UNIT

02

Logarithms

Introduction

Logarithms are powerful mathematical tools used to simplify complex calculations, particularly those involving exponential growth or decay. They are widely applicable across various fields, including banking, science, engineering and information technology.

Scientific Notation

A number in scientific notation is written as: $a \times 10^n$, where $1 \le a < 10$ and $n \in Z$

Here "a" is called the coefficient or base number.

Conversion of Numbers from Ordinary Notation to Scientific Notation

Example 1: Convert 78,000,000 to scientific notation. 09302001

Solution:

Step 1: Move the decimal to get a number between 1 and 10: 7.8

Step 2: Count the number of places you moved the decimal: 7 places

Step 3: Write in scientific notation:

$$78{,}000{,}000 = 7.8 \times 10^7$$

Since we moved the decimal to the **left**, the exponent is **positive**.

Example 2: Convert 0.0000000315 to scientific notation. 09302002

Solution:

Step 1: Move the decimal to get a number between 1 and 10: 3.15

Step 2: Count the number of places you moved the decimal: 8 places

Step 3: Write in scientific notation: 8 places

$0.0000000315 = 3.15 \times 10^{-8}$

Since we moved the decimal to the **right**, the exponent is **negative**.

Conversion of Numbers from Scientific Notation to Ordinary Notation

Example 3: Convert 3.47×10^6 to ordinary notation.

09302003

Solution:

Step 1: Identify the parts:

Coefficient: 3.47

Exponent: 10^6

Step 2: Since the exponent is **positive** 6, move the decimal point 6 places to the right.

$$3.47 \times 10^6 = 3{,}470{,}000$$

Example 4: Convert 6.23×10^{-4} to ordinary notation. 09302004

Solution:

Step 1: Identify the parts:

Coefficient: 6.23

Exponent: 10^{-4}

Step 2: Since the exponent is **negative** 4, move the decimal point 4 places to the **left.**

$$6.23 \times 10^{-4} = 0.000623$$

Exercise 2.1

Q.1 Express the following numbers in scientific notation.

(i) 2000000 09302005

Solution:

$2000000 = 2_{\wedge}000000.$

$= 2.0 \times 10^6$

Move decimal point 6 places to the left.

(ii) 48900 09302006

Solution:

$48900 = 4_{\wedge}8900.$

$= 4.89\times10^4$

Move decimal point 4 places to the left.

(iii) 0.0042

09302007

Solution:

$0.0042 = 0.004_{\wedge}2$

$= 4.2\times10^{-3}$

Move decimal point 3 places to the right.

(iv) 0.0000009

09302008

Solution:

$0.0000009 = 0.0000009_{\wedge}0$

$= 9.0\times10^{-7}$

Move decimal point 7 places to the right.

(v) 73×10^3

09302009

Solution:

$73\times10^3 = 7_{\wedge}3.\times10^3$

$= 7.3\times10^1\times10^3$

$= 7.3\times10^{1+3} = 7.3\times10^4$

Move decimal point 1 place to the left.

(vi) 0.65×10^2

09302010

Solution:

$0.65\times10^2 = 0.6_{\wedge}5\times10^2$

$= 6.5\times10^{-1}\times10^2 = 6.5\times10^{-1+2}$

$= 6.5\times10^1$

Move decimal point 1 place to the right.

Q.2 Express the following numbers in ordinary notation.

(i) 8.04×10^2

09302011

Solution:

8.04×10^2

Since exponent is positive 2, move decimal point 2 places to the right.

$= 804\times10^{-2}\times10^2$

$= 804 \times10^{-2+2}$

$= 804\times10^0 \qquad \because 10^0 = 1$

$= 804\times1 = 804$

(ii) 3×10^5

09302012

Solution

3×10^5

Since exponent is positive 5, move decimal point 5 places to the right.

$= 3.0\times10^5 = 300000.0\times10^{-5}\times10^5$

$= 300000\times10^0$

$= 300000\times1 = 300000$

(iii) 1.5×10^{-2}

09302013

Solution:

1.5×10^{-2}

Since exponent is negative 2, move decimal point 2 places to the left.

$= 0.015\times10^2\times10^{-2}$

$= 0.015\times10^{2-2}$

$= 0.015\times10^0$

$= 0.015\times1 = 0.015$

(iv) 1.77×10^7

09302014

Solution:

1.77×10^7

Since exponent is positive 7, move decimal point 7 places to the right.

$= 17700000.0\times10^{-7}\times10^7$

$= 17700000\times10^{-7+7}$

$= 17700000\times10^0$

$= 17700000\times1 = 17,700,000$

(v) 5.5×10^{-6}

09302015

Solution:

Since exponent is negative 6, move decimal point 6 places to the left.

$= 0.0000055\times10^6\times10^{-6}$

$= 0.0000055\times10^{6-6}$

$= 0.0000055\times10^0$

$= 0.0000055\times1 \qquad \because 10^0 = 1$

$= 0.0000055$

(vi) 4×10^{-5}

09302016

Solution:

Since exponent is negative 5, move decimal point 5 places to the left.

$= 0.00004\times10^5\times10^{-5}$

$= 0.00004\times10^{5-5}$

$= 0.00004\times10^0$

$= 0.00004\times1$

$= 0.00004$

Q.3 The speed of light is approximately 3×10^8 meters per second. Express it in standard form.

09302017

Solution:

$$3\times10^8\ \text{m}/\text{s} = 300{,}000{,}000\times10^{-8}\times10^8\ \text{m}/\text{s}$$

$$= 300{,}000{,}000\times10^{-8+8}\ \text{m}/\text{s}$$

$$= 300{,}000{,}000\times10^0\ \text{m}/\text{s}$$

$$= 300{,}000{,}000\times1\ \text{m}/\text{s}$$

$$= 300{,}000{,}000\ \text{m}/\text{s}$$

Q.4 The circumference of the Earth at the equator is about 40075000 metres. Express this number in scientific notation. **09302018**

Solution:

$$4\,00\,75000\text{m} = 4.0075\times10^7\text{m}$$

Move decimal point 7 places to the left.

Q.5 The diameter of Mars is 6.779×10^3 km. Express this number in standard form. (Correction) **09302019**

Solution:

The diameter of Mars $= 6.779\times10^3$ km

$= 6779\times10^{-3}\times10^3$km

$= 6779\times10^{-3+3}$km

$= 6779\times10^0$km

$= 6779\times1$km

$= 6779$ km

Q.6 The diameter of Earth is about 1.2756×10^4 km. Express this number in standard form. **09302020**

Solution:

1.2756×10^4km $= 12756\times10^{-4}\times10^4$ km

$= 12756\times10^{-4+4}$ km

$= 12756\times10^0$ km

$= 12756\times1$ km

$= 12756$ km

Logarithm

A logarithm is based on two greek words: logos and arithmos which means ratio or proportion. john napier, a scottish mathematician, invented the word logarithm. it is a way to simplify complex calculations, especially those involving multiplication and division of large numbers.

Logarithm of Real number

The logarithm of a number tells us how many times one number must be multiplied by itself to get another number.

The general form of a logarithm is:

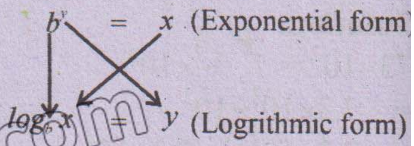
$$\log_b(x) = y$$

Where:

- b is the **base**,
- x is the **result** or the number whose logarithm is being taken,
- y is the **exponent** or the logarithm of x to the base b.

