UNIT 12 Information Handling

Introduction

In statistics, information handling is also known as data handling. "Data Handling" plays vital role to represent the information in a manageable way.

The word "Data Handling was first used by Sir Ronald Fisher. (17 February 1890–29 July 1962)

Information and data

To know about something is known as "Information" and to represent that information in a manageable way so that useful conclusions can be draw is called information handling. So, the collection of meaningful information in the form of facts and numerical figures is known as data.

Information Handling

Information handling is the process of collecting, organizing, summarizing, analyzing and interpreting numerical data.

Data further classified into two categories.

(i) Discrete Data: It can take only some specific values. Whole numbers are used to write discrete data. e.g., number of books sold by a shopkeeper, number of patients visited a hospital in a week etc. This data is only obtained by counting.

(ii) Continuous data: It can take every possible value in a given interval. Decimal numbers are used to write continuous data. The data is only obtained by measuring e.g., the mass students in class i.e., 28.5kg 26.5kg,27.5kg etc.

Ungrouped and Grouped Data

Data which is not arranged in any systematic order (groups or classes) is called ungrouped data. For example, the number of toys sold by a shopkeeper in a month is given below:

10, 5, 8, 12, 15, 20, 25, 30, 23, 15, 23, 21, 18, 15, 17, 23, 22, 15, 20, 21,24, 18, 16, 21, 23, 21,17, 19, 21, 23. This data is called ungrouped data. If we arrange the above given data in groups or classes, then it is called grouped data.

Classes	Tally marks	No. of toys sold
5–9	||	2
10–14	||	2
15–19	~~||||~~ ~~||||~~	10
20–24	~~||||~~ ~~||||~~ ||||	14
25–29	|	1
30–34	|	1

In above grouped data, 5, 10, 15, 25 and 30 are lower class limits and 9, 14, 19, 24, 29 and 34 are upper class limits.

Frequency Distribution

A distribution or table that represents classes or groups along with their respective class frequencies is called frequency distribution.

Think

If the size of class limits is 6. The greatest value is 80 and the smallest value is 25. Can you find the number of class limits for the data?

Solution:

Size of class limits = h = 6

Maximum value = X_{max} = 80

Minimum value = X_{min} = 25

No. of class limits = ?

We know that

$$\text{No. of class limits} = \frac{\text{Range}}{\text{Size}} = \frac{X_{max} - X_{min}}{h}$$

$$= \frac{80-25}{6}$$

$$= \frac{55}{6}$$
$$= 9.1 \approx 10$$
(Round to next number)

Following are the major steps to construct frequency distribution:

(i) Find the range of the data, range is the difference between the greatest value and the smallest value i.e., Range = $X_{max} - X_{min}$.

(ii) Find the size of the class by dividing the range by the number of classes or groups you wish to make.

For example, the greatest value is 136, the smallest value is 30 and if we have to make 10 classes or groups, then the size of class limits is found by the given formula.

$$\text{Size of class} = \frac{\text{Range}}{\text{Number of classes}}$$
$$= \frac{X_{max} - X_{min}}{\text{Number of classes}}$$
$$= \frac{136-30}{10} = \frac{106}{10} = 10.6 \approx 11$$

Keep in mind!
The number of times a value occurs in a data is called the frequency of that value. It is denoted by "f"

So, size of class limits = 11

(ii) Prepare four columns.

(a) Class limits (b) Tally marks
(c) Frequencies (d) Class Boundaries

(iv) Make classes having size of 11. Start from the smallest value.

For example, 30–40, 41–51, 52–62 and so on.

(v) Look for the class in which each element of ungrouped data falls. Draw small tally marks(|) against that class and also tick the element concerned with a sign (✔). In this way you can remember that you have counted for the element. Continue this way with the next element that upto the last element of the data set. If 5 or more tallies appear in any class, mark every 5th tally diagonally as 卌.

(vi) Class boundaries usually are found by the following method:

- Choose the upper class limit of the 1st class and lower class limit of the 2nd class.
- Find the difference between these two limits.
- The difference is divided by 2 and subtract it from the lower class limit and add it to the upper class limit.

Do you know?
Class boundaries may also be obtained from the midpoints (x) as $\left[x \mp \frac{h}{2}\right]$, Where h is the difference between any two consecutive values of x.

Example 1: Following are the number of telephone calls made in a week 30 teachers of a high school. **09312001**

5, 8, 11, 25, 13, 16, 20, 17, 15, 16, 30, 21, 14, 18, 19, 6, 22, 26 15, 19, 35, 29, 31, 23, 25, 20, 10, 9, 7, 26

Construct a frequency distribution with number of classes 7.

Solution:

(i) Find range

Greatest value (maximum value) = 35, smallest value (minimum value) = 5

Rang = $X_{max} - X_{min} = 35 - 5 = 30$

(ii) Size class limits = $\frac{\text{Range}}{\text{Number of Classes}}$

$$= \frac{30}{7} = 4.28 \approx 5$$

(iii) Make class limits having size 5. For example, 5–9, 10–14, 15–19 and so on. (see 1st column of table: 1).

(iv) Tally marks are used to count the values, fall in the given class limits. (See 2nd column of table:1).

(v) Now, count the number of tally marks and write the number as frequency in the third column (see 3rd column of table: 1)

(vi) **Class boundaries**

The difference between lower class limit of the second class and upper class limit of the first class is 1. i.e. 10–9 = 1. Now, divide the difference of the limits by 2 i.e. $\frac{1}{2} = 0.5$.

Lower class boundaries are obtained by "subtracting 0.5" from the lower class limits.

Upper class boundaries are obtained by "adding 0.5" to the upper class limits.

Lower class boundaries	Upper class boundaries
5 – 0.5 4.5 and so on.	9 + 0.5 9.5 and so on.

(see 4th column the table: 1

Table

Class limits	Tally marks	Frequency (f)	Class boundaries (C.B).
5–9	$\cancel{||||}$	5	4.5–9.5
10–14	$||||$	4	9.5–14.5
15–19	$\cancel{||||}\ |||$	8	14.5–19.5
20–24	$\cancel{||||}$	5	19.5–24.5
25–29	$\cancel{||||}$	5	24.5–29.5
30–34	$||$	2	29.5–34.5
35–39	$|$	1	34.5–39.5

Graph of Frequency Distribution

The following are the types of graph which can be used to represent a frequency distribution on a graph.

(a) Histogram (b) Frequency polygon

(a) Histogram (with equal class limits)

This is a graph of adjacent rectangles constructed on *xy* plane. A histogram is similar to bar graph but it is constructed for a frequency distribution. In a histogram, the values of the data (classes) are represented along the horizontal axis and the frequencies are shown by bars perpendicular to the horizontal axis.

Hostogram (with unequal class limits)

The procedure for making histogram is explained below:

(i) Draw lines as x–axis and y–axis on a graph paper perpendicular to each other.

(ii) Class boundaries are marked on x–axis and a rectangle is made against each group with its width proportional to the size of class limits and height proportional to the class frequencies.

(iii) This can be achieved by adjusting the heights of rectangle. The height of each rectangle is obtained by dividing each class frequency on its class limit size.

Example 2: The frequency distribution of ages (in years) of 76 members of a locality is available. Draw a histogram for this data.

09312002

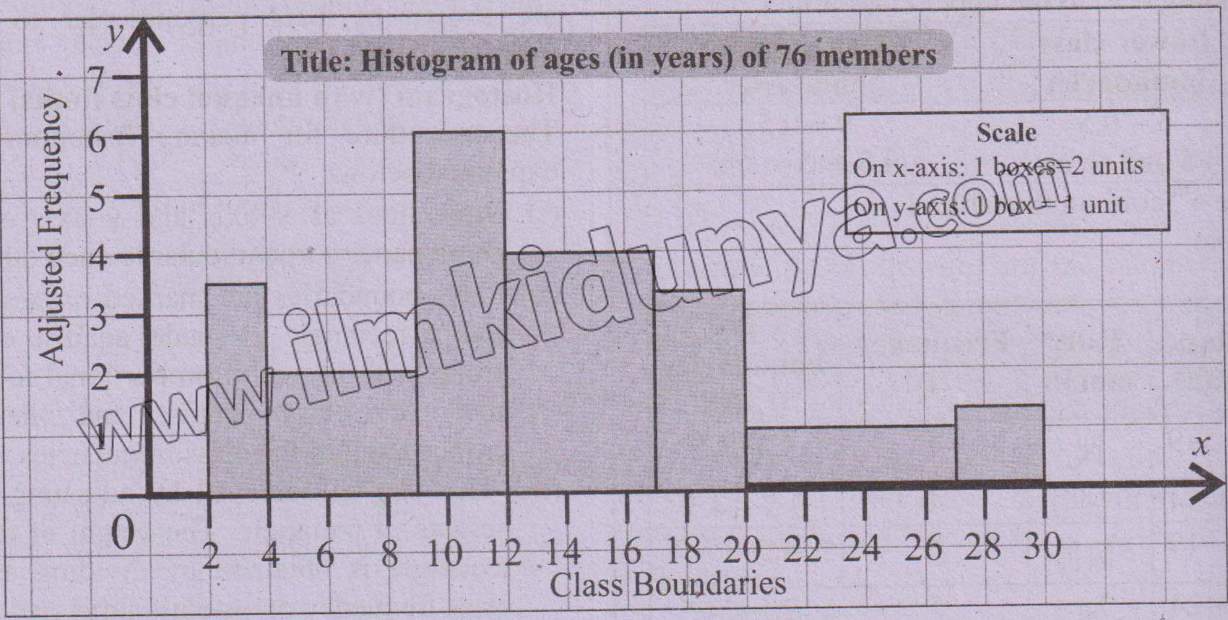
Class limits	2–4	4–9	9–12	12–17	17–20	20–27	27–30
Frequency (f)	7	10	18	20	10	7	4

Solution: Look at table, indicates that the width of the class limits is not equal as first class has width 2, second has 5, the third has 3, the fourth has 5, the fifth has 3, sixth class has 7, seventh class has width 3. So, there is need to adjust the heights of the rectangle i.e., for he first class we have 2 as width of class and 7 as a frequency, so the height of the first class is $\frac{7}{2} = 3.5$, similarly for the other $\frac{10}{5} = 2, \frac{18}{3} = 6, \frac{20}{5} = 4, \frac{10}{3} = 3.3,$ $\frac{7}{7} = 1, \frac{4}{3} = 1.3.$

These proportional heights are also called adjusted frequencies.

Class limits	Frequency (f)	Width of class	Height of rectangle (Adjusted frequency)
2–4	7	4–2 = 2	$\frac{7}{2} = 3.5$
4–9	10	9–4 = 5	$\frac{10}{5} = 2$
9–12	18	12–9 = 3	$\frac{18}{3} = 6$
12–17	20	17–12= 5	$\frac{20}{5} = 4$
17–20	10	20–17= 3	$\frac{10}{3} = 3.3$
20–27	7	27–20= 7	$\frac{7}{7} = 1$
27–30	4	30–27= 3	$\frac{4}{3} = 1.3$

Taking class boundaries along $x-$ axis and corresponding adjusted frequencies along y–axis, rectangles are drawn and the histogram is given below.



Frequency Polygon

A frequency polygon is a closed geometrical figure used to display a frequency distribution graphically. A line graph of a frequency distribution is known as frequency polygon in which frequencies are plotted against their midpoints. Midpoint is the average value of the lower and upper class limits. Midpoint is also known as class mark. Midpoint is calculated by the given formula.

Midpoint

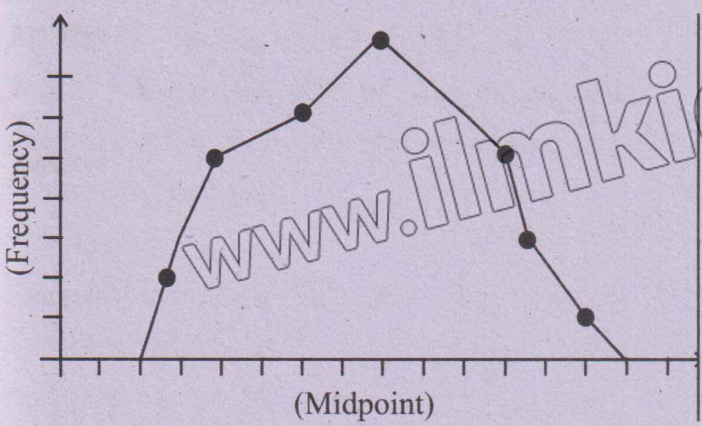
$$= \frac{\text{Lower class limit} + \text{Upper class limit}}{2}$$

The following steps are followed to draw a frequency polygon for a frequency distribution:

(i) Draw lines as x–axis and y–axis perpendicular to each other.

