UNIT 07

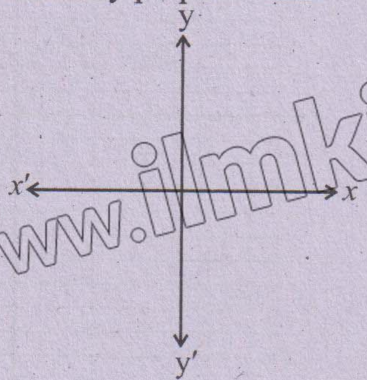
Coordinate Geometry

Introduction

Geometry is one of the most ancient branches of mathematics. The Greeks systematically studied it about four centuries B.C. Most of the geometry taught in schools is due to Euclid who expounded thirteen books on the subject (300 B.C.). A French philosopher and mathematician Rene Descartes (1596-1650 A.D.)

Coordinate Plane

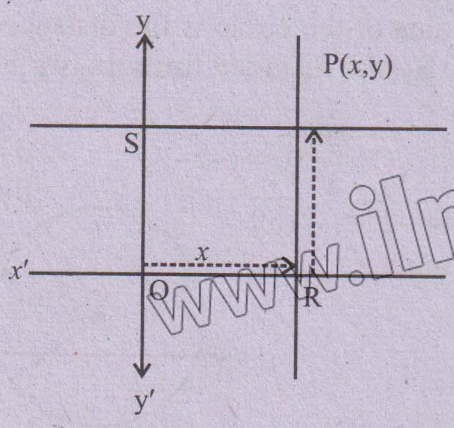
Draw in a plane two mutually perpendicular number lines $x'x$ and $y'y$ one horizontal and the other vertical. Let O be their point of intersection called origin and the real number 0 of both the lines is represented by O.



The two lines are called the **coordinate axes.** The horizontal line $x'Ox$ is called the **x-axis** and the vertical line $y'oy$ is called the **y-axis.**

The points lying on Ox are +ve and on Ox' are –ve.

The points lying on Oy are +ve and Oy′ are –ve.



Suppose P is any point in the plane. Then P can be located by using an ordered pair of real numbers. Through P draw lines parallel to the coordinates axes meeting x-axis at R and y-axis at S. Let the directed distance $\overline{OR} = x$ and the directed distance $\overline{OS} = y$.

The ordered pair (x, y) gives us enough information to locate the point P. Thus P has coordinates (x, y). The first component of the ordered pair (x, y) is called x-coordinate or abscissa and the second component is called y-coordinate or ordinate of P.

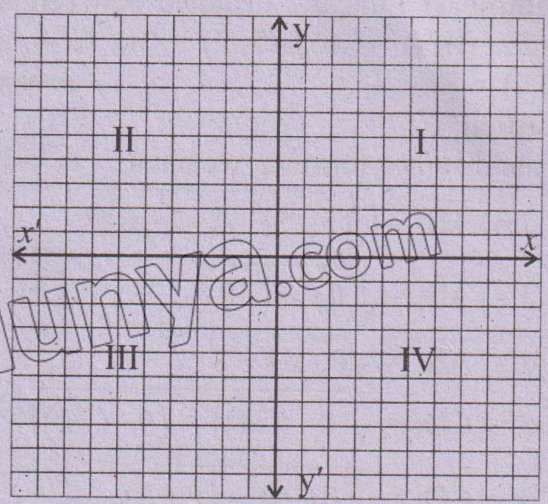
The coordinate axes divide the plane into four equal parts called quadrants. They are defined as follows:

Quadrant I: All points (x, y) with $x > 0$, $y > 0$

Quadrant II: All points (x, y) with $x < 0$, $y > 0$

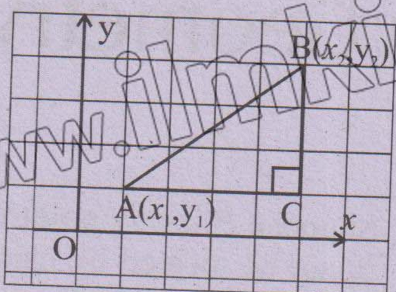
Quadrant III: All points (x, y) with $x < 0$, $y < 0$

Quadrant IV: All points (x, y) with $x > 0$, $y < 0$



The point P in the plane that corresponds to an ordered pair (x, y) is called the graph.

The Distance Formula



Let A (x_1, y_1) and B (x_2, y_2) be two points in the plane. To find the distance d = $|\overline{AB}|$, we draw a horizontal line from A to a point C lies directly below B, forming a right triangle ABC. So that $|\overline{AC}| = |x_2 - x_1|$ and $|\overline{BC}| = |y_2 - y_1|$. By using Pythagoras theorem, we have

$$d^2 = |\overline{AB}|^2 = |\overline{AC}| + |\overline{BC}|^2$$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

or $d = |\overline{AB}| = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$ ...(i)

The distance is always taken to be non-negative. It is not a directed distance from A to B.

If A and B lie on a line parallel to one of the coordinate axes, then by the formula (i), the distance $\overline{AB}$ is absolute value of the directed distance $\overline{AB}$.

The formula (i) shows that any of the two points can be taken as first point.

Example 1: Find the distance between the points: (i) A (5,6),B(5,-2) (ii) C(-4,-2), D(0,9)

09307001

Solution:

By the distance formula, we have

(i) $d = |\overline{AB}| = \sqrt{(5-5)^2 + (-2-6)^2}$

$d = |\overline{AB}| = \sqrt{(0)^2 + (-8)^2}$

$d = |\overline{AB}| = \sqrt{0+64} = 8$

(ii) $d = |\overline{CD}| = \sqrt{(0-(-4))^2 + (9-(-2))}$

$d = |\overline{CD}| = \sqrt{(0+4)^2 + (9+2)^2}$

$d = |\overline{CD}| = \sqrt{4^2 + 11^2}$

$d = |\overline{CD}| = \sqrt{16+121} = \sqrt{137}$

Example 2: Show that the points $A(-1,2)$, $B(7, 5)$ and $C(2, -6)$ are vertices of a right triangle.

09307002

Solution

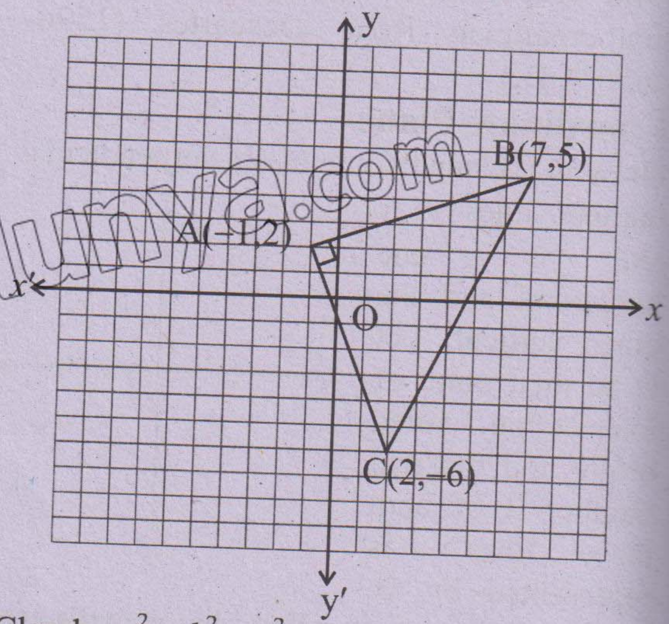
Let a, b and c denote the lengths of the sides $\overline{BC}$, $\overline{CA}$ and $\overline{AB}$ respectively.

By the distance formula, we have

$c = |\overline{AB}| = \sqrt{(7-(-1))^2 + (5-2)^2} = \sqrt{73}$

$a = |\overline{BC}| = \sqrt{(2-7)^2 + (-6-5)^2} = \sqrt{146}$

$b = |\overline{CA}| = \sqrt{(2-(-1))^2 + (-6-2)^2} = \sqrt{73}$



Clearly: $a^2 = b^2 + c^2$

Therefore, ABC is a right triangle with right angle at A.

Example 3: The point C (−5, 3) is the centre of a circle and P (7, −2) lies on the circle. What is the radius of the circle?

09307003

Solution:

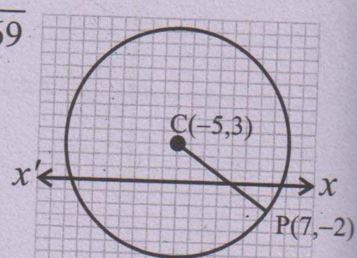
The radius of the circle is the distance from C to P. By the distance formula, we have

Radius =

$r = |\overline{CP}| = \sqrt{(7-(-5))^2 + (-2-3)^2}$

$r = \sqrt{144+25} = \sqrt{169}$

r = 13 units



Mid Point Formula

This formula is particularly useful when you need to divide a line segment into two equal halves or parts.

Derivation of the Midpoint Formula

Consider two points $A(x_1, y_1)$ and $B(x_2, y_1)$ on a two-dimensional plane. The line segment joining these two points has a midpoint $M(x, y)$, where x and y are the coordinates of the midpoint.

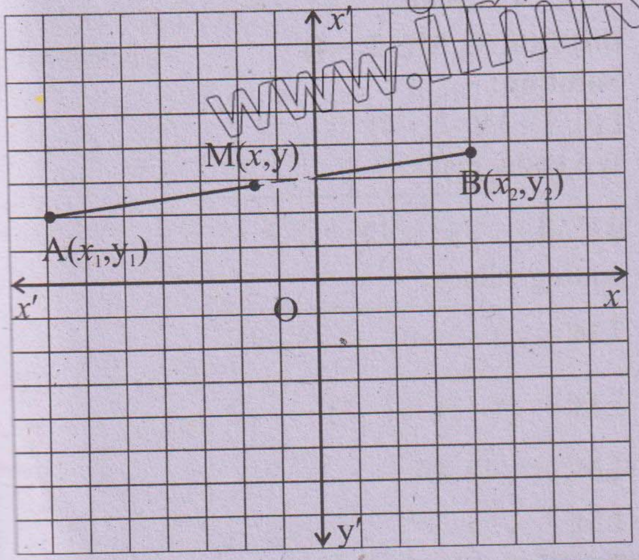
To derive the formula for $M(x, y)$ we need to average the x-coordinates and y-coordinates of points A and B separately.

1. *x*-Coordinate of the Midpoint

The x-coordinate of the midpoint is the average of the x-coordinates of points A and B.

i.e,

$$x = \frac{x_1 + x_2}{2}$$



2. y-Coordinate of the Midpoint

Similarly, the y-coordinate of the midpoint is the average of the y-coordinates of points A and B.

$$y = \frac{y_1 + y_2}{2}$$

Thus, the coordinates of the midpoint $M(x, y)$ are:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Example 4: Find the midpoint of the line segment joining the points A(2,3) and B(8,7). **09307004**

Solution:

Using the midpoint formula:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute $x_1 = 2$, $y_1 = 3$, $x_2 = 8$ and $y_2 = 7$, into the midpoint formula

$$M = \left(\frac{2+8}{2}, \frac{3+7}{2}\right)$$

$$M = \left(\frac{10}{2}, \frac{10}{2}\right)$$

$$= (5, 5)$$

EXERCISE 7.1

Q.1 Describe the location in the plane of the point $P(x, y)$, for which

(i) $x > 0$ **09307005**

Solution:

$x > 0$

The open right half of Cartesian plane.

(ii) $x > 0$ and $y > 0$ **09307006**

Solution

$x > 0$ and $y > 0$

The Set of all the points in 1st quadrant

(iii) $x = 0$ **09307007**

Solution:

$x = 0$, set of all points on y-axis

(iv) $y = 0$ 09307008

Solution:

$y = 0$, set of all points on x-axis.

x-axis

(v) $x > 0$ and $y \leq = 0$ 09307009

Solution:

$x > 0$ and $y \leq 0$

set of all points in 4th quadrant.

The 4th quadrant including negative y-axis.

(vi) $y = 0, x = 0$ 09307010

Solution:

$y = 0, x = 0$

The origin

vii) $x = y$ 09307011

Solution:

$\Rightarrow x = y$

It is a line bisecting the 1st and 3rd quadrant.

(viii) $x \geq 3$ 09307012

Solution:

$x \geq 3$

The set of points lying on and right side of the line $x = 3$ in Cartesian plane.

(ix) $y > 0$

Solution

$y > 0$

Set of all the points lying above the line of x-axis.

(x) x and y have opposite signs.

Solution

The set of all the points in the 2nd and 4th quadrants.

Q.2 Find the distance between the points.

