UNIT 04

Factorization and Algebraic Manipulation

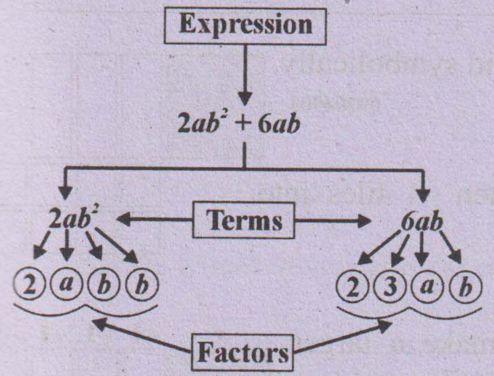
www.ilmkidunya.com

Common Factors

A common factor is an expression that divides two or more expressions exactly. For example,

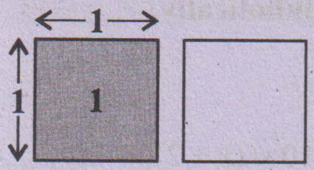
$$2x - 6 = 2(x - 3)$$

Here 2 is the common factor which is exactly divisible by both terms $2x$ and 6.



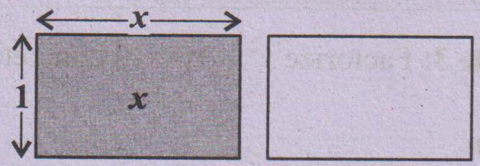
Unit Tiles

Here one grey unit tile represent 1 and one white unit tile represents –1. Both grey and white unit tiles form a zero pair.



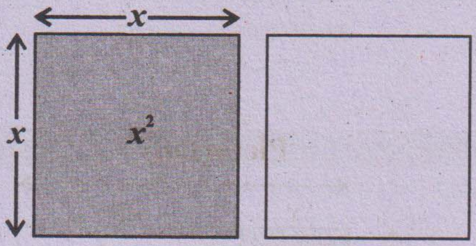
Rectangular Tiles

The grey rectangular tile represents x and the white rectangular tile represents – x. Both grey and white rectangular tiles also form a zero pair.



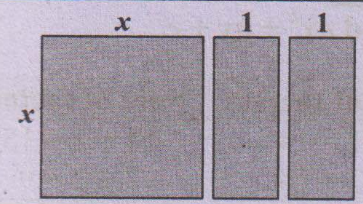
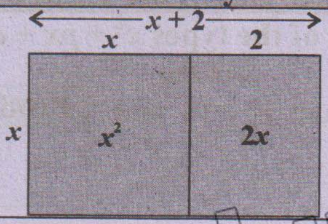
Square Tiles

The grey squared tile measure x units on each side and it has an area of $x \times x = x^2$ units. This tile is labeled as x^2 tile.. The white squared tile represens–x^2. Both grey and white squared tiles form a zero pair.



Example 1: Find common factor of $x^2 + 2x$ concretely, pictorially and symbolically 09304001

Solution: We arrange one x^2 tile and two x tiles into a rectangle.

Concretely	Pictorially	Symbolically
		$x^2 + 2x = x(x + 2)$

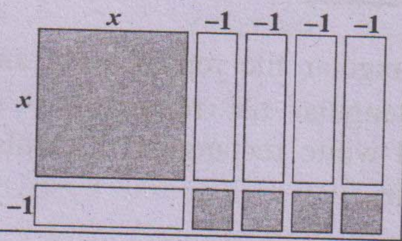
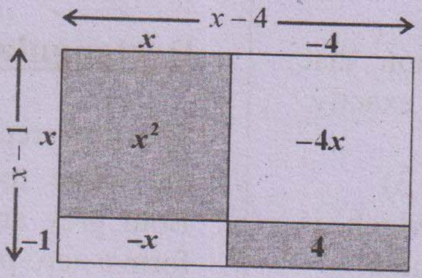
www.ilmkidunya.com

Trinomial Factoring

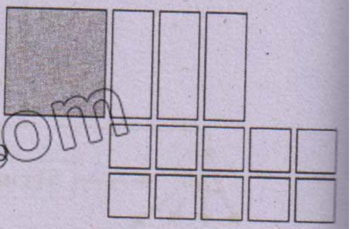
Trinomial factoring is converting trinomial expression as a product of two binomial expressions. A trinomial is an expression with three terms and binomial is an expression with two terms.

Example 2: Factorize $x^2 - 5x + 4$ concretely, pictorially and symbolically. **09304002**

Solution:

Concretely	Pictorially	Symbolically
We arrange one x^2 tile, five $-x$ tiles and four unit tiles into a rectangle. 		$x^2 - 5x + 4$ $= (x-1)(x-4)$

Example 3: Factorize $x^2 - 3x - 10$ concretely, pictorially and symbolically. **09304003**

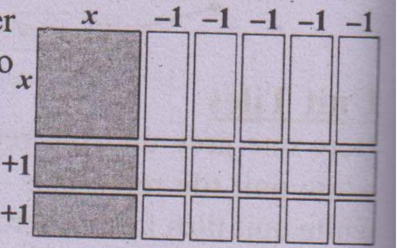


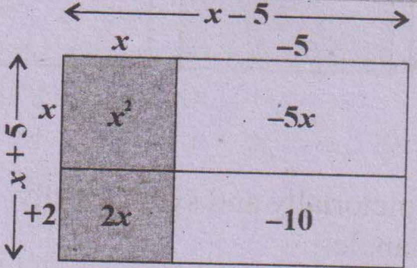
Solution

Concretely we arrange one x^2 tile, three $-x$ tiles and ten -1 tiles into rectangle.

www.ilmkidunya.com

We see that there are not enough rectangular tiles to make a larger rectangle. To fix this issue, we add zero pair. Adding two x tiles and two $-x$ tiles does not change the given expression because $2x - 2x = 0$.



Pictorially	Symbolically
	$x^2 - 3x - 10 = (x+2)(x-5)$

Factorizing Quadratic and Cubic Algebraic Expressions

Type – I: Factorization of expression of the types $x^2 + px + q$ and $ax^2 + bx + c$

Example 4: Factorize: $x^2 + 9x + 14$ **09304004**

Solution: Two numbers whose product is +14 and their sum is 9 are +2, +7.

Product of factors	Sum of factors
$14 \times 1 = 14$	$14 + 1 = 15$
$7 \times 2 = 14$	$7 + 2 = 9$

So, $\quad x^2 + 9x + 14$

$= x^2 + 2x + 7x + 14$

www.ilmkidunya.com

$= x(x + 2) + 7(x + 2)$

$= (x + 2)(x + 7)$

Example 5: Factorize: $x^2 - 11x + 24$

09304005

Solution: Two numbers whose product is +24 and their sum is –11 are –8, –3.

So, $x^2 - 11x + 24$

$= x^2 - 8x - 3x + 24$

$= x(x - 8) - 3(x - 8)$

$= (x - 8)(x - 3)$

Product of factors	**Sums of factors**
24 × 1 = 24	24 + 1 = 25
8 × 3 = 24	8 + 3 = 11
(–8) × (–3) = 24	–8 – 3 = –11
6 × 4 = 24	6 + 4 = 10
12 × 2 = 24	12 + 2 = 14

Example 6: Factorize: $p^2 + 11p + 18$

Solution: $p^2 + 11p + 18$

$= p^2 + 9p + 2p + 18$

$\boxed{\because 9 + 2 = 11\ ,\ 9 \times 2 = 18}$

$= p(p+9) + 2(p + 9)$

$= (p + 9)(p + 2)$

Consider cases where the coefficient of x^2 is not 1.

09304006

Example 7: Factorize: $2x^2 + 17x + 26$

09304007

Solution:

Step – I: Multiply the coefficient of x^2 with constant term. i.e.,

$2 \times 26 = 52$

Step – II: List all the factors of 52:

1, 52	–1, – 52
2, 26	–2, – 26
4, 13	–4, –13

Remember!
An expression having degree 2 is called a quadratic expression.

Step – III: Sum of factors equals middle term (17)

1 + 52 = 53	–1 – 52 = – 53
2 + 26 = 28	–2 – 26 = – 28
4 + 13 = 17	–4 – 13 = – 17

Try Yourself

Factorize the following expressions

(i) $x^2 + 7x - 18$

(ii) $t^2 - 5t - 24$

(iii) $6y^2 - y - 12$

Step – IV: Change the middle term in the given expression

$2x^2 + 17x + 26$

$= 2x^2 + 4x + 13x + 26$

Step – V: Take out commons from first two terms and last two terms

$= 2x(x + 2) + 13(x + 2)$

Step – VI: Again, take common from both terms

$= (x + 2)(2x + 13)$

Example 8: Factorize: $3x^2 - 4x - 4$

Solution: $3x^2 - 4x - 4$

$= 3x^2 + 2x - 6x - 4$

$\boxed{\because 2 \times (-6) = -12\ ,\ +2 - 6 = -4}$

$= x(3x + 2) - 2(3x + 2)$

$= (3x + 2)(x - 2)$

www.ilmkidunya.com

Exercise 4.1

Q.1 Factorize by identifying common factors.

(i) $6x + 12$ 09304008

Solution

$6x + 12$

$= 6x + 6\times 2$

$= 6(x+2)$ (6 is a common factor)

(ii) $15y^2 + 20y$ 09304009

Solution:

$15y^2 + 20y$

$= \textcircled{5}\times 3\times \textcircled{y}\times y + \textcircled{5}\times 4\times \textcircled{y}$

$= 5y\,(3y + 4)$ (5y is a common factor)

(iii) $-12x^2 - 3x$ 09304010

Solution:

$-12x^2 - 3x$

$= -\textcircled{3}\times 4\times \textcircled{x}\times x - \textcircled{3}\,\textcircled{x}\,\textcircled{1}$

$= -3x\,(4x+1)$ ($-3x$ is a common factor)

(iv) $4a^2b + 8ab^2$ 09304011

Solution:

$4a^2b + 8ab^2$

$= \textcircled{4a}\,a\,\textcircled{b} + 2\ \textcircled{4\,a}\,\textcircled{b}\,b$

$= 4ab\,(a+2b)$ (4 ab is a common factor)

(v) $xy - 3x^2 + 2x$

Solution:

$xy - 3x^2 + 2x$

$= \textcircled{x}\ y - 3x\ \textcircled{x} + 2\textcircled{x}$

$= x\,(y-3x + 2)$ (x is common factor)

(vi) $3a^2b - 9ab^2 + 15ab$ 09304012

Solution:

$3a^2b - 9ab^2 + 15ab$

$= \textcircled{3\times a}\times a\times \textcircled{b} - 3\times 3a\times \textcircled{b}\times b + \textcircled{3}\times 5\,\textcircled{a}\times \textcircled{b}$

$= 3ab(a-3b+5)$ ($3ab$ is a common factor)

Q.2 Factorize and represent pictorially:

(i) $5x + 15$ 09304013

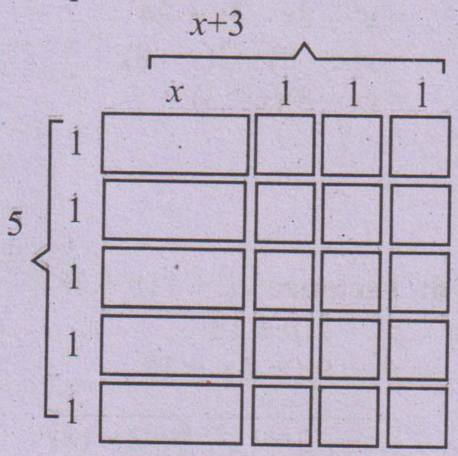
Solution:

$5x+15$

$= 5(x+3)$

Here, $(x+3)$ is 5 times. Draw the picture of $(x+3)$ and repeat it 5 times.

