UNIT 06

Trigonometry

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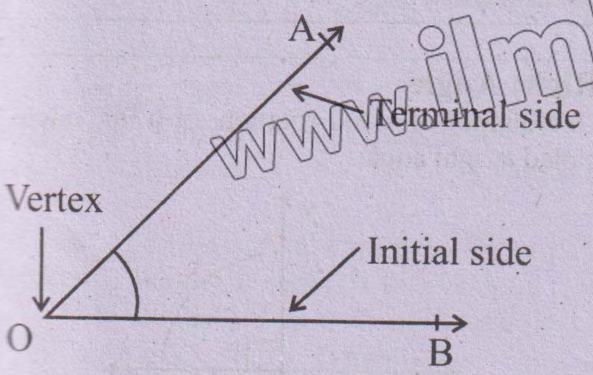
Introduction

Trigonometry is a branch of mathematics that deals with the relationships between the angles and sides of a triangle, especially right-angled triangle. It plays a vital role in various fields such as physics, engineering, architecture and astronomy.

Angle

A plane figure which is formed by two rays sharing a common end point is called an angle. The two rays are known as the **sides** of the angle and the amount of rotation or opening between these rays is called an angle. In the figure $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are rays and angle is AOB. Written as $\angle AOB$ or $A\hat{O}B$.



Identifying Angles in Standard Position (Degrees and Radians)

The angle is said to be in standard position if:

(a) Its **vertex** is located at the origin of the coordinate plane.
(b) One of its rays (the **initial side**) lies along the positive x-axis.
(c) The other ray (the **terminal side**) determines the direction of the angle.

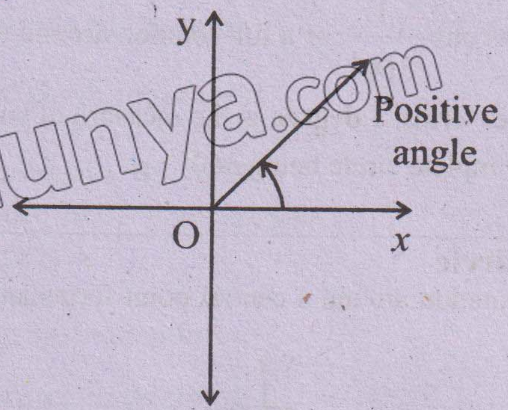
Types of angles are:

- Acute angle $0 < \theta < 90°$
- Obtuse angle $90° < \theta < 180°$
- Right angle $\theta = 90°$
- Straight angle $\theta = 180°$
- Reflex angle $180° < \theta < 360°$

An angle is measured from the initial side to the terminal side. It is usually represented by Greek symbols θ, α, β, γ etc.

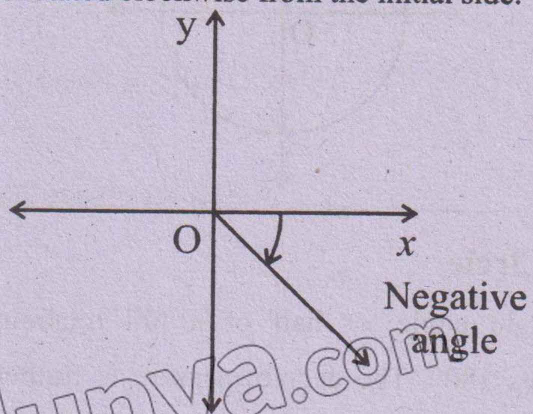
Positive angles

The angle will be positive if the terminal side is rotated counterclockwise from the initial side.



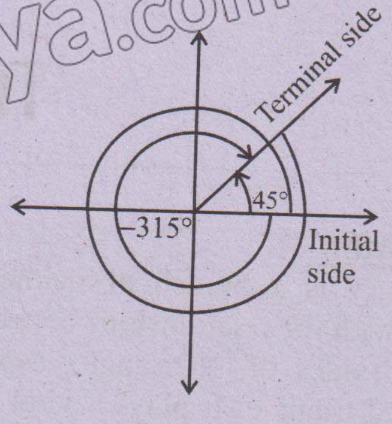
Negative angles

The angle will be negative if the terminal side is rotated clockwise from the initial side.



Co-Terminal Angles

Co- terminal angles are angles that share the same initial side and terminal side in standard position, but they may have different measures. These angles differ by a multiple of 360° or 2π rad. For example, 45°, 405° and −315° are co-terminal angles because 405° = 45° + 360° and −315° = 45° − 360°.



Degree Measurement

Degree

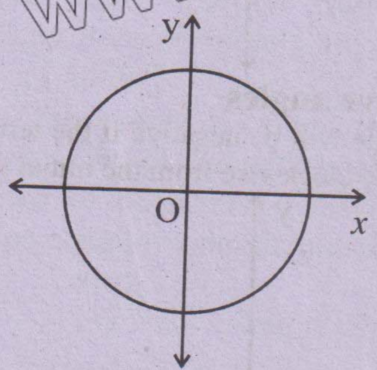
A degree (°) is a unit of measurement of angles. It represents one $\frac{1}{360°}$ of a full rotation around a point.

In simpler terms, a **degree** is the measure of an angle, with a complete circle being 360°.

Why 360° Historically? The choice of 360° to divide a circle dates back to the **Babylonians**, who used a base-60 number system (sexagesimal system). They were among the first to formalize the concept of angle measurement, and 360 was chosen likely because it is a highly composite number (it can be divided by 2, 3, 4, 5, 6, 9, 10, 12, 15, and more), making calculations easier. This system persisted throughout ancient times and degrees became entrenched in various cultures and mathematical traditions.

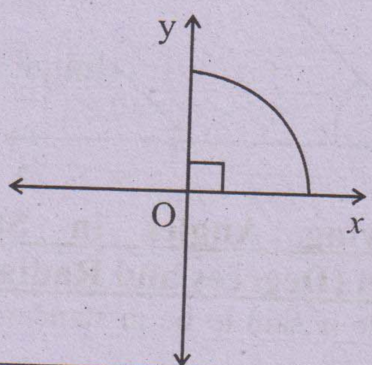
Full Circle

A full rotation around a central point forms an angle of 360°.



Right Angle

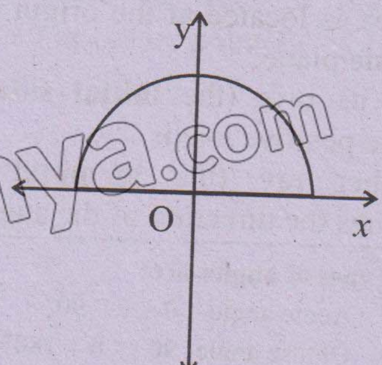
One-quarter of a full rotation, or a 90° angle, is called a right angle.



Half Circle

A straight angle, or half of a full rotation, measures 180°. The degree measure is further divided into minutes (′) and seconds (″).

1° = 60′ (60 minutes)



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$1' = 60''$ (60 seconds)
$1° = 3600''$ (60×60 seconds)

Converting Degrees to Minutes and Seconds

To convert decimal degrees to degrees, minutes, and seconds (DMS), follow the steps:

- Separate the whole number part (degrees) of the decimal.
- Multiply the decimal part by 60 to get the minutes.
- The whole number part of the result is the minutes. Multiply the decimal part of the minutes by 60 to get the seconds.

Example 1: Convert 73.12° to degrees, minutes, and seconds. **09306001**

Solution:

Degrees: The whole number part is 73°.

Minutes: Take the decimal part (0.12) and multiply by 60: $0.12 \times 60 = 7.2$. The whole number part is 7, so it's 7 minutes.

Seconds: Now take the decimal part (0.2) and multiply by 60: $0.2 \times 60 = 12$. So, the seconds are 12 seconds.

Final result: $73°7'\,12''$.

Example 2: Convert 109.42° to degrees, minutes, and seconds. **09306002**

Solution:

Degrees: The whole number part is 109.

Minutes: Take the decimal part (0.42) and multiply by 60: $0.42 \times 60 = 25.2$. The whole number part is 25, so it's 25 minutes.

Seconds: Now take the decimal part (0.2) and multiply by 60: $0.2 \times 60 = 12$. So, the seconds are 12 seconds.

Final result: **109° 25' 12''**

Converting from Degrees, Minutes, and Seconds to Decimal Degrees

To convert from degrees, minutes, and seconds (DMS) to decimal degrees, follow the steps:

- Keep the degrees as they are.
- Convert minutes to decimal degrees: Divide the number of minutes by 60.
- Convert seconds to decimal degrees: Divide the number of seconds by 3600.
- Add all the values together.

Example 3 Convert 45°45'45" to decimal degrees. **09306003**

Solution:

Degrees: Keep 45°.

Minutes to decimal: $\frac{45}{60} = 0.75°$

Seconds to decimal: $\frac{45}{3600} = 0.0125°$

Add them together:
$45° + 0.75° + 0.0125° = 45.7625°$

Final result: **45.7625°**

Example 4: Convert 94° 27' 54" to decimal degrees. **09306004**

Solution:

Degrees: Keep 94°

Minutes to decimal: $\frac{27}{60} = 0.45°$

Seconds to decimal: $\frac{54}{3600} = 0.015°$

Add them together: $94°+0.45°+0.015°=95.465°$

Final result: **94.465°**

Circular Measure (Radian)

There is another system of angular measurement called circular system.

The **radian**, denoted by the symbol **rad**, is the unit of angle.

A **radian** is a unit of angular measure in mathematics, particularly in trigonometry. It is defined as, "the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle". Unlike degrees, which are based on dividing a circle into 360 parts, the radian is inherently related to the circle's geometry and arc length.

Historical Background of the Radian

The concept of radian measure, as we know it today, was first formalized by mathematicians in the 18th century, but the principles behind it had been understood much earlier.

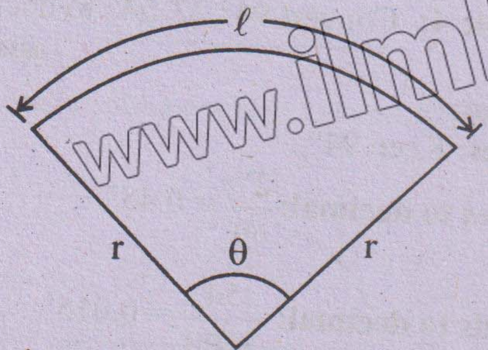
The word "radian" comes from the radius of a circle, as the radian is fundamentally related to the ratio between the arc length and the radius.

The first known use of the term **radian** in the context of angular measurement was by Scottish mathematician **James Thomson** in 1873. His brother, **William Thomson,** also known as Lord Kelvin, was a prominent physicist and both were influential in establishing radians as a standard unit.

Although the name and formalization of radians occurred in the 19th century, the relationship between arc length and angle (which is the core concept behind radians) was understood by ancient Greek mathematicians like **Euclid** and **Archimedes.**

A circle with a radius r, and an arc length ℓ is equal to the radius of the circle, then the angle θ subtended by that arc is 1 radian:

$$\theta = \frac{r}{r} = 1 \text{ radian}$$



A complete circle has an arc length equal to the circumference, $2\pi r$, so the angle subtended by the entire circle (the full rotation) is 2π radians. This means:

- One full revolution of a circle is 2π radians, or 360°.
- Therefore, 1 radian is approximately 57.2958°.

Conversion between degree and radian

Radians to Degrees: $1 \text{ rad} = \frac{180°}{\pi}$ degrees

Degrees to Radians: $1° = \frac{\pi}{180°}$ rad

Example 5: Convert radians to degree

(i) $\frac{5\pi}{3}$ rad 09306005 (ii) $\frac{7\pi}{6}$ rad 09306006

(iii) $\frac{11\pi}{6}$ 09306007

(iv) 1.2 rad

Solution:

(i) $\frac{5\pi}{3} \text{ rad} = \frac{5\pi}{3} \times \frac{180°}{\pi} = 300°$

$(1 \text{ rad} = \frac{180°}{\pi})$

(ii) $\frac{7\pi}{6}$ rad

$= \frac{7\pi}{6} \times \frac{180°}{\pi} = 210°$

(iii) $\frac{11\pi}{6}$ rad

$= \frac{11\pi}{6} \times \frac{180°}{\pi} = 330°$

(iv) $1.2 \text{ rad} = 1.2 \times \frac{180°}{\pi} = 68.75°$

Example 6: Convert degree to radian

(i) 15° 09306008 (ii) 75° 09306009

(iii) 315° 09306010 (iv) 15° 15′ 09306011

(i) $15° = 15 \times \frac{\pi}{180} = \frac{5\pi}{12}$ rad or 0.262 rad

(ii) $75° = 75 \times \frac{\pi}{180} = \frac{5\pi}{12}$ rad or 1.309 rad

(iii) $315° = 315 \times \frac{\pi}{180} = \frac{7\pi}{4}$ rad or 5.498 rad

(iv) $15°\ 15' = 15° + \frac{15}{60} = 15.25° = 15.25 \times \frac{\pi}{180}$ or 0.266 rad

Turns	0 turn	$\frac{1}{12}$ turn	$\frac{1}{8}$ turn	$\frac{1}{6}$ turn	$\frac{1}{4}$ turn	$\frac{1}{2}$ turn	1 turn
Radians	0 rad	$\frac{\pi}{6}$ rad	$\frac{\pi}{4}$ rad	$\frac{\pi}{3}$ rad	$\frac{\pi}{2}$ rad	π rad	2π rad
Degrees	0°	30°	45°	60°	90°	180°	360°

Arc Length and Area of sector

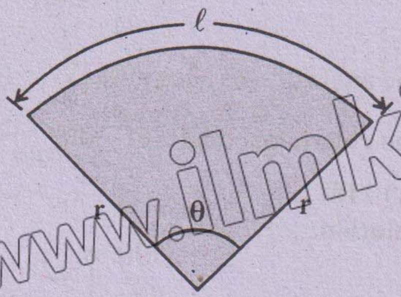
If r is the radius and θ(rad) is the angle subtended by the Arc of length' ℓ, then

Arc length $= \ell = r\theta$ and

Area of sector $= A = \frac{1}{2}r^2\theta$

Proof:

We know that:



$\ell = \frac{\theta}{360°} \times 2\pi r$

$\ell = \frac{\theta}{2\pi} \times 2\pi r$

$\ell = r\theta$

Proof: We know that

$A = \frac{\theta}{360°} \times \pi r^2$

$A = \frac{\theta}{2\pi} \times \pi r^2 \qquad (2\pi \text{ radians} = 360°)$

$A = \frac{1}{2}r^2\theta$

Hence arc length, $\ell = r\theta$ and area of sector

$A = \frac{1}{2}r^2\theta$

Example 7: Find the arc length of a sector with radius $r = 10$ cm and central angle $\theta = 60°$.

09306012

Solution

Convert $\theta = 60°$ to radians:

$\theta = 60° \times \frac{\pi}{180°} = \frac{\pi}{3}$ radians.

$\ell = r\theta = 10 \times \frac{\pi}{3} \approx 10.47$ cm

The arc length is approximately 10.47 cm

Example 8: Find the area of a sector with radius r = 8 cm and central angle θ = 45°.

09306013

Solution: Convert θ = 45° to radians:

$\theta = 45° \times \frac{\pi}{180°} = \frac{\pi}{4}$ radians.

$A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times \frac{\pi}{4} = \frac{1}{8} \times 64\pi \approx 25.12\ \text{cm}^2.$

The area of the sector is approximately $25.12\ \text{cm}^2$.

Example 9: If arc length of a sector of radius 7 cm is 11 cm, find the angle subtended by the arc in radians and degrees.

09306014

Solution

$r = 5$ cm ; $\ell = 11$ cm, ; $\theta = ?$

$\therefore \ \ell = r\theta$

$11 = 5\theta \quad \Rightarrow \theta = \frac{11}{5} = 2.2$ rad

$\theta = 2.2 \times \frac{180}{\pi} = 126.1°$

Thus, the angle subtended by the arc in radians is
2.2 rad and degrees is 126.1°

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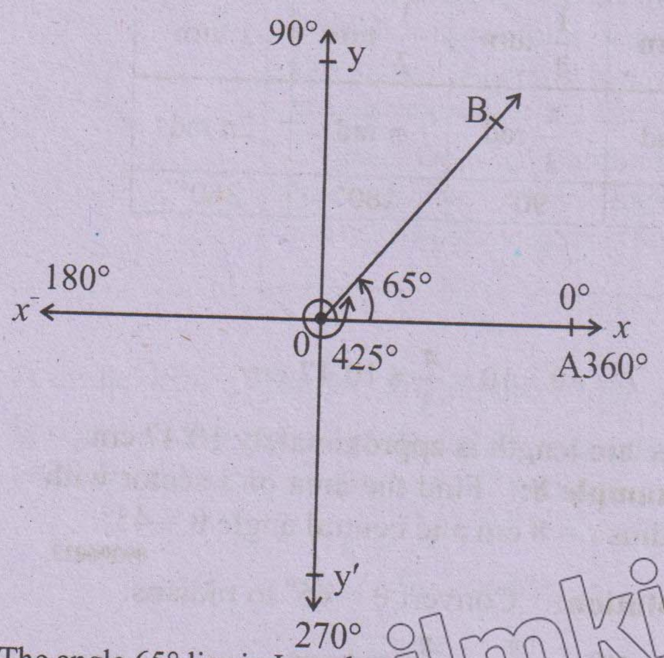
Exercise 6.1

Q.1 Find in which quadrant the following angles lie. Write a co-terminal angle for each.

(i) 65° 09306015

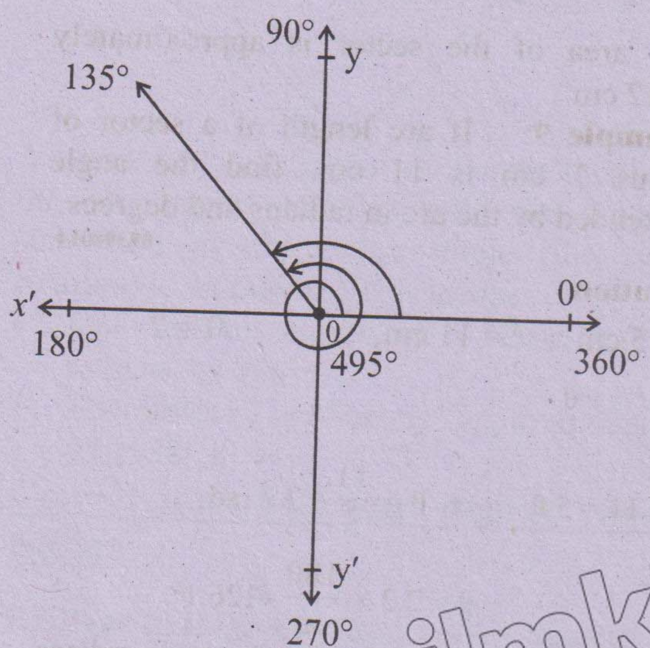
Solution



The angle 65° lies in I quadrant.
The coterminal angle of 65° = 65°+360° = 425°

(ii)135° 09306016

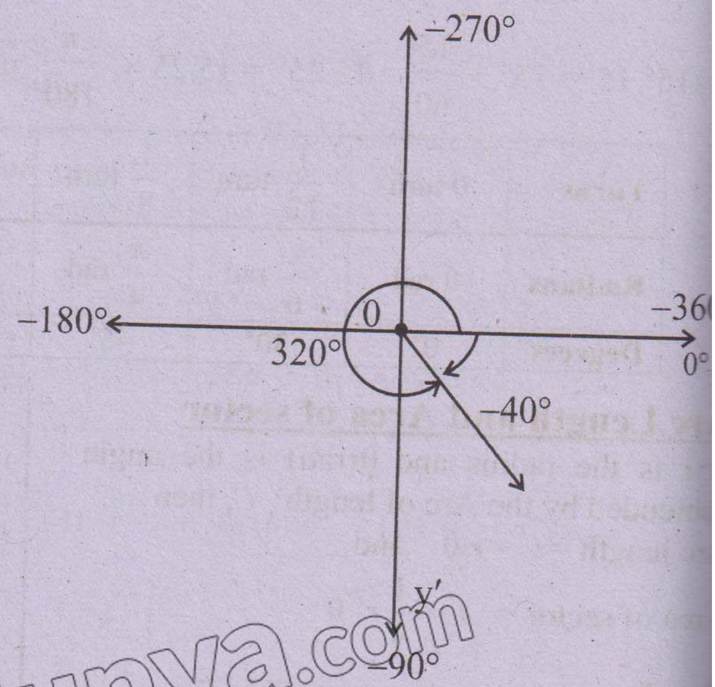
Solution:



The angle of 135° lies in Q.II.
The coterminal angle of 135°=135° +360°=495°

(iii) –40° 09306017

Solution:

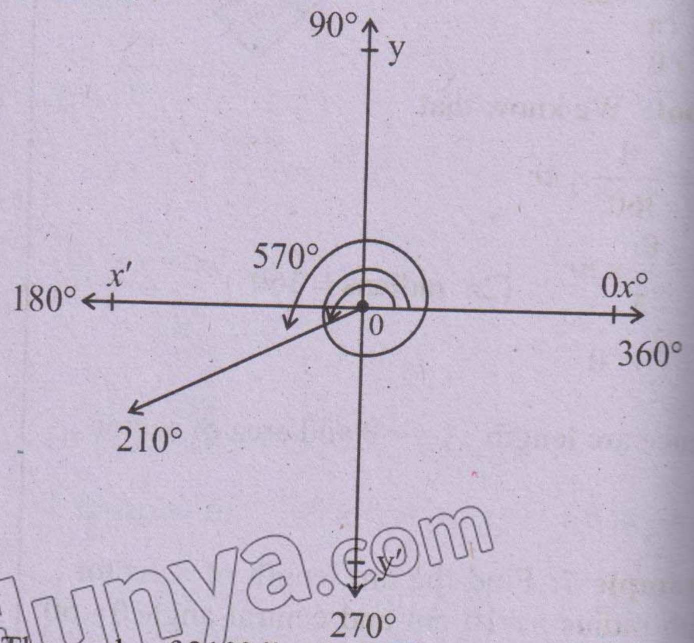


The angle of –40° lies in Q.IV.