(ii) Take midpoints on x–axis and class frequencies on y–axis.

(iii) Put a dot mark against each midpoint corresponding to its class frequency. Join all the dotted marks by straight lines to get the required frequency polygon.



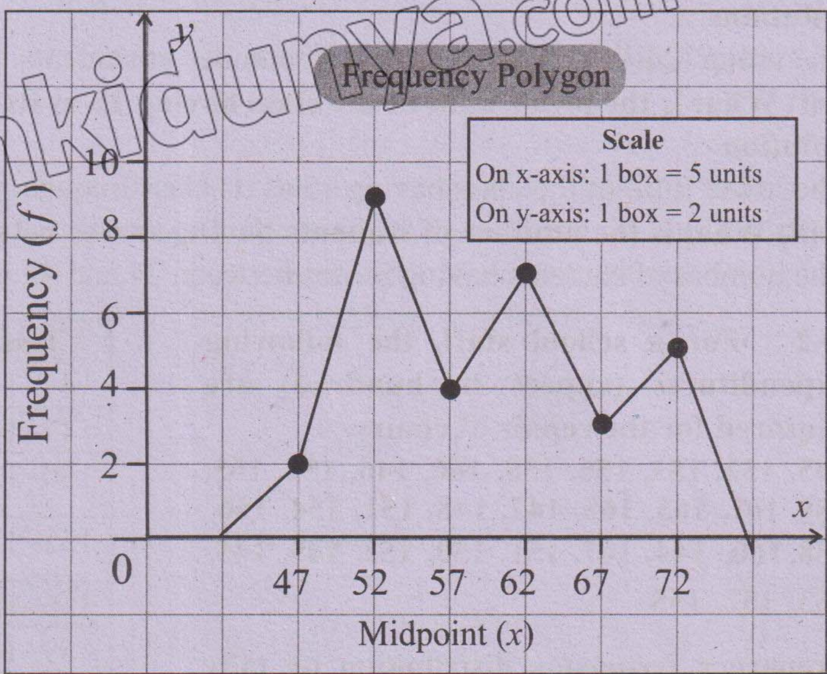
(iv) The lines at both ends are joined together with the next midpoints to touch the bases of x–axis.

Example 3: The following are the marks obtained by 30 students out of 100 in the subject mathematics at their final examination. Construct frequency polygon for the following frequency table. **09312003**

Marks	45–49	50–54	55–59	60–64	–65–69	70–74
Frequency	2	9	4	7	3	5

Solution:

Marks	f	Midpoints
45–49	2	$\frac{45+49}{2}=47$
50–54	9	$\frac{50+54}{2}=52$
55–59	4	$\frac{55+59}{2}=57$
60–64	7	$\frac{60+64}{2}=62$
65–69	3	$\frac{65+69}{2}=67$
70–74	5	$\frac{70+74}{2}=72$



Exercise 12.1

Q.1 The following distribution represent the achieved by a group of chemistry students in the chemistry laboratory.

Scores	24–28	29–33	34–38	39–43	44–48	49–53	**Total**
No. of students	3	6	12	23	15	6	**65**

(i) What is the upper limit of the last class? 09312004

Solution:

The upper limit of the last group is 53.

(ii) What is the lower limit of the class 39–43? 09312005

Solution:

The lower limit of the class (39-43) is 39. i.e.

(iii) What is the midpoint of the class (34–38)? 09312006

Solution:

The midpoint of the class (34–38) 36. i.e.

$$\text{Midpoint } (x) = \frac{34+38}{2} = \frac{72}{2} = 36$$

(iv) What are the class frequencies of the classes 29-33 and 44-48? 09312007

Solution

The class frequencies of the classes (29–33) and (44–48) are 6 and 15 respectively.

(v) What is the size of the class limits in the above frequency distribution? 09312008

Solution

The size of class intervals of the given frequency distribution is 5.

(vi) In which class or groups does minimum number of students fall? 09312009

Solutions

The group (24-28) contains minimum number of students.

(vii) What is the lower limit of the class having 15 as its class frequency? 09312010

Solution

The lower limit of the class having 15 as its class frequency is 44.

(viii) What is the number of students having scores between 24 and 43? 09312011

The number of students having scores between 24 and 43 is 44.

Q.2 For a school staff, the following expenditures (rupees in hundred) are required for the repair of chairs.
145, 152, 153, 156, 158, 160, 146, 152, 155, 159 161, 163, 165, 147, 148, 151, 154, 156, 158, 160, 144, 167, 151, 150, 152, 149, 145, 153, 152, 155

Prepare a frequency distribution by tally bar method using 3 as the size of class limits and also write down what are the frequencies of the last three classes? 09312012

Solution:

No. of observations: n = 30

Smallest value = 144

Largest value = 167

Size of class intervals = 3

Class limits	Tally marks	(f)
144–146	IIII	4
147–149	III	3
150–152	~~IIII~~ II	7
153–155	~~IIII~~	5
156–158	IIII	4
159–161	IIII	4
162–164	I	1
165–167	II	2
Total		$\Sigma f = 30$

The frequencies of last three classes are 4, 1 and 2 respectively.

Q.3 Given below are the weights in kg of 30 students of a high school. 09312013
30, 33, 24, 21, 15, 39, 37, 44, 42, 33
33, 28, 29, 32, 31, 28, 26, 32, 34, 35

38, 36, 41, 30, 35, 41, 23, 26, 18, 34

Taking 5 as the size of the class limit, prepare a frequency table and construct a frequency polygon.

Solution:

No. of observation = n = 30

Smallest value = 15

Largest value = 44

Size of class limits = 5

Class limits	Tally marks	(f)
15–19	||	2
20–24	|||	3
25–29	~~||||~~	5
30–34	~~||||~~ ~~||||~~	10
35–39	~~||||~~ |	6
40–44	||||	4
Total		Σf = 30

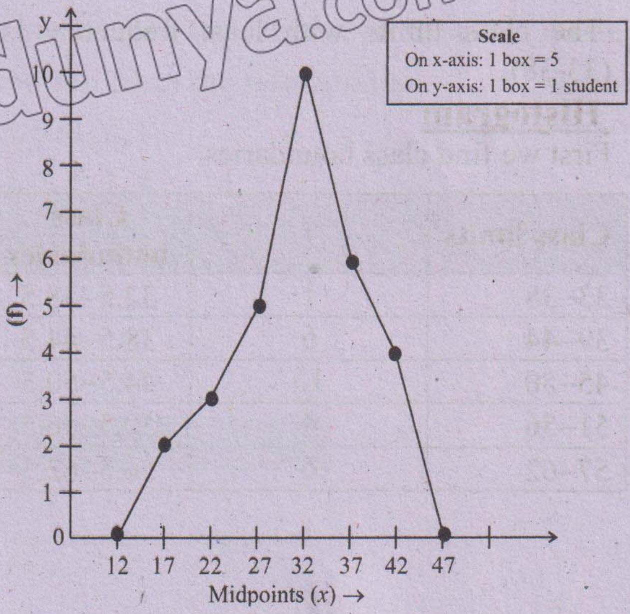
Frequency Polygon

Solution:

Take two additional groups with zero frequency one before the very first group and second after the very last group.

Also find the midpoints of all the groups.

Class limits	(f)	Midpoints
10–14	0	12
15–19	2	17
20–24	3	22
25–29	5	27
30–34	10	32
35–39	6	37
40–44	4	42
45–49	0	47



Q.4 A group of Grade-10 students obtained the following marks out of 100 marks in English test. **09312014**

58, 59, 58, 33, 40, 58, 45, 46, 43, 45, 45, 50, 52, 49, 50, 57, 52, 55, 49, 50, 62, 49, 48, 44, 42, 47, 46, 47, 46, 53, 40, 44.

Classify the data into a frequency distribution by (direct method) taking 6 as the size of class limit. Also find the class limit with least class frequency and construct histogram for the data.

Solution:

No. of observations = n = 32

Smallest value = 33

Largest value = 62

Size of class limits = 6

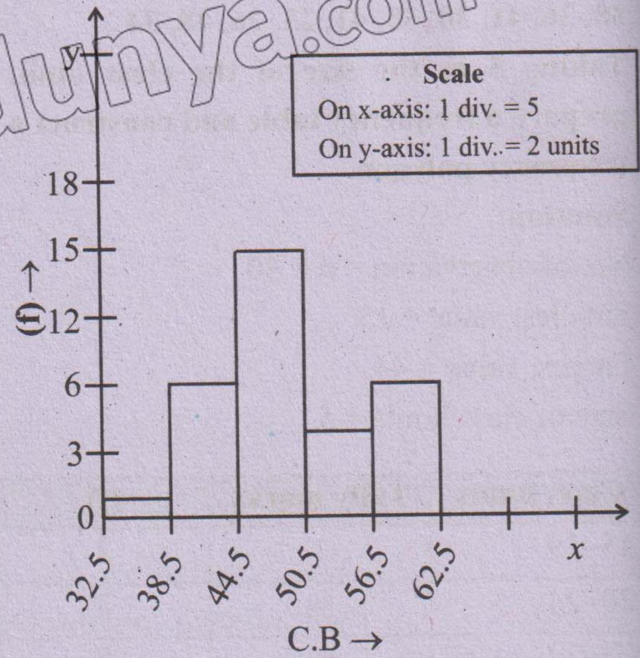
Class limits	Values	(f)
33–38	33,	1
39–44	40,43,44,42,40,44	6
45–50	45,46,45,45,50, 49, 50, 49, 50, 49, 48, 47, 46, 47, 46	15
51–56	52, 52, 55, 53	4
57–62	58, 59, 58, 58, 57, 62	6
		Σf = 32

The class limits with least frequency is (33-38).

Histogram

First we find class boundaries

Class limits	f	Class boundaries
33–38	1	32.5–38.5
39–44	6	38.5–44.5
45–50	15	44.5–50.5
51–56	4	50.5–56.5
57–62	6	56.5–62.5

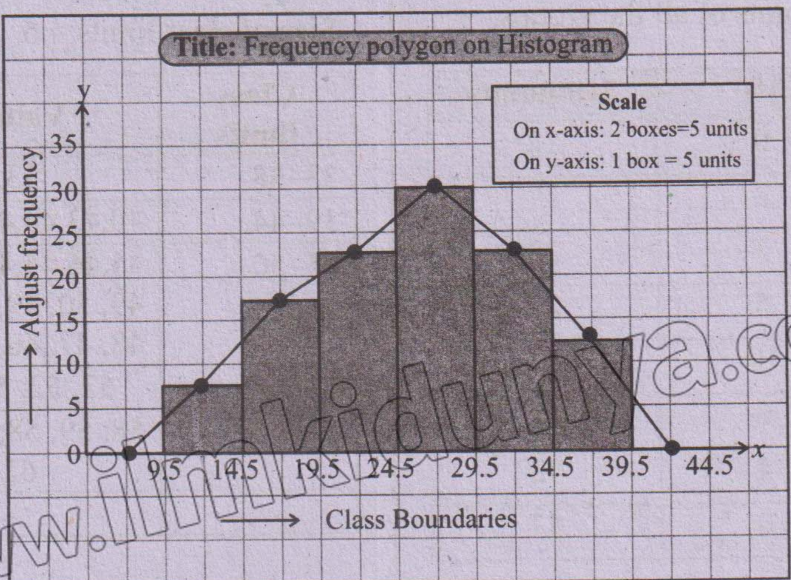


Q.5 From the table given below. Draw a frequency polygon on histogram for the given frequency distribution. **09312015**

Weight (kg)	10–14	15–19	20–24	25–29	30–34	35–39
Frequency (f)	06	17	23	30	22	13

Solution:

Weight	(f)	Midpoint (x)	Class Boundaries
10–14	6	12	9.5–14.5
15–19	17	17	14.5–19.5
20–24	23	22	195.5–24.5
25–29	30	27	24.5–29.5
30–34	22	32	29.5–34.5
35–39	13	37	34.5–39.5



Q.6 **The following data shows the number of heads in an experiment of 50 sets of tossing a coin 5 times. Make a discrete frequency distribution from the information.** 09312016

3, 3, 4, 0, 5, 4, 3, 3, 1, 2, 4, 5, 0, 3, 2, 4, 4, 0, 0, 0, 5, 5, 3, 2, 1

4, 3, 2, 5, 3, 2, 1, 3, 5, 4, 3, 2, 1, 3, 2, 1, 3, 1, 3, 1, 4, 3, 2, 2, 4

Solution:

