UNIT 03

Set and Functions

Introduction

Basic definitions

A **set** is described as a well-defined collection of distinct objects, numbers or elements, so that we may be able to decide whether the object belongs to the collection or not. Capital letters A, B, C, X, Y, Z etc., are generally used as names of sets and small letters a, b, c, x, y, z etc., are used as members or elements of sets.

There are three different ways of describing a set, the descriptive method, the tabular method and set builder method.

For example,

(i) The Descriptive form: A set may be described in words. For instance, the set of all vowels of the English alphabet.

(ii) The Tabular form: A set may be described by listing its elements within brackets. If A is the set mentioned above, then we may write:

$A = \{a, e, i, o, u\}$

The tabular form is also known as the Roster form.

(iii) Set-builder method: In set-builder notation a set is specified. By using a symbol or letter for an arbitrary set member and stating the property common to all the members. For example:

$A = \{x | x$ is a vowel of the English alphabets$\}$

This is read as A is the set of all x such that x is a vowel of the English alphabets. The symbol used for membership of a set is $a \in$ A means a is an element of A or a belongs A. $c \notin$ A means c does not belong to A or c is not a member of A. Elements of a set can be anything: people, countries, rivers, objects of our thought. In algebra, we usually deal with sets of numbers. Such sets, along with their names are given below:-

N = The set of natural numbers
$= \{1, 2, 3, \ldots\}$

W = The set of whole numbers
$= \{0, 1, 2, \ldots\}$

Z = The set of integers
$= \{0, \pm 1, \pm 2, \ldots\}$

O = The set of odd integers
$= \{\pm 1, \pm 3, \pm 5, \ldots\}$

E = The set of even integers
$= \{0, \pm 2, \pm 4, \ldots\}$

P = The set of prime numbers
$= \{2, 3, 5, 7, 11, 13, 17, \ldots\}$

Q = The set of all rational numbers

$$= \{x \mid x = \frac{p}{q} \text{ where } p, q \in Z \text{ and } q \neq 0\}$$

Q′ = The set of all irrational numbers

$$= \{x \mid x \neq \frac{p}{q} \text{ where } p, q \in Z \text{ and } q \neq 0\}$$

R = The set of real numbers

$$= Q \cup Q'$$

Singleton set

A set with only one element is called a singleton set. For example, $\{3\}$, $\{a\}$, and {Saturday} are singleton sets.

Empty set

The set with no elements (zero number of elements) is called an **empty set, null set** or **void set**.

The empty set is denoted by the symbol ϕ or $\{\}$.

> **Remember!**
> The set $\{0\}$ is a singleton set having zero as its only element, and not the empty set.

Equal sets

Two sets A and B are equal if they have exactly the same elements or if every element of set A is an element of set B. If two sets A and B are equal, we write A = B. Thus, the set {1, 2, 3} and {2, 1, 3} are equal.

Equivalent sets: Two sets A and B are equivalent if they have the same number of elements. For example, if A = {a, b, c, d, e} and B = {1, 2, 3, 4, 5}, then A and B are equivalent sets. The symbol ~ is used to represent equivalent sets. Thus, we can write A~B.

Subset

If every element of a set A is an element of set B, then A is a subset of B. Symbolically this is written as $A \subseteq B$ (A is a subset of B). In such a case, we say B is a superset of A. Symbolically this is written as:

$B \supseteq A$ (B is a superset of A).

> **Remember!**
> The subset of a set can also stated as follows:
> $A \subseteq B$ if $\forall x \in A \Rightarrow x \in B$

Proper subset: If A is a subset of B and B contains at least one element that is not an element of A, then A is said to be a proper subset of B. In such a case, we write:

$A \subset B$ (A is a proper subset of B).

Improper subset: If A is a subset of B and A = B then we say that A is an improper subset of B. From this definition, it also follows that every set A is a subset of itself and is called an improper subset.

For example, let A = {a, b, c}, B = {c, *a*, b} and C = {a, b, c, d}, then clearly.

$A \subset C$, $B \subset C$ but A = B

> **Remember!**
> When we do not want to distinguish between proper and improper subsets, we may use the symbol $\subseteq$ for the relationship. It is easy to see that:
> $N \subset W \subset Z \subset Q \subset R$

Notice that each of sets A and B is an improper subset of the other because A = B.

Universal set: The set that contains all objects or elements under consideration is called the universal set or the universe of discourse. It is denoted by U.

Power set: The power set of a set S denoted by P(S) is the set containing all the possible subsets of S. For Example:

(i) If C = {*a*, b, c, d}, then
P (C) = {ϕ, {*a*}, {b}, {c}, {d}, {*a*, b}, { *a*, c}, { *a*, d}, {b, c}, {b, d}, {c, d}, { *a*, b, c}, {*a*, b, d}, { *a*, c, d}, {b, c, d}, { *a*, b, c, d}}

(ii) If D = {*a*}, then P(D) = {ϕ, {*a*}}
If S is a finite set with n(S) = m representing the number of elements of the set S, the n{P(S)} = 2^m is the number of the elements of the power set.

EXERCISE 3.1

Q.1 Write the following sets in builder notation:

(i) {1, 4, 9, 16, 25, 36,…,484} 09303001

Solution:

{1, 4, 9, 16, 25, 36,…,484}

Set builder notation:

$\{x | x = n^2, n \in N \wedge 1 \le n \le 22\}$

(ii) {2, 4, 8, 16, 32,…,1024} 09303002

(expected correction)

Solution

let B = {2,4,6,16,32,…,1024}

Set builder notation:

$\{x | x = 2^n, n \in N \wedge 1 \le n \le 10\}$

(iii){0, ± 1, ± 2,…, ±1000} 09303003

Solution:

{0, ± 1, ± 2,…, ±1000}

Set builder notation:

$\{x|x \in Z \wedge -1000 \le x \le 1000\}$

(iv) {6, 12, 18, …,120} 09303004

Solution:

{6, 12, 18, …,120}

Set builder notation:

$\{x|x = 6n, n \in N \wedge 1 \le n \le 20\}$

(v) {100, 102, 104,…,400} 09303005

Solution:

= {100, 102, 104,…,400}

Set builder notation:

= $\{x|x = 2n, n \in N \wedge 50 \le n \le 200\}$

(vi) {1, 3, 9, 27, 81, …} 09303006

Solution:

{1, 3, 9, 27, 81, …}

Set builder notation:

$\{x|x = 3^n, n \in W\}$

(vii) {1, 2, 4, 5, 10, 20, 25, 50, 100} 09303007

Solution:

{1, 2, 4, 5, 10, 20, 25, 50, 100}

Set builder notation:

$\{x|x$ is a divisor of $100\}$

(viii) {5, 10, 15, …, 100} 09303008

Solution:

{5, 10, 15, …, 100}

Set builder notation:

$\{x|x = 5n, n \in N \wedge 1 \le n \le 20\}$

(ix) The set of all integers between – 100 and 1000 09303009

Solution:

The set of all integers between – 100 and 1000

Set builder notation:

$\{x|x \in Z \wedge -100 < x < 1000\}$

Q.2 Write each of the following sets in tabular forms:

(i) $\{x|x$ is a multiple of $3 \wedge x \le 36\}$ 09303010

Solution: (Expected correction)

$\{x|x$ is a multiple of $3 \wedge x \le 36\}$

{3, 6, 9, …, 36}

(ii) $\{x|x \in R \wedge 2x + 1 = 0\}$ 09303011

Solution:

$\{x|x \in R \wedge 2x + 1 = 0\}$

Tabular form:

We know that

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

$\left\{-\frac{1}{2}\right\}$

(iii) $\{x|x \in P \wedge x < 12\}$ 09303012

Solution:

$\{x|x \in P \wedge x < 12\}$

Tabular form:

{2, 3, 5, 7, 11}

(iv) $\{x|x$ is a divisor of $128\}$ 09303013

Solution:

$\{x|x$ is a divisor of $128\}$

Tabular form:

{1, 2, 4, 8, 16, 32, 64, 128}

(v) $\{x|x = 2^n, n \in N \wedge n < 8\}$ 09303014

Solution:

$\{x|x = 2^n, n \in N \wedge n < 8\}$

Tabular form:

{2, 4, 8, 16, 32, 64, 128}

(vi) $\{x|x \in N \wedge x + 4 = 0\}$ 09303015

Solution:

$\{x|x \in N \wedge x + 4 = 0\}$

Tabular form:

$x + 4 = 0 \Rightarrow x = -4$

$\{\ \} \because -4 \notin N$

(vii) $\{x|x \in N \wedge x = x\}$ 09303016

Solution:

$\{x|x \in N \wedge x = x\}$

Tabular form:

{1, 2, 3, 4, 5,…}

(viii) $\{x|x \in Z \wedge 3x + 1 = 0\}$ 09303017

Solution:

$\{x|x \in Z \wedge 3x + 1 = 0\}$

$\because 3x + 1 = 0 \Rightarrow x = -\frac{1}{3}$

$\{\ \} \because -\frac{1}{3} \notin Z$

Tabular form { }

Q.3 Write two proper subsets of each of the following sets:

(i) {a, b, c} 09303018

Solution:

{a, b, c}

Two proper subsets are $\{a\}$ and $\{b\}$

(ii) {0, 1} 09303019

Solution:

$\{0, 1\}$

Two proper subsets are $\{0\}$ and $\{1\}$

(iii)N 09303020

Solution:

$N = \{1, 2, 3, 4, 5 \ldots\}$

Two proper subsets:

$\{1, 3, 5, 7, \ldots\}$ and $\{2, 4, 6, 8 \ldots\}$

(iv)Z 09303021

Solution:

$Z = \{0, \pm1, \pm2, \pm3, \pm4, \ldots\}$

Two proper subsets:

$\{1, 2, 3, 4, \ldots\}$ and $\{-1, -2, -3, -4. \ldots\}$

(v) Q 09303022

Solution:

Two subsets: N and W

(vi)R 09303023

Solution:

Since, $R = Q \cup Q'$

Two proper subsets: Q and Q′

(vii) $\{x | x \in Q \wedge 0 < x \leq 2\}$ 09303024

Solution:

$\{x | x \in Q \wedge 0 < x \leq 2\}$

Take any two positive rational number up to 2.

Two proper subsets: $\left\{\frac{1}{2}\right\}\left\{\frac{2}{3}\right\}$

Q.4 Is there any set which has no proper subset? If so, name that set. 9303025

Solution 9303025

Yes, there is empty set has no proper subset.

Q.5 What is the difference between $\{a, b\}$ and $\{\{a,b\}\}$? 09303026

Solution

$\{a, b\}$ is a set containing two elements a and b while $\{\{a,b\}\}$ is a singleton set having only one element $\{a, b\}$.

Q.6 What is the number of elements of the power set of each of the following sets?

(i) { } 09303027

Solution:

$\{\ \}$

No. of elements = n = 0

No. of elements in power set $2^n = 2^0 = 1$

(ii) {0, 1} 9303028

Solution:

$\{0, 1\}$

No. of elements in given set = n = 2

No. of elements in power set $2^n = 2^2 = 4$

(iii){1, 2, 3, 4, 5, 6, 7} 09303029

Solution:

$\{1, 2, 3, 4, 5, 6, 7\}$

No. of elements in given set = n = 7

No. of elements in power set = $2^n = 2^7 = 128$

(iv){0, 1, 2, 3, 4, 5, 6, 7} 09303030

Solution:

$\{0, 1, 2, 3, 4, 5, 6, 7\}$

No. of elements in given set = n = 8

No. of elements in power set = $2^n = 2^8 = 256$

(v) {*a*, {b, c}}

Solution:

$\{a, \{b, c\}\}$

No. of elements in given set = n = 2

No. of elements in power set = $2^n = 2^2 = 4$

(vi){{*a*, b}, {b, c}, {d, e}}

Solution:

$\{\{a, b\}, \{b, c\}, \{d, e\}\}$

No. of elements in given set = n = 3

No. of elements in power set = $2^n = 2^3 = 8$

Q.7 Write down the power set of each of the following sets: 09303031

(i) {9, 11}

Solution

$\{9, 11\}$

No. of elements in given set = n = 2

No. of elements in power set = $2^n = 2^2 = 4$

Power set: $\{\phi, \{9\}, \{11\}, \{9,11\}\}$

(ii) {+, −, ×, ÷} 09303032

Solution:

{+, −, ×, ÷}

No. of elements in power set = $2^n = 2^4 = 16$

Power set:

$\{\phi, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+,-\}, \{+,\times\}, \{+,\div\},$
$\{-,\times\}, \{-,\div\}, \{\times,\div\}, \{+,-,\times\}, \{+,-,\div\},$
$\{+,\times,\div\}, \{-,\times,\div\}, \{+,-,\times,\div\}\}$

(iii) $\{\phi\}$ **09303033**

Solution:

$\{\phi\}$

No. of elements in power set = $2^n = 2^1 = 2$

Power set:

$\{\phi, \{\phi\}\}$

(iv) $\{a, \{b, c\}\}$ **09303034**

Solution:

$\{a, \{b, c\}\}$

No. of elements in power set = $2^n = 2^2 = 4$

Power set:

$\{\phi, \{a\}, \{b, c\}, \{a, \{b, c\}\}\}$

Operations on Sets

Union of two sets:

The union of two sets A and B, denoted by $A \cup B$, is the set of all elements which belong to A or B. Symbolically;

$A \cup B = \{x \mid x \in A \vee x \in B\}$

Thus if A= {1, 2, 3}, B = {2, 3, 4, 5} then $A \cup B = \{1,2,3,4,5\}$

Intersection of two sets:

The intersection of two sets A and B, denoted by $A \cap B$, is the set of all elements that belong to both A and B. Symbolically:

$A \cap B = \{x \mid x \in A \wedge x \in B\}$.

Thus, If A = {1,2,3}, B = {2,3,4,5},

$A \cap B = \{2, 3\}$.

Disjoint sets:

If the intersection of two sets is the empty set, the sets are said to be disjoint. For example, if

S_1 = The set of odd natural numbers

S_2 = The set of even natural numbers,

then S_1 and S_2 are disjoint sets.

Overlapping sets: If the intersection of two sets is non-empty but neither is a subset of the other, the sets are called overlapping sets, e.g., if

L = {2, 3, 4, 5, 6} and M = {5, 6, 7, 8, 9, 10} then L and M are overlapping sets.

Difference of two sets: The difference between the sets A and B denoted by A–B, consists of all the elements that belong to A but do not belong to B.

Symbolically, $A - B = \{x \mid x \in A \wedge x \notin B\}$ and $B - A = \{x \in B \wedge x \notin A\}$

For example, if A = {1, 2, 3, 4, 5} and B = {4, 5, 6, 7, 8, 9, 10},

then A–B = {1,2 3} and B–A={ 6,7,8,9,10}

Notice that: $A - B \neq B - A$.

Complement of a set: The complement of a set A, denoted by A' or A^c relative to the universal set U is the set of all elements of U, which do not belong to A.

Symbolically:

$$A' = \{x \mid x \in U \wedge x \notin A\}$$

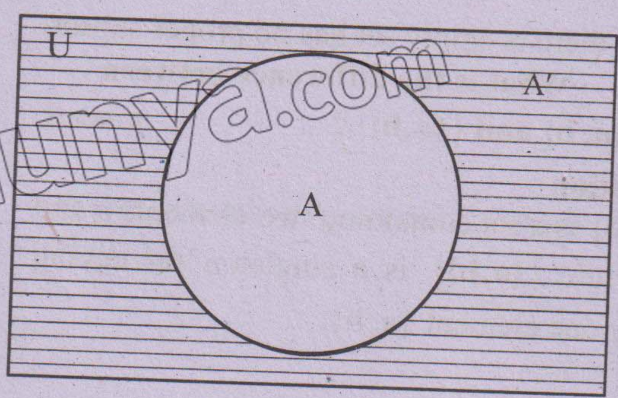
For example, if U = Z, then $E' = O$ and $O' = E$

For example, if U = set of alphabets of English language, C = set of consonants, W = Set of vowels, then $C' = W$ and $W' = C$.

Identification of Sets using Venn Diagram

Venn diagrams are very useful in depicting visually the basic concepts of sets and relationships between sets. These diagrams were first used by an English logician and mathematician John Venn (1834 to 1883 A.D).

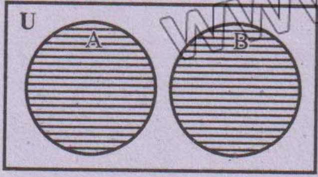
In the adjoining figures, the rectangle represents the universal set U and the shaded circular region represents a set A and the remaining portion of the rectangle represents the A' or U–A.



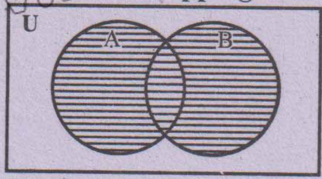
Below are given some more diagrams illustrating basic operations on two sets in different cases (the lined region represents the result of the relevant operation in each case shown below).

