UNIT 13

Probability

Introduction

Probability is the chance of occurrence of particular event.

Probability is calculated by using the given formula:

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

It is written as: $P(A) = \frac{n(A)}{n(S)}$

P(A) = Probability of an event A

n(A) = Number of favourable outcomes

n(S) = Total number of possible outcomes

Basic Concepts of Probability

Experiment: The process which generates results e.g. tossing a coin, rolling a dice, etc. is called an experiment.

Outcomes: the result of an experiment are called outcomes e.g., the possible outcomes of tossing a coin are head or tail, the possible outcomes of rolling a dice are 1,2,3,4,5, or 6

Favourable Outcomes: An outcome which represents how many times we expect the things to be happened e.g., while tossing a coin, there is 1 favourable outcome of getting head or tail. While rolling a dice, there are 3 favourable outcomes of getting multiples of 2 i.e. {2,4,6}

> **Remember!**
> Each element of the sample space is called sample point.

Sample Space: the set of all possible outcomes of an experiment is called sample space. It is denoted by 'S' e.g.,

While tossing a coin, the sample space will be S = {H,T}

While rolling a dice, the sample space will be S = { 1,2,3,4,5,6}

Event: The set of results of an experiment is called an event e.g., while rolling a dice getting even number is an event i.e., A= {2,4,6};n(A)=3.

> **Recall: Types of Events**
> - **Certain event:** An event which is sure to occur. The probability of sure event is 1.
> - **Impossible event:** An event cannot occur in any trial. The probability of this event is 0.
> - **Likely event:** An event which will probably occur. It has greater chance to occur.
> - **Unlikely event:** An event which will not probably occur. It has less chance to occur.
> - **Equality likely events:** The events which have equal chance of occurrence. The probability of these events is 0.5.

Impossible event	Unlikely event	Equally Likely event	Likely event	Certain event
0 or 0%	25%	50%	75%	100%

Probability of Single Event

Example 1: Abdul Raheem rolls a fair dice, what is the probability of getting the number divisible by 3? 09313001

Solution

When a dice is rolled, the sample space will be:

S = { 1,2,3,4,5,6} ;

n(S) =6

> **Keep in mind**
> The range of probability for an event is:
> $0 \le P(A) \le 1$

Let "A" be the event of getting the number divisible by 3.

A={3,6} ; n(A) =2

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

The probability of getting the number divisible by 3 is $\frac{1}{3}$.

Example 2: If Zeeshan rolled two fair dice, find the probability of getting:

(i) Even numbers on both dice. **09313002**

(ii) Multiples of 3 on both dice. **09313003**

(iii) Even number on the first dice and the number 3 on the second dice. **09313004**

(iv) At least the number 3 on the first dice and number 4 on the second dice. **09313005**

Solution:

When a pair of fair dice is rolled, the sample will be:

2nd \ 1st	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

(i) Even numbers on both dice.

Let "A" be the event of getting even numbers on both dice.

A= {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)}

n(A) = 9; n(S)=36

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{4}$$

Thus, the probability of getting even numbers on both dice is $\frac{1}{4}$.

(ii) Multiple of 3 on both dice.

Let "B" be the event of getting multiples of 3 on both dice.

B = {(3,3), (3,6), (6,3), (6,6)}

n(B) = 4; n(S) = 36

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Thus, the probability of getting multiples of 3 on both dice is $\frac{1}{9}$.

(iii) Even number on the first dice and the number 3 on the second dice.

Let "C" be the event of getting even numbers on the first dice and the number 3 on the second dice.

C = {(2,3), (4,3), (6,3)}

n(C) = 3; n(S) = 36

$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

Thus, the probability of getting an even number on the first dice and the number 3 on the 2^{nd} dice is $\frac{1}{12}$.

(iv) At least the number 3 on the first dice and number 4 on the second dice.

Let "D" be the vent of getting least the number 3 on the first dice and number 4 on the second dice.

D = {(3,4), (4,4), (5,4), (6,4)}

n(D) = 4; n(S) = 36

$$P(D) = \frac{n(D)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Thus, the probability of getting at least the number 3 on the first dice and number 4 on the 2^{nd} dice is $\frac{1}{9}$.

Probability of an Event Not Occurring

Sometimes, we are interested in the probability that the head will not occur while tossing a coin.

Let "A" be the event of getting head while tossing a coin, then the event "A" be the event of not getting head while tossing a coin.

The probability of not getting head while tossing a coin is known as the complement of that event. It is written as $P(A')$ or $P(A^c)$.

The complement of an event "A" is calculated by the given formula:

$$P(A') = 1 - P(A)$$

For example, while tossing a coin, the probability of getting a head is:

$$P(A) = \frac{1}{2}$$

Teachers' Note:
The compliment rule states that the sum of the probability of an event and its complement must be equal to 1

and the probability of not getting a head is:

$P(A') = 1 - P(A)$

$= 1 - \frac{1}{2} = \frac{1}{2}$

Thus, the complement of the event of getting a head is $\frac{1}{2}$.

Example 3: Zubair rolls a dice, what is the probability of not getting the number 6? 09313006

Solution:

Let "A" be the event of getting the number 6.

The sample space while rolling a dice is S = {1, 2, 3, 4, 5, 6}

n(S); n(A) = 1

$P(A) = \frac{n(A)}{n(S))} = \frac{1}{6}$

Remember!
The sum of the probability of an event "A" and the probability of an event not occurring "A"is always "1"
$P(A) + P(A') = 1$

To find out probability of not getting the number 6, we have

$P(A') = 1 - P(A)$

$= 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$

Thus, the probability of not getting the number 6 is $\frac{5}{6}$.