No of observations = n = 50

Smallest value = 0

Largest value = 5

No. of Heads	Tally Marks	(f)
0	~~||||~~	5
1	~~||||~~ ||	7
2	~~||||~~ ||||	9
3	~~||||~~ ~~||||~~ ||||	14
4	~~||||~~ ||||	9
5	~~||||~~ |	6
		$\Sigma f = 50$

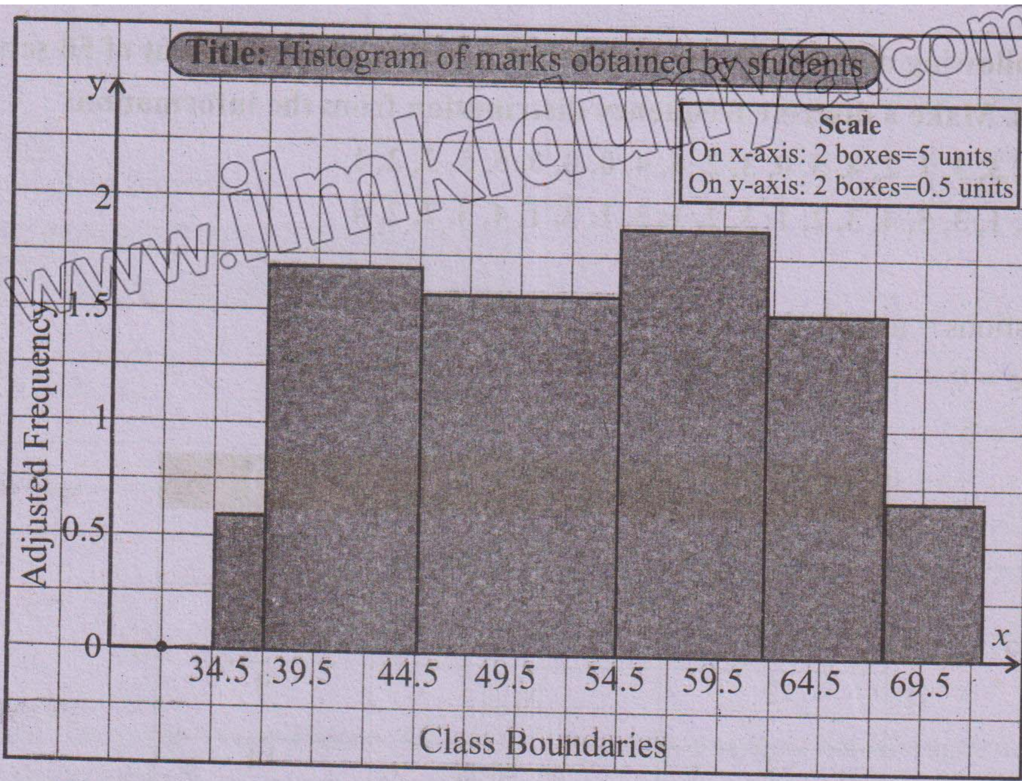
Q.7 **The marks obtained by the students of Grade-10 in mathematics test were grouped into the following frequency distribution.** 09312017

Marks	35–37	38–44	45–54	55–61	62–67	68–72
Frequency	2	12	16	13	9	3

Draw a histogram for the above distribution.

Solution:

Marks	(f)	Class boundaries	h	f/h
35–37	2	34.5–37.5	3	0.67
38–44	12	37.5–44.5	7	1.7
45–54	16	44.5–54.5	10	1.6
55–61	13	54.5–61.5	7	1.86
62–67	9	61.5–67.5	6	1.5
68–72	3	67.5–72.5	5	0.6



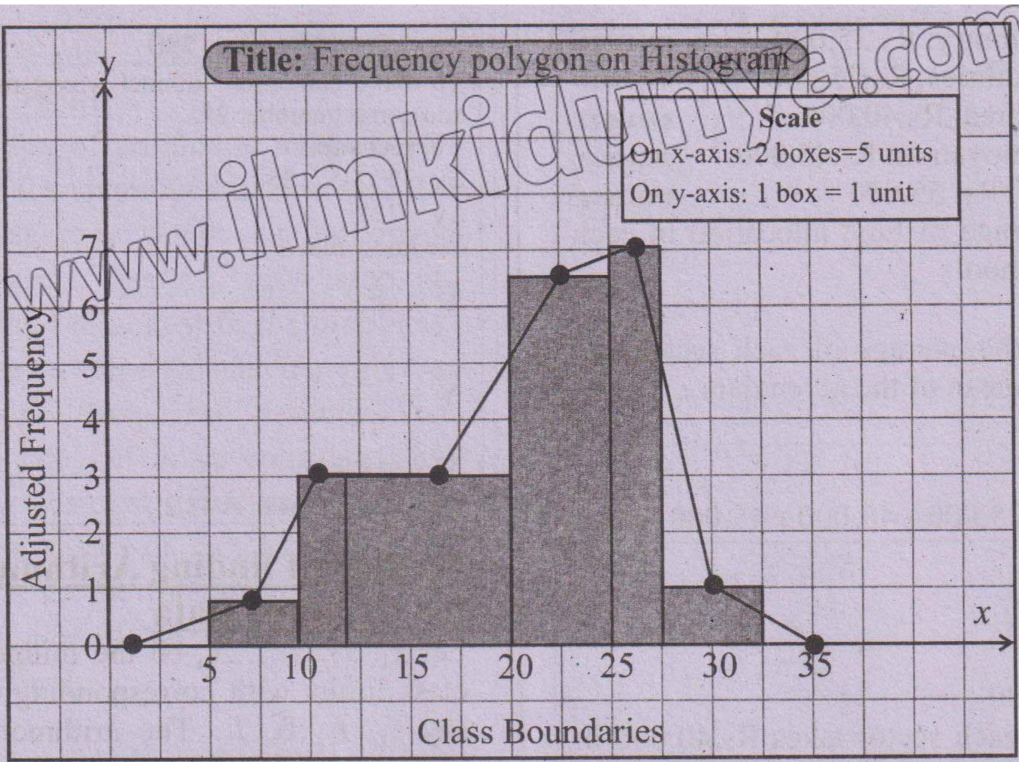
Q.8 Make a frequency polygon on histogram for the following grouped data: 09312018

Marks	5–8	8–12	12–20	20–25	25–27	27–32
Frequency	2	12	25	32	14	5

Solution:

Class limits	(f)	Midpoints (x)	Width of class (h)	Height of rectangle f/h
5–8	2	6.5	8–5 = 3	$\frac{2}{3} = 0.67$
8–12	12	10	12–8 = 4	$\frac{12}{4} = 3$
12–20	25	16	20–12 = 8	$\frac{25}{8} = 3.1$
20–25	32	22.5	25–20 = 5	$\frac{32}{5} = 6.4$
25–27	14	26	27–25 = 2	$\frac{14}{2} = 7$
27–32	5	28.5	32–27 = 5	$\frac{5}{5} = 1$

Frequency on Histogram.



Measures of Location (Central Tendency)

The measure that gives the centre of the data is called measure of central tendency.

The following measure of central tendency will be discussed in this section:

(i) Arithmetic Mean (A.M.)

(ii) Median

(iii) Mode

(iv) Weighted mean

Arithmetic Mean (A.M.)

It is defined as a value of variable which is obtained by dividing the sum of all the values (observations) by their number of observations. Thus, the arithmetic mean of a set of values $x_1, x_2, x_3 \ldots x_n$ is denoted by $\overline{X}$ (read as X-bar) and is calculated as:

$$\overline{X} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\sum x}{n} \text{ (Direct method)}$$

Where, the sign Σ stands for the sum and n is the number of observations.

Example 4: The marks of a student in five examinations were 64, 75, 81, 87, 90. Find the arithmetic mean of the marks. **09312019**

Solution:

$$\textbf{A.M.} = \overline{X} = \frac{\sum x}{n}$$

$$\overline{X} = \frac{64 + 75 + 81 + 87 + 90}{5}$$

$$\text{or } \overline{X} = \frac{397}{5} = 79.4$$

Try yourself

Data X = 10, 30, 40, x, 67, 81

No. of observation = n = 6

Mean = $\overline{X}$ = 50

We know that

$$\overline{X} = \frac{\sum X}{n}$$

$$50 = \frac{10 + 30 + 40 + x + 67 + 81}{6}$$

$50 \times 6 = 228 + x$

$300 - 228 = x$

$72 = x$

$\Rightarrow \boxed{x = 72}$

Example 5: A government allocates finds of Rs.200,000 to five sectors of a school i.e.,

(i) School Library: Rs.35,000 09312020
(ii) Sports facilities: Rs.25,000 09312021
(iii) Parking area: Rs.40,000 09312022
(iv) Room renovation: Rs.45,000 09312023
(v) Furniture: Rs. 55,000 09312024

Find the average of fund allocation in each sector of a school.

Solution

To find out the average of each sector, we will find the mean of the given data.

$$\overline{X} = \frac{\sum X}{n}$$

$$\overline{X} = \frac{35,000 + 25,000 + 40,000 + 45,000 + 55,000}{5}$$

$$\overline{X} = \frac{200,000}{5}$$

$\overline{X}$ = Rs. 40,000

On average, each sector takes Rs.40,000 in funding.

Try yourself

The mean of 15 values was 50. It was found on rechecking that the value 25 w2as wrongly copied as 52. Find the correct mean.

Solution:

No. of observation = n = 15

Mean = $\overline{X}$ = 50

Sum : ΣX = ?

$$\overline{X} = \frac{\sum X}{n}$$

$n\overline{X} = \Sigma X$

$\Rightarrow \Sigma X = 15 \times 50 = 750$

To make correction subtract wrong number 52 and add correct number 25.

Correct sum:

$\Sigma y = 570 - 52 + 25$

$\Sigma y = 723$

We know that

$$\overline{Y} = \frac{\sum y}{n}$$

$$\overline{Y} = \frac{723}{15}$$

$\overline{Y} = 48.2$

Thus correct mean is 48.2.

Method of finding Arithmetic Mean for Grouped Data

Let $x_1, x_2, x_3, \ldots, x_n$ be the midpoints of the class limits with corresponding frequencies say f_1, f_2, f_3, f_n. The arithmetic mean is obtained by dividing sum of the products of f and x by the sum of the frequencies.

$$\overline{X} = \frac{f_1 x_1 + f_2 x_2 + \ldots\ldots\ldots + f_n x_n}{f_1 + f_2 + \ldots.. + f_n} = \frac{\sum fx}{\sum f}$$

Example 6: Given below are the marks out of 100 obtained by 100 students in a examination. Find the average marks of the students. 09312025

Marks	30–35	35–40	40–45	45–50	50–55	55–60
No. of students	14	16	18	23	18	11

Solution:

Marks	Midpoint (x)	Frequency (f)	fx
30–35	32.5	14	455.0
35–40	37.5	16	600.0
40–45	42.5	18	765.0
45–50	47.5	23	1092.5
50–55	52.5	18	945.0
55–60	57.5	11	632.5
		$\Sigma f = 100$	$\Sigma fx = 4490$

$$\overline{X} = \frac{\sum fx}{\sum f} = \frac{4490}{100}$$

or $\overline{X}$ = 44.9 marks

Hence, the average marks is 44.9 of the students.

Short Formula for Computing Arithmetic Mean

The computation of arithmetic mean using direct method for ungrouped data as well as for grouped data is no doubt easy for small values. If x and f become very large, it becomes difficult to deal with the problems so to minimize our time and calculations we take deviations from an assumed or provisional mean. Let A be considered as assumed or provisional mean (may be any value from the values of x or any number) and D denotes the deviations of x from A i.e., $D = x - A$. For $x = D+A$, the formula of arithmetic becomes;

$\overline{X} = A + \frac{\sum D}{\sum n}$ (for ungrouped data) ...(i)

$\overline{X} = A + \frac{\sum fD}{\sum f}$ (for ungrouped data) ...(ii)

Example 7: Find he arithmetic mean using short formula for the runs made by a batsman. **09312026**

Runs: 40, 45, 50, 52, 50, 60, 56, 70.

Solution

Taking deviations from A = 52 (assumed mean)

x	40	45	50	52	50	60	56	70
$D=x-A$	−12	−7	−2	0	−2	8	4	18

Now: $\Sigma D = -23+30 = 7$

$\because \quad \overline{X} = A + \frac{\sum D}{n}$

So, $\overline{X} = 52 + \frac{7}{8}$

$= 52+0.875 = 52.88$ or 53 runs.

Try yourself!

If $\overline{X} = 120$; $A = 85$ and $n = 25$, then can you find the value of ΣD?

