UNIT 05

Linear Equations And Inequalities

Linear Equation

An equation of the form $ax + b = 0$ where 'a' and 'b' are constants, $a \neq 0$ and 'x' is a variable, is called a linear equation in one variable. In linear equation, the highest power of the variable is always 1.

Solving a Linear Equation in One Variable

Solving a linear equation in one variable means finding the value of the variable that makes the equation true. To solve the equation, the goal is to isolate the variable on one side of the equation and determine its value.

Steps to Solve a Linear Equation in One Variable

Simplify Both Sides (if necessary):

- Combine like terms on each side of the equation.
- Simplify expressions, including distributing any multiplication over parentheses.

Isolate the Variable Term:

- Move all terms containing the variable to one side of the equation and all constant terms (numbers) to the other side. You can do this by adding or subtracting terms from both sides of the equation.

Solve for the Variable:

- Once the variable term is isolated, solve for the variable by dividing or multiplying both sides of the equation by the co-efficient of the variable (the number in front of the variable).

Check Your Solution

- Substitute the solution back into the original equation to ensure that both sides of the equation are equal.

Example 1: Solve the following equations and represent their solutions on real line.

(i) $3x - 5 = 7$ **09305001**

(ii) $\dfrac{x-2}{5} - \dfrac{x-4}{2} = 2$ **09305002**

Solution:

(i)
$$3x - 5 = 7$$
$$3x - 5 + 5 = 7 + 5$$
$$3x = 12$$
$$x = \frac{12}{3} = 4$$

> **Remember!**
> A linear equation in one variable has only one solution.

Cheek: Substitute $x = 4$ back into the original equation

$$3(4) - 5 = 7$$
$$12 - 5 = 7$$

$$7 = 7$$

So, $x = 4$ is a solution because it makes the original equation true.

Representation of the solution on a number line:

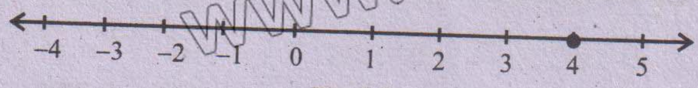


Fig. 5.1

(ii) $\dfrac{x-2}{5} - \dfrac{x-4}{2} = 2$

$$\frac{2(x-2)-5(x-4)}{10} = 2$$

$$\frac{2x-4-5x+20}{10} = 2$$

$$\frac{-3x+16}{10} = 2$$

$$-3x+16 = 2\times 10$$

$$-3x+16 = 20$$

$$-3x+16-16 = 20-16$$

$$-3x = 4$$

$$x = \frac{-4}{3} = -1\frac{1}{3}$$

> **Remember!**
> We check the solution after solving linear equations to ensure the accuracy of our work.

Check: Substitute $x = -\dfrac{4}{3}$ back into the original equation

$$\frac{-\frac{4}{3}-2}{5} - \frac{-\frac{4}{3}-4}{2} = 2$$

$$\Rightarrow \quad \frac{\frac{-4-6}{3}}{5} - \frac{\frac{-4-12}{3}}{2} = 2$$

$$\Rightarrow \quad \frac{-10}{15} - \frac{-16}{6} = 2$$

$$\Rightarrow \quad -\frac{2}{3} + \frac{8}{3} = 2$$

$$\Rightarrow \quad \frac{-2+8}{3} = 2$$

$$\Rightarrow \quad \frac{6}{3} = 2$$

$$\Rightarrow \quad 2 = 2$$

So, $x = -\dfrac{4}{3}$ is the solution of given equation.

Representation of the solution on a number line:

$\boxed{\dfrac{-4}{3}} = -1\dfrac{1}{3}$

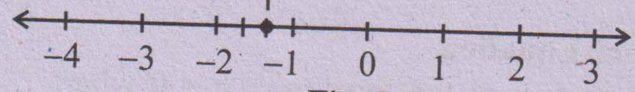


Fig. 5.2

> **Remember!**
> Moving terms from one side of the equation to the other to solve for a variable is called transposition. When we move a term, we change its sign: if it is positive, it becomes negative, and if it is negative, it becomes positive.

Linear Inequalities

Inequalities are expressed by the following four symbols:

$>$ (greater than), $<$ (less than), $\geq$ (greater than or equal to), $\leq$ less than or equal to)

For example,

(i) $ax < b$ (ii) $ax + b \geq c$

(iii) $ax + by > c$ (iv) $ax + by \leq c$

Example 2: Find solution of $\dfrac{2}{3}x - 1 < 0$ and also represent it on a real line. **09305003**

Solution:

$$\frac{2}{3}x - 1 < 0 \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{2}{3}x < 1$$

$$\Rightarrow \quad 2x < 3$$

$$\Rightarrow \quad x < \frac{3}{2}$$

> **History**
> The inequality signs < and > were introduced by the English mathematician Thomas Harriot in the early 17^{th} century.
> For further information, you can use the following link:
> https://en.wikipedia.org/wiki/Thomas_Harriot

It means that all real numbers less than $\frac{3}{2}$ are in the solution of (i)

Thus the interval $(-\infty, \frac{3}{2})$ or $-\infty < x < \frac{3}{2}$ is the solution of the given inequality which is shown in figure 5.3

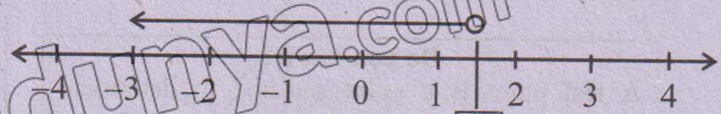
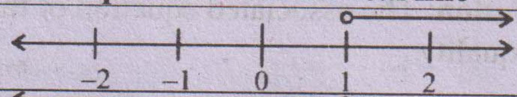
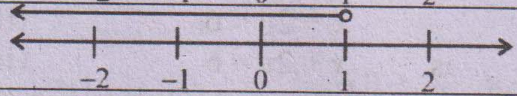
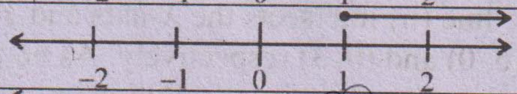
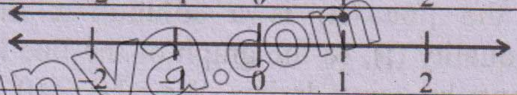


Fig. 5.3 $\boxed{\frac{3}{2}}$

We conclude that the solution set of an inequality consists of all solutions of the inequality.

Following are the inequalities and their solutions on a real line:

Inequality	Solution	Representation on real line
$x > 1$	$(1, \infty)$ or $1 < x < \infty$	
$x < 1$	$(-\infty, 1)$ or $-\infty < x < 1$	
$x \geq 1$	$[1, \infty)$ or $1 \leq x < \infty$	
$x \leq 1$	$(-\infty, 1]$ or $-\infty < x \leq 1$	

Solution of a Linear Inequality in Two Variables

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Generally, a linear inequality in two variables x and y can be one of the following forms:

$ax+by < c$; $ax+by > c$;

$ax+by \leq c$; $ax+by \geq c$

Where a, b, c are constants and a, b are not both zero. We know that the graph of liner equation of the form $ax+by = c$ is a line which divides the plane into two disjoint regions as stated below:

(i) The set of ordered pairs (x, y) such that $ax + by < c$

(ii) The set of ordered pairs (x, y) such that $ax+by > c$

The regions (i) and (ii) are called half planes and the lie $ax + by = c$ is called the boundary of each half plane.

Note that a **vertical line** divides the plane into **left and right half planes** while a non-vertical line divides the plane into **upper and lower half planes.**

> A solution of a linear inequality in x and y is an ordered pair or numbers which satisfies the inequality.
> For example, the ordered pair (1, 1) is a solution of the inequality $(x+2y < 6)$ because $1+2(1) = 3 < 6$ which is true.

There are infinitely many ordered pairs that satisfy the inequality $x+2y < 6$, so its graph will be a half plane. Note that the linear equation $ax+by = c$ is called **"associated or corresponding equation"** of each the above-mentioned inequalities.

Procedure for Graphing a linear inequality in two variables

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(i) The corresponding equation of the inequality is firs graphed by using 'dashes' if the inequality involves the symbols > or < and a solid line is drawn if the inequality involves the symbols ≥ or ≤.

(ii) A test point (not on the graph of the corresponding equation) is chosen which determines on which side of the boundary line the half lane lie.

Do you know?

A test point is a point selected to determine which side of the boundary line represents the solution region for an inequality. Usually, we take origion (0,0) as a test point.
If the inequality holds true with the test point, the region containing this point is part of the solution
If the inequality is false, the opposite region is the solution region.

Example 3: Graph the inequality $x + 2y < 6$.

Solution: The associated equation of the inequality

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$$x + 2y < 6 \quad \text{(i)}$$

is
$$x + 2y = 6 \quad \text{(ii)}$$

The line (ii) intersects the X-axis and Y-axis at (6, 0) and (0, 3) respectively. As no point of the line (ii) is a solution x of the inequality (i), so the graph of the line (ii) is shown by using dashes. We take O (0, 0) as a test point because it is not on the line (ii).

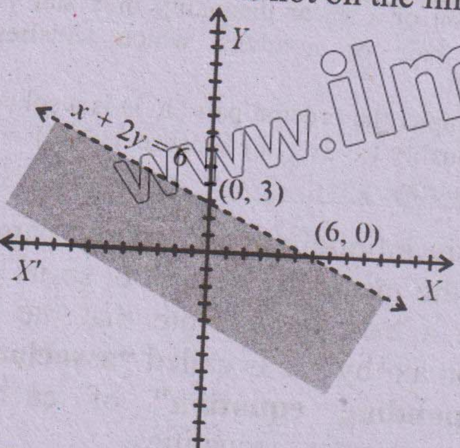


Fig. 5.4 (a)

Substituting x=0, y=0 in the expression $x + 2y$ gives $0 - 2(0) = 0 < 6$. So, the point (0, 0) satisfies the inequality (i). Any other point below the line (ii) satisfies the inequality (i), that is all points in the half plane containing the point (0,0) satisfy the inequality (i).
Thus, the graph of the solution set of inequality (i) is a region on the origin-side of the line (ii), that is, the region below the line (ii). A portion of the open half plane below the line (ii) is shown as shaded region in figure 5.4(a)

Note: All points above the dashed line satisfy the inequality $x + 2y > 6$ (iii)
A portion of the open half plane above the line (ii) is shown by shading in figure 5.4(b)

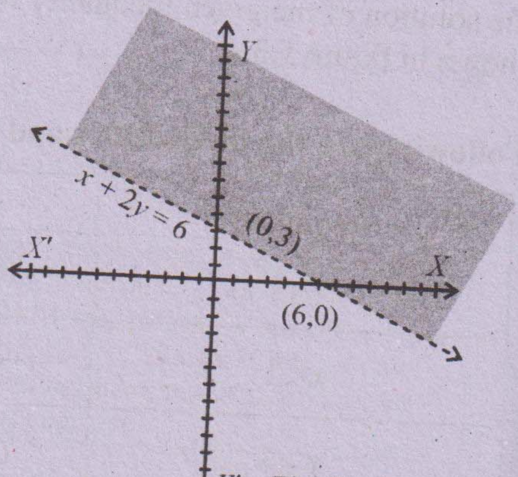


Fig. 5.4 (b)

Note: 1.
The graph of the inequality $x+2y \le 6$... (iv) includes the graph of the line (ii), so the open half-plane below the line (ii) including the graph of the line (ii) is the graph of the inequality (iv). A portion of the graph of the inequality (iv) is shown by shading in fig. 5.4 (c).

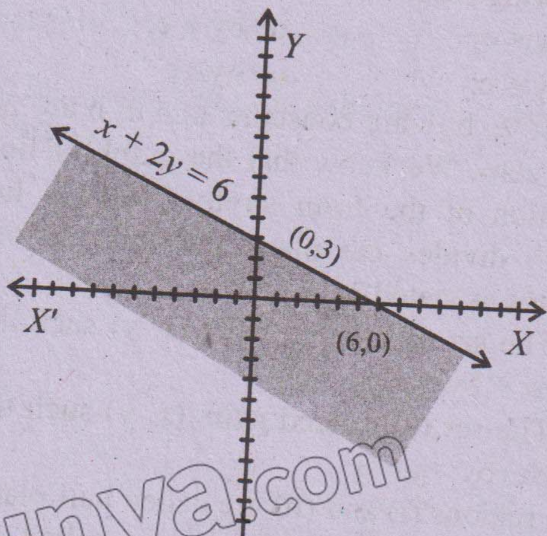


Fig. 5.4 (c)

Note: 2 All points on the line (ii) and above the line (ii) satisfy the inequality $x + 2y \ge 6$ .... (v). This means that the solution set of the inequality (v) consists of all points above the line (ii) and all points on the lines (ii).

The graph of the inequality (v) is partially shown as shaded region in fig. 5.4 (d).

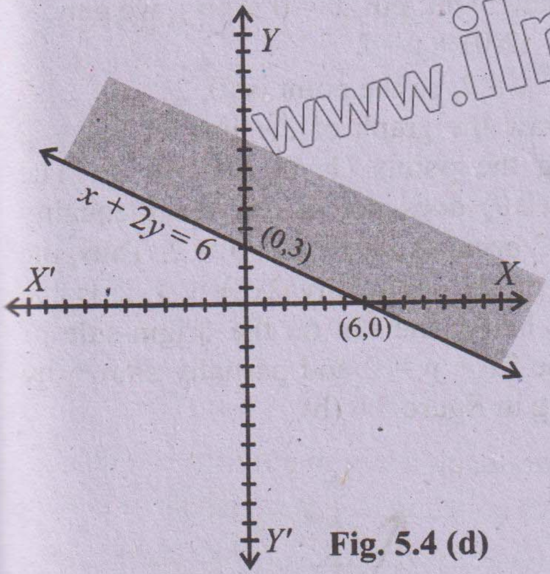


Fig. 5.4 (d)

Note: 3 The graphs of $x+2y \le 6$ and $x+2y \ge 6$ are closed half planes.

Example 4. Graph the following linear inequalities in xy-plane:

(i) $2x \ge -3$ 09305006 (ii) $y \le 2$ 09305007

Solution:

(i) The inequality (i) in xy-plane is considered as $2x+0y \ge -3$ and its solution set consists of all point (x, y) such that $x, y \in R$ and $x \ge -\frac{3}{2}$

The corresponding equation of the inequality (i) is $2x = -3$ (1)

which is a vertical line (parallel to the y-axis) and its graph is drawn in figure 5.5(a).

The graph of the inequality $2x > -3$ consists of boundary line and the open half plane to the right of the line (1).

Thus, the graph of $2x \ge -3$ is the closed half-plane to the right of the line (1).

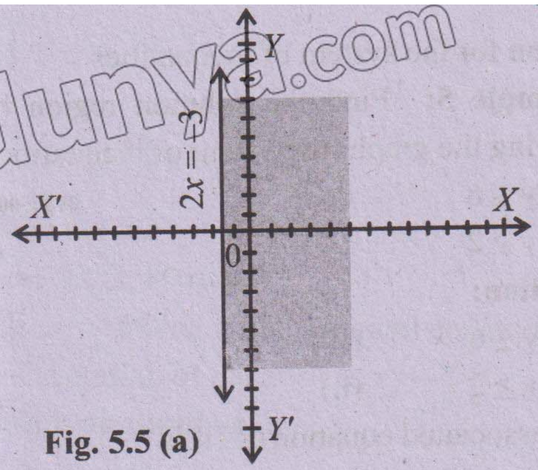


Fig. 5.5 (a)

Solution:

(ii) The associated equation of the inequality $y \le 2$ is $y = 2$ (2)

which is a horizontal line (parallel to the x-axis) and its graph is shown in figure 5.5 (b). Here the solution set of the inequality $y < 2$ is the open half plane below the boundary line $y = 2$. Thus, the graph of $y \le 2$ consists of the boundary line and the open half plane below it.

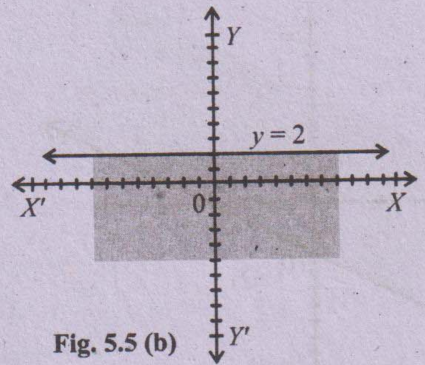


Fig. 5.5 (b)

Solution of Two Linear Inequalities in Two Variables

The graph of a system of linear inequalities consists of the set of all ordered pairs (x, y) in the xy-plane which simultaneously satisfies all the inequalities in the system. To find the graph of such a system, we draw the graph of each inequality in the system on the same coordinate axes and then take intersection of all the graphs. The common region so obtained is called the **solution**

region for the system of inequalities.

Example 5: Find the solution region by drawing the graph the system of inequalities

$x - 2y \le 6$ 09305008

$2x + y \ge 2$

Solution:

$x - 2y \le 6$ ...(i)

$2x + y \ge 2$ ...(ii)

The associated equation of (i) is

$$x - 2y = 6 \qquad \text{...(iii)}$$

For x-intercept, put $y = 0$ in (iii), we get

$x - 2(0) = 6$

$x - 0 = 6$

$\Rightarrow \quad x = 6$, so the point is $(6, 0)$

For y-intercept, put $x = 0$ in (iii), we get

$0 - 2y = 6$

$\Rightarrow \; -2y = 6$

$\Rightarrow \; y = \dfrac{6}{-2} = -3$, so the point is $(0,-3)$

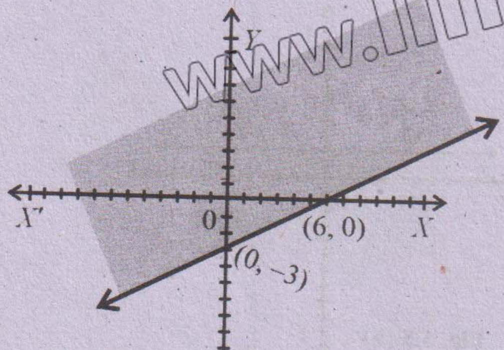


Fig. 5.6 (a)

The graph of the line $x - 2y = 6$ is drawn by joining the point $(6, 0)$ and $(0, -3)$. The point $(0, 0)$ satisfies the inequality $x - 2y < 6$ because $0 - 2(0) = 0 < 6$. Thus, the graph of $x - 2y \le 6$ is the upper half-plane including the graph of the line $x - 2y = 6$. The closed half-plane is partially shown by shading in figure 5.6 (a).

The associated equation of (ii) is

$2x + y = 2$ ...(iv)

For x-intercept, put $y = 0$ in (iv), we get

$2x + 0 = 2$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = 1$, so the point is $(1, 0)$

For y-intercept, put $x = 0$ in (iv), we get

$2(0) + y = 2$

$\Rightarrow \quad y = 2$, so the point is $(0, 2)$

We draw the graph of the line $2x + y = 2$ joining the points $(1, 0)$ and $(0, 2)$. The point $(0,0)$ does not satisfy the inequality $2x+y>2$ because $2(0) + 0 = 0 \not> 2$. Thus, the graph of the inequality $2x + y \ge 2$ is the closed half-plane not on the origin-side of the line $2x + y = 2$ and partially shown by shading in figure 5.6 (b).