This means that:

$b^y = x$



In words, "the logarithm of x to the base b is y, means that when b is raised to the power y, it equals x. The relationship between logarithmic form and exponential form is given below:

$\log_b(x) = y \Leftrightarrow b^y = x$ where $b > 0$, $x > 0$ and $b \neq 1$

Example 5: Convert $\log_2 8 = 3$ to exponential form. **09302021**

Solution: $\log_2 8 = 3$

Its exponential form is: $2^3 = 8$

Example 6: Convert $\log_{10}100 = 2$ to exponential form. **09302022**

Solution: $\log_{10}100 = 2$

Its exponential form is: $10^2 = 100$

Example 7: Find the value of x in each case:

(i) $\log_5 25 = x$ **09302023**

Solution:

$\log_5 25 = x$

Its exponential form is:

$5^x = 25$

$\Rightarrow 5^x = 5^2$

By comparing exponent, we get

$\Rightarrow x = 2$

(ii) $\log_2 x = 6$ 09302024

Solution:

$\log_2 x = 6$

Its exponential form is: $2^6 = x$

$x = 2\times2\times2\times2\times2\times2$

$\Rightarrow x = 64$

Example 8: Convert the following in logarithmic form:

(i) $3^4 = 81$ 09302025

Solution:

$3^4 = 81$

Its logarithmic form is:

$\log_3 81 = 4$

(ii) $7^0 = 1$

Solution:

$7^0 = 1$

Its logarithmic form is:

$\log_7 1 = 0$

Exercise 2.2

Q.1 Express each of the following in logarithmic form:

(i) $10^3 = 1000$ 09302026

Solution:

$10^3 = 1000$

$\Rightarrow \log_{10} 1000 = 3$

(ii) $2^8 = 256$ 09302027

Solution:

$2^8 = 256$

$\Rightarrow \log_2 256 = 8$

(iii)) $3^{-3} = \frac{1}{27}$ 09302028

Solution:

$3^{-3} = \frac{1}{27}$

$\Rightarrow \log_3 \frac{1}{27} = -3$

iv. $20^2 = 400$ 09302029

Solution:

$20^2 = 400$

$\Rightarrow \log_{20} 400 = 2$

(v) $16^{-\frac{1}{4}} = \frac{1}{2}$ 09302030

Solution:

$16^{-\frac{1}{4}} = \frac{1}{2}$

$\Rightarrow \log_{16} \frac{1}{2} = -\frac{1}{4}$

(vi) $11^2 = 121$ 09302031

Solution:

$11^2 = 121$

$\Rightarrow \log_{11} 121 = 2$

(vii) $p = q^r$

Solution:

$p = q^r$

$\Rightarrow \log_q p = r$

(viii) $(32)^{\frac{-1}{5}} = \frac{1}{2}$ 09302032

Solution:

$(32)^{\frac{-1}{5}} = \frac{1}{2}$

$\Rightarrow \log_{32} \frac{1}{2} = -\frac{1}{5}$

Q.2 Express each of the following in exponential form:

(i) $\log_5 125 = 3$ 09302033

Solution:

$\log_5 125 = 3$

$\Rightarrow 5^3 = 125$

(ii) $\log_2 16 = 4$ 09302034

Solution:

$\log_2 16 = 4$

$\Rightarrow 2^4 = 16$

(iii) $\log_{23} 1 = 0$ 09302035

Solution:

$\log_1 23 = 0$

$\Rightarrow 23^0 = 1$

(iv) $\log_5 5 = 1$ 09302036

$\log_5 5 = 1$

$\Rightarrow 5^1 = 5$

(v) $\log_2 \frac{1}{8} = -3$ 09302037

Solution:

$\log_2 \frac{1}{8} = -3$

$\Rightarrow 2^{-3} = \frac{1}{8}$

(vi) $\frac{1}{2} = \log_9 3$ 09302038

Solution:

$\frac{1}{2} = \log_9 3$

$\Rightarrow 9^{\frac{1}{2}} = 3$

(vii) $5 = \log_{10} 100000$ 09302039

Solution:

$5 = \log_{10} 100000$

$\Rightarrow 10^5 = 100000$

(viii) $\log_4 \frac{1}{16} = -2$ 09302040

Solution:

$\log_4 \frac{1}{16} = -2$

$\Rightarrow 4^{-2} = \frac{1}{16}$

Q.3 Find the value of x in each of the following:

(i) $\log_x 64 = 3$ 09302041

Solution:

$\log_x 64 = 3$

In exponential form

$x^3 = 64$

$x^3 = 4^3$

By comparing bases, we get

$x = 4$

(ii) $\log_5 1 = x$ 09302042

Solution:

$\log_5 1 = x$

In exponential form

$(5)^x = 1$

$5^x = 1$

$5^x = 5^0$

By comparing exponents, we get

$x = 0$

(iii) $\log_x 8 = 1$ 09302043

Solution:

$\log_x 8 = 1$

In exponential form

$(x)^1 = 8$

$\Rightarrow x = 8$

(iv) $\log_{10} x = -3$ 09302044

Solution:

$\log_{10} x = -3$

In exponent form

$10^{-3} = x$

$\frac{1}{10^3} = x$

$\Rightarrow x = \frac{1}{1000}$

$\Rightarrow x = 0.001$

(v) $\log_4 x = \frac{3}{2}$ 09302045

Solution:

$\log_4 x = \frac{3}{2}$

In exponential form

$4^{\frac{3}{2}} = x$

$\left(2^2\right)^{\frac{3}{2}} = x$

$2^3 = x$

$8 = x$

$\Rightarrow \boxed{x = 8}$

(vi) $\log_2 1024 = x$ 09302046

Solution:

$\log_2 1024 = x$

In exponential form

$2^x = 1024$

$2^x = 2^{10}$

By comparing exponents, we get.

$x = 10$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Common Logarithm

The **common logarithm** is the logarithm with a base of 10. It is written as $\log_{10}$ or simply as log (when no base is specified, it is usually assumed to be base 10).

For example:

$$10^1 = 10 \Leftrightarrow \log 10 = 1$$

$$10^2 = 100 \Leftrightarrow \log 100 = 2$$

$$10^3 = 1000 \Leftrightarrow \log 1000 = 3 \text{ and so on.}$$

$$10^{-1} = \frac{1}{10} = 0.1 \Leftrightarrow \log 0.1 = -1$$

$$10^{-2} = \frac{1}{100} = 0.01 \Leftrightarrow \log 0.01 = -2$$

$$10^{-3} = \frac{1}{1000} = 0.001 \Leftrightarrow \log 0.001 = -3 \text{ and so on.}$$

History

English mathematician Henry Briggs built Napier's work and developed the common logarithm. He also invented logarithmic table.

For further information, you can use the following link:

https://www.britannica.com/biography/Henry-Briggs

Characteristic and Mantissa of Logarithms

A logarithm of a number consists of two parts: **the characteristic** and **the mantissa**. Here is a simple way to understand them:

Characteristic

The characteristic is the integral part of the logarithm. It tells us how big or small the number is.

Rules for Finding the Characteristic

(i) For a number greater than 1:
Characteristic = **number of digits** to the left of the decimal point – 1

For example, in log 567 the characteristic = 3 – 1 = 2

(ii) For a number less than 1:
Characteristic = – (**number of zeros** between the decimal point and the first non-zero digit + 1)

For example, in log 0.0123 the characteristic = – (1+1) = –2 or $\bar{2}$

Example 9: Find characteristic of the followings:

(i) log 725 **09302047** (ii) log 9.87 **09302048**

(iii)log 0.00045 **09302049** (iv) log 0.54 **09302050**

(i) log 725

Solution

Characteristic = 3 – 1 = 2

(ii)log 9.87

Solution:

Remember!
log (Number) = Characteristic + Mantissa

Characteristic = 1 – 1 = 0

(iii) log 0.00045

Characteristic = – (3 + 1) = $\bar{4}$

(iv) log 0.54

Characteristic = – (0+1) = $\bar{1}$

Mantissa

The mantissa is the **decimal part** of the logarithm. It represents the "fractional" component and is always positive.

For example, in log 5000 = 3.698 the mantissa is 0.698

Finding Common Logarithm of a Number

Suppose we want to find the common logarithm of 13.45. The step-by-step procedure to find the logarithm is given below:

Step 1: Separate the integral and decimal parts.

Integral part = 13
Decimal part = 45

Step 2: Find the characteristic of the number

Characteristic = **number of digits** to the left of the decimal point – 1

= 2 – 1 = 1

Step 3: In common logarithm table (Complete table is given at the end of the book), check the intersection of row number 13 and column number 4 which is 1271.

Step 4: Find mean difference: Check the intersection of row number 13 and column number 5 in the mean difference which is 16.

Logarithm Table

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27

Step 5: Add the numbers found in step 3 and step 4. i.e. 1271 + 16 = 1287 which is the mantissa.