Solution:

We know that

$\overline{X} = 120$

$n = 25$, $A = 85$

$\Sigma D = ?$

We know that

$$\overline{X} = A + \frac{\sum D}{n}$$

$$120 = 85 + \frac{\sum D}{25}$$

$$120 - 85 = \frac{\sum D}{25}$$

$$35 = \frac{\sum D}{25}$$

$$35 \times 25 = \Sigma D$$

$$\Rightarrow \Sigma D = 875$$

Example 8: Deviations from 12.5 of ten different values are 6, −2,3.5, 9, 8.7, −5.5, 14, 11.3, −6.8, −4.2, find the arithmetic mean. **09312027**

Solution:

Deviations from 12.5 are:

6, −2, 3.5, 9, 8.7, −5.5, 14, 11.3, −6.8, −4.2

Now, $\Sigma D = 34$, Also, $A = 12.5$,

using the formula we have.

$\overline{X} = A + \frac{\sum D}{n}$

$= 12.5 + \frac{34}{10}$

or $\overline{X} = 12.5+3.4 = 15.9$

Example 9: The height (in inches) of 200 students are recorded in the following frequency distribution. Find the mean height of the student by short formula. **09312028**

Height (x) (in inches)	51	52	53	54	55	56	57	58	59	60
Frequency (f)	2	5	8	24	55	45	38	16	6	1

Solution:

Let assumed mean = A = 55

Heights (x) (n inches)	Frequency (f)	A = 55 D = x − A	fD
51	2	−4	−8
52	5	−3	−15
53	8	−2	−16
54	24	−1	−24
A←55	55	0	0
56	45	1	45
57	38	2	76
58	16	3	48
59	6	4	24
60	1	5	5
Total	**Σf = 200**		**ΣfD = 135**

Now, using the formula (ii), we get

$$\overline{X} = A + \frac{\sum fD}{\sum f}$$

$$\overline{X} = 55 + \frac{135}{200}$$

or $\overline{X} = 55 + 0.675$

$\because \overline{X} = 55.68$ inches approx.

Hence, the mean height of the students is 55.68 inches.

Example 10: Ten students each from Grade-V section A and B of a well reputed school were taken randomly. Their weights were measured in kg. and recorded as given below:

Weights (kg) Section A	30	28	32	29.5	35	34	31	33	40	37.5
Weights (kg) Section B	35	31.5	34.5	35	32.8	38	29.5	36	36.5	34

(i) Compute the mean weight for section A and B. **09312029**

(ii) Conclude which section is better on Average. **09312030**

Solution:

(i) We find arithmetic mean for both the sections by direct method.

As number of observations n = 10

and $\overline{X}_{(A)} = \frac{\sum X_{(A)}}{n}$

$\because \quad \overline{X}_{(A)} = \frac{330}{10} = 33\text{kg}$

and $\quad \overline{X}_{(B)} = \frac{\sum X_{(B)}}{n}$

$\because \quad \overline{X}_{(B)} = \frac{342.8}{10} = 34.28\text{kg}$

$X_{(A)}$	$X_{(B)}$
30	35
28	31.5
32	34.5
29.5	35
35	32.8
31	29.5
33	36
40	36.5
37.5	34
ΣX(A) = 330	**ΣX(B) = 342.8**

(ii) We have seen from the results that $\overline{X}_{(B)}$ is greater than $\overline{X}_{(A)}$. Therefore, we conclude that section B is better on the average.

Median

Median is the middle most value in an arranged (ascending or descending order) data set. Median is the value which divides the data into two equal parts i.e., 50% data is before the median and 50% data after it. Median is denoted by $\tilde{X}$.

Median for ungrouped data

The median of n observations $x_1, x_2 \ldots x_n$ is obtained as:

(when n is odd number)

$$\text{Median}(\tilde{X}) = \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

$\because$ When n is even number

$$\text{Median}(\tilde{X}) = \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n+2}{2}\right)\text{observation}\right)$$

Example 11: The following are the scores made by a batsman. Find the median of the data. 8, 12, 18, 13, 16, 5, 20. 09312031

Solution:

Writing the scores in an ascending order, we have 5, 8, 12, 13, 16, 18, 20

Since, number of observations is odd i.e.,n=7

$$\text{Median }(\tilde{X}) = \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

$$= \left(\frac{7+1}{2}\right)^{th} \text{ observation}$$

$= 4^{th}$ observation = 13

= Hence, 13 is the median of the given data.

Example 12: Following are the marks out of 100 obtained by 10 students in English. 23, 15, 35, 48, 41, 5, 8, 9, 11, 51. Find the median of the data. 09312032

Solution

Arranging the data in an ascending order.

5, 9, 9, 11, 15, 23, 35, 41, 48, 51

Since, number of observation is even, i.e., n = 10

$\therefore$ Median $(\tilde{X})$

$$= \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n+2}{2}\right)^{th}\right)\text{observation}$$

As, $\frac{n}{2} = \frac{10}{2} = 5$ and $\frac{n+2}{2} = \frac{12}{2} = 6$

$\therefore$ Median $= \frac{1}{2}$ [5^{th} observation+6^{th} observation]

or Median $= \frac{1}{2}[15+23] = \frac{38}{2} = 19$

Hence, 19 is the median of the data.

Median for Grouped Data

The median for grouped data is obtained by the following formula:

$$\text{Median }(\tilde{X}) = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

Where, ℓ = Lower class boundary of median class,

H = The size of class limits of median class,

f = Frequency of the median class,

n = Total frequency i.e., Σf,

and c = Cummulative frequency preceding the median class.

Remember the following points:

(i) The group of classes must be in a continuous form i.e., we need class boundaries.

(ii) Make the column of cumulative frequencies (c.*f.*) from the column of frequencies.

(iii) Locate median class i.e., $\left(\frac{n}{2}\right)^{th}$ see value in c.f. column wherever it lies.

(iv) Underline median class, then take the values of f and h of the median class thus obtained.

Example 13: The heights of 100 athletes, measured to the nearest (inches) are given in the following table. Find the median. 09312033

Heights (in inches)	62.5–63.5	63.5–64.5	64.5–65.5	65.5–66.5	66.5–67.5	67.5–68.5	68.5-69.5	69.5–70.5	70.5–71.5
No. of students	4	6	10	20	30	13	12	3	2

Solution:
In the above data, class boundaries have already been given.

Heights (inches)	Frequency (f)	c.f.	
62.5–63.5	4	4	
63.5–64.5	6	6+4 = 10	
64.5–65.5	10	10+10 = 20	
65.5–66.5	20	20+20 = 40 →c	
ℓ →66.5–67.5	f→30f	30+40=70→	Median group
67.5–68.5	13	13+70 = 83	
68.5–69.5	12	12+83 = 95	
69.5–70.5	3	3+95 = 98	
70.5–71.5	2	2+98 = 100	
Total	n = Σf = 100	---	

Here, n = 100

So, $\frac{n}{2} = \frac{100}{2} = 50$

50^{th} item lies in the class boundaries (66.5–67.5.)

Put ℓ = 66.5, h = 1, f = 30, c = 40

$\because$ Median = $\ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$

$\tilde{X} = 66.5 + \frac{1}{30}(50-40)$ (Putting the values)

$= 66.5 + \frac{10}{30}$

$= 66.5 + 0.33$

$\therefore$ Median = 66.83 inches

Example 14: Following are the weights (in kg) of 50 men. Find the median weight. **09312034**

Weights (kg)	110–114	115–119	120–124	125–129	130–134
No. of men (f)	5	12	23	6	4

Solution:
As class boundaries are not given so, first of all we make class boundaries by the usual procedure.

Weight (kg)	Frequency (f)	Class Boundaries	c.f.	
110–114	5	109.5–114.5	5	
115–119	12	114.5–119.5	17	
120–124	23→f	119.5–124.5	40→c	Median group
125–129	6	124.5–129.5	46	
130–134	4	129.5–134.5	50→n	
Total	**Σf = 50**	---	---	

Here n = 50 so, $\frac{n}{2}=\frac{50}{2}=25$, 25^{th} item lies in(119.5−124.5)

$\ell = 119.5$, $h = 5$, $f = 23$, $c = 17$

$$\text{Median} = \ell + \frac{h}{f}\left(\frac{n}{2}-c\right)$$

$$= 119.5+\frac{5}{23}(25-17) \quad \text{(Putting he values)}$$

$$= 119.5 + \frac{40}{23} = 119.5+1.74 = 121.14 \text{ kg}$$

Mode

In a data the values (observation) which appears or occurs most often is called mode of the data. It is the most common value. Mode is denoted by $\hat{X}$.

Mode for Ungrouped Data

Example 15: The marks in mathematics of Jamal in eight monthly tests were 75, 76, 80, 80, 82, 82, 82, 85. Find the mode of the marks.

09312035

Solution

As 82 is repeated more than any other number so, clearly mode is 82.

Example 16: Ten students were asked about the number of questions they have solved out of 20 questions last week. Records were 13, 14, 15, 11, 16, 10, 19, 20, 18, 17. Find he mode of the data. **09312036**

Solution:

It is obvious that the given data contains no mode. It is ill-defined. Sometimes data contains several modes. If the data is: 10, 15, 15, 15, 20, 20, 20, 25, 32, then data contains two modes i.e., 15 and 20.

Example 17: A survey was conducted from the 15 students of a school and asked the students about their favourite colour.

The responses are: purple, yellow, purple, yellow, yellow, red, blue, green, yellow, yellow, red, blue, yellow, purple, green. Find mode of the data. **09312037**

Solution:

Mode is the most frequency colour.

Mode = yellow

So, the colour "yellow" is the mode of the given data.

Remember!

A data can has more than one mode.
A data may or may not have a mode.

Mode for Grouped Data

Mode can be calculated by the following formula:

$$\text{Mode: } \ell+\frac{(f_m-f_1)}{(f_m-f_1)+(f_m-f_2)}\times h$$

Where,

ℓ = Lower class boundary of the model class

f = Frequency of the modal class.

f_1 = Frequency preceding the modal class.

f_2 = Frequency following the modal class and

h = Size of the modal class.

Remember

A data can has move than one mode. A data may or may not have a mode.

Note

Mode cannot be easily calculated from the data presented in a frequency distribution. As it has no individual values, so we do not know which value appears most frequently. We only assume he class with the highest frequency as a modal class.

Example 18: Following are the height in (inches) of 40 students in Grade-8. **09312038**

Heights (inches)	48–50	50–52	52–54	54–56	56–58	58–60
No. of students (f)	5	7	10	9	6	3

Heights (inches)	Frequency (f)
48–50	5
50–52	$7 \to f_1$
$\boxed{\ell\ 52–54}$	$\boxed{10 \to f_m}$, here h = 2
54–56	$9 \to f_2$
$56 \to 58$	6
$58 \to 60$	3
Total	$\Sigma f = 40$

Solution:
In the above data, class boundaries have already been given. Using the formula for grouped data we find mode as:

$$\text{Mode} = \ell + \frac{(f_m - f_1)\times h}{(f_m - f_1) + (f_m - f_2)}$$

Put $\ell = 52$, h = 2, $f_m = 10$, $f_1 = 7$, $f_2 = 9$

$$\text{or} \quad \text{Mode} = 52 + \frac{(10-7)\times 2}{(10-7)+(10-9)}$$

$$\text{or} \quad \text{Mode} = 52 + \frac{3\times 2}{3+1} = 52 + \frac{6}{4}$$

$$\text{or} \quad \text{Mode} = 52 + 1.5 = 53.5 \text{ (inches)}$$

Weighted Mean

Arithmetic Mean is used when all the observations are given equal importance/ weight but there are certain situations in which the different observations get different weights. In this situation, weighted mean denoted by $\overline{X}_w$ is preferred. The weighted mean of $X_1, X_2, X_3, \ldots, X_n$ with corresponding weights $W_1, W_2, W_3, \ldots, W_n$ is calculated as:

$$\overline{X}_w = \frac{W_1X_1 + W_2X_2 + W_3X_3 + \ldots + W_nX_n}{W_1 + W_2 + W_3 + \ldots W_n}$$

$$= \frac{\sum_{i-1}^{n} WiXi}{\sum_{i-1}^{n} Wi} = \frac{\Sigma WX}{\Sigma W}$$

Example 19: The following data describes the marks of a student in different subjects and weights assigned to these subjects are also given: **09312039**

Marks (x)	74	78	74	90
Weights (w)	4	3	5	6
wx	296	234	370	540

Find its weighted mean.