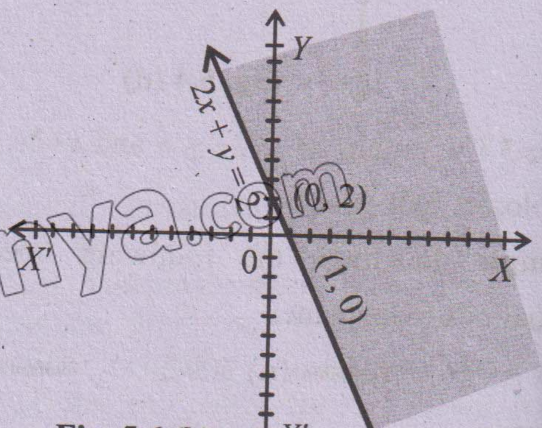


Fig. 5.6 (b)

The solution region of the given system of inequalities is the intersection of the graphs indicated in figures 5.6 (a) and 5.6(b) which is shown as shaded region in figure 5.6 (c).

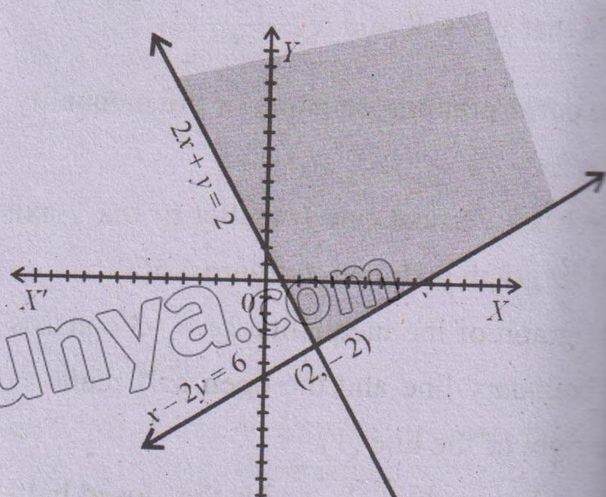


Fig. 5.6 (c)

Exercise 5.1

Q.1 Solve and represent the solution set on a real line.

(i) $12x+30=-6$ **09305009**

Solution:

$12x+30=-6$ ________ (i)

$12x=-6-30$

$12x=-36$

$x=\frac{-36}{12}$

$\boxed{x=-3}$

So, $x=-3$ is a solution of given equation because it makes the original equation true.

Check : Substitute $x=-3$ in equation (i)

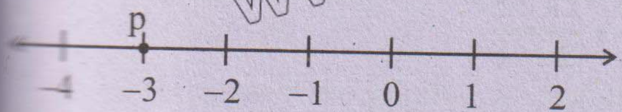
$12(-3)+30=-6$

$-36+30=-6$

$-6=-6$ (It is true)

So, $x=-3$ is solution set of given equation.

Solution on Number Line



Point P on the number line represents $x=-3$.

(ii) $\frac{x}{3}+6=-12$ **09305010**

Solution:

$\frac{x}{3}+6=-12$______ (i)

$\frac{x}{3}=-12-6$

$\frac{x}{3}=-18$

$x=-18\times3$

$x=-54$

Check : Substitute $x=-54$ in equation (i)

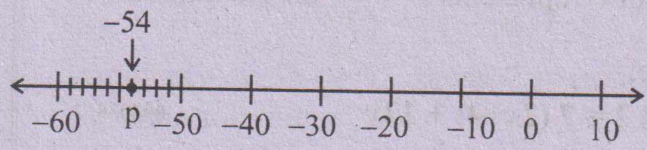
$\frac{-54}{3}+6=-12$

$-18+6=-12$

$-12=-12$ (It is true)

Since, $x=-54$ makes the original equation true so solution of equation is $x=-54$

Solution on number line



Point P on the number line represents −54.

(iii) $\frac{x}{2}-\frac{3x}{4}=\frac{1}{12}$ **09305011**

Solution

$\frac{x}{2}-\frac{3x}{4}=\frac{1}{12}$ ________ (i)

$\frac{2x-3x}{4}=\frac{1}{12}$ (L.C.M)

$\frac{-x}{4}=\frac{1}{12}$

$-x=\frac{1}{12}\times4$

$-x=\frac{1}{3}\Rightarrow\boxed{x=\frac{-1}{3}}$

Check: Substitute $x=\frac{-1}{3}$ in equation (i)

$(x)\times\frac{1}{2}-\frac{3}{4}x=\frac{1}{12}$

$\left(\frac{-1}{3}\times\frac{1}{2}\right)-\left(\frac{3}{4}\times\frac{-1}{3}\right)=\frac{1}{12}$

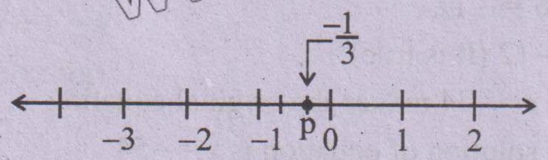
$\frac{-1}{6}+\frac{1}{4}=\frac{1}{12}$

$\frac{-2+3}{12}=\frac{1}{12}$

$\frac{1}{12} = \frac{1}{12}$ (It is true).

So, $x = \frac{-1}{3}$ is a solution of given equation.

Solution on number line



Point P represents $x = \frac{-1}{3}$ on the number line.

(iv) $2 = 7(2x+4) + 12x$ 09305012

Solution

$2 = 7(2x+4) + 12x$ _______(i)

$2 = 14x+28+12x$

$2 = 26x + 28$

$2 - 28 = 26x$

$-26 = 26x$

$\frac{-26}{26} = x$

$-1 = x$

$\Rightarrow \boxed{x = -1}$

Check: Substitute $x = -1$ in equation (i)

$2 = 7[2(-1) + 4] + 12(-1)$

$2 = 7[-2+4] - 12$

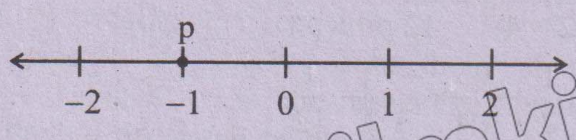
$2 = 7[2] - 12$

$2 = 14 - 12$

$2 = 2$ (It is true)

So, $x = -1$ is a solution of given equation.

Solution on number line



Point P on the number line represents the solution.

(v) $\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$ 09305013

Solution:

$\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$ ________ (i)

$\frac{4(2x-1) - 3(3x)}{12} = \frac{5}{6}$ (L.C.M)

$\frac{8x-4-9x}{12} = \frac{5}{6}$

$-x - 4 = \frac{5}{6} \times 12$

$-x - 4 = \frac{60}{6}$

$-x - 4 = 10$

$-x = 10 + 4$

$-x = 14$

$\Rightarrow \boxed{x = -14}$

Check: Put $x = -14$ in equation (i)

$\frac{2(-14)-1}{3} - \frac{3(-14)}{4} = \frac{5}{6}$

$\frac{-28-1}{3} + \frac{42}{4} = \frac{5}{6}$

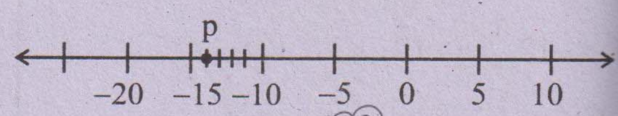
$\frac{-29}{3} + \frac{21}{2} = \frac{5}{6}$

$\frac{-58+63}{6} = \frac{5}{6}$ (L.C.M)

$\frac{5}{6} = \frac{5}{6}$ (It is true)

So, $x = -14$ is a solution of given equation.

Solution on number line



Point P on the number line represents $x = -14$.

(vi) $\frac{-5x}{10} = 9 - \frac{10}{5}x$ 09305014

Solution:

$\frac{-5x}{10} = 9 - \frac{10}{5}x$ ________ (i)

$$-\frac{1}{2}x = 9 - 2x$$

$$-\frac{1}{2}x + \frac{2x}{1} = 9$$

$$\frac{-x + 4x}{2} = 9$$

$3x = 9 \times 2$

$3x = 18$

$$x = \frac{18}{3}$$

$\boxed{x = 6}$

Check: Substitute $x = 6$ in equation (i)

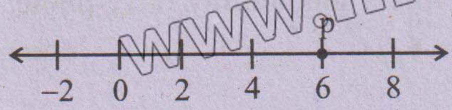
$$\frac{-5(6)}{10} = 9 - \frac{10}{5}(6)$$

$$\frac{-30}{10} = 9 - 2(6)$$

$-3 = 9 - 12$

$-3 = -3$ (It is true)

Solution on number line:



Point P represent the solution $x = 6$ on the number line.

Q.2 Solve each inequality and represent the solution on a real line.

(i) $x - 6 \le -2$ **09305015**

Solution:

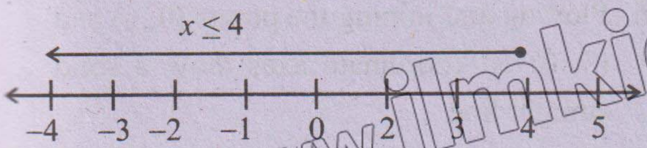
$x - 6 \le -2$

$x \le -2 + 6$

$x \le 4$,

The solution of inequality is $(-\infty, 4]$

Solution on number line



Filled circle on 4 means 4 is included in the solution

(ii) $-9 > -16 + x$ **09305016**

Solution

$-9 > -16 + x$

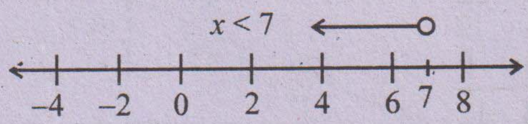
$-9 + 16 > x$

$7 > x$

$\Rightarrow$ $\boxed{x < 7}$

The solution of given inequality is $(-\infty, 7)$ or $-\infty < x < 7$.

Solution on number line



An empty circle on 7, means 7 is excluded form the solution.

(iii) $3 + 2x \ge 3$ **09305017**

Solution:

$3 + 2x \ge 3$

$2x \ge 3 - 3$

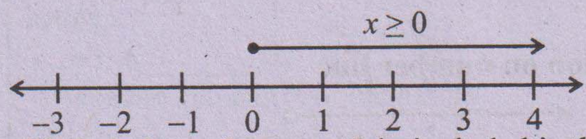
$2x \ge 0$

$$x \ge \frac{0}{2}$$

$\boxed{x \ge 0}$

Thus the solution of given inequality is $[0, \infty)$ or $0 \le x < \infty$.

Solution on number line



Filled circle on 0, means 0 in included is the solution.

(iv) $6(x+10) \le 0$ **09305018**

Solution:

$6(x+10) \le 0$

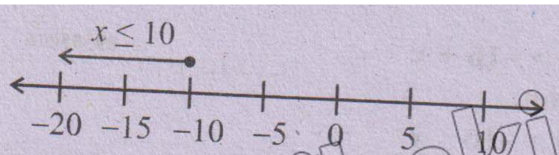
$$x + 10 \le \frac{0}{6}$$

$x + 10 \le 0$

$x \le -10$

The solution of inequality is $(-\infty, -10]$ or $-\infty < x \le -10$.

Solution on number line



Filled circle on −10 means −10 is included in the solution.

(v) $\dfrac{5}{3}x-\dfrac{3}{4}<\dfrac{-1}{12}$ **09305019**

Solution:

$$\frac{5}{3}x-\frac{3}{4}<\frac{-1}{12}$$

$$\frac{5}{3}x<\frac{3}{4}-\frac{1}{12}$$

$$\frac{5}{3}x<\frac{9-1}{12} \quad \text{(L.C.M)}$$

$$\frac{5}{3}x<\frac{8}{12}$$

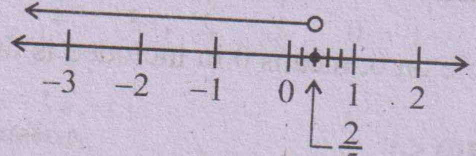
$$\frac{5}{3}x<\frac{2}{3}$$

$$x<\frac{2}{3}\times\frac{3}{5}$$

$$x<\frac{2}{5}$$

The solution of inequality is $(-\infty,\frac{2}{5})$ or $-\infty < x\ \frac{2}{5}$.

Solution on number line



An empty circle on $\frac{2}{5}$ shows that $\frac{2}{5}$ is not included in the solution.

(vi) $\dfrac{1}{4}x-\dfrac{1}{2}\le -1+\dfrac{1}{2}x$ **09305020**

Solution:

$$\frac{1}{4}x-\frac{1}{2}\le -1+\frac{1}{2}x$$

Multiplying both sides by "4"

$$4\left(\frac{1}{4}x-\frac{1}{2}\right)\le 4(-1+\frac{1}{2}x)$$

$$4\times\frac{1}{4}x-4\times\frac{1}{2}\le 4\times(-1)+4\times\left(\frac{1}{2}x\right)$$

$$x-2\le -4+2x$$

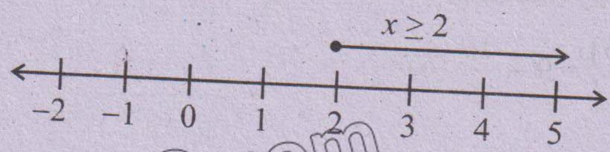
$$-2+4\le 2x-x$$

$$2\le x$$

$$\Rightarrow \boxed{x\ge 2}$$

The solution of inequality is $[2,\infty)$ or $2\le x<\infty$.

Solution on number line



A filled circle on the 2 shows that 2 is included in the solution.

Q.3 Shade the solution region for the following linear inequalities in *xy*-plane:

(i) $2x+y<6$ **09305021**

Solution:

$2x+y<6$

i. Associated Eq. of inequality is
$2x + y = 6$ ______ (ii)

ii. Getting two points of line:
For y intercept, put x = 0 in equation (ii)
$2(0)+y=6 \Rightarrow \boxed{y=6}$ so, (0,6)
For x intercept, put y = 0 in equation (ii)
$2x+0=6$
$2x=6 \Rightarrow \boxed{x=3}$ so, (3, 0)

iii. Plotting and joining the points (0, 6) and (3, 0) on coordinate axes draw a solid straight line.

iv. Since the line does not pass through the origin 0(0, 0), so we can take it as test point.

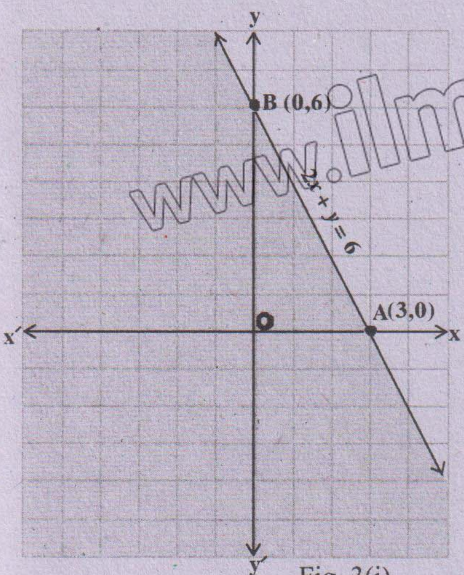


Fig. 3(i)

Put $x = 0$, y = 0 in inequality (i)

$2(0) + 0 \leq 6 \Rightarrow 0 \leq 6$ (which is true)

Since the point (0,0) satisfy the inequality $2x+y \leq 6$ therefore the graph of its solution is closed lower half plane on the origin side of line of equation (ii).

A portion of closed lower half plane is shown by the shaded region in he figure-3(i).

(ii) $3x + 7y \geq 21$ 09305022

Solution

$3x + 7y \geq 21$_______ (i)

i. Associated equation of inequality is
 $3x + 7y = 21$ _______ (ii)

ii. Getting two points of line:
 Put $x = 0$ in equation (ii),

$3(0) + 7y = 21 \Rightarrow 7y = 21 \Rightarrow \boxed{y = 3}$ so, (0,3)

Put y = 0 in equation(ii),

$3x+7(0) = 21$

$\Rightarrow 3x = 21$

$x = \dfrac{21}{3}$

$x = 7$

iii. Plotting and joining the points (0, 3) and (7, 0) on coordinate axes draw a solid straight line.

iv. Since origin O(0, 0), does not lie on the line so we can take it as test point.

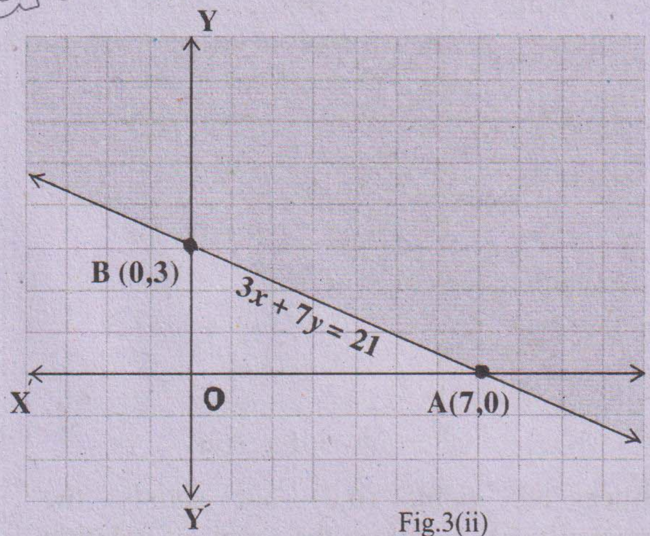


Fig.3(ii)

Put $x = 0$, y = 0 in inequality (i)

$3(0) + 7(0) \geq 21 \Rightarrow 0 \geq 21$ (not true)

Since the point (0,0) not satisfy the inequality(i) So the graph of its solution of in equality $3x + 7y \geq 21$ is the region on the opposite side of origin including the line of eq. (ii)

A portion of required closed half plane is shown by shaded region in the figure 3(ii).

(iii) $3x - 2y \geq 6$ 09305023

Solution:

$3x - 2y \geq 6$_______ (i)

i. Associated equation of inequality
 $3x-2y = 6$ _________ (ii)

ii. Getting two points of line:
 Put $x = 0$, in equation (ii)

$3(0) - 2y = 6 \Rightarrow -2y = 6 \Rightarrow y = -3$ so (0, −3)

Put y = 0 in equation (ii)

$3x - 2(0) = 6 \Rightarrow 3x = 6 \Rightarrow x = 2$, so (2, 0)

iii. Plotting and joining the points (0, 3) and (2, 0) on coordinate axes draw a solid straight line.

iv. Since origin (0, 0), does not lie on the line of equation (ii), so we take it as test point.

Put $x = 0$, y = 0 in inequality (i)

$3(0) + 2(0) \geq 6 \Rightarrow 0 \geq 6$ (not true)

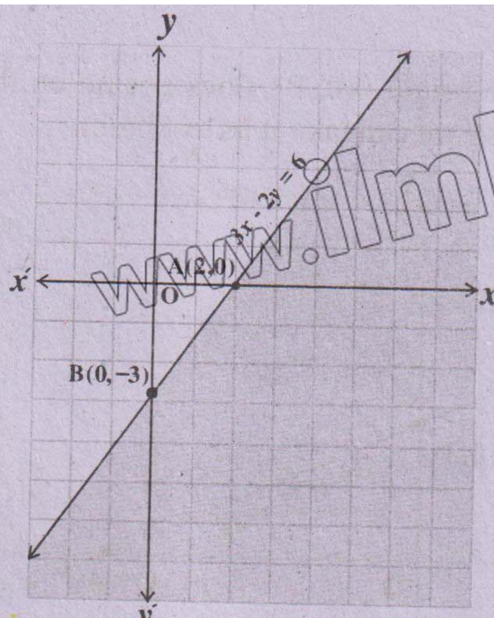


Fig. 3(iii)

Since the point (0,0) not satisfy the inequality(i), therefore the graph of solution of in equality $3x - 2y \geq 6$ is the closed lower half plane on opposite to origin side the line of equation (ii) including the line of eq. (ii).
A portion of the closed lower half plane is shown as shaded region in the figure 3(iii).