Step 6: Finally, combine the characteristic and mantissa parts found in step 2 and step 5 respectively. We get 1.1287

So, the value of log 13.45 is 1.1287

Example 10: Find logarithm of the following numbers:

(i) log 345 (ii) log 5.678

(iii)log 0.0036 (iv) log 0.0478

(i) log 345 **09302051**

Solution:

Characteristic = 3 – 1 = 2

Mantissa = 0.5378

(Look for 34 in the row and 5 in the column of the log table)

So, log(345) = 2 + 0.5378 = 2.5378

(ii) log 5.678 **09302052**

Solution:

Characteristic = 1 – 1 = 0

Mantissa = 0.7542 = (7536 + 6 = 7542)

So, log(5.678) = 0 + 0.7542 = 0.7542

Do you know?

log (0) = undefined

log (1) = 0

$log_a(a) = 1$

(iii) log 0.0036 **09302053**

Characteristic = – (2 + 1) = –3

Mantissa = 0.5563

(Look for 36 in the row and 0 in the column of the log table)

So, log(0.0036) = – 3 + 0.5563 = $\bar{3}$.5563

(iv) log 0.0478 **09302054**

Characteristic = – (1 + 1) = – 2 = $\bar{2}$

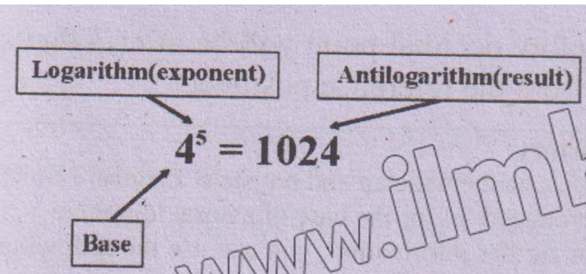
Mantissa = 0.6794

(Look for 47 in the row and 8 in the column of the log table)

So, log(0.0478) = – 2 + 0.6794 = $\bar{2}$.6794

Concept of Antilogarithm

An **antilogarithm** is the inverse operation of a logarithm. While a logarithm tells you the exponent to which a base (usually 10 for common logarithms) must be raised to obtain a particular number, the antilogarithm tells you what that number is, when you have the logarithm.



In simple terms:
If $log_b(x) = y \Leftrightarrow b^y = x$ then the process of finding x is called antilogarithm of y.

Find Antilogarithm of a number using tables:

Let us find the antilogarithm of 2.1245.
The step-by-step procedure to find the antilogarithm is given below:

Step 1: Separate the characteristic and mantissa parts:
Characteristic = 2
Mantissa = 0.1245

Step 2: Find corresponding value of mantissa from antilogarithm table (given at the end of the book):
Check the intersection of row number 12 and column number 4 which provides the number 1330.

> **Remember!**
> The word antilogarithm is another word for the number or result. For example, in $4^3 = 64$, the result 64 is the antilogarithm.

Step 3: Find the mean difference:
Check the intersection of row number 12 and the column number 5 of the mean difference in the antilogarithm table which gives 2.

Antilogarithm Table

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3

Step 4: Add the numbers found in the step 2 and step 3, we get 1330 + 2 = 1332

Step 5: Insert the decimal point:
Since characteristic is 2, therefore the decimal point will be after 2 digits right from the reference position. So, we get 133.2.
Thus, the antilog (2.1245) = $1_{\wedge}33.2$

Example 11: Find the value of x in the followings:
(i) $\log x = 0.256$
(ii) $\log x = -1.4567$
(iii) $\log x = -2.1234$

> **Remember!**
> The place between the first non-zero digit from left and its next digit is called reference position. For example, in 1332, the reference position is between 1 and 3

(i) **$\log x = 0.2568$** 09302055

Solution:
Characteristic = 0
Mantissa = .2568
Table value = 1803 + 3 = 1806
So, x = antilog (0.2568) = 1.806
(Insert the decimal point at reference position).

(ii) $\log x = -1.4567$ 09302056

Solution:
Since mantissa is negative, so we make it positive by adding and subtracting 2

$$\log x = -2 + 2 - 1.4567$$
$$= -2 + 0.5433 = \bar{2}.5433$$

Here characteristic = $\bar{2}$ and
Mantissa = 0.5433
Table value = 3491 + 2 = $3_{\wedge}493$

So, x = antilog ($\bar{2}$.5433)
= 0.03493

Since characteristic is $\bar{2}$, therefore decimal point will be after 2 digits left from the reference position

(iii)log $x = -2.1234$ 09302057

Solution

Since mantissa is negative, so we make it positive by adding and subtracting 3

$\log x \quad = -3 + 3 - 2.1234$

$\qquad = -3 + 0.8766 = \bar{3}.8766$

Here characteristic = $\bar{3}$, mantissa = 0.8766

Table value = 7516 + 10 = 7‸526

So, $\quad x = \text{antilog}(\bar{3}.8766)$

= 0.007526 (Since characteristic $= \bar{3}$ therefore decimal point will be after 3 digits left from the reference position.)

History:
Swiss mathematician and physicist Leonhard Euler introduced 'e' for the base of natural logarithm.
For further information, you can use the following link:
https://www.britannica.com/biography/Leonhard-Euler

Natural Logarithm

The natural logarithm is the logarithm with base *e*, where *e* is a mathematical constant approximately equal to 2.71828. it is denoted as ℓ_n.

Difference between Common and Natural Logarithms

Common Logarithm	Natural Logarithm
i. The base of a common logarithm is 10.	i. The base of a natural logarithm is *e*.
ii. It is written as $\log_{10}(x)$ or simply log (x) when no base is specified.	ii. It is written as $ln(x)$
iii. Common logarithms are widely used in everyday calculations, especially in scientific and engineering applications.	iii. Natural logarithms are commonly used in higher level mathematics particularly calculus and applications involving growth processes.

Exercise 2.3

Q.1 Find characteristic of the following numbers:

(i) 5287 09302058

Solution:

5287

Number of digits before decimal place = 4

Characteristic = 4−1 = 3

(ii) 59.28 09302059

Solution:

59.28

No of digits before decimal place = 2

Characteristic = 2−1 =1

(iii) 0.0567 09302060

Solution:

0.0567

No.of zeros between decimal point and 1st non-zero digit=1

Characteristic = − (1+1) = −2 or $\bar{2}$.

(iv) 234.7 09302061

Solution:

234.7

No. of digits before decimal point = 3

Characteristic = 3−1 = 2

(v) 0.000049 09302062

Solution:

0.000049

No. of zeros between decimal point and 1st non-zero digit = 4

Characteristic = − (4+1) = −5 or $\bar{5}$.

(vi) 145000 09302063

Solution:

145000

No. of digits before decimal point = 6

Characteristic = 6–1 = 5

Q.2 Find logarithm of the following numbers:

(i) 43 09302064

Solution:

log 43

Characteristic = 2–1 = 1

Mantissa = .6335

log 43 = 1 + .6335

log 43 = 1.6335

(ii) 579 09302065

Solution:

log 579

Characteristic = 3–1 = 2

Mantissa = .7627

log 579 = 2 + .7627

log 579 = 2.7627

(iii) 1.982 09302066

Solution:

log 1.982

Characteristic = 1–1 = 0

Mantissa = .2971

log 1.982 = 0 + .2971

log 1.982 = 0.2971

(iv) 0.0876 09302067

Solution:

log 0.0876

Characteristic = –(1+1) = –2 or $\bar{2}$

Mantissa = .9425

log 0.876 = $\bar{2}$ +.9425

log 0.0876 = $\bar{2}$. 9425

(v) 0.047 09302068

Solution:

log 0.047

Characteristic = –(1+1) = –2 or $\bar{2}$

Mantissa = .6721

log 0.876 = $\bar{2}$ +.6721

log 0.0876 = $\bar{2}$ .6721

(vi) 0.000354 09302069

Solution:

log 0.000354

Characteristic = –(3+1) = –4 or $\bar{4}$

Mantissa = .5490

log 0.000354 = $\bar{4}$ +.5490

log 0.000354 = $\bar{4}$ .5490

Q.3 If log 3.177 = 0.5019, then find:

(i) log 3177 09302070

Solution:

Characteristics = 4–1 = 3

Mantissa= .5019

log 3177 = 3+ .5019 = 3.5019

(ii) log 31.77 09302071

Solution:

Characteristics = 2–1 = 1

Mantissa = .5019

log 31.77 = 1+ .5019 = 1.5019

(iii) log 0.03177 09302072

Solution:

Characteristics = –(1+1) = –2 or $\bar{2}$

Mantissa = .5019

log 0.03177= $\bar{2}$ + .5019

= $\bar{2}$ .5019

Q.4 Find the value of *x*.