Solution: Weighted mean $\left(\overline{X}_w\right) = \dfrac{\Sigma WX}{\Sigma W}$

$$\overline{X}_w = \frac{4(74)+3(78)+5(74)+6(90)}{4+3+5+6}$$

$$= \frac{296+234+370+540}{18} = \frac{1440}{18}$$

$$\overline{X}_w = 80$$

Example 20: A medicine company started marketing of a sample of medicine in seven different areas of a city. The company distributed the packets of medicine in each area of the city and the weight of each area based on the demand of the medicine. Find the mean and weighted mean of the given data. **09312040**

Area	Number of package	Weights (kg)
A	15	5
B	25	4
C	18	3
D	23	4
E	15	2
F	10	1
G	8	2

Solution:

$$\text{Mean} = \frac{\text{Total number of package}}{\text{Total number of area}}$$

$$= \frac{15+25+18+23+15+10+8}{7}$$

$$= \frac{114}{7} = 16.29 \approx 16 \text{ packets}$$

So, the average number of packets of the medicine distributed by the company per area is 16.

Weighted mean =

$$\frac{\sum[\text{Number of packets} \times \text{weight}]}{\sum \text{Weights}}$$

$$= \frac{15(5)+25(4)+18(3)+23(4)+15(2)+10(1)+8(2)}{5+4+3+4+2+1+2}$$

$$= \frac{377}{21} = 17.95 \approx 18\text{kg}$$

Real life situation involving mean, Weighted mean, Median and mode

Sales and Marketing

Example 21: A toy factory sold toys in a month. Consider the following data: **09312041**

Class limits	10–20	20–30	30–40	40–50	50–60
f	15	28	45	29	20

(i) Calculate mean, median and mode of the number of toys sold by the factory.

(ii) Also tell the modal class of the distribution.

Class limits	f	X	fX	c.f.	
10–20	15	15	225	15	
20–30	28	25	700	28+15=43	
30–40	45	35	1575	45+43=88	Modal class
40–50	29	45	1305	29+88=117	Median class
50–60	20	55	1100	20+117=137	
Total	Σf=137		4905		

$$\text{Mean} = (\bar{X}) = \frac{\sum fx}{\sum f} = \frac{4905}{137} = 35.8 \approx 36$$

Average sale of the toys is 36.

For median: Here, n = 137, so, $\frac{137}{2} =$ 68.5; 68.5 lies in 30–40.

$\ell = 30$, h = 10, f = 45 , 137, c = 48

$$\text{Median } (\tilde{X}) = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$= 30 + \frac{10}{45}\left(\frac{137}{2} - 43\right)$$

$$= 30 + \frac{10}{45}(68.5 - 43)$$

$$= 30 + \frac{10}{45}(25.5)$$

$$= 30 + 5.7$$

$$\text{Median} = 35.7 \approx 36$$

Thus, median of the sold toys by the factory is 47.07.

For mode:

$\ell = 30$, h = 10, $f_m = 45$, $f_1 = 28$, $f_2 = 29$

$$\text{Mode } (\hat{X}) = \ell + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 30 + \frac{(45-28)}{(45-28)+(45-29)} \times 10$$

$$= 30 + \frac{17}{17+16} \times 10$$

$$= 30 + \frac{17}{33} \times 10$$

$$= 30 + 5.15$$

$$\text{Mode } (X) = 35.15 \approx 35$$

Thus, mode of the sold toys by the factory is 35.

(iii) The modal class of sold toys by the factory is (30–40).

Exercise 12.2

Q.1 Find the arithmetic mean in each of the following:

(i) 4, 6, 10, 12, 15, 20, 25, 28, 30 09312042

Solution:

$x = 4, 6, 10, 12, 15, 20, 25, 28, 30$

No. of observations = n = 9

We know that

$$\overline{X} = \frac{\sum x}{n}$$

$$\overline{X} = \frac{4+6+10+12+15+20+25+28+30}{9}$$

$$\overline{X} = \frac{150}{9}$$

$$\overline{X} = 16.67$$

(ii) 12, 18, 19, 0, −19, −18, −12 09312043

Solution:

X = 12, 18, 19, 0, −19, −18, −12

No. of observations = n = 7

We know that

$$\overline{X} = \frac{12+18+19+0+(-19)+(-18)+(-12)}{7}$$

$$= \frac{49-49}{7}$$

$$\overline{X} = \frac{0}{7}$$

$$\overline{X} = 0$$

(iii) 6.5, 11, 12.3, 9, 8.1, 16, 18, 20.5, 25 09312044

Solution:

X = 6.5, 11, 12.3, 9, 8.1, 16, 18, 20.5, 25

No. of observations = n = 9

We know that

$$\overline{X} = \frac{\sum x}{n}$$

$$\overline{X} = \frac{6.5+11+12.3+9+8.1+16+18+20.5+25}{9}$$

$$\overline{X} = \frac{126.4}{9}$$

$$\overline{X} = 14.04$$

(iv) 8, 10, 12, 14, 16, 20, 22 09312045

Solution:

$\overline{X} = 8, 10, 12, 14, 16, 20, 22$

No. of observations = n = 7

We know that

$$\overline{X} = \frac{\sum x}{n}$$

$$\overline{X} = \frac{8+10+12+14+16+20+22}{7}$$

$$\overline{X} = \frac{102}{7}$$

$$\overline{X} = 14.57$$

Q.2 Following are the heights in (inches) of 12 students. Find the median height. 09312046

55, 53, 54, 58, 60, 61, 62, 56, 57, 52, 51, 63

Solution:

Arranged data:

X = 51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63

No. of observations = n = 12 (Even)

We know that

$$\text{Median}(\tilde{x})$$

$$= \frac{1}{2}\left[\frac{12}{2}\text{th observation} + \frac{12+2}{2}\text{th observation}\right]$$

$$\tilde{X} = \frac{1}{2}[6^{th} \text{ observation} + 7^{th} \text{ observation}]$$

$$\tilde{X} = \frac{1}{2}[56+57]$$

$$\tilde{X} = \frac{1}{2}[113]$$

$$\tilde{X} = 56.5 \text{ inches}$$

Thus median height is 56.5 inches.

Q.3 Following are the earnings (in Rs.) of ten workers.

88, 70, 72, 125, 115, 95, 81, 90, 95, 90.

Calculate

(i) Arithmetic Mean 09312047

Solution:

Arithmetic Mean:

X = 88, 70, 72, 125, 115, 95, 81, 90, 95, 90

No. of observations = n = 10

We know that

$$\overline{X} = \frac{\sum x}{n}$$

$$\overline{X} = \frac{88+70+72+125+115+95+81+90+95+90}{10}$$

$$X = \frac{921}{10} = \text{Rs. } 92.1$$

(ii) Median 09312048

Arranged data

X = 70, 72, 81, 88, 90, 90, 95, 95, 115, 125

No. of observations = n = (Even)

$$\text{Median} = \tilde{X} = \frac{1}{2}\left[\frac{n}{2}\text{th obs.} + \frac{n+2}{2}\text{th obs.}\right]$$

$$\tilde{X} = \frac{1}{2}\left[\frac{10}{2}\text{th obs.} + \frac{10+2}{2}\text{th obs.}\right]$$

$$\tilde{X} = \frac{1}{2}[5\text{th obs.} + 6\text{th obs.}]$$

$$\tilde{X} = \frac{1}{2}[90+90]$$

$$\tilde{X} = \frac{1}{2}(180)$$

$$\tilde{X} = \text{Rs. } 90$$

(iii) Mode

The most repeated values are 90 and 95 so, mode = 90 and 95.

Q.4 The marks obtained by the students in the subject of English are given below.

Marks obtained	15–19	20–24	25–29	30–34	35–39
Frequency	9	18	35	17	5

Find

(i) Arithmetic mean of their marks by direct and short formula. 09312049

Solution:

Marks obtained	(f)	Midpoints (x)	fx
15–19	9	17	153
20–24	18	22	396
25–29	35	27	945
30–34	17	32	544
35–39	5	37	185
	Σf=84		Σfx = 2223

We know that

$$\overline{X} = \frac{\sum fx}{\sum f} = \frac{2223}{84} = 26.46$$

Arithmetic Mean by short formula

Let assumed mean = A = 27

Marks	(f)	Midpoints (x)	D=x–A	fD
15–19	9	17	17–27=–10	–90
20–24	18	22	22–27=–5	–90
25–29	35	27	27–27 = 0	0
30–34	17	32	32–27 = 5	85
35–39	5	37	37–27=10	50

Σf =84 ΣfD = –45

We know that

$$\tilde{X} = A + \frac{\sum fD}{\sum f}$$

$$\tilde{X} = 27 + \frac{-45}{84}$$

$$\tilde{X} = 27 - (0.54)$$

$$\tilde{X} = 26.46$$

(ii) (Median of their marks) 09312050

Solution:

Marks	(f)	Class Boundaries	C.F
15–19	9	14.5–19.5	9
20–24	18	19.5–24.5	9+18=27
25–29	35	24.5–29.5	27+35=62
30–34	17	29.5–34.5	62+17=79
35–39	5	34.5–39.5	79+5=84
	Σf = 84		

Median class = class having $\left(\frac{n}{2}\right)$th value.

= class having $\left(\frac{84}{2}\right)$th value.

= class having 42th value.

Since, 42th value lies in class (24.5–29.5)

We know that

$$\tilde{X} = \ell + \frac{h}{f}\left(\frac{n}{2} - C\right)$$

Put $\ell = 24.5$, h = 5, f = 35, n = 84, c = 27

$$\tilde{X} = 24.5 + \frac{5}{35}(42-27)$$

$$\tilde{X} = 24.5 + \frac{5\times15}{35}$$

$$\tilde{X} = 24.5 + \frac{75}{35}$$

= 24.5+2.14

= 26.64

Q.5 Given below is a frequency distribution. 09312051

Class Interval	5–9	10–14	15–19	20–24	25–29
Frequency	1	8	18	11	2

Find the mode of the frequency distribution.

Solution:

Class intervals	(f)	Class Boundaries
5–9	1	4.5–9.5
10–14	$8\rightarrow f_1$	9.5–14.5
$\boxed{15\text{–}19}$	$18\rightarrow f_m$	14.5–19.5
20–24	$11\rightarrow f_2$	19.5–24.5
25–29	2	24.5–29.5

Modal class is a class having maximum frequency so, modal class = (14.5–18.5)

We know that

$$\text{Mode}(\hat{X}) = \ell + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$\text{Mode } \hat{X} = 14.5 + \frac{(18-8)\times5}{(18-8)+(18-11)}$$

$$\hat{X} = 14.5 + \frac{10\times5}{10+7}$$

$$\hat{X} = 14.5 + \frac{50}{17} = 14.5+2.94 = 17.44$$

Q.6 (a) Ten boys work on a petrol pump station. They get weekly wages as follows: 09312052

Wages (in Rs.) 4250, 4350, 4400, 4250, 4350, 4410,4500, 4300, 4500, 4390. Find the arithmetic mean by short formula, median and mode of their wages.

Solution:

Arranged data

X = 4250, 4250, 4300, 4350, 4350, 4390, 4400, 4500, 4500

No. of obs = n = 10

(a) Arithmetic mean short formula

Let assumed mean = A = 4350

X	$D = x - A$
4250	4250–4350 = –100
4250	4250–4350 = –100
4300	4300–4350 = –50
4350	4350–4350 = 0
4350	4350–4350 = 0
4390	4390–4350 = 40
4400	4400–4350 = 90
4410	4410–4350 = 60
4500	4500–4350 = 150
4500	4500–4350 = 150
	$\Sigma = 200$

We know

$$\bar{X} = A + \frac{\Sigma D}{n}$$

$$\bar{X} = 4350 + \frac{200}{10} = 4350+20$$

$$\bar{X} = 4370$$

(b) Median 09312053

Solution:

Arranged data

X = 4250, 4250, 4300, 4350, 4350, 4390, 4400, 4410, 4500, 4500.

No. of observation that = n = 10 (Even)

We know that

$$\widetilde{X} = \frac{1}{2}\left[\left(\frac{10}{2}\right)\text{th value} + \left(\frac{n+2}{2}\right)\text{th value}\right]$$

$$\widetilde{X} = \frac{1}{2}\left[\left(\frac{10}{2}\right)\text{th value} + \left(\frac{10+2}{2}\right)\text{th value}\right]$$

$$\widetilde{X} = \frac{1}{2}[5^{th}\text{ value} + 6^{th}\text{ value}]$$

$$\widetilde{X} = \frac{1}{2}[4350 + 4390]$$

$$\widetilde{X} = \frac{1}{2}[8{,}740]$$

$$\widetilde{X} = 4{,}370$$

(C) Mode:

Since, most repeated value of data are 4250, 4350 and 4500, so

Mode = 4250, 4350 and 4500.