Pictorial represents



(ii) $x^2 + 4x + 3$ 09304014

Solution:

$x^2 + 4x + 3$

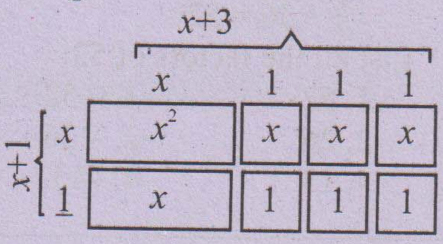
Factorization : x^2+4x+3

$= x^2+3x+x+3$

$= x(x+3) + 1(x+3)$

$= (x+3)\,(x+1)$

Pictorial Representation:



(iii) $x^2 + 6x + 8$ 09304015

Solution:

Factorization: x^2+6x+8

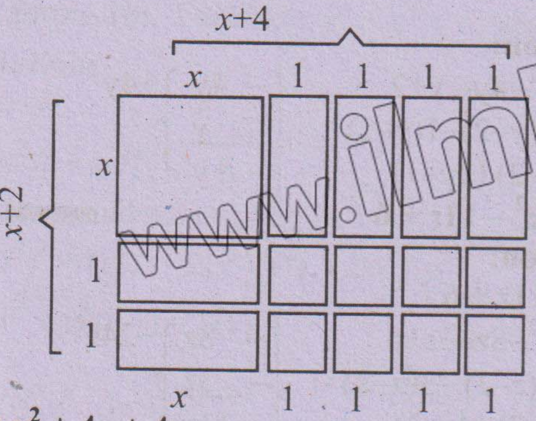
$= x^2+2x+4x+8$

$= x(x+2)+4(x+2)$

$= (x+2)\,(x + 4)$

Pictorial representation:

www.ilmkidunya.com



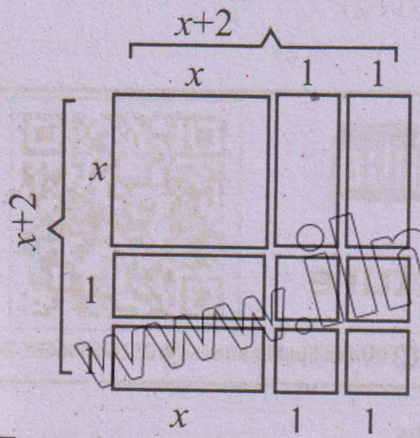
(iv) x^2+4x+4 09304016

Solution:

Factorization: x^2+4x+4

$$\begin{array}{ll} = & x^2+2x+2x+4 \\ = & x(x+2)+2(x+2) \\ = & (x+2)(x+2) \end{array}$$

Pictorial representation:



Q.3 Factorize:

(i) x^2+x-12 09304017

Solution:

x^2+x-12

$$\begin{array}{ll|l} = & x^2-3x+4x-12 & +4x \;\} \;\; -12x^2 \\ = & x(x-3)+4(x-3) & \underline{-3x} \\ = & (x-3)(x+4) & +\;x \end{array}$$

(ii) $x^2+7x+10$ 09304018

Solution:

$x^2+7x+10$

$$\begin{array}{ll|l} = & x^2+5x+2x+10 & +5x \;\} \;\; +10x^2 \\ = & x(x+5)+2(x+5) & \underline{+\;2x} \\ = & (x+2)(x+5) & +7x \end{array}$$

(iii) x^2-6x+8 09304020

Solution:

x^2-6x+8

$$\begin{array}{ll|l} = & x^2-4x-2x+8 & -4x \;\} \;\; +8x^2 \\ = & x(x-4)-2(x-4) & \underline{-2x} \\ = & (x-2)(x-4) & -6x \end{array}$$

(iv) x^2-x-56 09304021

Solution:

x^2-x-56

$$\begin{array}{ll|l} = & x^2-8x+7x-56 & -8x \;\} \;\; -56x^2 \\ = & x(x-8)+7(x-8) & +\underline{7x} \\ = & (x-8)(x+7) & -\;x \end{array}$$

(v) $x^2-10x-24$ 09304022

Solution:

$x^2-10x-24$

$$\begin{array}{ll|l} = & x^2-12x+2x-24 & -12x \;\} \;\; -24x^2 \\ = & x(x-12)+2(x-12) & +\underline{2x} \\ = & (x-12)(x+2) & -10x \end{array}$$

(vi) $y^2+4y-12$ 09304023

Solution:

$y^2+4y-12$

$$\begin{array}{ll|l} = & y^2-2y+6y-12 & +6y \;\} \;\; -12y^2 \\ = & y(y-2)+6(y-2) & -\;\underline{2y} \\ = & (y-2)(y+6) & +4y \end{array}$$

(vii) $y^2+13y+36$ 09304024

Solution:

$y^2+13y+36$

$$\begin{array}{ll|l} = & y^2+9y+4y+36 & 9y \;\} \;\; 36y^2 \\ = & y(y+9)+4(y+9) & +\underline{4y} \\ = & (y+9)(y+4) & +13y \end{array}$$

(viii) x^2-x-2 09304025

Solution:

x^2-x-2

$$\begin{array}{ll|l} = & x^2-2x+x-2 & -2x \;\} \;\; -2x^2 \\ = & x(x-2)+1(x-2) & \underline{+\;x} \\ = & (x-2)(x+1) & -\;x \end{array}$$

Q.4 Factorize:

(i) $2x^2+7x+3$ 09304026

Solution:

$2x^2+7x+3$

$$\begin{array}{ll|l} = & 2x^2+6x+x+3 & +6x \;\} \;\; 6x^2 \\ = & 2x(x+3)+1(x+3) & +\;\underline{x} \\ = & (x+3)(2x+1) & +7x \end{array}$$

(ii) $2x^2+11x+15$ 09304027

Solution:

$2x^2+11x+15$

$$\begin{array}{ll|l} = & 2x^2+6x+5x+15 & +6x \;\} \;\; +30x^2 \\ = & 2x(x+3)+5(x+3) & +\underline{5x} \\ = & (x+3)(2x+5) & +11x \end{array}$$

www.ilmkidunya.com

(iii) $4x^2 + 13x + 3$ 09304028

Solution:

$4x^2 + 13x + 3$

$= 4x^2+12x+x+3$

$= 4x(x+3)+1(x+3)$

$= (x+3)(4x+1)$

$$\left.\begin{array}{l}+12x\\ +\underline{\;\;x\;}\\ +13x\end{array}\right\}+12x^2$$

(iv) $3x^2 + 5x + 2$ 09304029

Solution:

$3x^2 + 5x + 2$

$= 3x^2+3x+2x+2$

$= 3x(x+1)+2(x+1)$

$= (3x+2)(x+1)$

$$\left.\begin{array}{l}+\;3x\\ +\underline{\;2\,x\;}\\ +5\,x\end{array}\right\}+6x^2$$

(v) $3y^2 - 11y + 6$ 09304030

Solution:

$3y^2 - 11y + 6$

$= 3y^2-9y-2y+6$

$= 3y(y-3)-2(y-3)$

$= (y-3)(3y-2)$

$$\left.\begin{array}{l}-\;\;9y\\ -\underline{\;\;2\,y\;}\\ -11\,y\end{array}\right\}+18\,y^2$$

(vi) $2y^2 - 5y + 2$ 09304031

Solution:

$= 2y^2-4y-y+2$

$= 2y(y-2)-1(y-2)$

$= (y-2)(2y-1)$

$$\left.\begin{array}{l}-\;4y\\ -\underline{\;\;y\;}\\ -5\,y\end{array}\right\}+4y^2$$

(vii) $4z^2 - 11z + 6$ 09304032

Solution:

$4z^2 - 11z + 6$

$= 4z^2-8z-3z+6$

$= 4z(z-2)-3(z-2)$

$= (z-2)(4z-3)$

$$\left.\begin{array}{l}-\;\;8z\\ -\underline{\;\;3z\;}\\ -11z\end{array}\right\}+24z^2$$

(viii) $6 + 7x - 3x^2$ 09304033

Solution:

$6+7x-3x^2$

$= 6-2x+9x-3x^2$

$= 2(3-x)+3x(3-x)$

$= (3-x)(2+3x)$

$= (3-x)(3x+2)$

$$\left.\begin{array}{l}+\;9x\\ -\underline{\;2x\;}\\ +7\,x\end{array}\right\}-18x^2$$

Factorization
YouTube
Online Lecture
publications.unique.edu.pk UGI.publications uniquenotesofficial @Uniquepublications 0324-6666661-2-3

Type–II: Factorization of the expression of the types

$a^4 + a^2b^2 + b^4$ or $a^4 + b^4$

Let's factorize: $a^4 + a^2b^2 + b^4$

$= a^4 + b^4 + a^2b^2$

$= (a^2)^2 + (b^2)^2 + a^2b^2$

$= (a^2)^2 + (b^2)^2 + a^2b^2$

$= (a^2)^2+(b^2)^2+2a^2b^2- 2a^2b^2+a^2b^2$

(Adding and subtracting $2a^2b^2$)

$= (a^2 + b^2)^2 - a^2b^2$

$= (a^2 + b^2)^2 - (ab)^2$

$= (a^2 + b^2 - ab)(a^2 + b^2 + ab)$

$= (a^2 - ab + b^2)(a^2 + ab + b^2)$

Example 9: Factorize: $x^4 + x^2 + 25$ 09304034

Solution:

$x^4 + x^2 + 25$

$= x^4 + 25 + x^2$

Factorize:

(x^4 +

($x^4 + \frac{1}{8\,x^4}$

$= (x^2)^2 + (5)^2 + 2(x^2)(5) - 2(x^2)(5) + x^2$

[(Adding and subtracting $2(x^2)(5)$]

$= (x^2 + 5)^2 - 10x^2 + x^2$

$= (x^2 + 5)^2 - 9x^2$

$= (x^2 + 5)^2 - (3x)^2$

$\boxed{\because a^2 - b^2 = (a+b)(a-b)}$

$= (x^2 + 5 - 3x)(x^2 + 5 + 3x)$

$= (x^2 - 3x + 5)(x^2 + 3x + 5)$

www.ilmkidunya.com

Example 10: Factorize: x^4+y^4 **09304035**

Solution:

x^4+y^4

$=(x^2)^2+(y^2)^2$

(Adding and subtracting $2x^2y^2$)