Example 4: If two fair dice are rolled. What is the probability of getting:

(i) not a double 09313007

(ii) not the sum of both dice is 8 09313008

Solution:

Sample space of two dice is given by:

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5),(2,6) (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2),(4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3),(5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}.

n(s) = 36

(i) not a double six.

Let "A′" be the event that a double six occurs.

A = {(6,6)}; n(A) = 1

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$

Let "A" be the event that not a double six occurs As we know that

$P(A') = 1 - P(A)$

$= 1 - \frac{1}{36} = \frac{36-1}{36} = \frac{35}{36}$

Thus, the probability of not getting the double six is $\frac{35}{36}$.

(ii) not the sum of the dice is 8.

Let "B" be the event that the sum of both dice is 8.

B = {(2,6), (3, 5), (4,4), (5,3), (6,2)}

$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

Let "B′" be the event not sum of both dice is 8.

$P(B') = 1 - P(B)$

$= 1 - \frac{5}{36} = \frac{36-5}{36} = \frac{31}{36}$

Thus, the probability of not the sum of both dice be 8 is $\frac{31}{36}$.

Real life problems involving probability

Example 5: Let, A, B and C are three missiles and they are fired at a target. If the probabilities of hitting the target are

$P(A) = \frac{1}{4}$, $P(B) = \frac{3}{7}$, $P(C) = \frac{5}{9}$, respectively.

Find the probabilities of

(i) Missile A does not hit the target. 09313009

(ii) Missile C does not hit the target. 09313010

(iii) Missile B does not hit the target 09313011

Solution:

(i) Missile A does not hit the target

Since $P(A) = \frac{1}{4}$

Let 'A′' be the event that missile a does not hit the target.

$P(A') = 1 - P(A)$

$= 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$

Thus, the probability of missile 'A' does not hit the target is $\frac{3}{4}$.

(ii) Missile 'B' does not hit the target.

Since, $P(B) = \frac{3}{7}$

Let 'B′' be the event missile B does not hit the target.

$P(B) = 1 - P(B)$

$= 1 - \frac{3}{7}$

$= \frac{7-3}{7} = \frac{4}{7}$

Thus, the probability of missile 'B' does not hit the target is $\frac{4}{7}$.

(iii) Missile 'C' does not hit the target.

Since, $P(C)\ \frac{5}{9}$

Let 'C′' be the event missile C of not hitting the target.

$P(C') = 1 - P(C)$

$= 1 - \frac{5}{9} = \frac{9-5}{9} = \frac{4}{9}$

Thus, the probability of missile 'C' does not hit the target is $\frac{4}{7}$.

Example 6: A bag contains 5 blue balls and 8 green balls. Find the probability of selecting at random:

(i) a blue ball 09313012

(ii) a green ball 09313013

(iii) not a green ball 09313014

Solution:

(i) a blue ball

Let 'A" be the event that the ball is blue

Blue balls = n(A) = 5

Total balls = n(S) = 5+8 = 13

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{5}{13}$

> **Try yourself**
>
> Can you find out the complement selecting of blue ball?
>
> Probability of complement of selecting blue ball is;
>
> **Solution:**
>
> $P(A') = 1 - P(A)$
>
> $P(A') = 1 - \frac{5}{13}$
>
> $P(A') = \frac{13-5}{13}$
>
> $P(A') = \frac{8}{13}$

Thus, the probability of selecting a blue ball is $\frac{5}{13}$.

(ii) a green ball

Let 'B′' be the event that ball is green

Green balls = n(B) = 8

Total balls = n(S) = 5+8 = 13

$P(B) = \frac{n(B)}{n(S)} = \frac{8}{13}$

Thus, the probability of selecting green ball is $\frac{8}{13}$.

(iii) Not a green ball

Let 'B′' be the event that ball is not green.

$P(B') = 1 - P(B)$

$= 1 - \frac{8}{13}$

$= \frac{13-8}{13} = \frac{5}{13}$

Thus, the probability of not selecting a green ball is $\frac{5}{13}$.

Example 7: A card drawn at random, from a pack of 52 playing cards. What is the probability of getting:
(i) a card of heart (ii) neither spade nor heart

Solution:

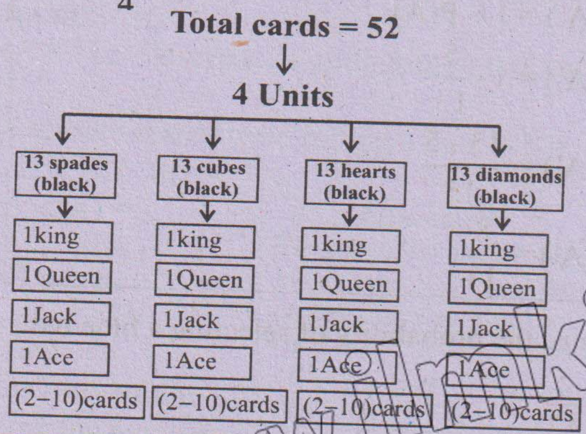
(i) a card of heart 09313015

Total number of cards = 52; n(S) = 52

Let 'A' be the event of selecting a card of heart. Number of heart cards = 13; n(A) = 13

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

Thus, the probability of getting a card of heart is $\frac{1}{4}$.



(ii) Neither spade nor heart 09313016

Let 'B' be the event of selecting a card of spade or heart

Number of spade and heart cards = 26;

n(B) = 26

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{26}{25}$$

$$= \frac{1}{2}$$

Let 'B′' be the event of selecting neither spade nor heart card.