(i) A(6,7), B(0,–2) 09307013

Solution

A(6,7), B(0,–2)

We know that

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

putting values

$$|AB| = \sqrt{(0-6)^2 + (-2-7)^2}$$

$$= \sqrt{(-6)^2 + (-9)^2}$$

$$= \sqrt{36+81}$$

$$= \sqrt{117}$$

$$|AB| = \sqrt{9 \times 13}$$

$$= 3\sqrt{13} \text{ units}$$

(ii) C(–5, –2), D(3, 2) 09307014

Solution:

C(–5, –2), D(3, –2)

We know that

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

putting values

$$|CD| = \sqrt{[3-(-5)]^2 + [2-(-2)]^2}$$

$$|CD| = \sqrt{(3+5)^2 + (2+2)^2}$$

$$|CD| = \sqrt{(8)^2 + (4)^2}$$

$$= \sqrt{64+16}$$

$$= \sqrt{80}$$

$$= \sqrt{16 \times 5}$$

$$= 4\sqrt{5} \text{ units}$$

(iii) L(0, 3), M(–2, –4) 09307015

Solution:

L(0, 3), M(–2, –4)

We know that

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

putting values

$$|LM| = \sqrt{(-2-0)^2 + (-4-3)^2}$$

$$|LM| = \sqrt{(-2)^2 + (-7)^2}$$

$$|LM| = \sqrt{4+49}$$

$$|LM| = \sqrt{53}$$

(iv) P(–8, –7), Q(0, 0) 09307016

Solution:

P(–8, –7), Q(0, 0)

We know that

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

putting values

$$|PQ| = \sqrt{[0-(-8)]^2 + [0-(-7)]^2}$$

$|PQ| = \sqrt{(8)^2 + (7)^2}$

$|PQ| = \sqrt{64 + 49}$

$|PQ| = \sqrt{113}$ units

Q.3 Find in each of the following:

(i) The distance between the two given points. 09307017

We know that

(a) A (3, 1), B(-2,-4) 09307018

Solution:

A (3, 1), B(−2,−4) we know that:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(-2-3)^2 + (-4-1)^2}$

$|AB| = \sqrt{(-5)^2 + (-5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50}$

$= \sqrt{25 \times 2}$

$= 5\sqrt{2}$ units

(b) A (-8,3), B(2,-1) 09307019

Solution

A (-8,3), B(2,-1) we know that:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{[2-(-8)]^2 + (-1-3)^2}$

$= \sqrt{(2+8)^2 + (-4)^2}$

$= \sqrt{(10)^2 + (-4)^2}$

$= \sqrt{100 + 16}$

$= \sqrt{116}$

$= \sqrt{4 \times 29}$ units

$= 2\sqrt{29}$ units

(c) $\mathbf{A\left(-\sqrt{5}, -\frac{1}{3}\right), B\left(-3\sqrt{5}, 5\right)}$ 09307020

Solution:

$A\left(-\sqrt{5}, -\frac{1}{3}\right), B\left(-3\sqrt{5}, 5\right)$ We know that

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{\left[-3\sqrt{5} - (-\sqrt{5})\right]^2 + \left[5 - \left(-\frac{1}{3}\right)\right]^2}$

$= \sqrt{\left[-3\sqrt{5} + \sqrt{5}\right]^2 + \left[5 + \frac{1}{3}\right]^2}$

$= \sqrt{(-2\sqrt{5})^2 + \left(\frac{15+1}{3}\right)^2}$

$= \sqrt{(-2)^2(\sqrt{5})^2 + \left(\frac{16}{3}\right)^2}$

$= \sqrt{4(5) + \frac{256}{9}}$

$= \sqrt{20 + \frac{256}{9}}$

$= \sqrt{\frac{180 + 256}{9}}$

$= \sqrt{\frac{436}{9}}$

$= \sqrt{\frac{4 \times 109}{9}}$

$|AB| = \frac{2\sqrt{109}}{3}$

(ii) Midpoint of the line segment joining the two points: 09307021

(a) A(3,1), B(-2,-4) 09307022

Solution:

A(3,1), B(−2, −4)

By formula of midpoint

$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= M\left(\frac{3+(-2)}{2}, \frac{1+(-4)}{2}\right)$

$= M\left(\frac{3-2}{2}, \frac{1-4}{2}\right)$

$= M\left(\frac{1}{2}, \frac{-3}{2}\right)$

(b) A (−8,3), B(2,−1) 09307023

Solution:

A (−8,3), B(2,−1)

By midpoint formula,

$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$= M\left(\frac{-8+2}{2},\frac{3+(-1)}{2}\right)$

$= M\left(\frac{-6}{2},\frac{3-1}{2}\right)$

$= M\left(-3,\frac{2}{2}\right)$

$= M(-3, 1)$

(c) $A\left(-\sqrt{5},-\frac{1}{3}\right), B\left(-3\sqrt{5},5\right)$ **09307024**

Solution:

$A\left(-\sqrt{5},-\frac{1}{3}\right), B\left(-3\sqrt{5},5\right)$

By midpoint formula,

$= M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$= M\left(\frac{-\sqrt{5}+(-3\sqrt{5})}{2},\frac{-\frac{1}{3}+5}{2}\right)$

$= M\left(\frac{-\sqrt{5}-3\sqrt{5}}{2},\frac{\frac{-1+15}{3}}{2}\right)$

$= M\left(\frac{-4\sqrt{5}}{2},\frac{14}{3\times 2}\right)$

$= M\left(-2\sqrt{5},\frac{7}{3}\right)$

Q.4 Which of the following points are at a distance of 15 units from the origin?

(i) $\left(\sqrt{176},7\right)$ **09307025**

Solution:

Given point

$\left(\sqrt{176},7\right)$, origin, $O(0,0)$

We know that.

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

putting values,

$|OA| = \sqrt{\left(\sqrt{176}-0\right)^2+(7-0)^2}$

$= \sqrt{\left(\sqrt{176}\right)^2+(7-0)^2}$

$= \sqrt{176+49}$

$= \sqrt{225}$

$=15$ unit

Thus the point $\left(\sqrt{176},7\right)$ is at 15 units from the origin.

(ii) (10, –10) **09307025a**

Solution:

Given point, (10, –10)

origin, O (0,0)

We know that

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

putting values.

$|OA| = \sqrt{(10-0)^2+(-10-0)^2}$

$= \sqrt{(10)^2+(-10)^2}$

$= \sqrt{100+100} = \sqrt{200}$

$= \sqrt{100\times 2} = 10\sqrt{2}$ units

The point (10,–10) is not at distance of 15 units from origin.

(iii) (1, 15) **09307025b**

Solution:

Given point origin

(1, 15)

O (0,0)

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|OA| = \sqrt{(1-0)^2+(15-0)^2} = \sqrt{(1)^2+(15)^2}$

$= \sqrt{1+225} = \sqrt{226}$

Thus distance of (1,15) from origin is not 15 units.

Q.5 Show that

(i) The points A(0, 2), B $(\sqrt{3},1)$ and C(0, –2) are vertices of a right triangle. **09307026**

Solution:

A(0, 2), B $(\sqrt{3},1)$ and C(0, –2)

Using distance formula we find the square of lengths of sides of Δ.

$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

$|AB|^2 = (\sqrt{3}-0)^2 + (1-2)^2$

$= (\sqrt{3})^2 + (-1)^2$

$= 3+1 = 4$ _____ (i)

$|BC|^2 = (0-\sqrt{3})^2 + (-2-1)^2$

$= (\sqrt{3})^2 + (-3)^2$

$= 3+9 = 12$ _____ (ii)

$|AC|^2 = (0-0)^2 + (-2-2)^2$

$= (0)^2+(-4)^2$

$= 0+16 = 16$ _____ (iii)

From (i), (ii) and (iii) we know that 16 = 12 +4

$|AC|^2 = |BC|^2 + |AB|^2$

Since, converse Pythagoras theorem is satisfied, so points A, B and C are vertices of a right-angled triangle.

(ii) The points *A*(3, 1), *B*(–2, –3) and *C*(2,2) are vertices of an isosceles triangle. **09307027**

Solution:

A(3, 1), B(–2, –3) and C(2, 2)

By using distance formula,

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|AB| = \sqrt{(-2-3)^2+(-3-1)^2}$

$= \sqrt{(-5)^2+(-4)^2}$

$= \sqrt{25+16} = \sqrt{41}$ _____ (i)

Now,

$|BC| = \sqrt{[2-(-2)]^2+[2-(-3)]^2}$

$= \sqrt{(2+2)^2+(2+3)^2}$

$= \sqrt{(4)^2+(5)^2}$

$= \sqrt{16+25}$

$= \sqrt{41}$ _____ (ii)

Now,

$|AC| = \sqrt{(2-3)^2+(2-1)^2}$

$= \sqrt{(-1)+(1)^2}$

$= \sqrt{(1)+(1)} = \sqrt{2}$ _____ (iii)

From (i) (ii) and (iii) we observe that

$|AB| = |BC|$

Which shows that ΔABC with given vertices is an isosceles triangle.

(iii) The points $A(5, 2)$, $B(-2, 3)$, $C(-3, -4)$ and $D(4, -5)$ are vertices of a parallelogram. 09307028

Solution:

A(5, 2), B(−2, 3), C(−3, −4) and D(4, −5)

By distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(-2-5)^2 + (3-2)^2}$$

$$= \sqrt{(-7)^2 + (1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} \quad \text{\_\_\_\_(i)}$$

$$|BC| = \sqrt{(-3+2)^2 + (-4-3)^2}$$

$$= \sqrt{(-1)^2 + (-7)^2}$$

$$= \sqrt{1+49} = \sqrt{50} \quad \text{\_\_\_(ii)}$$

$$|CD| = \sqrt{(4+3)^2 + (-5+4)^2}$$

$$= \sqrt{(7)^2 + (-1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} \quad \text{\_\_\_(iii)}$$

$$|AD| = \sqrt{(4-5)^2 + (-5-2)^2}$$

$$= \sqrt{(-1)^2 + (-7)^2}$$

$$= \sqrt{1+49} = \sqrt{50} \quad \text{\_\_\_(iv)}$$

From (i), (ii), (iii) and (iv)

$$\left|\overline{AB}\right| = \left|\overline{CD}\right| \text{ and } \left|\overline{BC}\right| = \left|\overline{AD}\right|$$

Opposite sides are equal in length.

Now, we find the midpoints of diagonals.

Midpoint point of diagonal $\overline{AC}$:

$$M_1(x, y) = M_1\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= M_1\left(\frac{5+(-3)}{2}, \frac{2+(-4)}{2}\right)$$

$$= M_1\left(\frac{5-3}{2}, \frac{2-4}{2}\right)$$

$$= M_1\left(\frac{2}{2}, \frac{-2}{2}\right)$$

$$= M_1(1, -1)$$

Midpoint of diagonal $\overline{BD}$:

$$M_2(x,y) = M_2\left(\frac{-2+4}{2}, \frac{3+(-5)}{2}\right)$$

$$= M_2\left(1, \frac{-2}{2}\right)$$

$$= M_2(1, -1)$$

Since, midpoints M_1 and M_2 of diagonals $\overline{AC}$ and $\overline{BD}$ are same. So diagonals bisects each other.

This, given points are vertices of a parallelogram.

Q.6 Find *h* such that the points A $\left(\sqrt{3}, -1\right)$, *B* (0, 2) and *C*(*h*, −2) are vertices of a right triangle with right angle at the vertex *A*. 09307029

Solution:

$A\left(\sqrt{3}, -1\right)$, B(0,2), C(h, −2)

We know that

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$|AB|^2 = \left(0 - \sqrt{3}\right)^2 + (2+1)^2 = \left(-\sqrt{3}\right)^2 + (3)^2$$

$$= 3+9 = 12$$

Now, $|\overline{BC}|^2 = (h-0)^2 + (-2-2)^2$

$$= h^2 + (-4)^2$$

$$|\overline{BC}|^2 = h^2 + 16$$

$$|AC|^2 = (h - \sqrt{3})^2 + (-2+1)^2$$

$$\boxed{\because (a-b)^2 = a^2 + b^2 - 2ab}$$

$$= (h)^2 + \left(\sqrt{3}\right)^2 - 2(h)\left(\sqrt{3}\right) + (-1)^2$$

$$= (h)^2 + 3 - 2\sqrt{3}\,h + 1$$

We know that right −triangle with right-angle at vertex A has side $\overline{BC}$ as hypotenuse, so, by Pythagoras theorem

$$|BC|^2 = |AB|^2 + |AC|^2$$

$$(h^2 + 16) = (12) + (h^2 - \sqrt{3}\,h + 4)$$

$$h^2 + 16 = 16 + h^2 - 2\sqrt{3} + h$$

$$= h^2 + 4 - 2\sqrt{3}\,h$$

$$\cancel{h^2} + \cancel{16} - \cancel{16} - \cancel{h^2} = -2\sqrt{3}h$$

$$0 = -2\sqrt{3}\,h$$

$\Rightarrow \quad \dfrac{0}{-2\sqrt{3}} = h$

$0 = h \Rightarrow h = 0$

Q.7 Find *h* such that *A*(–1, *h*), *B*(3, 2) and C(7, 3) are collinear. 09307030

Solution:

A(–1, h), *B*(3, 2)C(7, 3)

The points A, B and C are collinear if

$$\begin{vmatrix} -1 & h & 1 \\ 3 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix} = 0$$

Expanding by 1st row,

+(–1)

[2(1)–3(1)]–h[(3(1)–7(1)]+1[3(3)–7(2]=0

–1(2–3) –h(3–7)+[9–14] = 0

–1(–1) –h(–4)+1(–5) = 0

1+4h–5 = 0

4h–4 = 0

4h = 4

$h = \dfrac{4}{4}$

$\boxed{h=1}$

Q.8 The points A(–5, –2) and B (5, –4) are ends of a diameter of a circle. Find the centre and radius of the circle. 09307031

Solution:

A(–5, –2) and B (5, –4)

The midpoint of diameter is centre of circle by midpoint formula.