$=(x^2)^2+(y^2)^2+2(x^2)(y^2)-2(x^2)(y^2)$

$=(x^2+y^2)^2-(\sqrt{2}xy)^2$

$=(x^2+y^2-\sqrt{2}\,xy)\,(x^2+y^2+\sqrt{2}\,xy)$

$=(x^2-\sqrt{2}\,xy+y^2)\,(x^2+\sqrt{2}\,xy+y^2)$

Example 11: Factorize: a^4+64 **09304036**

Solution:

a^4+64

$=\ (a^2)^2+(8)^2$

$=(a^2)^2+(8)^2+2(a^2)(8)-2(a^2)(8)$

(Adding and subtracting $2(a^2)(8)$)

$=\ (a^2+8)^2-16a^2$

$\boxed{\because a^2-\text{b}^2=(a+\text{b})(a-\text{b})}$

$=\ (a^2+8)^2-(4a)^2$

$=\ (a^2+8-4a)\,(a^2+8+4a)$

$=\ (a^2-4a+8)\,(a^2+4a+8)$

Type–III: Factorization of the expression of the types

- $(ax^2+bx+c)(ax^2+bx+d)+k$
- $(x+\text{a})(x+b)(x+c)(x+d)+k$
- $(x+a)(x+b)(x+c)(x+d)+kx^2$

Example 12: Factorize: **09304037**

$(x^2+5x+4)(x^2+5x+6)-3$

Solution:

$(x^2+5x+4)(x^2+5x+6)-3$

Let $\text{y}=x^2+5x$

$=\ (y+4)(y+6)-3$

$=\ y^2+6y+4y+24-3$

$=\ y^2+10y+21$

$=\ y^2+7y+3y+21$

$=\ y(y+7)+3(y+7)$

$=\ (y+7)(y+3)$ $\boxed{\because y^2=x^2+5x}$

$=\ (x^2+5x+7)\,(x^2+5x+3)$

Example 13: Factorize: **09304038**

$(x+2)(x+3)(x+4)(x+5)-15$

Solution:

$(x+2)(x+3)(x+4)(x+5)-15$

Re-arrange the given expression because

$\boxed{2+5=3+4}$

$[(x+2)\,(x+5)][(x+3)(x+4)]-15$

$=(x^2+5x+2x+10)\,(x^2+4x+3x+12)-15$

$=(x^2+7x+10)(x^2+7x+12)-15$

Let $y=x^2+7x$

$=(y+10)(y+12)-15$

$=y^2+12y+10y+120-15$

$=y^2+22y+105$

$=y^2+15y+7y+105$

$=y(y+15)+7(y+15)$

$=(y+15)(y+7)$ $\boxed{\because \text{y}=x^2+7x}$

$=(x^2+7x+15)(x^2+7x+7)$

Example 14: Factorize: **09304039**

$(x-2)(x+2)(x+1)(x-4)+2x^2$

Solution:

$(x-2)(x+2)(x+1)(x-4)+2x^2$

$=\ [(x-2)(x+2)][(x+1)(x-4)]+2x^2$

$[\because\ (-2)\times 2=1\times(-4)]$

$=\ (x^2-2^2)(x^2-4x+x-4)+2x^2$

$=\ (x^2-4)(x^2-3x-4)+2x^2$

$=\ (x^2-4)(x^2-4-3x)+2x^2$

Let $y=x^2-4$

$=\ y(y-3x)+2x^2$

$=\ y^2-3xy+2x^2$

$=\ y^2-2xy-xy+2x^2$

$=\ y(y-2x)-x(y-2x)$

www.ilmkidunya.com

$= \quad (y-2x)(y-x)$

$= \quad (x^2-4-2x)(x^2-4-x)$

$= \quad (x^2-2x-4)(x^2-x-4)$

Type–IV: Factorization of the expression of the forms

- $a^3+3a^2b+3ab^2+b^3$
- $a^3-3a^2b+3ab^2-b^3$

Remember!

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

Example 15: Factorize: $8x^3+60x^2+150x+125$

09304040

Solution: $8x^3+60x^2+150x+125$

$= (2x)^3+3(2x)^2(5)+3(2x)(5)^2+(5)^3$

$= (2x+5)^3$

$= (2x+5)(2x+5)(2x+5)$

Example 16: Factorize: $x^3-18x^2+108x-216$

09304041

Solution:

$x^3-18x^2+108x-216$

$= (x)^3-3(x)^2(6)+3(x)(6)^2-(6)^3$

$= (x-6)^3$

$= (x-6)(x-6)(x-6)$

Type – V: Factorization of the expression of the form $a^3 \pm b^3$

The expression a^3+b^3 is a sum of cubes and it can be factorized using the following identity:

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

The expression a^3-b^3 is a difference of cubes and it can be factorized using the following identity:

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

Example 17: Factorize: $8x^3+27$ 09304042

Solution:

$8x^3+27$

$= (2x)^3+(3)^3$

$= (2x+3)[(2x)^2-(2x)(3)+(3)^2]$

$= (2x+3)(4x^2-6x+9)$

Example 18: Factorize: x^3-27y^3 09304043

Solution:

x^3-27y^3

$= (x)^3-(3y)^3$

$= (x-3y)[(x)^2+(x)(3y)+(3y)^2]$

$= (x-3y)(x^2+3xy+9y^2)$

Do you know?

$(a+b)^2 \neq a^2+b^2$

$(a-b)^2 \neq a^2-b^2$

$(a+b)^3 \neq a^3+b^3$

$(a-b)^3 \neq a^3-b^3$

Exercise 4.2

Q.1 Factorize each of the following expressions:

(i) $4x^4+81y^4$ 09304044

Solution:

$4x^4+81y^4$

$= (2x^2)^2+(9y^2)^2+2(2x^2)(9y^2)-2(2x^2)(9y^2)$

$= (2x^2+9y^2)^2-36x^2y^2$

$= (2x^2+9y^2)^2-(6xy)^2$

$= (2x^2+9y^2+6xy)(2x^2+9y^2-6xy)$

$\boxed{\because a^2-b^2 = (a+b)(a-b)}$

$= (2x^2+6xy+9y^2)(2x^2-6xy+9y^2)$

(ii) a^4+64b^4 09304045

Solution:

a^4+64b^4

$= (a^2)^2+(8b^2)^2+2(a^2)(8b^2)-2(a^2)(8b^2)$

$= (a^2+8b^2)^2-16a^2b^2$

$= (a^2+8b^2)^2-(4ab)^2$

www.ilmkidunya.com

$$\boxed{[\because a^2-b^2=(a+b)(a-b)]}$$

$= \quad (a^2+8b^2+4ab)\ (a^2+8b^2-4ab)$

$= \quad (a^2+4ab+8b^2)\ (a^2-4ab+8b^2)$

(iii) x^4+4x^2+16 09304046

Solution:

x^4+4x^2+16

Rearrang

$= \quad x^4+16+4x^2$

$= \quad (x^2)^2+(4)^2+4x^2$

By adding and subtracting $2(x^2)$ (4)

$= (x^2)^2+(4)^2+2(x^2)(4)-2(x^2)(4)+4x^2$

$= (x^2+4)^2-8x^2+4x^2$

$= (x^2+4)^2-4x^2$

$= (x^2+4)^2-(2x)^2$

$= (x^2+4+2x)\ (x^2+4-2x)$

$= (x^2+2x+4)(x^2-2x+4)$

(iv) x^4-14x^2+1 09304047

Solution:

x^4-14x^2+1

Rearrang

$= \quad x^4+1-14x^2$

$= \quad (x^2)^2+(1)^2-14x^2$

By adding and subtracting $2(x^2)$ (1)

$= \quad (x^2)^2+(1)^2+2(x^2)\ (1)-2(x^2)(1)-14x^2$

$= \quad (x^2+1)^2-2x^2-14x^2$

$= \quad (x^2+1)^2-16x^2$

$= \quad (x^2+1)^2-(4x)^2$

$$\boxed{\because a^2-b^2=(a+b)(a-b)}$$

$= \quad (x^2+1+4x)\ (x^2+1-4x)$

$= \quad (x^2+4x+1)\ (x^2-4x+1)$

(v) $x^4-30x^2y^2+9y^4$ 09304048

Solution:

$x^4-30x^2y^2+9y^4$

$= \quad x^4+9y^4-30x^2y^2$

$= \quad (x^2)^2+(3y^2)^2+2(x^2)\ (3y^2)-2(x^2)(3y^2)-30x^2y^2$

$= \quad (x^2+3y^2)^2-6x^2y^2-30x^2y^2$

$= \quad (x^2+3y^2)^2-36x^2y^2$

$= \quad (x^2+3y^2)^2-(6xy)^2$

$$\boxed{[\because a^2-b^2=(a+b)(a-b)]}$$

$= \quad (x^2+3y^2+6xy)\ (x^2+3y^2-6xy)$

$= \quad (x^2+6xy+3y^2)\ (x^2-6xy+3y^2)$

(vi) $x^4-7x^2y^2+y^4$ (Correction) 09304049

Solution:

$x^4-7x^2y^2+y^4$

Add and subtract $2(x^2)$ (y^2)

$= (x^2)^2+(y^2)^2+2(x^2)\ (y^2)-2(x^2)(y^2)-7x^2y^2$

$= (x^2+y^2)^2-2x^2y^2-7x^2y^2$

$= (x^2+y^2)^2-9x^2y^2$

$= (x^2+y^2)^2-(3xy)^2$

$$\boxed{\because a^2-b^2=(a+b)(a-b)}$$

$= (x^2+y^2-3xy)\ (x^2+y^2+3xy)$

$= (x^2-3xy+y^2)\ (x^2+3xy+y^2)$

Q.2 Factorize each of the following expressions:

(i) $(x+1)(x+2)(x+3)(x+4)+1$ 09304050

Solution:

$(x+1)(x+2)(x+3)(x+4)+1$

By Rearranging the expressions

$= \quad (x+1)(x+4)(x+2)(x+3)+1\ (\because 1+4=2+3)$

$= \quad (x^2+4x+x+4)(x^2+3x+2x+6)+1$

$= \quad (x^2+5x+4)(x^2+5x+6)+1$ ……… (i)

Let $x^2+5x=y$ ______ (ii)

Now, expression (i) can be written as

$= \quad (y+4)\ (y+6)+1$

$= \quad y^2+6y+4y+24+1$

$= \quad y^2+10y+25$

$= \quad (y)^2+2(y)\ (5)+(5)^2$

$$\boxed{\because a^2+2ab^2+b^2=(a+b)^2}$$

$= \quad (y+5)^2$

$= \quad (x^2+5x+5)^2\ (\because y=x^2+5x)$

(ii) $(x+2)(x-7)(x-4)(x-1)+17$ 09304051

Solution:

$(x+2)(x-7)(x-4)(x-1)+17$

$$\boxed{\because 2-7=-4-1}$$

$= \quad (x^2-7x+2x-14)\ (x^2-x-4x+4)+17$ …….. (i)

$(x^2-5x-14)(x^2-5x+4)+17$

www.ilmkidunya.com

Let $x^2-5x = \text{y}$ _______ (ii)

Now, expression (i) can be written as:

$= (\text{y}-14)(\text{y}+4) + 17$

$= \text{y}^2+4\text{y}-14\text{y}-56+17$

$= \text{y}^2-10\text{y}-39$

$= \text{y}^2-13\text{y}+3\text{y}-39$

$= \text{y}(\text{y}-13)+3(\text{y}-13)$

$= (\text{y}-13)(\text{y}+3)$

$(\because \text{y} = x^2+5x)$

$= (x^2-5x-13)(x^2-5x+3)$

(iii) $(2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1$ 09304052

Solution:

$(2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1$……(i)

Let $2x^2+7x = \text{y}$ ………...(ii)

Now, expression (i) can be written as:

$= (\text{y}+3)(\text{y}+5) + 1$

$= \text{y}^2+5\text{y}+3\text{y}+15+1$

$= \text{y}^2+8\text{y}+16$

$= (\text{y})^2+2(\text{y})(4) + (4)^2$

$\boxed{\because a^2 + 2a\text{b} + \text{b}^2 = (a + \text{b})^2}$

$= (\text{y}+4)^2$

Now,

$= (2x^2+7x+4)^2$ $(\because \text{y} = 2x^2+7x)$

(iv) $(3x^2+5x+3)(3x^2 + 5x + 5) - 3$ …….. (i) 09304053

Solution:

$(3x^2+5x+3)(3x^2 + 5x + 5)-3$ …….. (i)

Let $3x^2+5\,x = \text{y}$ …….. (ii)

Now, expression (i) can be written as:

$= (\text{y}+3)(\text{y}+5)-3$

$= \text{y}^2+5\text{y}+3\text{y}+15-3$

$= \text{y}^2+8\text{y}+12$

$= \text{y}^2+6\text{y}+2\text{y}+12$

$= \text{y}(\text{y}+6) + 2(\text{y}+6)$

$= (\text{y}+6)(\text{y}+2)$

$\boxed{\because \text{y} = 3x^2 + 5x}$

Now,

$= (3x^2+5x+6)(3x^2+5x+2)$

(v) $(x+1)(x+2)(x+3)(x + 6) - 3x^2$ 09304054

Solution:

$(x+1)(x+2)(x+3)(x + 6) - 3x^2$

By rearranging the terms,

$= (x+1)(x+6)(x+2)(x+3)-3x^2$

$= (x^2+6x + x+6)(x^2+3x+2x+6) - 3x^2$

$= (x^2+7x +6)(x^2+5x+6) - 3x^2$

$= (x^2+6+7x)(x^2+6+5x) - 3x^2$ ………(i)

let $x^2+6 = \text{y}$ ………...(ii)

$= (\text{y}+7x)(\text{y}+5x) - 3x^2$

$= \text{y}^2+5x\text{y}+7x\text{y}+35x^2-3x^2$

$= \text{y}^2+12x\text{y}+32x^2$

$= \text{y}^2+8x\text{y}+4xy+32x^2$

$= \text{y}(\text{y}+8x)+4x(\text{y}+8x)$

$= (\text{y}+8x)(\text{y}+4x)$

$\boxed{\because \text{y} = x^2 + 6}$

Now,

$= (x^2+6+8x)(x^2+6+4x)$

$= (x^2+8x+6)(x^2+4x+6)$

(vi) $(x+1)(x-1)(x+2)(x-2)+5x^2$ 09304055

Solution: (Excepted correction)

$(x+1)(x-1)(x+2)(x-2)+5x^2$

$=(x^2-1^2)(x^2-2^2)+5x^2$

$=(x^2-1)(x^2-4)+5x^2$

$= x^4-4x^2-x^2+4+5x^2$

$= x^4-5x^2+4+5x^2$

$= x^4+4$

$= (x^2)^2+(2)^2 +2(x^2)(2) - 2(x^2)(2)$

$=(x^2+2)^2-4x^2$

$=(x^2+2)^2-(2x)^2$

$=(x^2+2+2x)(x^2+2-2x)$

$=(x^2+2x+2)(x^2-2x+2)$

Q.3 Factorize: 09304056

(i) $8x^3 + 12x^2 + 6x + 1$

Solution:

$8x^3 + 12x^2 + 6x + 1$

$= (2x)^3+3(2x)^2(1)+3(2x)(1)^2+(1)^3$

$\boxed{\because a^2 + 3a^2\text{b} + 3a\text{b}^2 + \text{b}^3 = (a + \text{b})^3}$

$= (2x+1)^3$

(ii) $27a^3 + 108a^2b + 144ab^2 + 64b^3$ 09304057

Solution:

$27a^3 + 108a^2b + 144ab^2 + 64b^3$

$= (3a)^3+3(3a)^2(4\text{b})+3(3a)(4\text{b})^2+(4\text{b})^3$

$= (3a+4\text{b})^3$

(iii) $x^3 + 48x^2y + 108xy^2 + 216y^3$ 09304058

(Wrong question, expected correction)

Solution

$x^3 + 18x^2y + 108xy^2 + 216y^3$

$= (x)^3+3(x)^2(6\text{y})+3(x)(6\text{y})^2+(6\text{y})^3$

www.ilmkidunya.com

www.ilmkidunya.com

$= \quad (x+6y)^3$

(iv) $8x^3 - 125y^3 + 150xy^2 - 60x^2y$ 09304059

Solution:

$8x^3 - 125y^3 + 150xy^2 - 60x^2y$

$= \quad 8x^3 - 60x^2y + 150xy^2 + 125y^3$

$= \quad (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3$

$= \quad (2x-5y)^3$

Q4. Factorize:

(i) $125a^3 - 1$ 09304060

Solution:

$125a^3 - 1$

$= \quad (5a)^3 - (1)^3$

$= \quad (5a-1)\,[(5a)^2+(5a)(1)+(1)^2]$

$= \quad (5a-1)\,(25a^2+5a+1)$

(ii) $64x^3 + 125$ 09304061

Solution:

$64x^3 + 125$

$= \quad (4x)^3+(5)^3$

$= \quad (4x+5)[(4x)^2-(4x)(5)+(5)^2]$

$= \quad (4x+5)(16x^2-20x+25)$

(iii) $x^6 - 27$ 09304062

Solution:

$x^6 - 27$

$(x^2)^3-(3)^3$

$= \quad (x^2-3)[(x^2)^2+(x^2)\,(3)+(3)^2]$

$= \quad (x^2-3)\,(x^4+3x^2+9)$

(iv) $1000a^3 + 1$ 09304063

Solution:

$1000a^3 + 1$

$= \quad (10a)^3+(1)^3$

$= \quad (10a+1)[(10a)^2-(10a)(1)+(1)^2]$

$= \quad (10a+1)(100a^2-10a+1)$

(v) $343x^3 + 216$ 09304064

Solution:

$343x^3 + 216$

$= \quad (7x)^3+(6)^3$

$= \quad (7x+6)[(7x)^2-(7x)(6)+(6)^2]$

$= \quad (7x+6)(49x^2-42x+36)$

(vi) $27 - 512y^3$ 09304065

Solution

$27 - 512y^3$

$= \quad (3)^3-(8y)^3$

$= \quad (3-8y)\,[(3)^2+(3)(8y)+(8y)^2]$

$= \quad (3-8y)\,(9+24y+64y^2)$

Highest Common Factor (HCF)

The HCF of algebraic expressions refers to the greatest algebraic expression that divides two or more algebraic expressions without leaving a remainder.

(a) By factorization **(b)** By division

Example 19: Find the HCF of $6x^2y$, $9xy^2$ 09304066

Solution:

$6x^2y = 2 \times \boxed{3} \times \boxed{x} \times x \times \boxed{y}$

$9xy^2 = 3 \times \boxed{3} \times \boxed{x} \times y \times \boxed{y}$

$\therefore$ HCF $= 3 \times x \times y = 3xy = 3xy$

$= 3xy$ (Product of common factors)

Example 20: Find the HCF by factorization method $x^2 - 27$, $x^2 + 6x - 27$, $x^2 - 9$ 09304067

Solution:

$x^3 - 27 = x^3 - 3^3$

$= (x-3)[(x)^2 + (3)(x) + (3)^2]$

$= (x-3)(x^2 + 3x + 9)$

$x^2 + 6x - 27 = x^2 + 9x - 3x - 27$

$= x(x+9) - 3(x+9)$

$= (x+9)(x-3)$

$x^2 - 9 = x^2 - 3^2$

$= \quad (x-3)(x+3)$

Hence, HCF $= x - 3$

(b) HCF by Division Method 09304068

Example 21: Find HCF of $6x^3 - 17x^2 - 5x + 6$ and $6x^3 - 5x^2 - 3x + 2$ by using division method.

Solution:

$$\begin{array}{r|l} & \quad 1 \\ \hline 6x^3+17x^3+5x+6 & 6x^3 - 5x^2 - 3x + 2 \\ & -6x^3 \underset{+}{-} 17x^2 \underset{+}{-} 5x \underset{+}{-} 6 \\ \hline & \quad 12x^2 + 2x - 4 \end{array}$$

Here, $12x^2 + 2x - 4 = 2(6x^2 + x - 2)$
2 is not common in both the given polynomials, so we ignore it and consider only $6x^2 + x - 2$.

$$\begin{array}{r|l} & \qquad\quad x - 3 \\ \hline 6x^2 + x - 2 & 6x^2 - 17x^2 - 5x + 6 \\ & \underset{-}{}6x^3 \underset{-}{+} x^2 \underset{+}{-} 2x \\ \hline & \qquad -18x^2 - 3x + 6 \\ & \qquad \underset{+}{-}18x^2 \underset{+}{-} 3x \underset{-}{+} 6 \\ \hline & \qquad\qquad 0 \end{array}$$

The least divisor that leave the remainder zero is HCF. Hence, HCF = $6x^2 + x - 2$

Least Common Multiple (LCM)

The LCM of two or more algebraic expressions is the smallest expression that is divisible by each of the given expressions.

To find the LCM by factorization, we use the formula.