	Disjoint sets	Overlapping sets	$A \subseteq B$	$B \subseteq A$

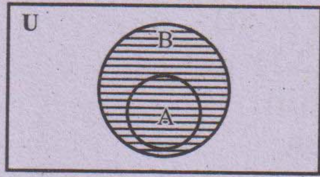
$A \cup B$



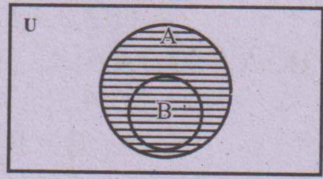
- $A \cup B \neq \phi$
- $n(A \cup B) = n(A) + n(B)$



- $A \cup B \neq \phi$
- $n(A \cup B) = n(A) + n(B) - (A \cap B)$

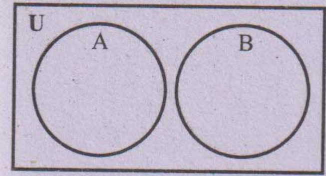


- $A \cup B = B$
- $n(A \cup B) = n(B)$

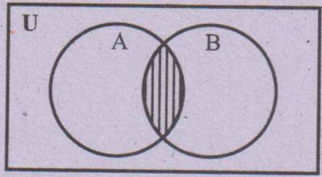


- $A \cup B = A$
- $n(A \cup B) = n(A)$

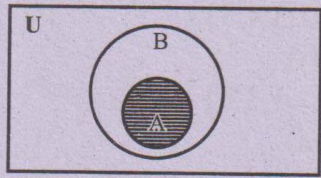
$A \cap B$



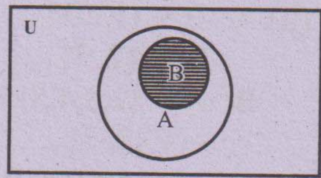
- $A \cap B = \phi$
- $n(A \cap B) = 0$



- $A \cap B \neq \phi$

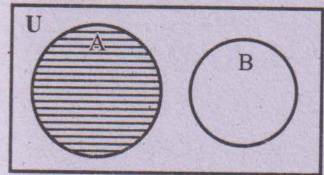


- $A \cap B = A$
- $n(A \cap B) = n(A)$

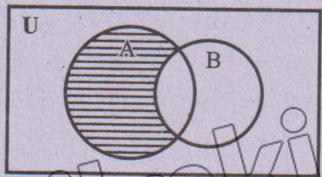


- $A \cap B = B$
- $n(A \cap B) = n(B)$

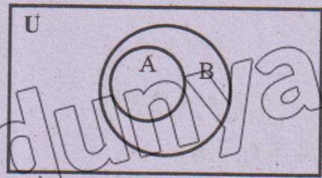
$A - B$



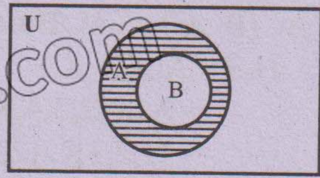
- $A - B = A$
- $n(A - B) = n(A)$



- $n(A - B) = n(A) - n(A \cap B)$

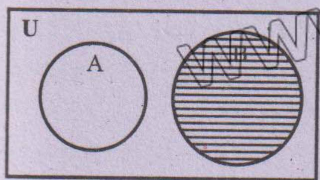


- $A - B = \phi$
- $n(A - B) = 0$

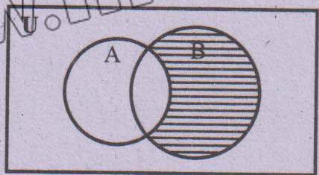


- $A - B \neq \phi$
- $n(A - B) = n(A) - n(B)$

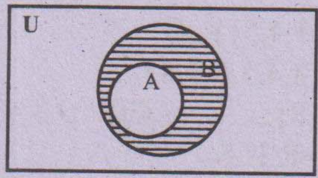
$B - A$



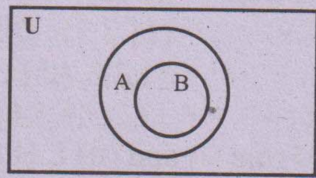
- $B - A = B$
- $n(B - A) = n(B)$



- $n(B - A) = n(B) - n(A \cap B)$



- $B - A \neq \phi$
- $n(B - A) = n(B) - n(A)$



- $B - A = \phi$
- $n(B - A) = 0$

Operations on three sets 9303035

If A, B and C are three given sets, operations of union and intersection can be performed on them in the following ways:

(i) $A \cup (B \cup C)$	(ii) $(A \cup B) \cup C$
(iii) $A \cap (B \cup C)$	(iv) $(A \cap B) \cap C$
(v) $A \cup (B \cap C)$	(vi) $(A \cap C) \cup (B \cap C)$
(vii) $(A \cup B) \cap C$	(viii) $(A \cap B) \cup C$
(ix) $(A \cup C) \cap (B \cup C)$	

Properties of union and intersection

9303036

We now state the fundamental properties of union and intersection of two or three sets.

Properties

(i) $A \cup B = B \cup A$
(Commutative property of Union)

(ii) $A \cap B = B \cap A$
(Commutative property of Intersection)

(iii) $A \cup (B \cup C) = (A \cup B) \cup C$
(Associative property of Union)

(iv) $A \cap (B \cap C) = (A \cap B) \cap C$
(Associative property of intersection)

(v) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(Distributivity of union over intersection)

(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(Distributivity of intersection over union)

(vii) $(A \cup B)' = A' \cap B'$ (De Morgan's Law)

(viii) $(A \cap B)' = A' \cup B'$ (De Morgan's Law)

Verification of the properties using sets

Let A = {1,2,3}, B={2,3,4,5} and C={3,4,5,6,7,8}

Commutative property of union:

$A \cup B = B \cup A$

(i) $A \cup B = \{1,2,3\} \cup \{2,3,4,5\}$

$= \{1,2,3,4,5\}$; .....(i)

$B \cup A = \{2,3,4,5\} \cup \{1,2,3\}$

$= \{1,2,3,4,5\}$......(ii)

So, $A \cup B = B \cup A$

Commutative of intersection:

$A \cap B = B \cap A$

(ii) $A \cap B = \{1,2,3\} \cap \{2,3,4,5\}$;

$= \{2, 3\}$...... (i)

$B \cap A = \{2,3,4,5\} \cap \{1,2,3\}$

$= \{2, 3\}$...... (ii)

So, $A \cap B = B \cap A$

(iii)Associative Property of Union:

$A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S = $A \cup (B \cup C)$

$= A \cup (\{2,3,4,5\} \cup \{3,4,5,6,7,8\})$

$= \{1,2,3\} \cup \{2,3,4,5,6,7,8\}$

$= \{1,2,3,4,5,6,7,8\}$..........(i)

R.H.S = $(A \cup B) \cup C$

$= (\{1,2,3\} \cup (\{2,3,4,5\}) \cup C$

$= \{1,2,3,4,5\} \cup \{3,4,5,6,7,8\}$

$= \{1,2,3,4,5,6,7,8\}$...........(ii)

From (i) and (ii) L.H.S=R.H.S

Hence proved $A \cup (B \cup C) = (A \cup B) \cup C$

(iv)Associative Property of Intersection:

$A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S = $A \cap (B \cap C)$

$= A \cap (\{2,3,4,5\} \cap \{3,4,5,6,7,8\})$

$= \{1,2,3\} \cap \{3,4,5\}$

$= \{3\}$...........(i)

R.H.S = $(A \cap B) \cap C$

$= (\{1,2,3\} \cap (\{2,3,4,5\}) \cup C$

$= \{2,3\} \cap \{3,4,5,6,7,8\}$

$= \{3\}$...........(ii)

From (i) and (ii) L.H.S=R.H.S

Hence proved $A \cup (B \cup C) = (A \cup B) \cup C$

(v) Distributive property of union over intersection:

$A \cup (B \cap C) = \{1,2,3\} \cup \{3,4,5\}$

$= \{1,2,3,4,5\}$ ...(i)

$(A \cup B) \cap (A \cup C) = [\{1,2,3\} \cup \{2,3,4,5\}] \cap [\{1,2,3\} \cup \{3,4,5,6,7,8\}]$

$= \{1,2,3,4,5\} \cap \{1,2,3,4,5,6,7,8\}$

$= \{1,2,3,4,5\}$ ...(ii)

From (i) and (ii), $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(vi) Distributive Property of Intersection Over Union:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C)$

$= A \cap (\{2,3,4,5\} \cup \{3,4,5,6,7,8\})$

$= \{1,2,3\} \cap \{2,3,4,5,6,7,8\}$

$= \{2,3\}$ ....(i)

R.H.S $= (A \cap B) \cup (A \cap C)$

$(A \cap B) = \{1,2,3\} \cap \{2,3,4,5\} = \{2,3\}$

$(A \cap C) = \{1,2,3\} \cap \{3,4,5,6,7,8\} = \{3\}$

R.H.S $= (A \cap B) \cup (A \cap C)$

$= \{2,3\} \cup \{3\}$

$= \{2,3\}$ ....(ii)

From (i) and (ii) L.H.S=R.H.S

Hence proved $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(vii) Let the universal set be U = {1,2,3,4,5,6,7,8,910}

$A \cup B = \{1,2,3\} \cup \{2,3,4,5\} = \{1,2,3,4,5\}$

$(A \cup B)' = U - (A \cup B) = \{6,7,8,9,10\}$ ...(i)

$A' = U - A = \{4, 5, 6, 7, 8, 9,10\}$

$B' = U - B = \{1, 6, 7, 8, 9,10\}$

$A' \cap B' = \{4,5,6,7,8,9,10\} \cap \{1,6,7,8,9,10\}$

$= \{6,7,8,9,10\}$ ...(ii)

From (i) and (ii), $(A \cup B)' = A' \cap B'$

(viii) De-Morgan's law $(A \cap B)' = A' \cup B'$

Let $U=\{1,2,3,4,\ldots,10\}$, $A=\{1,2,3\}$, $B=\{2,3,4,5\}$, $C=\{3,,5,6,7,8\}$

L.H.S$=(A \cap B)'$

$(A \cap B) = \{1,2,3\} \cap \{2,3,4,5\} = \{2,3\}$

$(A \cap B)' = U - (A \cap B)$

$= \{1,2,3,4,\ldots,10\} - \{2,3\}$

$= \{1,4,5,6,7,8,9,10\}$.........(i)

R.H.S$= A' \cup B'$

$A' = U - A$

$A' = \{1,2,3,4,\ldots,10\} - \{1,2,3\}$

$A' = \{4,5,6,7,8,9,10\}$

Now, $B' = U - B$

$B' = \{1,2,3,4,\ldots,10\} - \{2,3,4,5\}$

$B' = \{1,6,7,8,9,10\}$

Now, $A' \cup B' = \{4,5,6,7,8,9,10\} \cup \{1,6,7,8,9,10\}$

$= \{1,4,5,6,7,8,9,10\} \rightarrow$ (ii)

From (i) and (ii)

L.H.S = R.H.S

Verification of the properties with the help of Venn diagrams

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(i) and (ii): Verification is very simple, therefore, do it by yourself.

(iii) In Fig. (1), set A is represented by a vertically lined region and B∪C is represented by a horizontally lined region. The set A∪(B ∪C) is represented by the region lined either in one way or both.

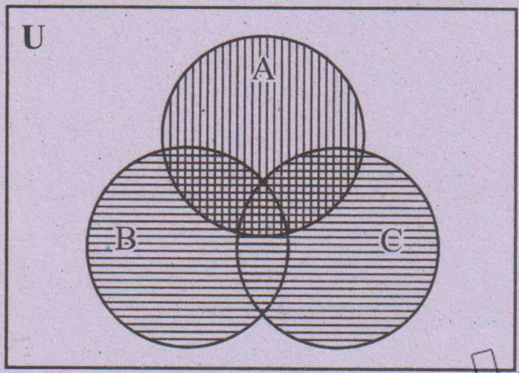


Fig.(1)

In Fig. (2) A ∪ B is represented by a horizontally lined region and C by a vertically lined region. (A∪B) ∪C is represented by the region lined in either one or both ways.

From Fig. (1) and (2) we can see that

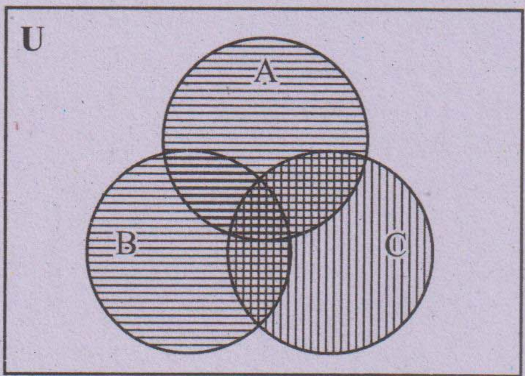


Fig.(2)

A ∪ (B∪C) = (A∪B) ∪C

(iv) In Fig. (3), the doubly lined region represents A∩(B∩C)

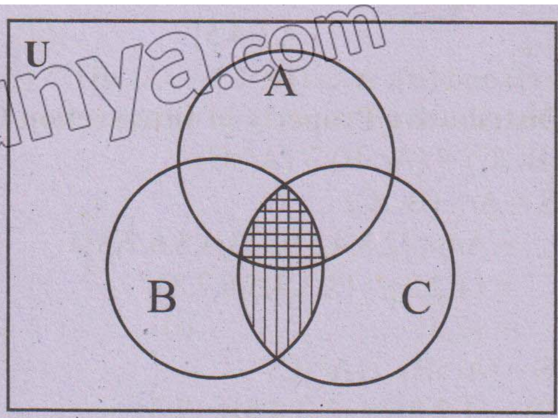


Fig.(3)

In Fig. (4) the doubly lined region represents (A∩B) ∩C. Since in Fig. (3) and Fig. (4), these regions are the same, therefore, A∩(B∩C) = (A∩B) ∩ C.

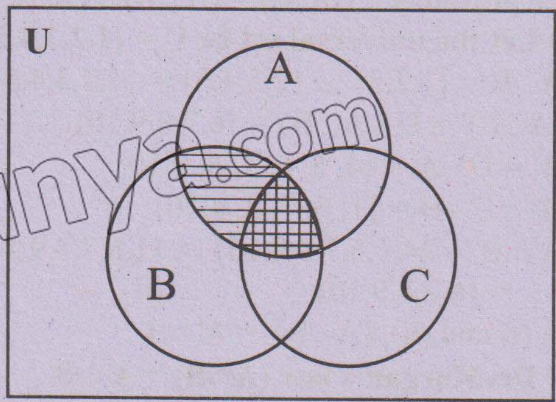


Fig.(4)

(v) In Fig. (5), A∪ (B∩C) is represented by the region which is lined horizontally or vertically or both ways.

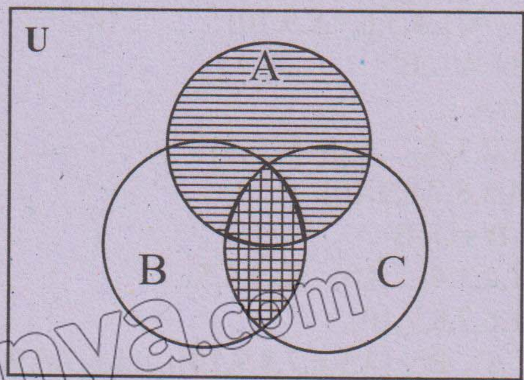


Fig.(5)

In Fig. (6), (A∪B) ∩ (A∪C) is represented by the doubly lined region.

Since the two regions in Fig. (5) and (6) are the same, therefore.

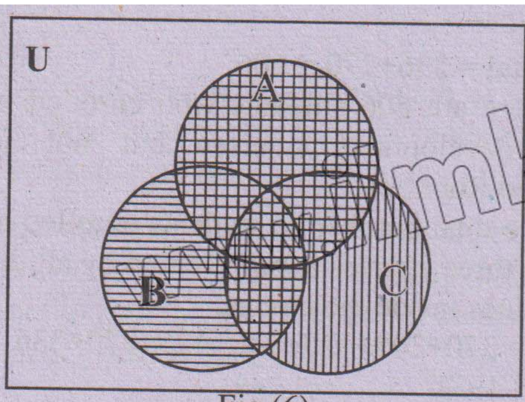


Fig.(6)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(vi) Verify yourselves.

(vii) In Fig. (7), $(A \cup B)'$ is represented by a vertically lined region.

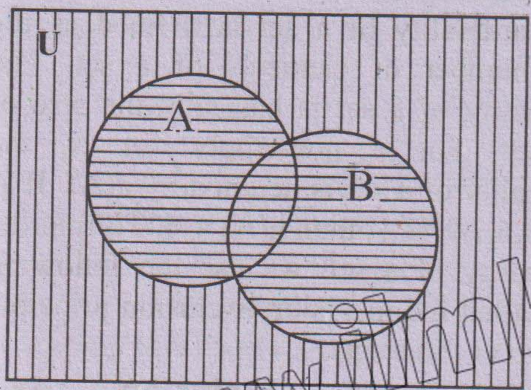


Fig.(7)

In Fig. (8), the doubly lined region represents $A' \cap B'$.