Q.7 The arithmetic mean of 45 numbers is 80. Find their sum. **09312054**

Solution:

No. of obs. N = 45

Arithmetic mean = $\overline{X}$ = 80

Sum = Σx = ?

We know that

$$\overline{X} = \frac{\Sigma x}{n}$$

$$80 = \frac{\Sigma x}{45}$$

$3,600 = \Sigma x$

$3,600 = \Sigma x$

$\Rightarrow \Sigma x = 3,600$

Q.8 Five numbers are 1,4,0,7,9. Find their mean, median and mode. **09312055**

Solution

Data = x = 1, 4, 0, 9, 7

(a) Mean: x = 1, 4, 0, 7, 9

n = 5

$$\overline{X} = \frac{\Sigma x}{n} = \frac{1+4+0+7+9}{5} = \frac{21}{5} = 4.2$$

(b) Median: Arranged data = 0, 1, 4, 7, 9

No. of obs. = n = 5 (odd)

$$\text{Median}(\widetilde{X}) = \frac{n+1}{2}\text{th observation}$$

$$= \frac{5+1}{2}\text{th observation}$$

$$= \frac{6}{2}\text{th observation}$$

$$= 3^{rd}\text{ observation}$$

Since, 3^{rd} observation in data set is 4.

Median ($\widetilde{X}$) = 4

(c) Mode

Data have no mode

Q.9 A set of data contains the values as 148, 145, 160. 157, 156, 160. Show that Mode > Median > Mean. **09312056**

Solution:

Data: X = 148, 145, 160, 157, 156, 160

No. of observations = n = 6

(a) Mean

$$\overline{X} = \frac{\Sigma x}{n}$$

$$\overline{X} = \frac{148+145+160+157+156+160}{6}$$

$$\overline{X} = \frac{926}{6}$$

$\overline{X}$ = 154.33 _______ (i)

(b) Median

Arranged data

X = 145, 148, 156, 157, 160, 160

No. of obs. = n = 6 (Even)

$$\widetilde{X} = \frac{1}{2}\left[\frac{n}{2}\text{th obs.} + \frac{n+2}{2}\text{th obs.}\right]$$

$$\widetilde{X} = \frac{1}{2}\left[\frac{6}{2}\text{th obs.} + \frac{6+2}{2}\text{th obs.}\right]$$

$$\widetilde{X} = \frac{1}{2}[3^{rd}\text{ obs.} + 4^{th}\text{ obs.}]$$

$$\widetilde{X} = \frac{1}{2}[156+157] = \frac{1}{2}[313]$$

$\widetilde{X}$ = 156.5 ________ (ii)

(c) Mode

The most repeat value is 160 so

Mode = 160 _________ (iii)

From (i), (ii) and (iii) 160 > 156.5>154.3 i.e. Mode > Median > Mode

Q.10 The monthly attendance of 10 students for their lunch in the hostel is recorded as: 21, 15, 16, 18, 14, 17, 15, 12, 13, 11. Find the median and mode of the attendance. Also find the mean if D=A−20.

09312057

Solution:

Data: x = 21, 15, 16, 18, 14, 17, 15, 12, 13, 11

No. of observations = n = 10

$$D = x - 20$$

Mean by short cut formula: $D = x - A$

$$\Rightarrow A = 20$$

X	D = x−20
11	11−20 = −9
12	12−20 = −8
13	13−20 = −7
14	14−20 = −6
15	15−20 = −5
15	15−20 = −5
16	16−20 = −4
17	17−20 = −3
18	18−20 = −2
21	21−20 = 1

$\Sigma D = -48$

We know that

$$\overline{X} = A + \frac{\Sigma D}{n}$$

$$\overline{X} = 20 + \left(\frac{-48}{10}\right)$$

$$\overline{X} = 20 - 4.8 = 15.2$$

$$\overline{X} = 15.2$$

(b): Median:

Arranged data:

X= 11,12,13,14,15,15,16,17,18,21

Number of observations = X =10 (even)

$$\text{Median}(\tilde{x}) = \frac{1}{2}\left[\frac{n}{2}\text{th value} + \frac{n+2}{2}\text{th value}\right]$$

$$= \frac{1}{2}\left[\frac{10}{2}\text{th value} + \frac{10+2}{2}\text{th value}\right]$$

$$= \frac{1}{2}[5\text{th value} + 6\text{th value}]$$

$$= \frac{1}{2}(15 + 15)$$

$$= \frac{1}{2}(30)$$

$$= 15$$

Thus median is 15.

(C) Mode:

Since, most frequent value is 15.

So, mode = 15.

Q.11 On a prize distribution day, 50 students brought pocket money as under:

Rupees	5–10	10–15	15–20	20–25	25–30
Frequency (f)	12	9	18	7	4

(i) Find the median and mode of the above data.

09312058

(ii) Find the arithmetic mean of the data given above using coding method.

09312059

Solution

(i) Median and mode

(a) Median

Rupees	(f)	C.F	
5–10	12	12	
10–15	9	12+9= 21⇒C	
15–20	f⇒18	21+18 = 39	Median Group
20–25	7	39+7 = 46	
25–30	4	46+4 = 50	

Σf= 50

Since, the class containing $\frac{50}{2}$th value = 25[th] value is a median class.

Median class = (15–20)

We know that

$$\text{Median}(\tilde{X}) = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$\tilde{X} = 15 + \frac{5}{18}\left(\frac{50}{2} - 21\right)$$

$$\tilde{X} = 15 + \frac{5}{18}(25-21) = 15 + \frac{5}{18}(24)$$

$$\tilde{X} = 15 + \frac{20}{18}$$

$$\tilde{X} = 16.11$$

(b) Mode

Rupees	(f)	
5–10	12	
10–15	$f_1 \to 9$	
15–20	$f_m \to 18$	Modal class
20–25	$f_2 \to 7$	
25–30	4	

Since, group having maximum frequency is a modal group. So modal class = (15–20)

We know that

$$\text{Mode}(\hat{X}) = \ell + \frac{(f_m - f_1) \times h}{(f_m - f_1) + (f_m - f_2)}$$

$$\hat{X} = 15 + \frac{(18-9) \times 5}{(18-9) + (18-7)}$$

$$\hat{X} = 15 + \frac{9 \times 5}{9 + 11}$$

$$\hat{X} = 15 + \frac{45}{20}$$

$$\hat{X} = 15 + 2.25$$

$$\hat{X} = 17.25$$

(ii) Arithmetic Mean by using coding

Let assumed mean = A = 17.5

Coding variable = $U = \frac{D}{h}$, Here, D = x−A, h = size of class interval.

Rupees	(f)	Midpoints (x)	$U = \frac{D}{h} = \frac{x-A}{h}$	U	fU
5–10	12	7.5	$\frac{7.5-17.5}{5} = \frac{-10}{5} =$	−2	−24
10–15	9	12.5	$\frac{12.5-17.5}{5} = \frac{-5}{5} =$	−1	−9
15–20	18	17.5	$\frac{17.5-17.5}{5} = \frac{0}{5} =$	0	0
20–25	7	22.5	$\frac{22.5-17.5}{5} = \frac{5}{5} =$	1	7
25–30	4	27.5	$\frac{27.5-17.5}{5} = \frac{10}{5} =$	2	8
	$\Sigma f = 50$				−18

We know that

$$\bar{X} = A + \frac{\Sigma fU}{\Sigma f} \times h$$

$$\bar{X} = 17.5 + \left(\frac{-18}{50}\right) \times 5$$

$$\bar{X} = 17.5 + \left(\frac{-90}{50}\right)$$

$$\bar{X} = 17.5 + (-1.8)$$

$$\bar{X} = 17.5 - 1.8$$

$$\bar{X} = 15.7$$

Q.12 The arithmetic mean of the ages of 20 boys is 13 years, 4 months and 5 days. Find the sum of heir ages. If one of the boys is of age exactly 15 years. What is the average of he remaining boys. **09312060**

Solution

No. of boys = n = 20

Mean of ages = $\bar{X}$ = 13y, 4 months, 5 days

Sum of ages = Σx = ?

We know that

$\overline{X} = \frac{\sum x}{n}$ $\begin{bmatrix} 1\text{month} = 30\text{Days} \\ 1\text{year} = 12\text{Months} \\ 1\text{year} = 365\text{Days} \end{bmatrix}$

$\Rightarrow n \times \overline{X} = \Sigma x$

$\Rightarrow \Sigma x = 20 \times (13y + 4M + 5D)$

$= 260y + 80M + 3M + 10D$

$= 260y + 83M + 10D$

$= 260y + 6y + 11M + 10D$

$= 266y + 11M + 10D$

Thus the sum of ages of 20 boys is 266y, 11m, 10 days.

If age of boys is 15 year, then

Sum of ages of

19 boys $= \Sigma Z = 266y + 11M + 10D - 15y$

$\Sigma Z = 251y + 11M + 10D$

$\Sigma Z = (251 \times 365)D + (11 \times 30)D + 10D$

$= (91{,}615D + 330D + 10D)$

$\Sigma Z = 91{,}955D$

Average of ages of 19 boys

$\overline{Z} = \frac{\Sigma Z}{19} = \frac{91{,}955}{19}D$

$= 4{,}839.74D$

$\overline{Z} = \frac{4{,}839.74}{365} = 13.26y$ (1year = 365 Days)

$\overline{Z} = 13y + 0.26y$

$\overline{Z} = 13y + (0.26y \times 12)M$

$\overline{Z} = 13y + 3.12M$

$\overline{Z} = 13y + 3M + 0.12M$

$\overline{Z} = 13y + 3M + (0.12 \times 30)D$

$\overline{Z} = 13y + 3M + 3.6D$

$\overline{Z} = 13y + 3M + 4D \quad (\because 3.6 \approx 4)$

Thus average of ages of 19 boys is 13 years, 3 months, 4D approximately.

Q.13 Calculate the arithmetic mean from the following information: 09312061

(i) If D = X − 140, ΣD = 500 and n = 10

Solution:

$D = X - 140$, $\Sigma D = 500$ and $n = 10$

We know that

$\overline{X} = A + \frac{\Sigma D}{n}$

$\because D = x - A$

$D = x - 140$

$\Rightarrow A = 140$

$\overline{X} = 140 + \frac{500}{10}$

$\overline{X} = 140 + 50$

$\overline{X} = 190$

(ii) If $U = \frac{x-130}{6}$, ΣU = − 150 and n = 15

09312062

Solution:

$U = \frac{x-130}{6}$, $\Sigma U = -150$ and $n = 15$

We know that

$U = \frac{x-A}{h}$

$U = \frac{x-130}{6}$

By comparing we get

$A = 130$, $h = 6$

We also know that

$\overline{X} = A + \frac{\Sigma U}{n} \times h$

Putting the values

$\overline{X} = 130 + \frac{-150}{15} \times 6$

$\overline{X} = 130 - 60 \Rightarrow \overline{X} = 70$

(iii) If D = x − 25, ΣfD = 300 and Σf = 20

09312063

Solution:

$D = x - 25$, $\Sigma fD = 300$ and $\Sigma f = 20$

Since, $D = x - A$,

Given that, $D = x - 25$

$\Rightarrow A = 25$

We know that

$\overline{X} = A + \frac{\Sigma fD}{\Sigma f}$

$\overline{X} = 25 + \frac{300}{20}$

$\overline{X} = 25 + 15$

$\overline{X} = 40$

(iv) If $U = \frac{x-120}{5}$, ΣfU = 60 and Σf = 100

Solution:

09312064

$U = \frac{x-120}{5}$, $\Sigma fU = 60$ and $\Sigma f = 100$

We know that $U = \frac{x - A}{h}$

Given that $U = \frac{x - 120}{5}$

By comparing we get

$A = 120$ and $h = 5$

We also know that

$$\overline{X} = A + \frac{\sum fU}{\sum f} \times h$$

$$\overline{X} = 120 + \frac{60}{100} \times 5$$

$$\overline{X} = 120 + \frac{300}{100}$$

$$\overline{X} = 120 + 3$$

$$\overline{X} = 123$$

Q.14 The three children Haris, Maham and Minal made the following scores in a game conducted by a group of teachers in the school. 09312065

Haris scores	50	55	70	85	90
Maham scores	75	60	60	45	53
Minal scores	80	77	66	42	48

It is decided that the candidate who gets the highest average score will be awarded rupees 1000. Who will get the awarded amount?