LCM = Common factors × Non-common factors

Example 22: Find the LCM of $4x^2y$, $8x^3y^2$.
09304069

Solution:

$4x^2y = \boxed{2} \times \boxed{2} \times \textcircled{x} \times \textcircled{x} \times \textcircled{y}$
$8x^3y^2 = \boxed{2} \times \boxed{2} \times 2 \times \textcircled{x} \times \textcircled{x} \times x \times y \times \textcircled{y}$

Common factors = $2 \times 2 \times x \times x \times y = 4x^2y$
Non-common factors = $2 \times x \times y = 2xy$
LCM = Common factors × Non-common factors
L.C.M = $4x^2y \times 2xy$
$= 8x^3y^2$

Example 23: Find the LCM of the polynomials $x^2 - 3x + 2$, $x^2 - 1$ and $x^2 - 5x + 4$.
09304070

Solution:

As $x^2 - 3x + 2$
$= x^2 - 2x - x + 2$
$= x(x - 2) - 1(x - 2)$
$= (x - 2)(x - 1)$

And $x^2 - 1$
$= (x - 1)(x + 1)$

$x^2 - 5x + 4$
$= x^2 - 4x - x + 4$
$= x(x - 4) - 1(x - 4)$
$= (x - 4)(x - 1)$

Common factors $= x - 1$
Non-common factors $= (x + 1)(x - 2)(x - 4)$
LCM = Common factors × Non-common factors
$= (x - 1) \times (x + 1)(x - 2)(x - 4)$

$= (x - 1)(x + 1)(x - 2)(x - 4)$

Relationship Between LCM and HCF

The relationship between LCM and HCF can be expressed as follows:

LCM × HCF = $p(x) \times q(x)$
Where, $p(x) = 1^{st}$ polynomial
$q(x) = 2^{nd}$ polynomial

Example 24: LCM and HCF of two polynomials are $x^3 - 10x^2 + 11x + 70$ and $x - 7$. If one of the polynomials is $x^2 - 12x + 35$, find the other polynomial.
09304071

Solution:

Given that:
LCM = $x^3 - 10x^2 + 11x + 70$
HCF = $x - 7$
$p(x) = x^2 - 12x + 35$
As we know that:

$$q(x) = \frac{\text{LCM} \times \text{HCF}}{p(x)}$$

$$= \frac{(x^3 - 10x^2 + 11x + 70)(x - 7)}{x^2 - 12x + 35}$$

$$\begin{array}{r|l} & \qquad\qquad x + 2 \\ \hline x^2 - 12x + 35 & x^3 - 10x^2 + 11x + 70 \\ & \underset{-}{x^3} \underset{+}{-} 12x^2 \underset{-}{+} 35x \\ \hline & \qquad\quad 2x^2 - 24x + 70 \\ & \qquad\quad \underset{-}{2x^2} \underset{+}{-} 24x \underset{-}{+} 70 \\ \hline & \qquad\qquad\quad 0 \end{array}$$

So, $q(x) = (x + 2)(x - 7)$
$= x^2 - 7x + 2x - 14$
$= x^2 - 5x - 14$

Example 25: The LCM of $x^2y + xy^2$ and $x^2 + xy$ is $xy(x + y)$. Find the HCF.
09304072

Solution:

Given that:
LCM = $xy(x + y)$
HCF = ?
1^{st} polynomial $= x^2y + xy^2$
2^{nd} polynomial $= x^2 + xy$

www.ilmkidunya.com

As we know that:

LCM × HCF = Product of two polynomials

$$HCF = \frac{\text{Product of two polynomials}}{\text{LCM}}$$

$$= \frac{(x^2y + xy^2)(x^2 + xy)}{xy(x+y)}$$

$$= \frac{xy(x+y)x(x+y)}{xy(x+y)}$$

$HCF = x(x+y)$

Exercise 4.3

Q.1 Find HCF by factorization method.

(i) $21x^2y, 35xy^2$ 09304073

Solution

$21x^2y, 35xy^2$

Factorization

$21x^2y = 3\times7\ x.\ x.y$

$35xy^2 = 5\times7\ x.y.y$

Common factors = 7, x, y

H.C.F = 7xy (product of common factors)

(ii) $4x^2 - 9y^2, 2x^2 - 3xy$ 09304074

Solution:

$4x^2 - 9y^2, 2x^2 - 3xy$

Factorization

$4x^2 - 9y^2 = (2x)^2 - (3y)^2$

$$\boxed{\because a^2 - b^2 = (a+b)(a-b)}$$

= $(2x+3y)(2x-3y)$

Now,

$2x^2-3xy = x(2x-3y)$

Common factor = $(2x-3y)$

H.C.F = $(2x-3y)$

(iii) $x^3 - 1, x^2 + x + 1$ 09304075

Solution:

$x^3 - 1, x^2 + x + 1$

Factorization

$x^3 - 1 \ = (x)^3 - (1)^3$

$$\boxed{\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$$

$= (x-1)\ [(x)^2 + (x)(1) + (1)^2]$

$= (x-1)\ (x^2 + x + 1)$

Now,

$x^2+x+1 = (x^2+x+1)$

Common factor = (x^2+x+1)

H.C.F = $(x^2+ x +1)$

(iv) $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$ 09304076

Solution:

$a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$

Factorization

$a^3 + 2a^2 - 3a \ = a\ (a^2+2a-3)$

$= a\,[a^2+3a-a-3]$

$= a\ [a(a+3) -1\ (a+3)]$

$= a(a+3)(a-1)$

Now, $2a^3+5a^2-3a = a(2a^2+5a-3)$

$= a[2a^2+6a-a-3)$

$= a[2a(a+3) -1(a+3)$

$= a\ (a+3)(2a-1)$

Common factors = $a\ (a+3)$

H.C.F = $a(a+3)$

(v) $t^2 + 3t - 4, t^2 + 5t + 4, t^2 - 1$ 09304077

Solution: (Correction)

$t^2 + 3t + 4, t^2 + 5t + 4, t^2 - 1$

Factorization

$t^2 + 3t + 4 = t^2 + 4t - t + 4$

$= t\ (t+4) -1(t+4)$

$= (t+4)\ (t+1)$

$t^2+5t+4 = t^2 + 4t + t + 4$

$= t(t+4) + 1(t+4)$

$[\because a^2-b^2 = (a+b)(a-b)]$

$= (t+4)\ (t+1)$

$t^2-1 \ = (t)^2-(1)^2$

$= (t+1)\ (t-1)$

Common factor = (t+1)

H.C.F= (t+1)

www.ilmkidunya.com

(vi) $x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x$

Solution: 09304078

$x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x$

Factorization

$x^2+15x+56 = x^2+7x+8x+56$

$= x(x+7)+8(x+7)$

$= (x+7)(x+8)$

$x^2+5x-24 = x^2+8x-3x-24$

$= x(x+8)-3(x+8)$

Now, $= (x+8)(x-3)$

$x^2+8x = x(x+8)$

Common factor $= (x+8)$

H.C.F $= (x+8)$

Q.2 Find HCF of the following expressions by using division method:

(i) $27x^3 + 9x^2 - 3x - 10, 3x - 2$(correction) 09304080

Solution:

$27x^3 + 9x^2 - 3x - 10,\ 3x - 2$

H.C.F by Division Method

$9x^2+9x+5$

$3x-2 \overline{)\ 27x^3+9x^2-3x-10}$

$\oplus 27x^3 \ominus 18x^2$

$-\quad +$

$27x^2-3x-10$

$\oplus 27x^2 \ominus 18x$

$-\quad +$

$15x-10$

$\ominus 15x \ominus 10$

$+\quad +$

0

Thus, H.C.F = $3x-2$

(ii) $x^3 - 9x^2 + 21x - 9\ ,\ x^2 - 4x + 3$ 09304081

Solution:

$x^3 - 9x^2 + 21x - 9\ ,\ x^2 - 4x + 3$

H.C.F by Division Method

$x - 5$

$x^2-4x+3 \overline{)\ x^3-9x^2+21x-9}$

$x^3 - 4x^2 + 3x$

$-\quad +\quad -$

$-5x^2+18x-9$

$-5x^2+20x-15$

$+\quad -\quad +$

$-2x+6$

$-2(x-3)$

$x-1$

$x-3 \overline{)\ x^2-4x+3}$

x^2-3x

$-\quad +$

$-x+3$

$-x+3$

$+\quad -$

0

H.C.F = $x -3$

(iii) $2x^3 + 2x^2 + 2x + 2,\ 6x^3 + 12x^2 + 6x+12$ 09304082

Solution:

$2x^3 + 2x^2 + 2x + 2,\ 6x^3 + 12x^2 + 6x + 12$

H.C.F by Division Method

3

$2x^3+2x^2+2x+2 \overline{)\ 6x^3+12x^2+6x+12}$

$\oplus 6x \oplus 6x^2 \oplus 6x \oplus 6$

$-\quad -\quad -\quad -$

$6x^2+6$

$3(2x^2+2)$

$x+1$

$2x^2+2 \overline{)\ 2x^3+2x^2+2x+2}$

$\oplus 2x^3 \quad \oplus 2x$

$-\quad -$

$2x^2+2$

$\oplus 2x^2 \oplus 2$

$-\quad -$

0

Thus H.C.F = $2x^2+2 = 2(x^2+1)$

(iv) $2x^3 - 4x^2 + 6x,\ x^3 - 2x,\ 3x^2 - 6x$ 09304083

Solution:

$2x^3 - 4x^2 + 6x,\ x^3 - 2x,\ 3x^2 - 6x$

First we find H.C.F of $x^3-2x, 3x^2-6x$

3

$(x^2-x) \overline{)\ 3x^2-6x}$

$\oplus 3x^2 \ominus 3x$

$-\quad +$

$-3x$

(Taking 6 as common) $6x^2-6x$

3

$(x^2-x) \overline{)\ 3x^2-6x}$

$\oplus 3x^2 \ominus 3x$

$-\quad +$

$-3x$

($\because -3$is not a common factor, we ignore it)

www.ilmkidunya.com

$$\begin{array}{r|l} & \quad x-1 \\ \hline x & x^2 - x \\ & \underset{-}{\oplus} x^2 \\ \hline & \qquad -x \\ & \qquad \underset{\oplus}{\mp} x \\ \hline & \qquad\ 0 \end{array}$$

Thus H.C.F is “x”.

Q.3 Find LCM of the following expressions by using prime factorization method.