(i) log *x* = 0.0065 09302073

Solution:

log *x* = 0.0065

characteristic = 0

Mantissa = .0065

x = antilog (0.0065)

x = 1.015

(ii) log *x* = 1.192 09302074

Solution:

log *x* = 1.192

Characteristics = 1

Mantissa = .192

x = antilog (1.192)

x = 1∧5.56 = 15.56

(iii) log *x* = –3.434 09302075

Solution:

log *x* = –3.434

To make mantissa positive add and subtract 4.

log *x* = –4+4–3.434

log *x* = –4 + .566 = $\bar{4}$.566

Characteristics = $\bar{4}$ or –4

Mantissa = .566

$x = \text{antilog}\,(\bar{4}.566)$

$x = 1\wedge 5.56 = 15.56$

$x = 0.0003\wedge 681$

$x = 0.0003681$

(iv) $\log x = -1.5726$ 09302076

Solution:

-1.5726

To make mantissa positive add and subtract 2.

$\log x = -2 + 2 - 1.5726$

$\log x = -2 + .4274$

$\log x = \bar{2}.4274$

Characteristics = $\bar{2}$ or -2

Mantissa = .4274

Table value
2673+3=2676

$x = \text{antilog}\,(\bar{2}.4274)$

$x = \text{antilog}\,(\bar{2}.4274)$

$x = 0.02\wedge 676$

$x = 0.02676$

(v) $\log x = 4.3561$ 09302077

Solution:

$\log x = 4.3561$

Characteristic = 4

Mantissa = .3561

$x = \text{antilog}\,(4.3561) = 2_{\wedge}2710. = 22710$

(vi) $\log x = -2.0184$ 09302078

Solution:

$\log x \;= -2.0184$

To make mantissa positive add and subtract 3.

$\log x \;= -3 + 3 - 2.0184$

$\quad\;\; = \bar{3} + .9816$

$x \;= \text{antilog}\,(\bar{3}.9816)$

$x \;= 0.009585$

Laws of Logarithm

Laws of logarithm are also known as rules or properties of logarithm. These laws help to simplify logarithmic expressions and solve logarithmic equations.

1. Product Law

$\log_b xy = \log_b x + \log_b y$

The logarithms of a product are the sum of the logarithms of the factors.

Proof: Let

$m = \log_b x \qquad \ldots(\text{i})$

and $n = \log_b y \qquad \ldots(\text{ii})$

In exponential form:

$x = b^m$ and $y = b^n$

Multiply x and y

$x.y = b^m . b^n = b^{m+n}$

In logarithmic form:

$\log_b xy = m + n$

$\log_b xy = \log_b x + \log_b y$ [From (i) and (ii)]

2. Quotient Law

$\log_b \left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

The logarithm of a quotient is the difference between the logarithms of the numerator and the denominator.

Proof: Let

$m = \log_b x \qquad \ldots(\text{i})$

and $n = \log_b y \qquad \ldots(\text{ii})$

In exponential form:

$x = b^m$ and $y = b^n$

Divide x and y

$$\frac{x}{y}=\frac{b^m}{b^n}=b^{m-n}$$

In logarithmic form:

$$\log_b\left(\frac{x}{y}\right)= m-n$$

$$\log_b\left(\frac{x}{y}\right)= \log_b x - \log_b y$$

3. Power Law

$\log_b x^n = n.\log_b x$

The logarithm of a number raised to a power is the product of the power and the logarithm of the base number.

Proof: Let

$m=\log_b x \qquad \ldots(i)$

In exponential form:

$$x=b^m$$

Raise both sides to the power n

$$x^n=(b^m)^n=b^{mn}$$

In logarithmic form:

$\log_b x^n = mn$

$\log_b x^n = n.\log_b x \qquad$ [From (i)]

4. Change of Base Law

$$\log_b x=\frac{\log_a x}{\log_a b}$$

This law allows to change the base of a logarithm from "b" to any other base "a".

Proof: Let

$m=\log_b x \qquad \ldots(i)$

In exponential form:

$b^m=x$

Taking $\log_a$ on both sides

$$\log_a b^m=\log_a x$$

$$m\log_a b=\log_a x$$

$$m=\frac{\log_a x}{\log_a b}$$

$\log_b x=\dfrac{\log_a x}{\log_a b} \qquad$ [From (i)]

Applications of Logarithm

Logarithms have a wide range of applications in many fields. Here some examples are given about the applications of logarithms.

Example 12: Expand the following logarithms:

(i) $\log_3(20)$ (ii) $\log_2(9)^5$

(iii) $\log_{32}27$

(i) $\log_3(20)$ 09302079

Solution:

$=\log_3(2\times 2\times 5)$

$=\log_3(2^2\times 5)$

$=\log_3(2)^2+\log_3 5$

$=2\log_3 2+\log_3 5$

(ii) $\log_2(9)^5$ 09302080

Solution:

$=\log_2(3^2)^5$

$=\log_2(3)^{10}$

$=10\log_2 3$

(iii) $\log_{32}27$ 09302081

Solution:

$\log_{32}27$

$$=\frac{\log 27}{\log 32}$$

$$=\frac{\log 3^3}{\log 2^5}$$

$$=\frac{3\log 3}{5\log 2}$$

Example 13: Expand the following logarithms:

(i) $\log_2\left(\dfrac{x-y}{z}\right)^3$ **(ii)** $\log_5\left(\dfrac{xy}{z}\right)^8$

Solution:

(i) $\log_2\left(\dfrac{x-y}{z}\right)^3$ 09302082

$$\log_2\left(\frac{x-y}{z}\right)^3$$

$$=3\log_2\left(\frac{x-y}{z}\right)$$

$$=3[\log_2(x-y)-\log_2 z]$$

(ii) $\log_5\left(\frac{xy}{z}\right)^8 = 8\log_5\left(\frac{xy}{z}\right)$ **09302083**

$$=8[\log_5(xy)-\log_5 z]$$

$$=8[\log_5 x+\log_5 y-\log_5 z]$$

Example 14: Write as a single logarithm:

(i) $2\log_3 10-\log_3 4$ **09302084**

(ii) $6\log_3 x+2\log_3 11$ **09302085**

Solution:

(i) $2\log_3 10-\log_3 4$

$$=\log_3(10)^2-\log_3 4$$

$$=\log_3 100-\log_3 4$$

$$=\log_3\left(\frac{100}{4}\right)$$

$$=\log_3 25$$

(ii) $6\log_3 x+2\log_3 11$

$$=\log_3 x^6+\log_3(11)^2$$

$$=\log_3 x^6+\log_3(121)$$

$$=\log_3(121x^6)$$

Example 15: The decibel scale measures sound intensity using the formula $L=40\log_{10}\left(\frac{I}{I_o}\right)$. If a sound has an intensity (I) of 10^6 times the reference intensity (I_o). What is the sound level in decibels? **09302086**

Solution:

Do you know?
$ln\,(0)$ = undefined
$ln\,(1)=0$
$ln\,(e)=1$

$$L=40\log_{10}\left(\frac{I}{I_o}\right)$$

Put $I=10^6 I_o$, we get

$$L=40\log_{10}\left(\frac{10^6 I_o}{I_o}\right)$$

$$L=40\log_{10}(10)^6$$

$$L=40\times 6\log_{10}10$$

$$L=40\times 6 \qquad (\because \log_{10}10=1)$$

$$L=240 \text{ decibels}$$

Exercise 2.4

Q.1 Without using calculator, evaluate the following:

(i) $\log_2 18-\log_2 9$ **09302087**

Solution:

$$\log_2 18-\log_2 9$$

$$=\ \log_2\left(\frac{18}{9}\right)$$

$$=\ \log_2 2 \qquad \because \log_a a=1$$

$$=\ 1$$

(ii) $\log_2 64+\log_2 2$ **09302088**

Solution:

$$\log_2 64+\log_2 2$$

$$=\ \log_2(64\times 2)$$

$$=\ \log_2(128)$$

$$=\ \log_2(2^7)$$

$$=\ 7\log_2 2 \qquad \because \log_2 2=1$$

$$=\ 7(1)=7$$

(iii) $\frac{1}{3}\log_3 8-\log_3 18$ **09302089**

Solution:

$$\frac{1}{3}\log_3 8-\log_3 18$$

$$=\ \log_3 8^{\frac{1}{3}}-\log_3 18$$

$$=\ \log_3\left(\frac{8^{\frac{1}{3}}}{18}\right)$$

$= \log_3 \dfrac{\left(2^3\right)^{\frac{1}{3}}}{18}$

$= \log_3 \dfrac{2}{18}$

$= \log_3 \dfrac{1}{9}$

$= \log_3 \dfrac{1}{3^2}$

$= \log_3 3^{-2}$

$= -2\log_3 3$

$= -2\ (1) \qquad (\because \log_a a = 1)$

$= -2$

(iv) $2\log 2 + \log 25$ **09302090**

Solution:

$2\log2+\log25$

$= \log2^2+\log25$

$= \log4 + \log 25$

$= \log (4\times25)$

$= \log 100$

$= \log 10^2$

$= 2 \log 10$

$= 2\ (1) = 2 \qquad \because \log10 = 1$

(v) $\dfrac{1}{3}\log_4 64 + 2\log_5 25$ **09302091**

Solution:

$\dfrac{1}{3}\log_4 64 + 2\log_5 25$

$= \dfrac{1}{3}\log_4\left(4^3\right) + 2\log_5 5^2$

$= \dfrac{1}{3}\times 3\log_4 4 + 2\times 2\log_5 5$

$= 1(1) + 4(1) \qquad (\because \log_a a = 1)$

$= 1+4$

$= 5$

vi) $\log_3 12 + \log_3 0.25$ **09302092**

Solution:

$\log_3 12+\log_3 0.25$

$= \log_3 (12\times0.25)$

$= \log_3 (3) = 1$

Q.2 Write the following as a single logarithm.

(i) $\dfrac{1}{2}\log 25 + 2 \log 3$ **09302093**

Solution:

$\dfrac{1}{2}\log 25 + 2 \log 3$

$= \log(25)^{\frac{1}{2}} + \log 3^2$

$= \log\left(5^2\right)^{\frac{1}{2}} + \log 9$

$= \log5 + \log 9$

$= \log(5\times9)$

$= \log 45$

(ii) $\log 9 - \log \dfrac{1}{3}$ **09302094**

Solution:

$\log 9 - \log \dfrac{1}{3}$

$= \log\left(\dfrac{9}{\frac{1}{3}}\right)$

$= \log (9\times3)$

$= \log 27$

(iii) $\log_5 b^2.\log_a 5^3$ **09302095**

Solution:

$\log_5 b^2.\log_a 5^3$

$= 2\log_5 b.3\log_a 5$

$= \dfrac{6\log b}{\cancel{\log 5}}\times\dfrac{\cancel{\log 5}}{\log a}$ (by change of base law)

$= \dfrac{6\log b}{\log a}$

$= 6 \log_a b$

(iv) $2\log_3 x + \log_3 y$ **09302096**

Solution:

$2\log_3 x + \log_3 y$

$= \log_3 x^2 + \log_3 y$

$= \log_3\left(x^2 y\right)$

(v) $4\log_5 x - \log_5 y + \log_5 z$ **09302097**

Solution:

$4 \log_5 x - \log_5 y + \log_5 z$

$$= \log_5 x^4 - \log_5 y + \log_5 z$$

$$= \log_5\left(\frac{x^4}{y}\right) + \log_5 z$$

$$= \log_5\left(\frac{x^4}{y} \times z\right)$$

$$= \log_5\left(\frac{x^4 z}{y}\right)$$

(vi) $2 \ln a + 3 \ln b - 4 \ell n c$ 09302098

Solution:

$$2 \ell n a + 3 \ell n b - 4 \ell n c$$

$$= \ell n a^2 + \ell n b^3 - \ell n c^4$$

$$= \ell n\left(a^2 \times b^3\right) - \ell n c^4$$

$$= \ell n\left(\frac{a^2 b^3}{c^4}\right)$$

Q.3 Expand the following using laws of logarithms:

(i) $\log\left(\frac{11}{5}\right)$ 09302099

Solution:

$$\log\left(\frac{11}{5}\right) = \log 11 - \log 5$$

(ii) $\log_5 \sqrt{8a^6}$ 09302100

Solution:

$$\log_5 \sqrt{8a^6}$$

$$= \log_5\left(8a^6\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\log_5\left(8a^6\right)$$

$$= \frac{1}{2}(\log_5 8 + \log_5 a^6)$$

$$= \frac{1}{2}[\log_5 2^3 + \log_5 a^6]$$

$$= \frac{1}{2}\ [3\log_5 2 + 6\log_5 a]$$

$$= \frac{1}{2} \times 3\log_5 2 + \frac{1}{2} \times 6\log_5 a]$$

$$= \frac{3}{2}\log_5 2 + 3\log_5 a$$

(iii) $\ell n\left(\frac{a^2 b}{c}\right)$ 09302101

Solution:

$$\ell n\left(\frac{a^2 b}{c}\right)$$

$$= \ell n a^2 b - \ell n c$$

$$= \ell n a^2 + \ell n b - \ell n c$$

$$= 2\ell n a + \ell n b - \ell n c$$

(iv) $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$ 09302102

Solution:

$$\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$$

$$= \frac{1}{9}\log\left(\frac{xy}{z}\right)$$

$$= \frac{1}{9}(\log xy - \log z)$$

$$= \frac{1}{9}(\log x + \log y - \log z)$$

$$= \frac{1}{9}\log x + \frac{1}{9}\log y - \frac{1}{9}\log z$$

(v) $\ell n\sqrt[3]{16x^3}$ 09302103

Solution:

$$\ell n\sqrt[3]{16x^3}$$

$$= \ell n\left(16x^3\right)^{\frac{1}{3}}$$

$$= \frac{1}{3}\left[\ell n 16x^3\right]$$

$$= \frac{1}{3}\left[\ell n 16 + \ell n x^3\right]$$

$$= \frac{1}{3}\left[\ell n 16 + 3\ell n x\right]$$

$$= \frac{1}{3}\ell n 16 + \frac{1}{\cancel{3}} \times \cancel{3}\ell n x$$

$$= \frac{1}{3}\ell n 2^4 + \ell n x$$

$$= \frac{1}{3} \times 4\ell n 2 + \ell n x$$

$= \frac{4}{3}\ell n2+\ell nx$

(vi) $\log_2\left(\frac{1-a}{b}\right)^5$ **09302104**

Solution:

$\log_2\left(\frac{1-a}{b}\right)^5$

$= 5\log_2\left(\frac{1-a}{b}\right)$

$= 5\left[\log_2(1-a)-\log_2 b\right]$

$= 5\log_2(1-a)-5\log_2 b$

Q.4 Find the value of *x* in the following equations:

(i) $\log 2+\log x=1$ **09302105**

Solution:

$\log 2+\log x=1$

$\log_{10} 2x=1$

In exponential form

$10^1=2x$

$\frac{10}{2}=x$

$5=x$

$\Rightarrow x=5$

(ii) $\log_2 x+\log_2 8=5$ **09302106**

Solution:

2	8
2	4
2	2
	1

$\log_2 x+\log_2 8=5$

$\log_2(x\times 8)=5$

$\log_2 8x=5$

In exponential form, we get

$2^5=8x$

$32=8x$

$\frac{32}{8}=x$

$4=x$

$\Rightarrow x=4$

(iii) $(81)^x=(243)^{x+2}$ **09302107**

Solution:

$(81)^x=(243)^{x+2}$

3	243
3	81
3	27
3	9
3	3
	1

$(3^4)^x=(3^5)^{x+2}$

$3^{4x}=3^{5x+10}$

By comparing exponents, we get

$4x=5x+10$

$-10=5x-4x$

$-10=x$

$\Rightarrow x=-10$

(iv) $\left(\frac{1}{27}\right)^{x-6}=27$ **09302108**

3	27
3	9
3	3
	1

Solution:

$\left(\frac{1}{27}\right)^{x-6}=27$

$(27^{-1})^{x-6}=(27)^1$

$(27)^{-x+6}=(27)^1$

By comparing exponents, we get

$-x+6=1$

$-x=1-6$

$-x=-5$

$\Rightarrow x=5$

(v) $\log(5x-10)=2$ **09302109**

Solution:

$\log_{10}(5x-10)=2$

In exponential form, we get

$10^2=5x-10$

$100=5x-10$

$100+10=5x$

$110=5x$

$\frac{110}{5}=x$

$22=x$

$\Rightarrow x=22$

(vi) $\log_2(x+1)-\log_2(x-4)=2$ **09302110**

Solution:

$\log_2(x+1)-\log_2(x-4)=2$

By quotient law

$\Rightarrow \log_2\left(\frac{x+1}{x-4}\right)=2$

In exponential form, we get

$2^2=\frac{x+1}{x-4}$

$$4 = \frac{x+1}{x-4}$$

$\Rightarrow 4(x-4) = x+1$

$4x - 16 = x+1$

$4x - x = 1+16$

$3x = 17$

$x = \frac{17}{3} = 5\frac{2}{3}$

$$\begin{array}{r} 5 \\ 3\overline{)17} \\ \underline{15} \\ 2 \end{array}$$

Q.5 Find the values of the following with the help of logarithm table:

(i) $\frac{\mathbf{3.68 \times 4.21}}{\mathbf{5.234}}$ **09302111**

Solution:

Let $x = \frac{3.68 \times 4.21}{5.234}$

Taking log of both sides.

$\log x = \log\left(\frac{3.68 \times 4.21}{5.234}\right)$

By applying laws of logarithm.

$\log x = \log 3.68 + \log 4.21 - \log 5.234$

$\log x = 0.5658 + 0.6243 - 0.7188$

$\log x = 1.1901 - 0.7188$

$\log x = 0.4713$

Characteristic = 0

Mantissa = .4713

$x = \text{antilog}\ (0.4713)$

$x = 2.960$

Thus $\frac{3.68 \times 4.21}{5.234} \approx 2.960$

(ii) 4.67 × 2.11 × 2.397

Solution:

Let $x = 4.67 \times 2.11 \times 2.397$

Taking log of both sides.

$\log x = \log\ (4.67 \times 2.11 \times 2.397)$

By applying law of logarithm.

$\log x = \log 4.67 + \log 2.11 + \log 2.397)$

$\log x = 0.6693 + 0.3243 + 0.3796$

$\log x = 1.3732$

Characteristic = 1

Mantissa =.3732

$x = \text{antilog}\ (1.3732)$

$x \approx 2_{\wedge}3.\ 62 \approx 2_{\wedge}3.62$

Thus $4.67 \times 2.11 \times 2.397 \approx 23.62$

(iii) $\frac{\mathbf{(20.46)^2 \times (2.4122)}}{\mathbf{754.3}}$ **09302112**

Solution:

Let $x = \frac{(20.46)^2 \times (2.412)}{754.3}$

$(\because 2.4122 \approx 2.412)$ (Round off)

Taking log of both sides.

$\log x = \log \frac{(20.46)^2 \times (2.412)}{754.3}$

By applying the laws of logarithm.

$\log x = \log(20.46)^2 + \log(2.412) - \log(754.3)$

$\log x = 2\ \log(20.46) + \log(2.412) - \log(754.3)$

$\log x = 2\ (1.3109) + (0.3824) - (2.8776)$

$\log x = 2.6218 + 0.3824 - 2.8776$

$\log x = 3.0042 - 2.8776$

$\log x = 0.1266$

Characteristic = 0

Mantissa = .1266

$x = \text{antilog}\ (0.1266)$

$x \approx 1_{\wedge}.339$

Thus $\frac{(20.46)^2 \times (2.412)}{754.3} \approx 1.339$

(iv) $\frac{\mathbf{\sqrt[3]{9.364} \times 21.64}}{\mathbf{3.21}}$ **09302113**

Solution:

Let $x = \frac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Taking log of both sides .

$\log x = \frac{\sqrt[3]{9.364} \times 21.64}{3.21}$

By applying the laws of logarithm.

$\log x = \log \sqrt[3]{9.364} + \log 21.64 - \log 3.21$

$\log x = \log(9.364)^{\frac{1}{3}} + \log 21.64 - \log 3.21$

$\log x = \frac{1}{3} \log\left(9_{\wedge}.364\right) + \log\ 2_{\wedge}1.64 - \log 3.21$

$\log x = \frac{1}{3}\ (0.9715) + (1.3353) - (0.5065)$

$\log x = 0.3238 + 1.3353 - 0.5065$

$\log x = 1.6591 - 0.5065$

$\log x = 1.1526$

Characteristic = 1

Mantissa =.1526

x = antilog (1.1526)

$x = 1_{\wedge}4.21$

Thus $\dfrac{\sqrt[3]{9.364}\times 21.64}{3.21} = 14.21$

Q.6 The formula to measure the magnitude of earthquakes is given by $M=\log_{10}\left(\dfrac{A}{A_o}\right)$. If amplitude ($A$) is 10,000 and reference amplitude (A_o) is 10. What is the magnitude of the earthquake?

09302114

Solution:

Given that $M = \log_{10}\left(\dfrac{A}{A_o}\right)$, $A = 10{,}000$,

$A_o=10$

Putting the values

$M = \log_{10}\left(\dfrac{10{,}000}{10}\right)$

$M = \log_{10}10^3$

$M = 3\ \log_{10}10$

$M = 3\ (1)$ $(\because\ \log_a a = 1)$

$M = 3$

Thus the magnitude of earthquake is 3.

Q.7 Abdullah invested Rs. 100,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after *t* years is Rs *y*. This is modelled by an equation $y = 100{,}000\ (1.05)^t,\ t \geq 0$. **09302115**
Find after how many years the investment will be double.

Solution:

Given that $y = 100{,}000\ (1.05)^t$ _____ (i)

Single investment = Rs.100,000/-

Double investment = Rs.100,000×2

= Rs.200,000/-

Let investment after t years is y.

y = Rs.200,000 putting the value in eq.(i).

$200{,}000 = 100{,}000\ (1.05)^t$

$\dfrac{200{,}000}{100{,}000} = (1.05)^t$

$2 = (1.05)^t$

Taking log of both sides

$\log 2 = \log(1.05)^t$

$\log 2 = t \times 0.0212$

$\dfrac{0.3010}{0.0212} = t$

$14.19 = t$

$\Rightarrow\ = 14.19$ years

On rounding off we get

$t \approx 14$ years

The investment will be double after 14 years.

Q.8 Huria is hiking up a mountain where the temperature decreases by 3% (or a factor of 0.97) for every 100 metres gained in altitude. The initial temperature at sea level is 20°C. Using the formula $T=T_i\times(0.97)^{\frac{h}{100}}$, calculate the temperature at an altitude of 500 metres.

09302116

Solution:

Given that $T = T_o \times\ (0.97)^{\frac{h}{100}}$ _____ (i)

Initial temperature at sea level = T_i = 20c°

Altitude or height = h = 500 m.

Putting the values in eq.(i)

$T = 20 \times\ (0.97)^{\frac{500}{100}}$

$T = 20 \times (0.97)^5$

Taking log of both sides

$\log T = \log 20\times(0.97)^5$

$\log T = \log 20+\log(0.97)^5$

$\log T = \log 20+5\log 0.97$

$\log T = 1.3010+5(\bar{1}.9867)$

$\log T = 1.3010+5(-1+.9867)$

$\log T = 1.3010+5(-0.0133)$

$\log T = 1.3010-0.0665$

$\log T = 1.2345$

Characteristic =1

Mantissa = .2345

T = antilog (1.2345)

$T \approx 17.16$

Thus temperature at an attitude of 500m will be 17.16°c approximately.

