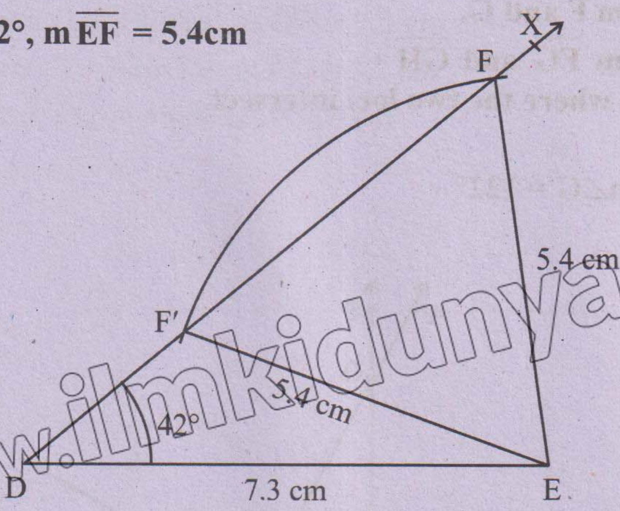
iv. Both rays, $\overrightarrow{AX}$ and $\overrightarrow{BY}$ intersect each other at point C. Thus, ΔABC is a required triangle.

v. Draw the right bisector $\overleftrightarrow{LM}$ of the side AB which is the locus of all the points that are equidistant from A and B.

Q.4 Construct a triangle with m $\overline{DE}$ = 7.3cm, m∠D = 42° and m $\overline{EF}$ = 5.4cm.

Solution: 09311087

m $\overline{DE}$ = 7.3cm, m∠D = 42°, m $\overline{EF}$ = 5.4cm



Steps of construction:

i. Draw $m\overline{DE} = 7.3\,cm$

ii. Construct $m\angle D = 42^o$ with the help of protractor and ruler and draw $m\overrightarrow{DX}$.

iii. With center E, draw an arc of radius 5.4 cm intersecting $\overrightarrow{DX}$ at F and F'.

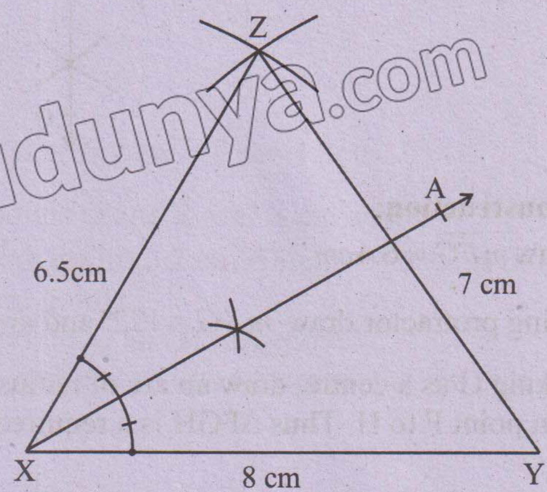
iv. Join E to F and E to F'..

Thus two triangles ΔDEF and $\Delta DEF'$ are constructed according to the given measurements.

Q.5 Construct a triangle XYZ with m $\overline{YX}$ = 8cm, m $\overline{YZ}$ = 7 cm and m $\overline{XZ}$ = 6.5 cm. Draw a locus of all points which are equidistant from $\overline{XY}$ and $\overline{XZ}$ 09311088

Solution:

m $\overline{XY}$ = 8 cm, m $\overline{YZ}$ = 7cm, m $\overline{XZ}$ = 6.5 cm



Steps of construction:

i. Draw $m\overline{XY} = 8\,cm$.

ii. Taking Y as a centre, draw an arc of radius 7 cm.

iii. Taking X as a centre, draw another arc of radius 6.5 cm which cuts the previous arc at point Z.