(i) $2a^2b,\ 4ab^2,\ 6ab$ 09304084

Solution:

$2a^2b,\ 4ab^2,\ 6ab$

$2a^2b = \textcircled{2}\ a \times \textcircled{a}\textcircled{b}$

$4ab^2 = \textcircled{2} \times 2 \times \textcircled{a}\textcircled{b} \times b$

$6ab = \textcircled{2} \times 3\textcircled{a}\textcircled{b}$

Product of common factors = 2ab

Product of non-common factors

$= (a)(2b)(3) = 6ab$

L.C.M = $(2ab)\ (6ab) = 12\ a^2\ b^2$

(ii) $x^2 + x,\ x^3 + x^2$ 09304085

Solution:

$x^2 + x,\ x^3 + x^2$

Factorization

$x^2+x = \textcircled{x}\ (x+1)$

$x^3+x^2 = x^2\ (x+1)$

$= \textcircled{x}\ .\ x\ (x+1)$

Product of common factors = $x(x+1)$

Product of non-common factors = x

L.C.M = $x(x+1) \times x$

L.C.M = $x^2(x+1)$

(iii) $a^2 - 4a + 4,\ a^2 - 2a$ 09304086

Solution:

$a^2 - 4a + 4,\ a^2 - 2a$

Factorization

$a^2-4a+4 = (a)^2-2(a)(2)+(2)^2$

$= (a-2)^2$

$= (a-2)\ (a-2)$

$a^2-2a\ = a(a-2)$

Product of common factors = $(a-2)$

Product of non-common factors = $a(a-2)$

L.C.M = $(a-2)\ .\ a(a-2)$

$= a(a-2)^2$

(iv) $x^4 - 16,\ x^3 - 4x$ 09304087

Solution:

$x^4 - 16,\ x^3 - 4x$

$x^4 - 16 = (x^2)^2-(4)^2$

$= (x^2+4)\ (x^2-4)$

$= (x^2+4)\ [x^2-2^2]$

$= (x^2+4)\ (x+2)(x-2)$

Now, $x^3-4x = x\ (x^2-4) = x(x^2-2^2)$

$= x(x+2)(x-2)$

L.C.M = $(x+2)(x-2)x(x^2+4)$

$= (x^2+2)(x-2)\ x(x^2+4)$

$= x(x^2-2^2)(x^2+4)$

$= x(x^2-4)(x^2+4)$

$= x(x^2)^2-(4)^2$

$= x(x^4-16)$

(v) $16 - 4x^2,\ x^2 + x - 6,\ 4 - x^2$ 09304088

Solution:

$16 - 4x^2,\ x^2 + x - 6,\ 4 - x^2$

Factorization

$16 - 4x^2 = 4\ (4-x^2)$

$= 4[2^2-x^2]$

$= 4(2+x)\ (2-x)$

$= 4(x+2)\ (-1)(-2+x)$

$= (-1)(4)(x+2)(x-2)$

$= -4(x+2)(x-2)$

$x^2+x-6\ = x^2+3x-2x-6$

$= x(x+3)-2(x+3)$

$= (x+3)(x-2)$

$4-x^2\ = (2)^2-(x)^2$

$= (2+x)\ (2-x)$

$= (x+2)(-1)(-2+x)$

$= -1(x+2)(x-2)$

Product of common factors

$(-1)\ (x+2)(x-2)$

$=\ (-1)\ (x^2-4) = (4-x^2)$

Product of non-common factors = $4(x+3)$

L.C.M = $4(4-x^2)(x+3)$

Q.4 The HCF of two polynomials is $y-7$ and their LCM is $y^3 - 10y^2 + 11y + 70$. If one of the polynomials is $y^2 - 5y - 14$, find the other. 09304089

Solution:

H.C.F = $y-7$

www.ilmkidunya.com
www.ilmkidunya.com
www.ilmkidunya.com

$\text{L.CM} = y^3 - 10y^2 + 11y + 70$

$p(y) = y^2 - 5y - 14$

We know that

$p(y)\, q(y) = \text{L.C.M} \times \text{H.C.F}$

$$q(y) = \frac{\text{L.C.M} \times \text{H.C.F}}{p(y)}$$

$$q(y) = \frac{(y^3 - 10y^2 + 11y + 70) \times (y - 7)}{y^2 - 5y - 14}$$

$q(y) = (y-5)\,(y-7)$

$q(y) = y^2 - 12y + 35$

www.ilmkidunya.com

Q.5 The LCM and HCF of two polynomial $p(x)$ and $q(x)$ are $36x^3(x+a)(x^3-a^3)$ and $x^2(x-a)$ respectively. If $p(x) = 4x^2(x^2 - a^2)$, find $q(x)$.

09304090

Solution:

$\text{L.C.M} = 36x^3(x+a)\,(x^3-a^3)$

$\text{H.C.F} = x^2(x-a)$

$P(x) = 4x^2(x^2-a^2)$

We know that

$p(x).q(x) = \text{L.C.M} \times \text{H.C.F}$

$$q(x) = \frac{\text{L.C.M} \times \text{H.C.F}}{p(x)}$$

$$q(x) = \frac{36x^3(x+a)(x^3-a^3) \times x^2(x-a)}{4x^2(x^2-a^2)}$$

$$q(x) = \frac{9x^3(x^3-a^3).x^2(x^2-a^2)}{x^2(x^2-a^2)}$$

$q(x) = 9x^3(x^3-a^3)$

Q.6 The HCF and LCM of two polynomials is $(x+a)$ and $12x^2(x+a)(x^2-a^2)$ respectively. Find the product of the two polynomials.

09304091

Solution:

$\text{H.C.F} = (x+a)$

$\text{L.C.M} = 12x^2(x+a)\,(x^2-a^2)$

Let $p(x)$ and $q(x)$ be required polynomials

We know that

$p(x) \times q(x) = \text{L.C.M} \times \text{H.C.F}$

$p(x) \times q(x) = 12x^2(x+a)(x^2-a^2) \times (x+a)$

www.ilmkidunya.com

$= 12x^2(x+a)^2(x+a) \times (x-a)$

$= 12x^2(x+a)^3(x-a)$

Square Root of an Algebraic Expression

There are following two methods for finding the square root of an algebraic expression:

(a) Square root by factorization method

(b) Square root by division method

(a) Factorization Method

Example 26: Find the square root of the expression $36x^4 - 36x^2 + 9$

09304092

Solution

$36x^4 - 36x^2 + 9$

$= 9(4x^4 - 4x^2 + 1 \qquad \because (a-b)^2 + a^2 - 2ab + b^2$

$= 9[(2x^2)^2 - 2(2x^2)(1) + (1)^2]$

$= 3^2(2x^2 - 1)^2$

Taking square root on both sides

$$\sqrt{36x^4 - 36x^2 + 9} = \sqrt{3^2(2x^2-1)^2}$$

$$= \sqrt{3^2} \cdot \sqrt{(2x^2-1)^2}$$

$$= \pm 3(2x^2 - 1)$$

www.ilmkidunya.com

(b) Division Method

When the degree of the polynomial is higher, division method in finding the square root is very useful.

Example 27: Find the square root of the polynomial $x^4 - 12x^3 + 42x^2 - 36x + 9$.

09304093

Solution: Multiply x^2 by x^2 to get x^4

Multiply the quotient (x^2) by 2, so we get $2x^2$. By dividing $-12x^3$ by $2x^2$, we get $-6x$. By continuing in this way, we get the remainder.

Hence, square root of $x^4 - 12x^3 + 42x^2 - 36x + 9$ is

$\pm (x^2 - 6x + 3)$

$$\begin{array}{r|l}
 & x^2-6x+3 \\ \hline
 & x^4-12x^3+42x^2-36x+9 \\
x^2 & \underset{-}{}x^4 \\ \hline
 & -12x^3+42x^2 \\
2x^2-6x & \underset{+}{-}12x^3\underset{-}{+}36x^2 \\ \hline
 & 6x^2-36x+9 \\
2x^2-12x+3 & \underset{-}{}6x^2\underset{+}{-}36x\underset{-}{+}9 \\ \hline
 & 0
\end{array}$$

Real World Problems Of Factorization

Example 28: Cost function for producing a part is modeled by:

$C(x) = 5x^2 - 25x + 30$

Where x is the width of the component and $C(x)$ is the cost. Find the value of x where $C(x)$ is minimum. **09304094**

Solution:

$C(x) = 5x^2 - 25x + 30$

$= 5(x^2 - 5x + 6)$

$= 5(x^2 - 2x - 3x + 6)$

$= 5[x(x-2) - 3(x-2)]$

$= 5(x-2)(x-3)$

Thus, the minimum cost occurs when $x = 2$ and $x = 3$.

Example 29: The potential energy $U(x)$ of a particle moving in a cubic potential is represented as: **09304095**

$$U(x) = x^3 - 6x^2 + 12x - 8$$

Factorize the expression to find the points where the energy in minimized.

Solution:

$U(x) \quad = x^3 - 6x^2 + 12x - 8$

$= (x)^3 - 3(x)^2(2) + 3(x)(2)^2 - (2)^3$

$= (x-2)^3$

$= (x-2)(x-2)(x-2)$

The factorized form of the potential energy function shows that the energy is minimized at $x = 2$.

Example 30: A company's profit $p(x)$ is modeled by the quadratic equation: **09304096**

$$P(x) \quad = -5x^2 + 50x - 120$$

Where x represents the number of units produced and $P(x)$ represents the profit in dollars. Find how many units should be produced to maximize profit.

Solution:

$P(x) = -5x^2 + 50x - 120$

$= -5(x^2 - 10x + 24)$

$= -5[x^2 - 4x - 6x + 24]$

$= -5[x(x-4) - 6(x-4)]$

$= -5(x-4)(x-6)$

We can see that profit will be 0 when $x = 4$ and $x = 6$. As coefficients of x^2 is negative, the maximum profit occurs at the midpoint between 4 and 6.

Which is: $\quad x = \frac{4+6}{2} = \frac{10}{2} = 5$

Thus, the company should produce 5 units to maximize profit.

Exercise 4.4

Q.1 Find the square root of the following polynomials by factorization method:

(i) $x^2 - 8x + 16$ **09304097**

Solution

$x^2 - 8x + 16$

By factorization

$x^2 - 8x + 16 = (x)^2 - 2(x)(4) + (4)^2$

$\boxed{\because (a-b)^2 = a^2 - 2ab + b^2}$

$= (x-4)^2$

Taking square root of both sides.

$\sqrt{x^2 - 8x + 16} = \pm\sqrt{(x-4)^2}$

$= \pm(x-4)$

(ii) $9x^2 + 12x + 4$ **09304098**

Solution

$9x^2 + 12x + 4$

www.ilmkidunya.com

By factorization

$9x^2+12x+4$ $\boxed{\because (a+b)^2=a^2+2ab+b^2}$

$= (3x)^2+2(3x)(2)+(2)^2$

Taking square root of both sides.