(iv) $5x-4y \leq 20$ **09305024**

Solution:

$5x-4y \leq 20$ ________ (i)

i. Associated equation of inequality is
$5x-4y = 20$ ________ (ii)

ii. Getting two points of line:
Put $x = 0$, in (ii)

$5(0) -5y = 20 \Rightarrow$

$-4y = 20 \Rightarrow \boxed{y = -5} \Rightarrow (0, -5)$

Put y = 0 in equation (ii)

$5x - 4(0) = 20$

$\Rightarrow 5x = 20 \Rightarrow \boxed{x = 4}$ so (4, 0)

iii. Plotting and joining the points (0, −5) and (4, 0) on coordinate axes draw a solid straight line.

iv. Since, the origin i.e. (0, 0), does not lie on the line, so we take it as test point.

Put $x = 0$, $y = 0$ in inequality (i)

$5(0) - 4(0) \leq 20 \Rightarrow 0 \leq 20$ (True).

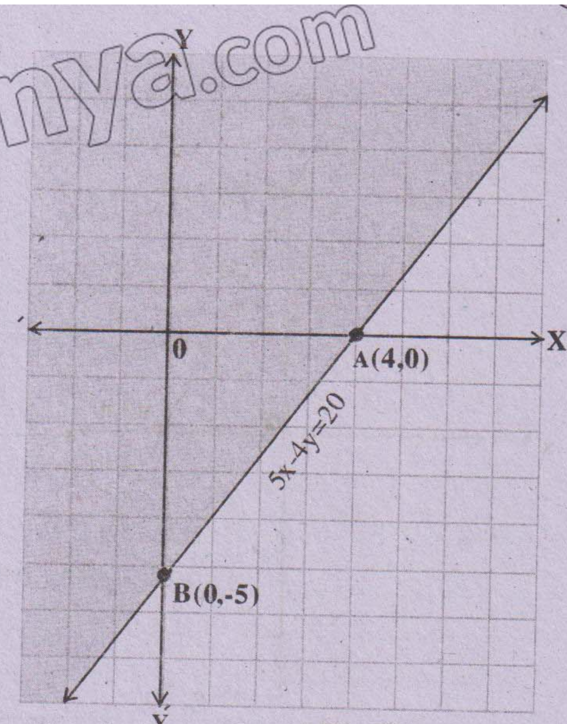


Fig. 3(iv)

Since the point (0,0) satisfy the inequality(i), therefore the graph of solution of in equality $5x - 4y \leq 20$ is the closed upper half plane on the of origin side of the line of equation (ii).
Including the line of eq. (ii)
A portion of the closed lower half plane is shown as shaded region in the figure 3 (iv).

(v) $2x + 1 > 0$ **09305025**

Solution:

$2x + 1 > 0$

$2x + 1 > 0$ ________ (i)

i. The corresponding (associated) equation of the inequality (i) is $2x+1 = 0$ ______ (ii)

$\Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$ i.e; $A\left(-\frac{1}{2}, 0\right)$

Which is a vertical line (Parallel to y-axis). The graph of equation (ii) is drawn in the figure 3 (v).

ii. Taking origin i.e., O(0, 0) as a test point. Clearly origin satisfies the inequality, (i) because

$2(0) + 1 > 0 \Rightarrow 1 > 0$ (True)

Thus the graph of inequality (i) is the closed right half-plane on the origin-side of the eq.(ii).
A portion of the closed half-plane on right side of the line (ii) is shown as shaded region in the figure 3(v).

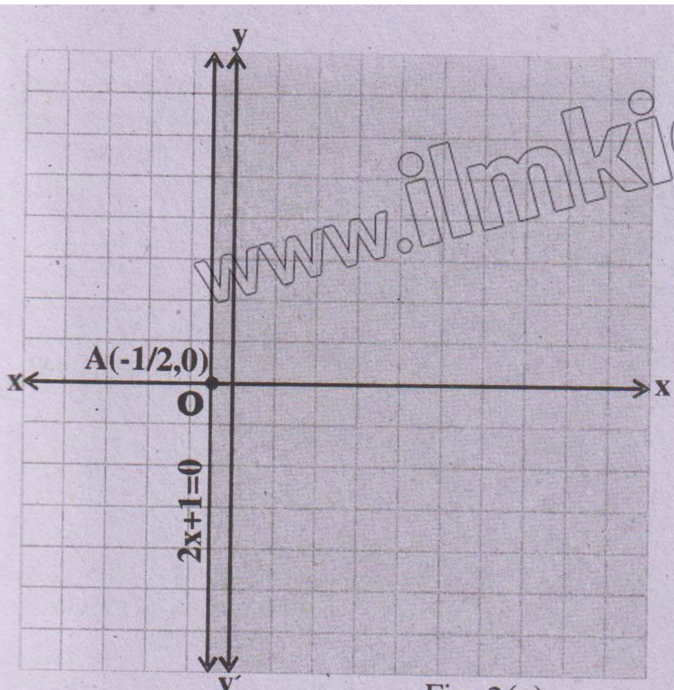


Fig. 3(v)

(vi) $3y - 4 \le 0$ 09305026

Solution:

$3y - 4 \le 0$

$3y - 4 \le 0$ ________ (i)

i. The associated equation of in equality is:
$3y - 4 = 0$ ______ (ii)

ii. By solving equation we get
$3y - 4 = 0$
$3y = 4$
$y = \frac{4}{3} = 1\frac{1}{3}$

iii. Draw the solid line of equation (ii) which is a horizontal line (parallel to x–axis) as shown in figure.

iv. Since, origin (0, 0) does not lie on the line so, we take it as a test point.

Put x = 0, y = 0 in inequality (i),
$3(0) - 4 \le 0 \Rightarrow -4 \le 0$ (true).

Since, point (0, 0) satisfy the inequality so its solution graph is on origin-side of line of equation (ii).

So the graph of solution of in equality $3y - 4 \le 0$ is the closed lower half-plane which is shown as shaded region in the figure 3(vi).

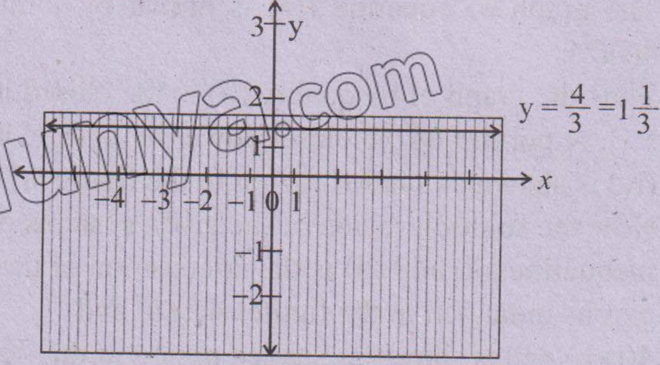


Fig. 3(vi)

Q.4 Indicate the solution region of the following linear inequalities by shading:

(i) $2x - 3y \le 6$ 09305027
$2x + 3y \le 12$

Solution:

$2x - 3y \le 6$____ (i)

$2x + 3y \le 12$ ____ (ii)

For $2x - 3y \le 6$

(a) The corresponding equation of the inequality (i) is
$2x - 3y = 6$ ______(iii)

For x–intercept, put $y = 0$ in eq. (iii), we have
$2x - 3(0) = 6 \Rightarrow 2x = 6 \Rightarrow x = 3$. So x-intercept is A(3,0)

For y-intercept, put x = 0 in eq. (iii), we have
$2(0) - 3y = 6 \Rightarrow -3y = 6 \Rightarrow y = -2$. So y-intercept is B(0, –2).

The graph of equation (iii) is drawn by joining the points A(3, 0) and (0, –2) in the figure 4(i)(a)

(b) Taking origin i.e., 0(0, 0) as a test point. Clearly

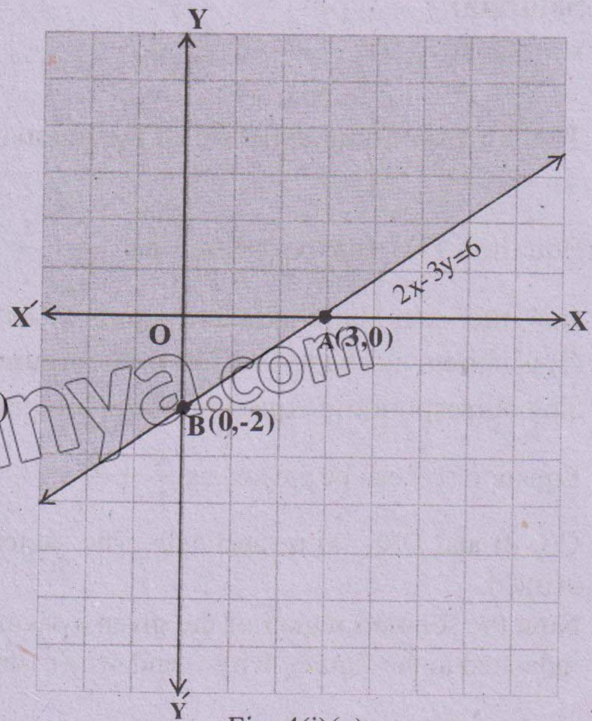


Fig. 4(i)(a)

origin satisfies the inequality (i) because $2(0) - 3(0) < 6$
$\Rightarrow 0 < 6$ (True)

Thus the graph of inequality (i) is the closed upper half-plane on the origin-side of the equation (iii). A portion of the closed lower half-plane above the line (iii) is shown as shaded region in the figure 4(i)(a).

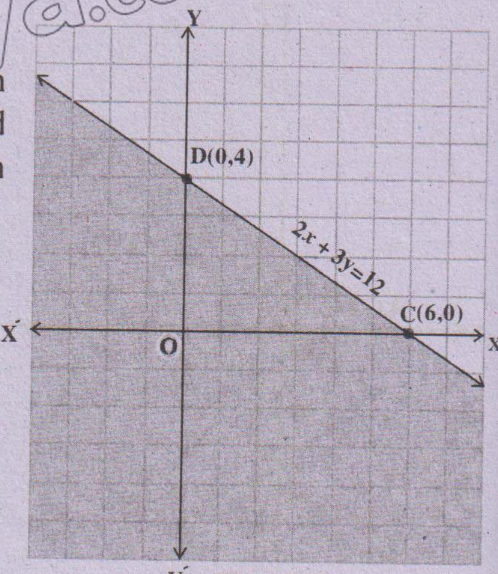


Fig. 4(i)(b)

For $2x + 3y \leq 12$

(a) The corresponding equation of the inequality (ii) is $2x + 3y = 12$... (iv)

For x-intercept, put $y = 0$ in eq.(iv), we have
$2x + 3(0) = 12 \Rightarrow 2x = 12 \Rightarrow x = 6$
So x-intercept is C(6, 0).

For y-intercept, put $x = 0$ in eq.(iv), we have
$2(0) + 3y = 12 \Rightarrow 3y = 12 \Rightarrow y = 4$
So y-intercept is D(0, 4).

The graph of equation (iv) is drawn by joining the points C(6, 0) and D(0, 4) in the figure 4(i)(b).

Thus the graph of inequality (ii) is the closed lower half-plane on the origin-side of the equation (iv). A portion of the closed lower half-plane below the line (iv) is shown as shaded region in the figure 4(i)(b).

Now the solution region of the given systems of inequalities (i) and (ii) is the intersection of the graphs indicated in the figures 4(i)(a) and 4(i)(b) and is shown as shaded region in the figure 4(i)(c).

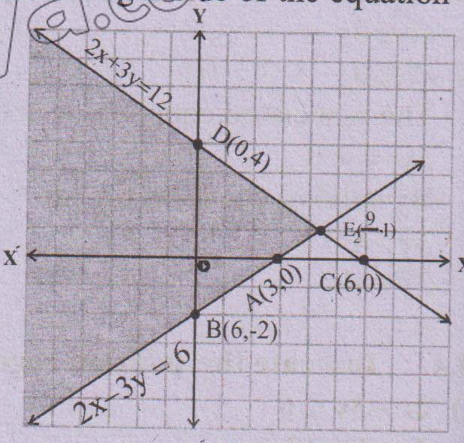


Fig. 4(i)(c)

(ii) $x + y \geq 5$, $-y + x \leq 1$ 09305028

Solution:

$x + y \geq 5$ ….. (i)

$-y + x \leq 1$ ….. (ii)

The corresponding equation of the inequality (i) is
$x + y = 5$….. (iii)

Equation (iii) can be written as $\frac{x}{5} + \frac{y}{5} = 1$. Thus the line (iii) intersects the x-axis and y-axis at A(5, 0) and B(0, 5) respectively. The sketch of inequality (i) is shown as shaded region in the figure 4(ii)(a).

The corresponding equation of the inequality (ii) is $x - y = 1$(iv)

Equation (iv) can be written as $\frac{x}{1} + \frac{y}{-1} = 1$. Thus the line (iv) intersects the x-axis and y-axis at C(l, 0) and D(0, -1) respectively. The sketch of inequality (ii) is shown as shaded region in the figure 4(ii)(b).

Now the solution region of the given systems of inequalities (i) and (ii) is the intersection of the graphs indicated in the figures 4(ii)(a) and 4(ii)(b) and is shown as shaded region in the figure 4(ii)(c).

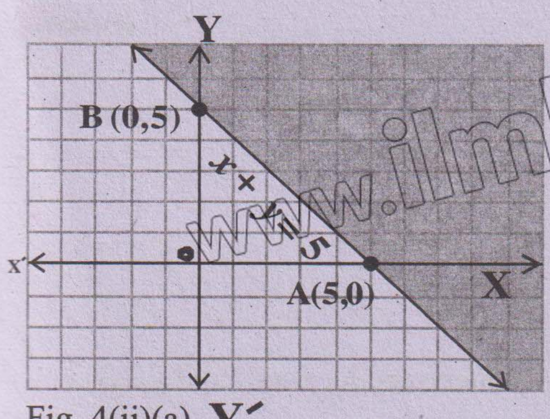


Fig. 4(ii)(a)

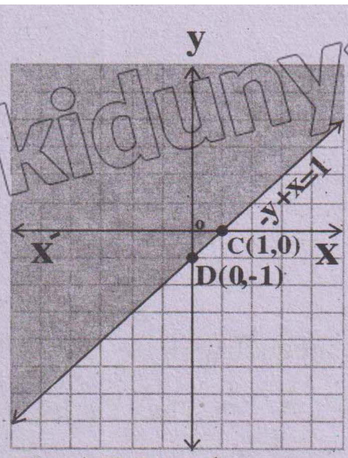


Fig. 4(ii)(b)

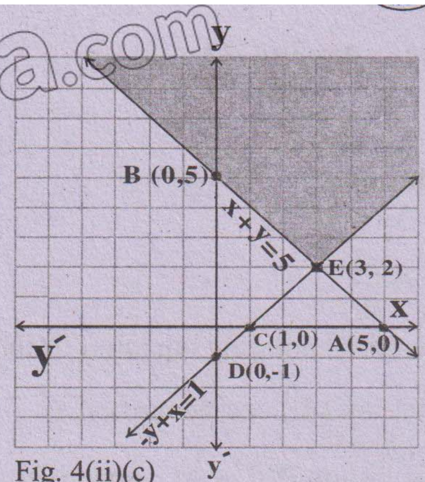


Fig. 4(ii)(c)

(iii) $\quad 3x + 7y \geq 21$ 09305029

$\quad\quad x - y \leq 2$

Solution:

$3x + 7y \geq 21$ .... (i)

$x - y \leq 2$ .... (ii)

The corresponding equation of the inequality (i) is $3x + 7y = 21$ (iii)

Equation (iii) can be written as $\frac{x}{7} + \frac{y}{3} = 1$. Thus line (iii) intersects the x-axis and y-axis at A(7,0) and B(0, 3) respectively. The sketch of inequality (i) is shown as shaded region in the figure 4(iii)(a).

The corresponding equation of the inequality (ii) is $x - y = 2$ (iv)

Equation (iv) can be written as $\frac{x}{2} + \frac{y}{-2} = 1$. Thus the line (iv) intersects the x-axis and y-axis at C(2, 0) and D(0, -2) respectively. The sketch of inequality (ii) is shown as shaded region in the figure 4(iii)(b).

Now the solution region of the given systems of inequalities (i) and (ii) is the intersection of the graphs indicated in the figures 4(iii)(a) and 4(iii)(b) and is shown as shaded region in the figure 4(iii)(c).

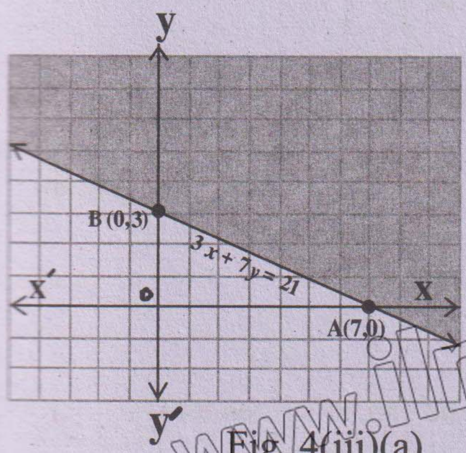


Fig. 4(iii)(a)

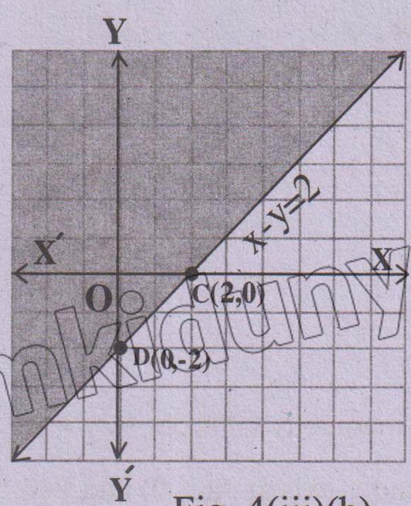


Fig. 4(iii)(b)

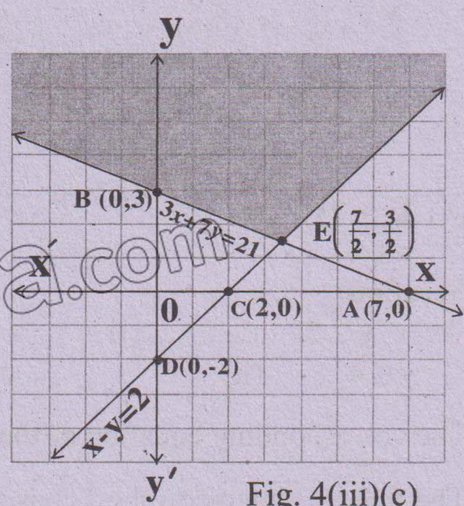


Fig. 4(iii)(c)

(iv) $4x-3y \le 12$

$x \ge \dfrac{-3}{2}$ 09305030

Solution:

$4x-3y \le 12$

$x \ge \dfrac{-3}{2}$

The corresponding equation of the inequality (i) is $4x - 3y = 12$ (iii)

Equation (iii) can be written as $\dfrac{x}{3} + \dfrac{y}{-4} = 1$. Thus the line (iii) intersects the x-axis and y-axis at A(3, 0) and B(0, − 4) respectively.