Solution:

No. of games: $n = 5$

Average of Haris scores:

$x = 50, 55, 70, 85, 90$

$n = 5$

$$\overline{X} = \frac{\sum x}{n} = \frac{50+55+70+85+90}{5}$$

$$\overline{X} = \frac{350}{5} = 70$$

$\overline{X} = 70$ ________ (i)

Average of Mahm's scores

$y = 75, 60, 60, 45, 53$

$n = 5$

$$\overline{Y} = \frac{\sum x}{n} = \frac{75+60+60+45+53}{5}$$

$$\overline{Y} = \frac{293}{5} = 58.6$$

$\overline{Y} = 58.6$ ________ (ii)

Average of Minal scores

$Z = 80, 77, 66, 42, 48$

$n = 5$

$$\overline{Z} = \frac{\sum x}{n}$$

$$\overline{Z} = \frac{80+77+66+42+48}{5}$$

$= \frac{313}{5} = 62.6$ ____ (ii)

From (i) (ii) and (iii) we observe that Average score of Haris is highest so Haris will get award of Rs. 1000.

Q.15 Given below is frequency distribution derived by making a substitution as D = X –20. Calculate the arithmetic mean. 09312066

Solution

$D = X - A$ (known)

$D = X - 20$ (Given)

D	f	fD
–6	1	–6
–4	3	–12
–2	6	–12
0	20	0
2	26	52
4	12	48
6	2	12
	$\Sigma f = 70$	$\Sigma fD = 82$

We know that

$$\overline{X} = A + \frac{\sum fD}{\sum f}$$

$$\overline{X} = 10 + \left(\frac{82}{70}\right)$$

$$\overline{X} = 20 + 1.17$$

$$\overline{X} = 21.17$$

Q.16 Being partners Hafsa and Fatima took part in a quiz programme. They made the following number of points 45, 51, 58, 61,74, 48, 46 and 50. Compute the average number of points using deviation D = *x* – 58. 09312067

Solution:

X= 45, 51, 58, 61, 74, 48, 46, 50

No. of obs.= n = 8

Given that D = X–58

We know that D = X–A

$\Rightarrow$ A = 58

X	D = X–A
45	45–58 = –13
51	51–58 = –7
58	58–58 = 0
61	61–58 = 3
74	74–58 = 16
48	48–58 = –10
46	46–58 = –12
50	50–58 = –8

$\Sigma D = -31$

We know that

$$\overline{X} = A + \frac{\Sigma D}{n}$$

$$\overline{X} = 58 + \frac{(-31)}{8}$$

$$\overline{X} = 58 + 3.85$$

$$\overline{X} = 54.125 \approx 54.13$$

Q.17 A person purchased the following food items: 09312068

Found item	Quantity (in Kg)	Cost per Kg (in Rs.)
Rice	10	96
Flour	12	48
Ghee	4	190
Sugar	3	49
Mutton	2	650

What is the weighted mean of cost of food items per kg?

Solution:

Food item	Quantity (in kg)W	Cost per kg (in Rs.) *X*	WX
Rice	10	96	960
Flour	12	48	576
Ghee	4	190	760
Sugar	3	49	147
Mutton	2	650	1300
	$\Sigma W = 31$		ΣWX=3,743

We know that weighted mean

$$\overline{X}w = \frac{\Sigma wx}{\Sigma w} = \frac{3,743}{31} = 120.74$$

Thus cost of food item per kg is Rs.120.74

Q.18 For the following data, find he weighted mean. 09312069

Solution:

Item	Quantity (W)	Cost of item (in thousands)
Washing Machine	5	35
Heater	3	5
Stove	2	13
Dispenser	6	18

Solution:

Weighted Mean

Item	Quantity (W)	Cost of item in thousand	WX
Washing machine	5	35	175
Heater	3	5	15
Stove	2	13	26
Dispenser	6	18	108
	$\Sigma W = 16$		ΣWX=324

We know that

$$\overline{X}_w = \frac{\Sigma wx}{\Sigma w}$$

$$\overline{X}_w = \frac{324}{16}$$

$\overline{X}_w = 20.25$ (in thousands)

Q.19 A company is planning its next year marketing budget across five years: yearly budgets (in million) are: 5, 7, 8, 6, 7. Find the average budget for the next year. 09312070

Solution:

$x = 5, 7, 8, 6, 7$ (in millions)

No. of observations n = 5

We know that

$$\overline{X} = \frac{\sum x}{n}$$

$$\overline{X} = \frac{5+7+8+6+7}{5} = \frac{33}{5} = 6.6$$

$\overline{X} = 6.6$ (in millions)

Thus the average budgets is 6.6 millions for next year.

Q.20 Ahmad obtained the following marks in a certain examination. Find the weighted mean if weights 5, 4, 2, 3, 2,4 respectively are allotted to the subjects.

$\overline{X}_w = 76.9$

09312071

Solution

Urdu	Eng.	Sci.	Math	Isl.	Comp.
78	65	80	90	85	72

Subjects	Marks (x)	Weights	Wx
Urdu	78	5	390
English	65	4	260
Science	80	2	160
Math	90	3	70
Islamyat	85	2	170
Computer	72	4	288
		ΣW = 20	ΣWX=1538

We know that

$$\overline{X}_w = \frac{\sum WX}{\sum w}$$

$$\overline{X}_w = \frac{1538}{20}$$

$\overline{X}_w = 76.9$

Review Exercise -12

Q.1 Choose the correct option.

i. Which data takes only some specific values? 09312072
(a) Continuous data (b) Discrete data
(c) Grouped data (d) Ungrouped data

ii. The number of times a value occurs in a data is called: 09312073
(a) Frequency (b) Relative frequency
(c) Class limit (d) Class mark

iii. Midpoint is also known as: 09312074
(a) Mean (b) Median
(c) Class limit (d) Class mark

iv. Frequency polygon is also drawn / constructed by using: 09312075
(a) Histogram (b) Bar graph
(c) Class boundaries
(d) Class limit

v. The difference between the greatest value and the smallest value is called: 09312076
(a) Class limits (b) Midpoint
(c) Relative frequency
(d) Range

vi. Measure of central tendency is used to find out the __________ of a data set. 09312077
(a) Class boundaries
(b) Cumulative frequency
(c) Middle or centre value
(d) Frequency

vii. If the mean of 5, 7, 8, 9 and x is 7.5, what will be the value of x? 09312078

(a) 10 (b) 8
(c) 8.5 (d) 5.8

viii. Find the mode of the given data: 2,5, 8, 9, 0, 1, 3, 7 and10. 09312079

(a) 5 (b) 7
(c) 0 (d) No mode

ix. In a data the values (observations) which appears or occurs most often is called: 09312080

(a) Mean (b) Mode
(c) Median (d) Weighted mean

x. Find the median of the given data: 110, 125, 122, 130, 124, 127 and 120: 09312081

(a) 124 (b) 120
(c) 125 (d) 127

Answers Key

i	b	ii	a	iii	d	iv	a	v	d
vi	c	vii	c	viii	d	ix	b	x	a

Multiple Choice Questions (Additional)

Frequency distribution

1. A data in the form of frequency distribution is also called: 09312082

(a) Grouped data (b) Ungrouped data
(c) Raw data (d) Dispersed data

2. The size of class interval (6–10) is. 09312083

(a) 4 (b) 5
(c) 8 (d) 10

3. The midpoint or class mark of the group (6 – 10), is: 09312084

(a) 4 (b) 6
(c) 8 (d) 10

4. A cumulative frequency means -----of frequencies. 09312085

(a) sum (b) difference
(c) product (d) quotient

Graphs of frequency distribution

5. A histogram is a graph of ----rectangles:

(a) adjacent (b) non-adjacent
(c) Parallel (d) equal height

09312086

6. A frequency polygon is geometrically: 09312087

(a) closed figure (b) open figure
(c) straight (d) curved

7. In a frequency polygon frequencies are plotted against: 09312088

(a) midpoints (b) class limits
(c) class boundaries (d) size of classes

Arithmetic Mean

8. The sum of all values divided by number of values is called: 09312089

(a) Mean (b) Median
(c) Mode (d) Range

9. Direct formula to find mean from ungrouped data. 09312090

(a) $\overline{X}=\frac{\sum X}{n}$ (b) $\overline{X}=\frac{\sum fX}{\sum f}$

(c) $\overline{X}=A+\frac{\sum D}{n}$ (d) $\overline{X}=A+\frac{\sum fD}{\sum f}$

10. Direct formula to find mean from grouped data is: 09312091

(a) $\overline{X}=\frac{\sum X}{n}$ (b) $\overline{X}=\frac{\sum fX}{\sum f}$

(c) $\overline{X}=A+\frac{\sum D}{n}$ (d) $\overline{X}=A+\frac{\sum fD}{\sum f}$

11. Short formula to find mean from ungrouped data is: 09312092

(a) $\overline{X}=\frac{\sum X}{n}$ (b) $\overline{X}=\frac{\sum fX}{\sum f}$

(c) $\overline{X}=A+\frac{\sum D}{n}$ (d) $\overline{X}=A+\frac{\sum fD}{\sum f}$

12. Short formula to find mean from grouped data is: 09312093

(a) $\overline{X}=\frac{\sum X}{n}$ (b) $\overline{X}=\frac{\sum fX}{\sum f}$

(c) $\overline{X}=A+\frac{\sum D}{n}$ (d) $\overline{X}=A+\frac{\sum fD}{\sum f}$

13. A deviation is a difference of any value of the variable from a: 09312094

(a) constant (b) variable
(c) sum (d) zero

Median

14. The middlemost observation in arranged data set is called: 09312095

(a) median (b) mode
(c) mean (d) range

15. The arrangement of data is necessary to find the value of: 09312096

(a) Mean (b) Median
(c) Mode (d) Range

16. Median from the data 1,4,0,7 and 9 is: 09312097

(a) 0 (b) 4
(c) 5 (d) 7

Mode

17. The observation that occurs most often is called: 09312098

(a) median (b) mode
(c) mean (d) range

18. The class having maximum frequency is called ……….class. 09312099

(a) Modal (b) Median
(c) Lower (d) Upper

Weighted mean

19. When all observations are not of equal importance then we find: 09312100

(a) Mean (b) Median
(c) Mode (d) weighted mean

20. Weighted mean $\overline{X}_w$ = – – – 09312101

(a) $\frac{\sum W}{\sum WX}$ (b) $\frac{\sum WX}{n}$

(c) $\frac{\sum X}{n}$ (d) $\frac{\sum WX}{\sum W}$

Answer Key

1	a	2	b	3	c	4	a	5	a	6	a	7	a	8	a	9	a	10	b
11	c	12	d	13	a	14	a	15	b	16	b	17	b	18	a	19	d	20	d

Q.2 Define the following:

(i) Frequency distribution 09312102

(ii) Histogram (unequal class limits) 09312103

(iii) Mean 09312104

(iv) Median 09312105

Q.3 Following are the weights of 40 students recorded to the nearest (Ibs). 138, 164, 150, 132, 144, 125, 149, 157, 146, 158,140, 147, 136, 148, 152, 144, 168, 126, 138, 176, 163, 119, 154, 165, 146, 173, 142, 147, 135, 153, 140, 135, 161, 145, 135, 142, 150, 156, 145, 128, make a frequency table taking size of class limits as 10. Also draw histogram and frequency polygon of the given data. 09312106

Solution:

(a) No. of observations = n =40

Smallest value = 119

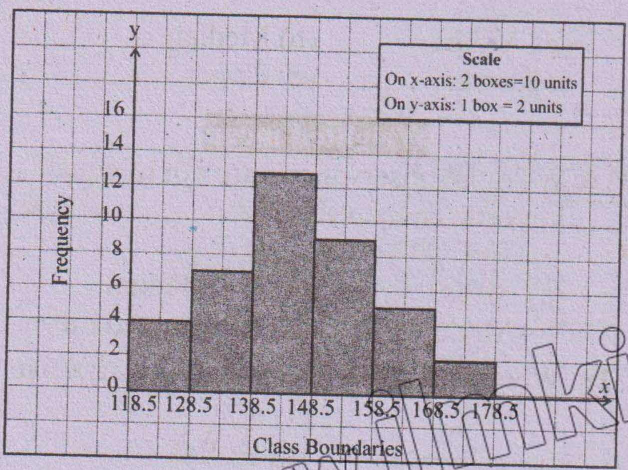
Largest value = 176

Size of class limits = 10

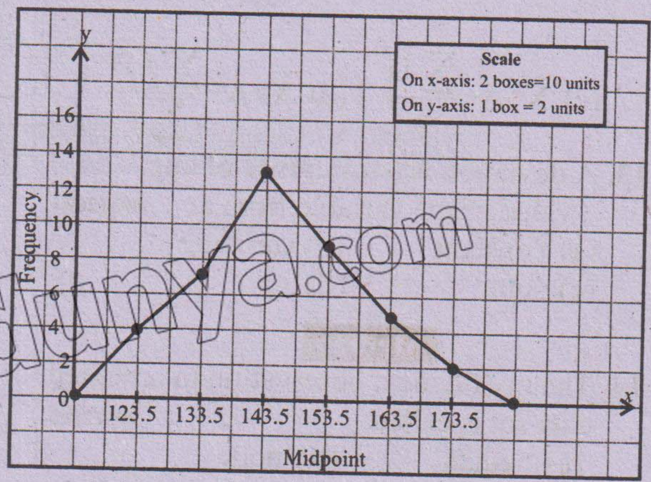
Frequency table taking size of class limits as 10.