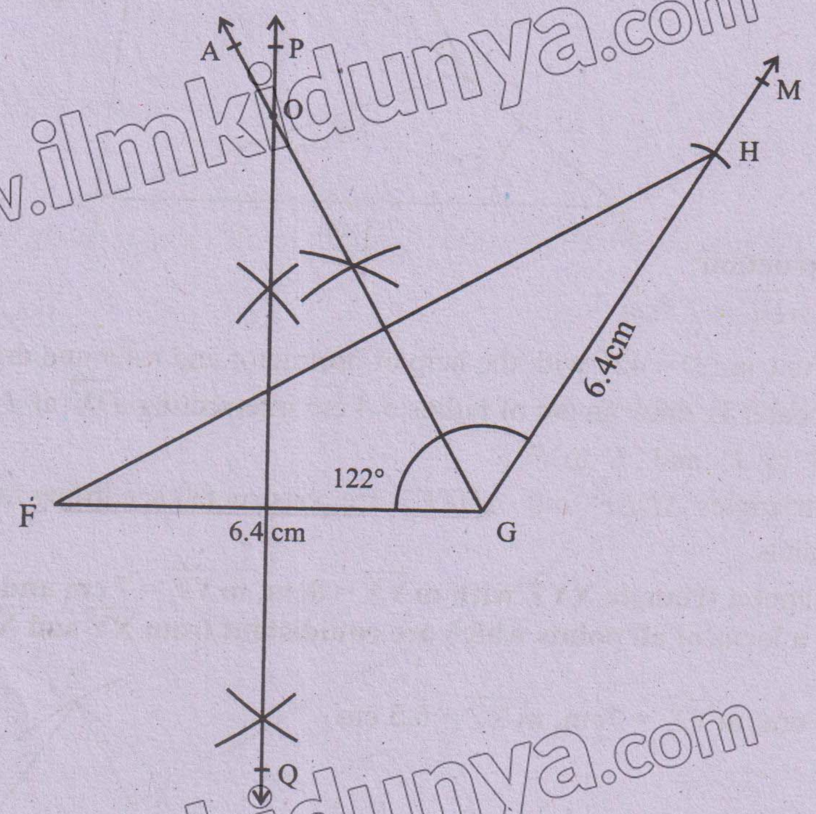
iv. Join point Z to X and Y. Thus ΔXYZ is a required triangle.

v. Draw angle bisector $\overrightarrow{XA}$ of $\angle X$ which is the locus of all the points that are equidistant from sides $\overline{XY}$ and $\overline{XZ}$.

Q.6 Construct a triangle FGH with $m\overline{FG} = m\overline{GH} = 6.4$ cm, $m\angle G = 122°$.
Draw the locus of all points which are:
(a) equidistant from F and G, 09311089
(b) equidistant from $\overline{FG}$ and $\overline{GH}$ 09311090
(c) Mark the point where the two loci intersect. 09311091

Solution:

$m\overline{FG} = m\overline{GH} = 6.4$ **cm,** $m\angle G = 122°$



Steps of construction:

i. Draw $m\overline{FG} = 6.4\,cm$.

ii. Using protractor draw $m\angle G = 122°$ and draw $\overrightarrow{GM}$.

iii. Taking G as a centre, draw an arc of radius 6.4 cm which cuts the $\overrightarrow{GM}$ at point H.

iv. Join point F to H. Thus ΔFGH is a required triangle.

v. Draw the right bisector $\overleftrightarrow{PQ}$ of the side FG which is the locus of all the points that are equidistant from F and G.

vi. Draw angle bisector $\overrightarrow{GA}$ of $\angle G$ which is the locus of all the points that are equidistant from sides $\overline{FG}$ and $\overline{GH}$.

vii. O is the point where two loci intersect each other.