The coterminal angle of –40° = –40°+360°=320°

(iv) 210° 09306018

Solution:



The angle of 210° lies in Q.III.
The coterminal angle of 210°
= 210° +360° = 570°

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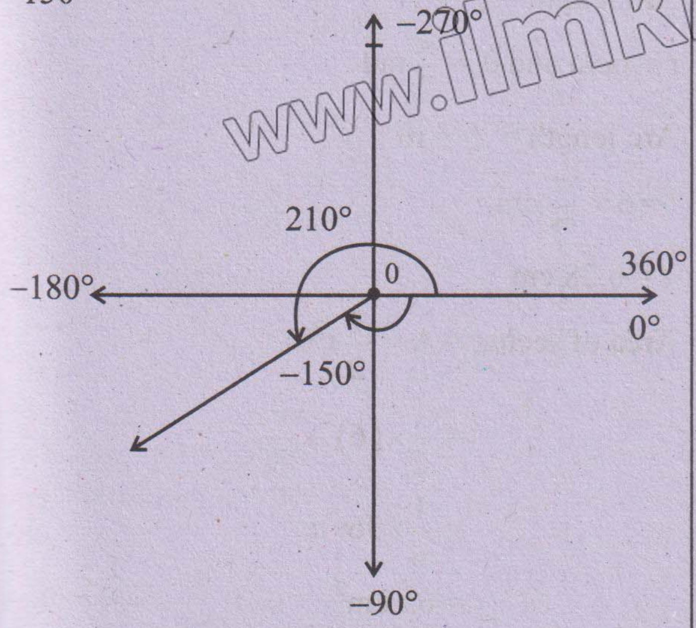
(v) –150° 09306019

Solution:

–150°



The angle of–150° lies in Q.III.

The coterminal angle of –150°

= –150°+360° = 210°

Q.2 Convert the following into to degrees, minutes, and seconds.

(i) 123.456° 09306020

Solution:

$123.456° = 123°+0.456°$

$= 123°+(0.456\times60)'$

$= 123°+27.36'$

$= 123°+27'+ 0.36'$

$= 123°+27'+ (0.36\times60)''$

$= 123°+27'+21.6''$

$= 123°27'21.6''$

(ii) 58.7891° 09306021

Solution:

$58.7891° = 58°+0.7891°$

$= 58°+(0.7891\times60)'$

$= 58°+47.346'$

$= 58°+47'+0.346'$

$= 58°+47'+(0.346\times60)''$

$= 58°+47'+21''$

$= 58°47'21''$

(iii) 90.5678°

Solution:

$= 90.5678° = 90°+0.5678°$

$= 90°+(0.5678\times60)'$

$= 90°+34.068'$

$= 90°+34'+0.068'$

$= 90°+34'+(0.068\times60)''$

$= 90°+34'+4.08''$

$= 90°34'4.08''$

Q.3 Convert the following into decimal degrees.

(i) 65° 32' 15" 09306022

Solution

65° 32' 15"

$= 65°+\left(\frac{32}{60}\right)^{o}+\left(\frac{15}{3600}\right)^{o}$

$= 65°+0.5333°+0.0042°$

$= 65.5375°$

(ii) 42° 18' 45" 09306023

Solution

42° 18' 45"

$= 42°+\left(\frac{18}{60}\right)^{o}+\left(\frac{45}{3600}\right)^{o}$

$= 42°+0.3°+0.0125°$

$= 42.3125°$

(iii) 78° 45' 36" 09306024

Solution:

78° 45' 36"

$= 78°+\left(\frac{45}{60}\right)^{o}+\left(\frac{36}{3600}\right)^{o}$

$= 78°+0.75°+0.01° = 78.76°$

Q.4 Convert the following into radian.

(i) 36° 09306025

Solution:

$36° = 36 \times 1°$ $\quad (\because 1° = \frac{\pi}{180}\text{rad})$

$= 36\times\frac{\pi}{180}\text{rad}$

$= \frac{\pi}{5}\text{rad}$

(ii) 22.5° 09306026

Solution:

$22.5° = 22.5\times1°$

$= 22.5 \times \frac{\pi}{180}\text{rad}$

$= \frac{\pi}{8}$ rad

(iii) 67.5° 09306027

Solution

$67.5° = 67.5 \times 1°$

$= 67.5 \times \frac{\pi}{180}$ rad $= \frac{3\pi}{8}$ rad

Q.5 Convert the following into degrees.

(i) $\frac{\pi}{16}$ rad 09306028

Solution:

$\frac{\pi}{16}$ rad

$= \left(\frac{\pi}{16} \times \frac{180}{\pi}\right)^{\circ}$

$= 11.25°$

(ii) $\frac{11\pi}{5}$ rad 09306029

Solution:

$\frac{11\pi}{5}$ rad

$= \frac{11\pi}{5}$ rad

$= \left(\frac{11\pi}{5} \times \frac{180}{\pi}\right)^{\circ}$

$= 396°$

(iii) $\frac{7\pi}{6}$ rad 09306030

Solution:

$\frac{7\pi}{6}$ rad

$= \left(\frac{7\pi}{6} \times \frac{180}{\pi}\right)^{\circ}$

$= 210°$

Q.6 Find the arc length and area of a sector with:

(i) r = 6cm and $\theta = \frac{\pi}{3}$ rad 09306031

Solution:

Area of sector = A = ?

Arc length = ℓ = ?

r = 6cm and $\theta = \frac{\pi}{3}$ rad

Arc length = $\ell = r\theta$

$\ell = 6 \times \frac{\pi}{3}$ cm

$\ell = 6.28$ cm

Area of sector = A = $\frac{1}{2} r^2\theta$

$= \frac{1}{2} \times (6)^2 \times \frac{\pi}{3}$

$= \frac{1}{6} \times 36 \times \pi$

$= 6\pi\ cm^2$

$= 18.85 cm^2$

(ii) $r = \frac{4.8}{\pi}$ cm and central angle $\theta = \frac{5\pi}{6}$ radians. 09306032

Solution:

$r = \frac{4.8}{\pi}$ cm and $\theta = \frac{5\pi}{6}$ rad.

Arc length = $\ell = r\theta$

$\ell = \frac{4.8}{\pi} \times \frac{5\pi}{6}$

$= 4$cm

Area of sector = A = $\frac{1}{2} r^2\theta$

$= \frac{1}{2}\left(\frac{4.8}{\pi}\right)^2 \frac{5\pi}{6}$

$= \frac{1}{12} \times \frac{23.04}{\pi^2} \times 5\pi$

$= \frac{1}{12} \times \frac{115.2}{\pi}$

$= 3.056\ cm^2$

Q.7 If the central angle of a sector is 60° and the radius of the circle is 12 cm, find the area of the sector and the percentage of the total area of the circle it

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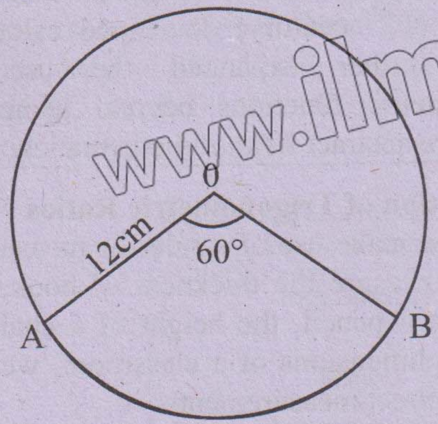
represents. 09306033

Solution



Radius = r = 12cm

Central angle = θ = 60°

$\theta = 60 \times \frac{\pi}{180}$ rad $= \frac{\pi}{3}$ rad

Area of sector $= A_1 = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times (12cm)^2 \times \frac{\pi}{3}$

$= \frac{1}{6} \times 144\pi$

$= 24\pi \qquad (\because \pi \approx 3.1416)$

$= 24 \times 3.1416$

$= 75.39 \approx 75.4 cm^2$

Area of circle $= A = \pi r^2$

$= 3.1416 \times (12cm)^2$

$= 3.1416 \times 144cm^2$

$= 452.39 \approx 452.4.cm^2$

Percentage of sector area $= \frac{A_1}{A} \times 100\%$

$= \frac{75.4}{452.4} \times 100\%$

$= 16.67\%$

Q.8 Find the percentage of the area of sector subtending an angle $\frac{\pi}{8}$ radians.

09306034

Solution:

Angle of sector $= \theta = \frac{\pi}{8}$ rad

$= \frac{\pi}{8}$ rad.

Let radius of circle be r units

Now area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2 \times \frac{\pi}{8}$ square unit

$= \frac{\pi}{16}r^2$ square unit

Area of circle $= \pi r^2$ square unit

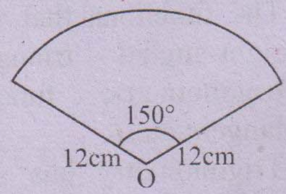
Percentage $= \frac{\text{Area of sector}}{\text{Area of circle}} \times 100\%$

$= \frac{\pi}{16}r^2 \div \pi r^2 \times 100\%$

$= \frac{\pi}{16}r^2 \times \frac{1}{\pi r^2} \times 100\%$

$= \frac{1}{16} \times 100\% = 6.25\%$

Q.9 A circular sector of radius r =12cm has an angle of 150°. This sector is cut out and then bent to form a cone. What is the slant height and radius of the base of cone.



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Solution:

Radius of sector = r = 12 cm

Angle of sector = 150°

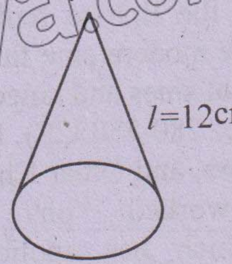
$= 150 \times \frac{\pi}{180}$ rad

$= \frac{5\pi}{6}$ rad

(i) The radius of sector is slant height of cone, so

Slant height of cone $= \ell = 12$ cm

Also



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(ii) We know that

The area of sector = curved surface area of cone ….. (i)

Now area of sector = $\frac{1}{2}r^2\theta$

$$= \frac{1}{2}(12)^2\frac{5\pi}{6}$$

$$= \frac{1}{2}\times 144\times\frac{5\pi}{6}$$

$$= 60\pi\ cm^2$$

Let radius of cone = R = ?

Curved surface area of cone = $\pi R\ell$

$\because$ using eq. (i)

$\Rightarrow\ \pi R\ell = 60\pi$

$$R = \frac{60\cancel{\pi}}{\cancel{\pi}\ell}$$

$$R = \frac{60}{12} \qquad (\because \ell = 12cm)$$

$$R = 5\,cm$$

Trigonometric Ratios

The functions that relate angles to side in right-angled triangle are known as trigonometric functions (sine, cosine, tangent etc.)

Trigonometry has since been extensively used in various scientific disciplines such as physics (especially wave theory) engineering, and computer graphics.

History of sine, Cosine and tangent

Hipparchus of Nieaea (c. 190-120 BC) is considered the "father of trigonomety." He was the first to compile a trigonometric table for solving problems related to astronomy, using chord functions. Hipparchus divided a circle into 360 degrees and used this system for measuring angles.

In Islamic golden age, Al-Battani (e. 88-929 CE) was among the first to replace chord functions with the modern sine function and calculated tables of sines and tangents.

Al-Khwarizimi (c. 780-850 CE), known for his work in algebra, and Omar Khayyam (c. 104-1131CE) worked on spherical trigonometry, which has applications in astronomy.

Isaac Newton and Gottfried Wilhelm Leibniz (17th century) developed calculus, which further expanded the use of trigonometric functions beyond geometry into more abstract field of mathematics.

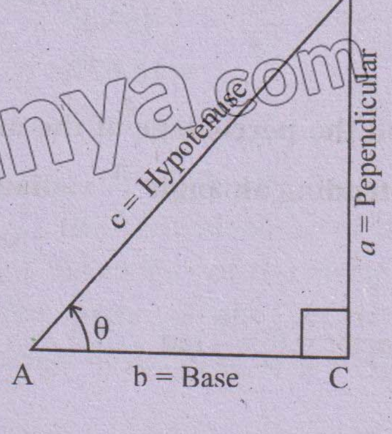
Application of Trigonometric Ratios

When we make use of a ruler or measuring tape to measure the thickness of book, the length of a pencil, the height of a chair or table or dimensions of a classroom, we are making direct measurements.

In some cases, it is not possible to obtain direct measurements, because these are difficult and dangerous. For example, it is difficult to climb upon a flag pole to measure its height. To measure the height of a cliff is also difficult and dangerous. These problems can be solved by indirect measurement with the help of trigonometry. For indirect measurements of distance or height it is very much useful. It also plays an important role in the field of surveying, navigation, , engineering and many other branches of physical sciences. We make use of these concepts of trigonometry to solve many of the problems in these fields.

Trigonometric ratios of an acute angle

The trigonometric ratios are applied to acute angle in a right-angled triangle, but the concepts extend to angles greater than 90° and are widely used in many areas of mathematics and science.



Let us consider a right-angled triangle ACB with respect to an angle θ (theta) = m∠CAB with m∠ACB = 90°. In the triangle ACB, the side BC is called perpendicular, which is opposite to an angle 'θ'.

The side AC is called the base and the side AB is called the hypotenuse. Let m $\overline{BC}$ = a, m $\overline{AC}$ = b and m $\overline{AB}$ = c. For this right angled triangle ACB, the trigonometric ratios of an angle "θ" defined as:

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a}{c} \quad : \quad \operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{c}{a}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{b}{c} \quad : \quad \sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{c}{b}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{a}{b} \quad : \quad \cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{b}{a}$$

The six trigonometric ratios described with reference to a right-angled triangle ACB are: sine (sin), cosine (cos), tangent(tan), cosecant (cosec), secant (sec) and cotangent (cot).

We note that: $\tan\theta = \frac{a}{b}$

$= \frac{a/c}{b/c}$ (Dividing by c)

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Similarly, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

We note that:

(i) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ (ii) $\sec\theta = \frac{1}{\cos\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta}$

Trigonometric ratios of complementary angles

We consider a right-angled triangle ACB, in which m∠A = θ, m∠C = 90° then, m∠B = 90° − θ. Using the trigonometric ratios of ∠B, we get.

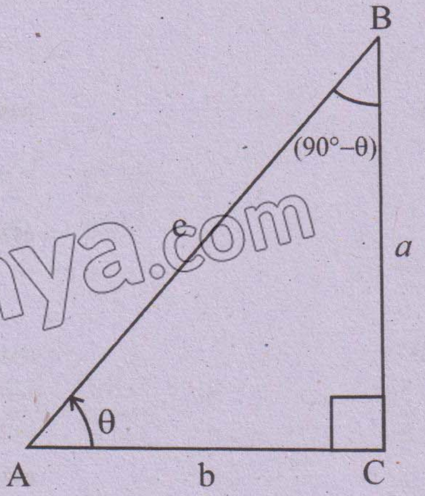
$$\text{Sin } m\angle B = \sin(90°-\theta) = \frac{m\overline{AC}}{m\overline{AB}} = \frac{b}{c}$$

…(i)

Using ratios of ∠A, we get

$$\text{Cos } m\angle A = \cos\theta = \frac{m\overline{AC}}{m\overline{AB}} = \frac{b}{c} \quad \ldots \text{(ii)}$$

From (i) and (ii), we get,



$\boxed{\sin(90°-\theta)=\cos\theta}$

Similarly, we have

$$\cos(90°-\theta)=\sin\theta\ ;\ \tan(90°-\theta)=\cot\theta;\ \cot(90°-\theta)=\tan\theta$$
$$\sec(90°-\theta)=\operatorname{cosec}\theta\ ;\ \operatorname{cosec}(90°-\theta)=\sec\theta$$

Exercise 6.2

Q.1 For each of the following right-angled triangles, find the trigonometric ratios

(i) $\sin\theta$ (ii) $\cos\theta$ (iii) $\tan\theta$

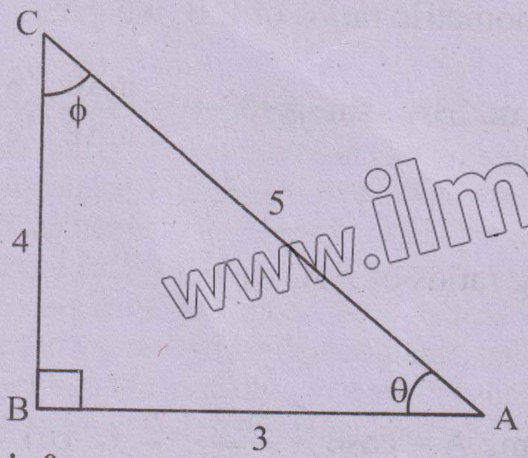
(iv) $\sec\theta$ (v) $\operatorname{cosec}\theta$ (vi) $\cot\phi$

(vii) $\tan\phi$ (viii) $\operatorname{cosec}\phi$ (ix) $\sec\phi$

(x) $\cos\phi$

Solution:

(a)



(i) sin θ 09306036

$\sin\theta=\dfrac{4}{5}$

(ii) cos θ 09306037

$\cos\theta=\dfrac{3}{5}$

(iii) tan θ 09306038

$\tan\theta=\dfrac{4}{3}$

(iv) sec θ 09306039

$\sec\theta=\dfrac{5}{3}$

(v) cosec θ 09306040

$\operatorname{cosec}\theta=\dfrac{5}{4}$

(vi) cot ϕ 09306041

$\cot\phi=\dfrac{4}{3}$

(vii) tan ϕ 09306042

$\tan\phi=\dfrac{3}{4}$

(viii) cosec ϕ 09306043

$\operatorname{cosec}\phi=\dfrac{5}{3}$

(ix) sec ϕ 09306044

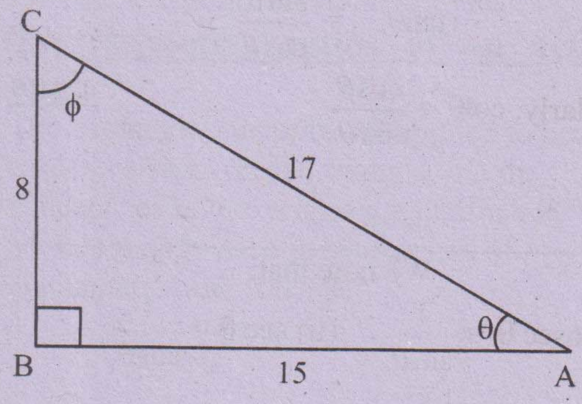
$\sec\phi=\dfrac{5}{4}$

(x) cos ϕ 09306045

$\cos\phi=\dfrac{4}{5}$

Solutions:

(b)



(i) sin θ

$\sin\theta=\dfrac{8}{17}$

(ii) cos θ

$\cos\theta=\dfrac{15}{17}$

(iii) tan θ

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$\tan\theta = \dfrac{8}{15}$

(iv) sec θ

$\sec\theta = \dfrac{17}{15}$

(v) cosec θ

$\operatorname{cosec}\theta = \dfrac{17}{8}$

(vi) cot ϕ

$\cot\phi = \dfrac{8}{15}$

(vii) tan ϕ

$\tan\phi = \dfrac{15}{8}$

(viii) cosec ϕ

$\operatorname{cosec}\phi = \dfrac{17}{15}$

(ix) sec ϕ

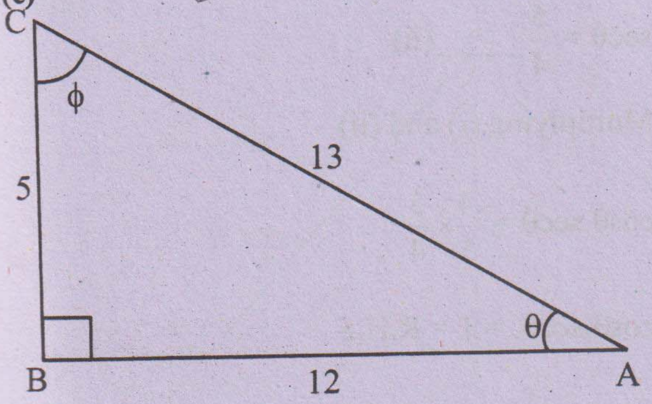
$\operatorname{cosec}\phi = \dfrac{17}{8}$

(x) cos ϕ

$\cos\phi = \dfrac{8}{17}$

Solutions:

(c)



(i) sin θ

$\sin\theta = \dfrac{5}{13}$

(ii) cos θ

$\cos\theta = \dfrac{12}{13}$

(iii) tan θ

$\tan\theta = \dfrac{5}{12}$

(iv) sec θ

$\sec\theta = \dfrac{13}{12}$

(v) cosec θ

$\operatorname{cosec}\theta = \dfrac{13}{5}$

(vi) cot ϕ

$\cot\phi = \dfrac{5}{12}$

(vii) tan ϕ

$\tan\phi = \dfrac{12}{5}$

(viii) cosec ϕ

$\operatorname{cosec}\phi = \dfrac{13}{12}$

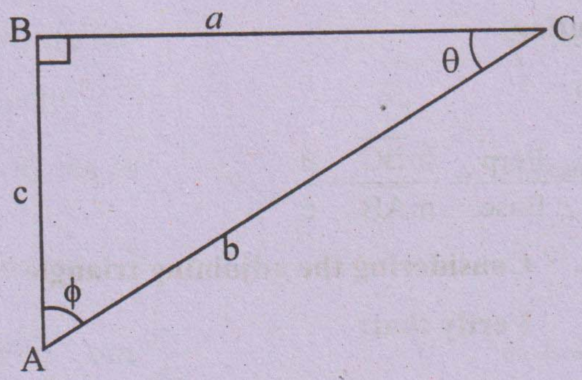
(ix) sec ϕ

$\sec\phi = \dfrac{13}{5}$

(x) cos ϕ

$\cos\phi = \dfrac{5}{13}$

Q.2 For the following right-angled triangles ABC find the trigonometric ratios for which $m\angle A = \phi$ and $m\angle C = \theta$



(i) sin θ **09306046**

Solution:

sin θ

$$\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{m\overline{AB}}{m\overline{AC}} = \frac{c}{b}$$

(ii) cos θ **09306047**

Solution:

cos θ

$$\cos\theta = \frac{\text{Base}}{\text{Hyp.}} = \frac{m\overline{BC}}{m\overline{AC}} = \frac{a}{b}$$

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(iii) tan θ 09306048

Solution:

tan θ

$$\tan\theta = \frac{\text{Perp.}}{\text{Base}} = \frac{m\overline{AB}}{m\overline{BC}} = \frac{c}{a}$$

(iv) sinϕ 09306049

Solution:

sinϕ

$$\sin\phi = \frac{\text{Perp.}}{\text{Hyp.}} = \frac{m\overline{BC}}{m\overline{AC}} = \frac{a}{b}$$

(v) cosϕ 09306050

Solution:

cosϕ

$$\tan\phi = \frac{\text{Base}}{\text{Hyp.}} = \frac{m\overline{AB}}{m\overline{AC}} = \frac{c}{b}$$

(vi) tanϕ 09306051

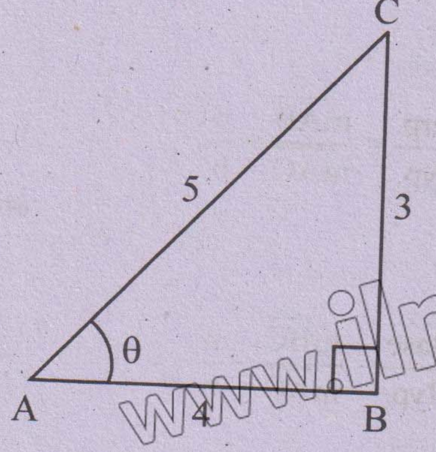
Solution:

tanϕ

$$\tan\phi = \frac{\text{Perp}}{\text{Base}} = \frac{m\overline{BC}}{m\overline{AB}} = \frac{a}{c}$$

Q.3 Considering the adjoining triangle

Verify that:



(i) sin θ cosec θ = 1 09306052

Solution:

sinθ.cosecθ =1

From right angled - triangle ABC

$\sin\theta = \frac{3}{5}$ ____(i)

$\text{cosec}\theta = \frac{5}{3}$ ____ (ii)

Multiplying (i) and (ii)

$$\sin\theta.\text{cosec}\theta = \frac{3}{5} \times \frac{5}{5}$$

$$\sin\theta.\sec\theta = 1 = \text{R.H.S}$$

(ii)cos θ sec θ =1 09306053

Solution:

cosθ.secθ =1

From right angled - triangle ABC

$\cos\theta = \frac{4}{5}$ ____(i)

$\sec\theta = \frac{5}{4}$ ____(ii)

Multiplying (i) and (ii)

$$\cos\theta.\sec\theta = \frac{4}{5} \times \frac{5}{4}$$

$$\cos\theta.\sec\theta. = 1 = \text{R.H.S}$$

(iii) tan θ cot θ =1 09306054

Solution:

tanθ.cotθ =1

From right angled - triangle ABC

$\tan\theta = \frac{3}{4}$ ____(i)

$\cot\theta = \frac{4}{3}$ ....(ii)

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Multiplying (i) and (ii)

$\tan\theta.\cot\theta = \frac{3}{4} \times \frac{4}{3}$

$\tan\theta.\cot\theta = 1$ R.H.S

Q.4 Fill in the blanks. 09306055

Solution:

(i) $\sin 30° = \sin(90° - 60°) = \cos 60°$

(ii) $\cos 30° = \cos(90° - 60°) = \sin 60°$

(iii) $\tan 30° = \tan(90° - 60°) = \cot 60°$

(iv) $\tan 60° = \tan(90° - 30°) = \cot 30°$

(v) $\sin 60° = \sin(90° - 30°) = \cos 30°$

(vi) $\cos 60° = \cos(90° - 30°) = \sin 30°$

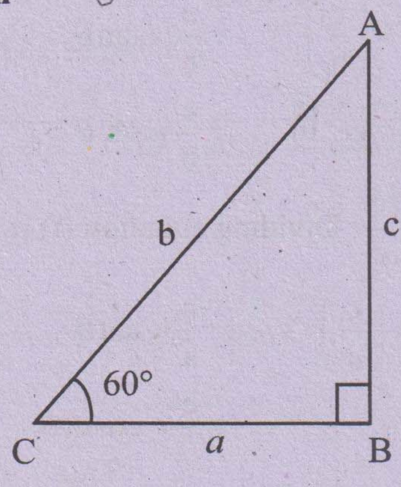
(vii) $\sin 45° = \sin(90° - 45°) = \cos 45°$

(viii) $\tan 45° = \tan(90° - 45°) = \cot 45°$

(ix) $\cos 45° = \cos(90° - 45°) = \sin 45°$

Q.5 If in a right angled triangle *ABC*, $m\angle B = 90°$ and *C* is an acute angle of $60°$. Also $\sin m\angle A = \frac{a}{b}$, then find the following trigonometric ratios. 09306056

Solution



(i) $\frac{m\overline{BC}}{m\overline{AB}}$ 09306057

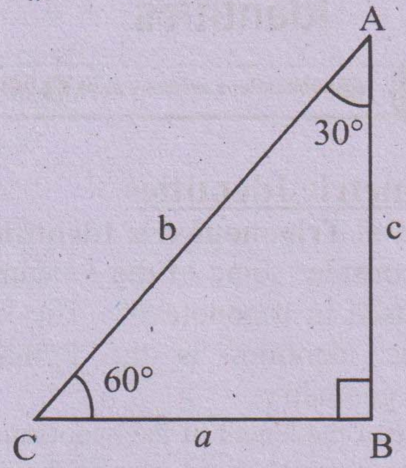
$\frac{m\overline{BC}}{m\overline{AB}} = \frac{a}{c}$

(ii) cos 60° 09306058

$\cos 60° = \frac{a}{b}$

(iii) tan 60° 09306059

$\tan 60° = \frac{c}{a}$



(iv) $\text{cosec}\,\frac{\pi}{3}$ = cosec60° 09306060

Solution:

$\text{cosec}\frac{\pi}{3} = \frac{b}{c}$ $\left(\because \frac{\pi}{3} = 60°\right)$

(v) cot 60° 09306061

$\cot 60° = \frac{a}{c}$

(vi) sin 30° 09306062

$\sin 30° = \frac{a}{b}$

(vii) cos 30° 09306063

$\cos 30° = \frac{c}{b}$

(viii) tan $\frac{\pi}{6}$ 09306064

$\tan 30° = \frac{a}{c}$

$\tan\frac{\pi}{6} = \frac{a}{c}$ $\left(\because \frac{\pi}{6} = 30°\right)$

Solution:

(ix) sec 30° 09306065

$\sec 30° = \frac{b}{c}$

(x) cot30° 09306066

$\text{Cot}\,30° = \frac{c}{a}$

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Trigonometric Identities

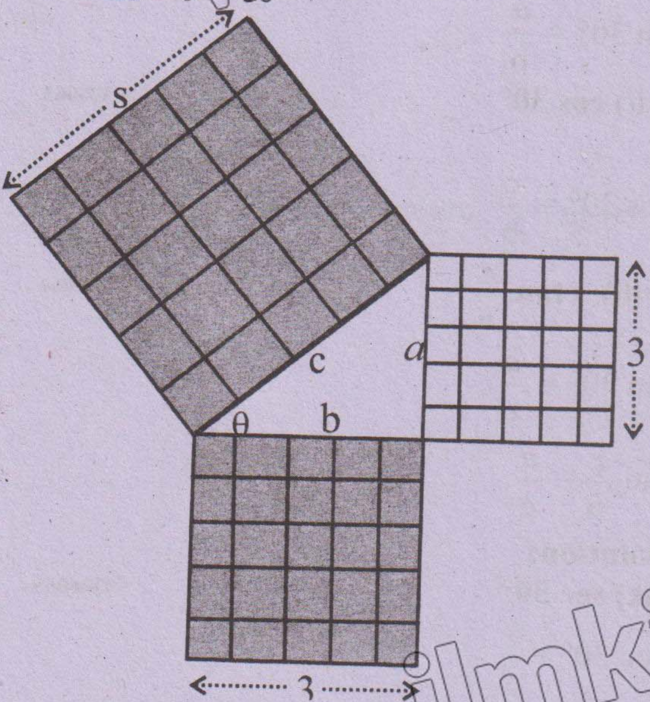
Fundamental Trigonometric Identities

We shall consider some of the fundamental identities used in trigonometry. The key to these basic identities is the Pythagoras theorem in geometry.

"The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the other two sides". (Fig. 1)

Note: Remember that the sum of the lengths of the two sides in a triangle is not equal to the length of the third side, but the sum of length of the two sides is greater than the length of the third side and this reality is the base for the construction of a triangle.

$$c^2 = a^2 + b^2$$
$$5^2 = 3^2 + 4^2$$
$$25 = 9 + 16$$



In figure (1) the opposite side equals to the length 'a' adjacent side equals to the length 'b', and hypotenuse equals to the length 'c'.

By Pythagoras Theorem, we have

$$a^2 + b^2 = c^2 \qquad \ldots\text{(i)}$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2} \quad \text{(Dividing by } c^2\text{)}$$

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1} \qquad \because \frac{a}{c} = \sin\theta$$

$$a^2 + b^2 = c^2 \qquad \because \frac{b}{c} = \cos\theta$$

Dividing equation (i) by b^2, we have

$$\frac{a^2}{b^2} + \frac{b^2}{b^2} = \frac{c^2}{b^2}$$

$$\because \left(\frac{a}{b}\right)^2 + 1 = \left(\frac{c}{b}\right)^2 \qquad \because \frac{a}{b} = \tan\theta$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta} \qquad \because \frac{c}{b} = \sec\theta$$

$$\frac{a^2}{a^2} + \frac{b^2}{a^2} = \frac{c^2}{a^2} \quad \text{Dividing equation (i) by } a^2\text{),}$$

$$1 + \left(\frac{b}{a}\right)^2 = \left(\frac{c}{a}\right)^2 \qquad \because \frac{b}{a} = \cot\theta$$

$$\because \frac{c}{a} = \operatorname{cosec}\theta$$

we have

$$\boxed{1 + \cot^2\theta = \operatorname{cosec}^2\theta} \qquad \ldots\text{(iv)}$$

Example 10:

Show that $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$ 09306067

Solution:

$$\text{L.H.S} = (\sec^2\theta - 1)\cos^2\theta$$
$$= \tan^2\theta \,.\, \cos^2\theta$$
$$(\because 1 + \tan^2\theta = \sec^2\theta)$$

$= \dfrac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta \quad \left(\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right)$

$= \sin^2\theta = \text{R.H.S}$

Hence, $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

Example 11:

Show that $\tan\theta + \cot\theta = \sec\theta\,\text{cosec}\,\theta$ **09306068**

Solution:

L.H.S $= \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin\theta\cdot\sin\theta + \cos\theta\cdot\cos\theta}{\cos\theta\cdot\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta} \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= \dfrac{1}{\cos\theta}\cdot\dfrac{1}{\sin\theta}$

$= \sec\theta\cdot\text{cosec}\,\theta = \text{R.H.S.}$

Hence, $\tan\theta + \cot\theta = \sec\theta\ \text{cosec}\,\theta$

Example 12: Show that

$$\frac{1}{\text{cosec}\,\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\text{cosec}\,\theta - \cot\theta}$$

09306069

Solution:

L.H.S $= \dfrac{1}{\text{cosec}\,\theta - \cot\theta} - \dfrac{1}{\sin\theta}$

$= \dfrac{1}{\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}} - \dfrac{1}{\sin\theta}$

$= \dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta}$

$= \dfrac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} - \dfrac{1}{\sin\theta}$

$= \dfrac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta} - \dfrac{1}{\sin\theta}$

$= \dfrac{\sin\theta(1+\cos\theta)}{\sin^2\theta} - \dfrac{1}{\sin\theta}$

$= \dfrac{1+\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}$

$= \dfrac{1+\cos\theta-1}{\sin\theta}$

$= \dfrac{\cos\theta}{\sin\theta} = \cot\theta$

R.H.S $= \dfrac{1}{\sin\theta} - \dfrac{1}{\text{cosec}\,\theta + \cot\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{1}{\dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}}$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{1+\cos\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{\sin\theta(1-\cos\theta)}{\sin^2\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{1-\cos\theta}{\sin\theta}$

$= \dfrac{1-1+\cos\theta}{\sin\theta}$

$= \dfrac{\cos\theta}{\sin\theta} = \cot\theta$

Hence, L.H.S = R.H.S

Example 13:

Show that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ **09306070**

Solution: L.H.S $= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

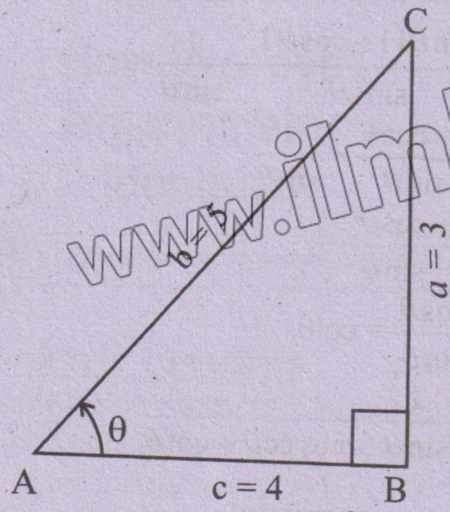
$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$= 1.[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta]$

$= 1 - 3\sin^2\theta\cos^2\theta = \text{R.H.S}$

Hence, $\sin^6\theta + \cos^6 = 1 - 3\sin^2\theta\cos^2\theta$

Example 14: If $\tan\theta = \dfrac{3}{4}$, find the remaining trigonometric ratios, when θ lies in first quadrant. **09306071**



Solution: Given: $\tan\theta = \dfrac{3}{4} = \dfrac{a}{c}$,

Where, $a = 3, c = 4$

By Pythagoras theorem, we have

$$b^2 = a^2 + c^2$$
$$b^2 = 9 + 16 = 25$$
$$\sqrt{b^2} = \sqrt{25}$$
$$\Rightarrow\ b = 5$$

Therefore,

$$\sin\theta = \frac{a}{b} = \frac{3}{5} \quad ; \quad \text{coses}\,\theta = \frac{b}{a} = \frac{5}{3}$$

$$\cos\theta = \frac{c}{b} = \frac{4}{5} \quad ; \quad \sec\theta = \frac{b}{c} = \frac{5}{4}$$

$$\cot\theta = \frac{c}{a} = \frac{4}{3}$$

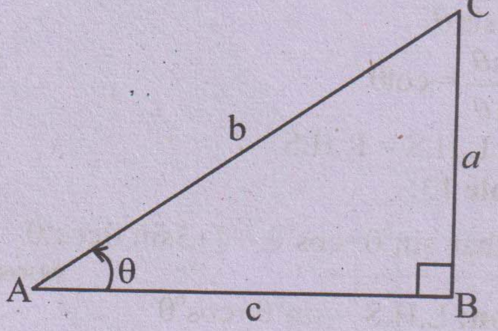
Exercise 6.3

Q.1 If θ lies in first quadrant, find the remaining trigonometric ratios of θ.