Let M(x_m, y_m) be midpoint of diameter $\overline{AB}$.

$x_m = \dfrac{x_1 + x_2}{2} = \dfrac{-5+5}{2} = \dfrac{0}{2}$

$x_m = 0$

$y_m = \dfrac{y_1 + y_2}{2}$

$y_m = \dfrac{-2+(-4)}{2} = \dfrac{-2-4}{2} = \dfrac{-6}{2}$

$y_m = -3$

Thus midpoint of diameter $\overline{AB}$ or centre of circle is M (0,–3)

(ii) Finding radius:

A (–5, –2), M (0, –3)

Since, radius of a circle is distance between centre and any point of circle, so we use distance formula.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

putting values

$|AM| = \sqrt{[0-(-5)]^2 + [-3-(-2)]^2}$

$= \sqrt{(0+5)^2 + (-3+2)^2}$

$= \sqrt{(5)^2 + (-1)^2}$

$= \sqrt{25+1}$

So, the radius of circle is = $\sqrt{26}$ units

Q.9 Find *h* such that the points *A*(*h* , 1), *B*(2, 7) and *C*(–6, –7) are vertices of a right triangle with right angle at the vertex *A*. 09307032

Solution:

A(h,1), B(2,7), c(–6, –7)

We know that

$d^2 = (x_1 - x_2)^2 + (y_2 - y_1)^2$

$|AB|^2 = (2-h)^2 + (7-1)^2$

$= (2)^2 + (h)^2 - 2(2)(h) + (6)^2$

$= 4 + h^2 - 4h + 36$

$= h^2 - 4h + 40$ _____ (i)

Now,

$|BC|^2 = (-6-2)^2 + (-7-7)^2$

$= (-8)^2 + (-14)^2$

$= 64 + 196$

$= 260$ _____ (ii)

Now,

$|AC|^2 = (-6-h)^2 + (-7-1)^2$

$= (-1)^2 (6+h)^2 + 64$

$= 1[6^2 + h^2 + 2(6)(h)] + 64$

$= 36 + h^2 + 12h + 64$

$= h^2 + 12h + 100$ ______ (iii)

We know that right-angled triangle with right angle at vertex A has side $\overline{BC}$ as its hypotenuse.

By using Pythagoras theorem.

$|BC|^2 = |AB|^2 + |AC|^2$

From (i), (ii) and (iii)

$260 = (h^2-4h+40)+(h^2+12h+100)$

$260 = 2h^2+8h+140$

$0 = 2h^2+8h+140-260$

$\Rightarrow\ 2h^2+8h-120 = 0$

$2(h^2+4h-60) = 0$

$\therefore\ h^2+4h-60 = 0 \qquad (\because 2 \neq 0)$

$h^2+10h-6h-60 = 0$

$h(h+10) -6(h+10) = 0$

$(h+10)(h-6) = 0$

$\Rightarrow\ h+10 = 0$ or $h-6 = 0$

$\Rightarrow\ h = -10$ or $h = 6$

Thus the value of h is either −10 or 6.

Q.10 A quadrilateral has the points *A*(9,3), *B*(−7, 7), *C*(−3, −7) and *D*(5, −5) as its vertices. Find the midpoints of its sides. Show that the figure formed by joining the midpoints consecutively is a parallelogram. 09307033

Solution:

A(9, 3), B(−7, 7), C(−3, −7) and D(5, −5)

First we find midpoints of all the sides of quadrilateral ABCD.

Finding midpoint of side $\overline{AB}$:

Let P(*x*, y) be midpoint of $\overline{AB}$.

$$P\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$= P\left(\frac{9+(-7)}{2},\frac{3+7}{2}\right) = p\left(\frac{9-7}{2},\frac{10}{2}\right) = \left(\frac{2}{2},5\right)$$

$= P\ (1, 5)$

Let Q(x, y) be midpoint of side $\overline{BC}$.

$$Q\left(\frac{x_1+x}{2},\frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{(-7)+(-3)}{2},\frac{7+(-7)}{2}\right)$$

$$= Q\left(\frac{-10}{2},\frac{0}{2}\right)$$

$Q\ (-5, 0)$

Let R(*x*,y) be midpoint of $\overline{CD}$

$$R\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$= R\left(\frac{-3+5}{2},\frac{(-7)+(-5)}{2}\right)$$

$$= R\left(\frac{2}{2},\frac{-12}{2}\right)$$

$= R\ (1, -6)$

Let S(*x*, y) be midpoint of DA.

$$S\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$= S\left(\frac{5+9}{2},\frac{-5+3}{2}\right)$$

$$= S\left(\frac{14}{2},\frac{-2}{2}\right) = S(7,-1)$$

Finding he midpoints of diagonals of PQRS

Let L(*x*,y) be midpoint of diagonal $\overline{PR}$: P(1,5), R(1,−6)

$$L\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$= L\left(\frac{1+2}{2},\frac{5-6}{2}\right) = L\left(\frac{2}{2},-\frac{1}{2}\right) = L\ (1,-0.5)$$

Let M(*x*, y) be midpoint of diagonal $\overline{QS}$: Q(−5,0), S(7,−1)

$$M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$= M\left(\frac{-5+7}{2},\frac{0-1}{2}\right) = \left(\frac{2}{2},\frac{-1}{2}\right) = M(1, -0.5)$$

Since, midpoints of diagonals coincide which proves that quadrilateral formed by joining the midpoints is a parallelogram.

Equations of Straight Lines

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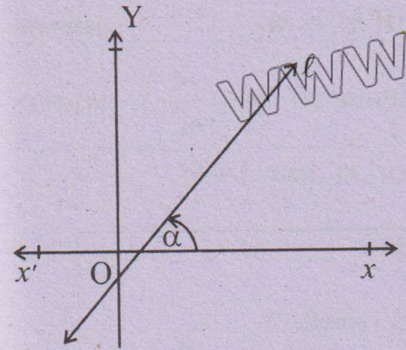
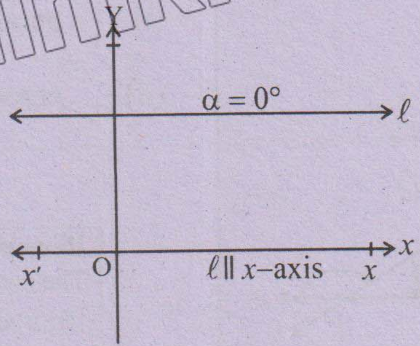
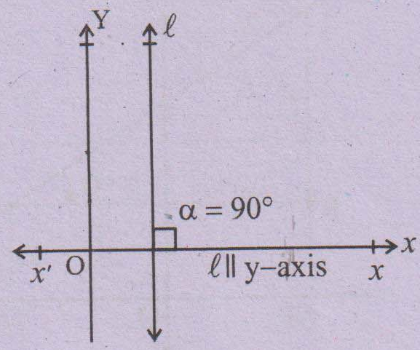
Inclination of a Line: The angle α ($0^\circ < \alpha < 180^\circ$) measured counterclockwise from positive x-axis to a non-horizontal straight line ℓ is called the inclination of ℓ.







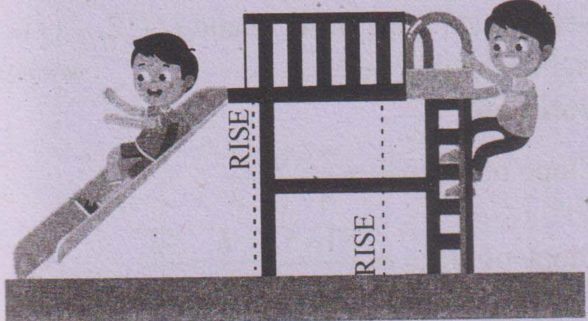
Observe that the angle α in the different positions of the line ℓ is α, 0° and 90° respectively.

> **Note:**
> (i) If l is parallel to x-axis, then $\alpha = 0^\circ$
> (ii) If l is parallel to y-axis, then $\alpha = 90^\circ$

Slope or Gradient of a Line

When we walk on an inclined plane, we cover horizontal distance (**run**) as well as vertical distance (**rise**) at the same time.
It is harder to climb a steeper inclined plane. The measure of steepness (ratio of rise to the run) is termed as slope or gradient of the inclined path and is denoted by m.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y}{x} = \tan\alpha$$



In analytical geometry, **slope or gradient m** of a non-vertical straight line with as its inclination is defined by: $m = \tan\alpha$.

If ℓ is horizontal its slope is zero and if l is vertical then its slope is undefined.
If $0^\circ < \alpha < 90^\circ$, m is positive and if $90^\circ < \alpha < 180^\circ$, then m is negative.

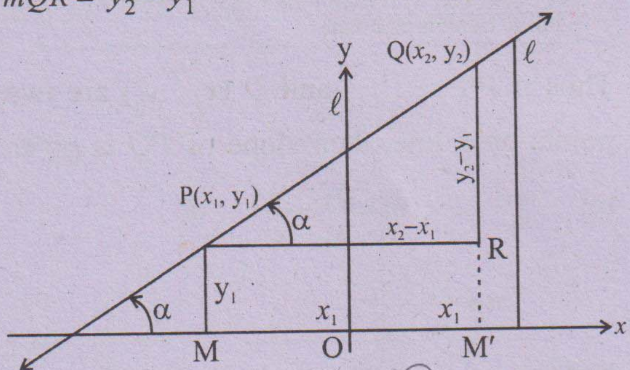
Slope or Gradient of a Straight Line Joining Two Points

09307035

Theorem 1: If a non-vertical line ℓ with inclination α passes through two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the slope or gradient m of ℓ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \tan\alpha$$

Proof: Let m be the slope of the line ℓ.
Draw perpendiculars PM and QM' on x-axis and a perpendicular PR on QM'.
Then $m\angle RPQ = \alpha$, $m\overline{PR} = x_2 - x$ and $m\overline{QR} = y_2 - y_1$



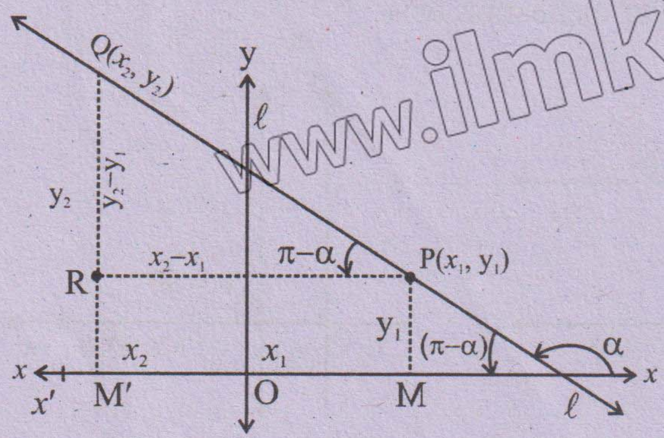
The slope or gradient of ℓ is defined as:

$$m = \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$$

Case (i). When $0 < \alpha < \frac{\pi}{2}$

In the right triangle PRQ, we have

$$m = \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$$



Case (ii). When $\frac{\pi}{2} < \alpha < \pi$

In the right triangle *PRQ*,

$$\tan(\pi - \alpha) = \frac{y_2 - y_1}{x_1 - x_2}$$

or $\quad -\tan\alpha = \frac{y_2 - y_1}{x_1 - x_2}$

or $\quad \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$

> **Why are slopes important?**
> The concept of slope is wisely used in engineering, architecture, and even sports like skiing, where understanding the steepness of a hill or ramp is essential.

Thus if $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points on a line, then slope of *PQ* is given by $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ or

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

Note:

(i) $m \neq \frac{y_2 - y_1}{x_1 - x_2}$ and $m \neq \frac{y_1 - y_2}{x_2 - x_1}$

(ii) ℓ is horizontal, iff $m = 0$ $(\because \alpha = 0^\circ)$

(iii) ℓ is vertical, iff m is not defined $(\because \alpha = 90^\circ)$

(iv) If slope of $\overline{AB}$ = slope of $\overline{BC}$, then the points *A*, *B* and *C* are collinear.