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

Thus, the probability of getting neither spade nor heart cards is $\frac{1}{2}$.

Exercise 13.1

Q.1 Arshad rolls a dice, with sides labelled L, M, N, O, P, U. What is the probability that the dice lands on consonant? 09313017

Solution:

When dice is rolled the sample space will be; S = {L, M, N, O, P, U}, n(S) = 6

Let 'A' be the event of getting consonant,

A = {L, M, N, P}, n(A) = 4

We know that

$$p(A) = \frac{n(A)}{n(S)} = \frac{4}{6}$$

$$= \frac{2}{3}$$

Thus, the probability of getting consonant is $\frac{2}{3}$.

Q.2 Shazia throws a pair of fair dice. What will be the probability of getting:

(i) Sum of dots is at least 4. 09313018

(ii) Product of both dots is between 5 to 10. 09313019

(iii) The difference between both the dots is equal to 4. 09313020

(iv) Number at least 5 on the first dice and the number at least 4 on the second dice. 09313021

Solution:

When a pair of fair dice is rolled, the sample space will be:

2nd \ 1st	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

(i) Sum of dots is at least 4.

Solution:

Probability of sum of dots is at least 4.

Let A be even of getting sum of dots is at least 4:

Since, there are only 3 outcomes where sum of dots is less than 4, so, there are 33 outcomes where sum of dots is either 4 or more than 4 so, n(A) = 33.

$$P(A) = \frac{n(A)}{n(S)} = \frac{33}{36} = \frac{11}{12}$$

Thus probability of getting sum of dots at least 4 is $\frac{11}{12}$.

(ii) Product of both dots is between 5 to 10.

Solution:

Probability of getting product of dots is between 5 and 10.

Let B be an event of getting product of dots from 5 to 10.

B={(1,5), (1,6), (2,3), (2,4), (2,5), (3,2), (3,3), (4,2), (5.1), (5,2), (6,1)}

$$P(B) = \frac{n(B)}{n(S)} = \frac{11}{36}$$

Thus probability of getting product of dots between 5 and 10 is $\frac{11}{36}$.

(iii) The difference between both the dots is equal to 4.

Solution:

Let D be event getting difference of dots equal to 4.

D = {1,5), (2,6), (5,1), (6,2)}, n(D) = 4

Probability of getting difference of dots equal to 4 is:

$$P(D) = \frac{n(D)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

(iv) Number at least 5 on the first dice and the number at least 4 on the second dice.

Solution:

Let E be an event getting at least 5 on the first dice and at least 4 on the second dice.

E={(5,4), (5,5), (5,6), (6,4), (6,5),(6,6)}

n(E) = 6

Probability of event E is:

$$p(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Q.3 One alphabet is selected at random from the word "MATHEMATICS". Find he probability of getting:

(i) vowel 09313022 **(ii) Consonant** 09313023

(iii) an E 09313024

(iv) an A 09313025 **(v) not M** 09313026

(vi) not T 09313027

Solution:

Word : Mathematics

Sample space: S = {M,A,T H, E, M, A, T,I,C,S}

n(S) =11

(i) Vowel

Solution:

Let "A" be an event getting a vowel.

A = {A, E, I, A}, n(A) = 4

Probability of getting a vowel is:

$$P(A) \frac{n(A)}{n(S)} = \frac{4}{11}$$

(ii) Consonant

Solution:

Let "B" be an event getting a consonant

B = {M, T, H, M, T, C, S}, n(B) = 7

Probability of getting a consonant is:

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{11}$$

(iii) an E

Solution:

Let "C" be can event of getting an E,

C = {E}, n(E) = 1

Probability of getting an E:

$$P(E)=\frac{n(E)}{n(S)}=\frac{1}{11}$$

(iv) an A

Solution

Let "D" be an event of getting an A.

D = {A, A}, n(D) = 2

Probability of getting an A:

$$P(D)=\frac{n(D)}{n(S)}=\frac{2}{11}$$

(v) not M

Let "M" be an event of getting M.

M = {M, M}, n(M) = 2

Probability of getting M:

$$P(M)=\frac{n(M)}{n(S)}=\frac{2}{11}$$

Probability of getting not M:

$$P(M')=1-P(M)$$

$$=1-\frac{2}{11}$$

$$=\frac{11-2}{11}=\frac{9}{11}$$

(vi) not T

Solution:

Let E be an event getting T.

E={T, T}, n(E) = 2

Probability of getting T:

$$P(E)=\frac{n(E)}{n(S)}=\frac{2}{11}$$

Probability of getting not T:

$$P(E')=1-p(E)=1-\frac{2}{11}=\frac{11-2}{11}=\frac{9}{11}$$

Q.4 Aslam rolled a dice. What is the probability of getting the numbers 3 or 4? Also find the probability of getting the numbers 3 or 4. 09313028

Solution:

When a dice is rolled, he sample space is:

S = {1, 2, 3, 4, 5, 6}

Let A be an event of getting 3 or 4.

n(A) = {3, 4}, n(A) = 2

Probability of getting 3 or 4:

$$P(\text{getting 3 or 4})=\frac{n(A)}{n(S)}=\frac{2}{6}=\frac{1}{3}$$

Probability of getting (not 3 or 4):

$$P(A')=1-P(A)=1-\frac{1}{3}$$

$$=\frac{3-1}{3}=\frac{2}{3}$$

Q.5 Abdul Hadi labelled cards from 1 to 30 and put them in a box. He selects a card at random. What is the probability that selected card containing:

(i) the number 25 09313029

(ii) number between 17 to 22 09313030

(iii) number at lest 20 09313031

(iv) number not 27and 29 09313032

(v) number not between 12 to 15 09313033

Solution

Sample space = S = {1,2,3,4,..., 30},

n(S) = 30

(i) The number 25

Solution:

Let "A" be an event of getting card containing 25, A={25}, n(A) = 1

Probability of getting a card containing 5:

$$P(A)=\frac{n(A)}{n(S)}=\frac{1}{30}$$

(ii) Number between 17 to 22

Solution:

Let "B" is an event of selecting a card from 17 to 22.