Class limits	Tally marks	f	C.B	Midpoints
119–128	||||	4	118.5–128.5	123.5
129–138	卌 ||	7	128.5–138.5	133.5
139–148	卌 卌 |||	13	138.5–148.5	143.5
149–158	卌 ||||	9	148.5–158.5	153.5
159–168	卌	5	158.5–168.5	163.5
169–178	||	2	168.5–178.5	173.5

(b) Histogram of weight 40 students



(c) Frequency Polygon of weight of 40 students



Q.4 From the table given below. Draw a frequency polygon on histogram for the given frequency distribution.

09312107

Weight (kg)	50–56	57–59	60–64	65–72	73–75	76–80
Frequency (f)	25	32	40	30	15	8

Solution:

Weight (kg)	(f)	Size of class (h)	Height of rectangle adjusted frequency f/h	C.B.
50–56	25	7	$25 \div 7 = 3.6$	49.5–56.5
57–59	32	3	$32 \div 3 = 10.7$	56.5–59.5
60–64	40	5	$40 \div 5 = 8$	59.5–64.5
65–72	30	8	$30 \div 8 = 3.8$	64.5–72.5
73–75	15	3	$15 \div 3 = 5$	72.5–75.5
76–80	8	5	$8 \div 5 = 1.6$	75.5–80.5

Title: Frequency polygon on histogram

Q.5 Given below are marks obtained by 45 students in the monthly test of Biology. 09312108

Marks	20–24	25–29	30–34	35–39	40–44	45–49
No. of students	05	08	12	15	03	02

With reference to the above table find the following:

(i) upper class boundary of the 5th class. 09312109

(ii) lower class boundaries of all the classes. 09312110

(iii) midpoint of all the classes 09312111

(iv) the class interval with the least frequency. 09312112

Solution:

Marks	No. of students (f)	Midpoints (x)	C.B
20–24	5	22	19.5–24.5
25–29	8	27	24.5–29.5
30–34	12	32	29.5–34.5
35–39	15	37	34.5–39.5
40–44	3	42	39.5–44.5
45–49	2	47	44.5–49.5

Answers:

(i) The upper class boundary of 5th class is 44.5.

(ii) The lower class boundaries of all classes are 19.5, 24.5, 29.5, 34.5, 39.5 and 44.5 respectively.

(iii) The midpoints of all classes are 22,27, 32, 37, 42 and 47 respectively. (see column 3)

(iv) The classes interval with least frequency is (45–49).

Q.6 Given below is frequency distribution. 09312113

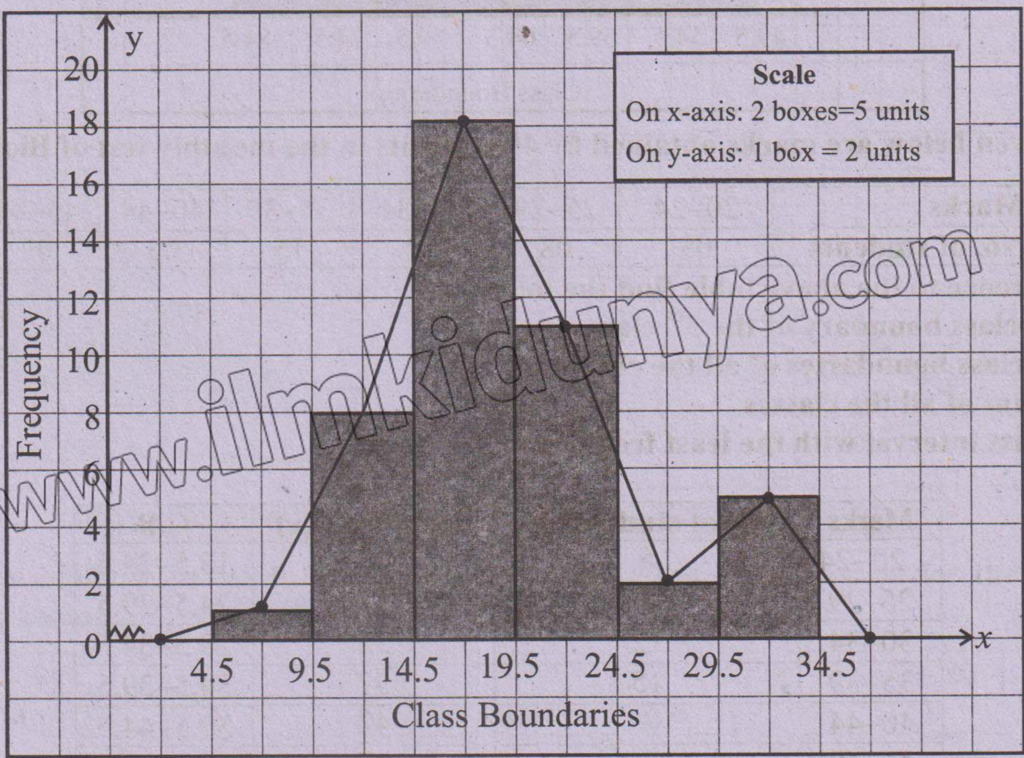
Draw frequency polygon and histogram for the distribution.

Class limits	5–9	10–14	15–19	20–24	25–29	30–34
Frequency	1	8	18	11	2	5

Solution:

Class lmits	f	Midpoints (x)	C.B
5–9	1	7	4.5–9.5
10–14	8	12	9.5–14.5
15–19	18	17	14.5–19.5
20–24	11	22	19.5–24.5
25–29	2	27	24.5–29.5
30–34	5	32	29.5–34.5

Histogram and frequency polygon.



Q.7 For the following data, find the weighted mean. 09312114

Item	Quantity	Cost of item (Rs.)
Chair	20	500
Table	20	400
Black board	10	750
Tube light	25	230
Cupboard	09	950

Solution:

Items	Quantity W	Cost of item S(Rs.)	WX
Chair	20	500	10,000
Table	20	400	8,000
Black board	10	750	7,500
Tube light	25	230	5,750
Cupboard	09	950	8,550
	$\Sigma W = 84$		$\Sigma WX =$ 39,800

We know that

$$\overline{X}_w = \frac{\sum WX}{\sum W} = \frac{39,800}{84} = 473.81$$

Thus the average price of each item is Rs.473.81.

Q.8 A principal of a school allocates funds of Rs.50,000 to five different sectors:

(i) chair Rs.15,000 **09312115**

(ii) tables Rs.12,000 **09312116**

(ii) black boards: Rs.6,000 **09312117**

(iv) room renovation: Rs.10,000 **09312118**

(v) Gardening: Rs. 7,000 **09312120**

Find the average of funds allocation in each sector of the school.

Solution:

Sectors	Allocated Fund (Rs.)
Chairs	15,000
Tables	12,000
Black boards	6,000
Room renovation	10,000
Gardening	7,000
Total	Rs.50,000

No. of sectors = 5

Total amount = Rs. 50,000

$$\overline{X} = \frac{\sum W}{n} = \frac{50,000}{5} = \text{Rs.}10,000$$

The average of fund allocation in each sector is Rs.10,000/-.

Q.9 The marks of a student Saad in six test were 84,91,72,68,87,78. Find the arithmetic mean of this marks. **09312121**

Solution:

X = 84, 91, 72, 68, 87, 78

No. of tests = n = 6

We know that

$$\overline{X} = \frac{\sum x}{n}$$

$$\overline{X} = \frac{84+91+72+68+87+78}{6}$$

$$\overline{X} = \frac{480}{6} = 80$$

$\overline{X}$ = 80 marks

Q.10 Adjoining distribution showed maximum load (in kg) supported by certain ropes. Find the mean load using short method. **09312122**

Max-Load kg	93–97	98–102	103–107	108–112	113–117	118–122
No. of ropes	2	5	8	12	6	2

Solution:

Let assumed mean = A = 100

Max. Load kg	No. of ropes (f)	Midpoints (x)	D = x–A	fD
93–97	2	95	95–100 = –5	–10
98–102	5	100	100–100 = 0	0
103–107	8	105	105–100 = 5	40
108 – 112	12	110	110–100 = 10	120
113–117	6	115	115–100 = 15	90
118–122	2	120	120–100 = 20	40
	Σw = 35			Σ*f*D = 280

We know that

$$\overline{X} = A + \frac{\sum fD}{\sum f} = 100 + \frac{280}{35} = 100 + 8$$

$\overline{X} = 108$ kg

Thus average load supported by certain ropes is 108kg.

Q.11 Usman rolled a fair dice eight times. Each time their sum was recorded as 8, 5, 6, 6, 9, 4, 3, 11. Find the median and mode of the sum. 09312123

Solution:

(a) Median ($\overline{X}$)

Arranged data:

X = 3, 4, 5, 6, 6, 8, 9, 11

No. of obs. = n = 8 (Even)

$$\widetilde{X} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{obs.} + \left(\frac{n+2}{2}\right)^{th} \text{obs.}\right]$$

$$\widetilde{X} = \frac{1}{2}\left[\left(\frac{8}{2}\right)^{th} \text{obs.} + \left(\frac{8+2}{2}\right)^{th} \text{obs.}\right]$$

$$\widetilde{X} = \frac{1}{2}[4^{th} \text{ obs.} + 5^{th} \text{ obs.}]$$

$$\widetilde{X} = \frac{1}{2}[6+6]$$

$$\widetilde{X} = \frac{1}{2}[12]$$

$$\widetilde{X} = 6$$

(b) Mode

Since, most repeated value is 6 so mode = 6.

Q.12 Two partners Mr.Aslam and Mrs. Kalsoom run a company. In the following data the weekly wages (in Rs.) of employees who work in the company are given: 09312124

Wages(Rs.)	600–700	700–800	800–900	900–1000	1000-1100
Employees	3	5	7	21	11

Find mean, median and mode of data. (correction)

Solution: 09312125

(a) Mean

Wages (Rs.)	Employees (f)	Midpoints (x)	fx
600–700	3	650	1,950
700–800	5	750	3,750
800–900	7	850	5,950
900–1000	21	950	19,950
1000–1100	11	1050	11,550
	$\Sigma f = 47$		$\Sigma fx = 43,150$

We know that

$$\overline{X} = \frac{\Sigma fx}{\Sigma f} = \frac{43,150}{47} = 918.09$$

(b) Median

Wages (Rs.)	Employees	Cummulative Frequency (C.F)
600–700	3	3
700–800	5	3+5 = 8
800–900	7	3+5 = 15 $\rightarrow$C

900–1000	$f \to 21$	$15 + 21 = 36$	Median group
1000–1100	11	$36 + 11 = 47$	

Since, class containing $\frac{n}{2}$th $= \left(\frac{47}{2}\right)$th observations $= 23.5^{th}$

Observation is a Median class, so median class (900–1000)
We know that

$$\tilde{X} = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

Put $\ell = 900$, $h = 100$, $f = 21$, $c = 15$

$$\tilde{X} = 900 + \frac{100}{21}\left[\frac{47}{2} - 15\right]$$

$$\tilde{X} = 900 + \frac{100}{21}(8.5)$$

$$\tilde{X} = 900 + \frac{850}{21}$$

$$\tilde{X} = 900 + 40.48$$

$$\tilde{X} = 940.48$$

(c) Mode

Wages (Rs.)	Employees (f)	
600–700	3	
700–800	5	
800–900	$f_1 \to 7$	
$\ell \to$ 900 – 1000	$f_m \to 21$	Modal group
1000–1000	$f_2 \to 11$	

Since the class (900–1000) has maximum frequency so modal class is (900–1000)
We know that

$$\hat{X} = \ell + \frac{(f_m - f_1) \times h}{(f_m - f_1) + (f_m - f_2)}$$

Put $\ell = 900$, $f_m = 21$, $f_1 = 7$, $f_2 = 11$ and $h = 100$

$$\hat{X} = 900 + \frac{(21-7) \times 100}{(21-7) + (21-11)}$$

$$\hat{X} = 900 + \frac{14 \times 100}{14 + 10}$$

$$\hat{X} = 900 + \frac{1400}{24}$$

$$\hat{X} = 900 + 58.33$$

$$\hat{X} = 958.33$$