Theorem 2: The two lines ℓ_1 and ℓ_2 with slopes m_1 and m_2 respectively are

(i) parallel iff $m_1 = m_2$ **09307036**

(ii) perpendicular iff $m_1 = \frac{-1}{m_2}$ **09307037**

or $m_1 m_2 + 1 = 0$

> **Remember!**
> The symbol:
> (i) || stand for "parallel".
> (ii) ∦ stands for "not parallel".
> (iii) ⊥stands for "perpendicular"

Example 5: Show that the points *A*(−3, 6), *B*(3, 2) and *C*(6, 0) are collinear.

Solution:

We know that the points *A*, *B* and *C* are collinear if the line *AB* and *BC* have the same slopes.

Here Slope of $\overline{AB} = \frac{2-6}{3-(-3)} = \frac{-4}{3+3} = \frac{-4}{6} = \frac{-2}{3}$

and

slope of $\overline{BC} = \frac{0-2}{6-3} = \frac{-2}{3}$

$\because$ Slope of *AB* = Slope of *BC*

Thus *A*, *B* and *C* are collinear.

Example 6: Show that the triangle with vertices *A* (1, 1), *B* (4, 5) and *C* (12, −1) is a right triangle. **09307038**

Solution:

Slope of $\overline{AB} = m_1 = \frac{5-1}{4-1} = \frac{4}{3}$

Slope of $\overline{BC} = m_2 \frac{-1-5}{12-4} = \frac{-6}{8} = \frac{-3}{4}$

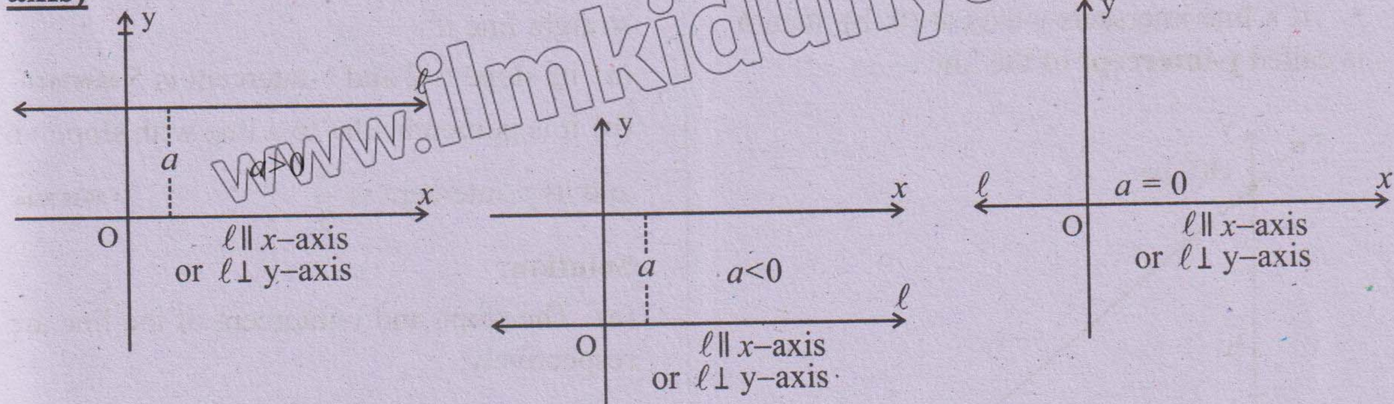
Since, $m_1 m_2 = \left(\frac{4}{3}\right)\left(-\frac{3}{4}\right) = -1,$

$\Rightarrow m_1 \times m_2 = -1$ therefore, $\overline{AB} \perp \overline{BC}$

So ΔABC is a right triangle.

Equation of a Straight Line Parallel to the *x*-axis (or perpendicular to the *y*-axis)

09307039



All the points on the line ℓ parallel to x-axis remain at a constant distance (say a) from x-axis. Therefore, each point on the line has its distance from x-axis equal to a, which is its y-coordinate (ordinate). So, all the points on this line satisfy the equation: $\boxed{y = a}$

Note:

(i) If $a > 0$, then the line ℓ is above the x-axis.

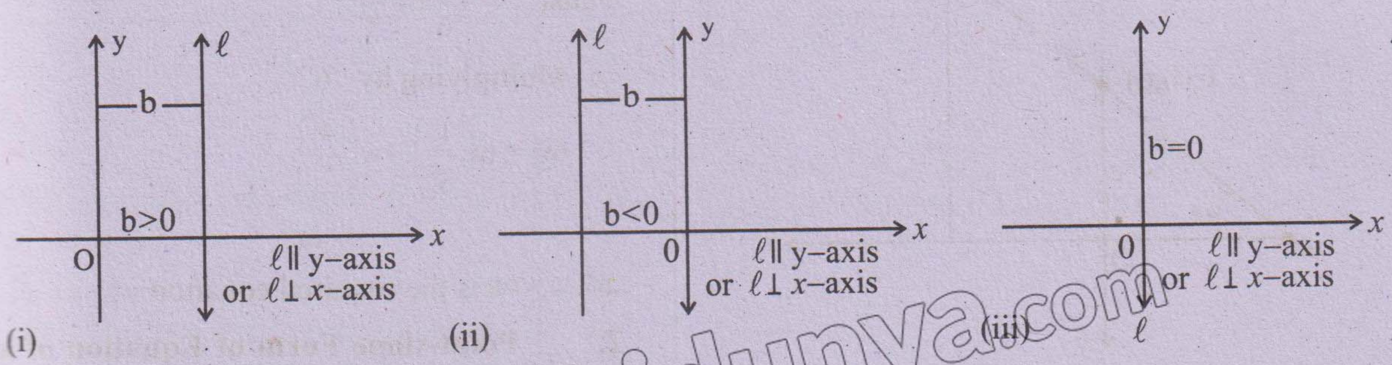
(ii) If $a < 0$, then the line ℓ is below the x-axis.

(iii) If $a = 0$, then the line ℓ becomes the x-axis.

Thus the equation of x-axis is $y = 0$

Equation of a straight Line Parallel to the *y*-axis (or perpendicular to the *x*-axis

09307040



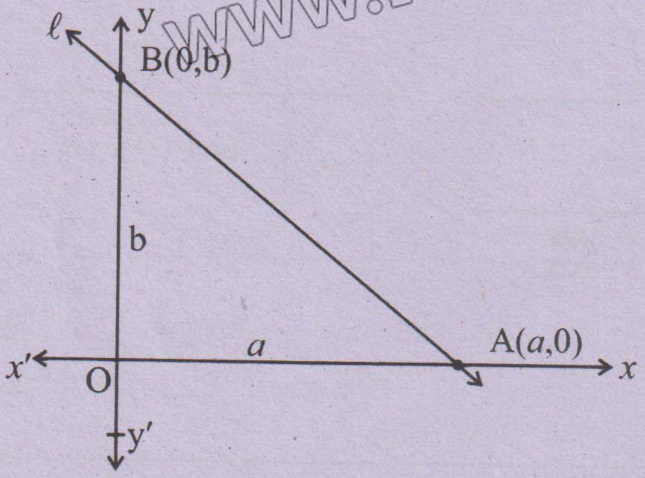
All the points on the line ℓ parallel to y-axis remain at a constant distance (say b) from y-axis. Each point on the line has its distance from y-axis equal to b, whicsh is its x-coordinate (abscissa). So, all the points on this line satisfy the equation: $\boxed{x = b}$

Derivation of Standard Forms of Equations of Straight Lines

09307041

Intercepts of a line

- If a line intersects x-axis at $(a, 0)$, then a is called **x-intercept** of the line.
- If a line intersects y-axis at $(0, b)$, then b is called **y-intercept** of the line.

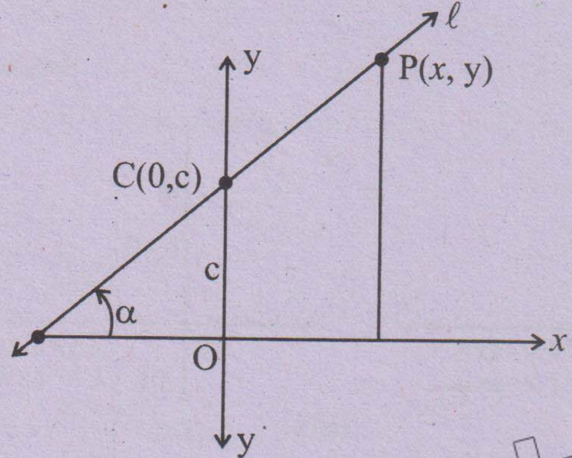


1. Slope-Intercept form of Equation of a Straight Line 09307042

Theorem 3: Equation of a non-vertical straight line with slope m and y-intercept c is given by:

$$\boxed{y = mx + c}$$

Proof: Let $P(x, y)$ be an arbitrary point of the straight line ℓ with slope m and y-intercept c. As $C(0, c)$ and $P(x, y)$ lie on the line, so the slope of the line is:



$m = \dfrac{y-c}{x-0}$ or $y - c = mx$ or $y = mx + c$ is an equation of ℓ.

The equation of the line for which $c = 0$ is $y = mx$. In this case the line passes through the origin.

Example 7: Find an equation of the straight line if

(a) its slope is 2 and y-intercept is 5 09307043

(b) it is perpendicular to a line with slope -6 and its y-intercept is $\dfrac{4}{3}$ 09307044

Solution:

(a) The slope and y-intercept of the line are respectively:

$$m = 2 \text{ and } c = 5$$

Thus $y = 2x + 5$

(Slope-intercept form:

$y = mx + c$ is the required equation.

(b) The slope of the given line is

$$m_1 = -6$$

$\therefore$ The slope of the required line is:

$$m_2 = -\frac{1}{m_1} = \frac{1}{6}$$

$$y = -\frac{1}{6}x + \frac{4}{3}$$

The slope and y-intercept of the required line are respectively:

$m_2 = \dfrac{1}{6}$ and $c = \dfrac{4}{3}$

Thus, $y = -\dfrac{1}{6}x + \dfrac{4}{3}$

$\Rightarrow$ Multiplying by "6"

$$6y = 6\left(\frac{1}{6}\right) + 6\left(\frac{4}{3}\right)$$

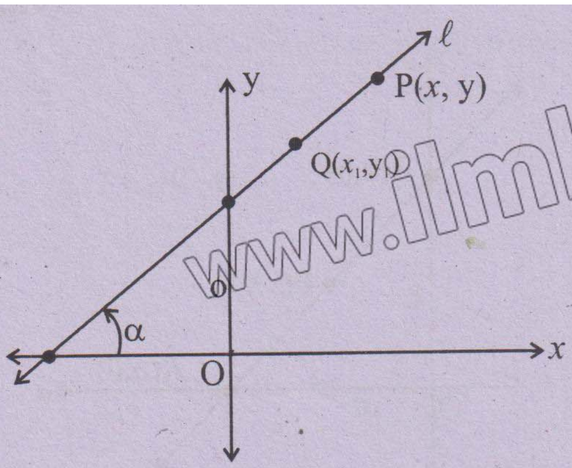
or

$6y = x + 8$ is the required equation.

2. Point-slope Form of Equation of a Straight Line 09307045

Theorem 4: Equation of a non-vertical straight line ℓ with slope m and passing through a point $Q(x_1, y_1)$ is given by:

$y - y_1 = m(x - x_1)$



Proof: Let $P(x, y)$ be an arbitrary point of the straight line with slope m and passing through $Q(x_1, y_1)$.

As $Q(x_1, y_1)$ and $P(x, y)$ both lie on the line, so the slope of the line is

$$m = \frac{y - y_1}{x - x_1} \text{ or } y - y_1 = m(x - x_1)$$

which is an equation of the straight line passing through (x_1, y_1) with slope m.

3. Symmetric Form of Equation of a Straight Line

We have $m = \frac{y - y_1}{x - x_1} = \tan\alpha$ where α is the inclination of the line.

$$\because \tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\text{or } \frac{y - y_1}{x - x_1} = \frac{\sin\alpha}{\cos\alpha} \qquad \text{or } \frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha} = r(\text{say})$$

This is called **symmetric** form of equation of the line.