The two regions in Fig. (7) and (8) are the same, therefore, $(A \cup B)' = A' \cap B'$

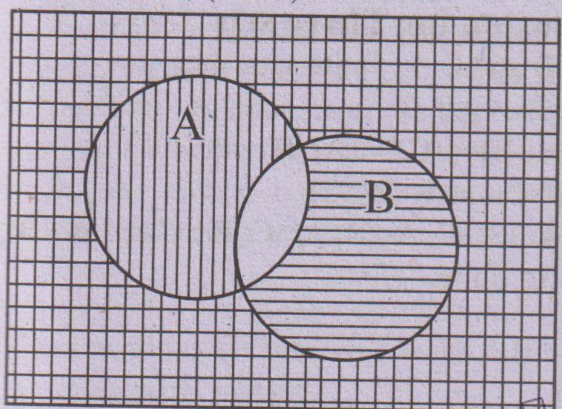


Fig.(8)

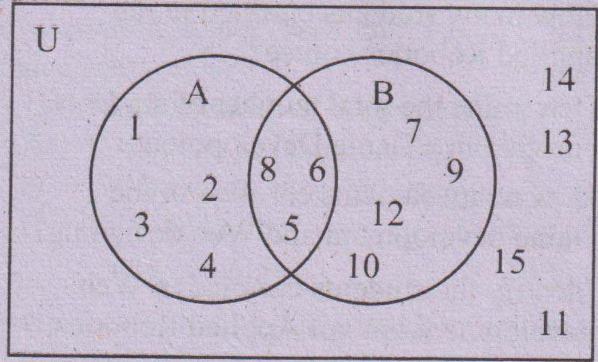
(viii) Verify yourselves.

Example 1: Consider the adjacent Venn diagram illustrating two non-empty sets, A and B.

(a) Determine the number of elements to common to sets A and B.

(b) Identify all the elements exclusively in set B and not in set A.

(c) Calculate the union of sets A and B.



Solution:

From the information provided in the Venn diagram, we have:

Let $U = \{1,2,3,4,5,6,7,8,910,11,12,13,14,15\}$

$A = \{1,2,3,4,5,6,8\}$

$B = \{5,6,7,8,9,10,12\}$

(a) The elements common to sets A and B are the intersection of the sets:

$A \cap B = \{5,6,8\}$

(b) The elements that are only in set B, not in set A, is the sets' differences.

$B - A = \{7, 9, 10, 12\}$

(c) $A \cup B = \{1,2,3,4,5,6,8\} \cup \{5,6,7,8,9,10,12\}$

$= \{1,2,3,4,5,6,7,8,9,10,12\}$

Example 2: Consider the adjacent Venn diagram representing the students enrolled in different courses in an IT institutions.

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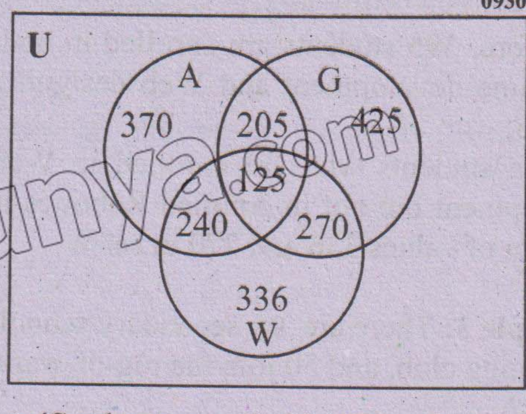


U = {Students enrolled in IT institutions)

A = {Students enrolled in an Applied Robotics}

G = {Students enrolled in Game Development}

W = {Students enrolled in a Web Designing}

(a) How many students enrolled in the applied Robotics course?

(b) Determine the total number of students enrolled in a Game Development.

(c) How many students enrolled in the Game development and Web designing?

(d) Identify the students enrolled in Web development but not Applied Robotics.

(e) How many students are enrolled IT institutions?

(f) How many students enrolled in all three courses?

Solution:

(a) Set A represents the total number of students enrolled in the Applied Robotics program.

Total = 370 + 205 + 125 + 240 = 940

So, the total number of students in the Applied Robotics course is 940.

(b) The total number of students enrolled in a Game Development is represented by the set G.

Total = 205 + 125 + 270 + 425 = 1025

Thus, the Students enrolled in a Game Development is 1025.

(c) Total students enrolled in both the Game development and Web designing is the intersection of G and W.

$G \cap W = 125 + 270 = 395$

Therefore, 395 students are enrolled in both the Game development and Web designing Course.

(d) The students who are enrolled in Web development but not in Applied Robotics is the sum of values 336 and 270 in set W.

Total = 336+270 = 606

So, there are 606 students who enrolled in Web development courses but not in Applied Robotics.

(e) The total number of students enrolled in all three courses is represented by all the values inside the circles.

Total = 370+205+125+240+425+270+336
= 1971

(f) The students, who enrolled in all three courses are the intersection of all the circles, are represented by the value 125.

Real world applications

Cardinality of a set

The cardinality of a set is defined as the total number of elements of a set. The cardinality of a set is basically the size of the set. For a non-empty set A, the cardinality of a set is denoted by n(A). If A = {1,3,5,7,9,11,}, then n(A) = 6. To find the cardinality of a set, we use the following rule called the inclusion-exclusion principle for two or three sets.

Principle of Inclusion and Exclusion for Two Sets

Let A and B be finite sets, then

$n(A \cup B) = n(A)+n(B) - n(A \cap B)$

and $A \cup B$ and $A \cap B$ are also finite.

Principle of Inclusion and Exclusion for Three sets

If A, B and C are finite sets, then

$n(A \cup B \cup C) =$
$n(A)+n(B)+n(C)-n(A \cap B)-n(A \cap C)$
$-n(B \cap C)+n(A \cap B \cap C)$

and $A \cup B \cup C$, $A \cap B$, $A \cap C$, $B \cap C$ and $A \cap B \cap C$ are also finite.

Example 3: There are 98 secondary school students in a sports club. 58 students join the swimming club, and 50 join the tug-of-war club. How many students participated in both games?

Solution:

Let U = {total student in a sports club of school}

A = {students who participated in swimming club}

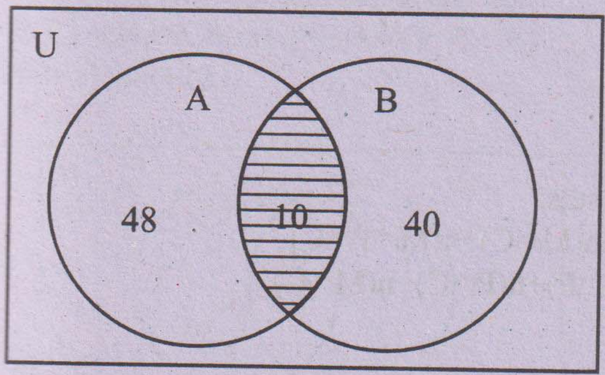
B = {students who participated in tug - of - war club}

From the statement of problems, we have

$n(U) = n(A \cup B) = 98$, $n(A) = 58$, $n(B) = 50$.

We want to find the total number of students who participated in both clubs.

$$n(A \cap B) = ?$$



Using the principles of inclusion and exclusion for two sets.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\Rightarrow n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 58 + 50 - 98$$

$$= 10$$

Thus, 10 students participated in both clubs. The adjacent venn diagram shows the number of students in each sports club.

Example 4: Mr. Saleem, a school teacher, has a small library in his house containing 150 books. He has two main categories for these books: Islamic and science. He categorized 70 books as Islamic books and 90 books as science books. There are 15 books that neither belong to the Islamic nor science books category. How many books are classified under both the Islamic and science categories? **09303040**

Let U = {total number of books in library}

A = {70 books in Islamic category}

B = {90 books in Science category}

C = {15 book that does not belong to any category}

x = number of books that belong to both the categories

The adjacent Venn diagram shows the number of books that are classified under both the Islamic and science categories.

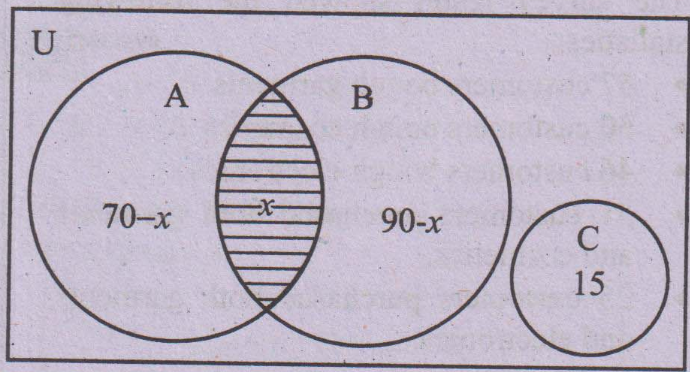
As, $n(U) = 150$

So, $70 - x + x + 90 - x + 15 = 150$

$\Rightarrow 175 - x = 150$

$\Rightarrow 175 - 150 = x$

$25 = x \Rightarrow x = 25$



Thus 25 books are classified under both Islamic and science categories.

Example 5: In a college, 45 teachers teach mathematics or physics or chemistry. Here is information about teachers who teach different subjects: **09303041**

- 18 teach mathematics
- 8 teach chemistry
- 12 teach physics
- 6 teach both mathematics and physics
- 4 teach both physics and chemistry
- 2 teach both mathematics and chemistry.

- How many teachers teach all three subjects?

Solution:

Let U = {total number of teachers in the college}, n(U) = 45

M = {teachers who teach mathematics}

P = {teachers who teach physics}

C = {teachers who teach chemistry}

From statements, we have $n(M \cup P \cup C) = 45$,

$n(M) = 18$

$n(P) = 12$

$n(C) = 8$

$n(M \cap P) = 6$,

$n(P \cap C) = 4$, $n(M \cap C) = 2$

Total number of teachers who teach all the subjects:

$n(M \cap P \cap C) = ?$

Using the principle of inclusion and exclusion for three sets:

$$n(M \cup P \cup C) = n(M)+n(P)+n(C)-n(M \cap P)-n(P \cap C)-n(M \cap C)+n(M \cap P \cap C)$$

$$\Rightarrow n(M \cap P \cap C) = n(M \cup P \cup C)-n(M)-n(P)-n(C)+n(M \cap P)+n(P \cap C)+n(M \cap C)$$

$$= 45-18-12-8+6+4+2$$

$$= 19$$

Therefore 19 teachers teach all subjects.

Example 6: A survey of 130 customers in a shopping mall was conducted in which they were asked about buying preferences.

The survey result showed the following statistics: **09303042**

- 57 customers bough garments
- 50 customers bough cosmetics
- 46 customers bough electronics
- 31 customers purchased both garments and cosmetics.
- 25 customers purchased both garments and electronics.
- 21 customers purchased both cosmetics and electronics
- 12 customers purchased all three products i.e. garments, cosmetics and electronics.

(a) How many of the customers bought at least one of the products: garments, cosmetics or electronics?

(b) How many bought only one of the products: garments, cosmetics or electronics?

(c) How many of the customers did not buy any of the three products.

Solution:

Let U = {total number of customers surveyed in the shopping mall}

G = {Customer who bought garments}

C = {Customer who bought cosmetics}

E = {Customer who bought electronics}

From the statement of problems, we have

$n(U)=130$, $n(G)=57$, $n(C) = 50$, $n(E) = 46$, $n(G \cap C) = 31$, $n(G \cap E) = 25$, $n(C \cap E)=21$ and $n(G \cap C \cap E)=12$.

(a) We want to find the total number of customer who have bought at least one of the products: garments, cosmetics or electronics. We are to find $n(G \cup C \cup E)$.

Using the principle of inclusion and exclusion for three sets:

$$n(G \cup C \cup E) = n(G)+n(C)+n(E)-n(G \cap C)-n(G \cap E)-n(C \cap E)+n(G \cap C \cap E)$$

$= 57+50+46-31-25-21+12$
$= 165 - 77$
$= 88$

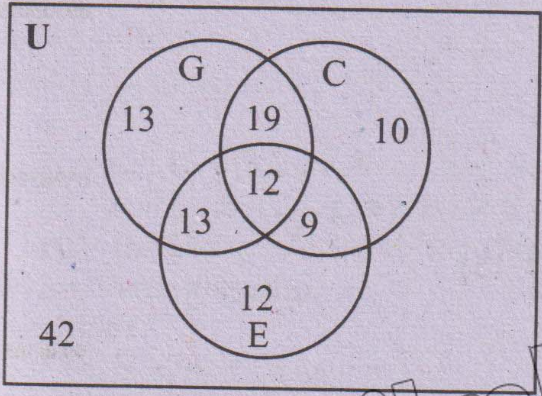
Thus, 88 customers bought at least one of the product: garments, cosmetics, or electronics.

(b) Customers who bought only garments

$= n(G) - n(G\cap C) - n(G\cap E) + n(G\cap C\cap E)$
$= 57-31-25+12$
$= 13$

Customers who bought only cosmetics

$= n(C) - n(G\cap C) - n(G\cap E) + n(G\cap C\cap E)$
$= 50-31-21+12$
$= 10$



Customers who bought only electronics

$= n(E) - n(G\cap E) - n(C\cap E) + n(G\cap C\cap E)$
$= 46 - 25 - 21 + 12 = 12$

Therefore, he customers bought only one of the products: garments, cosmetics, or electronics = 13+10+12 = 35

(c) Since the total number of Customers surveyed was 130, and 88 customers bought at least one of the products: garments, cosmetics or electronics. The customers who did not buy any of the three products can be calculated as:

$n(G\cup C\cup E)^c = n(U) - n(G\cup C\cup E)$
$= 130 - 88 = 42$

So, 42 customers did not buy any of the products.

Exercise 3.2

Q.1 Consider the universal set
$U = \{x: x$ is multiple of 2 and $0 < x \le 30\}$.
$A = \{x:x$ is a multiple of 6) and
$B = \{x:x$ is a multiple of 8$\}$
(i) List all elements of sets A and B in tabular form. 09303043

(i) Tabular form:

Solution: 09303044

$U = \{2, 4, 6, 8, \ldots, 30\}$
$A = \{6, 12, 18, 24, 30\}$
$B = \{8, 16, 24\}$

(ii) Find $A \cap B$

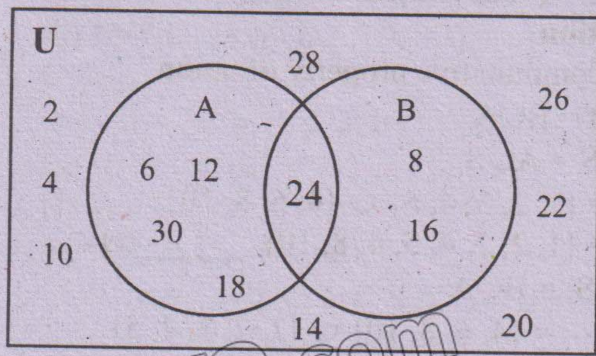
Solution:

$A \cap B$

$A\cap B = \{6, 12, 18, 24, 30\} \cap \{8, 16, 24\}$
$= \{24\}$

(iii) Draw a Venn diagram

Venn diagram:



Q.2 Let, $U = \{x:x$ is an integer and $0 < x \le 150\}$,
$G = \{x:x = 2^m$ for integer m and $0 \le m \le 12\}$ and
$H = \{x:x$ is a square$\}$
(i) List all elements of sets G and H in tabular form 09303045

Solution:

(i) Tabular form: 09303046

U = {1, 2, 3, 4, …,150}
G = {1, 2, 4, 8, 16, 32, 64, 128}
H = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144}

(ii) Find G∪H 09303047

Solution

G∪H = {1, 2, 4, 8, 16, 32, 64, 128}∪{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144}
G∪H = {1, 2, 4, 8, 9, 16, 25, 32, 36, 49, 64, 81, 100, 121, 128, 144}

(iii) Find G∩H 09303048

Solution:

G∩H = {1, 2, 4, 8, 16, 32, 64, 128}∩{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144}
G∩H = {1, 4, 16, 64}

Q.3 Consider the sets P = {x:x is a prime number and 0 < x < 20} and Q = {x:x is a divisor of 210 and 0 < x < 20} 09303049

Solution:

P = {2, 3, 5, 7, 11, 13, 17, 19}
Q = {1, 2, 3, 5, 6, 7, 10, 14, 15}

(i) Find P ∩ Q 09303050

Solution:

P∩Q = {2, 3, 5, 7, 11, 13, 17, 19} ∩ {1, 2, 3, 5, 6, 7, 10, 14, 15}
P∩Q = {2, 3, 5, 7}

(ii) Find P ∪ Q 09303051

Solution:

P∪Q = {2, 3, 5, 7, 11, 13, 17, 19} ∪ {1, 2, 3, 5, 6, 7, 10, 14, 15}
P∪Q = {1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19}

Q.4 Verify the commutative properties of union and intersection for the following pairs of sets:

Solution:

(i) A = {1,2,3,4,5}, B = {4, 6, 8, 10} 09303052

Solution

(a) Commutative property of union

A∪B = B∪A

L.H.S = A∪B
= {1, 2, 3, 4, 5} ∪ {4, 6, 8, 10}
= {1, 2, 3, 4, 5, 6, 8, 10} ______ (i)

R.H.S = B∪A
= {4, 6, 8, 10} ∪ {1, 2, 3, 4, 5}
= {1, 2, 3, 4, 5, 6, 8, 10} _______ (ii)