B = {17, 18, 19, 20, 21, 22}, n(B) = 6

Probability of selecting number from 17 to 22 is:

$$P(B)=\frac{n(B)}{n(S)}=\frac{6}{30}=\frac{1}{5}$$

(iii) Number at least 20

Solution:

Let "C" be an event of selecting a card containing number at least 20.

C = {20, 21, 22,...30}, n(C) = 11

Probability of selecting a card containing a number at least 20.

$P(C) = \frac{n(C)}{n(S)} = \frac{11}{30}$.

(iv) Number not 27 and 29

Solution:

Let "D" be an event of selecting a card containing 27 and 29..

D= {27, 29}, n(D) = 2

Probability of selecting a card containing 27 and 29.

$P(D) = \frac{2}{30} = \frac{1}{15}$.

Probability of selecting a card containing a number not 27 and 29.

$P(D') = 1 - P(D)$

$= 1 - \frac{1}{15}$

$= \frac{15-1}{15} = \frac{14}{15}$

(v) Numbers not between 12 to 15.

Solution:

Let 'E' be an event of selecting a card containing number '12' to '15'.

E = {12, 13, 14, 15}, n(E) = 4

Now, let E′ be an event of selecting a card containing number not between 12 to 15.

n(E′) = n(S) – n(E)=30 – 4 = 26

$P(E) = \frac{n(S)}{n((E')} = \frac{26}{30} = \frac{13}{15}$

So, probability that the selected cards contains a number not between 12 to 15 is $\frac{13}{15}$.

Q.6 The probability that Ayesha will pass the examination is 0.85. What will be the probability that Ayesha will not pass the examination? 09313034

Solution:

Let "A" be an event of Ayesha will pass examination. The probability of Ayesha will not pass examination p(Á) = 1– P(A)

P((A′) = 1– 0.85

P((A′) = 0.15

Q.7 Taabish tossed a fair coin and rolled a fair dice once. Find the probability of the following events:

(i) tail on con and at least 4 on dice. 09313035

(ii) head on coin and the number 2,3 on dice. 09313036

(iii) head and tail on coin and the number 6 on dice. 09313037

(iv) not tail on coin and the number 5 on dice. 09313038

(v) not head on coin and the number 5 and 2 on dice. 09313039

Solution:

When a fair coin is tossed along with rolling a fair dice one, the possible outcomes are:

S = {(H,1), (H,2), (H,3), (H,4), (H,5), (H,6) (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)}

n(S) = 12

(i) Tail on coin and at least 4 on dice.

Solution:

Let A be an event of occurring tail on coin and at least 4 an dice;

A={T,4), (T,5), (T,6)}, n(A) = 3

Probability of event A:

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{12} = \frac{1}{4}$

(ii) Head on coin and the number 2,3 on dice.

Let "B" is an event of selecting a Head coin and number 2,3 on dice:

B ={H,2), (H,3)}, n(B) = 2

Probability of event B.

$P(B) = \frac{n(C)}{n(S)} = \frac{2}{12} = \frac{1}{6}$

(iii) Head and tail on coin and the number of 6 on dice.

Solution:

Let "C" be an event of occurring head and tail on coin and number 6 on dice

$C=\{(H,6), (T,6)\}$, $n(C) = 2$

Probability of event C:

$$P(C)\ \frac{n(C)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

(iv) Not tail on coin and the number 5 on dice.

Solution:

Let "D" be an event of occurring a tail on coin and number 5 on dice.

$D = \{(T,5)\}$, $n(D) = 1$

Probability event D:

$$P(D) = \frac{n(D)}{n(S)} = \frac{1}{12}$$

The probability of not tail on coin and number 5 on dice is:

$$P(D') = 1 - P(D)$$

$$= 1 - \frac{1}{12}$$

$$= \frac{12-1}{12} = \frac{11}{12}$$

(v) Not head on coin and the number 5 and 2 on dice.

Solution:

Let "E" be an event of occurring head on coin and number 5 and 2 on dice.

$E = \{(H,2), (H,5)\}$, $n(E) = 2$

Probability event E:

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

The probability of not occurring head on coin and number 5 and 2 on dice.

$$P(E') = 1 - P(E)$$

$$= 1 - \frac{1}{6}$$

$$= \frac{6-1}{6} = \frac{5}{6}$$

Q.8 A card is selected at random from a well shuffled pack of 52 plying cards. What will be the probability of selecting.

(i) a queen **09313040**

(ii) neither a queen nor a jack **09313041**

Solution:

Since total cards are 52, so $n(S) = 52$

(i) A queen

Solution:

Let A be an event of getting a queen.

$n(A) = 4$, there are four cards of queen.

Probability of getting a queen:

$$p(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(ii) Neither a queen nor a jack

Let "B" be an event of getting a queen or jack, Since, there are 4 cards of queen and 4 cards of Jack, so,

$n(B) = 8$

Probability of event B.

$$P(B) = \frac{n(B)}{n(S)} = \frac{8}{52} = \frac{2}{13}$$

The probability of getting neither queen nor jack.

$$P(B') = 1 - \frac{2}{13}$$

$$= \frac{13-2}{13} = \frac{11}{13}$$

Q.9 A card is chosen at random from a pack of 52 playing cards. Find the probability of getting.