Example 8: Write down an equation of the straight line passing through (5, 1) and parallel to a line passing through the points (0,–1), (7, –15). 09307046

Solution:

Let m be the slope of the required straight line, then

$$m = \frac{-15-(-1)}{7-0} = -2$$

($\because$ Slopes of parallel lines are equal) $m = -2$

As the point (5, 1) lies on the required line having slope –2 so, by point-slope form of equation of the straight line, we have

$$y - (1) = -2(x - 5)$$
$$y - 1 = -2x + 10$$
$$y = -2x + 10 + 1$$
$$\text{or} \quad y = -2x + 11$$
$$\text{or} \quad 2x + y - 11 = 0$$

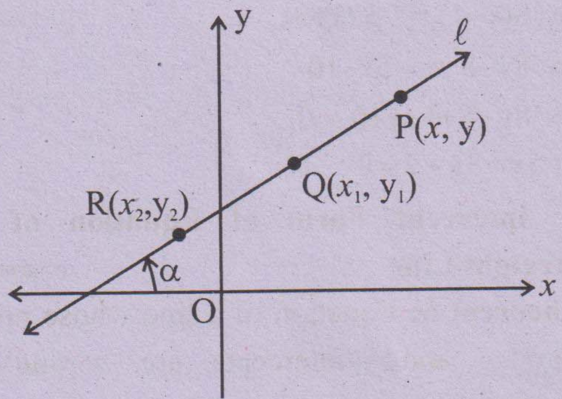
is an equation of the required line.

4. Two-point Form of Equation of a Straight Line 09307047

Theorem 5: Equation of a non-vertical straight line passing through two points $Q(x_1, y_1)$ and $R(x_2, y_2)$ is:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\text{or} \quad y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2)$$



Proof: Let $P(x, y)$ be an arbitrary point of the line passing through $Q(x_1, y_1)$ and $R(x_2, y_2)$. So

$$\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

(P, Q and R are collinear points)

We take

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

or $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$, the required equation of the line PQ.

or

$$(y_2 - y_1)x - (x_2 - x_1)y + (x_1y_2 - x_2y_1) = 0$$

We may write this equation in determinant form as: $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Example 9: Find an equation of line through the points (–2,1) and (6,–4).

09307048

Solution:

Using two-points form of the equation of straight line, the required equation is

$$y - 1 = \frac{-4-1}{6-(-2)}[x-(-2)]$$

or $y - 1 = \dfrac{-5}{8}(x+2)$

$\Rightarrow$ $8(y-1) = -5(x+2)$

$\Rightarrow$ $8y - 8 = -5x - 10$

$\Rightarrow$ $8y - 8 + 5x + 10 = 0$

or $5x + 8y + 2 = 0$

5. Intercept Form of Equation of a Straight Line

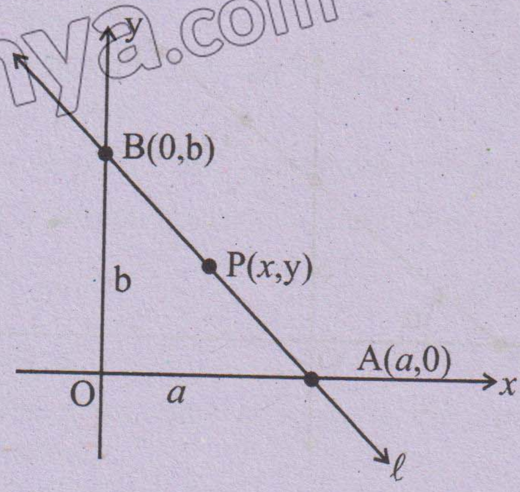
09307049

Theorem 6: Equation of a line whose non-zero x and y-intercepts are a and b respectively is:

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

Proof: Let $P(x, y)$ be an arbitrary point of the line whose non-zero x and y-intercepts are a and b respectively. Obviously, the points $A(a, 0)$ and $B(0, b)$ lie on the required line. So, by the two-point form of the equation of line,

we have



$$y - 0 = \frac{b-0}{0-a}(x-a)$$

(P, A and B are collinear)

or $-ay = b(x-a)$

or

$\Rightarrow$ $-ay = bx - ab$

$bx + ay = ab$

or $\dfrac{bx}{ab} + \dfrac{ay}{ab} = \dfrac{ab}{ab}$ (dividing by ab)

$\dfrac{x}{a} + \dfrac{y}{b} = 1$

Hence the result.

Example 10: Write down an equation of the line which cuts the x-axis at (2, 0) and y-axis at (0, –4).

09307050

Solution:

As 2 and –4 are respectively x and y-intercepts of the required line, so by two-intercepts form of equation of a straight line, we have

$$\frac{x}{2} + \frac{y}{-4} = 1 \Rightarrow 4\left(\frac{x}{2} + \frac{y}{-4}\right) = 4(1)$$

or $2x - y = 4$ $\quad 2x - y - 4 = 0$

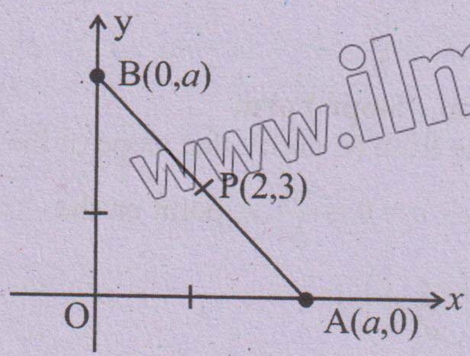
Which is the required equation.

Example 11: Find an equation of the line through the point $P(2, 3)$ which forms an isosceles triangle with the coordinate axes in the first quadrant.

09307051

Solution: Let OAB be an isosceles triangle so that the line AB passes through A $(a, 0)$

and $B(0, a)$, where a is some positive real number.



Slope of $\overline{AB} = \frac{a-0}{0-a} = -1$. But $\overline{AB}$ passes through $P(2, 3)$.

Equation of the line through $P(2, 3)$ with slope -1 is

$y - 3 = -1(x - 2) \Rightarrow y-3 = -x+2$ or

$y-3+x-2=0 \qquad x + y - 5 = 0$

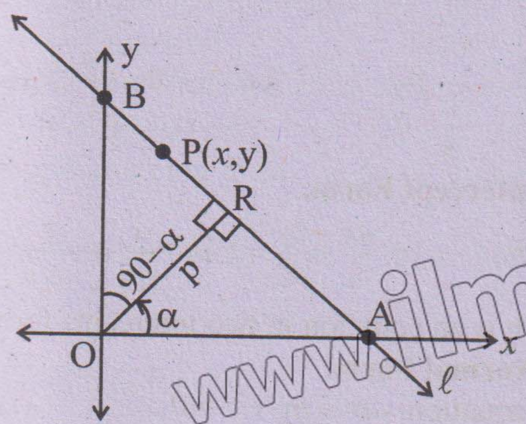
6. Normal Form of Equation of a Straight Line 09307052

Theorem 7: An equation of a non-vertical straight-line l, such that length of the perpendicular from the origin to l is p and α is the inclination of this perpendicular, is

$$\boxed{x\cos\alpha + y\sin\alpha = p}$$

Proof: Let the line ℓ meet the x-axis and y-axis at the points A and B respectively. Let $P(x, y)$ be an arbitrary point of line AB and let $\overline{OR}$ be perpendicular to the line ℓ. Then $|\overline{OR}| = P$

From the right triangles ORA and ORB, we have



$\cos\alpha = \frac{p}{\overline{OA}}$ or $\overline{OA} = \frac{P}{\cos\alpha}$

and $\cos(90^\circ - \alpha) = \frac{p}{\overline{OB}} = \sin\alpha = \frac{p}{\overline{OB}}$

$= \overline{OB} = \frac{p}{\sin a}$

$[\because \cos(90^\circ - a) = \sin\alpha]$

As $\overline{OA}$ and $\overline{OB}$ are the x and y-intercepts of the line AB, so equation of AB is:

$\frac{x}{p/\cos\alpha} + \frac{x}{p/\sin\alpha} = 1$ (Two-intercept form)

$\frac{x\cos\alpha}{p} + \frac{x\sin\alpha}{p} = 1$

$\Rightarrow x\cos\alpha + x\sin = p$

That is $x\cos a + y\sin a = p$ is the required equation.

Example 12: The length of perpendicular from the origin to a line is 5 units and the inclination of this perpendicular is 120°. Find the slope and y-intercept of the line. 09307053

Solution:

Here $p = 5$, $\alpha = 120°$.

Equation of the line in normal form is

$$x\cos 120^\circ + y\sin 120^\circ = 5$$

$\Rightarrow -\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 5$

$\Rightarrow -x + \sqrt{3}y = 10$

$\Rightarrow x - \sqrt{3}y + 10 = 0$

To find the slope of the line, we re-write (1) as: $y = \frac{x}{\sqrt{3}} + \frac{10}{\sqrt{3}}$

which is slope-intercept form of the equation.

Here $m = \frac{1}{\sqrt{3}}$ and $c = \frac{10}{\sqrt{3}}$

A Linear Equation in two Variables Represents a Straight Line

Theorem 8: The linear equation $ax + by + c = 0$ in two variables x and y represents a straight line. A linear equation in two variables x and y is:

$$\boxed{ax+by+c=0} \quad \ldots(i)$$

where a, b and c are constants and a and b are not simultaneously zero.

Proof: Here a and b cannot be both zero. So the following cases arise:

Case I: $a \neq 0$, $b = 0$

In this case equation (1) takes the form:

$$ax+c=0 \text{ or } x=-\frac{c}{a}$$

which is an equation of the straight line parallel to the y-axis at a directed distance $-\frac{c}{\alpha}$ from the y-axis.

Case II: $a = 0$, $b \neq 0$

In this case equation (i) takes the form:

$$by+c= \text{ or } y=-\frac{c}{b}.$$

which is an equation of the straight line parallel to x-axis at a directed distance $\frac{-c}{b}$ from the x-axis.

Case III: $a \neq 0$, $b \neq 0$

In this case equation (1) takes the form:

$$by=-ax-c \text{ or } y=\frac{-a}{b}x-\frac{c}{b}=mx+c$$

which is the slope-intercept form of the straight line with slope $\frac{-a}{b}$ and y-intercept $\frac{-c}{b}$

Thus the equation $ax + by + c = 0$, always represents a straight line.

> **Remember!**
> The equation (i) represents a straight line and is called the **general equation of a straight line.**

Transform the General Linear Equation to Standard Forms

Lets transform the equation $ax + by + c = 0$ into the standard form

i. Slope-Intercept Form

We have:

$$by=-ax-c \text{ or } y=\frac{-a}{b}x-\frac{c}{b}=mx+c_1 \text{ where}$$

$$m=\frac{-a}{b}, c_1=\frac{-c}{b}$$

ii. Point - Slope Form

We note from (i) above that slope of the line $ax+by+c=0$ is $\frac{-a}{b}$. A point on the line is $\left(\frac{-c}{a},0\right)$.

Equation of the line becomes

$$y-0=-\frac{a}{b}\left(x+\frac{c}{a}\right)$$

which is in the point-slope form.

iii. Symmetric Form

$$m=\tan\alpha=\frac{-a}{b}, \sin\alpha=\frac{a}{\pm\sqrt{a^2+b^2}}, \cos\alpha\frac{b}{\pm\sqrt{a^2+b^2}}$$

A point on $ax+by+c=0$ is, $\left(\frac{-c}{a},0\right)$

Equation in the symmetric form becomes

$$\frac{x-\left(-\frac{c}{a}\right)}{b\div\pm\sqrt{a^2+b^2}}=\frac{y-0}{a\div\pm\sqrt{a^2+b^2}}=r(say)$$

is the required transformed equation. Sign of the radical to be properly chosen.

iv. Two -Point Form

We choose two arbitrary points on $ax + by + c = 0$. Two such points are $\left(\frac{-c}{a},0\right)$ and $\left(0,\frac{-c}{b}\right)$. Equation of the line through these points is:

$$\frac{y-0}{0+\frac{c}{b}}=\frac{x+\frac{c}{a}}{-\frac{c}{a}-0} \quad \text{i.e., } y-0=\frac{-a}{b}\left(x+\frac{c}{a}\right)$$

v. Intercept Form.

$$ax+by=-c \text{ or } \frac{ax}{-c}+\frac{by}{-c}=1 \text{ i.e. } \frac{x}{c/a}+\frac{y}{-c/a}=1$$

which is an equation in two intercepts form.

vi. Normal Form.

The equation: $ax+by+c=0$ …(i)

can be written in the normal form as:

$$\frac{ax+by}{\pm\sqrt{a^2+b^2}}=\frac{-c}{\pm\sqrt{a^2+b^2}} \quad \ldots(ii)$$

The sign of the radical to be such that the right hand side of (ii) is positive.

Proof. We know that an equation of a line in normal form is

$x\cos\alpha + y\sin\alpha = p$.....(iii)

(3) If (i) and (iii) are identical, we must have

$$\frac{a}{\cos\alpha}+\frac{b}{\sin\alpha}=\frac{-c}{p}$$

i.e., $\frac{p}{-c}=\frac{\cos\alpha}{a}=\frac{\sin\alpha}{b}$

$$=\frac{\sqrt{cos^2\alpha+\sin^2\alpha}}{\pm\sqrt{a^2+b^2}}$$

$$=\frac{1}{\pm\sqrt{a^2+b^2}}$$

Hence, $\cos\alpha=\frac{a}{\pm\sqrt{a^2+b^2}}$ and $\sin\alpha$

$$\sin\alpha=\frac{b}{\pm\sqrt{a^2+b^2}}$$

$$p=\frac{-c}{\pm\sqrt{a^2+b^2}}$$

Substituting for $\cos\alpha$, $\sin\alpha$ and p into (iii), we have

$$\frac{ax+by}{\pm\sqrt{a^2+b^2}}=\frac{-c}{\pm\sqrt{a^2+b^2}}$$

Thus (i) can be reduced to the form (ii) by dividing it by $\pm\sqrt{a^2+b^2}$. The sign of the radical to be chosen so that the right hand side of (ii) is positive.