$\sqrt{9x^2+12x+4}=\pm\sqrt{(3x+2)^2}$

$=\pm(3x+2)$

(iii) $36a^2+84a+49$ 09304099

Solution

$36a^2+84a+49$

$=(6a)^2+2(6a)(7)+(7)^2$

$=(6a+7)^2$

By factorization

$\boxed{\because (a+b)^2=a^2+2ab+b^2}$

Taking square root of both sides

$\sqrt{36a^2 84a+49}=\pm\sqrt{(6a+7)^2}$

$=\pm(6a+7)$

(iv) $64y^2-32y+4$ 09304100

Solution

$64y^2-32y+4$ $\boxed{\because (a-b)^2=a^2-2ab+b^2}$

$=(8y)^2-2(8y)(2)+(2)^2$

$=(8y-2)^2$

Taking square root of both sides

$\sqrt{64y^2-32y+4}=\pm\sqrt{(8y-2)^2}$

$=\pm(8y-2)$

(v) $200t^2-120t+18$ 09304101

Solution

$200t^2-120t+18$

$= 2(100t^2-60t+9)$

$\boxed{\because (a-b)^2=a^2-2ab+b^2}$

$= 2[(10t)^2-2(10t)(3)+(3)^2]$

$= 2(10t-3)^2$

$= 2(10t-3)^2$

Taking square root of both sides

$\sqrt{200t^2-120t+18}=\pm\sqrt{2(10t-3)^2}$

$=\pm\sqrt{2}\sqrt{(10t-3)^{\cancel{2}}}$

$=\pm\sqrt{2}(10t-3)$

(vi) $40x^2+120x+90$ 09304102

Solution:

$40x^2+120x+90$

$= 10(4x^2+12x+9)$

$\boxed{\because (a+b)^2=a^2+2ab+b^2}$

$= 10[(2x)^2+2(2x)(3)+(3)^2]$

$= 10(2x+3)^2$

Taking square root of both sides

$\sqrt{40x^2+12x+90}=\pm\sqrt{10(2x+3)^2}$

$=\pm\sqrt{10}\sqrt{(2x+3)^{\cancel{2}}}$

$=\pm\sqrt{10}(2x+3)$

Q.2 Find the square root of the following polynomials by division method:

(i) $4x^4-28x^3+37x^2+42x+9$ 09304103

Solution:

$4x^4-28x^3+37x^2+42x+9$

Square root by division method:

$$\begin{array}{r|l} & 2x^2-7x-3 \\ \cline{2-2} 2x^2 & 4x^4-28x^3-37x^2+42x+9 \\ & \underset{-}{\oplus}4x^4 \\ \hline 4x^2-7x & -28x^3+37x^2+42x+9 \\ & \underset{-}{\oplus}28x^3\underset{-}{\oplus}49x^2 \\ \cline{2-2} 4x^2-14x-3 & -12x^2+42x+9 \\ & \underset{+}{\ominus}12x^2\underset{-}{\oplus}42x\underset{-}{\oplus}9 \\ \cline{2-2} & 0 \end{array}$$

Thus required square root is ± $(2x^2-7x-3)$.

(ii) $121x^4-198x^3-183x^2+216x+144$

09304104

www.ilmkidunya.com

Solution

$121x^4 - 198x^3 - 183x^2 + 216x + 144$

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & \quad 11x^2-9x-12 \\ \hline
11x^2 & 121x^4-198x^3-183x^2+216x+144 \\
 & \underset{-}{\oplus}\,121x^4 \\ \hline
22x^2-9x & \quad -198x^3-183x^2+216x+144 \\
 & \quad \underset{+}{\ominus}198x^3\underset{-}{\oplus}81x^2 \\ \cline{2-2}
22x^2-18x-12 & \qquad -264x^2+216x+144 \\
 & \qquad \underset{+}{\ominus}264x^2\underset{-}{\oplus}216x\underset{-}{\oplus}144 \\ \cline{2-2}
\multicolumn{1}{r}{} & \qquad\qquad 0
\end{array}$$

Thus required square root is $\pm(11x^2-9x-12)$

(iii) $\boldsymbol{x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4}$ **09304105**

Solution:

$x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & \quad x^2-5xy+y^2 \\ \hline
x^2 & x^4-10x^3y+27x^2y^2-10xy^3+y^4 \\
 & \underset{-}{\oplus}\,x^4 \\ \hline
2x^2-5xy & \quad -10x^3y+27x^2y^2-10xy^3+y^4 \\
 & \quad \underset{+}{\ominus}10x^3y\underset{-}{\oplus}25x^2y^2 \\ \cline{2-2}
2x^2-10xy+y^2 & \qquad 2x^2y^2-10xy^3+y^4 \\
 & \qquad \underset{-}{\oplus}2x^2y^2\underset{+}{\ominus}10xy^3\underset{-}{\oplus}y^4 \\ \cline{2-2}
\multicolumn{1}{r}{} & \qquad\qquad 0
\end{array}$$

Thus required square root is $\pm(x^2-5xy+y^2)$.

(iv) $\boldsymbol{4x^4 - 12x^3 + 37x^2 - 42x + 49}$ **09304106**

Solution:

$4x^4 - 12x^3 + 37x^2 - 42x + 49$

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & \quad 2x^2-3x+7 \\ \hline
2x^2 & 4x^4-12x^3+37x^2-42x+49 \\
 & \underset{-}{\oplus}\,4x^4 \\ \hline
4x^2-3x & \quad -12x^3+37x^2 \\
 & \quad \underset{+}{\ominus}12x^2\underset{-}{\oplus}9x^2 \\ \cline{2-2}
4x^2-6x+7 & \qquad 28x^2-42x+49 \\
 & \qquad \underset{-}{\oplus}28x^2\underset{+}{\ominus}42x\underset{-}{\oplus}49 \\ \cline{2-2}
\multicolumn{1}{r}{} & \qquad\qquad 0
\end{array}$$

Thus required square root is $\pm(2x^2-3x+7)$.

Q.3 An investor's return $R(x)$ in rupees after investing x thousand rupees is given by quadratic expression: **09304107**

$$R(x) = -x^2 + 6x - 8$$

Factor the expression and find the investment levels that result in zero return.

Solution

Investor's return: $R(x) = -x^2+6x-8$

$\Rightarrow R(x) = -(x^2-6x+8)$

$= -(x^2-2x-4x+8)$

$= -[x(x-2)-4(x-2)]$

$R(x) = -(x-2)(x-4)$

The following form of return function. Shows that return is zero at $x = 2$ or $x = 4$.

Q.4 A company's profit $P(x)$ in rupees from selling x units of a product is modeled by the cubic expression:

$$P(x) = x^3 - 15x^2 + 75x - 125$$

Find the break-even point(s), where the profit is zero. **09304108**

Solution

Note: Break even point means no profit no loss.

$P(x) = x^3-15x^2+75x-125$

$P(x) = (x)^3-3(x)^2(5)+3(x)+3(x)(5)^2-(5)^3$

$P(x) = (x-5)^3$

$P(x) = (x-5)(x-5)(x-5)$

The factorized form of profit function shows that profit is zero at $x = 5$.

Q.5 The potential energy $V(x)$ in an electric field varies as a cubic function of distance x, given by: **09304109**

$$V(x) = 2x^3 - 6x^2 + 4x$$

Determine where the potential energy is zero.

Solution

$V(x) = 2x^3 - 6x^2 + 4x$

$=2x(x^2-3x+2)$

$=2x(x^2-x-2x+2)$

$=2x[x(x-1)-2(x-1)]$

$=2x(x-1)(x-2)$

The factorized form of electric potential energy function shows that potential energy is zero at.

$x = 0$, or $x = 1$ or $x = 2$

Q.6 In structural engineering, the deflection $y(x)$ of a beam is given by:

$$Y(x) = 2x^2 - 8x + 6$$

This equation gives the vertical deflection at any point x along the beam. Find the points of zero deflection. **09304110**

Solution

$$\begin{aligned} Y(x) &= 2x^2 - 8x + 6 \\ &= 2(x^2 - 4x + 3) \\ &= 2(x^2 - 3x - x + 3) \\ &= 2[x(x-3) - 1(x-3)] \\ &= 2(x-3)(x-1) \end{aligned}$$

The factorized form of function shows that deflection is zero at $x = 1$ and $x = 3$

Review Exercise 4

Q.1 Choose the correct option.

i. The factorization of $12x + 36$ is: **09304111**

(a) $12(x + 3)$ (b) $12(3x)$
(c) $12(3x + 1)$ (d) $x(12 + 36x)$

ii. The factors of $4x^2 - 12y + 9$ are: **09304112**

(a) $(2x + 3)^2$
(b) $(2x - 3)^2$
(c) $(2x - 3)(2x + 3)$
(d) $(2 + 3x)(2 - 3x)^2$

iii. The HCF of a^3b^3 and ab^2 is: **09304113**

(a) a^3b^3 (b) ab^2
(c) a^4b^5 (d) a^2b

iv. The LCM of $16x^2$, $4x$ and $30xy$ is: **09304114**

(a) $480x^3y$ (b) $240xy$
(c) $240x^2y$ (d) $120x^4y$

v. Product of LCM and HCF = ___________ of two polynomials. **09304115**

(a) sum (b) difference
(c) product (d) quotient

vi. The square root of $x^2 - 6x + 9$ is: **09304116**

(a) $\pm(x - 3)$ (b) $\pm(x + 3)$
(c) $x - 3$ (d) $x + 3$

vii. The LCM of $(a - b)^2$ and $(a - b)^4$ is: **09304117**

(a) $(a - b)^2$ (b) $(a - b)^3$
(c) $(a - b)^4$ (d) $(a - b)^6$

viii. Factorization of $x^3 + 3x^2 + 3x + 1$ is: **09304118**

(a) $(x + 1)^3$
(b) $(x - 1)^3$
(c) $(x + 1)(x^2 + x + 1)$
(d) $(x - 1)(x^2 - x + 1)$

ix. Cubic polynomial has degree: **09304119**

(a) 1 (b) 2
(c) 3 (d) 4

x. One of the factors of $x^3 - 27$ is:

(a) $x - 3$ (b) $x + 3$
(c) $x^2 - 3x + 9$ (d) Both *a* and c

i	a	ii	b	iii	b	iv	c	v	c
vi	a	vii	c	viii	a	ix	c	x	a

Multiple Choice Questions (Additional)

Factorization

1. The degree of quadratic polynomial is: **09304120**

(a) 1 (b) 2
(c) 3 (d) –2

2. The factor of $x^2 - 5x + 6$ are: **09304121**

(a) $x + 1, x - 6$ (b) $x - 2, x - 3$
(c) $x + 6, x - 1$ (d) $x + 2, x + 3$

www.ilmkidunya.com

3. Factors of $3x^2 - x - 2$ are: **09304122**
(a) $(x+1)(3x-2)$ (b) $(x+1)(3x+2)$
(c) $(x-1)(3x-2)$ (d) $(x-1)(3x+2)$

4. Factors of $x^4 - y^4$ are: **09304123**
(a) $(x-y)(x+y)(x^2+y^2)$
(b) $(x-y)(x^2+y^2)$
(c) $(x-y)(x+y)(x^2-y^2)$
(d) $(x+y)(x^2+y^2)$

5. What will be added to complete the square of $9a^2 - 12ab$? **09304124**
(a) $-16b^2$ (b) $16b^2$
(c) $4b^2$ (d) $-4b^2$

6. Find m so that $x^2 + 8x + m$ is a complete square: **09304125**
(a) 8 (b) −8
(c) 4 (d) 16

7. What should be added to complete the square of $y^4 + 81$? **09304126**
(a) $18y^2$ (b) $-18y^2$
(c) $9y^2$ (d) $18y$

8. Let $5x^2 - 17xy - 12y^2 = A \times B$ if $A = (x-4y)$ then B is: **09304127**
(a) $(5x+3y)$ (b) $(5x-3y)$
(c) $(5x+3y)$ (d) $(5x-4y)$

9. Factors of $8x^3 - y^3$ are: **09304128**
(a) $(2x+y)(4x^2+2xy-y^2)$
(b) $(2x+y)(4x^2+2xy+y^2)$
(c) $(2x-y)(4x^2-2xy+y^2)$
(d) $(2x-y)(4x^2+2xy+y^2)$