(i) $\sin\theta = \dfrac{2}{3}$ 09306072

Solution:

$\sin\theta = \dfrac{2}{3}$ ________ (i)



Let ΔABC be a right angled triangle in which $m\angle B = 90°$, and $m\angle A = \theta$

In right-angled ΔABC,

$\sin\theta = \dfrac{a}{b}$ ________ (ii)

From (i) and (ii) $\dfrac{a}{b} = \dfrac{2}{3} \Rightarrow \boxed{a = 2, b = 3}$

By Pythagoras, theorem

$b^2 = a^2 + c^2$

$(3)^2 = (2)^2 + c^2$

$9 = 4 + c^2$

$9 - 4 = c^2$

$5 = c^2$

$\sqrt{c^2} = \sqrt{5}$

$c = \sqrt{5}$

Remaining ratios:

$\cos\theta = \dfrac{c}{b} = \dfrac{\sqrt{5}}{3}$

$\tan\theta = \dfrac{a}{c} = \dfrac{2}{\sqrt{5}}$

$\text{cosec}\theta = \dfrac{b}{a} = \dfrac{3}{2}$

$\sec\theta = \dfrac{b}{c} = \dfrac{3}{\sqrt{5}}$

$\cot = \dfrac{c}{a} = \dfrac{\sqrt{5}}{2}$

(ii) $\cos\theta = \dfrac{3}{4}$ ______ (i) 09306073

Let ΔABC be a right angled triangle in which $m\angle B = 90°$, $m\angle A = \theta$,

In ΔABC,

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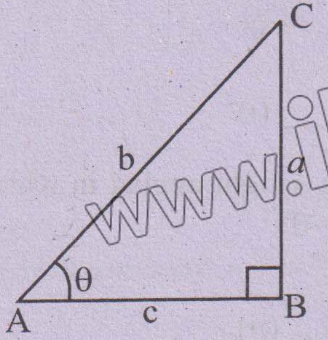


$\cos\theta = \frac{c}{b}$ ______ (ii)

From (i) and (ii)

$\frac{c}{b} = \frac{3}{4}$

$\Rightarrow \boxed{c = 3, b = 4}$

By pythagoras theorem

$b^2 = c^2 + a^2$

$4^2 = 3^2 + a^2$

$16 = 9 + a^2$

$16 = 9 = a^2$

$a^2 = 7$

$\sqrt{a^2} = \sqrt{7}$

$\boxed{a = \sqrt{7}}$

Remaining ratios

$\sin\theta = \frac{a}{b} = \frac{\sqrt{7}}{4}$

$\tan\theta = \frac{a}{c} = \frac{\sqrt{7}}{3}$

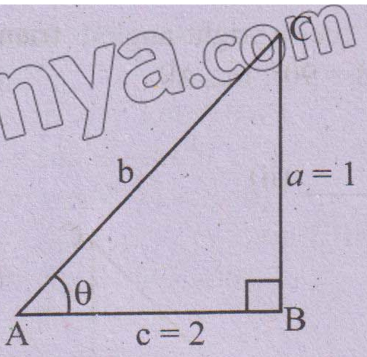
$\operatorname{cosec}\theta = \frac{b}{a} = \frac{4}{\sqrt{7}}$

$\sec\theta = \frac{b}{c} = \frac{4}{3}$

$\cot\theta = \frac{c}{a} = \frac{3}{\sqrt{7}}$

(iii) $\tan\theta = \frac{1}{2}$ **09306074**

Solution:



$\tan\theta = \frac{1}{2}$

Let ΔABC is a right – angled triangle in which $m\angle B = 90°$, $m\angle A = \theta$

In ΔABC, $\tan\theta = \frac{a}{c} =$ ______ (ii)

By comparing (i) & (ii)

$\frac{a}{c} = \frac{1}{2}$

$\Rightarrow \boxed{a = 1, c = 2}$

By Pythagoras theorem

$b^2 = c^2 + a^2$

$b^2 = 2^2 + 1^2$

$b^2 = 4 + 1$

$b^2 = 5$

$\sqrt{b^2} = \sqrt{5}$

$\Rightarrow b = \sqrt{5}$

Remaining Ratios

$\sin\theta = \frac{a}{b} = \frac{1}{\sqrt{5}}$

$\cos\theta = \frac{c}{b} = \frac{2}{\sqrt{5}}$

$\operatorname{cosec}\theta = \frac{b}{a} = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\sec\theta = \frac{b}{c} = \frac{\sqrt{5}}{2}$

$\cot\theta = \frac{c}{a} = \frac{2}{1} = 2$

(iv) $\sec\theta = 3$ **09306075**

Solution:

$\sec\theta = 3$

$\sec\theta = \frac{3}{1}$ ______ (i)

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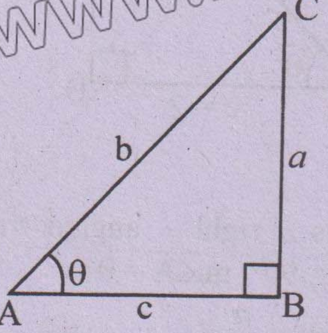
Let ΔABC be a right-angled triangle in which $m\angle B = 90°$, $m\angle A\theta$

In ΔABC,

$\sec\theta = \dfrac{b}{c}$ ______ (ii)



From (i) and (ii)

$\dfrac{b}{c} = \dfrac{3}{1}$

$\Rightarrow \boxed{b = 3, c = 1}$

By Pythagoras theorem

$b^2 = a^2 + c^2$

$3^2 = a^2 + (i)^2$

$9 = a^2 + 1$

$9 - 1 = a^2$

$8 = a^2$

$\sqrt{a^2} = \sqrt{8}$

$a = \sqrt{4 \times 2}$

$\boxed{a = 2\sqrt{2}}$

Remaining Ratios

$\sin\theta = \dfrac{a}{b} = \dfrac{2\sqrt{2}}{3}$

$\cos\theta = \dfrac{c}{b} = \dfrac{1}{3}$

$\tan\theta = \dfrac{a}{c} = \dfrac{2\sqrt{2}}{1} = 2\sqrt{2}$

$\operatorname{cosec}\theta = \dfrac{b}{a} = \dfrac{3}{2\sqrt{2}} = \dfrac{3}{2\sqrt{2}}$

$\cot\theta = \dfrac{c}{a} = \dfrac{1}{2\sqrt{2}}$

(v) $\cot\theta = \sqrt{\dfrac{3}{2}}$

09306076

Solution:

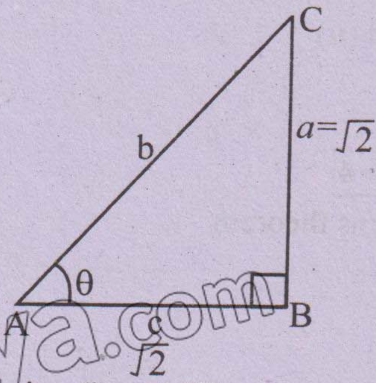
$\cot\theta = \sqrt{\dfrac{3}{2}}$

$\cot\theta = \dfrac{\sqrt{3}}{\sqrt{2}}$ _____ (i)

Let ΔABC is *a* right angled in which $m\angle B = 90°$ $m\angle B = \theta$

In ΔABC

$\cot\theta = \dfrac{c}{a}$ ______ (ii)



By comparing (i) and (iii)

$\dfrac{c}{a} = \dfrac{\sqrt{3}}{\sqrt{2}}$

$\Rightarrow \boxed{\begin{array}{l} c = \sqrt{3} \\ a = \sqrt{2} \end{array}}$

By Pythagoras theorem

$b^2 = c^2 + a^2$

$b^2 = \left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2$

$b^2 = 3 + 2 = 5$

$\sqrt{b^2} = \sqrt{5}$

$\boxed{b = \sqrt{5}}$

Remaining Ratios

$\sin\theta = \dfrac{a}{b} = \dfrac{\sqrt{2}}{\sqrt{5}} = \sqrt{\dfrac{2}{5}} \quad \because \dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

$\cos\theta = \dfrac{c}{b} = \dfrac{\sqrt{3}}{\sqrt{5}} = \sqrt{\dfrac{3}{5}}$

$\tan\theta = \dfrac{a}{c} = \dfrac{\sqrt{2}}{\sqrt{3}} = \sqrt{\dfrac{2}{3}}$

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$$\text{cosec}\theta = \frac{b}{a} = \frac{\sqrt{5}}{\sqrt{2}} = \sqrt{\frac{5}{2}}$$

$$\sec\theta = \frac{b}{c} = \frac{\sqrt{5}}{\sqrt{3}} = \sqrt{\frac{5}{3}}$$

Prove the following trigonometric Identities.

Q.2 $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$ **09306077**

Solution:

$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

L.H.S $= (\sin\theta + \cos\theta)^2$

$[\because (a+b)^2 = a^2+b^2+2ab]$

$= (\sin\theta)^2+(\cos\theta)^2+2\sin\theta\cos\theta$

$= \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta$

$= 1+2\sin\theta\cos\theta$

$=$ R.H.S $[\because \sin^2\theta+\cos^2\theta = 1]$

Hence proved

Q.3 $\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$ **09306078**

Solution:

$$\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

L.H.S $= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$= \frac{1}{\tan\theta}$

L.H.S = R.H.S

Q.4 $\frac{\sin\theta}{\text{cosec}\,\theta} + \frac{\cos\theta}{\sec\theta} = 1$ **09306079**

Solution:

$$\frac{\sin\theta}{\text{cosec}\,\theta} + \frac{\cos\theta}{\sec\theta} = 1$$

L.H.S $= \frac{\sin\theta}{\text{cosec}\,\theta} + \frac{\cos\theta}{\sec\theta}$

$= \sin\theta.\frac{1}{\text{cose}\theta} + \cos\theta.\frac{1}{\text{Sec}\theta}$

$= \sin\theta.\sin\theta.+\cos\theta.\cos\theta$

$= \sin^2\theta+\cos^2\theta$

$= 1$ $[\because \sin^2\theta+\cos^2\theta=1]$

$=$ R.H.S

Hence proved

Q.5 $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$ **09306080**

Solution:

$\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

L.H.S $= \cos^2\theta - \sin^2\theta$

$= \cos^2\theta-(1-\cos^2\theta)$

$[\because \sin^2\theta+\cos^2\theta=1)$

$= \cos^2\theta-1+\cos^2\theta$

$= 2\cos^2\theta-1$

L.H.S = R.H.S

Q.6 $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$ **09306081**

$\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

L.H.S $= \cos^2\theta - \sin^2\theta$

$= 1-\sin^2\theta-\sin^2\theta$ $[\because \sin^2\theta+\cos^2\theta=1]$

$= 1-2\text{Sin}^2\theta$

L.H.S = R.H.S

Q.7 $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$ **09306082**

Solution:

$$\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$$

L.H.S $= \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1-\sin\theta}{\text{Cos}\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$

$= \frac{(1)^2-(\sin\theta)^2}{\cos\theta(1+\sin\theta)}$

$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$ $[\because \sin^2\theta+\cos^2\theta=1]$

$= \frac{\cos\theta}{1+\sin\theta}$

L.H.S = R.H.S

Hence proved

Q.8 $(\sec\theta-\tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$ **09306083**

Solution:

$$(\sec\theta-\tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

L.H.S $= (\sec\theta-\tan\theta)^2$

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$$= \left[\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right]^2 \qquad \because \sec\theta = \frac{1}{\cos\theta}$$

$$\left(\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right)$$

$$= \left[\frac{1-\sin\theta}{\cos\theta}\right]^2$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} \qquad (\because \cos^2\theta = 1-\sin^2\theta)$$

$$= \frac{(1-\sin\theta)^2}{(1)^2-(\sin\theta)^2}$$

$$[\because a^2-b^2 = (a+b)(a-b)]$$

$$= \frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta} = \text{R.H.S}$$

Hence proved.

Q.9 $(\tan\theta+\cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$ **09306084**

Solution:

$(\tan\theta+\cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$

L.H.S $= (\tan\theta+\cot\theta)^2$

$$= \left[\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right]^2$$

$$= \left[\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right]^2$$

$$= \left(\frac{1}{\cos\theta\sin\theta}\right)^2$$

$= (\sec\theta.\operatorname{cosec}\theta)^2$

$= \sec^2\theta.\operatorname{cosec}^2\theta$

= R.H.S

Hence proved

Q.10 $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1} = \tan\theta+\sec\theta$ **09306085**

Solution:

$$\text{L.H.S} = \frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$$

$$= \frac{(\tan\theta+\sec\theta)-(1)}{\tan\theta-\sec\theta+1}$$

$$= \boxed{\begin{aligned}&\because 1+\tan^2\theta = \sec^2\theta\\ &1 = \sec^2\theta-\tan^2\theta\end{aligned}}$$

$$\therefore \text{L.H.S} = \frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$

$$= \frac{(\tan\theta+\sec\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{(\tan\theta-\sec\theta+1)}$$

$$= \frac{(\tan\theta+\sec\theta)[1-(\sec\theta-\tan\theta)]}{(\tan\theta-\sec\theta+1)}$$

$$= \frac{(\tan\theta+\sec\theta)[1-\sec\theta+\tan\theta]}{(\tan\theta-\sec\theta+1)}$$

$$= \frac{(\tan\theta+\sec\theta)[\tan\theta-\sec\theta+1]}{(\tan\theta-\sec\theta+1)}$$

L.H.S = $\tan\theta+\sec\theta$

L.H.S = R.H.S

Hence proved.

Q.11 $\sin^3\theta-\cos^3\theta = (\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$ **09306086**

Solution:

$\sin^3\theta-\cos^3\theta = (\sin\theta-\cos\theta)(1+\sin\theta\cos\theta)$

L.H.S $= \sin^3\theta-\cos^3\theta$

$\because a^3-b^3 = (a-b)(a^2+ab+b^2)$

$= (\sin\theta-\cos\theta)[\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta]$

$= (\sin\theta-\cos\theta)(1+\sin\theta.\cos\theta)$

L.H.S = R.H.S

Q.12 $\sin^6\theta-\cos^6\theta$

$=(\sin^2\theta-\cos^2\theta)(1-\sin^2\theta\cos^2\theta)$ **09306087**

Solution:

$\sin^6\theta-\cos^6\theta = (\sin^2\theta-\cos^2\theta)(1-\sin^2\theta\cos^2\theta)$

L.H.S $= \sin^6\theta-\cos^6\theta$

$= (\sin^2\theta)^3-(\cos^2\theta)^3$

$\boxed{\because a^3-b^3 = (a-b)(a^2+ab+b^2)}$

$= (\sin^2\theta-\cos^2\theta)$

$[(\sin^2\theta)^2+(\cos^2\theta)^2+\sin^2\theta.\cos^2\theta]$

$= (\sin^2\theta-\cos^2\theta)$

$[(\sin^2\theta)^2+(\cos^2\theta)^2+2\sin^2\theta.\cos^2\theta-\sin^2\theta$

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$\cos^2\theta]$
$= (\sin^2\theta - \cos^2\theta)[(\sin^2\theta + \cos^2\theta]^2 - \sin^2\theta\cos^2\theta]$
$= (\sin^2\theta - \cos^2\theta)\ [(1)^2 - \sin^2\theta.\cos^2\theta]$
$= (\sin^2\theta - \cos^2\theta)\ (1 - \sin^2\theta.\cos^2\theta)$
$=$ R.H.S
Hence proved.

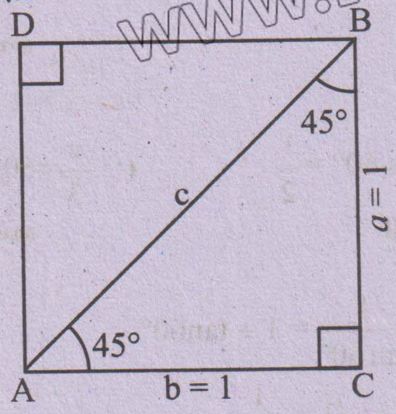
Values of Trigonometric Ratios of Angles

09306088

Trigonometric ratios of 45° $\left(\frac{\pi}{4}\text{radian}\right)$

Consider a square *ACBD* of side length 1 unit.
We know that the diagonal bisect the angles.
So in the triangle *ABC*
$m\angle A = m\angle B = 45°$ and $m\angle C = 90°$.
In ΔABC,
Using Pythagoras theorem

$$c^2 = a^2 + b^2$$
$$c^2 = 1 + 1$$
$$c^2 = 2$$
$$\Rightarrow\ c = \sqrt{2}$$



The trigonometric ratio are:

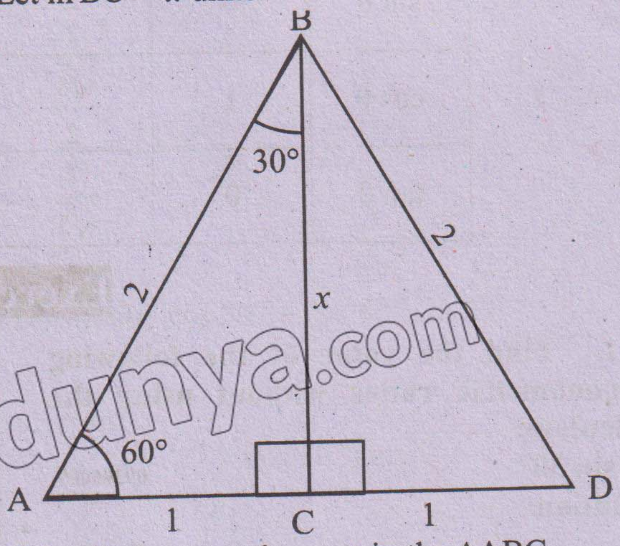
$\sin 45° = \frac{a}{c} = \frac{1}{\sqrt{2}}$; $\operatorname{cosec} 45° = \frac{c}{a} = \sqrt{2}$

$\cos 45° = \frac{b}{c} = \frac{1}{\sqrt{2}}$; $\sec 45° = \frac{c}{b} = \sqrt{2}$

$\tan 45° = \frac{a}{b} = 1$; $\cot 45° = \frac{b}{a} = 1$

Trigonometric ratios of 30° $\left(\frac{\pi}{6}\text{radian}\right)$ and 60° $\left(\frac{\pi}{3}\text{radian}\right)$

Consider an equilateral triangle ABD of side 2 units. Draw a perpendicular bisector $\overline{BC}$ on $\overline{AD}$. The point C is the midpoint of $\overline{AD}$.
So, $m\overline{AC} = m\overline{CD}$ in which $m\angle BAC = 60°$, $m\angle ABC = 30°$, $m\angle ACB = 90°$.
Let $m\overline{BC} = x$ units.



Using Pythagoras theorem in the ΔABC.

$$2^2 = 1^2 + x^2$$
$$4 = 1 + x^2$$
$$x^2 = 4 - 1$$
$$\Rightarrow\ x^2 = 3$$
$$\Rightarrow\ x = \sqrt{3}$$

$m\overline{BC} = \sqrt{3}$ units

Trigonometric ratios of 30° $\left(\frac{\pi}{6}\text{ radian}\right)$:

In the triangle, ABC with $\angle ABC = 30°$

$\sin 30° = \frac{1}{2}$; $\operatorname{cosec} 30° = 2$

$\cos 30° = \frac{\sqrt{3}}{2}$; $\sin 30° = \frac{2}{\sqrt{3}}$

$\tan 30° = \frac{1}{\sqrt{3}}$; $\cot 30° = \sqrt{3}$

Trigonometric Ratios of 60° $\left(\frac{\pi}{3}\text{radian}\right)$

In right angled triangle *ABC*, with $m\angle A = 60°$

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$\sin 60° = \frac{\sqrt{3}}{2}$; $\text{cosec } 60° = \frac{1}{2}$; $\tan 60° = \sqrt{3}$ $\text{cosec } 60° = \frac{2}{\sqrt{3}}$; $\sec 60° = 2$; $\cot 60° = \frac{1}{\sqrt{3}}$

These results in the form of a table can be written as:

θ	0	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \frac{\pi}{2}$
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Exercise 6.4

Q.1 Find the value of the following trigonometric ratios without using the calculator.