Example 13: Transform the equation $5x-12y+39=0$ into

(i) Slope intercept form **09307054**

(ii) Two-intercept form **09307055**

(iii) Normal form **09307056**

(iv) Point-slope form **09307057**

(v) Two-point form **09307058**

(vi) Symmetric form **09307059**

Solution:

Slope intercept form

(i) We have $12y=5x+39$ or

$$y=\frac{5}{12}x+\frac{39}{12},$$

$$m=\frac{5}{12} \qquad (\because y=mx+c)$$

$$y-\text{intercept } c=\frac{39}{12}$$

(ii) $5x-12y=-39$ or $\frac{5x}{-39}+\frac{12y}{39}=1$ or

$\frac{x}{-39/5}+\frac{y}{39/12}=1$ is the required equation.

(iii) $5x-12y=-39$. Divide both sides by $\pm\sqrt{5^2+12^2}=\pm13$. Since R.H.S is to be positive, we have to take negative sign.

Hence $\frac{5x}{-13}+\frac{12y}{13}=3$ is the normal form of the equation.

(iv) A point on the line is $\left(\frac{-39}{5},0\right)$ and its slope is $\frac{5}{12}$.

Equation of the line can be written as:

$$y-0=\frac{5}{12}\left(x+\frac{39}{5}\right)$$

(v) Another point on the line is $\left(0,\frac{39}{12}\right)$.

Line through $\left(\frac{-39}{5},0\right)$ and $\left(0,\frac{39}{12}\right)$ is

$$\frac{y-0}{0+\frac{39}{12}}=\frac{x+\frac{39}{5}}{\frac{-39}{5}-0}$$

(vi) We have $\tan\alpha=\frac{5}{12}=m$,

so $\sin\alpha=\frac{5}{13}$, $\cos\alpha=\frac{12}{13}$.

A point of the line is $\left(\frac{-39}{5},0\right)$

Equation of the line in symmetric form is:

$$x+\frac{\frac{39}{5}}{12}=\frac{\frac{y-0}{5}}{13}=r(\text{say})$$

EXERCISE 7.2

Q.1 Find the slope and inclination of the line joining the points:

(i) (–2, 4) ; (5, 11) 09307060

Solution:

(–2, 4) ; (5, 11)

$$\text{Slope} = m = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{11-4}{5-(-2)}$$

$$m = \frac{7}{5+2}=\frac{7}{7}=1$$

Indication: $\tan\alpha = m$

$\alpha = \tan^{-1}(m)$

$\alpha = \tan^{-1}(1)$

$\alpha = 45°$

(ii) (3, –2) ; (2, 7) 09307061

Solution:

(3, –2), (2, 7)

$$\text{Slope} = m\ \frac{y_2-y_1}{x_2-x_1}$$

$$m = \frac{7-(-2)}{2-3}=\frac{7+2}{-1}=\frac{9}{-1}=-9$$

Inclination: $\tan\alpha = m$

$\alpha = \tan^{-1}(m)$

$\alpha = \tan^{-1}(-9)$

$\alpha = -83.65$

Making angle positive:

$\alpha = 180°-83.65°$

$\alpha = 96.34$

(iii) (4, 6) ; (4, 8) 09307062

Solution:

(4, 6) , (4, 8)

$$\text{Slope} = m = \frac{y_2-y_1}{x_2-x_1}$$

$$= \frac{8-6}{4-4}=\frac{2}{0}=\infty$$

$m = \infty$ (undefined)

Inclination: $\tan\alpha = m$

$\tan\alpha = \infty \quad \therefore \alpha = 90°$

Q.2 By means of slopes, show that the following points lie on the same line:

(i) A(–1, –3); B(1,5); C(2,9) 09307063

Solution:

A(–1, –3); B(1,5); C(2,9)

If slope of $\overline{AB}$ = slope of $\overline{BC}$ then points A, B and C are collinear.

Slope of $\overline{AB}$:

$$m_1=\frac{y_2-y_1}{x_2-x_1}$$

$$m_1=\frac{5-(-3)}{1-(-1)}=\frac{5+3}{1+1}=\frac{8}{2}=4$$

Slope of $\overline{BC}$: $m_2=\frac{y_2-y_1}{x_2-x_1}$

$$m_2=\frac{9-5}{2-1}=\frac{4}{1}=4$$

Since, slope of $\overline{AB}$ = slope of $\overline{BC}$, so points A, B and C are collinear.

(ii) P(4, –5), Q(7,5), R(10,15) 09307064

Solution:

P(4, –5); Q(7,5); R(10,15)

Slope of $\overline{PQ}$:

$$m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{5-(-5)}{7-4}$$

$$=\frac{5+5}{3}=\frac{10}{3}$$

Slope of $\overline{QR}$ = $m_2=\frac{y_2-y_1}{x_2-x_1}$

$$=\frac{15-5}{10-7}=\frac{10}{3}$$

Since, slope of $\overline{PQ}$ = slope of $\overline{QR}$, so points P,Q and R are collinear points.

(iii) L(–4, 6); M(3,8); N(10,10) 09307065

Solution:

L(–4, 6); M(3,8); N(10,10)

Slope of $\overline{LM} = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{8-6}{3-(-4)} = \frac{8-6}{3+4} = \frac{2}{7}$

Slope of $\overline{MN} = m_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_2 = \frac{10-8}{10-3} = \frac{2}{7}$

Since, slope of $\overline{LM}$ = slope of $\overline{MN}$, so points L,M and N are collinear points.

(iv) *X*(*a*, 2b); Y(c, *a*+b); Z(2c–*a*,2*a*) 09307066

Solution:

X(*a*, 2b); Y(c, *a*+b); Z(2c–*a*,2*a*)

Slope of $\overline{XY} = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{(a+b)-(2b)}{c-a}$

$m_1 = \frac{a+b-2b}{c-a}$

$= \frac{a-b}{c-a}$

Slope of $\overline{YZ} = m_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_2 = \frac{2a-(a+b)}{(2c-a)-(c)}$

$m_2 = \frac{2a-a-b}{2c-a-c}$

$m_2 = \frac{a-b}{c-a}$

Since, slope of $\overline{YZ}$ = slope of $\overline{YZ}$, so points X,Y and Z are collinear points.

Q.3 Find *k* so that the line joining *A*(7, 3);*B*(*k*, –6) and the line joining *C*(–4,5) ; *D* (–6, 4) are:

(i) parallel 09307067

(ii) perpendicular 09307067a

Solution:

Slope of $\overline{AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{-6-3}{k-7} = \frac{-9}{k-7}$

Slope of $\overline{CD} = m_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_2 = \frac{4-5}{-6(-4)} = \frac{-1}{-6+4} = \frac{-1}{-2} = \frac{1}{2}$

(i) When line segments are parallel. 09307068

Solution:

Slopes of parallel lines are equal.

If $\overline{AB} \parallel \overline{CD}$, then

$$m_1 = m_2$$

$$\frac{-9}{k-7} = \frac{1}{2}$$

$$-18 = k-7$$

$$-18+7 = k \Rightarrow -11 = K$$

$$\Rightarrow \boxed{k = -11}$$

(ii) When line segments are perpendicular. 09307069

If $\overline{AB} \perp \overline{CD}$, then

$m_1 \times m_2 = -1$

$\frac{-9}{k-7} \times \frac{1}{2} = -1$

$\frac{-9}{2k-14} = 1$

$$9 = 2k-14$$

$$9+14 = 2k$$

$$23 = 2k$$

$$\frac{23}{2} = k$$

$$\Rightarrow k = \frac{23}{2}$$

Q.4 Using slopes, show that the triangle with its vertices *A*(6, 1), *B*(2, 7) and *C*(–6, –7) is a right triangle. 09307070

Solution:

A(6, 1), B(2, 7) and C(−6, −7)

First we find the slopes of sides of ΔABC.

Slope of side $\overline{AB}$:

$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-1}{2-6} = \frac{6}{-4} = \frac{-3}{2}$

Slope of side $\overline{BC}$:

$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$

Slope of side $\overline{AC}$:

$m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-1}{-6-6} = \frac{-8}{-12} = \frac{2}{3}$

We observe that

$m_1 \times m_3 = \frac{-3}{2} \times \frac{2}{3}$

$m_1 \times m_3 = -1$

This shows that side $\overline{AB} \perp$ side $\overline{AC}$

Hence, ΔABC is a right angled triangle with 90° at vertex A.

Q.5 Two pairs of points are given. Find whether the two lines determined by these points are:

(i) parallel **09307071** (ii)perpendicular **09307072**

(iii) none **09307073**

(a) (1, −2), (2, 4) and (4, 1) (−8, 2) 09307074

Solution:

Let A(1, −2), B(2, 4) and C(4, 1),D(−8, 2)

Slope of $\overline{AB}$:

$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{2-1} = \frac{4+2}{1} = \frac{6}{1} = 6$

Slope of $\overline{CD}$:

$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{-8-4} = \frac{1}{-12}$

Multiplying these slopes:

$m_1 \times m_2 = 6 \times \frac{1}{-12} = -\frac{1}{2}$

These slopes are neither equal nor their product is −1, so the lines determined by given points are neither parallel nor perpendicular to each other.

(b) (−3, 4), (6, 2) and (4, 5), (−2, −7) 09307075

Solution

A(−3, 4), B(6, 2) and C(4, 5), D(−2, −7)

Slope of $\overline{AB}$:

$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{6-(-3)} = \frac{-2}{6+3} = \frac{-2}{9}$

Slope of $\overline{CD}$:

$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-5}{-2-4} = \frac{-12}{-6} = 2$

Multiplying these slopes,

$m_1 \times m_2 = \frac{-2}{9} \times 2$

$m_1 \times m_2 = \frac{-4}{9}$

We observe that the slopes are neither equal nor their product is −1, so the lines determined by these points are neither parallel nor perpendicular to each other.

Q.6 Find an equation of:

(a) the horizontal line through (7, −9) 09307076

Solution:

The point slope form of equation is:

$y - y_1 = m(x - x_1)$ ______ (i)

The slope of horizontal line: m = 0

Given point $(x_1, y_1) = (7, -9)$

Put $x_1 = 7$ and $y_1 = -9$ in eq. (1)

$y - (-9y) = 0(x - 7)$

$y + 9 = 0$

(b) the vertical line through (−5, 3) 09307077

Solution:

Equation of vertical line through (−5,3)

Equation of line in point sloe form is

$y - y_1 = m(x - x_1)$ _______ (i)

Slope of vertical line = $m = \infty = \frac{1}{0}$

Given point $(x_1, y_1) = (-5, 3)$ putting the value in eq. (i)

$y - 3 = \frac{1}{0}[x - (-5)]$

$\Rightarrow 0(y-3) = 1(x+5)$

$0 = x + 5$

$\Rightarrow x + 5 = 0$

(c) through $A(-6, 5)$ having slope 7 09307078

Solution:

A(−6, 5), slope = m = 7

The equation of line in point slope form is:

$y-y_1 = m(x-x_1)$ ______ (i)

Put $x_1 = -6$, $y_1 = 5$ and m = 7 in eq. (i)

$y - 5 = 7[x-(-6)]$

$y - 5 = 7(x+6)$

$y - 5 = 7x+42$

$\Rightarrow\ 0 = 7x + 42 - y + 5$

$\Rightarrow\ 7x - y + 47 = 0$

(d) through (8, −3) having slope 0 09307079

Solution:

Point P(8, −3) and slope = m = 0

The equation of line in point slope form is

$y-y_1 = m(x-x_1)$ ______ (i)

Put $x_1 = 8$, $y_1 = -3$ and m = 0 in eq. (i)

$y - (-3) = 0(x-8)$

$y + 3 = 0$

(e) through (−8 , 5) having slope undefined 09307080

Solution:

Point p(−8, 5), slope = $m = \infty = \frac{1}{0}$ (undefined)

The equation of line in point slope form is

$y-y_1 = m(x-x_1)$ ______ (i)

Put $x_1 = 8$, $y_1 = 5$ and $m = \frac{1}{0}$

$y - 5 = \frac{1}{0}[x-(-8)]$

$0(y - 5) = 1(x+8)$

$0 = x + 8$

$\Rightarrow\ x + 8 = 0$

(f) through (−5, −3) and (9, −1) 09307081

Solution:

Points A(−5, −3), B(9, −1)