The sketch of inequality (i) is shown as shaded region in the figure 4(iv)(a).

The sketch of inequality (ii) is shown as shaded region in the figure 4(iv)(b). The boundary line is drawn at $\left(\dfrac{-3}{2}, 0\right)$.

Now the solution region of the given systems of inequalities is the intersection of the graphs indicated in the figure 4(iv)(a) and 4(iv)(b) and is shown as shaded region in the figure 4(iv)(c).

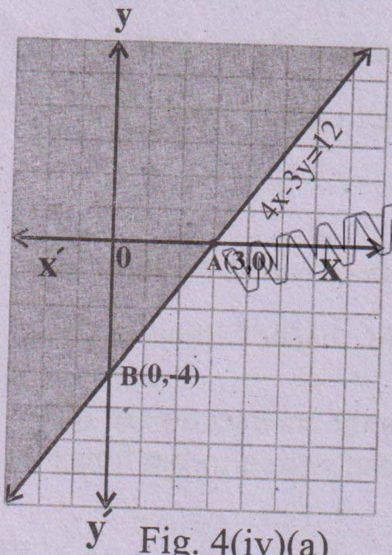


Fig. 4(iv)(a)

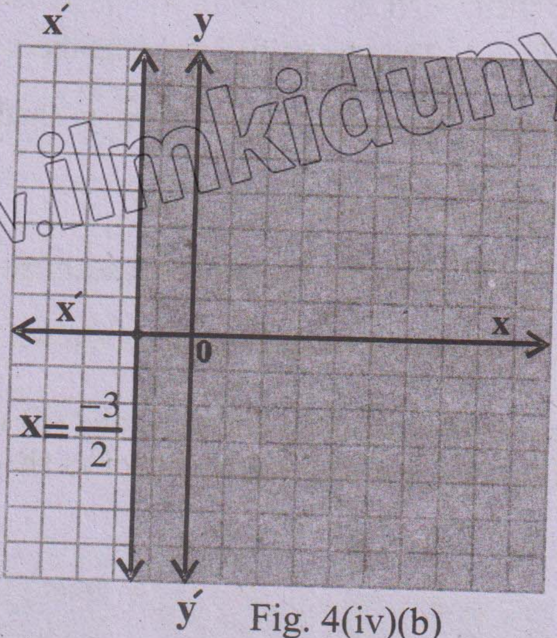


Fig. 4(iv)(b)

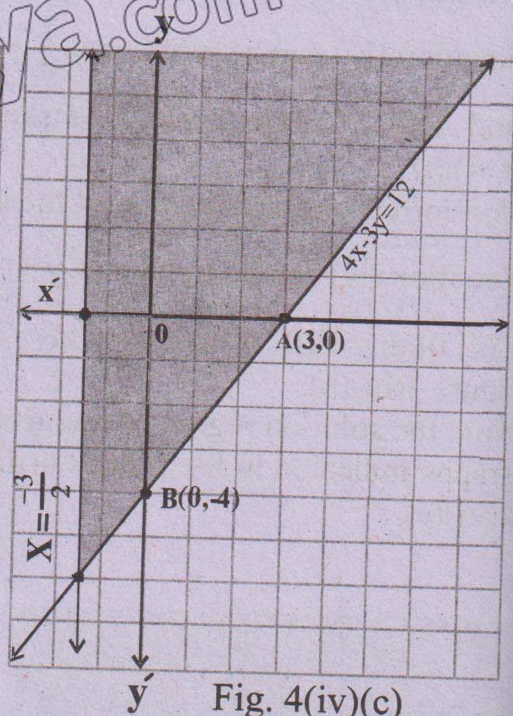


Fig. 4(iv)(c)

(v) $3x + 7y \ge 21$

$y \le 4$ 09305031

Solution:

$3x + 7y \ge 21$ ....(i)

$y \le 4$ (ii)

The corresponding equation of the inequality (i) is $3x + 7y = 21$ and can be written as $\dfrac{x}{7} + \dfrac{y}{3} = 1$

The line (iii) intersects the x-axis and y-axis at A(7, 0) and B(0, 3) respectively.

The sketch of inequality (i) is shown as shaded region in the figure 4(v)(a). The sketch of inequality (ii) is shown as shaded region in the figure 4(v)(b). The boundary line is drawn at c(0,4).

Now the solution region of the given systems of inequalities is the intersection of the graphs indicated in the figure 4(v)(a) and 4(v)(b) and is shown as shaded region in the figure 4(v)(c).

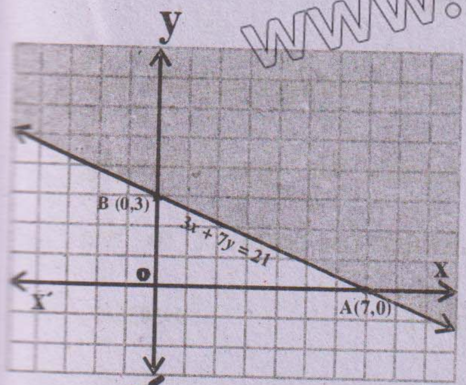


Fig. 4(v)(a)

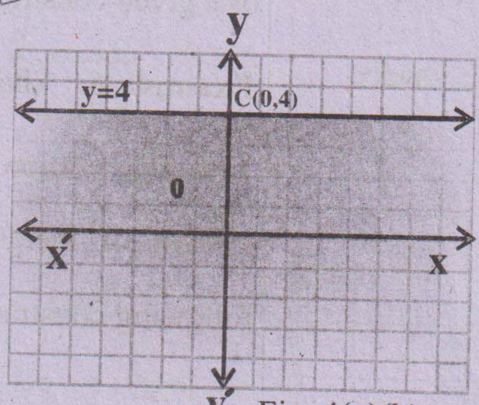


Fig. 4(v)(b)

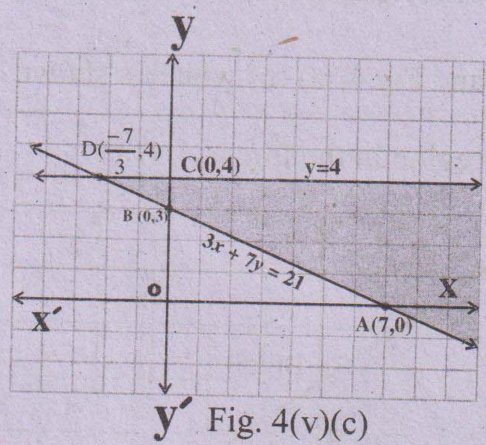


Fig. 4(v)(c)

(vi) **$5x+7y \le 35$ ....(i)** 09305032

$x-2y \le 2$ ...... (ii)

The associated equation of inequality (i) is $5x+7y=35$ and it can be written in the form $\frac{x}{7}+\frac{y}{5}=1$. This line intersects the x-axis and y-axis at point A(7,0) and B(0,5) respectively. The sketch of inequality (i) is shown as shaded region in the Fig.4(vi) (a).

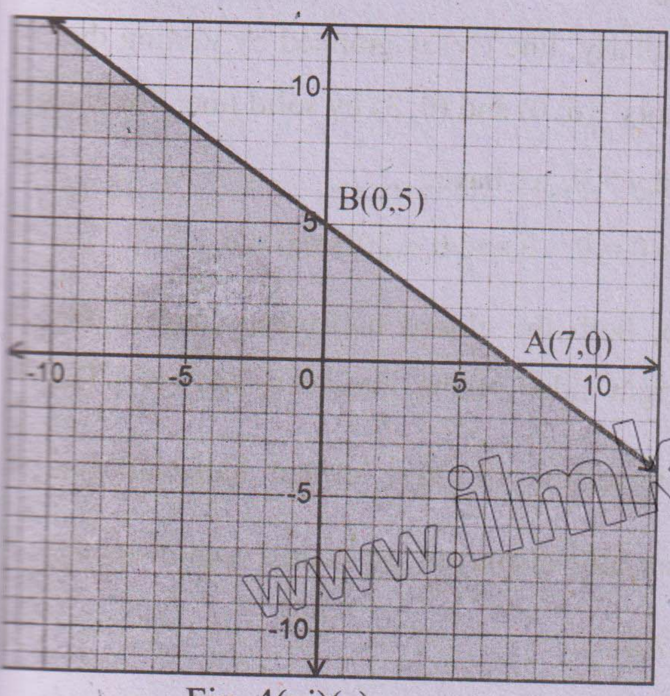


Fig. 4(vi)(a)

The associated equation of inequality (ii) is $x-2y=2$ and it can be written in the form $\frac{x}{2}+\frac{y}{-1}=1$. This line intersects the x-axis and y-axis at point C(2,0) and D(0,–1) respectively. The sketch of inequality (ii) is shown as shaded region in the fig.4(vi) (b).

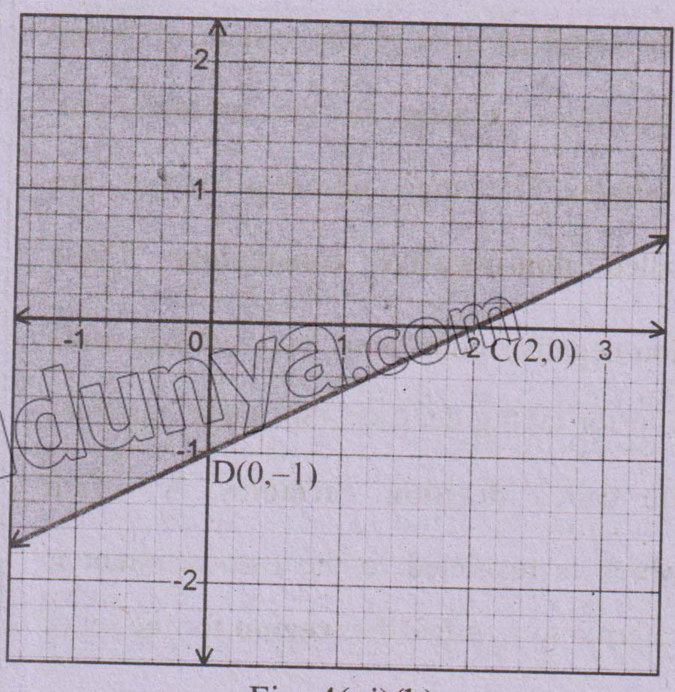


Fig. 4(vi)(b)

Now the solution region of the given system of inequalities (i) and (ii) is the intersection of the graphs indicated in the Fig.4(vi) (a) and Fig.4(vi) (b) which is shown as shaded region in the Fig.4(vi) (c)

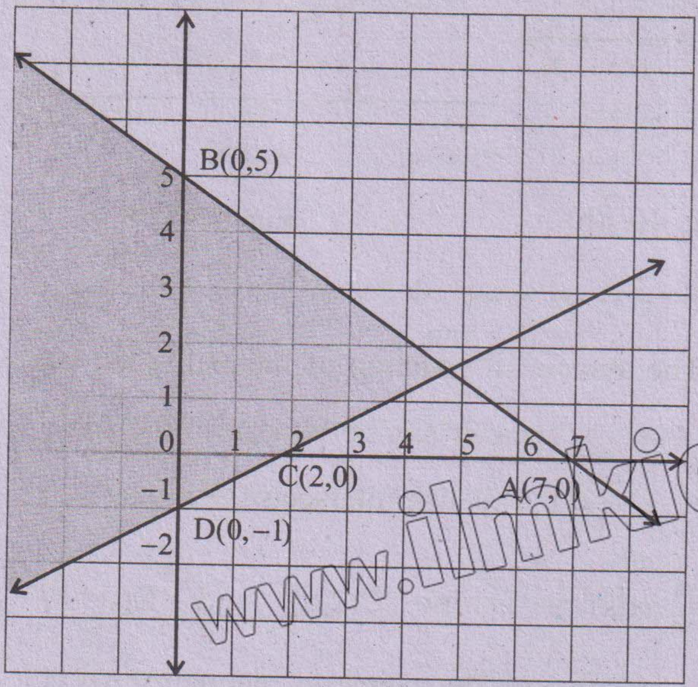


Fig.4(vi) (c)

Feasible Solution

The variables used in the system of linear inequalities relating to the problems of everyday life are non-negative and are called **non-negative constraints**. These non-negative constraints play an important role for taking decision. So, these variables are called **decision variables**. A region which is restricted to the first quadrant is referred to as a **feasible region** for the set of given constraints. Each point of the feasible region is called a **feasible solution** of the system of linear inequalities (or for the set of a given constraints).

Example 6: Shade the feasible region and find the corner points for the following system of inequalities (or subject to the following constraints). **09305033**

$x - y \leq 3$

$x + 2y \leq 6, \quad x \geq 0, \quad y \geq 0$

Solution:

The associated equations for the inequalities

$x - y \leq 3$ (i) and $x + 2y \leq 6$ (ii) are $x - y = 3$ (iii) and $x + 2y = 6$ (iv)

As the point (3, 0) and (0,–3) are on the line (iii), so the graph of $x - y = 3$ is drawn by joining the points (3, 0) and (0,–3) by solid line.

Similarly, line (iv) is graphed by joining the points (6, 0) and (0, 3) by solid line. For $x = 0$ and $y = 0$, we have;

$0 - 0 = 0 < 3$ and $0 + 2(0) = 0 < 6$

So, both the closed half-planes are on the origin sides of the lines (iii) and (iv). The intersection of these closed half-planes is partially displayed as shaded region in fig. 5.7(a).

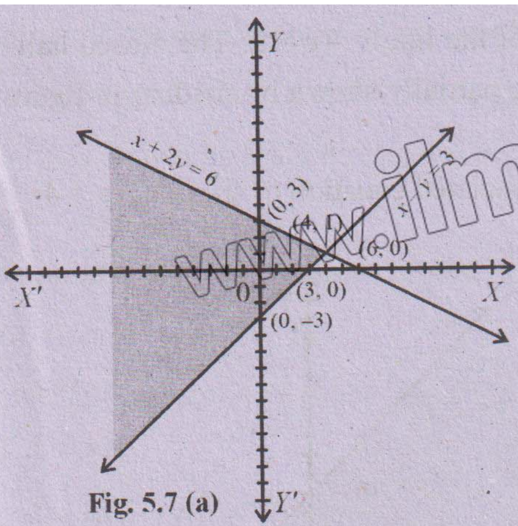


Fig. 5.7 (a)

The graph of $y \geq 0$, will be the closed upper half plane.

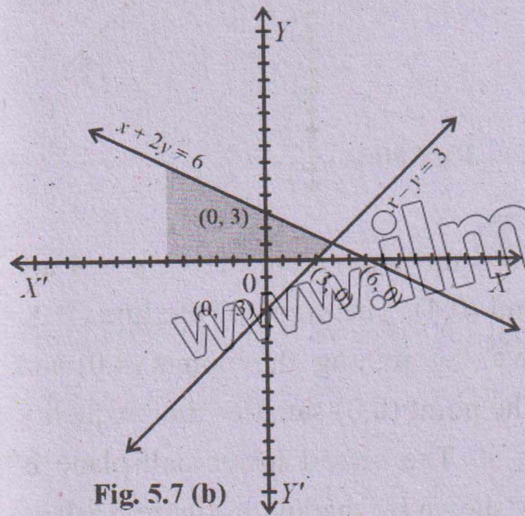


Fig. 5.7 (b)

The intersection of graph shown in figure 5.7(a) and closed upper half plane is partially displayed as shaded region in figure 5.7 (b).

The graph of $x \geq 0$ will be closed right half plane.

The intersection of the graph shown in fig. 5.7(a) and closed right half plane is graphed in fig. 5.7 (c).

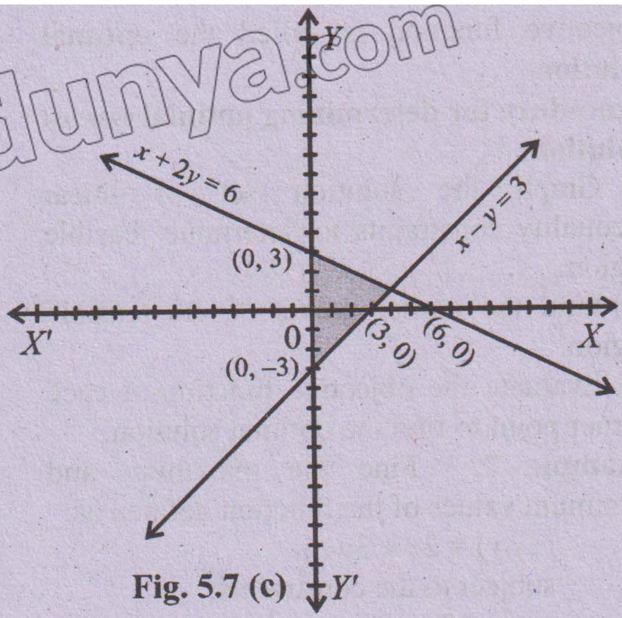


Fig. 5.7 (c)

Finally, the graph of the given system of linear inequalities is displayed in figure 5.7 (d) which is the feasible region for the given system of linear inequalities. The points (0,0), (3, 0), (4, 1) and (0, 3) are corner points of the feasible region.

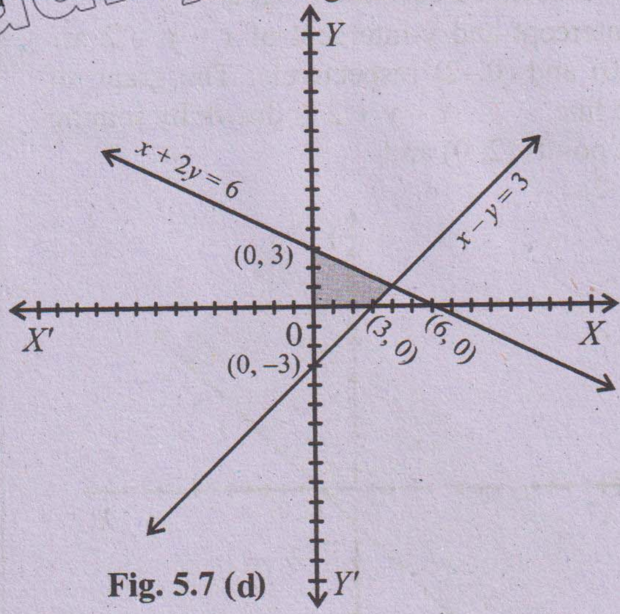


Fig. 5.7 (d)

Maximum and Minimum Values of a Function in the Feasible Solution

A function which is to be maximized or minimized is called an **objective function**. Note that there are infinitely many feasible solutions in the feasible region. The feasible solution which maximizes or minimizes the objective function is called the **optimal solution**.

Procedure for determining optimal Solution: 09305034

(i) Graph the solution set of linear inequality constraints to determine feasible region.

(ii) Find the corner points of the feasible region.

(iii) Evaluate the objective function at each corner point to find the optimal solution.