(i) sin 30° 09306089

Solution:

$\sin 30° = \frac{1}{2}$

(ii) cos 30° 09306090

Solution:

$\cos 30° = \frac{\sqrt{3}}{2}$

(iii) tan $\frac{\pi}{6}$

Solution:

$\tan 30° = \frac{1}{\sqrt{3}}$ $\quad (\because \frac{\pi}{6} = 30°)$

(iv) tan 60°

Solution

tan 60°

$\tan 60° = \sqrt{3}$

(v) sec 60° 09306091

Solution:

$\sec 60° = \frac{1}{\cos 60°} = 1 \div \cos 60°$

$= 1 \div \frac{1}{2}$

$= 1 \times \frac{1}{2} = 2$

Sec60° = 2

(vi) cos $\frac{\pi}{3}$ 09306092

Solution:

$\cos\frac{\pi}{3} = \cos 60° = \frac{1}{2}$ $\quad (\because \frac{\pi}{3} = 60°)$

(vii) cot 60° 09306093

Solution:

$\cot 60° = \frac{1}{\tan 60°} = 1 \div \tan 60°$

$= 1 \div \sqrt{3} = \frac{1}{\sqrt{3}}$

cot 60°

$\cot 60° = \frac{1}{\sqrt{3}}$

(viii) sin 60° 09306094

Solution:

sin 60°

$\sin 60° = \frac{\sqrt{3}}{2}$

(ix) sec 30° 09306095

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Solution:

$$\text{Sec}30° = \frac{1}{\cos 30°} = 1 \div \cos 30°$$

$$= 1 \div \frac{\sqrt{3}}{2} = 1 \times \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$\sec 30° = \frac{2}{\sqrt{3}}$$

(x) cosec 30° 09306096

Solution:

$$\text{cosec } 30° = \frac{1}{\sin 30°} = 1 \div \sin 30°$$

$$= 1 \div \frac{1}{2} = 1 \times \frac{2}{1} = 2$$

$$\text{cosec } 30° = 2$$

(xi) sin 45° 09306097

Solution:

$$\sin 45° = \frac{1}{\sqrt{2}}$$

(xii) cos $\frac{\pi}{4}$ 09306098

Solution

$$\cos \frac{\pi}{4} = \cos 45° = \frac{1}{\sqrt{2}} \qquad \left(\because \frac{\pi}{4} = 45°\right)$$

Q.2 Evaluate

(i) 2 sin 60° cos60° 09306099

Solution:

2sin 60° cos 60°

$$= 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2}$$

(ii) 2cos $\frac{\pi}{6}$ sin $\frac{\pi}{6}$

Solution:

$$2\cos \frac{\pi}{6} \sin \frac{\pi}{6}$$

$$= 2\cos 30° \sin 30° \qquad \left(\because \frac{\pi}{6} = 30°\right)$$

$$= 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2}$$

(iii) 2 sin 45° + 2cos 45° 09306100

Solution:

2 sin 45°+2cos 45°

$$= 2 \times \frac{1}{\sqrt{2}} + 2 \times \frac{1}{\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}$$

$$= \frac{2+2}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}} = 2 \times \frac{2}{\sqrt{2}}$$

$$= 2\sqrt{2} \qquad \left(\because \frac{a}{\sqrt{a}} = \sqrt{a}\right)$$

(iv) sin 60° cos 30° + cos 60° sin 30° 09306101

Solution:

sub 60° cos 30° + cos 60° sin 30°

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{\left(\sqrt{3}\right)^2}{4} - \frac{1}{4}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{3+1}{4}$$

$$= \frac{4}{4}$$

$$= \mathbf{1}$$

(v) cos 60° cos 30° – sin 60° sin 30° 09306102

Solution:

cos 60° cos 30° – sin 60° sin 30°

$$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

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$= \frac{\sqrt{3}-\sqrt{3}}{4}$

$= \frac{0}{4} = 0$

$= 0$

(vi) sin 60° cos 30° – cos 60° sin 30° 09306103

Solution:

sin 60° cos 30° – cos 60° sin 30°

$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} - \frac{1}{2}.\frac{1}{2}$

$= \frac{(\sqrt{3})^2}{4} - \frac{1}{4}$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{3-1}{4}$

$= \frac{2}{4}$

$= \frac{1}{2}$

(vii) cos 60° cos 30° + sin 60° sin 30°

09306104

Solution:

cos 60° cos 30° + sin 60° sin 30°

$= \frac{1}{2}.\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}.\frac{1}{2}$

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$= \frac{\sqrt{3}+\sqrt{3}}{4}$

$= \frac{\cancel{2}\sqrt{3}}{\cancel{4}\,2}$

$= \frac{\sqrt{3}}{2}$

(viii) tan $\frac{\pi}{6}$ cot $\frac{\pi}{6}$ +1

09306105

Solution:

tan $\frac{\pi}{6}$ cot $\frac{\pi}{6}$ +1 $\left(\because \frac{\pi}{6} = 30°\right)$

$= \frac{1}{\sqrt{3}}.\sqrt{3}+1$

$= 1+1$

$= 2$

Q.3 If sin 45° and cos 45° equal to $\frac{1}{\sqrt{2}}$ each, then find the value of the followings:

(i) 2 sin 45° – 2 cos 45° 09306106

Solution:

2 sin 45° – 2 cos 45°

$= 2\times\frac{1}{\sqrt{2}} - 2\times\frac{1}{\sqrt{2}}$

$= \frac{2-2}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$

(ii) 3 cos 45° + 4 sin 30° 09306107

Solution:

3 cos 45° + 4 sin 30°

$= 3\times\frac{1}{\sqrt{2}} + 4\times\frac{1}{\sqrt{2}}$

$\frac{3}{\sqrt{2}} + \frac{4}{\sqrt{2}}$

$= \frac{3+4}{\sqrt{2}}$

$= \frac{7}{\sqrt{2}}$

(iii) 5 cos 45° – 3 sin 45° 09306108

Solution:

5 cos 45° – 3 sin 45°

$= 5\times\frac{1}{\sqrt{2}} - 3\times\frac{1}{\sqrt{2}}$

$= \frac{5}{\sqrt{2}} - \frac{3}{\sqrt{2}}$

$= \frac{5-3}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$ $\left(\because \frac{a}{\sqrt{a}} = \sqrt{a}\right)$

$= \sqrt{2}$

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Solution of a Triangle

We know that there are three sides and three angles in a triangle. Out of these six elements, if we know three of them including at least one side, then we can find the measures of the remaining elements. Finding the measures of the remaining elements is called the solution of a triangle.

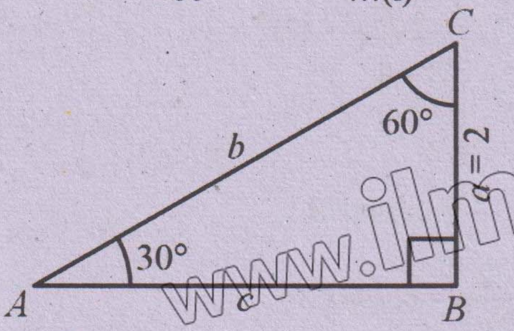
Case I: When measures of one side and one angle are given.

Example 15: Solve triangle ABC, in which $m\angle B = 90° = m\angle A = 30°$, $a = 2$

Solution:

We are required to find b, c and $m\angle A$.

Now $m\angle C = m\angle B - m\angle A$
$= 90° - 30°$
$= 60°$ ...(i)



and $\dfrac{a}{b} = \sin 30°$

$\Rightarrow \dfrac{2}{b} = \sin 30°$ $(\because a = 2)$

$\Rightarrow \dfrac{2}{b} = \dfrac{1}{2}$ $\left(\because \sin 30° = \dfrac{1}{2}\right)$

$\Rightarrow b = 4$ ...(ii)

and $\dfrac{a}{c} = \tan 30°$

$\Rightarrow \dfrac{2}{c} = \dfrac{1}{\sqrt{3}}$ $\left(\because a = 2, \tan 30° = \dfrac{1}{\sqrt{3}}\right)$

thus $c = 2\sqrt{3}$ ...(iii)

(i), (ii) and (iii) are the required results.

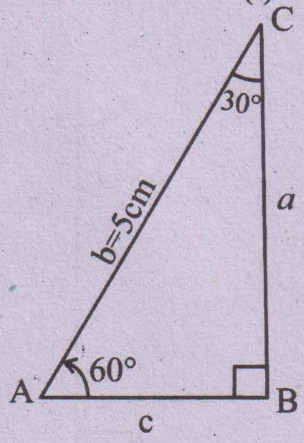
Case II: When measure of the hypotenuse and an angle are given.

Example 16: Solve triangle ABC, when $m\angle A = 60°$, $b = 5$ cm, $m\angle B = 90°$

Solution:

We are required to find a, c and $m\angle C$

$m\angle A = 60°$
$m\angle B = 90°$
$m\angle C = m\angle B - m\angle A$
$= 90° - 60°$
$= 30°$ ...(i)



Now $\dfrac{a}{b} = \sin 60°$

$\dfrac{a}{5} = \dfrac{\sqrt{3}}{2}$

$\left(\because b = 5, \sin 60° = \dfrac{\sqrt{3}}{2}\right)$

$\Rightarrow a = \dfrac{5\sqrt{3}}{2}$

$\Rightarrow a = 4.33$ cm ...(ii)

and $\dfrac{c}{b} = \cos 60°$

$\dfrac{c}{5} = \dfrac{1}{2}$

$\left(\because b = 5, \cos 60° = \dfrac{1}{2}\right)$

$\Rightarrow c = \dfrac{5}{2}$

$\Rightarrow c = 2.5$ cm ...(iii)

(i), (ii) and (iii) are the required results.

Case III: When measure of two sides are given.

Example 17: Solve triangle ABC, when

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$a = \sqrt{2}$ cm, c = 1 cm and m∠B = 90° **09306109**

Solution: We are required to find b, m∠A.

By Pythagoras theorem, we have

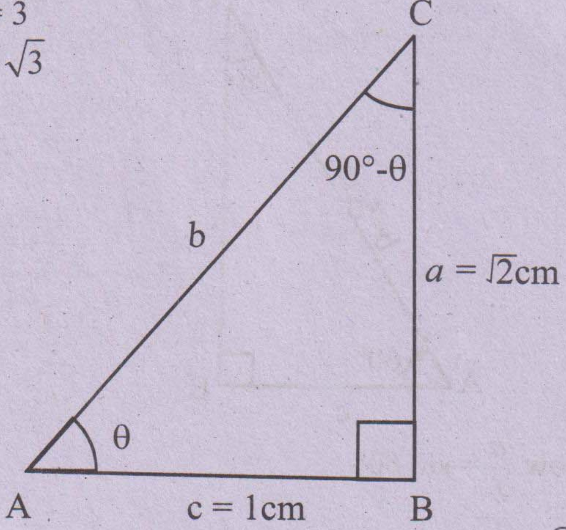
$$b^2 = c^2 + a^2$$

$$b^2 = (1)^2 + \left(\sqrt{2}\right)^2$$

$$b^2 = 1+2$$

$$b^2 = 3$$

$$b = \sqrt{3}$$



Now $\quad \sin m\angle A = \dfrac{a}{b} = \dfrac{\sqrt{2}}{\sqrt{3}}$

$\Rightarrow m\angle A = \sin^{-1}\sqrt{\dfrac{2}{3}} = 54.7°$

$\Rightarrow \; m\angle A = 54.7° \qquad \ldots$(ii)

and $\quad m\angle C = m\angle B - m\angle A$

$= 90° - 54.7°$

$= 35.3° \qquad \ldots$(iii)

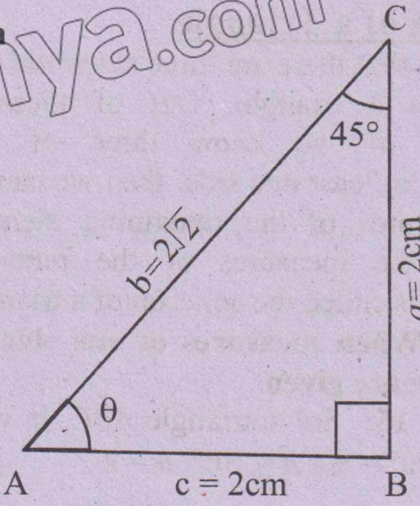
(i), (ii) and (iii) are the required results.

Case IV: When measure of one side and hypotenuse are given.

Example 18: Solve triangle *ABC*, when a

$a = 2$cm, $b = 2\sqrt{2}$ cm and $m\angle B = 90°$ **09306110**

Solution



We are require to find $m\angle A$, $m\angle C$ and c.

By Pythagoras theorem, we have

$$b^2 = a^2 + c^2$$

or $\quad c^2 = b^2 - a^2$

$$c^2 = (2\sqrt{2})^2 - (2)^2$$

$$c^2 = 8 - 4$$

$$c^2 = 4$$

or $\quad c = 2 \qquad \ldots$(i)

Now $\quad \dfrac{c}{b} = \cos m\angle A$

$\Rightarrow \quad \dfrac{2}{2\sqrt{2}} = \cos m\angle A$

$\cos m\angle A = \dfrac{1}{\sqrt{2}} \qquad \left(\because \cos 45° = \dfrac{1}{\sqrt{2}}\right)$

$m\angle A = 45°$

Thus, $\quad m\angle C = m\angle B - m\angle A$

$= 90° - 45°$

$m\angle C = 45° \qquad \ldots$(iii)

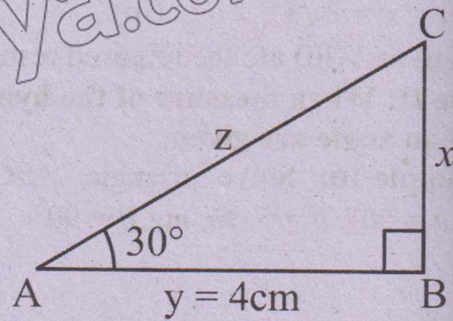
Hence (i), (ii) and (iii) are the required results.

Exercise 6.5

Q.1 Find the values of x, y and z from the following right angled triangles.

(i) **09306111**

Solution:



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From the figure in right angle ΔABC

$m\angle B = 90^\circ$

$m\angle A = \theta = 30^\circ$

$y = 4cm$

$x = ?$ and $z = ?$

Using $\tan\theta = \dfrac{\text{Perp.}}{\text{Base}}$

$$\tan 30^\circ = \frac{x}{y}$$

$$\Rightarrow x = y \tan 30^\circ$$

$$x = 4 \times \frac{1}{\sqrt{3}}$$

$$x = \frac{4}{\sqrt{3}}\text{ cm}$$

Now,

$$\cos\theta = \frac{\text{Base}}{\text{Hyp.}}$$

$$\cos 30^\circ = \frac{y}{z}$$

$$\frac{\sqrt{3}}{2} = \frac{4}{z} \qquad \left(\because \cos 30^\circ = \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow z\sqrt{3} = 2 \times 4$$

$$\Rightarrow z\sqrt{3} = 8$$

$$\Rightarrow z = \frac{8}{\sqrt{3}}\text{ cm}$$

(ii) 09306112

Solution:

In right angle ΔABC

$m\angle B = 90^\circ$

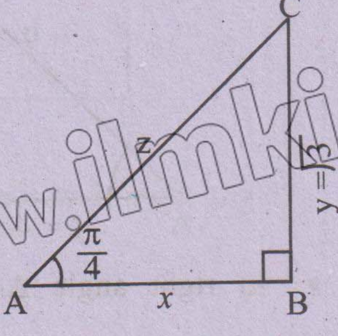
$m\angle A = \dfrac{\pi}{4}$

or

$m\angle A = 45^\circ$

$y = \sqrt{3}$ cm

$x = ?$ and $z = ?$



By using

$$\tan\theta = \frac{\text{Perp.}}{\text{Base}}$$

$$\tan\frac{\pi}{4} = \frac{y}{x}$$

$$\tan 45^\circ = \frac{y}{x} \qquad \left(\because \frac{\pi}{4} = 45^\circ\right)$$

$$\Rightarrow 1 = \frac{\sqrt{3}\text{cm}}{x} \qquad (\because \tan 45^\circ = 1)$$

$$x = \sqrt{3}\text{ cm}$$

now, $\sin\theta = \dfrac{\text{Per.}}{\text{Hyp.}}$

$$\sin\frac{\pi}{4} = \frac{y}{z}$$

$$\sin 45^\circ = \frac{y}{z} \qquad \left(\because \sin 45^\circ = \frac{1}{\sqrt{2}}\right)$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{z}$$

$$\Rightarrow z = \sqrt{2} \times \sqrt{3}$$

$$\Rightarrow z = \sqrt{2 \times 3}$$

$$\Rightarrow z = \sqrt{6}\text{ cm}$$

(iii) 09306113

Solution:

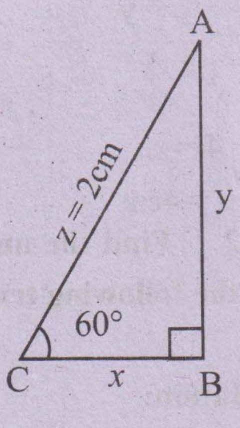
In right angle ΔABC

$m\angle B = 90^\circ$

$m\angle C = 60^\circ$

$z = 2$ cm

$x = ?$ and $y = ?$



Using $\cos\theta = \dfrac{\text{Base}}{\text{Hyp.}}$

$$\cos 60^\circ = \frac{x}{z}$$

$$\Rightarrow x = z \cos 60^\circ$$

$$\Rightarrow x = 2 \times \frac{1}{2} \qquad \left(\because \cos 60^\circ = \frac{1}{2}\right)$$

$$x = 1\text{cm}$$

now, $\sin\theta = \dfrac{\text{Perp.}}{\text{Hyp.}}$

$$\sin 60^\circ = \frac{y}{z}$$

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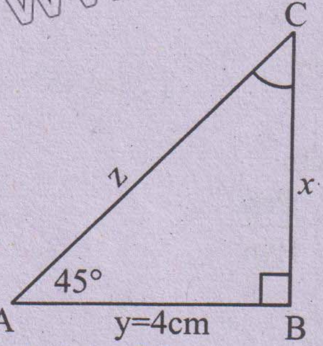
$\Rightarrow \quad y = z \sin 60°$

$\Rightarrow \quad y = 2 \times \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad y = \sqrt{3}$ cm

(iv) **09306114**

Solution:



- In right angled Δ ABC,

$\cos 45° = \dfrac{y}{z}$

$\dfrac{1}{\sqrt{2}} = \dfrac{4}{z} \qquad \left(\because \cos 45° = \dfrac{1}{\sqrt{2}}\right)$

$\Rightarrow z = 4\sqrt{2}$ cm

- In right-angled ΔABC,

$\tan 45° = \dfrac{x}{y}$

$1 = \dfrac{x}{4}$

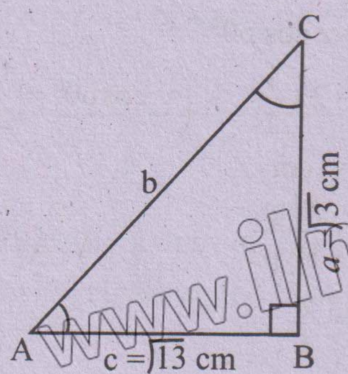
$\Rightarrow 4 = x \qquad (\because \tan 45° = 1)$

$\Rightarrow x = 4$cm

Q.2 Find the unknown side and angles of the following triangles.

(i) **09306115**

Solution:



- In right angled ΔABC, by Pythagoras theorem

$b^2 = c^2 + a^2$

$b^2 = \sqrt{(13)^2} + \sqrt{(3)^2}$

$b^2 = 13 + 3$

$b^2 = 16$

$\sqrt{b^2} = \sqrt{16}$

$\Rightarrow \boxed{6 = 4\text{cm}}$

- In right ΔABC,

$\tan m\angle A = \dfrac{a}{c}$

$\tan m\angle A = \dfrac{\sqrt{3}}{\sqrt{13}}$

$m\angle A = \tan^{-1}\left(\dfrac{\sqrt{3}}{\sqrt{13}}\right) = 25.658°$

$\boxed{m\angle A = 25.7°}$

- In right-angled ΔABC,

$m\angle A + m\angle C = 90°$

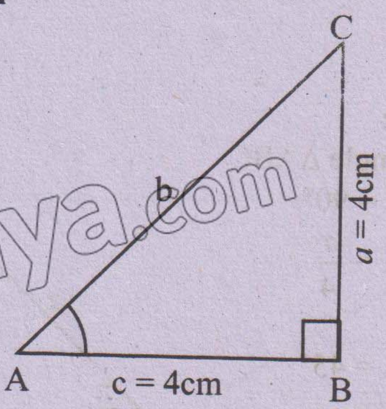
$m\angle C = 90° - m\angle A$

$m\angle C = 90° - 25.7°$

$m\angle C = 64.3°$

(ii) **09306116**

Solution



- In right angle ΔABC by Pyghgora's

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theorem $(\text{Hyp.})^2 = (\text{Base})^2 + (\text{Per.})^2$

$b^2 = c^2 + a^2$

$b^2 = (4)^2 + (4)^2$

$b^2 = 16 + 16$

$b^2 = 32$

Taking square root

$b = \sqrt{32}$

$b = \sqrt{16 \times 2}$

$b = 4\sqrt{2}$ cm

- In right ΔABC,

$\tan m\angle A = \dfrac{a}{c} = \dfrac{4}{4} = 1$

$\tan m\angle A = \tan^{-1}(1)$

$\angle A = 45^\circ$

We know that in right ΔABC

$m\angle A + m\angle C = 90 = 90^\circ$

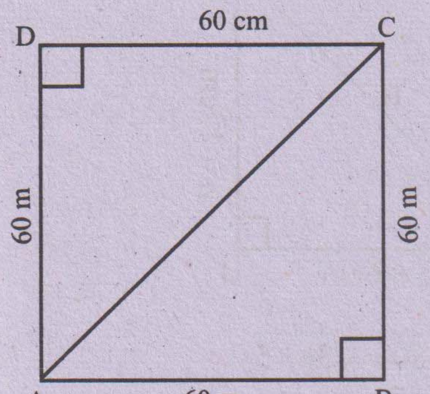
$45^\circ + m\angle C = 90^\circ$

$m\angle A = 90^\circ - 45^\circ = 45$

Q.3 Each side of a square field is 60m long. Find the lengths of the diagonals of the field. **09306117**

Solution:

Let ABCD be the square filed with each side 60m long. Let AC is the one of diagonals.