Review Exercise 2

Q.1 Choose the correct option.

i. The standard form of 5.2×10^6 is: **09302117**

(a) 52,000 (b) 520,000

(c) 5,200,000 (d) 52,000,000

ii. Scientific notation of 0.00034 is: **09302118**

(a) 3.4×10^3 (b) 3.4×10^{-4}

(c) 3.4×10^4 (d) 3.4×10^{-3}

iii. The base of common logarithm is: **09302119**

(a) 2 (b) 10

(c) 5 (d) *e*

iv. $\log_2 2^3 =$ __________ **09302120**

(a) 1 (b)2

(c) 5 (d) 3

v. log 100 = __________ **09302121**

(a) 2 (b) 3

(c) 10 (d) 1

vi. If log 2 = 0.3010, then log 200 is: **09302122**

(a) 1.3010 (b) 0.6010

(c) 2.3010 (d) 2.000

vii. log (0) = **09302123**

(a) positive (b) negative

(c) zero (d) undefined

viii. log 10,000 = **09302124**

(a) 2 (b) 3

(c) 4 (d) 5

ix. log 5 + log 3 = __________. **09302125**

(a) log 0 (b) log 2

(c) $\log\left(\frac{5}{3}\right)$ (d) log 15

x. $3^4 = 81$ in logarithmic form is: **09302126**

(a) $\log_3 4 = 81$ (b) $\log_4 3 = 81$

(c) $\log_3 81 = 4$ (d) $\log_4 81 = 3$

Answers Key

i	c	ii	b	iii	b	iv	d	v	a
vi	c	vii	d	viii	c	ix	d	x	c

Multiple Choice Questions (Additional)

Scientific Notation

1. If a = b × 10^n is written in scientific notation then: **09302127**

(a) $0 \le b \le 10$ (b) $0 \le b < 10$

(c) $1 \le b \le 10$ (d) $1 \le b < 10$

Characteristic

2. For the log 0.00327, characteristic is: **09302128**

(a) −2 (b) −3

(c) 3 (d) 0

3. Characteristic of log 4.9×10^{-5} is: **09302129**

(a) −5 (b) 10

(c) 4 (d) 9

Logarithmic and exponential form:

4. If $a^x = n$, then: **09302130**

(a) $a = \log_x n$ (b) $x = \log_n a$

(c) $x = \log_a n$ (d) $a = \log_n x$

5. The relation of $y = \log_z x$ implies: **09302131**

(a) $x^y = z$ (b) $z^y = x$

(c) $x^z = y$ (d) $y^z = x$

Basic concept of logarithms

6. The logarithm of unity to any base is: **09302132**

(a) 1 (b) 10

(c) e (d) 0

7. The logarithm of any number to itself as base is: **09302133**

(a) 1 (b) 0

(c) −1 (d) 10

8. The base of natural logarithm is: **09302134**

(a) 0 (b) 1

(c) 10 (d) e

Finding the value of unknown:

9. If log $(x + 3)$ = log $(15x - 4)$ then x is: **09302135**

(a) 0.5 (b) 7
(c) 17 (d) 2

10. If $\log_{\sqrt{x}} 25 = 4$ then x is: **09302136**

(a) +5 (b) −5
(c) ±5 (d) mpossible

Laws of logarithms

11. $\log_b g^h$ is: **09302137**

(a) $g \log_b h$ (b) $\log_b (gh)$
(c) $(\log_b g) \times h$ (d) $h \log_g b$

12. $\log_b x - \log_b y$ is: **09302138**

(a) $\dfrac{\log_b x}{\log_b y}$ (b) $\log_b \dfrac{x}{y}$
(c) $\log_y x$ (d) $\dfrac{\log_b y}{\log_b x}$

13. $\log_b a \times \log_c b$ can be written as: **09302139**

(a) $\log_c a$ (b) $\log_a c$
(c) $\log_a b$ (d) $\log_b c$

14. $\text{Log}_y x$ will be equal to: **09302140**

(a) $\dfrac{\log_z x}{\log_y z}$ (b) $\dfrac{\log_x z}{\log_y z}$
(c) $\dfrac{\log_z x}{\log_z y}$ (d) $\dfrac{\log_z y}{\log_z x}$

15. $\text{Log}\left(\dfrac{1}{n}\right)$ = ------- **09302141**

(a) log1 (b) log n
(c) log (1 − n) (d) − logn

16. If $\log\dfrac{a}{b} + \log\dfrac{b}{a} = \log(a+b)$,then: **09302142**

(a) $a+b=1$ (b) $a-b=1$
(c) $a=b$ (d) $a^2-b^2=1$

Finding the value of logarithms

17. log e = ____ where e ≈ 2. 718

(a) 0 (b) 0.4343
(c) ∞ (d) 1

18. $\log_e 10$ = ____ where e ≈ 2. 718 **09302143**

(a) 2.3026 (b) 0.4343
(c) e^{10} (d) 10

19. $\log_9 \dfrac{1}{81}$ = ______

(a) −1 (b) −2
(c) 2 (d)does not exist

20. $\text{Log}_7 7^{-3} + \log_2 4^3$ is: **09302144**

(a) 0 (b) −3
(c) 3 (d) ±3

21. $\log_{\sqrt{10}} 100^2$ is: **09302145**

(a) 2 (b) 1
(c) 4 (d) 8

22. $\log_{10} 10^0$ is: **09302146**

(a) 2 (b) 0
(c) 1 (d) impossible

23. The value of $\log 4 + \log 25$ is: **09302147**

(a) 2 (b) 3
(c) 4 (d) 5

24. Evaluate $\log_7 \dfrac{1}{\sqrt{7}}$. **09302148**

(a) −1 (b) $-\dfrac{1}{2}$
(c) $\dfrac{1}{2}$ (d) $\dfrac{1}{7}$

25. If $\log_b x = 4$ and $\log_b y = 2$ then value of $\log_b\left(x \times y^3\right)$ is: **09302149**

(a) 5 (b) 7
(c) 9 (d) 10

Answer Key

1	**d**	2	**b**	3	**a**	4	**c**	5	**b**	6	**d**	7	**a**	8	**d**	9	**a**	10	**a**
11	**c**	12	**b**	13	**a**	14	**c**	15	**d**	16	**a**	17	**b**	18	**a**	19	**b**	20	**c**
21	**d**	22	**b**	23	**a**	24	**b**	25	**d**										

Q.2 Express the following numbers in scientific notations.

(i) 0.000567 09302150

Solution:

0.000567

$= 0\,.\,0005_{\wedge}67 = 5.67\times10^{-4}$

Move decimal point 4 places to the right.

(ii) 734 09302151

Solution:

734

$7_{\wedge}34\,. = 7.34\times10^{2}$

Move decimal point 2 places to the left.

(iii) 0.33×10^{3} 09302152

Solution

0.33×10^{3}

$0\,.\,3_{\wedge}3\times10^{3} = 3.3\times10^{-1}\times10^{3}$

$= 3.3\times10^{-1+3}$

$= 3.3\times10^{2}$

Move decimal point 1 place to the right.

Q.3 Express the following numbers in ordinary notation.

(i) 2.6×10^{3} 09302153

Solution:

2.6×10^{3}

Since, exponent is positive 3, move decimal point 3 places to the right.

$= 2600\times10^{-3}\times10^{3}$

$= 2600\times10^{0}$ $\quad(\because 10^{0}=1)$

$= 2600\times1 = 2600$

(ii) 8.794×10^{-4} 09302154

Solution:

8.794×10^{-4}

Since, exponent is negative four, move decimal point 4 places to the left.

$= 0.0008794\times10^{4}\times10^{-4}$

$= 0.0008794\times10^{4-4}$

$= 0.0008794\times10^{0}$

$= 0.0008794\times1$ $\quad(\because 10^{0}=1)$

$= 0.0008794$

(iii) 6×10^{-6} 09302155

Solution:

Since, exponent is negative six, move decimal point 6 places to the left.