From (i) and (ii), L.H.S = R.H.S.
Hence, A∪B = B∪A

(b) Commutative property of intersection

A∩B = B∩A

L.H. S = A∩B
= {1, 2, 3, 4, 5}∩{4, 6, 8, 10}
= {4} __________ (i)

Now, R.H.S = B∩A
= {4, 6, 8, 10} ∩ {1,2,3,4, 5}
= {4} ________ (ii)

From (i) and (ii) L.H.S = R.H.S
Hence (A∩B) = (B∩A)

(ii) N, Z 9303053

Solution

For N, Z We know that
N ⊂ Z

(a) Commutative property of union:

N∪Z = Z∪N
L.H.S = N∪Z = Z ______ (i)
R.H.S = Z∪N = Z ______ (ii)
From (i) and (ii) L.H.S = R.H.S

(b) Commutative property of intersection:

N∩Z = Z∩N
L.H.S = N∩Z = N ______ (i)
R.H.S = Z∩N = N ______ (ii)

From (i), & (ii) L.H.S = R.H.S

(iii) $A = \{x | x \in R \wedge x \geq 0\}$, B = R 09303054

Solution

$A = \{x | x \in R \wedge x \geq 0\}$, B = R

$\Rightarrow A \subset B$

(a) Commutative property of union:

$A \cup B = B \cup A$

L.H.S = $A \cup B$ = B __________ (i)

R.H.S = $B \cup A$ = B __________ (ii)

From (i) and (ii) L.H.S = R.H.S

(b) Commutative property of intersection:

$A \cap B = B \cap A$

L.H.S = $A \cap B$ = A __________ (i)

R.H.S = $B \cap A$ = A __________ (ii)

From (i) and (ii) L.H.S = R.H.S

Q.5 Let U = {*a*, b, c, d, e, , f, g, h, i, j}
A = {*a*, b, c, d, g, h}, B={c,d,e,f,j}
Verify De Morgan's Laws for these sets, Draw Venn diagram. 09303055

Solution:

U = {*a*, b, c, d, e, , f, g, h, i, j}

A = {*a*, b, c, d, g, h}

B = {c, d, e, f, j}

(i) De-Morgan's law: $(A \cup B)' = A' \cap B'$

L.H.S = $(A \cup B)'$ 09303056

$(A \cup B)$ = {*a*, b, c, d, g, h} $\cup$ {c, d, e, f, j}

$(A \cup B)$ = {*a*, b, c, d, e, f, g, h, j}

Now, $(A \cup B)' = U - (A \cup B)$

= {*a*,b,c ,d ,e, f, g, h, i, j}−{*a*,b,c,d,e,f,g,h,j}

= { i } __________ (i)

Now, R.H.S = $A' \cap B'$

$A' = U - A$

A' = {*a*, b, c, d, e, f, g, h, i, j}− {a,b,c,d,g,h}

A' = {e, f, i, j}

Now, $B' = U - B$

B' = {*a*, b, c, d, e, f, g, h, i, j, }−{c, d, e, f, j}

B' = {*a*, b, g, h , i}

Now, R.H.S = $A' \cap B'$

= {e, f, i, j} $\cap$ {*a*, b, g, h, i}

= {i} __________ (ii)

From (i) and (ii)

L.H.S = R.H.S

$(A \cup B)' = A' \cap B'$

Verification by Venn Diagram:

Since, $A \cap B$ = {c, d} $\neq \phi$, So, A and B are overlapping sets.

L.H.S = $(A \cup B)'$

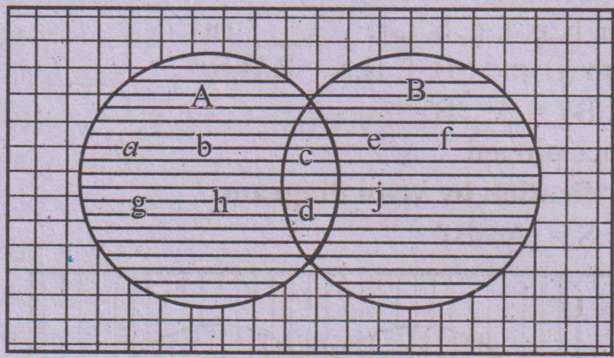


Fig. 1

$(A \cup B)$ = region of horizontal line segments

$(A \cup B)'$ = region of squares.

R.H.S = $A' \cap B'$

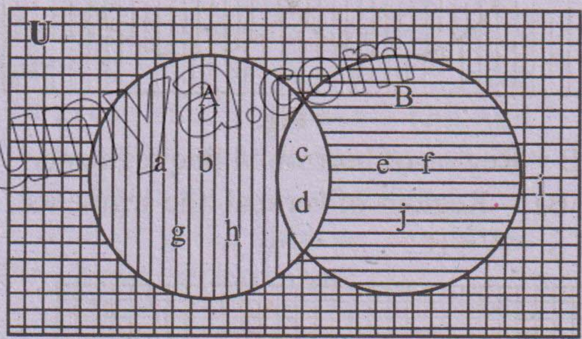


Fig. 2

$A' \cap B'$ = Region of squares.

From Fig.1 and Fig. 2,

Region showing $(A \cup B)'$ and $A' \cap B'$ are same which prove that $(A \cup B)' = A' \cap B'$

(ii) De-Morgan's law: $(A \cap B)' = A' \cup B'$

L.H.S = $(A \cap B)'$ 09303057

$(A \cap B)$ = {*a*, b, c, d, g, h} $\cap$ {c, d, e, f, j}

$(A \cap B)$ = {c, d,}

Now,

$(A \cap B)' = U - (A \cap B)$

= {*a*, b, c, d, e, f, g, h, i, j}−{c, d}

= {*a*, b, e, f, g, h, i, j} ________ (i)

R.H.S = $A' \cup B'$

$A' = U - A$

A'={a, b, c, d, e, f, g, h ,i , j} −{a, b, c, d, g, h}

A' = {e, f, i, j}

Now, $B' = U - B$

$B' = \{a, b, c, d, e, f, g, h, i, j\} - \{c, d, e, f, j\}$

$B' = \{a, b, g, h, i\}$

Now, R.H.S = $A' \cup B'$

$= \{e, f, i, j\} \cup \{a, b, g, h, i\}$

$= \{a, b, e, f, g, h, i, j\}$ ______ (ii)

From (i) and (ii), L.H.S = R.H.S

$(A \cap B)' = A' \cup B'$

Hence proved

Verification by venn diagram

L.H.S = $(A \cap B)'$

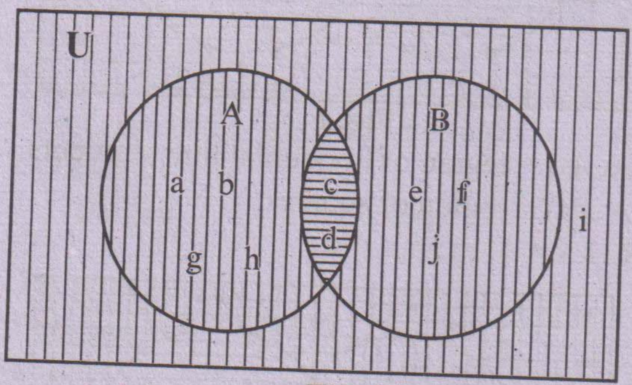


Fig. 1

$A \cap B$ = Region of horizontal line segments

$(A \cap B)'$ = Regions of vertical line segments.

R.H.S = $A' \cup B'$

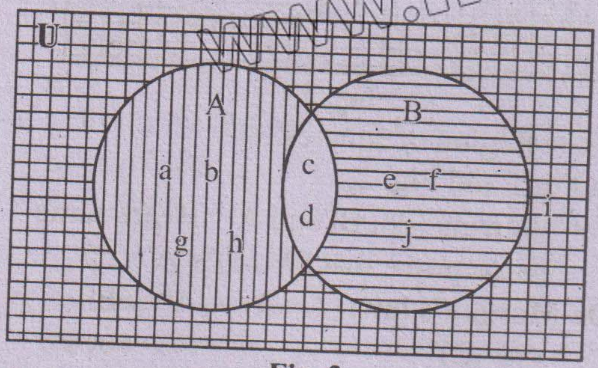


Fig. 2

$A' \cup B'$ = Regions of squares, horizontal and vertical line segments.

From Fig. 1 and fig. 2, regions showing $(A \cap B)'$ and $A' \cup B'$ are same which prove that $(A \cap B)' = A' \cup B'$.

Q.6 If $U=\{1,2,3,\ldots,20\}$ and $A=\{1,3,5,\ldots,19\}$, verify the following:

Solution:

$U = \{1, 2, 3, 4, \ldots, 20\}$

$A = \{1, 3, 5, \ldots, 19\}$

(i) $A \cup A' = U$ 09303058

Solution:

$A \cup A' = U$

$U = \{1, 2, 3, 4, \ldots, 20\}$

$A' = U - A$

$A' = \{1, 2, 3, 4 \ldots, 20\} - \{1, 3, 5, \ldots, 19\}$

$A' = \{2, 4, 6, 8, \ldots, 20\}$

Now,

$A \cup A' = \{1, 3, 5, \ldots, 19\} \cup \{2, 4, 6, 8, \ldots, 20\}$

$A \cup A' = \{1, 2, 3, 4, 5, \ldots, 20\}$ ______ (ii)

From (i) and (ii)

$A \cup A' = U$ (Hence proved)

(ii) $A \cap U = A$

Solution:

$A \cap U = A$

Proving $A \cap U = A$:

$A = \{1, 3, 5, \ldots, 19\}$ ______ (i)

Now,

$A \cap U = \{1, 3, 5, \ldots, 19\} \cap \{1, 2, 3, 4, 5 \ldots 20\}$

$A \cap U = \{1, 3, 5, \ldots, 19\}$ ______ (ii)

From (i) and (ii)

$A \cap U = A$

Hence proved

(iii) $A \cap A' = \phi$: 09303059

Solution:

$A \cap A' = \phi$

$A = \{1, 3, 5, \ldots 19\}$

$U = \{1, 2, 3, 4, 5, \ldots, 20$

$A' = U - A$

$A' = \{1, 2, 3, 4, 5, \ldots, 20\} - \{1, 3, 5, \ldots, 19\}$

$A' = \{2, 4, 6, 8, \ldots, 20\}$

Now,

$A \cap A' = \{1, 3, 5, \ldots, 19\} \cap \{2, 4, 6, 8, \ldots, 20\}$

$A \cap A' = \{\ \}$

$A \cap A' = \phi$ Hence proved

Q.7 In a class of 55 students, 34 like to play cricket and 30 like to play hockey. Also each students likes to play at least one of the two games. How many students like to play both games?

Solution :

Let A and B are two sets showing the students playing cricket and hockey respectively.

Total student = n(A∪B) = 55

34 like to play cricket = n(A) = 34

30 like to play hockey = n(B) =30

Let x students like to play both games, n(A∩B)= x

Using principle of inclusion and exclusion for two sets.

n(A∪B) = n(A) + n (B) – n(A∩B)

$55 = 34 + 30 - x$

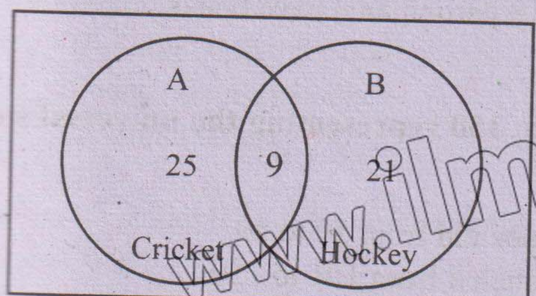
$55 = 64 - x$

$\Rightarrow x = 64 - 55$

$x = 9$

Thus 9 students like to play both games.

Venn diagram:



Q.8 In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak both Urdu and English, 30 can speak both English and Punjabi and 10 can speak both Urdu and Punjabi. How many can speak all three languages? 09303060

Solution:

Let A, B and C be three sets representing the employees who can speak Urdu, English and Punjabi respectively.

Total employees = 500

250 can speak Urdu, i.e. n(A) = 250

150 can speak English, i.e. n(B) = 150

50 can speak Punjabi, i.e. n(C) = 50

40 can speak Urdu and English i.e. n(A∩B) = 40

30 can speak English and Punjabi, i.e. n(B∩C) = 30

10 can speak Urdu, and Punjabi, i.e. n(A∩C) = 10

Let x can speak all the three languages, n(A∩B∩C) = x

Using principle of inclusion and exclusion for three sets.

n(A∪B∪C) = n(A) + n(B) +n(C)–n(A∩B)–n(B∩C) –n(A∩C) + n(A∩B∩C)

$500 = 250 + 150 + 50 - 40 - 30 - 10 + x$

$500 = 450 - 80 + x$

$500 = 370 + x$

$500 - 370 = x$

$130 = x$

$\Rightarrow \boxed{x = 130}$

Thus 130 employees can speak all the three languages.

Q.9 In sports events, 19 people wear blue shirts, 15 wear green shirts, 3 wear blue and green shirts, 4 wear a cap and blue shirts, and 2 wear a cap and green shirts. The Total number of people with either a blue or green shirt or cap is 25. How many people are wearing caps? 09303061

Solution: (Correction)

Let A, B and C be three showing people who wear green, blue shirts and cap respectively.

15 wear green shirts = n(A) = 15

19 wear blue shirts = n(B) = 19

Let x wear caps = n(c) = x

3 wear blue and green shirts = n(A∩B) = 3

4 wear cap and blue shirts = n(B∩C) = 4

2 wear cap and green shirts = n(A∩C) = 2

34 Wear either blue, green or cap n(A∪B∪C) =34

According to information no one wear all the three items, so n(A∩B∩C) = 0

Using principle of inclusion and exclusion:

n(A∪B∪C) = n(A)+n(B)+n(C)–n(A∪B)–n(B∩C)–n(A∩C)+n(A∩B∩C)

$34 = 15+19+x-3-4-2+0$

$34 = 34+x-9$

$\cancel{34} - \cancel{34} + 9 = x$

$9 = x$

$\Rightarrow \boxed{x = 9}$

Thus 9 people are wearing cap.

Q.10 In a training session, 17 participants have laptop, 11 have tablets, 9 have laptops and tablets, 6 have laptops and books and 4 have both tablets and books. Eight participants have all three items. The total number of participants with laptops, tablets, or books is 35. How many participants have books? 09303062

Solution:

Let A, B and C be the sets representing the participants having laptops, tablets and books respectively.

17 participants have laptops, n(A) = 17

11 participants have tablets, n(B) = 11

x participants have books, n(C) = x

9 have laptops and tablets, n(A∩B) = 9

6 have laptops and books, n(A∩B) = 6

4 have tablets and book, n(B∩C) = 4

8 have all these items = n(A∩B∩C) = 8

35 have laptops, tablets or books = n(A∩B∩C) = 35

Using principle of inclusion and exclusion for three sets.

n(A∪B∪C) = n(A) + n(B) + n(C) – n(A∩B) – n(B∩C) – n(A∩C)+n(A∩B∩C)

$35 = 17 + 11 + x - 9 - 4 - 6 + 8$

$35 = 36 - 19 + x$

$35 = 17 + x$

$35 - 17 = x$

$18 = x$

$\Rightarrow \boxed{x = 18}$

Thus 18 participants have books.

Q.11 A shopping mall has 150 employees, labeled 1 to 150 representing the universal set U. The employees fall into the following categories: 09303064

- Set-A: 40 employees with a salary range of 30k-45k, labeleld from 50 to 89.
- Set-B: 50 employees with a salary range of 50k-80k, labeled from 101 to 150.
- Set-C: 60 employees with a salary range of 100k-150k, labeleld from 1 to 49 and 90 to 100.

(a) Find (A′∪B′)∩C (b) Find n{A∩(B^c∩C^c)}

Solution:

U = {1, 2, 3, 4,..., 150}, n(U) = 150

A = {50, 51, 52, …, 89}, n(A) = 40

B = {101, 102, 103,…, 150}, n(B) = 50

C = {1,2,3,4,…49, 90,91,92,…,100}, n(C) = 60

(a) Find (A′∪B′)∩C

A′ = U–A

A′ = {1,2,3,4,5…150} – {50,51,52,…,89}

A′ = {1, 2, 3, 4…, 49, 90, 91, 92, 93,…,150}

Now, B′ = U–B

= {1, 2, 3, 4…,150}– {101, 102, 103,…150}

B′ = {1, 2, 3, 4…,100}

Now,

A′∪ B′ = {1,2,3,4,…,49,90,91,92,93,…,150}∪{1, 2, 3, 4, …,100}

A′∪ B′ = {1, 2, 3, 4, …,150}

Now,

(A′∪B′) ∩C = {1,2,3,4,…,150} ∩{1,2,3,4,…,49, 90, 91, 92, …,100}

$(A' \cup B') \cap C = \{1, 2, 3, 4, \ldots, 49, 90, 91, 93, \ldots, 100\}$

(b) Find $n\{A \cap (B^c \cap C^c)\}$

$B^c = U - B$

$B^c = \{1, 2, 3, 4, 5, \ldots, 150\} - \{101, 102, 103, \ldots, 150\}$

$B^c = \{1, 2, 3, 4, \ldots, 100\}$

Now, $C^c = U - C$

$C^c = \{1, 2, 3, 4, \ldots, 150\} - \{1, 2, 3, 4, \ldots, 49, 90, 91, 92, \ldots, 150\}$

$C^c = \{50, 51, 52, \ldots, 89, 101, 102, 103, \ldots, 150\}$

Now

$B^c \cap C^c = \{50, 51, 52, \ldots, 89\} = A$

Now

$A \cap (B^c \cap C^c) = A \cap A$

$A \cap (B^c \cap C^c) = A$

$n\{A \cap (B^c \cap C^c)\} = n(A) = 40$

Q.12 In a secondary school 125 students participate in at least one of the following sports: cricket, football, or hockey. 09303067

- 60 students play cricket.
- 70 students play football.
- 40 students play hockey.
- 25 students play both cricket and foot ball.
- 15 students play both football and hockey.
- 10 students play both cricket and hockey.