(i) a Jack 09313042 (ii) no diamond 09313043

Solution:

Since total cards are 52,

so, $n(S) = 52$

(i) a jack

Let A be an event of getting a jack.

$n(A) = 4$, (There are four cards of jack)

probability of event A:

$$p(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(ii) no diamond

Let "B" is an event of getting diamond.

Since there are 13 cards of diamond so,

$n(B) = 13$

Probability of event B.

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

Probability of getting no diamond.

$$P(B') = 1 - p(B)$$

$$= 1 - \frac{1}{4}$$

$$= \frac{4-1}{4} = \frac{3}{4}$$

Relative Frequency as an estimate of probability

Relative frequency tells us how often a specific event occurs relative to the total number of frequency event or trials. It is calculated by using the following method:

$$\text{Relative frequency} = \frac{\text{Frequency of specific event}}{\text{Total frequency}}$$

$$= \frac{x}{N}, \text{ where } N = \Sigma f$$

Example 8: Find the relative frequency of the given date: **09313044**

x	2	3	4	5	6	7	8
f	3	5	6	9	10	8	2

x	f	$\frac{f}{\Sigma f}$
2	3	$\frac{3}{43} = 0.07$
3	5	$\frac{5}{43} = 0.12$
4	6	$\frac{6}{43} = 0.14$
5	9	$\frac{9}{43} = 0.21$
6	10	$\frac{10}{43} = 0.23$
7	8	$\frac{8}{43} = 0.19$
8	2	$\frac{2}{43} = 0.04$
Total: $\Sigma f = 43$		

Real life application of relative frequency

Example 9: A survey was conducted on 80 students of Grade–IX and asked about their favourite colour. The responses are:

> **Remember!**
> Relative frequency is an estimated probability of an event occurring when an experiment is repeated a fixed number of times.

(i) Red colour = 23 students **09313045**

(ii) Green colour = 15 students **09313046**

(iii) Pink colour = 25 students **09313047**

(iv) Blue colour = 10 students **09313048**

(v) White colour = 7 students **09313049**

Find the relative frequency for each colour

Solution:

Total number of students = 80

(i) Relative frequency for red colour $= \frac{23}{80}$

$= 0.29$

It means that 29% students prefer red colour.

(ii) Relative frequency for green colour

$= \frac{15}{80}$

$= 0.19.$

It means that 19% students prefer green colour.

(iii) Relative frequency for pink colour $= \frac{25}{80}$

$= 0.31.$

It means that 31% students prefer pink colour.

(iv) Relative frequency for blue colour $= \frac{10}{80}$

$= 0.12.$

(v) Relative frequency for white colour $= \frac{7}{80}$

$= 0.09.$

Try yourself!

Out of 200 students in a school, 80 play cricket, 50 play football, 25 play volleyball and 45 do not play any game. Can you find out the probability of the students who do not play any game and relative frequency of the students who play cricket?

Solution:

Total students = n(S) = 200

80 play cricket = (C) = 80

50 play football = n(F) = 50

25 play volleyball = n(V) = 25

45 do not play any game = n(N)

(i) Probability of students Not playing any game:

$$P(N) = \frac{n(N)}{n(S)} = \frac{45}{200} = \frac{9}{40}$$

(ii) $r.f = \frac{f}{\Sigma f} = \frac{80}{200} = \frac{2}{5}$

Example 10: Abdul Rehman obtained student marks in different subjects out of 100 marks. The detail is as under:

Find the relative frequency of above given data. **09313050**

Solution:

Keep in mind

The sum of all the relative frequencies is always equal to or approximately equal to 1.

Subject	Marks obtained	Relative frequency
Urdu	75	$\frac{75}{488} = 0.15$
English	80	$\frac{80}{488} = 0.16$
Islamiat	72	$\frac{72}{488} = 0.15$
Mathematics	95	$\frac{95}{488} = 0.19$
Science	81	$\frac{81}{488} = 0.17$
Computer science	85	$\frac{85}{488} = 0.17$
Total	$\Sigma f = 488$	

Expected Frequency

Expected frequency is a measure that estimate how often an event should be occurred depended on probability. Expected frequency is found by using the following method:

Expected frequency = total number of trials × Probability of the event.

Subject	Urdu	Eng.	Isl.	Math	Sci.	Comp.
Marks obtained	75	80	72	95	81	85

Example 11: Six fair dice are rolled 50 times. The probability of occurrence of different number of sixes are given below. Find the expected frequency of the following data: **09313051**

x	0	1	2	3	4	5	6
$P(x)$	0.09	0.10	0.12	0.24	0.10	0.20	0.15

Find he expected frequency of occurrence of each six.

Solution:

No. of sixes (x)	$P(x)$	Expected frequency = $N \times P((x) = 50 \times P(x)$
0	0.09	$50 \times 0.09 = 4.5$
1	0.10	$50 \times 0.10 = 5$
2	0.12	$50 \times 0.12 = 6$
3	0.24	$50 \times 0.10 = 12$
4	0.10	$50 \times 0.10 = 5$
5	0.20	$50 \times 0.20 = 10$
6	0.15	$50 \times 0.15 = 4.5$

Remember!
Sum of all expected frequencies is always equal to or approximately equal to a fixed number of trials.