Example 7: Find the maximum and minimum values of the function defined as:

$f(x,y) = 2x + 3y$

subject to the constraints;

$x - y \leq 2$

$x + y \leq 4$

$x \geq 0, y \geq 0$

09305035

Solution:

$x - y \leq 2 \quad \ldots(i)$

$x + y \leq 4 \quad \ldots(ii)$

The associated equation of (i) is x-intercept and y-intercept of $x - y = 2$ are (2,0) and (0,–2) respectively. The graph of the line $x - y = 2$ is drawn by joining the points (2, 0) and (0,–2).

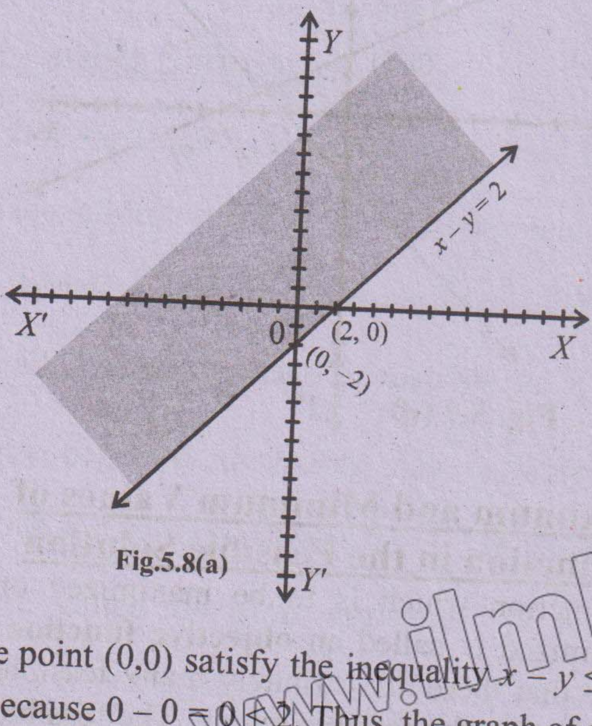


Fig.5.8(a)

The point (0,0) satisfy the inequality $x - y \leq 2$ because $0 - 0 = 0 < 2$. Thus, the graph of $x - y \leq 2$ is the upper half-plane including the graph of the line $x - y = 2$. The closed half-plane is partially shown by shading in figure 5.8(a).

The associated equation of (ii) is $x + y = 4$

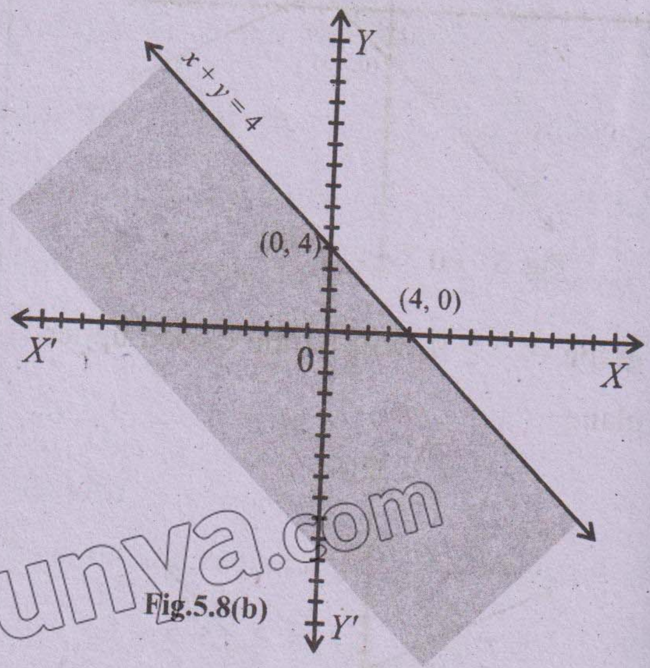


Fig.5.8(b)

x-intercept and y-intercept of $x + y = 4$ are (4, 0) and (0,4), The graph of the line $x + y = 4$ is drawn by joining the points (4,0) and (0,4). The point (0,0) satisfies the inequality $x + y \leq 4$. The closed lower half-plane is partially shown by shading in figure 5.8(b).

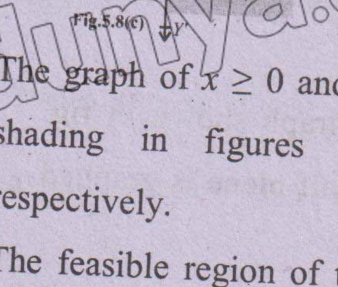


Fig.5.8(c)

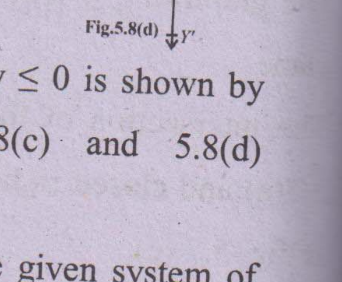


Fig.5.8(d)

The graph of $x \geq 0$ and $y \leq 0$ is shown by shading in figures 5.8(c) and 5.8(d) respectively.

The feasible region of the given system of inequalities is the intersection of the graphs indicated in figures 5.8(a), 5.8(b), 5.8(c) and 5.8(d) and is shown as shaded region ABCD in figure 5.8(e).

Corner points of the feasible region are (0,0), (2, 0), (3, 1) and (0, 4).

Now, we find values of $f(x,y) = 2x + 3y$ at the corner points.

$$f(0,0) = 2(0) + 3(0) = 0$$

$$f(2,0) = 2(2) + 3(0) = 4$$

$$f(3,1) = 2(3) + 3(1) = 9$$

$$f(0,4) = 2(0) + 3(4) = 12$$

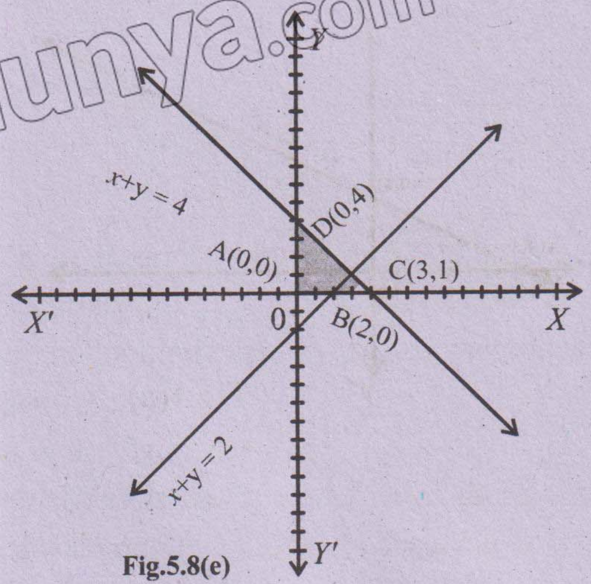


Fig.5.8(e)

Thus, the minimum value of f is 0 at the corner point (0,0) and maximum value of f at corner point (0,4) is 12.

Exercise 5.2

Q.1 Maximize $f(x, y) = 2x + 5y$; subject to the constraints 09305036

$2y - x \leq 8;$ $x - y \leq 4;$ $x \geq 0; y \geq 0$

Solution:

The constraints are

$2y - x \leq 8$ .....(i)

$x - y \leq 4$ .....(ii)

$x \geq 0, y \geq 0$ .....(iii)

The sketch of inequality (i) is shown as shaded region in the figure 1(a) by joining the points A(−8,0) and B(0,4) which are x and y intercepts respectively.

The sketch of inequality (ii) is shown as shaded region in the figure 1(b) by joining the points C(4,0) and D(0, −4) which are x and y intercepts respectively.

The sketch of inequalities $x \geq 0, y \geq 0$ (iii) is shown as shaded region in the figure 1(c)

Now the solution region of the given systems of inequalities is the intersection of the graphs indicated in the figure 1(a), 1(b), 1(c) and is shown as shaded region OCEB in the figure 1(d).

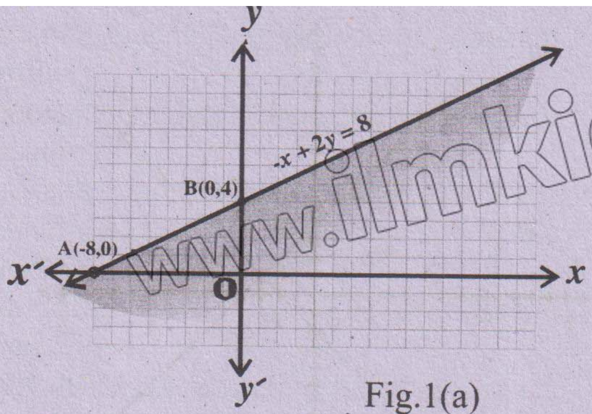


Fig.1(a)

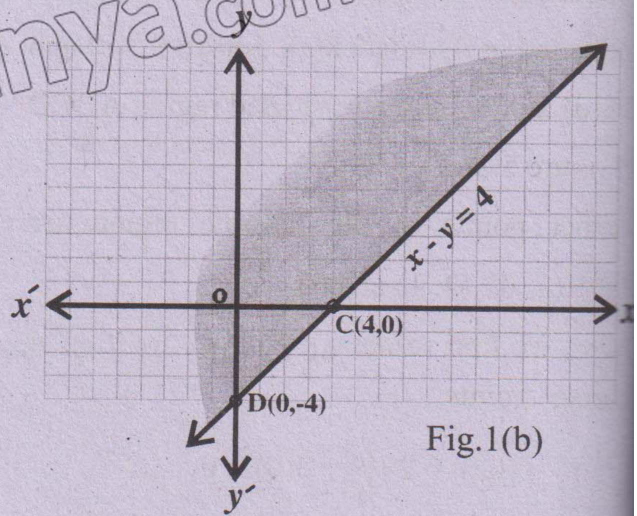


Fig.1(b)

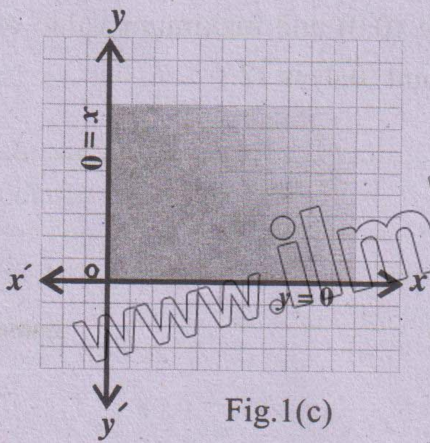


Fig.1(c)

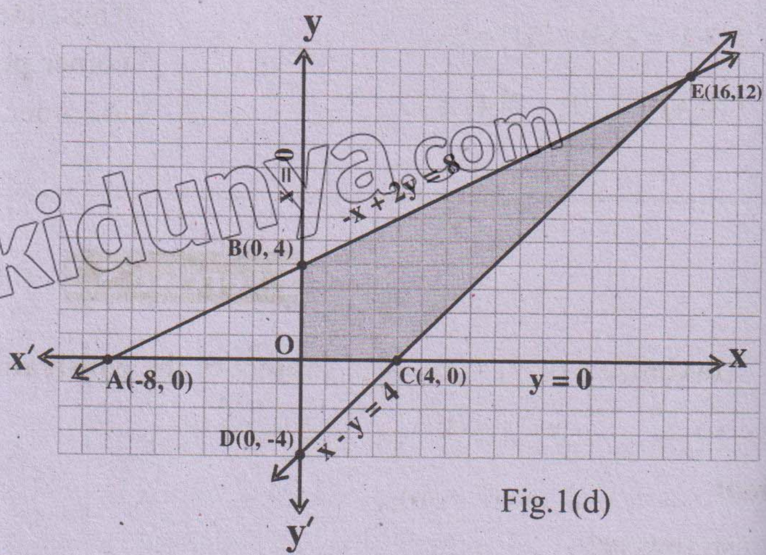


Fig.1(d)

So the corner points are O, C, E and B.

For the corner point E, solving the lines 2y - x = 8 and x - y = 4 simultaneously, we have the intersection point E(16, 12). It can also be taken from the graph directly.

Thus corner points are 0(0, 0), C(4, 0), E(16,12) and B(0, 4).

Now find the value of $f(x, y)$ at the corner points, we have

Corner points	$f(x,y) = 2x + 5y$
0(0, 0)	$f(0, 0) = 2(0) + 5(0) = 0$
C(4, 0)	$f(4,0) = 2(4) + 5(0) = 8$
E(16,12)	$f(16,12) = 2(16) + 5(12) = 92$
B(0, 4)	$f(0, 4) = 2(0) + 5(4) = 20$

We observe that the maximum value of $f(x, y)$ is 92 at the corner point E(16,12).

Q.2 Maximize $f(x, y) = x + 3y$ subject to the constraints

$$2x + 5y \leq 30 \; ; \; 5x + 4y \leq 20 \; ; \; x \geq 0 ; y \geq 0$$

Solution:

The constraints are

$$2x + 5y \le 30 \text{ .....(i)}$$

$$5x + 4y \le 20 \text{ .....(ii)}$$

$$x \ge 0,\ y \ge 0 \text{ .....(iii)}$$

The sketch of inequality (i) is shown as shaded region in the figure 2(a) by joining the points A(15,0) and B(0,6) which are x and y intercepts respectively

The sketch of in equalities (ii) is shown as shaded region in the figure 2(b) by joining the points C(4,0) and D(0, 5) which are x and y intercepts respectively

The sketch of inequality (iii) is shown as shaded region in the figure (c)

Now the solution region of the given systems of inequalities is the intersection of the graphs indicated in the figure 2(a), 2(b), 2(c) and is shown as shaded region OCD in the figure 2(d).

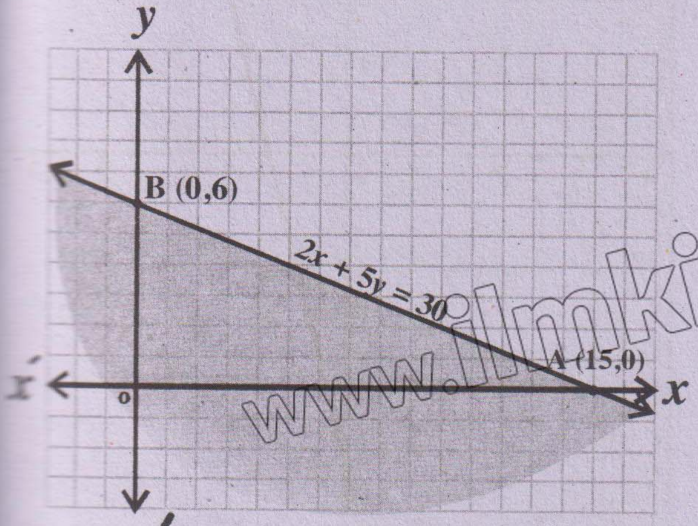


Fig.2(a)

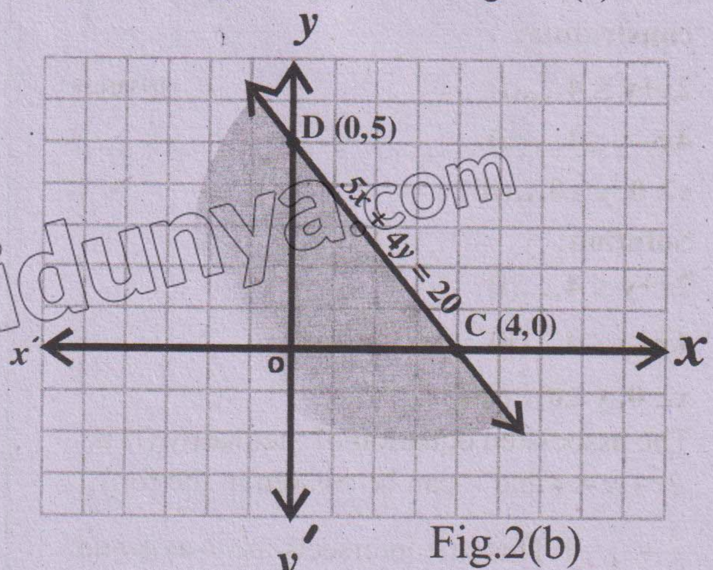


Fig.2(b)

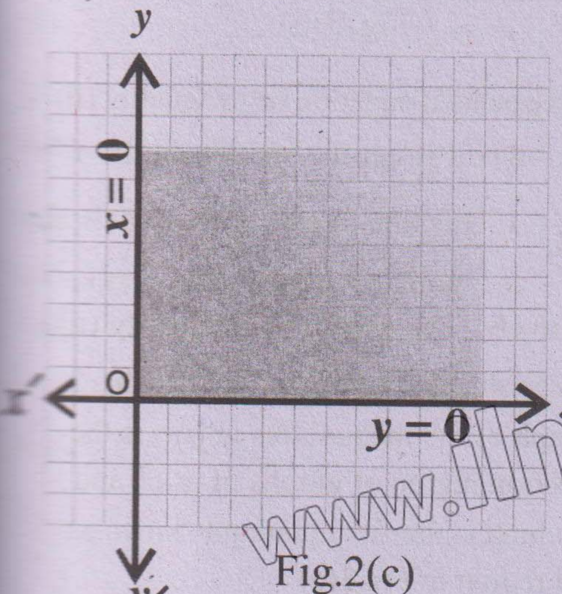


Fig.2(c)

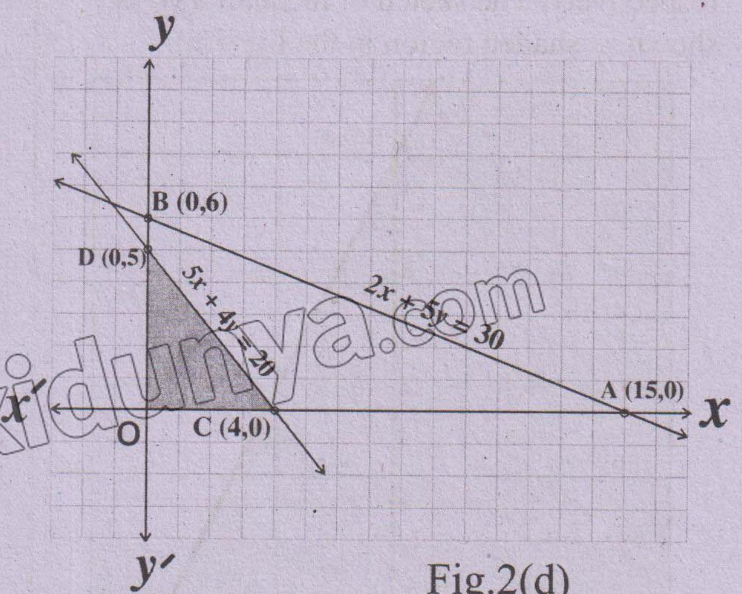


Fig.2(d)

Thus corner points are O(0, 0), C(4, 0) and D(0, 5).

Now find the value of $f(x, y)$ at the corner points, we have

Corner points	$f(x,y)=x + 3y$
O(0, 0)	$f(0, 0) = 0 + 3(0) = 0$
C(4, 0)	$f(4, 0) = 4 + 3(0) = 4$
D(0, 5)	$f(0, 5) = 0 + 3(5) = 15$

We observe that the maximum value of $f(x, y)$ is 15 at the corner point D(0, 5).