(a) How many students play all three sports?

(b) Draw a Venn diagram showing the distribution of sports participation in all the games.

Solution:

Let A, B and C be the sets representing the students who play cricket, football and hockey respectively.

60 students play cricket $= n(A) = 60$

70 students play football $= n = (B) = 70$

40 students play hockey $= n(C) = 40$

25 students play both cricket and football $=$ $n(A \cap B = 25$

15 students play both football and hockey $=$ $n(B \cap C) = 15$

10 students play both cricket and hockey $=$ $n(A \cap C) = 10$

09303066

(a) Let students playing all three games be $n(A \cap B \cap C) = x$,

Using principle of inclusion and exclusion.

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$ $-(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$125 = 60 + 70 + 40 - 25 - 15 - 10 + x$

$125 = 170 - 50 + x$

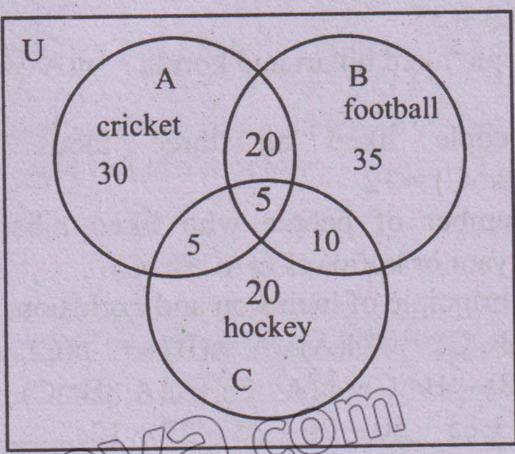
$125 = 120 + x$

$125 - 120 = x$

$5 = x \Rightarrow \boxed{x = 5}$

Thus 5 students play all the three games.

(b) Venn Diagram



Q.13 A survey was conducted in which 130 people were asked about their favourite foods. The survey results showed the following information:

09303068

- 40 people said they like nihari
- 65 people said they like biryani

- 50 people said they like korma
- 20 people said they liked nihari and biryani.
- 35 people said they like biryani and korma
- 27 people said they like nihari and korma.
- 12 people said they like all three foods nihari, biryani and korma.

(a) At least how many people like nihari, biryani or korma?

(b) How many people did not like nihari, biryani or korma?

(c) How many people like only one of the following foods: nihari, biryani or korma?

(d) Draw a Venn diagram.

Solution

Let A, B and C be the sets representing people who like nihari, biryani and korma respectively.

130 total people suryed, $n(U) = 130$

40 people liked nihari = $n(A) = 40$

65 people liked biryani = $n(B) = 65$

50 people liked korma = $n(C) = 50$

20 people liked nihari and biryani = $n(A \cap B) = 20$

35 people liked biryani, and korma = $n(B \cap C) = 35$

27 people liked nihari and korma = $n(A \cap C) = 27$

12 people liked all three foods = $n(A \cap B \cap C) = 12$

(a) Number of people who liked nihari, biryani or korma is $n(A \cup B \cup C)$.

Using principle of inclusion and exclusion.

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - (B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 40 + 65 + 50 - 20 - 35 - 27 + 12$

$= 167 - 82$

$= 85$

Thus 85 people like atleast one of foods item.

(b) Number of people who did not like any food item.

$N(A \cup B \cup C)' = n(U) - n(A \cup B \cup C)$

$= 130 - 85 = 45$

Thus 45 people did not like any food item.

(c) People who like only nihari =

$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$= 40 - 20 - 27 + 12$

$= 52 - 47 = 5$

People who like only biryany:

$= n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

$= 65 - 20 - 35 + 12$

$= 77 - 55$

$= 22$

People who like only korma:

$= n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

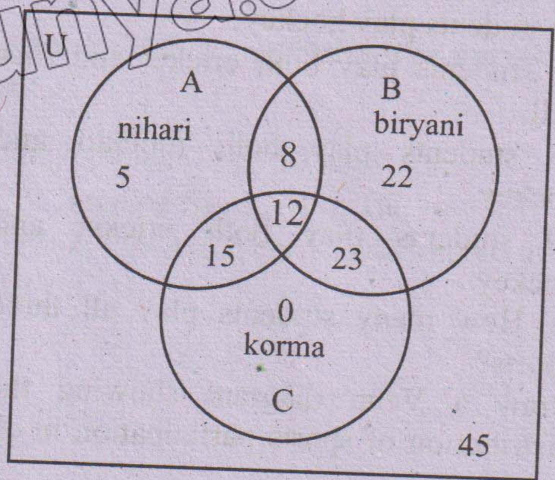
$= 50 - 27 - 35 + 12$

$= 62 - 62$

$= 0$

Thus number of people who like only one of the foods item = 5+22+0 = 27

(d) Venn Diagram



Binary Relations

(i) Let A and B be two non-empty sets, then the Cartesian product is the set of all ordered pairs (x, y) such that $x \in A$ and $y \in B$ and is denoted by:

$A \times B = \{(x, y) | x \in A \text{ and } y \in B\}$.

(ii) Any subset of $A \times B$ is called a binary relation, or simply a relation, from A to B. Ordinarily a relation will be denoted by the letter r.

(iii) The set of the first elements of the ordered pairs forming a relation is called its domain. The domain of any relation r is denoted as Dom r.

(iv) The set of the second elements of the ordered pairs forming a relation is called its range. The range of any relation r is denoted as Ran r.

(v) If A is a non-empty set, any subset of A×A is called a relation in A.

Example 7: Let c_1, c_2, c_3 be three children and m_1, m_2 be two men such that he father of both c_1, c_2 is m_1 and father of c_3 is m_2. Find the relation {(child, father)}

09303069

Solution:

C = Set of children = $\{c_1, c_2, c_3\}$ and

F = set of fathers = $\{m_1, m_2\}$

The Cartesian product of C and F:

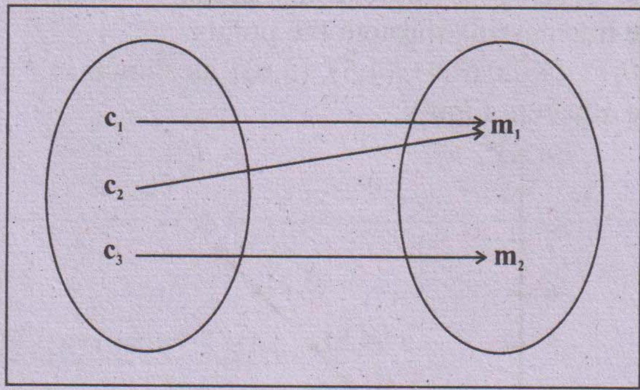
$C\times F = \{(c_1,m_1), (c_1,m_2), (c_2,m_1), (c_2,m_2), (c_3,m_1), (c_3,m_2)\}$

r = set of ordered pairs (child, father).

$= \{(c_1, m_1), (c_2, m_1), (c_3, m_2)\}$

Dom R = $\{c_1,c_2,c_3\}$, Range r= $\{m_1, m_2\}$

The relation is shown diagrammatically in adjacent figure.



Example 8: Let A = {1, 2, 3}. Determine the relation r such that x r y if f $x < y$. **09303070**

Solution:

A×A = {(1,1), (1, 2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)}

Clearly, required relation is:

R = {(1,2), (1,3), (2,3)}, DomR = {1,2}, Range R= {2,3}

Relation as Table, Ordered Pair and Graphs

Ordered pairs

A relation can be represented by a set of ordered pairs. For example, consider a water tank that starts with 1 litre of water already inside. Each minute, 1 additional litre of water is added to the tank. The situation can be represented by the relation $r = \{(x, y)| y = x+1\}$. Where x is the number of minutes (time) that have passed since the filling started and y is the total amount of water (in litres) in the tank.

When $x = 0$, y = 1 and x = 1, y = 2

In order pair this relation is represented as:

{(0,1), (1,2), (2,3), (3,4), (4,5), (5,6)}

The above relation in table form can be represented as given below:-

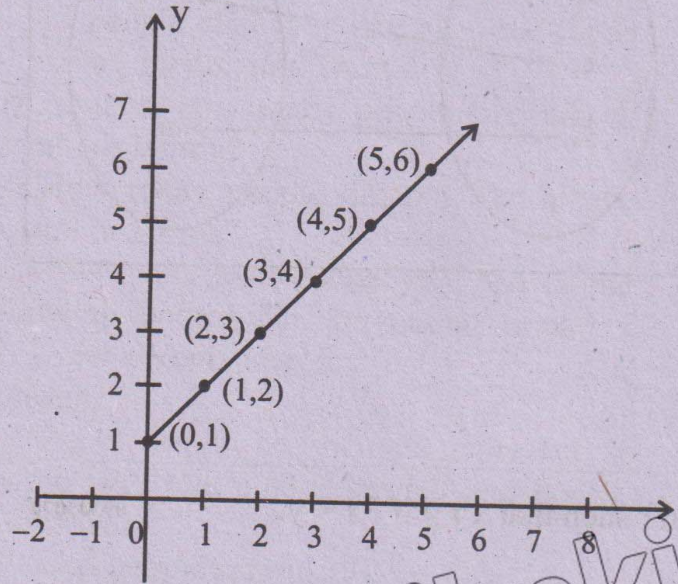
Table

x(time in minutes)	Y = *x*+1 (water in litres
0	y = 0 + 1 = 1
1	y = 1 + 1 = 2
2	y = 2 + 1 = 3
3	y = 3 + 1 = 4
4	y = 4 + 1 = 5
5	y = 5 + 1 = 6

Graph:

We can also represent the relations visually by drawing a graph. To draw the diagram, we use ordered pairs. Each ordered pair(x, y) is plotted as a point in the coordinate plane, where x is the first element and y is the second element of the ordered pair.

The relation is represented graphically by the line passing through the points, $\{(0,1), (1,2), (3,4), (4,5), (5,6)\}$ as shown in the adjacent Figure.



Function and its Domain and Range

Functions

A very important particular type of relation is a function defined as below:

(i) f is a relation from A to B, that is, f is a subset of $A\times B$

(ii) Domain $f = A$

(iii) First element of no two pairs of f are equal, then f is said to be a function from A to B.

The function f is also written as:

$$f: A \rightarrow B$$

Which is read as f is a function from A to B. The set of all first elements of each ordered pair represents the domain of f, and set of all second elements represent the range of f. Here, the domain of f is A, and the range of f is B.

If (x, y) is an element of f when regarded as a set of ordered pairs. We write $y = f(x)$. y is called the value of f for x or the image of x under f.

Example 9: If $A = \{0, 1, 2, 3,4\}$ and $B = \{3, 5, 7, 9, 11\}$, define a function f: $A\rightarrow B$, $f = \{(x, y) \mid y = 2x+3, x\in A \text{ and } y \in B\}$, Find the value of function f, its domain, co-domain and range.

09303071

Solution

Given: $y = 2x + 3$: $x \in A$ and $y \in B$, then value of function,

$f = \{(0, 3), (1,5), (2,7), (3,9), (4,11)\}$

Dom $f = \{0,1,2,3,4\} = A$

$\Rightarrow$ Co-domain $f = B$ and

$\Rightarrow$ Range $f = \{3,5,7,9,11\} \subseteq B$

Types of Functions

In this section we discuss different types of functions:

(i) Into Function

If a function f: $A\rightarrow B$ is such that Range $f\subset B$ i.e., Range $f \neq B$, then f is said to be a into function from A into B. In Fig. (i), f is clearly a function. But range $f \neq B$. Therefore, f is a function from A into B.

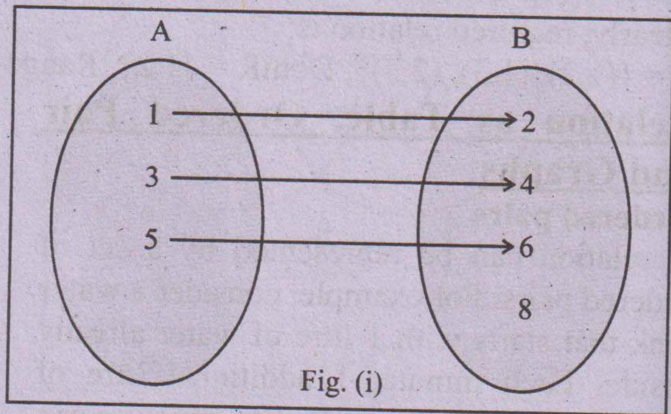


Fig. (i)

$f = \{(1,2), (3,4), (5,6)\}$

(ii) (One-One) Function (or Injective Function

If a function f from A into B is such that second elements of no two of its ordered pair are same, then it is called an injective function; the function shown in Fig. (iii) is such a function.

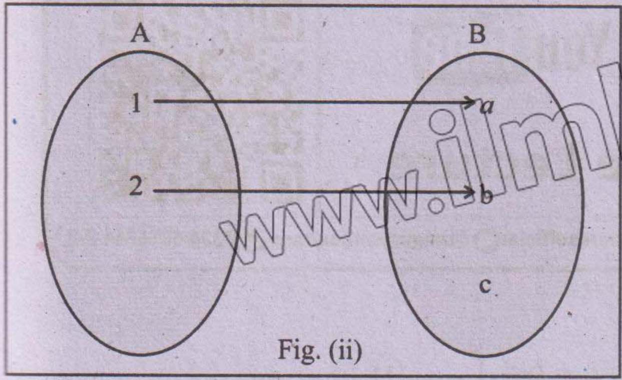


Fig. (ii)

$f = \{(1,a), (2,b)\}$

(iii)Onto function (or surjective function)

If a function f:A→B is such that Range f =B i.e., every element of B is the image of some element of A, then f is called an **onto** function or a surjective function.

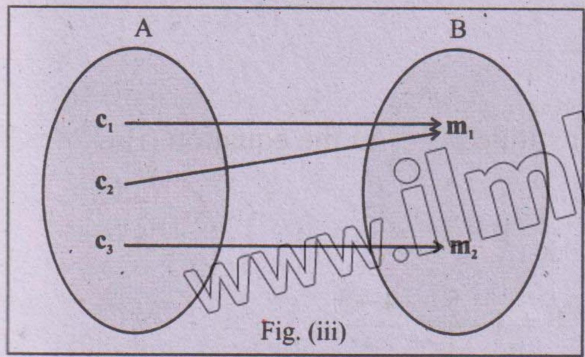


Fig. (iii)

$f = \{(c_1,m_1), (c_2,m_1), (c_3,m_2)\}$

(iv)(One-One) and onto Function (or Bijective Function)

A function f from A to B is said to be a Bijective function if it is both one-one and onto. Such a function is also called (1−1) correspondence between the sets A and B.

(a, z), (b, x) and (c, y) are the pairs of corresponding elements i.e., in this case f = {(a, z), (b, x), (c, y)} which is a bijective function or (1−1) correspondence between the sets A and B.

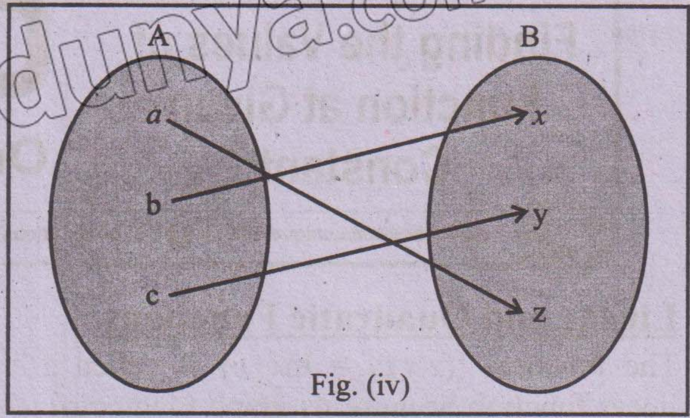


Fig. (iv)

$f = \{(a, z), (b,x), (c, y)\}$

Notation of Function

We know that set-builder notation is more suitable for infinite sets. So is the case with respect to a function comprising an infinite number of ordered pairs. Consider for instance, the function.

$f = \{(-1, 1), (0, 0) (1, 1), (2, 4),$
$(3, 9), (4, 16), \ldots\}$

Dom $f = \{-1, 0, 1, 2, 3, 4,\ldots\}$ and

Range $f = \{0, 1, 4, 9, 16\ldots\}$

This function may be written as:

$f = \{(x, y) \mid y = x^2, x \in N\}$

The mapping diagram for the function is shown in the Fig. (v).