Q.7 Two houses Q and R are 73 metres apart. Using a scale of 1 cm to represent 10m, construct the locus of a point P which moves such that it is: **09311092**

(i) at a distance of 32 metres form Q

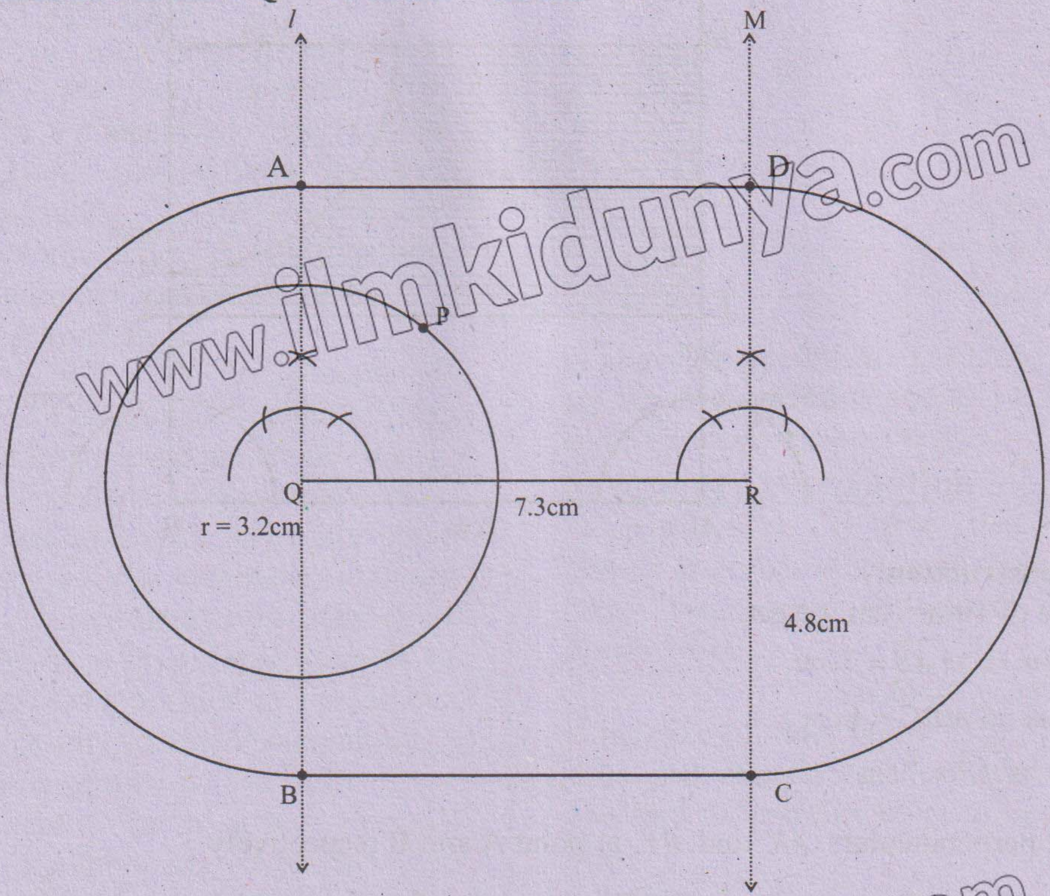
(ii) at a distance of 48 metres from the line joining Q and R.

Solution:

Distance of R from Q = 73m i.e. m $\overline{QR}$ = 7.3 cm

Distance of P from Q = 32m i.e. m $\overline{PQ}$ = 3.2 cm

Distance of P from $\overline{PQ}$ = 48 m i.e. = 4.8 cm



Steps of construction:

Using scale of 10m=1cm we get 73m =7.3 cm, 32m=3.2 cm, 48 m= 4.8cm

i. Draw $m\overline{QR} = 7.3\,cm$ showing that two points Q and R are7.3 cm apart

ii. Taking point Q as a centre, draw a circle of radius 3.2 cm which is the locus of a moving point P at a distance of 3.2 cm (32m) from point Q.

iii. Draw perpendicular lines *l* and *m* at endpoints Q and R respectively.

iv. Taking point Q as a centre, draw a semi- circle of radius 4.8 cm on the left side of Q cutting the line "*l*" at A and B.

v. Taking point R as a centre, draw a semi- circle of radius 4.8 cm on the right side of R cutting the line "*m*" at C and D.

vi. Join A to D and B to C.