Applying Pythagora's theorem in right angle ΔABC.

$(\text{Hyp.})^2 = (\text{Base})^2 + (\text{Per.})^2$

$\left(\overline{AC}\right)^2 = (60)^2 + (60)^2$

$\left(\overline{AC}\right)^2 = 3600 + 3600$

$= 7200$

Taking square root

$\left(\overline{AC}\right)^2 = \sqrt{7200}$

$\overline{AC} = \sqrt{3600 \times 2}$

$\overline{AC} = 60\sqrt{2}$ m

as length each diagonal of square is same.

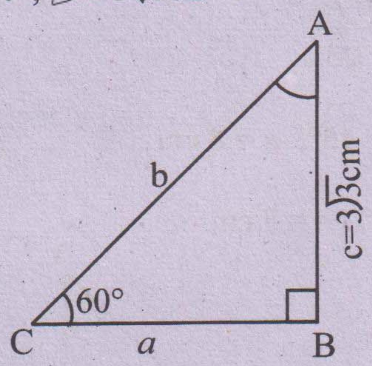
So length of each diagonal is $60\sqrt{2}$ m

Q.4 Solve the following triangles when $m\angle B = 90^\circ$:

(i) $m\angle C = 60^\circ$, $c = 3\sqrt{3}$cm **09306118**

Solution

$m\angle C = 60^\circ$, $c = 3\sqrt{3}$cm



In right angle ΔABC

$m\angle B = 90^\circ$ and

$m\angle C = 60^\circ$

$c = 3\sqrt{3}$ cm

$a = ?$ $b = ?$

and $m\angle A = ?$

As $m\angle A + m\angle B + m\angle C = 180^\circ$

$m\angle A + 90^\circ + 60^\circ = 180^\circ$

$m\angle A + 150^\circ = 180^\circ$

$m\angle A = 180^\circ - 150^\circ = 30^\circ$

$m\angle A = 30^\circ$

Now, using

$$\tan\theta = \frac{\text{Perp.}}{\text{Base}}$$

$$\tan 60° = \frac{c}{a}$$

$$\sqrt{3} = \frac{3\sqrt{3}}{a} \qquad (\because \tan 60° = \sqrt{3})$$

$$\Rightarrow \sqrt{3}a = 3\sqrt{3}$$

$$a = \frac{3\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}$$

$a = 3\text{cm}$

Now, $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}}$

$$\sin 60° = \frac{c}{b}$$

$$\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{b} \qquad \left(\because \sin 60° = \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \sqrt{3}b = 2 \times 3\sqrt{3}$$

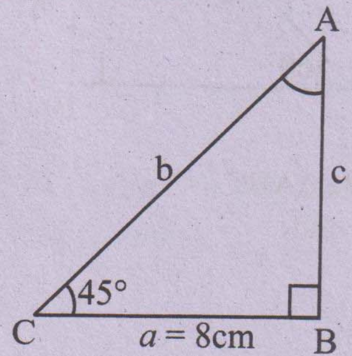
$$\Rightarrow \quad b = \frac{6\sqrt{3}}{\sqrt{3}}$$

$b = 6\text{cm}$

(ii) $m\angle C = 45°$, $a = 8$ cm 09306119

Solution

$m\angle C = 45°$, $a = 8$ cm



- In right angle ΔABC

$m\angle B = 90°$ and $m\angle C = 45°$

$a = 8\text{cm}$

$m\angle A = ?$

$b = ?$ and $c = ?$

We know that in a ΔABC

$m\angle A + m\angle B + m\angle C = 180°$

$m\angle A + 90° + 45° = 180°$

$m\angle A + 135° = 180$

$m\angle B = 180° - 135°$

$m\angle A = 45°$

Now, using

$$\tan\theta = \frac{\text{Perp.}}{\text{Base}}$$

$$\tan 45° = \frac{c}{a} \qquad (\because \tan 45° = 1)$$

$$\Rightarrow \quad 1 = \frac{c}{8\text{cm}}$$

$$\Rightarrow \quad c = 8\text{cm}$$

$$\text{Now}, \cos\theta = \frac{\text{Base}}{\text{Hyp.}}$$

$$\cos 45° = \frac{a}{b}$$

$$\frac{1}{\sqrt{2}} = \frac{8\text{cm}}{b} \qquad \left(\because \cos 45° = \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow b = 8\sqrt{2}\text{ cm}$$

(iii) $a = 12$ cm, $c = 6$ cm 09306120

Solution:

$m\angle B = 90°$, $a = 12\text{cm}$, $c = 6\text{cm}$

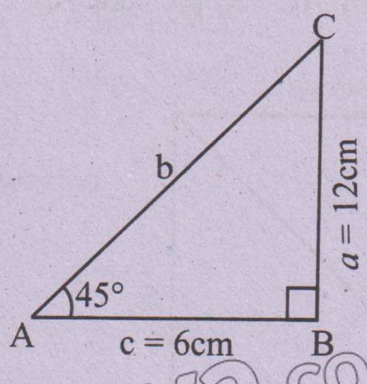
- In right angled ΔABC by Pythagoras theorem

$b^2 = c^2 + a^2$

$b^2 = (6)^2 + (2)^2$

$b^2 = 36 + 144$

$b^2 = 180$



$\sqrt{b} = \sqrt{180} = \sqrt{36 \times 5}$

$\boxed{b = 6\sqrt{5}\text{cm}}$

- In right angled ΔABC,

$$\tan m\angle A = \frac{a}{c}$$

$\tan m\angle A = \dfrac{12}{6} = 2$

$\Rightarrow m\angle A = \tan^{-1}(2)$

$m\angle A = 63.4°$

- In right-angled ΔABC,

$m\angle A + m\angle C = 90°$

$m\angle C = 90° - m\angle A$

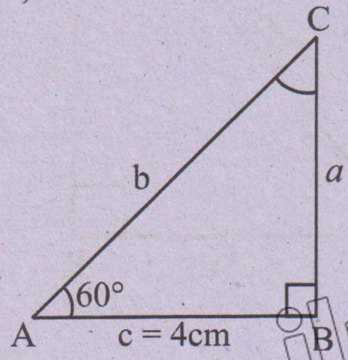
$m\angle C = 90° - 63.43°$

$m\angle C = 26.6°$

(iv) $m\angle A = 60°$, $c = 4$ cm **09306121**

Solution

$m\angle A = 60°$, $c = 4$ cm



- In right angle ΔABC $m\angle B = 90°$ and $m\angle A = 60°$

$c = 4$cm

$m\angle C = ?$

$a = ?$ and $b = ?$

We know that

As $m\angle A + m\angle B + m\angle C = 180°$

$60° + 90° + m\angle C = 180°$

$150° + m\angle C = 180°$

$m\angle C = 180° - 150°$

$m\angle C = 30°$

- Now using

$\tan\theta = \dfrac{\text{Perp.}}{\text{Base}}$

$\Rightarrow \tan 60° = \dfrac{a}{c}$

$\Rightarrow \sqrt{3} = \dfrac{a}{4}$ $\quad (\because \tan 60° = \sqrt{3})$

$a = 4\sqrt{3}$ cm

Now $\cos\theta = \dfrac{\text{Base}}{\text{Hyp.}}$

$\cos 60° = \dfrac{c}{b}$

$\dfrac{1}{2} = \dfrac{4}{b}$

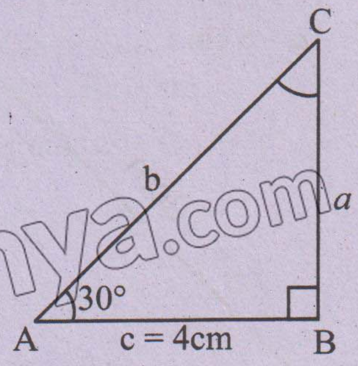
$\Rightarrow b \times 1 = 4 \times 2$ $\quad \left(\because \cos 60° = \dfrac{1}{2}\right)$

$b = 8$cm

(v) $m\angle A = 30°$, $c = 4$ cm **09306122**

Solution:

$m\angle A = 30°$, $c = 4$ cm



- In right angle ΔABC $m\angle B = 90°$ and $m\angle A = 30°$

$c = 4$cm

$m\angle C = ?$

$a = ?$ and $b = ?$

We know that

$m\angle A + m\angle B + m\angle C = 180°$

$30° + 90° + m\angle C = 180°$

$120° + m\angle C = 180°$

$m\angle C = 180° - 120°$

$m\angle C = 60°$

Now using

$\tan\theta = \dfrac{\text{Per.}}{\text{Base}}$

$\tan 30° = \dfrac{a}{c}$

$\dfrac{1}{\sqrt{3}} = \dfrac{a}{4}$

$\dfrac{1}{\sqrt{3}} \times 4 = a$

$\Rightarrow \ a = \dfrac{4}{\sqrt{3}}$ cm

Now $\quad \cos\theta = \dfrac{\text{Base}}{\text{Hyp.}}$

$\cos 30° = \dfrac{c}{b}$

$\dfrac{\sqrt{3}}{2} = \dfrac{4}{b}$

$\Rightarrow \ \sqrt{3}\,b = 2 \times 4$

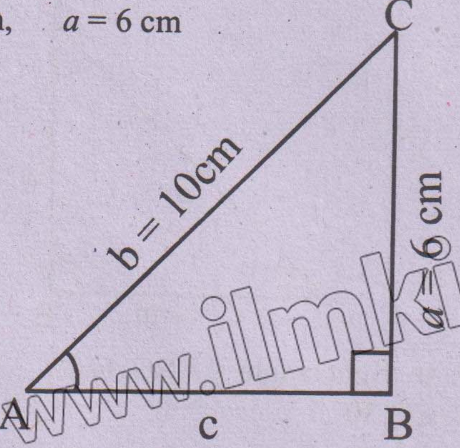
$\sqrt{3}\,b = 8$

$b = \dfrac{8}{\sqrt{3}}$ cm

(vi) b = 10 cm, $\quad$ *a* = 6 cm $\qquad$ **09306123**

Solution:

b = 10 cm, $\quad a = 6$ cm



- In right – angled ΔABC,

By Pythagoras theorem

$b^2 = a^2 + c^2$

$(10)^2 = (6)^2 + c^2$

$100 = 36 + c^2$

$100 - 36 = c^2$

$64 = c^2$

$\Rightarrow \ c^2 = 64$

$\sqrt{c^2} = 64$

$\boxed{c = 8\text{cm}}$

- In right angled ΔABC,

$\sin m\angle A = \dfrac{a}{b}$

$\sin m\angle A = \dfrac{6}{10} = \dfrac{3}{5}$

$m\angle A = \sin^{-1}\left(\dfrac{3}{5}\right)$

$m\angle A = 36.869$

$m\angle A = 36.9°$

- In right-angled ΔABC,

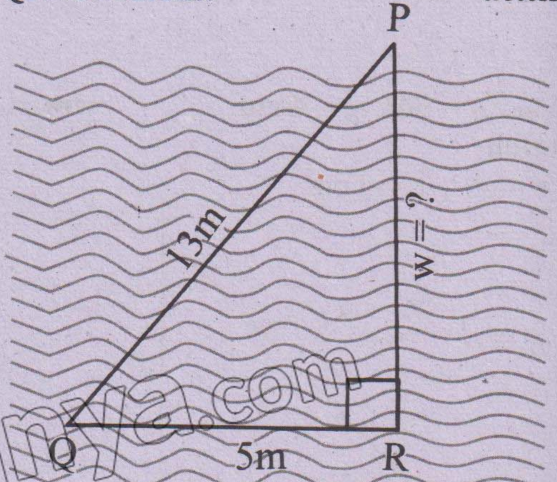
$m\angle A + m\angle C = 90°$

$m\angle C + = 90° - m\angle A$

$m\angle C + = 90° - 36.9°$

$m\angle C = 53.1°$

Q.5 $\quad$ Let *Q* and *R* be the two points on the same bank of a canal. The point *P* is placed on the other bank straight to point *R*. Find the width of the canal and angle PQR in radians. $\qquad$ **09306124**



Solution

From the figure width of canal = $\overline{PR} = w = ?$

$m\overline{PQ} = 13$m and $m\overline{QR} = 5$m

In right angle Δ PQR By Pythagora's theorem $\left(m\overline{PR}\right)^2 + \left(m\overline{QR}\right)^2 = \left(m\overline{PQ}\right)^2$

$\left(m\overline{PR}\right)^2 + (5)^2 = (13)^2$

$\left(m\overline{PR}\right)^2 = 169 - 25$

$\left(m\overline{PR}\right)^2 = 144$

Taking square root of both sides

$m\overline{PR} = 12$cm

In Δ PQR

$\cos\theta = \dfrac{\overline{QR}}{\overline{QP}}$

$\cos\theta = \dfrac{5}{13}$

$\theta = \cos^{-1}\left(\dfrac{5}{13}\right)$

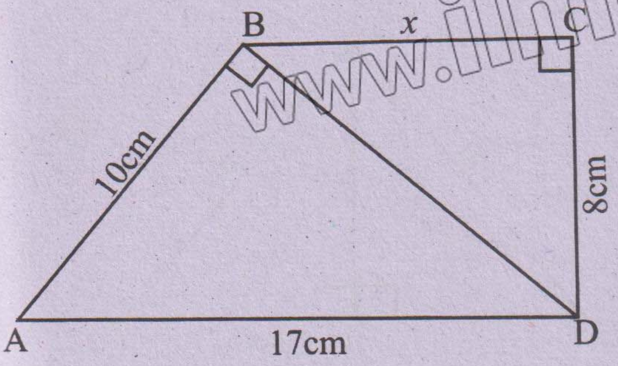
$\theta = 67.38°$

$\theta = 67.38 \times \dfrac{\pi}{180}$ rad

$\theta = 1.8$ rad

So, width of canal is w = 12 m

Q.6 Calculate the length x in the adjoining figure. 09306125



Solution:

In right angle ΔABD By Pythagora's theorem.

$$\left(m\overline{BD}\right)^2+\left(m\overline{AB}\right)^2=\left(m\overline{AD}\right)^2$$

$$\left(m\overline{BD}\right)^2+(10)^2=(17)^2$$

$$\left(m\overline{BD}\right)^2+100=289$$

$$\left(m\overline{BD}\right)^2=289-100=189$$

$$\Rightarrow \boxed{\left(m\overline{BD}\right)^2=189}$$

Now in ΔBCD, by Pythagoras theorem

$$\left(m\overline{BC}\right)^2+\left(m\overline{CD}\right)^2=\left(m\overline{BD}\right)^2$$

$x^2+(8)^2=189$

$x^2=189-64$

$x^2=125$

$\sqrt{x^2}=\sqrt{125}=\sqrt{25\times 5}=5\sqrt{5}\text{cm}$

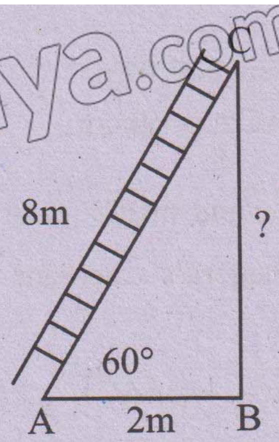
Taking square root

$x=\sqrt{125}$

$=11.18$ cm

Q.7 If the ladder is placed along the wall such that the foot of the ladder is 2 m away from the wall. If the length of the ladder is 8 m, find the height of the wall? 09306126

Solution



Let length of ladder = $m\overline{AC}=8\text{m}$

Distance of ladder from wall = $\overline{AB}=2\text{m}$

Height of wall = $m\overline{BC}=?$

In right angle Δ ABC

By Pythagora's theorem

$$\left(m\overline{BC}\right)^2+\left(m\overline{AB}\right)^2=\left(m\overline{AC}\right)^2$$

$$\left(m\overline{BC}\right)^2+(2)^2=(8)^2$$

$$\left(m\overline{BC}\right)^2+4=64$$

$$\left(\overline{BC}\right)^2=64-4$$

$$\left(m\overline{BC}\right)^2=60$$

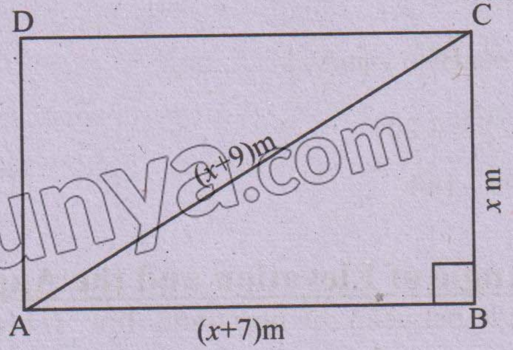
Taking square root

$$\sqrt{\left(m\overline{BC}\right)^2}=\sqrt{60}$$

$m\overline{BC}=7.75\text{m}$

So, height of wall is 7.75m

Q.8 The diagonal of a rectangular field *ABCD* is $(x+9)m$ and the sides are $(x+7)$m and x m. Find the value of x. 09306127



Solution:

ABCD is a rectangular filed.

Diagonal = $m\overline{AC} = (x+9)$ cm

Sides are

$m\overline{AB} = (x+7)$cm and $m\overline{BC} = x$ cm

Applying Pythagora's theorem in right angled ΔABC.

$$\left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 = \left(m\overline{AC}\right)^2$$

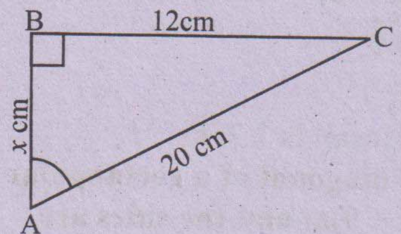
$$(x+7)^2+(x)^2 = (x+9)^2$$
$$(x)^2+2(x)(7)+(7)^2+x^2 = (x^2)+2(x)(9)+(9)^2$$
$$x^2+14x+49+x^2 = x^2+18x+81$$
$$2x^2+14x+49-x^2-18x-81 = 0$$
$$x^2-4x-32 = 0$$
$$x^2-8x+4x-32 = 0$$
$$x(x-8)+4(x-8) = 0$$
$$(x-8)(x+4) = 0$$
$$\Rightarrow x-8 = 0 \text{ or } x+4 = 0$$
$$\boxed{x=8} \quad \text{or} \quad \boxed{x=-4}$$

Since x is a length of a side of rectangle which cannot be negative, so we ignore $x = -4$.

So required values is $x = 8$cm.