Q.3 Maximize $z = 2x + 3y$; subject to the constraints:

$2x+y \le 4$....(i) 09305038

$4x- y \le 4$...(ii)

$x\ge 0, y \ge 0$...(iii)

Solution:

$2x+y \le 4$....(i)

$4x- y \le 4$...(ii)

$x\ge 0, y \ge 0$...(iii)

The associated equation of inequality (i) is $2x+ y= 4$ and it can be written in the form $\frac{x}{2}+\frac{y}{4}=1$. This line intersects the x-axis and y-axis at point A(2,0) and B(0,4) respectively. The sketch of inequality (i) is shown as shaded region in the Fig.3(a).

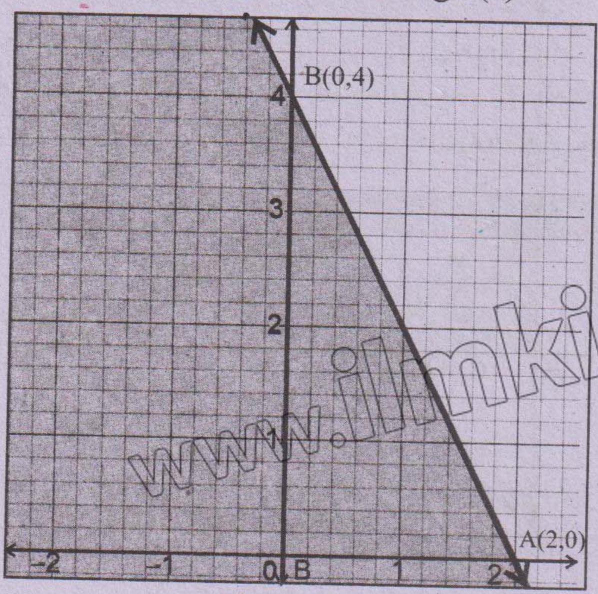


Fig. 3(a)

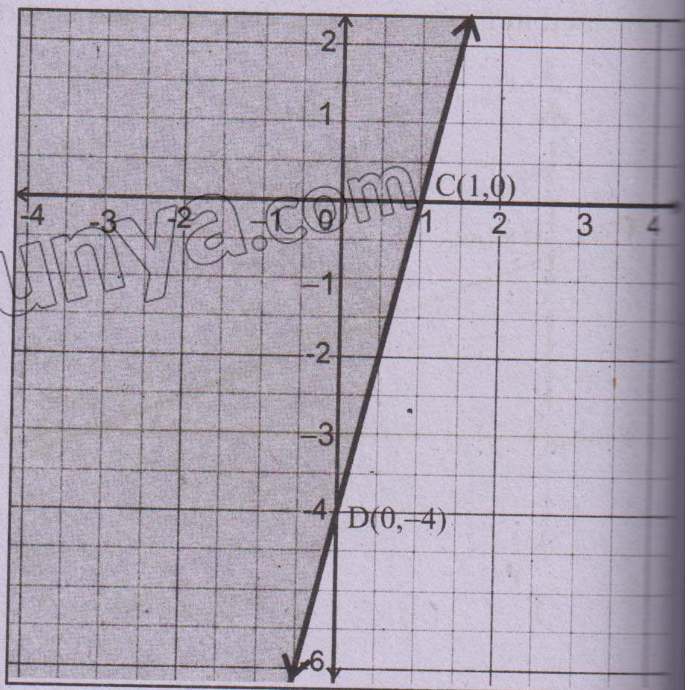


Fig. 3(b)

Now associated equation of in equality (ii) is $4x - y= 4$ and it can be written in the form $\frac{x}{1}+\frac{y}{-4}=1$. This line intersects the x-axis and y-axis at point C(1,0) and D(0,−4) respectively. The sketch of inequality (ii) is shown as shaded region in the Fig.3(b).

The graph of the inequalities in (iii) is the intersection region of graphs of $x\ge 0$ and $y\ge 0$ which is the first quadrant shown as shaded region in Fig.3(c).

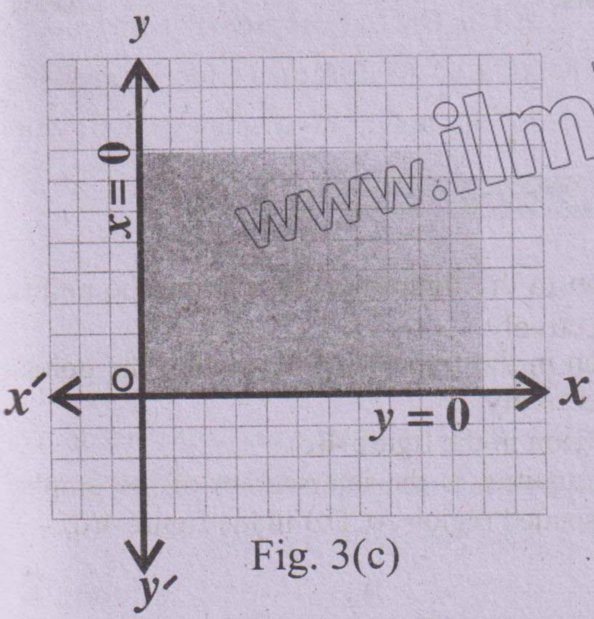


Fig. 3(c)

Now the solution region of the given system of inequalities is the intersection of the graphs indicated in the Fig.3(a) ,3(b) and 3(c) which is shown as shaded region in the Fig.3(d)

We observe that solution region is bounded region and its corner points are B(0,4), A(1,0), O(0,0) and E $\left(\frac{4}{3},\frac{4}{3}\right)$.

The point of intersection of corresponding lines of inequalities (i) and (ii) which can be obtained by solving the corresponding lines simultaneously.

$2x + y = 4$ ....(iv)

$4x - y = 4$ ....(v)

Adding (iv) and (v) we get

$2x + y + 4x - y = 4 + 4$

$6x = 8 \Rightarrow x = \frac{8}{6} = \frac{4}{3} = 1\frac{1}{3}$

Put it in equation (iv)

$2\left(\frac{4}{3}\right) + y = 4 \Rightarrow y = 4 - \frac{8}{3} = \frac{12-8}{3} = \frac{4}{3} = 1\frac{1}{3}$

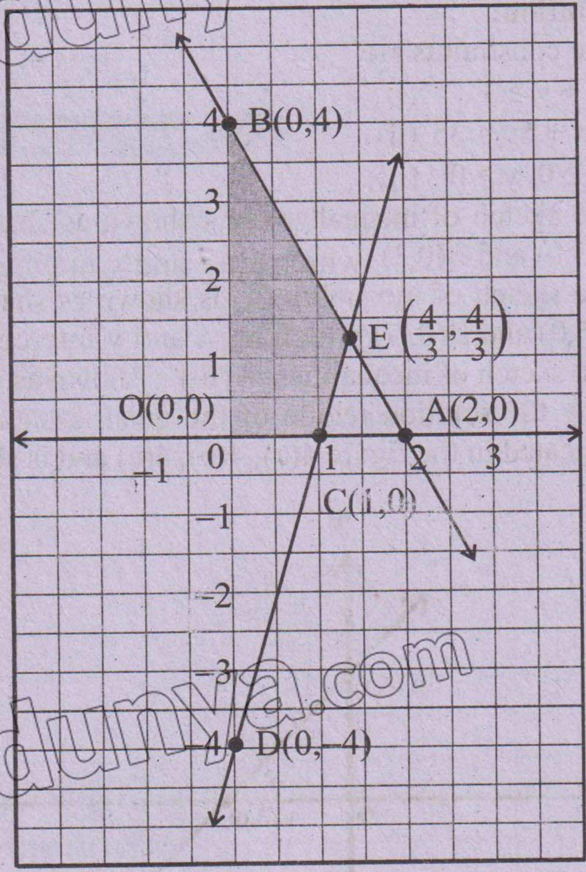


Fig. 3(d)

Now, we find the value of $f(x, y) = 2x+3y$ at corner points.

Corner points	$f(x,y)=2x+3y$
B(0,4),	$f(0,4)=2(0)+3(4)=0+12=12$
A(1,0)	$f(1,0)=2(1)+3(0)=2+0=2$
O(0,0)	$f(0,0)=2(0)+3(0)=0+0=0$
E$\left(\frac{4}{3},\frac{4}{3}\right)$	$f(\frac{4}{3},\frac{4}{3})=2(\frac{4}{3})+3(\frac{4}{3})$ $=\frac{8}{3}+4=\frac{8+12}{3}=\frac{20}{3}=6.666$

We observe that the value of function is maximum 12 at corner point B(0,4).

09305039

Q.4 **Minimize $z = 2x + y$: subject to the constraints:**
$x + y \geq 3,\ 7x + 5y \leq 35,\ x \geq 0,\ y \geq 0$

Solution:

The constraints are

$x + y \geq 3$ (i)

$7x + 5y \leq 35$ (ii)

$x \geq 0,\ y \geq 0$ (iii)

The sketch of inequality (i) is shown as shaded region in the figure 4(a) by joining the points A(3,0) and B(0,3) which are x and y intercepts respectively

The sketch of inequality (ii) is shown as shaded region in the figure 4(b) by joining the points C(5,0) and D(0, 7) which are x and y intercepts respectively

The sketch of inequalities in (iii) is shown as shaded region in the figure 4(c)

Now the solution region of the given systems of inequalities is the intersection of the graphs indicated in the figure 4(a), 4(b), 4(c) and is shown as shaded region ACDB in the figure 4(d).

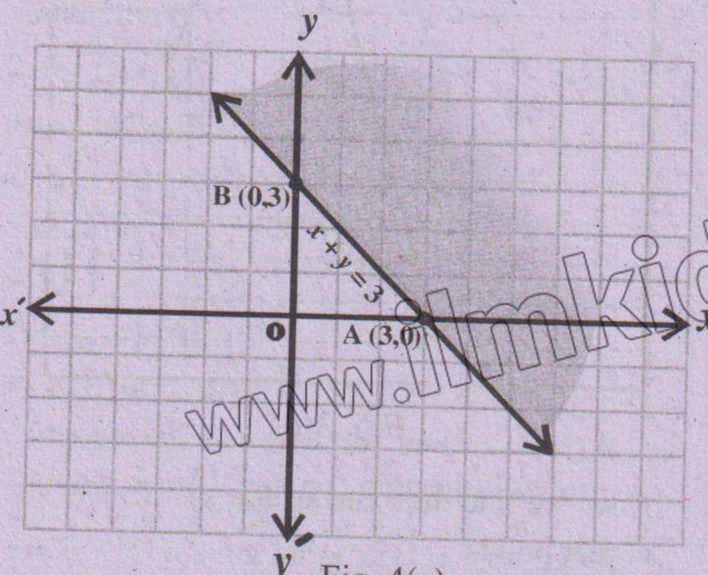


Fig. 4(a)

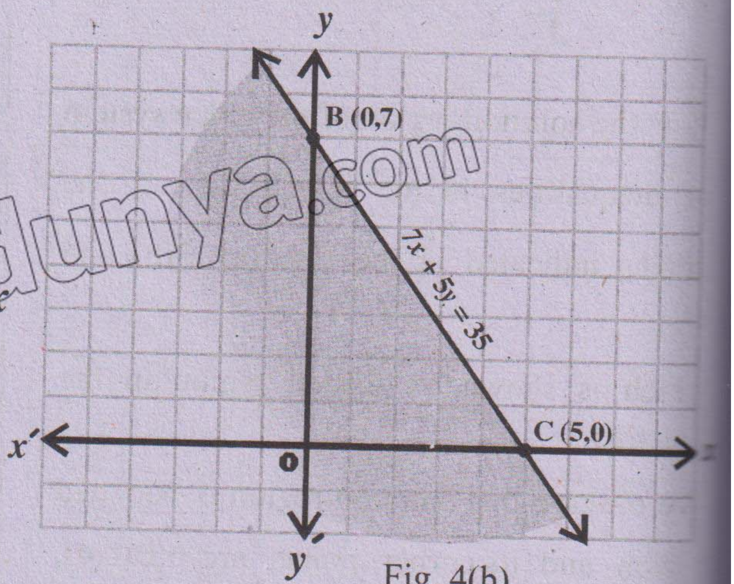


Fig. 4(b)

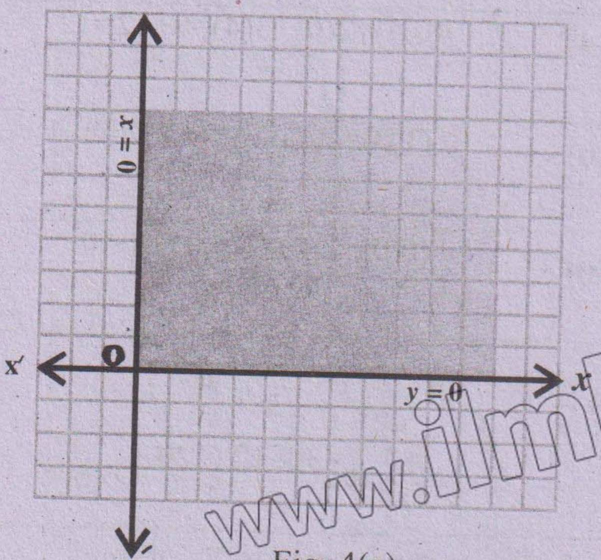


Fig. 4(c)

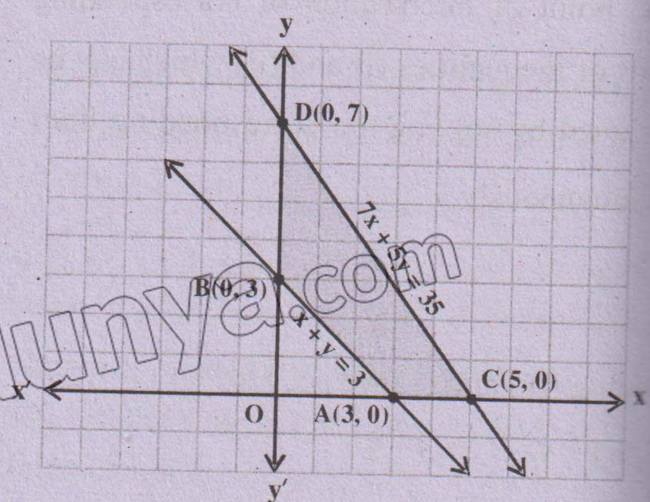


Fig. 4(d)

So the corner points are A, C, D and B.

Thus corner points are A(3, 0), C(0, 5), D(0, 7) and B(0, 3).

Now find the value of $f(x, y)$ at the corner's points, we have

Corner Points	$f(x,y) = 2x+y$
A(3, 0)	$f(3,0) = 2(3) + 0 = 6$
C(5, 0)	$f(5, 0) = 2(5) + 0 = 10$
D(0, 7)	$f(0, 7) = 2(0)+ 7 = 7$
B(0, 3)	$f(0, 3) = 2(0) + 3 = 3$

We observe that the minimum value of $f(x, y)$ is 3 at the corner point B(0,3).

Q.5 Maximize the function defined as; $f(x, y) = 2x + 3y$ subject to the constraints:

$2x + y \leq 8$; $x + 2y \leq 14$; $x \geq 0$; $y \geq 0$ 09305040

Solution:

The constraints are

$2x + y \leq 8$ (i)

$x + 2y \leq 14$ (ii)

$x \geq 0, y \geq 0$ (iii)

The sketch of inequality (i) is shown as shaded region in the figure 5(a) by joining the points A(4,0) and B(0,8) which are x and y intercepts respectively

The sketch of inequality (ii) is shown as shaded region in the figure 5(b) by joining the points C(14,0) and D(0,7) which are x and y intercepts respectively

The sketch of in equalities in (iii) is shown as shaded region in the figure 5(c)

Now the solution region of the given systems of inequalities is the intersection of the graphs indicated in the figure 5(a), 5(b), 5(c) and is shown as shaded region OAED in the figure 5(d).

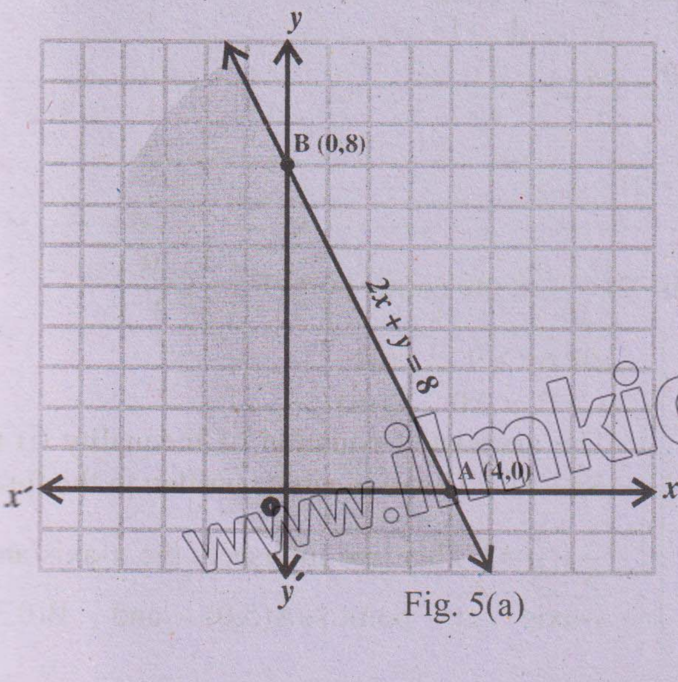


Fig. 5(a)

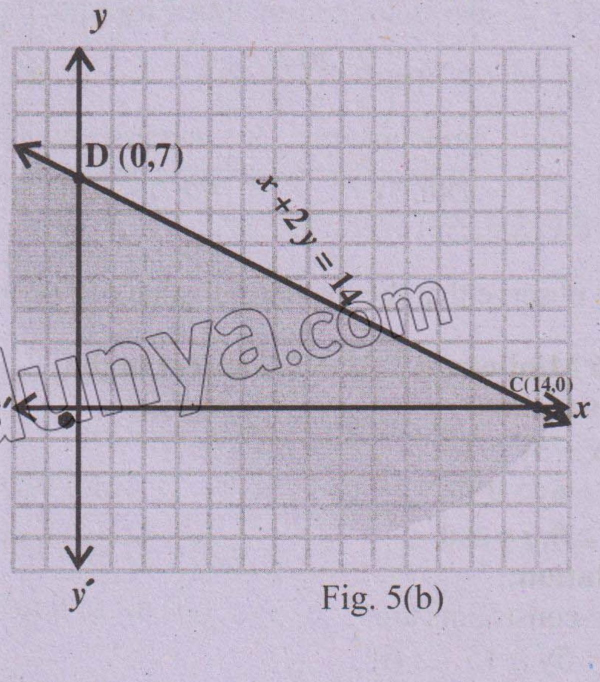


Fig. 5(b)

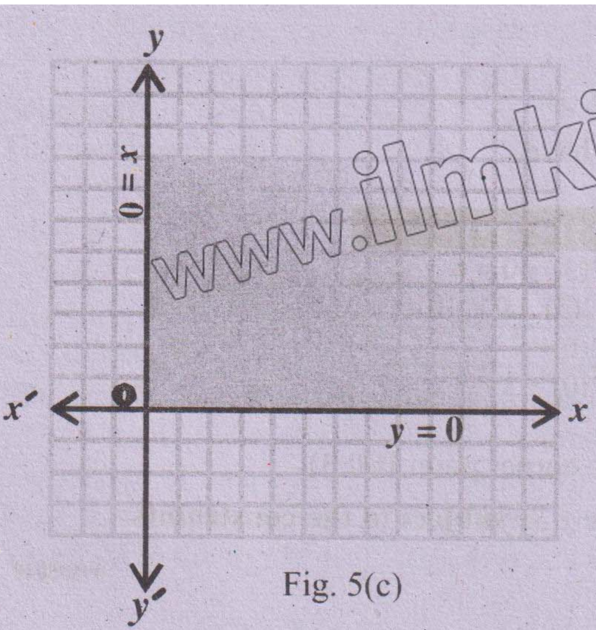


Fig. 5(c)

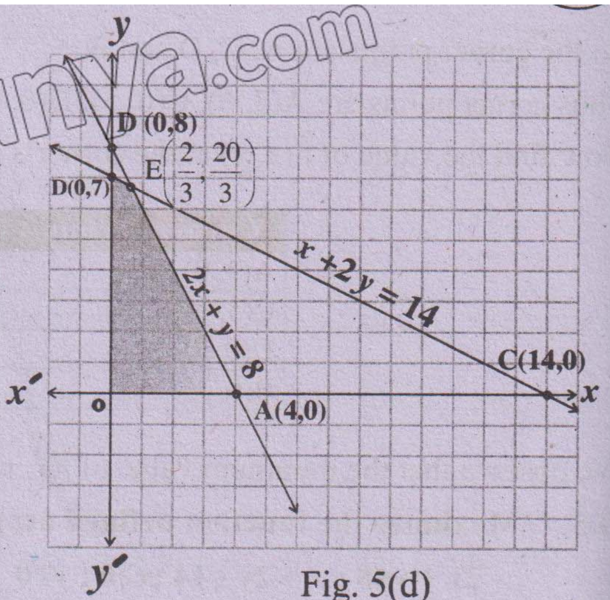


Fig. 5(d)

So the corner points are O, A, E and D.
For the corner point E, solving the lines $2x + y = 8$ and $x + 2y = 14$ simultaneously, we have the intersection point E $\left(\frac{2}{3}, \frac{20}{3}\right)$.