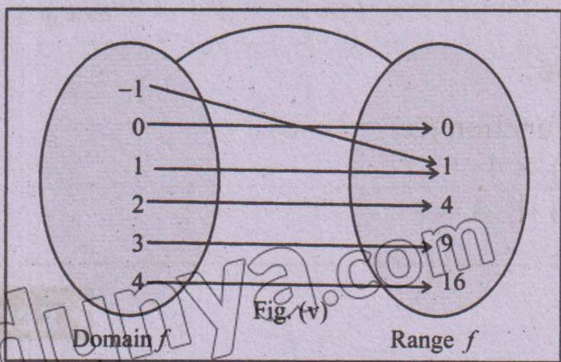


Fig. (v)

Linear and Quadratic Functions

The function $\{(x,y)|y = mx+c\}$ is called a linear function because its graph (geometric representation) is a straight line. We know that an equation of the form $y = mx+c$ represents a straight line. The function $\{(x, y) \mid y = ax^2+bx+c\}$ is called a quadratic function. We will study their geometric representation in the next chapter.

Example 10: If $f(x) = 2x-1$ and $g(x) = x^2-3$, then find:

(i) $f(1)$ 09303072 (ii) $f(-3)$ 09303073 (iii) $f(7)$ 09303074

(iv) $g(1)$ (v) $g(-3)$ (vi) $g(4)$

Solution:

(i) $f(1) = 2\times1-1 = 2-1 = 1$
(ii) $f(-3) = 2\times(-3)-1 = -6-1 = -7$
(iii) $f(7) = 2\times7-1 = 14-1 = 13$
(iv) $g(1) = (1)^2-3 = 1-3 = -2$
(v) $g(-3) = (-3)^2-3 = 9-3 = 6$
(vi) $g(4) = (4)^2-3 = 16-3 = 13$

Example 11: Consider $f(x) = ax+b+3$, where a and b are constant numbers. If $f(1) = 4$ and $f(5) = 9$, then find the value of a and b. 09303075

Solution:

Given function $f(x) = ax+b+3$
If $f(1) = 4$
$\Rightarrow a\times1+b+3 = 4$
$\Rightarrow a+b = 1$ ...(i)
Similarly, $f(5) = 9$
Then $a\times5+b+3 = 9$
$\Rightarrow 5a+b = 6$ ...(ii)
Subtract equation (i) from equation (ii), we get,
$(5a+b)-(a+b) = 6-1$
$5a+b-a-b = 5$
$4a = 5 \Rightarrow$
$a = \frac{5}{4}$
Substitute $a = \frac{5}{4}$ in the equation (i)
$\frac{5}{4}+b = 1$
$b = 1-\frac{5}{4} = \frac{4-5}{4}$
$\Rightarrow b = \frac{-1}{4}$
Thus, $a = \frac{5}{4}$, $b = -\frac{1}{4}$

Exercise 3.3

Q.1 For A = {1,2,3,4}, find the following relations in A. State the domain and range of each relation. 09303076

Solution:

$A = \{1, 2, 3, 4\}$
$A\times A = \{1, 2, 3, 4\}\times\{1, 2, 3, 4\}$
$A\times A = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

(i) $\{(x,y) \mid y = x\}$ 09303077

Solution:

$R = \{x, y) \mid y = x\}$

$R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Dom (R) = {1, 2, 3, 4}

range (R) = {1, 2, 3, 4}

(ii) $\{(x,y) \mid y+x = 5\}$ 09303078

Solution:

$\{(x,y) \mid y+x = 5\}$

R = {1, 4), (2, 3), (3, 2)(4, 1)}

Dom (R) = {1,2,3,4}

Range (R) = {4, 3, 2, 1}

(iii) $\{(x,y) \mid x+y < 5\}$ 09303079

Solution:

$\{(x,y) \mid (x+y < 5\}$

$R = \{x, y), \mid x+y < 5\}$

R = {1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3,1)}

Dom (R) = {1, 2, 3}

Range (R) = {1, 2, 3}

(iv) $R = \{(x,y) \mid x+y > 5\}$ 09303080

Solution:

$R = \{(x,y) \mid x+y > 5\}$

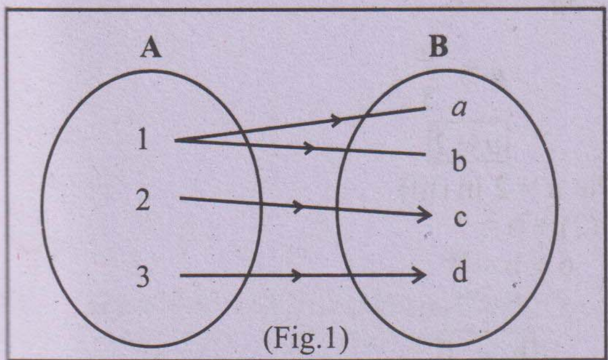
R = {(2,4), (3,3), (3,4),(4,2), (4,3), (4,4)}

Dom (R) = {2,3,4}

Range (R) = {4, 3, 2}

Q.2 Which of the following diagrams represent functions and of which type?

(i) 09303081



(Fig.1)

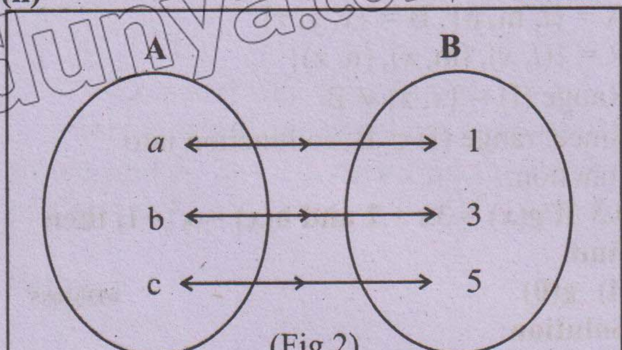
Solution:

A = {1,2,3}, B = {*a*, b, c, d},

R={(1,*a*), (1,b), (2,c), (3,d)}

Since, first elements in first two ordered pairs are same i.e. 1 , 1, so the relation is Not a function.

(ii) 09303082



(Fig.2)

Solution:

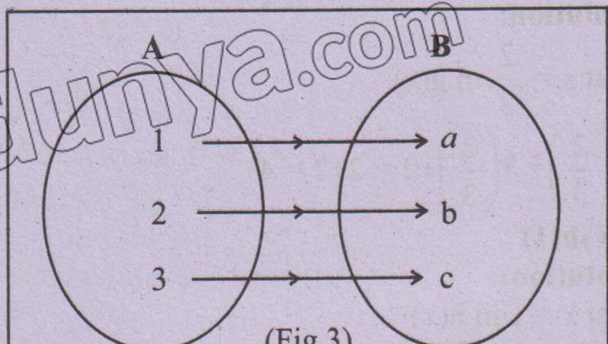
A = {*a*, b, c}, B = {1, 3, 5}

R = {(*a*, 1), (b, 3), (c, 5)}

Range (R) = {1, 3, 5} = B

Since relation is an onto and one-one function, so relation is a bijective function.

(iii) 09303083



(Fig.3)

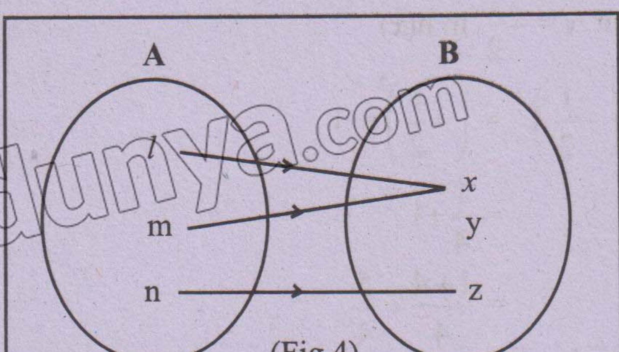
R = {(1,*a*), (2,b), (3,c)}

A = {1, 2, 3}, B ={*a*, b, c}

Range = {*a*, b, c} = B

Since, the relation is onto function and one-one function. So relation is a bijective function.

(iv) 09303084



(Fig.4)

Solution:

$A = \{l, m, n\}$, $B = \{x, y, z\}$

$R = \{(l, x), (m, x), (n, z)\}$

Range (f) $= \{x, z\} \neq B$

Since, range (f) $\subseteq B$, so function into function.

Q.3 If $g(x) = 3x + 2$ and $h(x) = x^2 + 1$, then find:

(i) g(0) 09303085

Solution:

Put $x = 0$ in $g(x)$

$g(0) = 3(0) + 2 = 0 + 2 = 2$

(ii) g(−3) 09303086

Solution

Put $x = -3$ in $g(x)$

$g(-3) = 3(-3) + 2 = -9 + 2 = -7$

(iii) $g\left(\frac{2}{3}\right)$

Solution:

Put $x = \frac{2}{3}$ in $g(x)$

$g\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + 2 = 2 + 2 = 4$

(iv) h(1)

Solution:

Put $x = 1$ in $h(x)$

$h(1) = (1)^2 + 1 = 1 + 1 = 2$

(v) h (−4)

Solution:

Put $x = -4$ in $h(x)$

$h(-4) = (-4)^2 + 1 = 16 + 1 = 17$

(vi) $h\left(-\frac{1}{2}\right)$

Solution:

Put $x = -\frac{1}{2}$ in $h(x)$

$$h\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + 1$$

$$= \frac{1}{4} + 1$$

$$= \frac{1+4}{4} = \frac{5}{4}$$

Q.4 Given that $f(x) = ax + b + 1$, where a and b are constant numbers. If $f(3) = 8$ and $f(6) = 14$, then find the values of a and b. 09303087

Solution:

Given that

$f(3) = 8$ ______ (i)

$f(6) = 14$ ______ (ii)

$f(x) = ax + b + 1$

Put $x = 3$ in $f(x)$

$f(3) = a(3) + b + 1$

$f(3) = 3a + b + 1$

From (i),

$8 = 3a + b + 1$

$8 - 1 = 3a + b$

$7 = 3a + b$

$\Rightarrow 3a + b = 7$ ______ (iii)

Now, put $x = 6$ in $f(x)$,

$f(6) = a(6) + b + 1$

$f(6) = 6a + b + 1$

From (ii) put $f(6) = 14$

$14 = 6a + b + 1$

$14 - 1 = 6a + b$

$13 = 6a + b$

$\Rightarrow 6a + b = 13$ ______ (iv)

Subtracting (iii), from (iv)

$$\begin{array}{r} 6a + \cancel{b} = 13 \\ \pm 3a \pm \cancel{b} = \pm 7 \\ \hline 3a \quad = 6 \end{array}$$

$$a = \frac{6}{3}$$

$$\boxed{a = 2}$$

Put $a = 2$ in (iii)

$3(2) + b = 7$

$6 + b = 7$

$b = 7 - 6$

$\boxed{b = 1}$

Thus $a = 2$, $b = 1$

Q.5 Given that $g(x) = ax + b + 5$, where a and b are constant numbers. If $g(-1) = 0$ and $g(2) = 10$, find the values of a and b. 09303088

Solution:

$g(-1) = 0$ ______ (i)

$g(2) = 10$ ______ (ii)

$g(x) = ax + b + 5$

Put $x = -1$ in $g(x)$

$g(-1) = a(-1) + b + 5$

$g(-1) = -a + b + 5$

From put (i) put $g(-1) = 0$

$\Rightarrow 0 = -a + b + 5$

$\Rightarrow a - b = 5$ _______ (iii)

Now, put $x = 2$ in $g(x)$

$g(2) = a(2) + b + 5$

$g(2) = 2a + b + 5$

From (ii) put $g(2) = 10$

$\Rightarrow 10 = 2a + b + 5$

$10 - 5 = 2a + b$

$5 = 2a + b$

$\Rightarrow 2a + b + 5$ _______ (iv)

Adding eq. (iii) and (iv)

$$\begin{aligned} a - \cancel{b} &= 5 \\ 2a + \cancel{b} &= 5 \\ \hline 3a &= 10 \end{aligned}$$

$$\Rightarrow \boxed{a = \frac{10}{3}}$$

Put it in equation (iii)

$$\frac{10}{3} - b = 5$$

$$\frac{10}{3} - 5 = b$$

$$\frac{10 - 15}{3} = b$$

$$\boxed{b = \frac{-5}{3}}$$

Thus $a = \frac{10}{3}$, $b = \frac{-5}{3}$

Q.6 Consider the function defined by $f(x) = 5x + 1$. If $f(x) = 32$, find the x value. 09303089

Solution:

$f(x) = 5x + 1$

$f(x) = 32$

By comparing, we get

$5x + 1 = 32$

$5x = 32 - 1$

$5x = 31$

$$x = \frac{31}{5}$$

Q.7 Consider the function $f(x) = cx^2 + d$, where c and d are constant numbers If $f(1) = 6$ and $f(-2) = 10$, then find the values of c and d. 09303090

Solution

$f(1) = 6$ _______ (i)

$f(-2) = 10$ _______ (ii

$f(x) = cx^2 + d$

Put $x = 1$ in $f(x)$

$f(1) = c(1)^2 + d$

$f(1) = c + d$ _______

From (i) put $f(1) = 6$

$6 = c + d$

$\Rightarrow c + d = 6$ _______ (iii)

Now, put $x = -2$ in $f(x)$

$f(-2) = c(-2)^2 + d$

$f(-2) = c(4) + d$

$f(-2) = 4c + d$

From (ii) put $f(-2) = 10$

$\Rightarrow 10 = 4c + d$

$\Rightarrow 4c + d = 10$ _____ (iv)

Subtracting (iii) from (iv)

$$\begin{aligned} 4c + \cancel{d} &= 10 \\ \pm c \pm \cancel{d} &= \pm 6 \\ \hline 3c &= 4 \end{aligned}$$

$$c = \frac{4}{3}$$

Put it in equation (iii)

$c + d = 6$

$$\frac{4}{3} + d = 6$$

$$d = 6 - \frac{4}{3}$$

$$d = \frac{18 - 4}{3}$$

$$d = \frac{14}{3}$$

Thus $c = \frac{4}{3}$, $d = \frac{14}{3}$

Review Exercise 3

Q.1 Choose the correct option.

i. The set builder form of the set $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, ...\right\}$ is: 09303091

(a) $\left\{x \mid x = \frac{1}{n}, n \in W\right\}$

(b) $\left\{x \mid x = \frac{1}{2n+1}, n \in W\right\}$

(c) $\left\{x \mid x = \frac{1}{n+1}, n \in W\right\}$

(d) $\{x \mid x = 2n+1, n \in W\}$

ii. If A = { }, then P(A) is: 09303092

(a) { } (b) {1}
(c) {{ }} (d) ϕ

iii. If U = {1,2,3,4,5}, A = {1,2,3} and B = {3,4,5}, then U−(A∩B) is: 09303093

(a) {1,2,4,5} (b) {2,3}
(c) {1,3,4,5}] (d) {1,2,3}

iv. If A and B are overlapping sets, then n(A−B) is equal to: 09303094

(a) n(A) (b) n(B)
(c) A ∩ B (d) n(A)−n(A∩B)

v. If A ⊆ B and B −A ≠ ϕ, then n(B−A) is equal to: 09303095

(a) 0 (b) n(B)
(c) n(A) (d) n(B) −n(A)

vi. If n(A∪B) = 50, n(A) = 30 n(B) = 35, then n(A∩B) =: 09303096

(a) 23 (b) 15
(c) 9 (d) 40

vii. If A = {1,2,3,4} and B = {x,y,z}, then Cartesian product of A and B contains exactly ___________ elements. 09303097

(a) 13 (b) 12
(c) 10 (d) 6

viii. If $f(x) = x^2 - 3x+2$, then the value of $f(a+1)$ is equal to: 09303098

(a) $a+1$ (b) a^2+1
(c) a^2+2a+1 (d) $a^2 - a$

ix. Given that $f(x) = 3x+1$, if $f(x) = 28$, then the value of x is: 09303099

(a) 9 (b) 27
(c) 3 (d) 18

x. Let A = {1, 2, 3} and B = {a, b} two non-empty sets and f:A→B be a function defined as f = {(1,a), (2,b), (3,b)}, then which of the following statement is true? 09303100

(a) f is injective
(b) f is surjective
(c) f is bijective
(d) f is into only

Answer Key

i	**b**	ii	**c**	iii.	**a**	iv	**d**	v	**d**
vi	**b**	vii	**b**	viii	**d**	ix	**a**	x	**b**

Multiple Choice Questions (Additional)