Q.9 Calculate the value of 'x' in each case. **09306128**

(i)



Solution:

In right angle ΔABC By Pythagora's theorem

$$\left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2 = \left(m\overline{AC}\right)^2$$

$$x^2 + (12)^2 = (20)^2$$

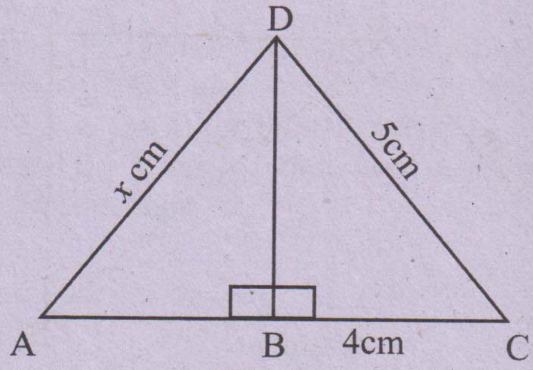
$$x^2 = 400 - 144$$

$$\sqrt{x^2} = \sqrt{256}$$

Taking square root

$x = 16$cm

(ii)



$m = \overline{AB} = \overline{BC}$

Solution

In Δ DBC

$$\left(m\overline{BD}\right)^2 + \left(m\overline{BC}\right)^2 = \left(m\overline{CD}\right)^2$$

$$\left(m\overline{BD}\right)^2 + (4)^2 = (5)^2$$

$$\left(m\overline{BD}\right)^2 + 16 = 25$$

$$\left(m\overline{BD}\right)^2 = 25-16$$

$$\left(m\overline{BD}\right)^2 = 9$$

Given that

$m\overline{AB} = m\overline{BC}$

So, $m\overline{AB} = 4$cm

In right angle ΔABD By Pythagora's theorem

$$\left(m\overline{AD}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BD}\right)^2$$

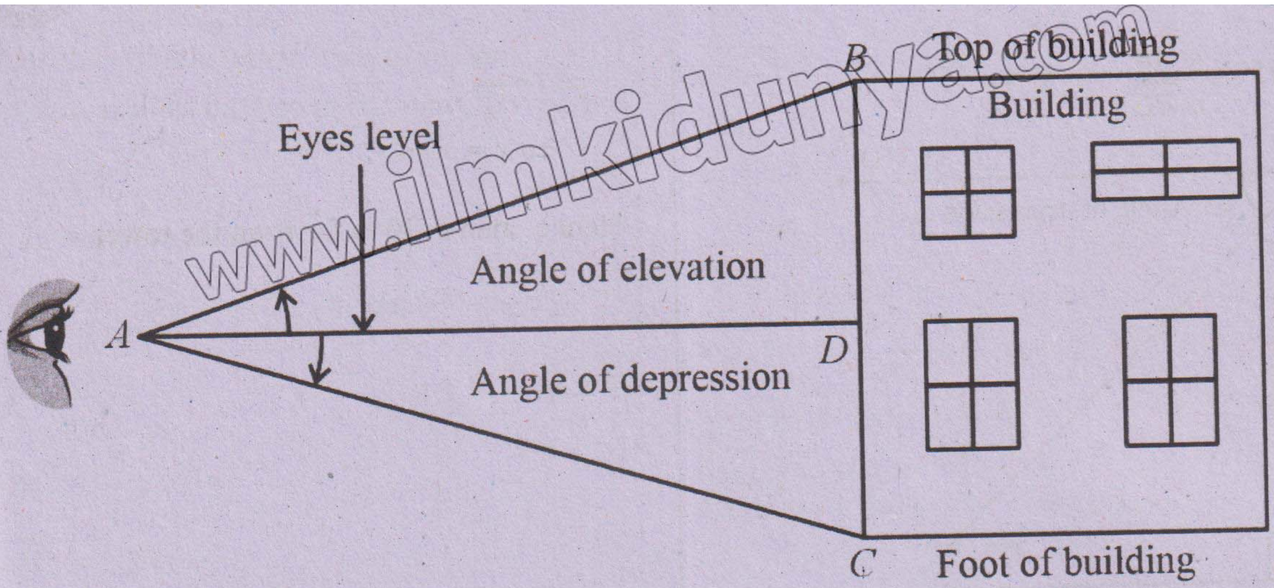
$$x^2 = (4)^2 + 9$$
$$= 16 + 9$$
$$x^2 = 25$$

Taking square root

$$\sqrt{x^2} = \sqrt{25}$$
$$x = 5\text{cm}$$

The Angle of Elevation and the Angle of Depression

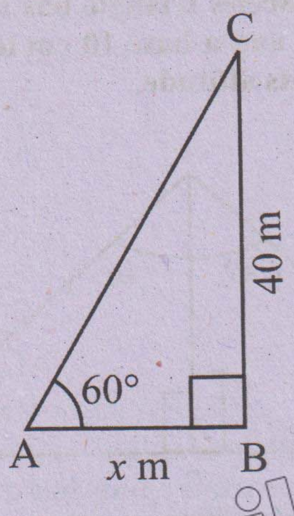
The angle between the horizontal line AD (eye level) and a line from the eye A to top of building (B) the is called an **angle of elevation.**



The angle made between the horizontal line AD (eye level) and of line from the eye 'A' to the bottom of the building (C) is called the angle of depression.

Example 19: The angle of elevation of the top of a pole 40 m high is 60° when seen from a point on the ground level. Find the distance of the point from the foot of the pole. **09306129**

Solution:



In the triangle ABC, we have

$m\overline{BC} = 40$ m

$m\angle A = 60°$

Let $m\overline{AB} = x$

In right angled triangle ABC,

$$\tan 60° = \frac{m\overline{BC}}{mAB}$$

$$\sqrt{3} = \frac{40}{x} \quad (\because \tan 60° = \sqrt{3})$$

$$\Rightarrow x = \frac{40}{\sqrt{3}}$$

$$\Rightarrow 23.09 \text{ m}$$

Hence, distance of the point from the foot of the pole = 23.09 m

Example 20: From the top of a lookout tower, the angle of depression of a building has on the ground level of 45°. How far is a man on the ground form the tower, if the height of the lower is 30 m? **09306130**

Solution: In the triangle ABC, we have

$m\overline{AB} = 30$m

$m\angle CAD = m\angle C = 45°$

$m\overline{BC} = x$m = ?

Let x be right angled triangle ABC,

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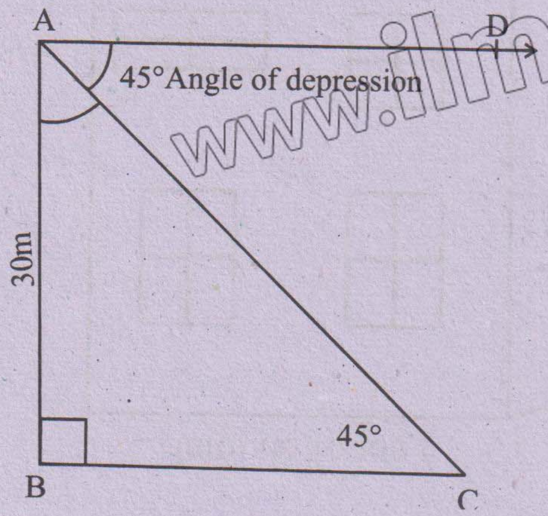
$$45^\circ = \frac{m\overline{AB}}{m\overline{BC}} \qquad (\because \tan 45^\circ = 1)$$



$$\Rightarrow 1 = \frac{30}{x}$$

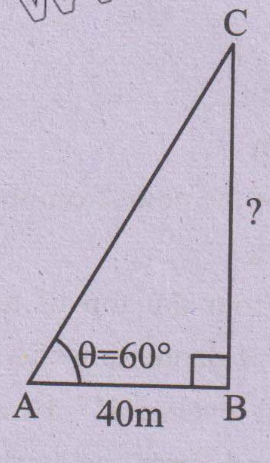
$$\Rightarrow x = 30\text{m}$$

Hence, man is 30 m far from the tower.

Exercise 6.6

Q.1 The angle of elevation of the top of a flag post from a point on the ground level 40 m away from the flag post is 60°. Find the height of the post. **09306131**

Solution



Height of flag post = $m\overline{BC}$ = ?

Distance of point from flag post = $m\overline{AB}$ = 40m

Angle of elevation = θ = 60°

In right-angled ΔABC,

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan 60^\circ = \frac{m\overline{BC}}{40}$$

$\Rightarrow \ m\overline{BC} = 40 \times \tan 60^\circ$

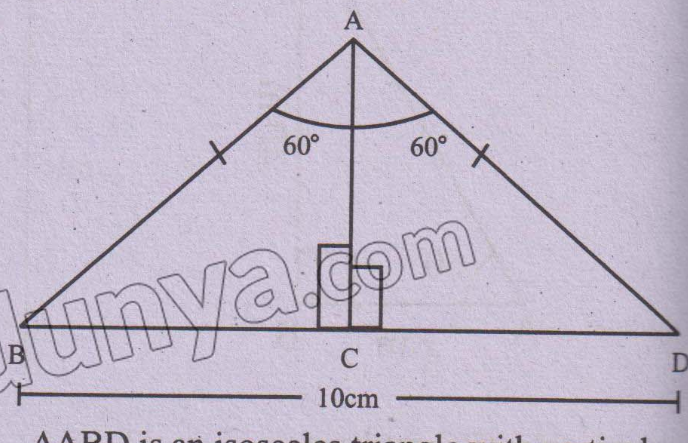
$m\overline{BC} = 40 \times \sqrt{3}$ m

$= 69.28$ m

So height of flag post is 69.28m

Q.2 An isosceles triangle bas a vertical angle of 120° and a base 10 cm long Find the length of its altitude. **09306132**

Solution:



ΔABD is an isosceles triangle with vertical angle 120° and base

$m\overline{BD}$ = 10cm.

$\overline{AC}$ is the altitude which bisects vertical angle, as well as base so $m\overline{BC} = m\overline{CD} = 5cm$

$m\ \overline{AC} = ?$

- In right angle ΔABC

$\tan 60° = \dfrac{m\overline{BC}}{m\overline{AC}}$ $\quad (\because \tan 60° = \sqrt{3})$

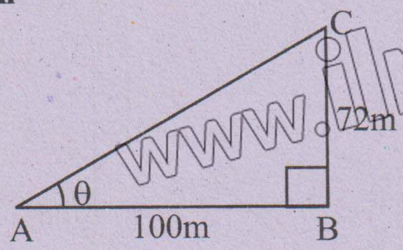
$\sqrt{3} = \dfrac{5}{m\overline{AC}}$

$m\overline{AC} = \dfrac{5}{\sqrt{3}}$

$m\overline{AC} = 2.89cm$

Q.3 A tree is 72 m high. Find the angle of elevation of its top 100 m away on the ground level. **09306133**

Solution



Height of tree $= m\overline{BC} = 72m$

Distance of point from tree $= m\overline{AB} = 100m$

Angle of elevation $= \theta = ?$

As $\tan\theta = \dfrac{m\overline{BC}}{m\overline{AB}}$

$\tan\theta = \dfrac{72}{100}$

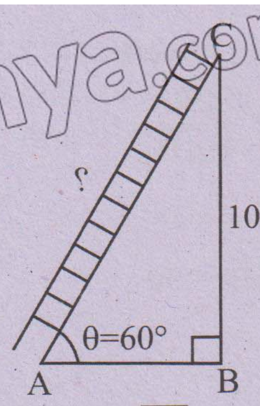
$\theta = \tan^{-1}\left(\dfrac{72}{100}\right)$

$\theta = 35.75°$

So angle of elevation is $35.75°$

Q.4 A ladder makes an angle of 60° with the ground and reaches a height of 10m along the wall. Find the length of the ladder. **09306134**

Solution:



Length of ladder $= m\overline{AC} = ?$

Height of wall $= \overline{BC} = 10m$

Angle made by ladder with ground

$= \theta = 60°$

As $\quad \sin\theta = \dfrac{m\overline{BC}}{m\overline{AC}}$

$\sin 60° = \dfrac{m10}{m\overline{AC}}$

$m\overline{AC} \times \sin 60^0 = 10m$

$\Rightarrow m\overline{AC} = \dfrac{10}{\sin 60°}$

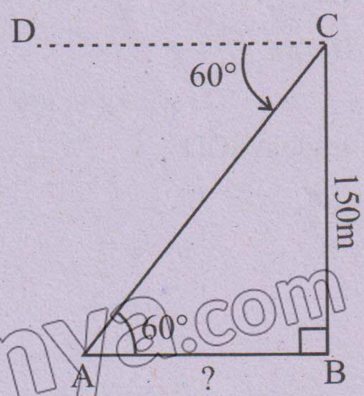
$= 11.55m$

So, Length of ladder is 11.55m

Q.5 A light house tower is 150 m high from the sea level. The angle of depression from the top of the tower to a ship is 60°. Find the distance between the ship and the tower.

09306135

Solution:



Height of light house tower $= m\overline{BC} = 150m$

Angle of depression to boat $= 60°$

Distance of boat from tower $= m\overline{AB} =$

As alternate angles of parallel lines are

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equal, so $\quad m\angle A = 60°$

Now $\quad \tan 60° = \dfrac{m\overline{BC}}{m\overline{AB}}$

$\sqrt{3} = \dfrac{150}{m\overline{AB}}$

$\Rightarrow = \quad \sqrt{3} \times m\overline{AB} = 150$

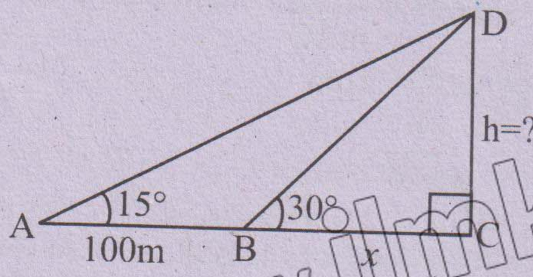
$m\overline{AB} = \dfrac{150}{\sqrt{3}}$

$= 86.60m$

So, required distance is 86.60m

Q.6 Measure of an angle of elevation of the top of a pole is 15° from a point on the ground, in walking 100 m towards the pole the measure of angle is found to be 30°. Find the height of the pole. 09306136

Solution:



Height of pole = $\overline{CD}$ = h=?

Angles of elevation are

$m\angle A = 15°$ and $m\angle B = 30°$

$m\overline{AB} = 100m$

Left $m\overline{BC} = x$

$m\overline{AC} = (x+100)m$

- In right angle ΔBCD

$\tan 30° = \dfrac{m\overline{CD}}{m\overline{BC}}$

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{x}$

$x = \sqrt{3}\,h$…….(i)

- In right angle $\Delta\ ACD$

$\tan 15° = \dfrac{m\overline{CD}}{m\overline{AC}}$

$\tan 15° = \dfrac{h}{100+x}$

$0.268 = \dfrac{h}{100+\sqrt{3}h}$ ….. from (i)

$0.268\,(100+\sqrt{3}\,h) = h$

$26.8+0.464h = h$

$26.8 = h - 0.464\,h$

$26.8 = (1-0.464)\,h$

$26.8 = 0.536\,h$

$h = \dfrac{26.8}{0.536}$

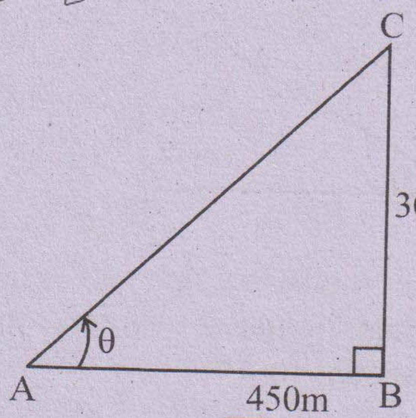
$h = 50m$

So, height of tower is 50m.

Q.7 Find the measure of an angle of elevation of the Sun, if a tower 300 m high casts a shadow 450 m long.

09306137

Solution



Height of tower = $m\overline{BC} = 300m$

$\tan\theta = \dfrac{m\overline{BC}}{m\overline{AB}} = \dfrac{300}{450}$

$\theta = \tan^{-1}\left(\dfrac{300}{450}\right)$

$\theta = 33.69°$

So, angle of elevation of sun is 33.69°

Q.8 Measure of angle of elevation of the top of a cliff is 25°, on walking 100 metres towards the cliff, measure of angle of elevation of the top is 45°. Find the height of the cliff.

09306139

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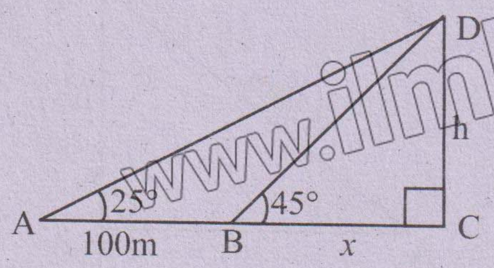
Solution:



From the figure

Height of cliff = $\overline{CD}$ = h =?

Angles of elevation are:

$m\angle A = 25°$

$m\angle B = 45°$

Let $m\overline{BC} = x$

$m\overline{AC} = 100 + x$

- In right angle Δ BCD

$$\tan 45° = \frac{\overline{CD}}{\overline{BC}}$$

$$1 = \frac{h}{x}$$

$\boxed{x = h}$ …….. (i)

- In right angle Δ ACD

$$\tan 25° = \frac{\overline{CD}}{\overline{AC}}$$

$$\tan 25° = \frac{h}{100 + x}$$

$$0.466 = \frac{h}{100 + h} \text{ from (i)}$$

$0.466° (100+h) = h$

$46.6° + 0.466h = h$

$46.6° = h - 0.466\ h$

$46.6° = (1 - 0.466)\ h$

$46.6° = 0.534\ h$

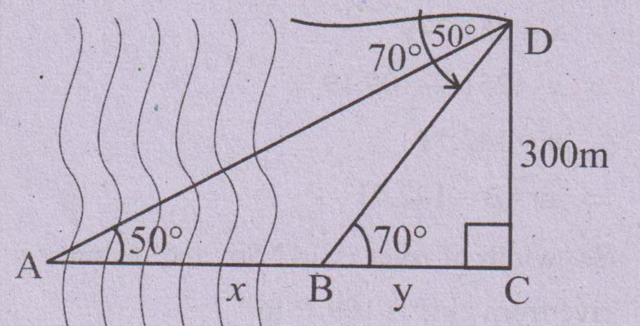
$$h = \frac{46.6}{0.534}$$

$h = 87.27m$

So, height of cliff is 87.27m.

Q.9 From the top of a hill 300 m high, the measure of the angle of depression of a point on the nearer shore of the river is 70^o and measure of the angle of depression of a point, directly across the river is 50^o. Find the width of the river How far is the river from the foot of the hill? **09306140**

Solution:



Height of hill = $\overline{CD}$ = 300m

Angles of depression of two shores from top of hill are 70° and 50° respectively.

As alternate angles of parallel lines are equal so $m\angle A = 50°$ and $m\angle B = 70°$

Width of river = $\overline{AB} = x$

Let distance of river from cliff = $\overline{BC}$ = y = ?

- In right angle Δ BCD

$$\tan 70° = \frac{\overline{CD}}{\overline{BC}}$$

$$\tan 70° = \frac{300}{y}$$

$$y = \frac{300}{\tan 70°}$$

$y = 109.19m$ …………(i)

- In right angle Δ ACD

$$\tan 50° = \frac{m\overline{CD}}{m\overline{AC}}$$

$$\Rightarrow \tan 50° = \frac{300}{x+y}$$

$$\tan 50° = \frac{300}{x+y}$$

$(x+y) \tan 50° = 300°$

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$x+y = \dfrac{300}{\tan 50°}$

$x+y = 251.7$

$x = 251.7-y$

$x = 251.7-109.19$

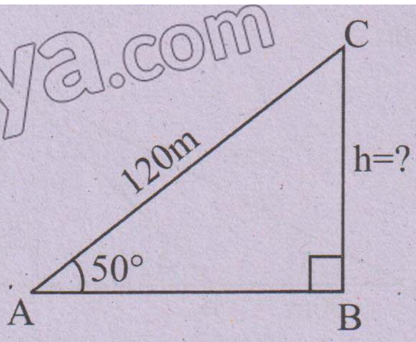
$x = 142.51$

$\Rightarrow\ m\overline{AB} = 142.51$

So, width of river is 142.5m and distance of rive from cliff is 109.19m

Q.10 A kite has 120 m of string attached to it when at an elevation of 50°. How far is it above the hand holding it? (Assume that the string is tight. **09306141**

Solution



Length of string = $\overline{AC} = 120m$

Angle of elevation = $\theta = 50°$

Height above = $\overline{BC} = h = ?$

As $\sin 50° = \dfrac{BC}{AC}$

$\text{Sin}50° = \dfrac{h}{120}$

$120\times\sin 50° = h$

$120\times 0.766 = h$

$91.92 = h$

$\Rightarrow h = 91.92m$

$= 91.92m$

So, required altitude is 91.92m.