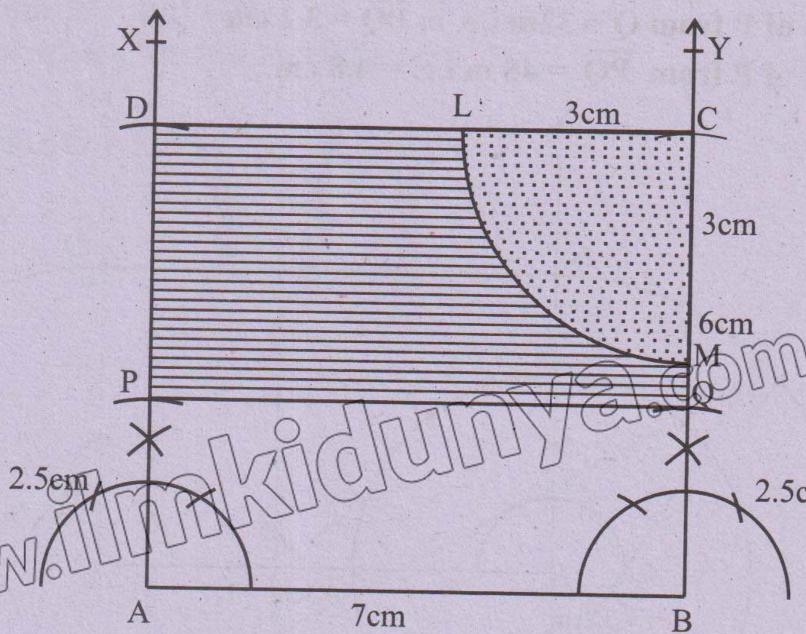
Thus the path ABCD including both semicircular paths around the $\overline{QR}$ is the locus of moving point P which is at a distance of 4.8cm (48m) from the $\overline{QR}$.

Q.8 The field is in the form a rectangle ABCD with $m\overline{AB} = 70m$ and $m\overline{BC} = 60m$. construct the rectangle ABCD using a scale of 1cm to represent 10m. Show the region inside the field which is less than 30m from C and farther than 25m from $\overline{AB}$. **09311093**

Solution:

$m\overline{AB} = 70$ m i.e. $m\overline{AB} = 7$ cm

$m\overline{BC} = 60$ m i.e. $m\overline{BC} = 6$ cm



Steps of construction:

Using scale of 10m=1cm we get

$m\overline{AB} = 70\,m \Rightarrow m\overline{AB} = 7\,cm$

$m\overline{BC} = 60\,m \Rightarrow m\overline{BC} = 6\,cm$

i. Draw $m\overline{AB} = 7\,cm$.

ii. Draw perpendiculars $\overrightarrow{AX}$ and $\overrightarrow{BY}$ at point A and B respectively.

iii. Taking A and B as a centres, draw two arcs of radius 6 cm which cuts the $\overrightarrow{AX}$ and $\overrightarrow{BY}$ at points D and C respectively.

iv. Join C to D and get a required rectangle ABCD of given measurements.

v. Taking point C as a centre, draw an arc of radius 3 cm from the side CD to CB which is a quarter of a circle and its interior is the region which is less than 3cm (30m) from point C.

vi. Taking A and B as a centres, draw two arcs of radius 2.5cm which cuts the sides $\overline{AD}$ and $\overline{BC}$ at points P and Q respectively.

vii. Join P to Q. The interior region of rectangle PQCD is the region inside the rectangle ABCD which is farther than 2.5 cm (25m) from the side AB.