The slope of line passing through given points is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-3)}{9-(-5)} = \frac{-1+3}{9+5} = \frac{2}{14} = \frac{1}{7}$$

The equation of line in point slope form is:

$y-y_1 = m(x-x_1)$ ________ (i)

Let $(x_1, y_1) = (-5, -3)$ [Take any one of two points)

Putting the values in eq. (i)

$y-(-3) = \frac{1}{7}[x-(-5)]$

$y+3 = \frac{1}{7}(x+5)$

$7(y+3) = x+5$

$7y+21 = x+5$

$0 = x+5-7y-21$

$\Rightarrow\ x-7y-16 = 0$

(g) *y*-intercept: −7 and slope: −5 09307082

Solution:

y−intercept: 7 $\Rightarrow$ c = −7

slope: −5 $\Rightarrow$ m = −5

The equation of line in slope-intercept form is

$y = mx + c$

$y = -5x + (-7)$ (putting values)

$y = -5x - 7$

(h) *x*-intercept: −3 and y-intercept: 4 09307083

Solution:

x− intercept be $a = -3$

y− intercept be b = 4

The equation of lien two-intercept form is:

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{-3} + \frac{y}{4} = 1$

$\frac{-4x + (3)y}{12} = 1$

$-4x + 3y = 12$

$\Rightarrow\ 0 = 4x - 3y = 12$

$4x - 3y + 12 = 0$

(i) *x*-intercept: −9 and slope: −4 09307084

Solution:

x−intercept = −9

Slope = m = −4

If *x*−intercept is −9, then line passes through point (−9, 0)

The equation of line in point slope form is:

$y-y_1 = m(x - x_1)$ ________ (i)

Putting the values

$y-0 = -4[x-(-9)]$

$y-0 = -4(x+9)$

$y = -4x - 36$

$4x + y + 36 = 0$

Q.7 Find an equation of the perpendicular bisector of the segment joining the points A (3 ,5) and B (9, 8). 09307085

Solution:

A (3 ,5), B (9, 8).

Perpendicular bisector passes through midpoint of a line segment perpendicularly.

Let midpoint of A and B be $M(x_m, y_m)$

$$M\left(\frac{x_1+x_2}{2}, \frac{y_2+y_1}{2}\right)$$

$$M\left(\frac{3+9}{2}, \frac{5+8}{2}\right)$$

$$= M\left(\frac{12}{2}, \frac{13}{2}\right)$$

$= M(6, 6.5)$

Slope of $\overline{AB}$: $m_1 = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{8-5}{9-3} = \dfrac{3}{6} = \dfrac{1}{2}$

Let m_2 be the slope of line $\perp \overline{AB}$

We know that for perpendicular lines

$m_1 \times m_2 = -1$

$\frac{1}{2} \times m_2 = -1$

$m_2 = -1 \times 2$

$\boxed{m_2 = -2}$

The point slope form of equation is:

$y-y_1 = m(x-x_1)$

Since line passes through midpoint. So put

$x_1 = 6, y_1 = \dfrac{13}{2}$

$y-\dfrac{13}{2} = -2(x-6)$

$\dfrac{2y-13}{2} = -2x+12$

$2y-13 = -4x+24$

$2y-13+4x-24 = 0$

$4x+2y-37 = 0$

Q.8 Find an equation of the line through (–4, –6) and perpendicular to a line having slope $\frac{-3}{2}$. 09307086

Solution:

Slope of line $= m_1 = \dfrac{-3}{2}$

Let slope of perpendicular line $= m_2$

We know that

$m_1 \times m_2 = -1$

$\dfrac{-3}{2} \times m_2 = -1$

$\Rightarrow m_2 = \dfrac{-1\times 2}{3}$

$\boxed{m_2 = \dfrac{2}{3}}$

Since line passes through (–4, –6), so we take $(x_1, y_1) = (-4, -6)$

The point slope form of equation is:

$y-y_1 = m(x-x_1)$

$y-(-6) = \dfrac{2}{3}[x-(-4)]$

$y+6 = \dfrac{2}{3}(x+4)$

$\Rightarrow 3(y+6) = 2(x+4)$

$3y+18 = 2x+8$

$\Rightarrow 0 = 2x+8-3y-18$

$\Rightarrow 2x-3y-10 = 0$

Q.9 Find an equation of the line through (11, –5) and parallel to a line with slope–24. 09307087

Solution:

Given point p(11, –5)

Let slope of line $\overline{AB} = m_1 = -24$

Let slope of line parallel to $\overline{AB} = m_2$

Since slopes of parallel lines are equal

$m_2 = m_1$

$m_2 = -24$

The point = slope form of an equation is:

$y - y_1 = m_2(x-x_1)$

put $x_1 = 11, y = -5$

$y-(-5) = -24(x-11)$

$y+5 = -24x+264$

$y+5+24x-264 = 0$

$24x+y-259 = 0$

Q.10 Convert each of the following equations into <u>slope intercept form</u>, two <u>intercept form</u> and <u>normal form</u>: 09307088

Solution

(a) $2x-4y+11=0$ 09307089

$2x-4y+11=0$ ______ (i)

(i) Slope intercept form

From eq. (i)

$2x-4y+11=0$

$2x+11+4y=0$

$\Rightarrow y=\frac{2x+11}{4}$

$y=\frac{2x}{4}+\frac{11}{4}$

$y=\frac{1}{2}x+\frac{11}{4}$ $(\because y=mx+c)$

(ii) Two intercept form

From (i)

$2x-4y+11=0$

$2x-4y=-11$

Dividing both side by -11, we get

$\frac{2x}{-11}-\frac{4y}{-11}=\frac{-11}{-11}$

$\Rightarrow \frac{x}{\frac{-11}{2}}-\frac{y}{\frac{11}{2}}=1$ $(\because \frac{x}{a}+\frac{y}{b}=1)$

(iii) Normal form

From (i)

$2x-4y+11=0$

$2x-4y=-11$

$\because \mp\sqrt{(2)^2+(-4)^2}$

$=\mp\sqrt{4+16}=\pm\sqrt{20}=\pm\sqrt{4\times 5}=\pm 2\sqrt{5}$

Since R.H.S is to be positive, we have to take negative sign.

Dividing both side by $-2\sqrt{5}$

$\frac{2x}{-2\sqrt{5}}-\frac{4y}{-2\sqrt{5}}=\frac{-11}{-2\sqrt{5}}$

$=\frac{11}{2\sqrt{5}}$

Comparing it with

$x\cos\alpha+y\sin\alpha=p$

$x\cos\alpha=\frac{-2}{2\sqrt{5}}<0$ and $\sin\alpha=\frac{4}{2\sqrt{5}}>0$

$\Rightarrow$ Angle α lies in 2^{nd} quadrant and $\alpha=116.57°$

So, $x\cos 116.57°+y\sin 116.57°=\frac{11}{2\sqrt{5}}$

(b) $4x+7y-2=0$ 09307090

(i) slope–intercept form

$4x+7y-2=0$

$7y=-4x+2$

$y=\frac{4x+2}{7}$ (Dividing B.S by 7)

$y=\frac{-4}{7}x+\frac{2}{7}$ $(\because y=mx+c)$

(ii) Two intercept form

$4x+7y-2=0$

$4x+7y=2$

Dividing both side by "2"

$\frac{4x}{2}+\frac{7y}{2}=\frac{2}{2}$

$\frac{2x}{1}+\frac{7y}{2}=1$

$\frac{x}{\frac{1}{2}}+\frac{y}{\frac{2}{7}}=1$ $(\because \frac{x}{a}+\frac{y}{b}=1)$

(iii) Normal form

$4x+7y-2=0$

$4x+7y=2$

$\because \sqrt{4^2+7^2}=\sqrt{16+49}=\sqrt{65}$

Dividing both side by $\sqrt{65}$

$\frac{4x}{\sqrt{65}}+\frac{7y}{\sqrt{65}}=\frac{2}{\sqrt{65}}$

Comparing with $x\cos\alpha+y\sin\alpha=p$

$\cos\alpha=\frac{4}{\sqrt{65}}>0$, $\sin\alpha=\frac{7}{\sqrt{65}}>0$

$\Rightarrow$ Angle lies in 1^{st} quadrant and $\alpha=60.26°$

(c) $15y-8x+3=0$ 09307091

(i) Slope–intercept form

$15y-8x+3=0$

$15y=8x-3$

$y=\frac{8x-3}{15}$

$y=\frac{8x}{15}-\frac{3}{15}$ $\because y=mx+c$

(ii) Two intercept form

$15y - 8x + 3 = 0$

$\Rightarrow -8x + 15y = -3$

Dividing by "−3"

$$\frac{-8x}{-3} + \frac{15y}{-3} = \frac{-3}{-3}$$

$$\frac{8x}{3} + \frac{15y}{-3} = 1$$

$$\Rightarrow \frac{8x}{3} + \frac{15y}{-1} = 1$$

$$\Rightarrow \frac{x}{\frac{3}{8}} + \frac{y}{\frac{-1}{5}} = 1$$

(iii) Normal form

$15y - 8x + 3 = 0$

$-8x + 15y = -3$

$$\left(\because \sqrt{(-8)^2 + (15)^2} = \pm\sqrt{64 + 225} = \pm\sqrt{289} = \pm 17\right)$$

To make R.H.S positive

Dividing both side by "−17"

$$\frac{-8x}{-17} + \frac{15y}{-17} = \frac{-3}{-17}$$

$$\frac{8x}{17} + \frac{15y}{-17} = \frac{3}{17}$$

Comparing it with $x\cos\alpha + y\sin\alpha = p$

$\Rightarrow \cos\alpha = \frac{8}{17} > 0$ and $\sin\alpha = \frac{-15}{17} < 0,$

$\Rightarrow$ Angle α lies in 4^{th} quadrant and α =298.070

$$\Rightarrow x\cos 298.07^\circ\ y\sin 298.07^\circ = \frac{3}{17}$$

Q.11 In each of the following check whether the two lines are:

(i) Parallel 09307092

(ii) Perpendicular 09307093

(iii) Neither parallel nor perpendicular

(a) $2x+y-3=0,\ 4x+2y+5=0$

Solution

$2x+y-3=0$ _______(i)

$4x+2y+5=0$ _______(ii)

Slope of line (i), $m_1 = \frac{-\text{coeff. of } x}{\text{coeff. of y}} = \frac{-2}{1} = -2$

Slope of line (ii),

$$m_2 = \frac{-\text{coeff. of } x}{\text{coeff. of y}} = -\frac{4}{2} = -2$$

We observe that $m_1 = m_2$

So given pair of lines are parallel. To each other.

(b) $3y = 2x + 5,\ 3x + 2y - 8 = 0$ 09307094

$2x - 3y + 5 = 0$ _______(i)

$3x + 2y - 8 = 0$ _______(ii)

Slope of line (i):

$$m_1 = \frac{-\text{coeff. of } x}{\text{coeff. of y}} = \_\_\_ \frac{-2}{-3} = \frac{2}{3}$$

Slope of line (ii)

$$m_2 = \frac{-\text{coeff. of } x}{\text{coeff. of y}} = -\frac{3}{2}$$

We observe that

$$m_1 \times m_2 = \frac{2}{3} \times \frac{-3}{2}$$

$m_1 \times m_2 = -1$

Hence the pair of lines are perpendicular to each other.

(c) $4y + 2x - 1 = 0,\ x - 2y - 7 = 0$ 09307095

Solution

$2x + 4y - 1 = 0$ _______ (i)

$x - 2y - 7 = 0$ _______ (ii)

Slope of line (i),

$$m_1 = \frac{-\text{coeff. of } x}{\text{coeff. of y}} = -\frac{2}{4} = -\frac{1}{2}$$

Slope of line (ii)

$$m_2 = \frac{-\text{coeff. of } x}{\text{coeff. of y}} = -\frac{1}{-2} = \frac{1}{2}$$

Multiplying m_1 & m_2

$$m_1 \times m_2 = \frac{-1}{2} \times \frac{1}{2} = \frac{-1}{4}$$

Hence the lines are neither parallel not perpendicular to each other.

Q.12 Find an equation of the line (−4,7) and parallel to the line $2x-7y+4=0$. 09307096

Solution:

$2x - 7y + 4 = 0$

Sot slope of line,

$$m_1 = \frac{-\text{coeff. of } x}{\text{coeff. of y}} = \frac{-2}{-7} = \frac{2}{7}$$

Since, slope of parallel line are equal so.

Slope of new line is $m = \frac{2}{7}$.

As line passes through point (−4, 7), so we can take $(x_1, y_1) = (-4, 7)$

Using point slope from of equation.