Real life application on expected frequency

Example 12: Find the average number of times getting 1 or 6, when a fair dice is rolled 300 times. **09313052**

Solution:

Let "S" be the sample space when a fair dice is rolled:

$S = \{1, 2, 3, 4, 5, 6\}; n(S) = 6$

Let "B" be the event that 1 or 6 comes up.

$B = \{1, 6\}; n(B) = 2$

So, $P(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Therefore, $E(B) = N \times P(B)$

$= 300 \times \frac{1}{3} = 100$

Thus, the average number of times 1 or 6 comes up is 100.

Example 13: If the probability of a defective bolt is 0.3. Find the number of non-defective bolts in a total of 800. **09313053**

Solution:

The probability of defective bolt is = 0.3

Probability of non-defective bolt

$= 1 - 0.3 = 0.7$

Number of non-defective bolts

$= 0.7 \times 800 = 560$

Thus, the non-defective bolts will be 560.

Exercise 13.2

Q.1 A researcher collected data on number of deaths from Horse-Ricks in Russian Army crops over to years. The table is as follows: **09313054**

No. of death	0	1	2	3	4	5	6
frequency	60	50	87	40	32	15	10

Find the relative frequency of the data.

Solution:

No. of death	f	Relative frequency $(r.f = \frac{f}{\Sigma f})$
0	60	$\frac{60}{294} = \frac{30}{147}$
1	50	$\frac{50}{294} = \frac{25}{147}$
2	87	$\frac{87}{294} = \frac{29}{98}$
3	40	$\frac{40}{294} = \frac{20}{147}$
4	32	$\frac{32}{294} = \frac{16}{147}$
5	15	$\frac{15}{294} = \frac{5}{98}$
6	10	$\frac{10}{294} = \frac{5}{147}$
	$\Sigma f = 294$	

Q.2 **The frequency of defective products in 750 samples are shown in the following table. Find the relative frequency for the given table.** **09313055**

No. of defectives per sample	0	1	2	3	4	5	6	7	8
No. of sample	120	140	94	85	105	50	40	66	50

Solution:

No. of defectives per sample	No. of sample (f)	Relative frequency $r.f = \frac{f}{\Sigma f}$
0	120	$\frac{120}{750} = \frac{4}{25}$
1	140	$\frac{140}{750} = \frac{14}{75}$
2	94	$\frac{94}{750} = \frac{47}{375}$
3	85	$\frac{85}{750} = \frac{17}{150}$
4	105	$\frac{105}{750} = \frac{21}{150}$
5	50	$\frac{50}{750} = \frac{1}{15}$
6	40	$\frac{40}{750} = \frac{4}{75}$
7	66	$\frac{66}{750} = \frac{11}{125}$
8	50	$\frac{50}{750} = \frac{1}{15}$
	$\Sigma f = 750$	

Q.3 **A quiz competition on general knowledge is conducted. The number of corrected answers out of 5 questions for 100 sets of questions is given below.**

09313056

X	0	1	2	3	4	5
f	10	23	15	25	18	9

Find the relative frequencies for the given data.

Solution:

x	f	$\frac{f}{\Sigma f}$
0	10	$\frac{10}{100} = \frac{1}{10}$
1	23	$\frac{23}{100}$
2	15	$\frac{15}{100} = \frac{3}{20}$
3	25	$\frac{25}{100} = \frac{1}{4}$
4	18	$\frac{18}{100} = \frac{9}{50}$
5	9	$\frac{9}{100}$
	$\Sigma f = 100$	

Q.4 A survey conducted from the 50 students of a class and asked about their favourite food. The responses are as under:

Name of food item	Biryani	Fresh juice	Chicken	Bar. B.Q	Sweets
No. of studetns	40	07	21	15	25

(i) How many percentage of students like biryani? 09313058

(ii) How many percentage of students like chicken? 09313059

(iii) Which food is the least like by the students? 09313060

(iv) Which food is the most prefer by the students? 09313061

Solution:

Name of food item	No. of stude nts (f)	Relative frequency $\frac{f}{\Sigma f}$	%age
Biryani	40	$\frac{40}{108}=0.37$	0.37×100=37%
Fresh juice	07	$\frac{7}{108}=0.065$	0.065×100=6.5 %
Chicken	21	$\frac{21}{108}=0.194$	0.19×100=19.4 %
Bar. B.Q	15	$\frac{15}{108}=0.14$	0.14×100=14%
Sweet	25	$\frac{25}{108}=0.23$	0.23×100=23%
Total	**$\Sigma f=$ 108**		

(i) 37% students like biryani.

(ii) 19.4% students like chicken.

(iii) fresh juice is liked by least students.

(iv) Biryani is most preferred by the students (6.5% only)

Q.5 In 500 trials of a thrown of two dice, what is expected frequency that the sum will be greater than 8? 09313062

Solution:

Total trailes = N = 500

We know that

Expected frequency = N×P(A)_____ (i)

When two dice are rolled once, then 36 outcomes are possible i.e. n(S) = 36

Let "A" be an event of occurring sum of dots greater than 8.

A = {3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)}

n(A) = 10

Probability of event A in one trail.

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

Probability of event A in 500 trails:

$$\text{Expected frequency} = 500\times\frac{5}{18}$$

Expected frequency = N × P(A)

$$= \frac{2500}{18} = 138.8 \approx 139$$

Q.6 What is the expectation of a person who is to get Rs. 120 if he obtains at least 2 heads in single toss of three coins? 09313063

Solution:

Amount for occurring an vent of two heads = Rs. 120

Let 'A' be an event of getting at least 2 heads on tossing three coins single time.