10. $(x+y)(x^2 - xy + y^2) =$ ___ **09304129**
(a) $x^3 - y^3$ (b) $x^3 + y^3$
(c) $(x+y)^3$ (d) $(x-y)^3$

11. Factors of $x^4 - 16$ is: **09304130**
(a) $(x-2)^2$
(b) $(x-2)(x+2)(x^2+4)$
(c) $(x-2)(x+2)$
(d) $(x+2)^2$

H.C.F (Highest common factor)

12. H.C.F of $x^3y - xy^3$ and $x^5y^2 - x^2y^5$ is: **09304131**
(a) $xy(x^2-y^2)$ (b) $xy(x-y)$
(c) $x^2y^2(x-y)$ (d) $xy(x^3-y^3)$

13. H.C.F. of $35a^2b^2$ and $20a^3b^3$ is: **09304132**
(a) $5a^2b^2$ (b) $20a^3b^3$
(c) $35a^5b^5$ (d) $5ab$

14. H.C.F of $m - 2$ and $m^2 + m - 6$ is:
09304133
(a) $m^2 + m - 6$ (b) $m + 2$
(c) $m - 2$ (d) $m + 3$

15. H.C.F of $a^3 + b^3$ and $a^2 - ab + b^2$ is: **09304134**
(a) $a + b$ (b) $a^2 - ab + b^2$
(c) $(a-b)^2$ (d) $a^2 + b^2$

16. H.C.F of $a^2 - b^2$ and $a^3 - b^3$ is: **09304135**
(a) $a - b$ (b) $a + b$
(c) $a^2 + ab + b^2$ (d) $a^2 - ab + b^2$

L.C.M (Least common Multiple)

17. L.C.M of $15x^2z, 45xy^2$ and $30yz^2$ is: **09304136**
(a) $90xyz$ (b) $90x^2y^2z^2$
(c) $90x^3y^3z^3$ (d) $15x^2yz$

18. L.C.M of $a^2 - b^2$ and $a^4 - b^4$ is: **09304136**
(a) $a^2 + b^2$ (b) $a^2 - b^2$
(c) $a^4 - b^4$ (d) $a - b$

19. The square root of $x^2 - 6x + 9$ is: **09304137**
(a) $\pm(x+3)$ (b) $\pm(x-3)$
(c) $x-3$ (d) $x+3$

20. The product of two polynomials is equal to the ___ of their H.C.F and L.C.M. **09304138**
(a) Sum (b) Difference
(c) Product (d) Quotient

Answer Key

1	b	2	b	3	d	4	a	5	c	6	d	7	a	8	c	9	d	10	b
11	b	12	b	13	a	14	c	15	b	16	a	17	b	18	c	19	b	20	c

www.ilmkidunya.com

Q.2 Factorize the following expressions:

(i) $4x^3 + 18x^2 - 12x$ 09304139

Solution

$4x^3 + 18x^2 - 12x$

$= 2x(2x^2+9x-6)$

$= 2x(2x^2+9x+6)$

(ii) $x^3 + 64y^3$ 09304140

Solution

$= x^3 + 64y^3$

$= (x)^3 + (4y)^3$

$$\boxed{\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)}$$

$= (x+4y)\ [(x)^2-(x)(4y)+(4y)^2]$

$= (x+4y)(x^2-4xy+16y^2)$

(iii) $x^3y^3 - 8$ 09304141

Solution

$= (xy)^3 - (2)^3$

$$\boxed{\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$$

$= (xy-2)\ [(xy)^2+(xy)(2)+(2)^2]$

$= (xy-2)\ (x^2y^2+2xy+4)$

(iv) $-x^2 - 23x - 60$ 09304142

Solution

$-x^2 - 23x - 60$

$= -1(x^2+23x+60)$

$= -1[x^2+20x+3x+60]$

$= -1[x(x+20)+3(x+20)]$

$= -1\ (x+20)\ (x+3)$

(v) $2x^2 + 7x + 3$ 09304143

Solution

$2x^2 + 7x + 3$

$= 2x^2+x+6x+3$

$= x(2x+1)+3(2x+1)$

$= (2x+1)\ (x+3)$

(vi) $x^4 + 64$ 09304144

Solution

$x^4 + 64$

$= (x^2)^2+(8)^2+2(x^2)(8)-2(x^2)(8)$

$$\boxed{\therefore (a+b)^2 = a^2 + 2ab + b^2}$$

$= (x^2+8)^2 - 16x^2$

$= (x^2+8)^2 - (4x)^2$

$$\boxed{\because a^2 - b^2 = (a+b)(a-b)}$$

$= (x^2+8+4x)\ (x^2+8-4x)$

$= (x^2+4x+8)\ (x^2-4x+8)$

(vii) $x^4 + 2x^2 + 9$ 09304145

Solution

$x^4 +9+2x^2$

$= (x^2)^2+(3)^2+2(x^2)(3)-2(x^2)(3)+2x^2$

$= (x^2+3)^2-6x^2+2x^2$

$= (x^2+3)^2 -4x^2$

$= (x^2+3)^2 -(2x)^2$

$= (x^2+3+2x)(x^2+3-2x)$

$= (x^2+2x+3)(x^2-2x+3)$

(viii) $(x + 3)(x + 4)(x + 5)(x + 6) - 360$ 09304146

Solution:

$(x + 3)(x + 4)(x + 5)(x + 6) - 360$

By rearranging the terms,

$= (x+3)\ (x+6)\ (x + 4)\ (x+5) - 360$

$(\because 3+6 = 4+5)$

$= (x^2+6x+3x+18)\ (x^2+5x+4x+20) -360$

$= (x^2+9x+18)\ (x^2+9x+20) -36$ ----- (i)

Let $x^2+9x = y$ ------- (ii)

Now, expression can be written as:

$= (y+18)\ (y+20) - 360$

$= y^2+20y+18y+360-360$

$= y^2+38y$

$= y(y+38)$

$(\because y = x^2+9x)$

$\Rightarrow = (x^2+9x)\ (x^2+9x+38)$

$= x(x+9)(x^2+9x+38)$

(ix) $(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$ 09304147

Solution:

$(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$

Let $x^2+6x = y$ ---------- (ii)

Now, expression (i) can be written as:

$= (y+3)\ (y-9)+36$

$= y^2-9y+3y-27+36$

$= y^2-6y+9$

$= (y)^2-2(y)(3)+(3)^2$

$= (y-3)^2$

$$\boxed{\because x^2 + 6x = y}$$

$= (x^2+6x-3)^2$

www.ilmkidunya.com

Q.3 Find LCM and HCF by prime factorization method:

(i) $4x^3 + 12x^2,\ 8x^2 + 16x$ 09304148

Solution:

$4x^3 + 12x^2,\ 8x^2 + 16x$

Factorization

$4x^3+12x^2 = 4x^2(x+3) \quad = \textcircled{2} \times \textcircled{2} \,.\, \textcircled{x} \,.x(x+3)$

$8x^2+16x = 8x\,(x+2) = 2\times\textcircled{2}\times\textcircled{2} \,.\, \textcircled{x}\,(x+2)$

H.C.F = $2\times2\times x = 4x$

L.C.M

Product of common factors = $2\times2\times x = 4x$

Product of non-common factors

$= x(x+3)\times2(x+2)$

$= 2x(x+3)\,(x+2)$

L.C.M = $(4x)\,[(2x(x+3)(x+2)]$

$= 8x^2(x+3)(x+2)$

(ii) $x^3+3x^2-4x,\ x^2-4x+3$ 09304149

Solution:

$x^3+3x^2-4x = x(x^2+3x-4)$

$= x(x^2+4x-x-4)$

$= x[x(x+4) -1(x+4)]$

$= x(x+4)(x-1)$

$x^4-4x+3 = x^2-3x-x+3$

$= x(x-3) -1(x-3)$

$= (x-3)\,(x-1)$

H.C.F $= (x-1)$

$= (x-1) \times x\,(x-3)\,(x+4)$

$= x(x-1)\,(x-3)\,(x+4)$

(iii) $x^2 + 8x + 16,\ x^2 - 16$ 09304150

Solution:

$x^2 + 8x + 16,\ x^2 - 16$

Solution:

$x^2+8x+16 = (x)^2+2(x)\,(4)+(4)^2$

$= (x+4)^2$

$= (x+4)(x+4)$

$x^2-16 \quad = (x)^2-(4)^2$

$= (x+4)(x-4)$

H.C.F = $(x+4)$

L.C.M=[common factor]×[non common factors]

$= x[(x+4)]\times[(x+4)\,(x-4)]$

$= (x+4)^2\,(x-4)$

(iv) $x^3 - 9x,\ x^2-x-6$ 09304151

Solution:

$x^3\text{--}9x = x(x^2-9)$

$= x[(x)^2 - (3)^2]$

$= x\,(x+3)\,(x-3)$

$x^2-x - 6 = x^2-3x+2x-6$

$= x(x-3) + 2(x-3)$

$= (x-3)\,(x+2)$

H.C.F = $(x-3)$

L.C.M= common factors × non common factors

$= (x-3) \times x(x+2)\,(x+3)$

$= x(x+2)\,(x-3)\,(x+3)$

$= x(x+2)\,[(x)^2-(3)^2]$

$= x(x+2)\,(x^2-9)$

Q.4 Find square root by factorization and division method of the expression $16x^4 + 8x^2 + 1$. 09304152

Solution

$16x^4 + 8x^2 + 1.$

Square root by factorization

$16x^4 + 8x^2 + 1$

$= \quad (4x^2)^2+2(4x^2)(1) + (1)^2$

$= \quad (4x^2+1)^2$

Taking square of both sides

$$\sqrt{16x^4+8x^2+1} \quad = \pm\sqrt{\left(4x^2+1\right)^2}$$

$$= \pm\,(4x^2+1)$$

Square root by division method:

$$\begin{array}{r|l}
 & \quad 4x^2+1 \\ \hline
4x^2 & 16x^4+8x^2+1 \\
 & \underset{-}{\oplus}16x^4 \\ \hline
8x^2+1 & \quad +8x^2+1 \\
 & \quad \underset{-}{\oplus}8x^2\underset{-}{\oplus}1 \\ \hline
 & \qquad 0
\end{array}$$

www.ilmkidunya.com

Thus $\sqrt{16x^4+8x^2+1} = \pm\left(4x^2+1\right)$

Q.5 Huria is analyzing the total cost of her loan, modeled by the expression $C(x) = x^2 - 8x + 15$, where x represents the number of years. What is the optimal repayment period for Huria's loan?

09304153

Solution

$C(x) = x^2 - 8x + 15$

$C(x) = x^2 - 5x - 3x + 15$

$C(x) = x(x-5) - 3(x-5)$

$= (x-5)(x-3)$

The factorized form of expression shows that cost of loan is zero at $x = 5$ and $x = 3$.

So, the optional repayment period is 3 years or 5 years.

www.ilmkidunya.com

www.ilmkidunya.com

www.ilmkidunya.com