Thus corner points are $0(0, 0)$, $A(4, 0)$, $E\left(\frac{2}{3}, \frac{20}{3}\right)$ and $D(0, 7)$.

Now find the value of $f(x, y)$ at the cornet points, we have

Corner points	$f(x,y) = 2x + 3y$
$0(0, 0)$	$f(0, 0) = 2(0) + 3(0) = 0$
$A(4, 0)$	$f(4, 0) = 2(4) + 3(0) = 8$
$E\left(\frac{2}{3}, \frac{20}{3}\right)$	$f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right) = \frac{4}{3} + \frac{20}{1}$ $= \frac{4+60}{3} = \frac{64}{3} = 21\frac{1}{3}$
$D(0, 7)$	$f(0,7) = 2(0) + 3(7) = 21$

We observe that the maximum value of $f(x, y)$ is "$21\frac{1}{3}$" at the corner point E $\left(\frac{2}{3}, \frac{20}{3}\right)$

Q.6 Minimize $z = 3x + y$; subject to the constraints: **09305041**

$3x + 5y \geq 15$;

$x + 6y \geq 9$;

$x \geq 0, ; y \geq 0$

Solution:

The constraints are

$3x + 5y \geq 15$….. (i)

$x + 6y \geq 9$ ….. (ii)

$x \geq 0, y \geq 0$ …..(iii)

The associated equation of inequality (i) is $3x + 5y = 15$ and it can be written in the form $\frac{x}{5} + \frac{y}{3} = 1$. This line intersects the x-axis and y-axis at point A(5,0) and B(0,3) respectively. The sketch of inequality (i) is shown as shaded region in the Fig.6(a).

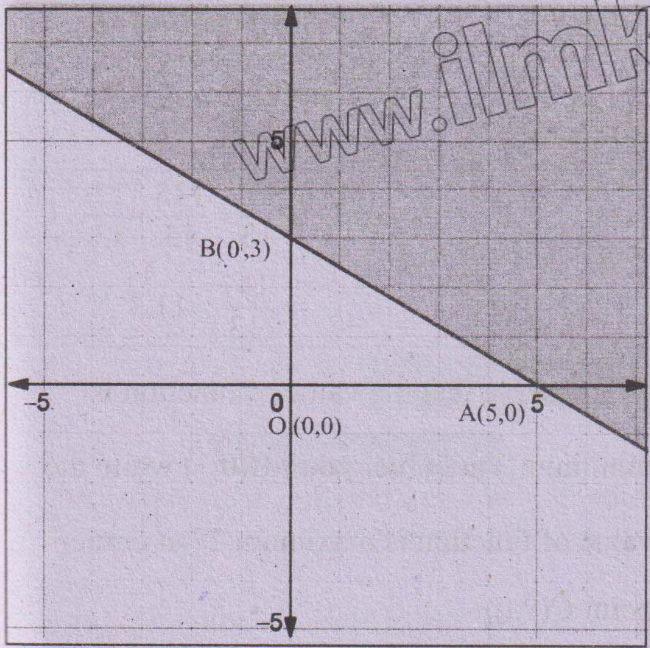


Fig. 6(a)

The associated equation of inequality (ii) is $x+6y=9$ and it can be written in the form $\frac{x}{9}+\frac{y}{3/2}=1$. This line intersects the x-axis and y-axis at point C(9,0) and D(0, $\frac{3}{2}$) respectively. The sketch of inequality (ii) is shown as shaded region in the Fig.6(b).

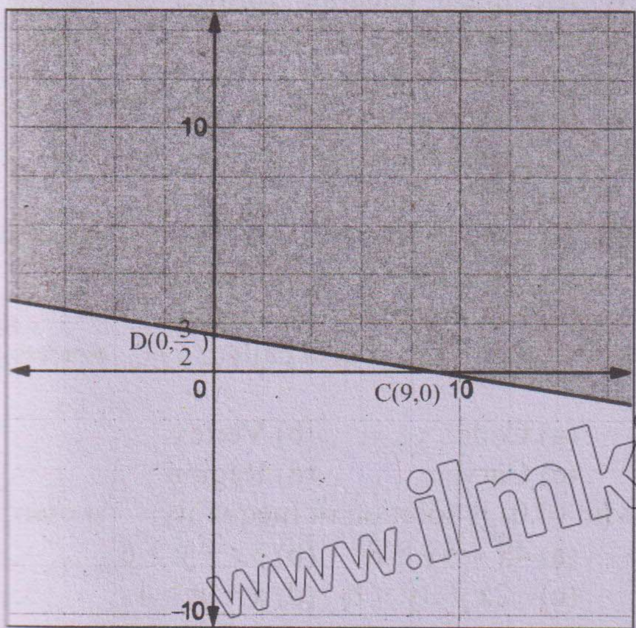


Fig. 6(b)

The graph of the inequalities in (iii) is the intersection region of graphs of $x \geq 0$ and $y \geq 0$ which is the first quadrant shown as shaded region in Fig.6(c).

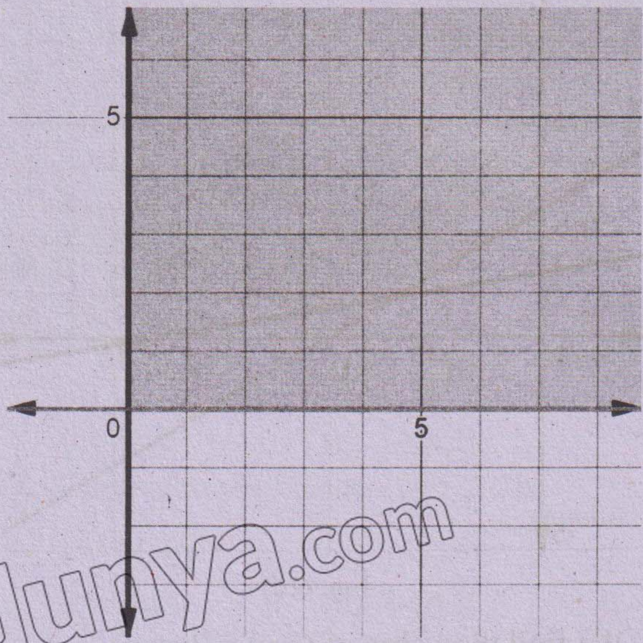


Fig. 6(c)

Now the solution region of the given system of inequalities is the intersection of the graphs indicated in the Fig.6(a) ,6(b) and 6(c)
which is shown as shaded region in the Fig.6(d)
We observe that solution region is unbounded and it corner points are B(0,3) C(9,0) and E$\left(\frac{45}{13},\frac{12}{13}\right)$. Point E is the point of intersection of corresponding lines of inequalities (i) and (ii) which can be obtained by solving the corresponding equations simultaneously.

$3x+5y=15$........(iii)
$x+6y=9$ ....(iv)

Subtract eq.(iii) from 3 times the eq.(iv)

$$3(x+6y)-(3x+5y)=3(9)-15$$

$$3x+18y-3x-5y=27-15$$

$$13y=12 \Rightarrow y=\frac{12}{13}$$

Put it in Eq.(iv)

$$x+6\left(\frac{12}{13}\right)=9 \Rightarrow x=9-\frac{72}{13} \Rightarrow x=\frac{117-72}{13}=\frac{45}{13}$$

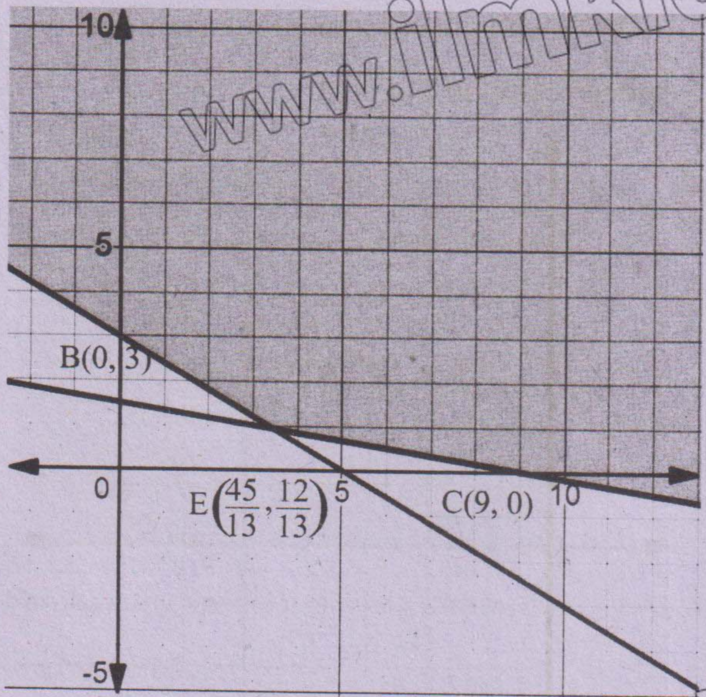


Fig. 6(d)

We find the value of $(x,y) = 3x + y$ at corner points.

Corner points	$f(x,y)=3x+y$
B(0,3)	$f(0,3)=3(0)+3=0+3=3$
C(9,0)	$f(9,0)=3(9)+0=27+0=27$
$E\left(\frac{45}{13},\frac{12}{13}\right)$.	$f\left(\frac{45}{13},\frac{12}{13}\right)=$
	$=3\left(\frac{45}{13}\right)+\frac{12}{13}=\frac{135}{13}+\frac{12}{13}$ $=\frac{147}{13}=11.3$

We observe that the value of function is minimum 3 at corner point B(0,3) while the value of function is maximum 27 at corner point C(9,0)

Review Exercise 5

Q.1 Choose the correct option.

i. In the following, linear equation is: **09305042**

(a) $5x > 7$ (b) $4x - 2 < 1$
(c) $2x + 1 = 1$ (d) $4 = 1 + 3$

ii. Solution of $5x - 10 = 10$ is: **09305043**

(a) 0 (b) 50
(c) 4 (d) –4

iii. If $7x + 4 < 6x + 6$, then x belongs to the interval: **09305044**

(a) $(2, \infty)$ (b) $[2,\infty)$
(c) $(-\infty, 2)$ (d) $(-\infty, 2]$

iv. A vertical line divides the plane into **09305045**

(a) left half plane
(b) right half plane
(c) full plane
(d) two half plane

v. The linear equation formed out of the linear inequality is called: **09305046**

(a) Linear equation
(b) Associated equation
(c) Quadratic equal
(d) None of these

vi. $3x + 4 < 0$ is: **09305047**

(a) Equation (b) Inequality
(c) Not inequality (d) identity

vii. Corner point is also called: **09305048**

(a) Code (b) Vertex
(c) Curve (d) Region

viii. (0,0) is solution of inequality: **09305049**

(a) $4x + 5y > 8$ (b) $3x + y > 6$
(c) $-2x + 3y < b$ (d) $x + y > 4$

ix. The solution region restricted to the first quadrant is called: **09305050**
(a) Objective region
(b) Feasible region
(c) Solution region
(d) Constraints region

x. A function that is to be maximized or minimized is called: **09305051**
(a) Solution function
(b) Objective function
(c) Feasible functioned)
(d) None of these

Answers Key

i	**c**	ii	**c**	iii	**c**	iv	**d**	v	**b**
vi	**b**	vii	**b**	viii	**c**	ix	**b**	x	**b**

Multiple Choice Questions (Additional)

Linear inequality

1. A statement involving any of the symbols <, > or ≤ or ≥ is called: **09305052**
(a) Equation (b) Identity
(c) Inequality (d) Linear equation

2. The degree of linear inequality is: **09305053**
(a) 1 (b) 2
(c) 3 (d) 4

Solution of linear inequality

3. Which of the following is the solution sof the inequality $3-4x \le 19$? **09305054**
(a) $x \ge -8$ (b) $x \ge -4$
(c) $x \ge \frac{-22}{4}$ (d) 16

4. x = _________ is not a solution of the inequality $x < -\frac{3}{2}$ **09305055**
(a) -1.5 (b) -2.5
(c) -3 (d) -2

5. x=0 is a solution of the inequality: 09305056
(a) $x > 0$ (b) $3x < 0$
(c) $x+2<0$ (d) $x-2<0$

6. The solution of inequality x>1 is: **09305057**
(a) $(1,\infty)$ (b) $(-\infty,1)$
(c) $[1,\infty)$ (d) $(-\infty,1]$

7. The solution of inequality x<1 is: **09305058**
(a) $(1,\infty)$ (b) $(-\infty,1)$
(c) $[1,\infty)$ (d) $(-\infty,1]$

8. The solution of inequality $x \le 1$ is: **09305059**
(a) $(1,\infty)$ (b) $(-1,\infty)$
(c) $[1,\infty)$ (d) $(-\infty,1]$

9. The solution of inequality $x \ge 1$is: **09305060**
(a) $(1,\infty)$ (b) $(-1,\infty)$
(c) $[1,\infty)$ (d) $(-\infty,1]$

Graph of linear inequality

10. The graph of inequality x>0 is half plane: **09305061**
(a)lower (b) upper
(c) right (d) left

11. The graph of inequality y>0 is half plane: **09305062**
(a) lower (b) upper
(c) right (d) left

12. The graph of inequality x<0 is half plane: **09305063**
(a) lower (b) upper
(c) right (d) left

13. The graph of inequality y<0 is half plane: **09305064**
(a) lower (b) upper
(c) right (d) left

14. Which of the following line does pass through the origin? **09305065**
(a) y=4 (b) y=4x
(c)y=4x+5 (d) $y \ge -2$

Solution region of linear inequality

15. The solution region of inequality x<1 is half plane: **09305066**

(a) Closed right (b) Closed left
(c) open right (d) open left

16. The solution region of inequality $x>1$ is half plane: 09305067

(a) closed right (b) closed left
(c) open right (d) open left

17. The solution region of inequality $x\geq 1$ is half plane: 09305068

(a) Closed right (b) Closed left
(c) open right (d) open left

18. The solution region of inequality $x\leq 2$ is half plane: 09305069

(a) Closed right (b) Closed left
(c) open right (d) open left

19. The solution region of inequality $y\leq 3$ is half plane: 09305070

(a) closed lower (b) closed upper
(c) open lower (d) open upper

20. The solution region of inequality $y\geq 3$ is half plane: 09305071

(a) closed lower (b) closed upper
(c) open lower (d) open upper

Answer Key

1	c	2	a	3	b	4	a	5	d	6	a	7	b	8	d	9	c	10	c
11	b	12	d	13	a	14	b	15	d	16	c	17	a	18	b	19	a	20	b

Q.2 Solve and represent their solutions on real line.

(i) $\dfrac{x+5}{3}=1-x$ 09305072

Solution:

$$\frac{x+5}{3}=1-x \quad \text{\_\_\_\_\_\_ (i)}$$

$$\cancel{3}\times\frac{x+5}{\cancel{3}}=3(1-x)$$

$x+5 = 3-3x$

$x+3x = 3-5$

$4x = -2$

$x = -\dfrac{2}{4}$

$x = -\dfrac{1}{2}$

Check: put $x = \dfrac{-1}{2}$ in equation (i)

$$\frac{-\frac{1}{2}+5}{3} = 1-\left(-\frac{1}{2}\right)$$

$$\frac{\frac{-1+10}{2}}{3} = 1+\frac{1}{2}$$

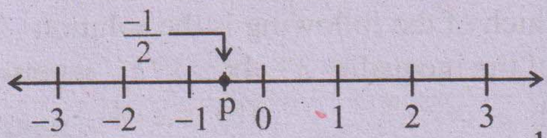
$$\frac{9}{2\times 3}=\frac{2+1}{2}$$

$$\frac{9}{6}=\frac{3}{2}$$

$$\frac{3}{2}=\frac{3}{2} \quad \text{(it is true)}$$

So $x=-\dfrac{1}{2}$ is solution of given equation.

Solution on number line:



Point p on the number line represents $-\dfrac{1}{2}$.

(ii) $\dfrac{2x+1}{3}+\dfrac{1}{2}=1-\dfrac{x-1}{3}$ 09305073

Solution:

$$\frac{2x+1}{3}+\frac{1}{2}=1-\frac{x-1}{3}$$

$$\frac{2x+1}{3}+\frac{1}{2}=1-\frac{x-1}{3} \quad \text{\_\_\_\_\_\_ (i)}$$

$$\frac{2(2x+1)+3}{6}=\frac{3-(x-1)}{3}$$

$$4x+2+3=\frac{(3-x+1)}{3}\times 6$$

$4x+5 = (-x+4)\times 2$

$4x+5 = -2x+8$

$4x+2x = 8-5$

$6x = 3$

$x = \dfrac{3}{6}$ $\boxed{x=\dfrac{1}{2}}$

Check: put $x = \frac{1}{2}$ in equation (i)

$$\frac{2\left(\frac{1}{2}\right)+1}{3}+\frac{1}{2}=1-\frac{\frac{1}{2}-1}{3}$$

$$\frac{1+1}{3}+\frac{1}{2}=1-\frac{\frac{1-2}{2}}{3}$$

$$\frac{2}{3}+\frac{1}{2}=1-\frac{-\frac{1}{2}}{3}$$

$$\frac{4+3}{6}=1+\frac{1}{2\times 3}$$

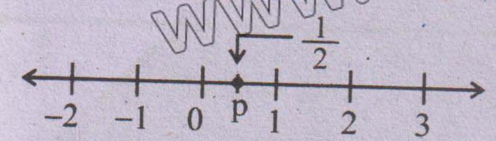
$$\frac{7}{6}=1+\frac{1}{6}$$

$$\frac{7}{6}=\frac{6+1}{6}$$

$$\frac{7}{6}=\frac{7}{6} \quad \text{(It is true)}$$

So $x = \frac{1}{2}$ is solution of given equation.