Total possible outcomes = $2^3=8$ i.e. n(s) = 8

A = {(H_1,H_2,H_3), (H_1,H_2,T_3), (H_1,T_2,H_3), (T_1,H_2,H_3)}, n(A) = 4

Probability of event A:

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

$$\text{Expected amount of event} = Rs.120\times\frac{1}{2}$$

$$= Rs.60$$

Q.7 Find the expected frequencies of the given data if the experiment is repeated 200 times. 09313064

x	0	1	2	3	4	5	6
P(x)	0.11	0.21	0.17	0.18	0.09	0.17	0.07

Solution:

Total no. of trails = N = 200

x	P(x)	Expected frequency = N × P(x)
0	0.11	200×0.11 = 22
1	0.21	200×0.21 = 42
2	0.17	200×0.17 = 34
3	0.18	200×0.18 = 36
4	0.09	200×0.09 = 18
5	0.17	200×0.17 = 34
6	0.07	200×0.07 = 14

Q.8 The probability of getting 5 sixes while tossing dice is $\frac{2}{5}$. The dice is rolled 200 times. How many times would you expect it to show 5 sixes?

Solution:

Let "A" be an event of getting 5 sixes while tossing dice.

Probability of getting 5 sixes = P(A) = $\frac{2}{5}$

Numbers of trails = N = 200

We know that

Expected frequency = N×P(A)

Expected frequency = $200\times\frac{2}{5}$

= 40×2

= 80

In 200 trails, we expect it will show 5 sixes

Review Exercise 13

Q.1 Choose the correct option.

i. Each element of the sample space is called: 09313065

(a) Event (b) Experiment
(c) Sample point (d) Outcomes

ii. An outcome which represent how many times we expect the things to be happened is called: 09313066

(a) Outcomes
(b) Favourable outcomes
(c) Sample space
(d) Sample point

iii. Which one tells us how often a specific event occurs relative to the total number of frequency event or trials? 09313067

(a) Expected frequency
(b) Sum of relative frequency
(c) Relative frequency
(d) Frequency

iv. Estimated probability of an event occurring is also known as: 09313068

(a) Relative frequency
(b) Expected frequency
(c) Class boundaries
(d) Sum of expected frequency

v. The sum of all expected frequencies is equal to the fixed number of: 09313069

(a) Trials (b) Relative frequencies
(c) Outcomes (d) Events

vi. The chance of occurrence of a particular event is called: 09313070

(a) Sample space
(b) Estimated probability
(c) Probability
(d) Expected frequency

vii. An event which will probably occur. It has greater chance to occur is called: 09313071

(a) Equally likely event
(b) Likely event
(c) Unlikely event
(d) Certain event

viii. Find out the total number of possible sample space when 4 dice are rolled: 9313072

(a) 6^2 (b) 6^3
(c) 6^4 (d) 6^6

ix. While rolling a pair of dice, what will be the probability of double 2? 09313073

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{5}{6}$ (d) $\frac{1}{36}$

x. A card is chosen from a pack of 52 playing cards, find the probability of getting no jack and king: 09313074

(a) $\frac{2}{3}$ (b) $\frac{11}{13}$

(c) $\frac{2}{52}$ (d) $\frac{11}{52}$

Answers Key

i	c	ii	b	iii	c	iv	a	v	a
vi	c	vii	b	viii	c	ix	d	x	b

Multiple Choice Questions (Additional)

1. The word probability is derived from: 09313075
   (a) English word (b) Latin word
   (c) French word (d) Greek word
2. Who is known as the father of probability? 09313076
   (a) Girolamo Cardano
   (b) Sir Ronald Fisher
   (c) George Cantor
   (d) John Venn
3. The process which generate the result is called: 09313077
   (a) Event (b) Experiment
   (c) out comes (d) probability
4. The set of all possible outcomes is called: 09313078
   (a) Event (b) Experiment
   (c) Sample space (d) probability
5. The probability of a certain Event is: 09313079
   (a) 0 (b) 1
   (c) 2 (d) not possible
6. The probability of an impossible event is: 09313080
   (a) 0 (b) 1
   (c) 2 (d) −1
7. The probability of an equally likely event is: 09313081
   (a) 0 (b) 1
   (c) 50 (d) 0.5
8. What is range of probability? 09313082
   (a) $0 \leq P(A) \leq 1$ (b) $0 < P(A) \leq 1$
   (c) $0 \leq P(A) \leq 1$ (d) $0 \leq P(A) \geq 1$
9. If Hadi rolled a fair dice then the probability of getting a prime number is: 09313083
   (a) 0.5 (b) 1
   (c) 0 (d) 0.6
10. If Fatima rolled two fair dice then the probability of getting a fractional number is: 09313084
    (a) 0.5 (b) 1
    (c) 0 (d) 2
11. The sum of the probability of an event and its complement must be: 09313085
    (a) 0.5 (b) 1
    (c) 0 (d) 2
12. If the probability of an event is $\frac{3}{7}$, then what is the probability of not occurring that event? 09313086
    (a) $\frac{6}{14}$ (b) $\frac{7}{3}$
    (c) 0 (d) $\frac{4}{7}$

13. The sum of all relative frequencies is always equal to: 09313087

(a) 0 (b) 1

(c) 1.5 (d) 2

14. If n(S)=12 and n(B)= 8 then P(B) is: 09313088

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$

(c) 20 (d) 4

15. If n(S)=18 and n(E′)= 4 then P(E′) is: 09313089

(a) $\frac{4}{18}$ (b) $\frac{2}{9}$

(c) $\frac{7}{9}$ (d) $\frac{18}{4}$

Answer Key

1	b	2	a	3	b	4	c	5	b	6	a	7	d	8	c	9	a	10	c
11	b	12	d	13	b	14	b	15	b										

Q.2 Define the following:

(i) Relative frequency 09313090

(ii) Expected frequency 09313091

(i) Solution:

Relative frequency tells us how often a specific event occurs relative to the total number of frequency event or trails. It is calculated by using the following method.