Basics of set

1. A collection of well-defined distinct objects is called: 09303101

(a) subset (b) power set
(c) set (d) Venn diagram

2. Which of the following is the set of first hundred whole number? 09303102

(a) {1,2,3,…,100}
(b) {1,2,3,…,99}
(c) {0,1,2,3,…,100}
(d) {0,1,2,3,…,99}

3. The different number of ways to describe a set are: **09303103**

(a) 1 (b) 2
(c) 3 (d) 4

4. A set with no element is called: **09303104**

(a) subset (b) Null set
(c) singleton set (d) super set

5. The set $\{x / x \in W \wedge x \geq 106\}$ is: **09303105**

(a) infinite set (b) subset
(c) supper set (d) finite set

6. The set having only one element is called: **09303106**

(a) Null set (b) power set
(c) singleton set (d) subset

Power set

7. The number of elements in power set $\{a,b,c,d\}$ is: **09303107**

(a) 4 (b) 8
(c) 16 (d) 32

8. Power set of an empty set is: **09303108**

(a) ϕ (b) $\{a\}$
(c) $\{\phi,\{a\}\}$ (d) $\{\phi\}$

Union and intersection of sets

9. If $P \subseteq Q$ then $P \cup Q$ is equal to: **09303109**

(a) P (b) Q
(c) ϕ (d) U

10. If $X \subseteq Y$ then $X \cap Y$ is equal to: **09303110**

(a) X (b) Y
(c) ϕ (d) U

11. If $A \subseteq B$ then $A - B$ is equal to: **09303111**

(a) A (b) B
(c) ϕ (d) U

12. $(A \cup B) \cup C$ is equal to: **09303112**

(a) $A \cap (B \cup C)$ (b) $(A \cup B) \cap C)$
(c) $A \cup (B \cup C)$ (d) $A \cap (B \cap C)$

13. $A \cup (B \cap C)$ is equal to: **09303113**

(a) $A \cup (B \cup C)$
(b) $A \cap (B \cap C)$
(c) $(A \cap B) \cup (A \cap C)$
(d) $(A \cup B) \cap (A \cup C)$

14. If A and B are disjoint sets, then $A \cup B$ is equal to: **09303114**

(a) A (b) B
(c) ϕ (d) $B \cup A$

15. If $A \cap B = \phi$, then set A and B are ….sets. **09303115**

(a) sub (b) over lapping
(c) disjoint (d) power

16. The complement of U is:

(a) U (b) ϕ
(c) impossible (d) union

17. The complement of ϕ is: **09303116**

(a) U (b) ϕ
(c) impossible (d) union

18. $A \cap A^c = \ldots\ldots$ **09303117**

(a) U (b) A
(c) A^c (d) ϕ

19. $A \cup A^c = \ldots\ldots$ **09303118**

(a) U (b) A
(c) A^c (d) ϕ

20. The set $\{ x \mid x \in B \text{ and } x \notin A \}$ is : **09303119**

(a) $A \cup B$ (b) $A \cap B$
(c) $A - B$ (d) $B - A$

21. The set $\{ x \mid x \in A \text{ and } x \in B \}$ is : **09303120**

(a) $A \cup B$ (b) $A \cap B$
(c) $A - B$ (d) $B - A$

22. $N \cup W = \ldots\ldots\ldots\ldots$ **09303121**

(a) ϕ (b) $\{0\}$
(c) N (d) W

23. $N - W = \ldots\ldots\ldots\ldots$ **09303122**

(a) ϕ (b) $\{0\}$
(c) N (d) W

Domain and Range of R

24. The domain of $R=\{(1,2),(2,3),(3,3)(3,4)\}$ is: 09303123
   (a) {1,3,4} (b) {1,2,3}
   (c) {1,2,4} (d) {2,3,4}

25. The Range of R $=\{(1,3),(2,2),(3,1)(4,4)\}$ is: 09303124
   (a) {1,2,4} (b) {3,2,4}
   (c) {1,2,3,4} (d) {1,3,4}

Ordered pairs

26. Point (–3,4) lies in the quadrant: 09303125
   (a) I (b) II
   (c) III (d) IV

27. The point (–4 , –5) lies in … quadrant 09303126
   (a) I (b) II
   (c) III (d) IV

Function

28. If $g(x)= 7x–2$ then $g(–1)=$…. 09303127
   (a) –2 (b) –7
   (c) –9 (d) –1

29. If Range (f) ⊂ B, then function is: 09303128
   (a) into (b) onto
   (c) bijective (d) injective

30. If $f(x)= 5x+12$ and $f(x)=32$ then $x=$---- 09303129
   (a) –4 (b) 4
   (c) 20 (d) 32

Answer Key

1	c	2	d	3	c	4	b	5	a	6	c	7	c	8	d	9	b	10	a
11	c	12	c	13	d	14	d	15	c	16	b	17	a	18	d	19	a	20	d
21	b	22	d	23	a	24	b	25	c	26	b	27	c	28	c	29	a	30	b

Q.2 Write each of the following sets in tabular forms:

(i) $\{x| x = 2n, n\in N\}$ 09303130

Solution:

$\{x| x = 2n, n\in N\}$

Tabular Form

{2,4,6,8, …}

(ii) $\{x| x = 2m+1, m\in N\}$ 09303131

Solution:

$\{x| x = 2m+1, m\in N\}$

Tabular Form

{3, 5, 7, 9, …}

(iii) $\{x| x = 11n, n\in W\wedge n<11\}$ 09303132

Solution:

$\{x| x = 11n, n\in W\wedge n<11\}$

Tabular Form

{0,11,22,33,44,55,66,77,88,99,110}

(iv) $\{x| x\in E \wedge 4 <x<6\}$ 09303133

Solution:

Tabular Form

ϕ or { }

(v) $\{x| x\in O \wedge 5\le x < 7\}$ 09303134

Solution:

$\{x| x\in O \wedge 5 \le x < 7\}$

Tabular Form

{5}

(vi) $\{x| x\in Q \wedge x^2 = 2\}$ 09303135

Solution:

$\{x| x\in Q \wedge x^2 = 2\}$

{ } or ϕ

$\left(\because \sqrt{x^2} = \sqrt{2},\quad x = \sqrt{2} \notin Q\right)$

(vii) $\{x| x\in Q\wedge x=-x\}$ 09303136

Solution:

$\{x| x\in Q\wedge x = -x \}$

Tabular Form

{0}

$(\because x+x = 0)$

$0+0 = 0$

(viii) $\{x| x\in R \wedge x\notin Q'\}$ 09303137

Solution:

$\{x| x\in R \wedge x\notin Q'\}$

$Q = R - Q'$

Q.3 Let U = {1,2,3,4,5,6,7,8,9,10}, A = {2,4,6,8,10}, B = {1,2,3,4,5} and C= {1,3,5,7,9}

List the members of each the following sets:

Solution:

$U = \{1,2,3,4,5,6,7,8,9,10\}$

$A = \{2, 4, 6, 8, 10\}$

$B = \{1, 2, 3, 4, 5\}$

$C = \{1, 3, 5, 7, 9\}$

(i) A' 09303138

Solution:

$A' = U - A$

$A' = \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$

$A' = \{1,3,5,7,9\}$

(ii) B' 09303139

Solution:

$B' = U - B$

$B' = \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5\}$

$B' = \{6,7,8,9,10\}$

(iii) $A \cup B$ 09303140

Solution:

$A \cup B$

$A \cup B = \{2, 4, 6, 8, 10\} \cup \{1, 2, 3, 4, 5\}$

$A \cup B = \{1,2,3,4,5,6, 8,10\}$

(iv) $A - B$ 09303141

Solution:

$A - B$

$A - B = \{2, 4, 6, 8, 10\} - \{1, 2, 3, 4, 5\}$

$A - B = \{6, 8, 10\}$

(v) $A \cap C$ 09303142

Solution:

$A \cap C$

$A \cap C = \{2, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\}$

$A \cap C = \phi$

(vi) $A' \cup C'$ 09303143

Solution:

$A' \cup C'$

$A' = U - A$

$A' = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2,4,6,8,10\}$

$A' = \{1, 3, 5, 7, 9\}$

Now, $C' = U - C$

$C' = \{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\}$

$C' = \{2,4,6,8,10\}$

Now, $A' \cup C' = \{1,3,5,7,9\} \cup \{2,4,6,8,10\}$

$A' \cup C' = \{1,2,3,4,\ldots,10\}$

(vii) $A' \cup C$ 09303144

Solution:

$A' = U - A$

$A' = \{1,2,3,4,5,67,8,9,10\} - \{2,4,6,8,10\}$

$A' = \{1, 3, 5, 7, 9\}$

Now, $A' \cup C = \{1,3,5,7,9\} \cup \{1,3,5,7,9\}$

$A' \cup C = \{1,3,5,7,9\}$

(viii) U' 09303145

Solution:

$U' = U - U$

$U' = \{1,2,3,4,5,67,8,9,10\} - \{1,2,3,4,\ldots,10\}$

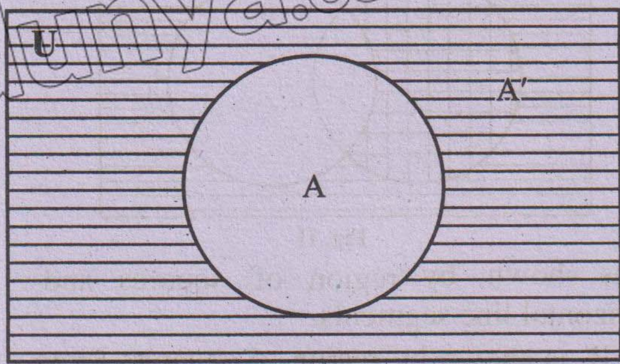
$U' = \phi$

Q.4 Using the Venn diagrams, if necessary, find the single sets equal to the following:

(i) A' 09303146

Solution:

$A' = U - A$



Horizontally lined region shows A'.

(ii) $A \cap B$ 09303147

Solution:

$A \cap B$

$A \cap U = A \qquad (\because A \subset U)$

(iii) $A \cup U$ 09303148

Solution:

$A \cup U = U \qquad (\because A \subset U)$

(iv) $A \cup \phi$ 09303149

Solution:

$A \cup \phi$

$A \cup \phi = A \qquad (\because \phi \subset A)$

(v) $\phi \cap \phi$ 09303150

Solution:

$\phi \cap \phi$

$\phi \cap \phi = \phi$

Q.5 Use Venn diagrams to verify the following:

(i) A−B = A∪B′ 09303151

Solution:

A−B = A∪B′

L.H.S = A−B

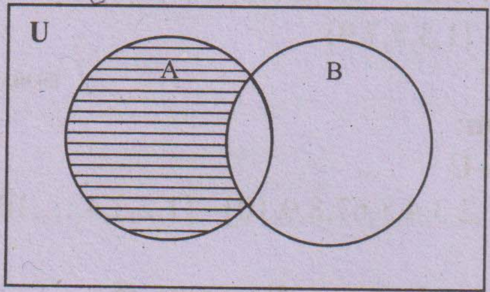


Fig. I

A−B is shown by region of horizontal line segments..

R.H.S = A∩B′

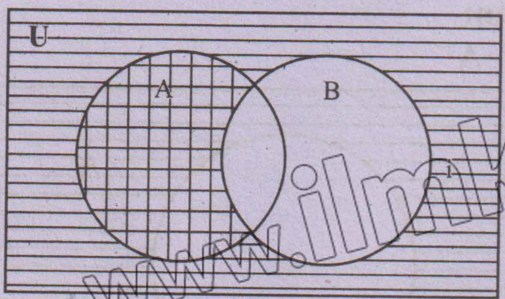


Fig. II

B′ is shown by region of squares and horizontal line segments.

A∩B′ is shown by region of squares. From fig. (I) and Fig. (ii) regions showing (A−B) and A∩B′ are same, so

A−B = A∩B′.

(ii) (A−B)′∩B = B 09303152

Solution:

L.H.S (A−B)′ ∩B

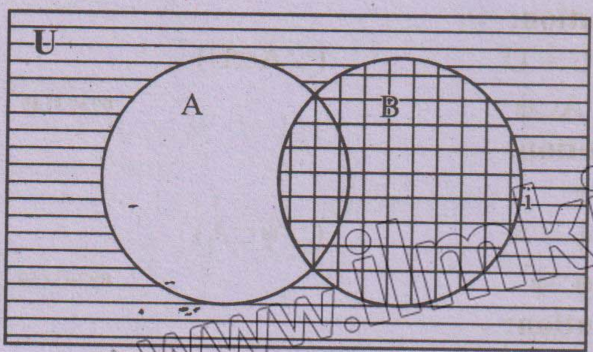


Fig. I

(A−B)′∩ B is shown by regions of squares

(A−B)′∩B is shown by region of square.

R.H.S = B

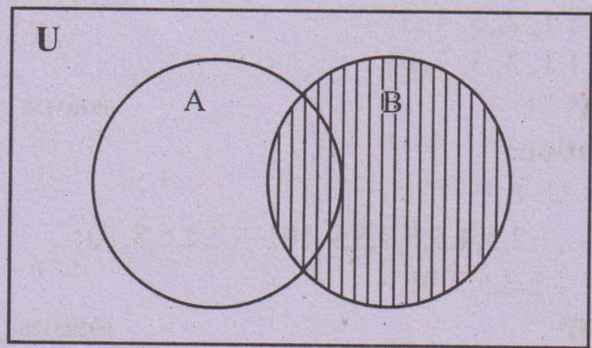


Fig. II

Regions showing (A−B)′∩B and B are same so,

(A−B)′∩B = B

Q.6 Verify the properties for the sets A, B and C given below: 09303153

Solution:

(a) A = {1,2,3,4}, B={3,4,5,6,7,8},
C= {5,6,7,9,10}

(i) Associativity of Union 09303154

Solution:

(A∪B) ∪C = A∪ (B∪C)

L.H.S = (A∪B) ∪C

= ({1,2,3,4} ∪ {3,4,5,6,7,8}) ∪C

= {1,2,3,4,5,6,7,8} ∪ {5,6,7,9,10}

= {1,2,3,4,5,6,7,8,9,10} ______ (i)

Now, R.H.S = A∪(B∪C)

= A∪({3,4,5,6,7,8}∪{5,6,7,9,10})

= {1,2,3,4}∪{3,4,5,6,7,8,910}

= {1,2,3,4,5,6,7,8,9,10}_______(ii)

From (i) and (ii) L.H.S = R.H.S

(A∪B) ∪C = A∪(B∪C) Hence proved

(ii) Associativity of intersection 09303155

Solution:

(A∩B)∩C = A∩(B∩C)

L.H.S = (A∩B) ∩C

= ({1,2,3,4} ∩ {3,4,5,6,7,8})∩C

= {3,4,}∩{5,6,7,9,10}

= { }______(i)

R.H.S = A∩(B∩C)

= A∩({3,4,5,6,7,8} ∩ {5,6,7,9,10})

= {1,2,3,4} ∩ {5,6,7}
= { } _____ (ii)
From (i) and (ii) L.HS = R.H.S
(A∩B) ∩C = A∩(B∩C) Hence proved.

(iii)Distributivity of Union over intersection 09303156

Solution:

A∪(B∩C) = (A∪B) ∩(A∪C)
L.H.S = A∪ (B∩C)
= A∪({3,4,5,6,7,8}∩{5,6,7,9,10})
= {1, 2, 3, 4} ∪ {5, 6, 7}
= {1,2,3,4,5,6,7} _____ (i)
R.H.S = (A∪B) ∩ (A∪C)
(A∪B) = {1,2,3,4} ∪ {3,4,5,6,7,8}
(A∪B) = {1,2,3,4,5,6,7,8}
Now, A∪C = {1,2,3,4} ∪ {5,6,7,9,10}
= {1, 2, 3, 4, 5, 6, 7, 9, 10}
Now,
(A∪B) ∩ (A∪C)
= {1,2,3,4,5,6,7,8} ∩{1,2,3,4,5,6,7,9,10}
= {1,2,3,4,5,6,7} _____(ii)
From (i) and (ii), L.H.S = R.H.S
A∪(B∩C) = (A∪B)∩(A∪C) Hence proved

(iv)Distributivity of intersection over union 09303157

(a) A = {1,2,3,4}, B = {3,4,5,6,7,8}, C={5,6,7,9,10}

Solution:

A∩ (B∪C) = (A∩B) ∪ (A∩C)
L.H.S = A∩(B∪C)
= A∩({3,4,5,6,7,8}∪{5,6,7,9,10})
= {1,2,3,4}∩{3,4,5,6,7,8,9,10}
= {3, 4} __________ (i)
Now, R.H.S = (A∩B) ∪ (A∩C)
(A∩B) = {1,2,3,4} ∩ {3,4,5,6,7,8}
(A∩B) = {3,4}
Now, (A∩C) = {1,2,3,4} ∩ {5,6,7,9,10}
= { }
Now, (A∩B) ∪ (A∩C) = {3, 4}∪{ }
= {3, 4} _____(ii)
From (i) and (ii)
L.H.S = R.H.S
A∩(B∪C) = (A∩B) ∪ (A∩C)

(b) A = ϕ, B = {0}, C = {0, 1, 2}

(i) Associvity of union 09303158

(A∪B) ∪C = A∪(B∪C)
L.H.S = (A∪B) ∪C
= ({ } ∪ {0}) ∪C
= {0} ∪ {0,1,2}
= {0, 1, 2}_____ (i)
R.H.S = A∪(B∪C)
= A∪({0} ∪{0, 1, 2})
= { } ∪{0, 1, 2}
= {0, 1, 2} ________ (i)
From (i) and (ii) L.H.S = R.H.S
(A∪B) ∪C = A∪(B∪C)
= {0,4,2}∪{0,1,2}
Hence proved.