$= 0.000006\times10^{6}\times10^{-6}$

$= 0.000006\times10^{6-6}$

$= 0.000006\times10^{0}$

$= 0.000006\times1$ $\quad(\because 10^{0}=1)$

$= 0.000006$

Q.4 Express each of the following in logarithmic form.

(i) $3^{7} = 2187$ 09302156

Solution:

$3^{7} = 2187$

$\Rightarrow \log_{3}2187 = 7$

(ii) $a^{b} = c$ 09302157

Solution

$a^{b} = c$

$\Rightarrow \log_{a}c = b$

(iii) $(12)^{2} = 144$ 09302158

Solution:

$(12)^{2} = 144$

$\Rightarrow \log_{12}144 = 2$

Q.5 Express each of the following is exponential form. 09302159

(i) $\log_{4}8 = x$

Solution

$\log_{4}8 = x$

$\Rightarrow 4^{x} = 8$

(ii) $\log_{9}729 = 3$ 09302160

Solution:

$\log_{9}729 = 3$

$\Rightarrow 9^{3} = 729$

(iii) $\log_{4}1024 = 5$ 09302161

Solution:

$\log_{4}1024 = 5$

$\Rightarrow 4^{5} = 1024$

Solution:

Q.6 Find value of x in the following.

(i) $\log_{9}x = 0.5$ 09302162

Solution:

$\log_{9}x = 0.5$

In exponential form

$9^{0.5} = x$

$9^{\frac{5}{10}} = x$

$9^{\frac{1}{2}} = x$

$\Rightarrow x = \sqrt{9}$

$x = 3$

(ii) $\left(\frac{1}{9}\right)^{3x} = 27$ **09302163**

3	27
3	9
3	3
	1

Solution:

$\left(\frac{1}{9}\right)^{3x} = 27$

$\Rightarrow \left(\frac{1}{3^2}\right)^{3x} = 3^3$

$\Rightarrow \left(3^{-2}\right)^{3x} = 3^3$

$\Rightarrow 3^{-6x} = 3^3$

By comparing exponents, we get

$-6x = 3$

$x = \frac{3}{-6}$

$x = \frac{1}{-2}$

$x = -\frac{1}{2}$

(iii) $\left(\frac{1}{32}\right)^{2x} = 64$ **09302164**

2	64
2	32
2	16
2	8
2	4
2	2
	1

Solution:

$\left(\frac{1}{32}\right)^{2x} = 64$

$\Rightarrow \left(\frac{1}{2^5}\right)^{2x} = 2^6$

$\left(2^{-5}\right)^{2x} = 2^6$

$2^{-10x} = 2^6$

By comparing exponents, we get.

$-10x = 6$

$x = \frac{6}{-10}$

$x = \frac{3}{-5}$

$\Rightarrow x = -\frac{3}{5}$

Q.7 Write the following as a single legerithm.

(i) $7 \log x - 3\log y^2$ **09302165**

Solution:

$7 \log x - 3\log y^2$

$\Rightarrow = \log x^7 - \log(y^2)^3$

$= \log x^7 - \log(y^6)$

$= \log\left(\frac{x^7}{y^6}\right)$

(ii) 3 log 4 – log 32 **09302166**

Solution:

$3 \log 4 - \log 32$

$= \log 4^3 - \log 32$

$= \log 64 - \log 32$

$= \log\left(\frac{64}{32}\right)$

$= \log 2$

(iii) $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$ **09302167**

Solution:

$\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

$= \frac{1}{3}\log_5 (8 \times 27) - \log_5 3$

$= \frac{1}{3}\log_5 216 - \log_5 3$

$= \log_5 (216)^{\frac{1}{3}} - \log_5 3$

$= \log_5 \sqrt[3]{216} - \log_5 3$

$= \log_5 \sqrt[3]{6^3} - \log_5 3$

$= \log_5 6 - \log_5 3$

$= \log_5\left(\frac{6}{3}\right)$

$= \log_5 2$

Q.8 Expand the following using laws of logarithms:

(i) $\log (x y z^6)$ 09302168

$\log (x y z^6)$

$= \log x+\log y+\log z^6$

$= \log x+\log y+6\log z$

(ii) $\log_3 \sqrt[6]{m^5n^3}$

Solution:

$\log_3 \sqrt[6]{m^5n^3}$

$= \log_3 \left(m^5n^3\right)^{\frac{1}{6}}$

$= \frac{1}{6} \log_3(m^5n^3)$

$= \frac{1}{6} (\log_3 m^5+\log_3 n^3)$

$= \frac{1}{6}[5 \log_3 m+3\log_3 n]$

(iii) $\log\sqrt{8x^3}$ 09302169

Solution:

$\log\sqrt{8x^3}$

$= \log\sqrt{2^3 x^3}$

$= \log\sqrt{(2x)^3}$

$= \log (2x)^{3\times\frac{1}{2}}$

$= \log(2x)^{\frac{3}{2}}$

$= \frac{3}{2}\log 2x$

$= \frac{3}{2}(\log 2+\log x)$

Q.9 Find values of the following with the help of logarithm table:

(i) $\sqrt[3]{68.24}$ 09302170

Solution:

Let $x = \sqrt[3]{68.24}$

Taking log of both sides

$\log x = \log \sqrt[3]{68.24}$

By Applying laws of logariths

$\log x = \log(68.24)^{\frac{1}{3}}$

$\log x = \frac{1}{3}\log(68.24)$

$\log x = \frac{1}{3}\ (1.8340)$

$\log x = 0.6113$

Characteristic = 0

Mantissa = .6113

$x =$ antilog (0.6113)

$x = 4.086$

Thus $\sqrt[3]{68.24} \approx 4.086$

(ii) 319.8 × 3.543 09302171

Solution:

let $x = 319.8 \times 3.543$

Taking log of both sides

$\log x = \log (319.8\times3.543)$

$\log x = \log 3_{\wedge}19.8+\log 3_{\wedge}.543$

$\log x = 2.5049+0.5493$

$\log x = 3.0542$

Characteristic =3

Mantissa = .0542

$x =$ antilog (3.0542)

$x = 1133$

Thus $319.8\times3.543 \approx 1133$

(iii) $\frac{36.12\times750.9}{113.2\times9.98}$ 09302172

Solution:

Let

$x=\frac{36.12\times750.9}{113.2\times9.98}$

Taking log of both sides.

$\log x = \log\frac{36.12\times750.9}{113.2\times9.98}$

By applying laws of logarithms.

$\log x = \log (36.12\times750.9)-\log(113.2\times9.98)$

$\log x=(\log 36.12+\log 750.9)-(\log 113.2+\log 9.98)$

$\log x$

$= \log 3_{\wedge}6.12+\log 7_{\wedge}50.9-\log 1_{\wedge}13.2-\log 9_{\wedge}.98$

$\log x = 1.5577+2.8756-2.0538-0.9991$

$\log x = 4.4333-3.0529$

$\log x = 1.3804$

Characteristic = 1

Mantissa = .3804

$x = \text{antilog}\ (1.3804)$

$x = 2_{\wedge}4.01 \approx 24.01$

Thus $\frac{36.12\times 750.9}{113.2\times 9.98}\approx 24.01$

Q.10 In the year 2016, the population of a city was 22 millions and was growing at a rate of 2.5% per year. The function $p(t) = 22(1.025)^t$ gives the population in millions, *t* years after 2016. Use the model to determine in which year the population will reach 35 millions. Round to the nearest year. **09302173**

Solution:

Given that

$P(t) = 22\ (1.025)^t$

Population for which time "t" is required 35 millions

$35 = 22\ (1.025)^t$

$\frac{35}{22} = (1.025)^t$

$1.5909 = (1.025)^t$

Taking log of both sides

$\log 1._{\wedge}5909 = \log\ (1.025)^t$

$0.2016 = t\times \log 1_{\wedge}.025$

$0.2016 = t\times 0.0107$

$\frac{0.2016}{0.0107} = t$

18.84

$\Rightarrow\ t = 18.84$ years

On rounding off we get

$t \approx 19$ years.

It means 19 years after 2016 i.e. 2016+19 = 2035. Thus the population of the city will be 35 millions in 2035 (19 years after 2016).