Solution on number line:



Point p on the number line represents $x = \frac{1}{2}$

(iii) $3x + 7 < 16$ 09305074

Solution

$3x + 7 < 16$

$3x < 16 - 7$

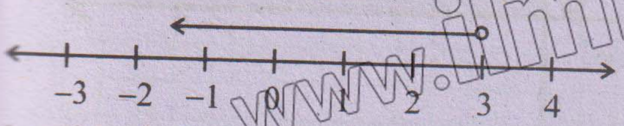
$3x < 9$

$x < \frac{9}{3}$

$\boxed{x < 3}$

The solution of given equality is $(-\infty, 3)$ or $-\infty < x < 3$

Solution on number line



A empty circle on 3 shows that 3 is not included in the solution.

(iv) $5(x-3) \geq 26x - (10x + 4)$ 09305075

Solution

$5(x-3) \geq 26x - (10x + 4)$

$5x - 15 \geq 26x - 10x - 4$

$5x - 15 \geq 16x - 4$

$-15 + 4 \geq 16x - 5x$

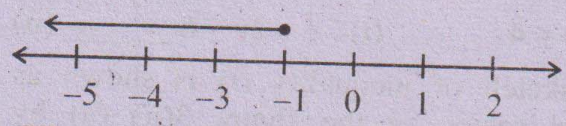
$-11 \geq 11x$

$\frac{-11}{11} \geq x$

$-1 \geq x \Rightarrow \boxed{x \leq -1}$

The given inequality is $(-\infty, -1]$ or $\infty < x \leq -1$.

Solution on number line



A filled circle on −1 shows that −1 is included in the solution.

Q.3 Find the solution region of the following linear equalities:

(i) $3x - 4y \leq 12$; $3x + 2y \geq 3$ 09305076

Solution:

$3x - 4y \leq 12$....(i)

$3x + 2y \geq 3$....(ii)

The sketch of inequality (i) is shown as shaded region in the figure 3(i) (a) by joining the points A(4,0) and B(0,−3)

The sketch of inequality (ii) is shown as shaded region in the figure 3(i) (b) by joining the points C(1,0) and D(0,3/2).

Now the solution region of the given systems of inequalities is the intersection of the graphs indicated in the figure 3(i) (a), and 3 (i) (b) is shown as shaded region in the figure 3(i) (c).

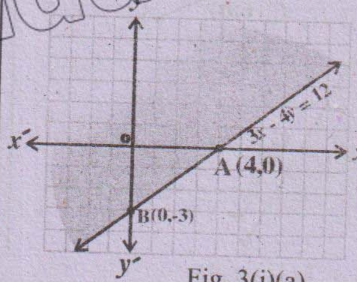


Fig. 3(i)(a)

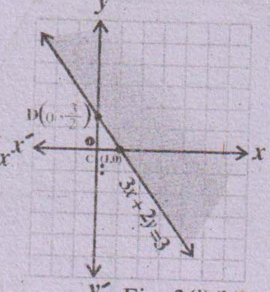


Fig. 3(i)(b)

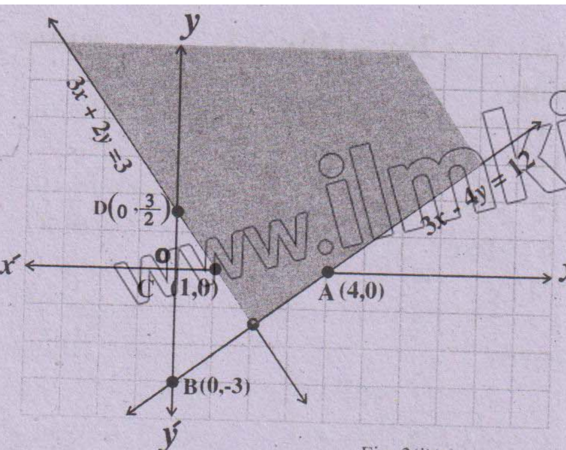


Fig. 3(i)(c)

(ii) $\quad 2x + y \leq 4\ ;\ x + 2y \leq 6$ **09305077**

Solution:

$2x + y \leq 4$ _______ (i) , $x + 2y \leq 6$ _______ (ii)

The sketch of inequality (i) is shown as shaded region in the figure 3(ii) (a) by joining the points A(2,0) and B(0,4).

The sketch of inequality (ii) is shown as shaded region in the figure 3 (ii) (b) by joining the points C(6,0) and D(0,3).

Now the solution region of the given systems of inequalities is the intersection of the graphs indicated in the figure 3(iii) (a), and 3(iii)(b) and is shown as shaded region in the figure 3(ii)(c).

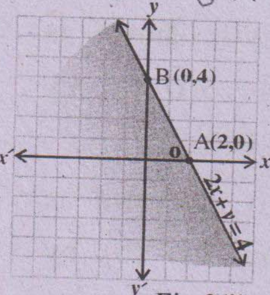


Fig.3(ii) (a)

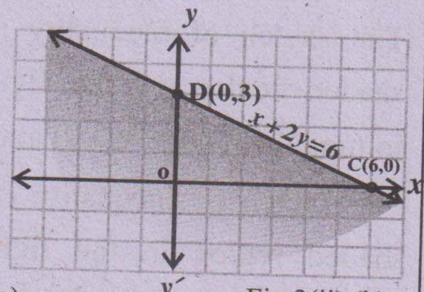


Fig.3(ii) (b)

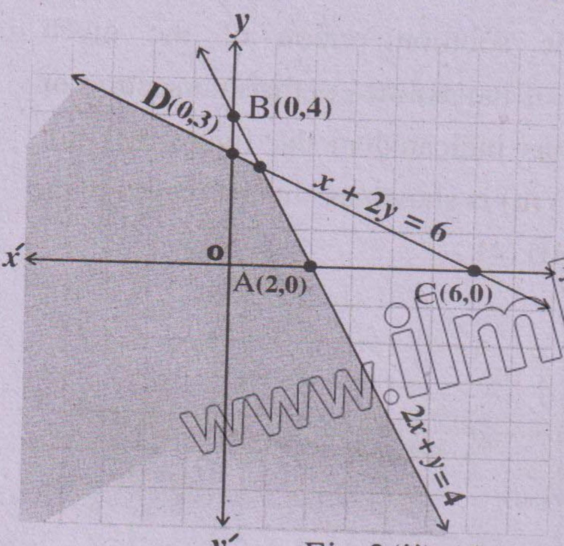


Fig.3(ii) (d)

Q.4 Find the maximum value of $g(x,y) = x + 4y$ subject to constraints. **09305078**

$x + y \leq 4$, $x \geq 0$ and $y \geq 0$.

Solution:

$g(x, y) = x + 4y$

The constraints

$x + y \leq 4$ ______ (i)

$x \geq 0$ _________ (ii)

$y \geq 0$ _________ (iii)

The associated equation of inequality(i) is $x + y = 4$__________ (iv)

Draw the line of equation (iv) using x and y–intercepts A(4,0) and B(0,4). The sketch of graph of inequality (i) is shown as shaded region in figure. 4*a*

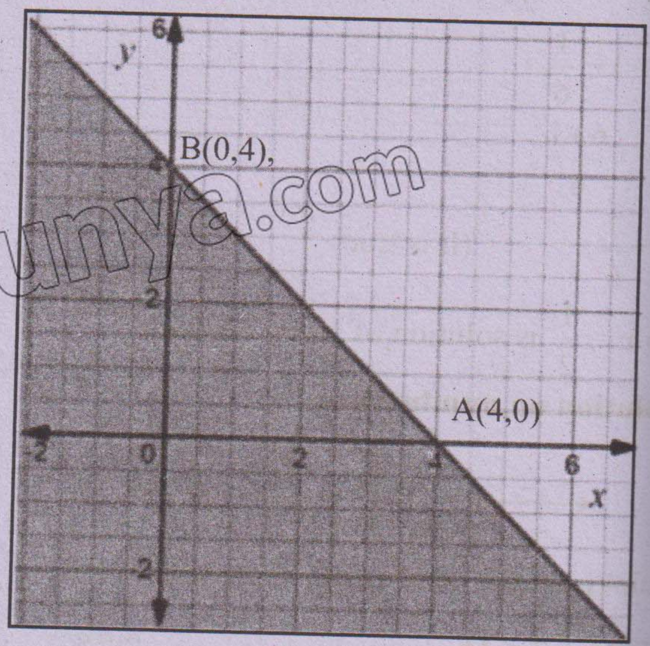


Fig.4(a)

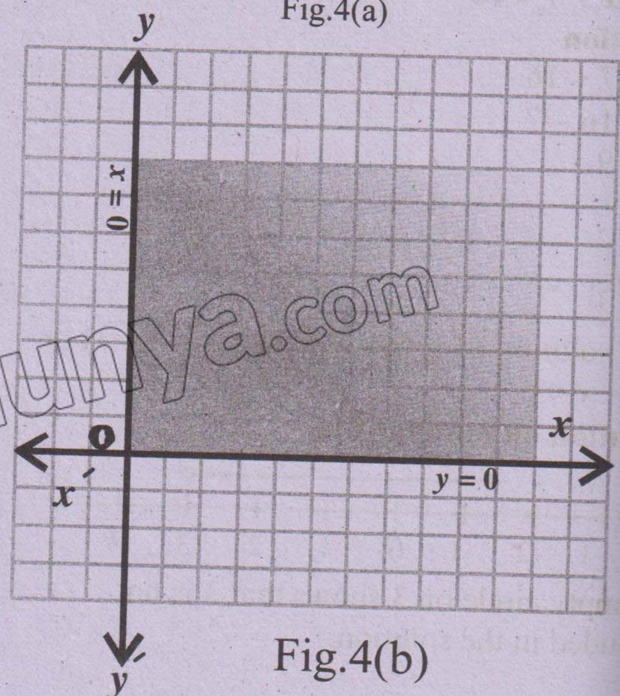


Fig.4(b)

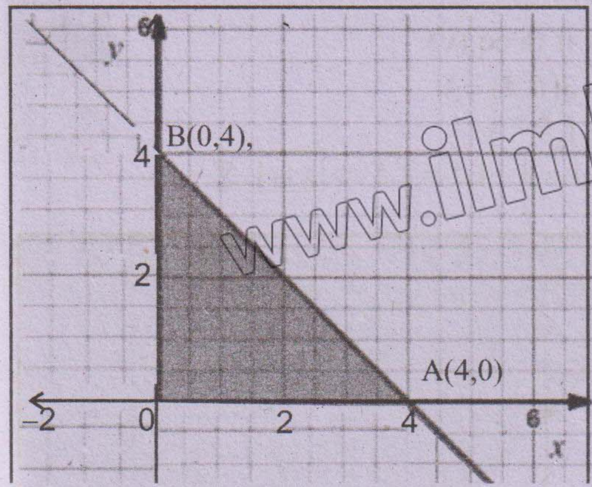


Fig.4(c)

The sketch of inequalities (ii) and (iii) is shown as shaded region in figure 4b. Now, the solution region of given systems of inequalities is the intersection of the graphs indicated in figure 4a and 4b, which is shown as shaded region in figure 4c. We observe that the corner points of solution region are O(0,0), A(4,0), B(0,4).

Now, we find the value of g(x,y) at the corner points.

Corner points	$g(x,y) = x+4y$
0(0,0)	$f(0,0) = 0+4(0) = 0$
A(4,0)	$f(4,0) = 4+4(0) = 4$
B(0,4)	$f(0,4) = 0+4(4) = 16$

Thus the given function is maximum at the corner point (0, 4).

Q.5 Find the minimum value of $f(x,y) = 3x + 5y$ subject to constraints.
$x + 3y \geq 3, \quad x + y \geq 2, \; x \geq 0, \; y \geq 0.$ 09305079

Solution:

$x + 3y \geq 3,$ .....(i)

$x + y \geq 2,$ .....(ii)

$x \geq 0, \; y \geq 0$......(iii)

The associated equation of inequality (i) is $x + 3y = 3$ and it can be written in the form $\frac{x}{3} + \frac{y}{1} = 1$. This line intersects the x-axis and y-axis at point A(3,0) and B(0,1) respectively. The sketch of inequality (i) is shown as shaded region in the Fig.5(a).

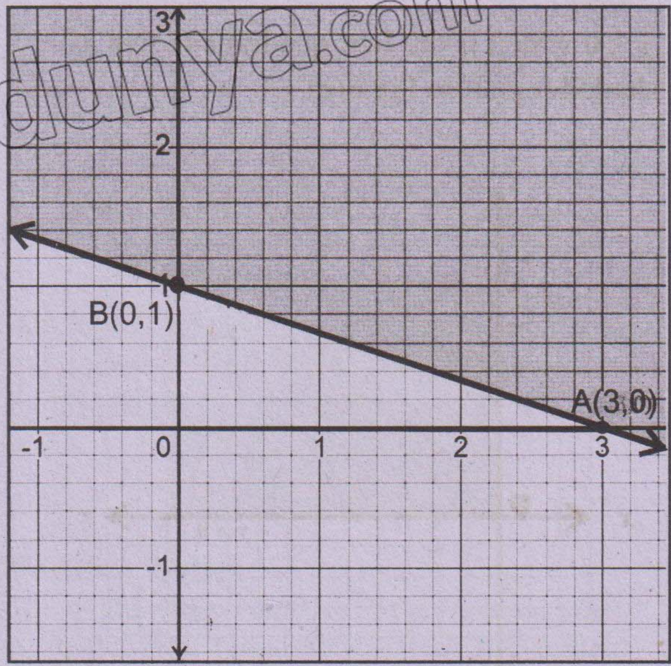
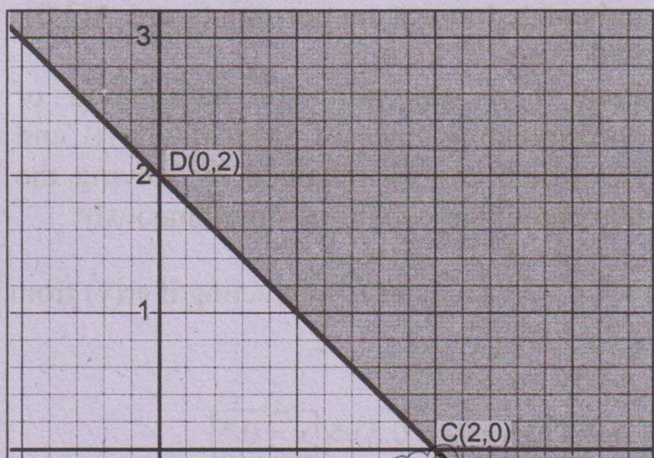


Fig. 5(a)

The associated equation of inequality (ii) is $x + y = 2$ and it can be written in the form $\frac{x}{2} + \frac{y}{2} = 1$. This line intersects the x-axis and y-axis at point C(2,0) and D(0,2) respectively. The sketch of inequality (i) is shown as shaded region in the Fig.5(b).



The graph of the inequalities in (iii) is the intersection region of graphs of $x \geq 0$ and

$y \geq 0$ which is the first quadrant shown as shaded region in Fig.6(c).

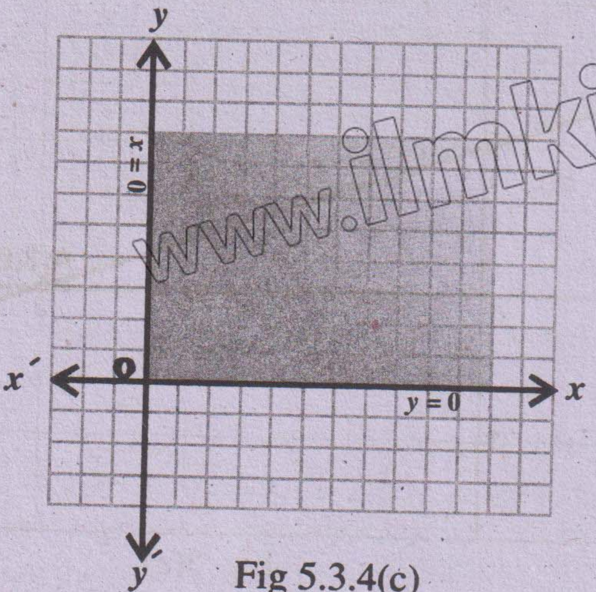


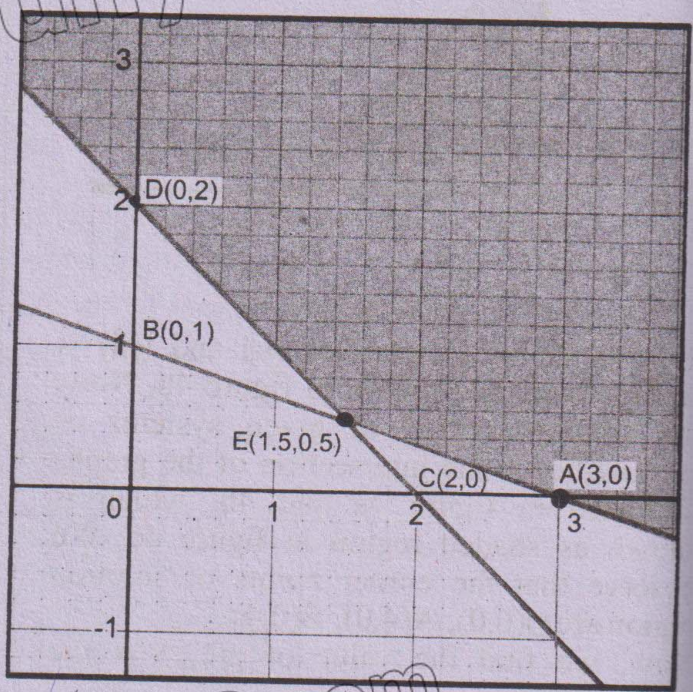
Fig 5.3.4(c)

Now the solution region of the given system of inequalities is the intersection of the graphs indicated in the Fig.5(a) ,5(b) and 5(c)
which is shown as shaded region in the Fig.5(d)
We observe that solution region is unbounded and it corner points are A(3,0), D(0,2) and E(1.5,0.5).
Point E is the point of intersection of corresponding lines of inequalities (i) and (ii) which can be obtained by solving the corresponding equations simultaneously.

$x+3y=3$............(iv)
$x+y=2$ ....(v) Subtracting if er(v) from (iv).

$(x+3y)-(x+y)=3-2$
$x+3y-x-y=1$
$2y=1 \Rightarrow y=\frac{1}{2}=0.5 \Rightarrow \boxed{y=0.5}$

Put it in eq.(v)
$x+3(0.5)=3$
$x+1.5=3$
$\Rightarrow x=3-1.5=1.5 \Rightarrow \boxed{x=1.5}$



Thus point of intersection is E(1.5,0.5).
Now, we find the values of $f(x,y) = 3x+5y$ at corner points.

Corner points	$f(x,y)=3x+5y$
A(3,0)	$f(3,0)=3(3)+5(0)$ $=9+0=9$
D(0,2)	$f(0,2)=0(3)+5(2)$ $=0+10=10$
E(1.5,0.5)	$f(1.5,0.5)=3(1.5)+5(0.5)$ $=4.5+2.5=7$

We observe that the value of function is minimum 7 at corner point E(1.5,0.5).