(ii) Associativity of intersection 09303159

(A∩B) ∩C = A∩(B∩C)
L.H.S = (A∩B) ∩C
= ({ } ∩{0}) ∩C
= { } ∩ {0, 1, 2)}
= { } ________ (i)
R.H.S = A∩(B∩C)
= A∩({0} ∩ {0, 1, 2})
= { } ∩ { 0}
= { } ______ (ii)
From (i) and (ii) L.H.S = R.H.S
(A∩B) ∩C = A∩(B∩C)
Hence proved.

(iii)Distributivity of Union over intersection 09303160

Solution:

A∪(B∩C) = (A∪B) ∩ (A∪C)
L.H.S = A∪(B∩C)
= A∪({0}∩ {0,1,2})
= { } ∪ {0}
= {0} ______ (i)
R.H.S = (A∪B) ∩(A∪C)
= ({ } ∪{0}) ∩ ({ } ∪ {0,1,2})
= {0}∩ {0,1,2}
= { 0 } ________ (ii)
From (i) and (ii), L.H.S = R.H.S
A∪(B∩C) = (A∪B) ∩ (A∪C)
Hence proved.

(iv) Distributivity of intersection over Union 09303161

Solution:

$A\cap(B\cup C) = (A\cap B) \cup (A\cap C)$

L.H.S $= A\cap(B\cup C)$

$= \{\ \}\cap(\{\ \}\cup\{0,1,2\})$

$= \{\ \}\cap\{0, 1, 2\}$

$= \{\ \}$ _____ (i)

R.H.S $= (A\cap B) \cup(A\cap C)$

$= (\{\ \}\cap\{0\})\cup(\{\ \}\cap\{0,1,2\})$

$= \{\ \} \cup \{\ \}$

$= \{\ \}$ _____ (ii)

From (i) and (ii) L.H.S = R.H.S

$A\cap(B\cup C) = (A\cap B) \cup(A\cap C)$

(c) $A = N, B = Z, C = Q$

Solution:

$N \subset Z \subset Q$

(i) Associativity of union 09303162

Solution

$(A\cup B) \cup C = A\cup(B\cup C)$

L.H.S $= (A\cup B) \cup C$

$= (N\cup Z) \cup Q$

$= Z\cup Q$

$= Q$ _____ (i)

R.H.S $= A\cup(B\cup C)$

$= N\cup(Z\cup Q)$

$= N\cup Q$

$= Q$ _____ (ii)

From (i) and (ii) L.H.S = R.H.S

$(A\cup B) \cup C = A\cup(B\cup C)$

Hence proved.

(ii) Associativity of intersection 09303163

Solution:

$(A\cap B)\cap C = A\cap(B\cap C)$

L.H.S $= (A\cap B) \cap C$

$= (N\cap Z) \cap Q$

$= N\cap Q$

$= N$ _____ (i)

R.H.S $= A\cap(B\cap C)$

$= N\cap(Z\cap Q)$

$= N\cap Z$

$= N$ _____ (ii)

From (i) and (ii) L.H.S = R.H.S

$(A\cap B) \cap C = A\cap(B\cap C)$

Hence proved

(iii) Distributivity of union over intersection 09303164

Solution:

$A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

L.H.S $= A\cup(B\cap C)$

$= N\cup(Z\cap Q)$

$= N\cup Z$

$= Z$ _____ (i)

R.H.S $= (A\cup B)\cap (A\cup C)$

$= (N\cup Z) \cap(N\cup Q)$

$= Z\cap Q$

$= Z$ _____ (ii)

From (i) and (ii), L.H.S = R.H.S

$A\cup(B\cap C) = (A\cup B) \cap (A\cup C)$

Hence proved.

(iv) Distributivity of intersection over union 09303165

Solution:

$A\cap(B\cup C) = (A\cap B) \cup (A\cap C)$

L.H.S $= A\cap(B\cup C)$

$= N\cap(Z\cup Q)$

$= N\cap Q$

$= N$ _____ (i)

R.H.S $= (A\cap B) \cup(A\cap C)$

$= (N\cap Z) \cup (N\cap Q)$

$= N\cup N$

$= N$ _____ (ii)

From (i) and (ii) L.H.S = R.H.S

$A\cap(B\cup C) = (A\cap B) \cup(A\cap C)$

Hence proved.

Q.7 Verify De Morgan's Laws for the following sets: 09303166

$U = \{1,2,3,\ldots,20\}$, $A = \{2,4,6,\ldots, 20\}$ and $B=\{1,3,5,\ldots 19\}$.

Solution:

$U = \{1, 2, 3, 4,\ldots,20\}$

$A = \{2, 4, 6, 8,\ldots, 20\}$

$B = \{1, 3, 5,\ldots,19\}$

(i) $(A\cup B)' = A'\cap B'$ 09303167

Solution:

$(A\cup B)' = A'\cap B'$

L.H.S $= (A\cup B)'$

$(A\cup B) = \{2,4,6,8,\ldots,20\}\cup\{1,3,5,7,\ldots,19\}$

$= \{1,2,3,4,5,6,\ldots,19,20\}$

Now, $(A\cup B)' = U-(A\cup B)$
$= \{1,2,3,4,\ldots,20\}-\{1,2,3,4,\ldots,20\}$
$= \{\ \}$ ________ (i)
Now, R.H.S $= A'\cap B'$
$A' = U-A$
$A' = \{1,2,3,4,5,\ldots,20\}-\{2,4,6,\ldots,20\}$
$A' = \{1,3,5,7,\ldots,19\}$
Now, $B' = U-B$
$B' = \{1,2,3,4,\ldots,20\}-\{1,3,5,\ldots,19\}$
$B' = \{2,4,6,8,\ldots,20\}$
R.H.S $= A'\cap B'$
$= \{1,3,5,\ldots,19\}\cap\{2,4,6,\ldots,20\}$
$= \{\ \}$ ______ (ii)
From (i) and (ii), L.H.S = R.H.S
$(A\cup B)' = A'\cap B'$ Hence proved.

(ii) $(A\cap B)' = A'\cup B'$ 09303168

L.H.S $= (A\cap B)'$
$(A\cap B) = \{2,4,6,\ldots,20\}\cap\{1,3,5,\ldots,19\}$
$(A\cap B) = \{\ \}$
Now, $(A\cap B)' = U-(A\cap B)$
$= \{1,2,3,4,\ldots,20\}-\{\ \}$
$= \{1,2,3,4,\ldots,20\}$ ______ (i)
R.H.S $= A'\cup B'$
$A' = U-A$
$A' = \{1,2,3,4,\ldots,20\}-\{2,4,6,\ldots,20\}$
$A' = \{1,3,5,\ldots,19\}$
Now, $B' = U-B$
$B' = \{1,2,3,4,\ldots,20\}-\{1,3,5,\ldots,19\}$
$B' = \{2,4,6,\ldots,20\}$
Now,
$A'\cup B' = \{1,3,5,\ldots,19\}\cup\{2,4,6,\ldots,20\}$
$= \{1,2,3,4,5,\ldots,20\}$ ______ (ii)
From (i) and (ii) L.H.S = R.H.S
$(A\cap B)' = A'\cup B'$

Q.8 Consider the set $P = \{x|x = 5m, m \in N\}$ and $Q = \{x|x = 2m, m\in N\}$. Find $P\cap Q$ 09303170

Solution:
$P = \{x|x = 5m, m\in N\}$
$Q = \{x|x = 2m, m\in N\}$
In tabular form:
$P = \{5, 10, 15, 20, 25,\ldots\}$
$Q = \{2, 4, 6, 8, 10,\ldots\}$
Finding $P\cap Q$:
$P\cap Q = \{5,10,15,20,25,\ldots\}\cap\{2,4,6,8,10,\ldots\}$
$P\cap Q = \{10, 20, 30, 40,\ldots\}$

Q.9 From suitable properties of union and intersection, deduce the following results:

(i) $A\cap(A\cup B) = A\cup(A\cap B)$ 09303171

Solution:
$A\cap(A\cup B) = A\cup(A\cap B)$
L.H.S $= A\cap(A\cup B)$ (by distributive property of intersection over union)
L.H.S $= (A\cap A)\cup(A\cap B)$ $(\because A\cap A = A)$
L.H.S $= A\cup(A\cap B)$
L.H.S = R.H.S

(ii) $A\cup(A\cap B) = A\cap(A\cup B)$ 09303172

L.H.S $= A\cup(A\cap B)$
$= (A\cup A)\cap(A\cup B)$ (by distributive property of union over intersection)
$= A\cap(A\cup B)$ $(\because A\cup A = A)$
L.H.S = R.H.S

Q.10 If $g(x) = 7x-2$ and $s(x) = 8x^2-3$ find: 09303173

Solution:
$g(x) = 7x-2$
$s(x) = 8x^2-3$

(i) g(0) 09303174

Solution
Put $x = 0$ in $g(x)$
$g(0) = 7(0)-2 = 0-2 = -2$

(ii) g(−1) 09303175

Solution:
Put $x = -1$ in $g(x)$
$p(-1) = 7(-1)-2 = -7-1 = -9$

(iii) $g\left(-\frac{5}{3}\right)$ 09303176

Solution:
Finding $g\left(-\frac{5}{3}\right)$
Put $x = \frac{-5}{3}$ in $g(x)$
$g\left(\frac{-5}{3}\right) = 7\left(\frac{-5}{3}\right)-2$
$= \frac{-35}{3}-2$

$$= \frac{-35-6}{3}$$

$$= \frac{-41}{3}$$

(iv) s(1) 09303177

Solution:

Put $x = 1$ in $s(x)$

$s(1) = 8(1)^2 - 3 = 8(1) - 3 = 8 - 3 = 5$

(v) s(−9) 09303178

Solution:

put $x = -9$ in $s(x)$

$s(-9) = 8(-9)^2 - 3$

$= 8(81) - 3$

$= 648 - 3$

$= 645$

(vi) $s\left(-\frac{7}{2}\right)$

Solution:

Finding $s\left(-\frac{7}{2}\right)$

Put $x = \frac{7}{2}$ in $s(x)$

$s\left(\frac{7}{2}\right) = 8\left(\frac{7}{2}\right)^2 - 3$

$= 8\left(\frac{49}{4}\right) - 3$

$= 2(49) - 3$

$= 98 - 3$

$= 95$

Q.11 Given that $f(x) = ax+b$, where a and b are constant numbers. If $f(-2) = 3$ and $f(4) = 10$, then find the values of a and b. 09303179

Solution:

$f(-2) = 3$ ______ (i)

$f(4) = 10$ ______ (ii)

$f(x) = a\,x + b$

To find $f(-2)$, put $x = -2$ in $f(x)$.

$f(-2) = a(-2) + b$

$f(-2) = -2a + b$

Putting value from (i)

$3 = -2a + b$

$\Rightarrow 2a - b = -3$ ______ (iii)

To find $f(4)$, put $x = 4$ in $f(x)$

$f(a) = a(4) + b$

$f(a) = 4a + b$

from (ii), putting value.

$10 = 4a + b$

$\Rightarrow 4a + b = 10$ ______ (iv)

Adding eq. (iii) (iv)

$2a - \cancel{b} = -3$

$4a + \cancel{b} = 10$

$6a = 7$

$$\boxed{a = \frac{7}{6}}$$

Put ii in eq. (iv)

$4\left(\frac{7}{6}\right) + b = 10$

$\frac{2\times 7}{3} + b = 10$

$\frac{14}{3} + b = 10$

$b = 10 - \frac{14}{3}$

$b = \frac{30-14}{3}$

$$\boxed{b = \frac{16}{3}}$$

Q.12 Consider the function defined by $k(x) = 7x - 5$. If $k(x) = 100$, find the value of x. 09303180

Solution:

$k(x) = 7x - 5$

$k(x) = 100$

By comparing, we get

$7x - 5 = 100$

$7x = 100 + 5$

$7x = 105$

$x = \frac{105}{7}$

$x = 15$

Q.13 Consider the function $g(x) = mx^2+n$, where m and n are constant numbers. If g(4) = 20 and g(0) = 5, find the values of m and n.

Solution: 09303181

$g(x) = mx^2 + n$

g(4) = 20 ______ (i)

g(0) = 5 ______ (ii)

Put x = 4 in g(x)

$g(4) = m(4)^2 + n$

$g(4) = m(16) + n$

$g(4) = 16m + n$

From (i) Put g(4) = 20 i.e.

20 = 16m+n

⇒ 16m + n = 20 ______ (iii)

Now, put x = 0 in g(x)

$g(0) = m(0)^2 + n$

$g(0) = m(0) + n$

$g(0) = n$

From (ii), g(0) = 5, so

5 = n

⇒ $\boxed{n = 5}$

Put it in eq. (iii)

16m + 5 = 20

16m = 20 − 5

16m = 15

$m = \frac{15}{16}$

Q.14 A shopping mall has 100 products from various categories labeled 1 to 100, representing the universal set U. The products are categorized as follows: 09303182

- Set A: Electronics, consisting of 30 products labeled from 1 to 30.
- Set B: Clothing comprises 25 products labeled from 31 to 55.
- Set C: Beauty Products, comprising 25 products labeled from 76 to 100. Write each set in tabular form, and find the union of all three sets.

Solution:

Tubular form:

U = {1, 2, 3, 4,…,100}, n(U) = 100

A = {1, 2, 3, 4, …, 30}, n(A) = 30

B = {31, 32, 33, …, 55}, n(B) = 25

C = {76, 77, 78, …, 100}, n(C) = 25

Finding union of all three sets

$(A \cup B) \cup C$

$= (\{1,2,3,4,\ldots,30\} \cup \{31,32,33,\ldots,55\}) \cup C$

$= \{1,2,3,4,\ldots,55\} \cup \{76,77,78,79,\ldots,100\}$

$= \{1,2,3,4,\ldots,55, 76,77,78,79,\ldots,100\}$

Q.15 Out of the 180 students who appeared in the annual examination, 120 passed the math test 90 passed the science test, and 60 passed both the math and science tests. 09303183

(a) How many passed either the math or science test?

(b) How many did not pass either of the two test?

(c) How many passed the science test but not the math test?

(d) How many failed the science test?

Solution:

Total students = 180

Let A and B are the sets representing the students who passed Math test and Science test respectively.

120 passed Math test, i.e. n(A) = 120

90 passed Science test i.e. n(B) = 90

60 passed both Math and Science test $n(A \cap B) = 90$

(a) Students who passed either the math or science test is $n(A \cup B)$.

We know that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

= 120 + 90 − 60

= 120 + 30

= 150

(b) Students who did not pass either of two test is $n(A \cup B)'$.

$n(A \cup B)' = n(U) - n(A \cup B)$

= 180 − 150

= 30

Thus 30 students could not pass any subject.

(c) Students only passed in science test.

$= n(B) - n(A \cap B)$

= 90 − 60

= 30

Thus 30 students passed only science test.

(d) Failed in science test

$N(B)' = n(U) - n(B)$

$= 180 - 90$

$= 90$

Thus 90 students failed in science test.

Q.16 In a software house of a city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survey revealed the following statistics: 09303184

- 150 developers like Python.
- 130 developers like Java.
- 120 developers like PHP.
- 70 developers like both Python and Java.
- 60 developers like both Python and PHP.
- 40 developers like all three languages: Python, Java and PHP.

(a) How many developers use at least one of these languages?

(b) How many developers use only of these languages?

(c) How many developers do not use any of these languages?

(d) How many developers use only PHP?

Solution:

Let A, B and C be three sets representing the software developers who like python, Java and PHP respectively.

Total soft ware developers = $n(U) = 300$

150 like python, $n(A) = 150$

130 like java , $n(B) = 130$

120 like PHP , $n(C) = 120$

70 like both python and Java = $n(A\cap B)=70$

50 like both Java and PHP = $n(A\cap C) = 50$

60 like both python and PHP = $n(A\cap C)= 60$

40 like all the three languages=$n(A\cap B\cap C)=40$

(a) Using principle of inclusion and exclusion.

$n(A\cup B\cup C) = n(A) + n(B) + n(c) - n(A\cap B) - n(B\cap C) - n(A\cap C)+n(A\cap B\cap C)$

$= 150 + 130 + 120 - 70 - 50 - 60 + 40$

$= 440 - 180$

$= 260$

(b) Developers who like python only:

$= n(A) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)$

$= 150-70-60+40$

$= 190-130 = 60$

Developers who like Java only:

$= n(B) - n(A\cap B) - n(B\cap C) + n(A\cap B\cap C)$

$= 130-70-50+40$

$= 170 - 120$

$= 50$

Developers who like PHP only.

$= n(C) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$= 120 - 50 - 60 + 40$

$= 160 - 110$

$= 50$

Developers who use only one of these.

Languages = $60 + 50 + 50 = 160$

(c) $n(A\cup B\cup C)' = n(U) - n(A\cup B\cup C)$

$= 300 - 260$

$= 40$

(d) developers who use only PHP only:

$= n(C) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$= 120 - 50 - 60 + 40$

$= 160 - 110$

$= 50$