Relative frequency

$$= \frac{\text{Frequency of specific event}}{\text{Total frequency}}$$

$$= \frac{x}{N}, \text{ where } N = \Sigma f$$

(ii) Expected Frequency is a measure that estimate how often an event should be occurred depended on probability. Expected frequency is found by using the following method:

Expected = Total number of trails × Probability of the event.

$= N \times P(A)$

Q.3 An urn contains 10 red balls, 5 green balls and 8 blue balls. Find the probability of selecting at random.

(i) a green ball 09313091

(ii) a red ball 09313091

(iii) a blue ball 09313092

(iv) not a red ball 09313093

(v) not a green ball 09313094

Solution

Red balls = 10, n(R) = 10

Green balls = 5, n(G) = 5

Blue balls = 8, n(B) = 8

Total balls =10+5+8=23,

n(S)=23

(i) Probability of selecting green ball:

$$P(G) = \frac{n(G)}{n(S)} = \frac{5}{23}$$

(ii) Probability of selecting red ball:

$$P(R) = \frac{n(R)}{n(S)} = \frac{10}{23}$$

(iii) Probability of selecting blue ball:

$$P(B) = \frac{n(B)}{n(S)} = \frac{8}{23}$$

(iv) Probability of selecting not red ball:

$$P(R') = 1 - \frac{10}{23}$$

$$= \frac{23-10}{23}$$

$$= \frac{13}{23}$$

(v) Probability of selecting not green ball:

$$P(G') = 1 - P(G)$$

$$P(G') = 1 - \frac{5}{23}$$

$$P(G') = \frac{23-5}{23}$$

$$P(G') = \frac{18}{23}$$

Q.4 Three coins are tossed together. What is the probability of getting.

(i) exactly three heads 09313095

(ii) at least two tails 09313096

(iii) not a t least two heads 09313097

(iv) not exactly two heads 09313098

Solution:

When two coins are tossed together, their all possible outcomes are given below.

S = {(HHH), (HHT), (HTH), (THH) (TTT), (TTH), (THT), (HTT)}

n(S) = 8

(i) Exactly three heads

Solution:

(i) Exactly three heads:

A={HHH)},n(A) =1

Probability of getting exactly three heads:

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{8}$$

(ii) At least two tails

Solution:

Let "B" an event of at least two tails.

B ={(TTT), (TTH), (THT), (HTT)}

n(B) = 4

Probability of getting at least two tails

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(iii)Not at least two heads

Solution

Let "C" be event of at least two heads

n(C) = 4

C = {(H,H,H), (H,H,T), (H,T,H), T,H,H)}

Probability of at least two heads.

$$P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Probability of getting at least not two heads

$$P(C') = 1 - P(C)$$

$$= 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$$

(iv)Not exactly two heads

Solution:

Let "D" be an event of exactly two heads

D ={(HHT), (HTH), (THH), n(D) = 3

Probability of getting exactly two heads.

$$P(D) = \frac{n(D)}{n(S)} = \frac{3}{8}$$

Probability of getting not exactly two heads.

$$P(D') = 1 - P(D)$$

$$= 1 - \frac{1}{8} = \frac{8-3}{8} = \frac{5}{8}$$

Q.5 A card is drawn from a well shuffled pack of 52 playing cards. What will be the probability of getting:

(i) King or Jack of red colour 09313099

Solution

Total cards = 52, n(S) = 52

Let A be an event of getting King or jack of red colour.

Number of king or jack red cards, n(A) = 4

Probability of event A,

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

(ii) Not "2" of club and spade 09313100

Let "B" be an event of getting "2" of club and spade cards.

Number of "2" of club and spade cards.

n(B) = 2

Probability of event B,

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

Probability of getting not "2" of club and spade cards,

$$P(B') = 1 - P(B)$$

$$\Rightarrow = 1 - \frac{1}{26}$$

$$= \frac{26-1}{26}$$

$$= \frac{25}{26}$$

Q.6 Six coins are tossed 600 times. The number of occurrence of tails are recorded and shown in the table given below. 09313101

No. of tails	0	1	2	3	4	5	6
Frequency	110	90	105	80	76	123	16

Find the relative frequency of given table.

Solution

No. of tails	f	Relative frequency $\frac{f}{\Sigma f}$
0	110	$\frac{110}{600}=\frac{11}{60}$
1	90	$\frac{90}{600}=\frac{3}{20}$
2	105	$\frac{105}{600}=\frac{7}{40}$
3	80	$\frac{80}{600}=\frac{2}{15}$
4	76	$\frac{76}{600}=\frac{19}{150}$
5	123	$\frac{123}{600}=\frac{41}{200}$
6	16	$\frac{16}{600}=\frac{2}{75}$
	$\Sigma f = 600$	

Q.7 From a lot containing 25 items, 8 items are defective. Find the relative frequency of non-defective items, also find the expected frequency of non-defective items.

Solution:

Total number of items = 25

Number of defective items = 8

Number of non-defective items = 25–8 = 17

Relative frequency of non-defective items

$= \frac{17}{25} = 0.68$

Expected frequency of non-defective items

$=$ N × R.f. of non-defective items.

$= 25 \times \frac{17}{25}$

$= 17$