Review Exercise 6

Q.1 Choose the correct option.

i. The value of $\tan^{-1}\left(\dfrac{1}{2}\right)$ in radians is: **09306142**

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{3\pi}{2}$

(c) $0.4636\ \pi$ (d) 0.4636

ii. In a right triangle, the hypotenuse is 13 units and one of the angles is $\theta = 30°$. What is the length of the opposite side? **09306143**

(a) 6.5 units (b) 7.5 units

(c) 6 units (d) 5 units

iii. A person standing 50 m away from a building sees the top of the building at an angle of elevation of 45°. Height of the building is: **09306144**

(a) 50 m (b) 25 m

(c) 35 m (d) 70 m

iv. $\text{Sec}^2\theta-\tan^2\theta =$ _________.

(a) $\sin^2\theta$ (b) 1

(c) $\cos^2\theta$ (d) $\cot^2\theta$

v. If $\sin\theta = \dfrac{3}{5}$, and θ is an acute angle, $\cos^2\theta =$ ___________. **09306145**

(a) $\dfrac{7}{25}$ (b) $\dfrac{24}{25}$

(c) $\dfrac{16}{25}$ (d) $\dfrac{4}{25}$

vi. $\dfrac{5\pi}{24}$ rad = _________ degrees. **09306146**

(a) 30° (b) 37.5°

(c) 45° (d) 52.5°

vii. $292.5° =$ __________ rad. **09306147**

(a) $\dfrac{17\pi}{6}$ (b) $\dfrac{17\pi}{4}$

(c) 1.6π (d) 1.625π

viii. Which of the following is a valid identity? 09306148

(a) $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$

(b) $\cos\left(\frac{\pi}{2}-\theta\right)=\cos\theta$

(c) $\cos\left(\frac{\pi}{2}-\theta\right)=\sec\theta$

(d) $\cos\left(\frac{\pi}{2}-\theta\right)=\operatorname{cosec}\theta$

ix. $\sin 60° =$ ___________. 09306149

(a) 1 (b) $\frac{1}{2}$

(c) $\sqrt{(3)^2}$ (d) $\frac{\sqrt{3}}{2}$

x. $\cos^2 100\pi + \sin^2 100\pi =$ ___________. 09306150

(a) 1 (b) 2

(c) 3 (d) 4

Answer Key

i	**d**	ii	**a**	iii	**a**	iv	**b**	v	**c**
vi	**b**	vii	**d**	viii	**a**	ix	**d**	x	**a**

Multiple Choice Questions (Additional)

Angle and it suits

1. The plane figure formed by two rays sharing a common endpoint is called: 09306151

(a) an angle (b) a degree

(c) triangle (d) a radian

2. In sexagesimal system of measurement, the angle is measured in: 09306152

(a) Radian (b) Gradian

(c) °C (d) $D^oM'S''$

3. $25° =$ ………. 09306153

(a) $360'$ (b) $630'$

(c) $1500'$ (d) $9000'$

4. $\frac{5\pi}{4}$ radians = 09306154

(a) 125° (b) 135°

(c) 150° (d) 225°

5. 2π radians = 09306155

(a) 0^o (b) 90^o

(c) 180^o (d) 360^o

6. π radians = 09306156

(a) 0^o (b) 90^o

(c) 180^o (d) 360^o

7. $\frac{\pi}{2}$ radians = 09306157

(a) 30^o (b) 45^o

(c) 60^o (d) 90^o

8. $\frac{\pi}{3}$ radians = 09306158

(a) 30^o (b) 45^o

(c) 60^o (d) 90^o

9. $\frac{\pi}{4}$ radians = 09306159

(a) 30^o (b) 45^o

(c) 60^o (d) 90^o

10. $\frac{\pi}{6}$ radians = 09306160

(a) 30^o (b) 45^o

(c) 60^o (d) 90^o

11. $\frac{3\pi}{2}$ radians = 09306161

(a) 90^o (b) 180^o

(c) 270^o (d) 360^o

Trigonometric ratios

12. Fundamental trigonometric ratios are: 09306162

(a) 3 (b) 4

(c) 5 (d) 6

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13. $\sin\theta\times\cos\theta\times\sec\theta\times\text{cosec}\theta$ =----

(a) $\sin\theta\times\cos\theta$ (b) $\tan\theta+\cot\theta$
(c) $\sec\theta\times\text{cosec}\theta$ (d) $\tan\theta\times\cot\theta$

Trigonometric identities

14. $1+\tan^2\theta=$
(a) $\sin^2\theta$ (b) $\sec^2\theta$ =
(c) $\cos^2\theta$ (d) $\text{cosec}^2\theta$

15. $\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}$ 09306163

(a) $2\sec^2\theta$ (b) $2\cos^2\theta$
(c) $2\text{cosec}^2\theta$ (d) $2\tan^2\theta$

16. $\dfrac{1}{2}$ cosec45°= 09306164

(a) $\dfrac{1}{2\sqrt{2}}$ (b) $\dfrac{1}{\sqrt{2}}$
(c) $\sqrt{2}$ (d) $\dfrac{\sqrt{3}}{2}$

17. $\sec\theta\cot\theta$ = 09306165
(a) $\sin\theta$ (b) $\text{cosec}\theta$
(c) $\sec\theta$ (d) $\tan\theta$

18. $\cot^2\theta-\text{cosec}^2\theta$ = 09306166
(a) -1 (b) 1
(c) 0 (d) $\tan\theta$

Values of Trigonometric Ratios

19. $\sin 45^\circ\cos 45^\circ$ = ---- = 09306167

(a) 1 (b) $\sqrt{2}$
(c) $\dfrac{1}{\sqrt{2}}$ (d) $\dfrac{1}{2}$

20. $\cos 30^\circ\tan 45^\circ$ = ---- = 09306168
(a) $\dfrac{2}{\sqrt{3}}$ (b) $\dfrac{1}{\sqrt{3}}$
(c) $\dfrac{1}{\sqrt{2}}$ (d) $\dfrac{\sqrt{3}}{2}$

21. $\sec 45^\circ\,\text{cosec}\,45^\circ$ = --- 09306169
(a) 2 (b) $\sqrt{2}$
(c) $\dfrac{1}{2}$ (d) $\dfrac{1}{\sqrt{2}}$

22. $\cot 45^\circ+\tan 45^\circ$ = --- 09306170
(a) $\sqrt{2}$ (b) 2
(c) $\dfrac{1}{\sqrt{2}}$ (d) 1

23. $\sin 30^\circ\cos 30^\circ$ = --- 09306171
(a) $\dfrac{1}{2}$ (b) $\dfrac{\sqrt{3}}{2}$
(c) $\dfrac{\sqrt{3}}{4}$ (d) $\dfrac{2}{\sqrt{3}}$

24. $\tan 30^\circ\sin 60^\circ$ = --- 09306172
(a) $\dfrac{1}{2}$ (b) $\dfrac{\sqrt{3}}{2}$
(c) $\sqrt{3}$ (d) $\dfrac{1}{\sqrt{3}}$

25. If $\tan\theta=\sqrt{3}$, then θ is equal to: 09306173
(a) 90° (b) 45°
(c) 60° (d) 30°

Answer Key

1	a	2	d	3	c	4	d	5	d	6	c	7	d	8	c	9	b	10	a
11	c	12	d	13	d	14	b	15	a	16	b	17	b	18	a	19	d	20	d
21	a	22	b	23	c	24	a	25	c										

Q.2 Convert the given angles from:

(a) degrees to radians giving answer in terms of π.

(i) 255° 09306174

Solution

255°

$= 255\times\dfrac{\pi}{180}$ rad

$=\dfrac{51\pi}{36}$ rad $=\dfrac{17\pi}{12}$ rad

(ii) 75° 45′ 09306175

Solution:

75° 45′

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$$= 75°+\left(\frac{45}{60}\right)^\circ$$
$$= 75.75°$$
$$= 75.75\times\frac{\pi}{180}\text{ rad}$$
$$= \frac{7575}{100}\times\frac{\pi}{180}\text{ rad}$$
$$= \frac{101}{240}\text{ rad}$$

(iii) 142.5° 09306176

Solution:

142.5°
$$= 142.5\times\frac{\pi}{180}\text{ rad}$$
$$= \frac{1425}{10}\times\frac{\pi}{180}\text{ rad}$$
$$= \frac{19\pi}{24}\text{ rad.}$$

(b) radians to degrees giving answer in degrees and minutes.

(i) $\frac{17\pi}{24}$ 09306177

Solution:

$$\frac{17\pi}{24}\text{ rad}$$
$$= \left(\frac{17\cancel{\pi}}{24}\times\frac{180}{\cancel{\pi}}\right)^\circ = 127°+0.5°$$
$$= 127.5° = 127°+(0.5\times60)'$$
$$= 127°30'$$

(ii) $\frac{7\pi}{12}$ 09306178

Solution

$$\frac{7\pi}{12}$$
$$= \frac{7\pi}{12}\text{ rad}$$
$$= \left(\frac{7\cancel{\pi}}{\cancel{12}}\times\frac{{}^{15}\cancel{180}}{\cancel{\pi}}\right)^\circ$$
$$= (7\times15)°$$
$$= 105°$$

(iii) $\frac{11\pi}{16}$ 09306179

Solution:

$$\frac{11\pi}{16}$$
$$= \frac{11\pi}{16}\text{ rad}$$
$$= \left(\frac{11\pi}{\cancel{16}_4}\times\frac{{}^{45}180}{\pi}\right)^\circ$$
$$= 123.75° = 123°+0.75°$$
$$= 123°+(0.75\times60)'$$
$$= 123°+45' = 123°45'$$

Q.3 Prove the following trigonometric identities: 09306180

(i) $\frac{\sin\theta}{1-\cos\theta}=\frac{1+\cos\theta}{\sin\theta}$

Solution

$$\frac{\sin\theta}{1-\cos\theta}=\frac{1+\cos\theta}{\sin\theta}$$
$$\text{L.H.S} = \frac{\sin\theta}{1-\cos\theta}$$
$$= \frac{\sin\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}$$
$$= \frac{\sin\theta(1+\cos\theta)}{(1)^2-(\cos\theta)^2}$$
$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$
$$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}\quad[\because \sin^2\theta+\cos^2\theta=1]$$
$$= \frac{\sin\theta(1+\cos\theta)}{\sin\theta\sin\theta}$$
$$= \frac{1+\cos\theta}{\sin\theta}$$
= R.H.S

(ii) $\sin\theta(\text{cosec}\,\theta-\sin\theta)=\frac{1}{\sec^2\theta}$ 09306181

Solution

$$\sin\theta(\text{cosec}\,\theta-\sin\theta)=\frac{1}{\sec^2\theta}$$
$$\text{L.H.S} = \sin\theta(\text{cosec}\theta-\sin\theta)$$

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$= \sin\theta.\text{cosec}\theta - \sin\theta.\sin\theta$

$(\because \sin\theta.\text{cosec}\theta = 1)$

$= 1 - \sin^2\theta$

$= \cos^2\theta \qquad [\because \sin^2\theta + \cos^2\theta = 1]$

$= \dfrac{1}{\sec^2\theta}$

= R.H.S

Hence proved.

(iii) $\dfrac{\textbf{cosec}\,\theta - \textbf{sec}\theta}{\textbf{cosec}\,\theta + \textbf{sec}\theta} = \dfrac{1 - \tan\theta}{1 + \tan\theta}$ **09306182**

Solution

$$\frac{\text{cosec}\,\theta - \sec\theta}{\text{cosec}\,\theta + \sec\theta} = \frac{1 - \cos\theta}{1 + \tan\theta}$$

$$\text{L.H.S} = \frac{\text{cosec}\theta - \sec\theta}{\text{cosec}\theta + \sec\theta}$$

$$= \frac{\dfrac{1}{\sin\theta} - \dfrac{1}{\cos\theta}}{\dfrac{1}{\sin\theta} + \dfrac{1}{\cos\theta}}$$

Multiplying and dividing by $\sin\theta$

$$= \frac{\sin\theta\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\cos\theta}\right)}{\sin\theta\left(\dfrac{1}{\sin\theta} + \dfrac{1}{\cos\theta}\right)}$$

$$= \frac{\dfrac{\sin\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}}$$

$$= \frac{1 - \tan\theta}{1 + \tan\theta}$$

= R.H.S

Hence proved.

(iv) $\textbf{tan}\theta + \textbf{cot}\theta = \dfrac{1}{\sin\theta\,\cos\theta}$ **09306183**

Solution

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\,\cos\theta}$$

L.H.S $= \tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

= R.H.S

Hence proved.

(v) $\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \dfrac{\cos\theta - \sin\theta}{\cos\theta - \sin\theta} = \dfrac{2}{1 - 2\sin^2\theta}$ **09306184**

Solution

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{2}{1 - 2\sin^2\theta}$$

$$\text{L.H.S} \quad \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

$$\boxed{\begin{array}{l} \because (a+b)^2 + (a-b)^2 = 2(a^2 + b^2) \\ (a-b)(a+b) = a^2 - b^2 \end{array}}$$

$$= \frac{2\left[(\cos\theta)^2 + (\sin\theta)^2\right]}{(\cos\theta)^2 - (\sin\theta)^2}$$

$$= \frac{2(\cos^2\theta + \sin^2\theta)}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2(1)}{(1 - \sin^2\theta) - \sin^2\theta} \qquad (\because \cos^2\theta = 1 - \sin^2\theta)$$

$$= \frac{2}{1 - \sin^2\theta - \sin^2\theta}$$

$$= \frac{2}{1 - 2\sin^2\theta}$$

= R.H.S. Hence proved.

(vi) $\dfrac{1 + \cos\theta}{1 - \cos\theta} = (\textbf{cosec}\,\theta + \textbf{cot}\theta)^2$ **09306185**

Solution

$$\frac{1 + \cos\theta}{1 - \cos\theta} = (\text{cosec}\,\theta + \cot\theta)^2$$

$$\text{L.H.S} = \frac{1 + \cos\theta}{1 - \cos\theta}$$

$= (1 + \cos\theta) \div (1 - \cos\theta)$

dividing by $\sin\theta$

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$$=\frac{(1+\cos\theta)}{\sin\theta}\div\frac{(1-\cos\theta)}{\sin\theta}$$

$$=\left(\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\right)\div\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)$$

$$\left(\because \frac{\cos\theta}{\sin\theta}=\cot\theta\right)$$

$$=(\text{cosec}\theta+\cot\theta)\div(\text{cosec}\theta-\cot\theta)$$

$$=\frac{\text{cosec}\theta+\cot\theta}{\text{cosec}\theta-\cot\theta}$$

$$=\frac{(\text{cosec}\theta+\cot\theta)}{(\text{cosec}\theta-\cot\theta)}\times\frac{(\text{cosec}\theta+\cot\theta)}{(\text{cosec}\theta+\cot\theta)}$$

$$=\frac{(\text{cosec}\theta+\cot\theta)^2}{\text{cosec}^2\theta-\cot^2\theta}$$

$$=\frac{(\text{cosec}\theta+\cot\theta)^2}{1}$$

$$\boxed{\because 1+\cot^2\theta=\text{cosec}^2\theta}$$

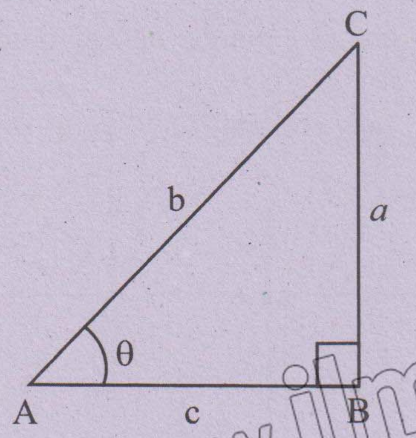
$$=(\text{cosec}\theta+\cot\theta)^2$$

$= \text{R.H.S}$

Hence proved.

Q.4 If $\tan\theta = \frac{3}{\sqrt{2}}$, then find the remaining trigonometric ratios when θ lies in first quadrant. **09306186**

Solution



$\tan\theta = \frac{3}{\sqrt{2}}$ ______ (i)

Let ΔABC is a right angled in which

$m\angle B = 90°$, $m\angle A = \theta = \tan\theta = \frac{a}{c}$ ______ (ii)

Comparing (i) and (ii),

$$\frac{a}{c}=\frac{3}{\sqrt{2}} \Rightarrow \boxed{a=3, c=\sqrt{2}}$$

By Pythagoras, theorem

$$b^2 = c^2 + a^2$$

$$b^2 = \left(\sqrt{2}\right)^2 + (3)^2$$

$$b^2 = 2 + 9$$

$$b^2 = 11$$

$$\sqrt{b^2} = \sqrt{11}$$

$$\boxed{b = \sqrt{11}\text{cm}}$$

Remaining ratios

$$\sin\theta = \frac{a}{b} = \frac{3}{\sqrt{11}}$$

$$\cos\theta = \frac{c}{b} = \frac{\sqrt{2}}{\sqrt{11}} = \sqrt{\frac{2}{11}}$$

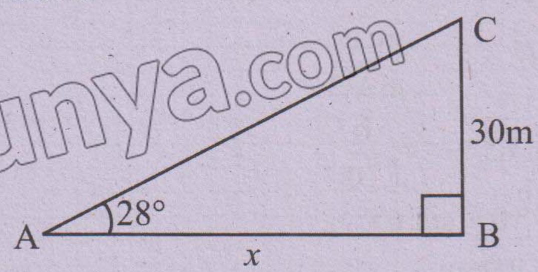
$$\text{cosec}\theta = \frac{b}{a} = \frac{\sqrt{11}}{3}$$

$$\sec\theta = \frac{b}{c} = \frac{\sqrt{11}}{\sqrt{2}} = \sqrt{\frac{11}{2}}$$

$$\cot\theta = \frac{c}{a} = \frac{\sqrt{2}}{3}$$

Q.5 From a point on the ground, the angle of elevation to the top of a 30-meter-high building is 28°. How far is the point from the base of the building? **09306187**

Solution:



Height of building = $m\overline{BC} = 30\text{m}$

Angle of elevation = $\theta = 28°$

Distance of point from building = $m\overline{AB} = ?$

As $\tan 28° = \dfrac{m\overline{BC}}{m\overline{AB}}$

$\Rightarrow \tan 28° = \dfrac{30}{x}$

$\Rightarrow x.\tan 28° = 30$

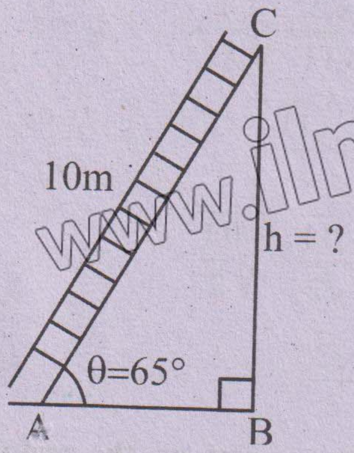
$\Rightarrow x = \dfrac{30}{\tan 28°}$

$x = 56.42\text{m}$

So, required distance is 56.42m

Q.6 A ladder leaning against a wall forms an angle of 65° with the ground. If the ladder is 10 meters long, how high does it reach on the wall? 09306188

Solution:



Length of ladder = $m\overline{AC} = 10\text{m}$

Height of wall = $m\overline{BC} = h = ?$

Angle made by ladder

With ground = $\theta = 65°$

As $\sin\theta = \dfrac{m\overline{BC}}{m\overline{AC}}$

$\sin 65° = \dfrac{h}{10}$

$\Rightarrow h = 10 \times \sin 65°$

$h = 9.06\text{m}$

So, height of wall is 9.06m

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