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{2}{7}[x - (-4)]$$

$$7(y-7) = 2(x+4)$$

$$7y - 49 = 2x + 8$$

$$\Rightarrow 0 = 2x + 8 - 7y + 49$$

$$\Rightarrow 2x - 7y + 57 = 0$$

Q.13 Find an equation of the line through (−5, 8) and perpendicular to the join of A(−15, −8), B(10, 7). **09307097**

Solution:

A(−15, −8), B(10, 7)

$$\text{Slope of } \overline{AB} = \frac{x_2 - x_1}{y_2 - y_1} = \frac{7-(-8)}{10(-15)}$$

$$= \frac{7+8}{10+15} = \frac{15}{25} = \frac{3}{5}$$

The Slope of line perpendicular to $\overline{AB} = m_2$ we know that for perpendicular lines,

$$m_1 \times m_2 = -1$$

$$\frac{3}{5} \times m_2 = -1$$

$$\boxed{m_2 = \frac{-5}{3}}$$

Since line passes through point (5, −8), so we can take $(x_1, y_1) = (5, -8)$

The point – slope of equation is:

$$y - y_1 = m_2(x - x)$$

$$y - (-8) = \frac{-5}{3}(x - 5)$$

$$3(y+8) = -5(x-5)$$

$$3y + 24 = -5x + 25$$

$$\Rightarrow 3y + 24 + 5x - 25 = 0$$

$$\Rightarrow 5x + 3y - 1 = 0$$

Applications of Coordinate Geometry in Real life Situation

Example 14: On a map, Town A is at coordinates (2, 3) and Town *B* is at (−4, −1). What is the distance between the two towns? **09307098**

Solution: Use the distance formula:

$$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$$

Substitute the values:

$$d = \sqrt{(-4-2)^2 + (-1-3)^2} = \sqrt{(-6)^2 + (-4)^2}$$

$$= \sqrt{36+16} = \sqrt{52} \approx 7.21 \text{ unit.}$$

Thus, the distance between Town A and Town B is approximately 7.21 units.

Example 15: Suppose two cities, City *A* and City *B*, are represented by the coordinates (3, 4) and (7, 1) on a map. Find the straight-line distance between the two cities. **09307099**

Solution:

We apply the distance formula:

$$d = \sqrt{(7-3)^2 + (1-4)^2}$$

$$d = \sqrt{(4)^2 + (-3)^2}$$

$$d = \sqrt{16+9}$$

$$d = \sqrt{25}$$

$$d = 5$$

Thus, the straight line distance between City A and City B is 5 units.

Example 16: An Engineer is building a bridge between two points on a riverbank. Suppose the coordinates of the two points where the bridge will start and end are (2,5) and (8,9). Find the coordinates of the midpoint, which will represent the center of the bridge. **09307100**

Solution:

We apply the midpoint formula:

$$M = \left(\frac{2+8}{2}, \frac{5+9}{2}\right)$$

$$M = \left(\frac{10}{2}, \frac{14}{2}\right)$$

$$M = (5, 7)$$

Thus, the center of the bridge is at the point (5, 7)

Example 17: A landscaper is designing a triangular garden with corners at points $A(2, 3)$, $B(5, 7)$, and $C(6, 2)$. Calculate the lengths of the sides of the triangle.

Solution:

Use the **distance formula** to find the length of each side:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$|\overline{AB}| = \sqrt{(5-2)^2+(7-3)^2}$$

$$|\overline{AB}| = \sqrt{(3)^2+(4)^2}$$

$$|\overline{AB}| = \sqrt{9+16}=\sqrt{25}=5 \text{ units}$$

Now,

$$|\overline{BC}| = \sqrt{(6-5)^2+(2-7)^2}$$

$$|\overline{BC}| = \sqrt{(1)^2+(-5)^2}$$

$$|\overline{BC}| \ \sqrt{1+25}=\sqrt{26}=5.10 \text{ units}$$

Now,

$$|\overline{AC}| = \sqrt{(6-2)^2+(2-3)^2}$$

$$|\overline{AC}| = \sqrt{(4)^2+(-1)^2}$$

$$|\overline{AC}| = \sqrt{16+1}=\sqrt{17}=4.12 \text{ units}$$

Thus, the lengths of the sides are:

$m\overline{AB} = 5$units, $m\overline{BC} \approx 5.10$ units,

$m\overline{AC} \approx 4.12$units

Example 18: A pilot needs to travel from city A(50, 60) to city B(120, 150), Determine the heading angle the plane should take relative to the east direction.

09307101

Solution

The heading angle can be calculated using the slope: $m = \dfrac{y_2-y_1}{x_2-x_1}$

$$m=\frac{150-60}{120-50}=\frac{90}{70}=\frac{9}{7}$$

Let θ be the required angle, then

> **Do you know?**
>
> **Aviation** is the operation and flight of aircraft, including airplanes, helicopters and drones.
>
> **Navigation** is the process of determining and controlling the route of a vehicle, such as an aircraft, from one place to another.

$$\tan\theta = m = \left(\frac{9}{7}\right)$$

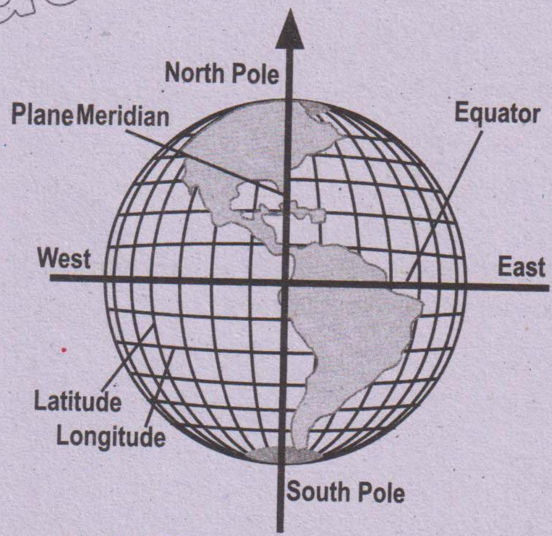
$$\theta = \tan^{-1} = \left(\frac{9}{7}\right)$$

$$\theta = \tan^{-1} = (1.2857)$$

$$\theta \approx 52.13°$$

Thus, the plane should take a heading angle of 52.13° north of east.

Latitude: Measures how far a location is from the equator. It ranges from 0° at the equator to 90° north (at the North Pole) or 90° south (at the South Pole).



Longitude: Measures how far a location is from the Prime Meridian to 180° east and 180° west.

Example 19: Abdul Hadi is traveling from point A (Latitude 10° N, Longitude 50°E) to point B (Latitude 20°N, Longitude 60°E). Find the midpoint of his journey in terms of latitude and longitude.

09307102

Solution:

Point A (Latitude 10° N, Longitude 50°E)

Point B (Latitude 20° N, Longitude 60°E)

Midpoint latitude = $\frac{10° + 20°}{2} = \frac{30°}{2} = 15°N$

Midpoint latitude = $\frac{50° + 60°}{2} = \frac{110°}{2} = 55°E$

Thus, the midpoint of Abdul Hadi's journey would be at Latiude 15° N, Longtitude 55°E.

Example20: A landscaper is designing a straight pathway from (P(2, 3) to Q(8, 9). What is the length of the pathway?

09307103

Solution

The length of the straight pathway can be found using the distance formula:

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8-2)^2 + (9-3)^2}$

$= \sqrt{(6)^2 + (6)^2}$

$= \sqrt{36 + 36}$

$= \sqrt{72}$

$= \sqrt{36 \times 2}$

$= 6\sqrt{2}$

So, the length of the pathway is approximately $6\sqrt{2}$ units.

EXERCISE 7.3

Q.1 If the houses of two friends are represented by coordinates (2, 6) and (9, 12) on a grid. Find the straight line distance between their houses if the grid units represent kilometes? **09307104**

Solution

Let house A(2,6)

House B(9, 12)

We know that

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

putting values

$|\overline{AB}| = \sqrt{(9-2)^2 + (12-6)^2}$

$|\overline{AB}| = \sqrt{(7)^2 + (6)^2}$

$|\overline{AB}| = \sqrt{49 + 36}$

$|\overline{AB}| = \sqrt{85} \approx 9.22 \quad (\because 1 \text{ unit} = \text{km})$

Thus distance between the houses is 9.22 Km.

Q.2 Consider a straight trail (represented by coordinate plane) that starts at point (5, 7) and ends at point (15, 3). What is the coordinate of the midpoint? **09307105**

Solution:

Let end points be A (5,7), B(15,3)

Let midpoint $M(x_m, y_m)$ =

$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= M\left(\frac{5+15}{2}, \frac{7+3}{2}\right)$

$= M\left(\frac{20}{2}, \frac{10}{2}\right)$

$= M(10, 5)$

Q.3 An architect is designing a park with two buildings located at (10, 8) and (4, 3) on the grid. Calculate the straight-line distance between the buildings. Assume the coordinates are in meters.

09307106

Solution:

Let A and B represents he locations of two buildings. A(10,8), B(4,3).

By using distance formula.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

putting values

$|\overline{AB}| = \sqrt{(4-10)^2 + (3-8)^2}$

$|\overline{AB}| = \sqrt{(-6)^2 + (-5)^2}$

$|\overline{AB}| = \sqrt{36+25}$

$|\overline{AB}| = \sqrt{61}$

$|\overline{AB}| = 7.81$ $\because$ 1unit = 1meter

Thus distance between two building is 7.81meters.

Q.4 A delivery driver needs to calculate the distance between two delivery locations. One location is at (7, 2) and the other at (12, 10) on the city grid map, where each unit represents kilometers. What is the distance between the two locations? 09307107

Solution:

Let A and B represent two locations.

A (7, 2), B(12, 10)

By using distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

putting values

$$|\overline{AB}| = \sqrt{(12-7)^2 + (10-2)^2}$$

$$|\overline{AB}| = \sqrt{(5)^2 + (8)^2}$$

$$|\overline{AB}| = \sqrt{25+64}$$

$$|\overline{AB}| = \sqrt{89} \approx 9.43 \text{ units}$$

Since, 1 grid unit = 1km.

So distance between two location is 9.43km

Q.5 The start and end points of a race track are given by coordinates (3, 9) and (9, 13). What is the midpoint of the track? 09307108

Solution:

Let start point be A(3,9)

end point be B(9,13)

Let midpoint of track be $M(x_m, y_m)$.

$$M(x_m, y_m) = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Putting values

$$= M\left(\frac{3+9}{2}, \frac{9+13}{2}\right)$$

$$= M\left(\frac{12}{2}, \frac{22}{2}\right)$$

$$= M(6, 11)$$

Thus coordinates of midpoint of track are M(6,11)

Q.6 The coordinates of two point on a road are A(3, 4) and B(7, 10). Find the midpoint of the road. 09307109

Solution:

Two points on the road A(3,4), B(7,10)

By using midpoint formula.

$$M(x_m, y_m) = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

putting values

$$= M\left(\frac{3+7}{2}, \frac{4+10}{2}\right)$$

$$= M\left(\frac{10}{2}, \frac{14}{2}\right)$$

$$= M(5, 7)$$

Thus midpoint of two points on the road is m(5,7)

Q.7 A ship is navigating from port A located at (12°N, 65°W) to port B at (20°N, 45°W). If the ship travels along the shortest path on the surface of the Earth, calculate the straight line distance between the points. 09307110

Solution

Location of port A(12°N, 65°W) = (x_1, y_1)

Location of port B(20°N, 45°W) = (x_2, y_2)

By using distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|\overline{AB}| = \sqrt{(20-12)^2 + (45-65)^2}$$

$$|\overline{AB}| = \sqrt{(8)^2 + (-20)^2} = \sqrt{64+400}$$

$$= \sqrt{464} = 21.54 \text{ units.}$$

Q.8 Farah is fencing around a rectangular field with corners at (0,0), (0,5), (8,5) and (8,0). How much fencing material will she need to cover the entire perimeter of the field? 09307111

Solution:

Let coordinates of corners of rectangular field be A(0,0), B(0,5), C(8,5), D(8,0).

First we find the length and width of rectangular field using the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Finding length $|\overline{AB}|$

$$|\overline{AB}| = \sqrt{(0-0)^2 + (5-0)^2} = \sqrt{(0)^2 + (5)^2}$$

$$= \sqrt{0+25} = \sqrt{25} = 5 \text{ units}$$

Finding width $|\overline{BC}|$

$$|\overline{BC}| = \sqrt{(8-0)+(5-5)^2} = \sqrt{(8)^2 + (0)^2}$$

$$= \sqrt{64+0} = \sqrt{64} = 8 \text{ units}$$

Finding perimeter

Perimeter = 2 [$|\overline{AB}| + |\overline{BC}|$] $\because P = 2(\ell + w)$

$= 2[5 + 8]$ units

$= (13)$ units

$= 26$ units

Thus 26 units facing material is required to cover the perimeter of the field.

Q.9 An airplane is flying from city X at (40° N, 100°W) to city Y at (50°N, 80°W). Use coordinate geometry, to calculate the shortest distance between these cities.

09307112

Solution

By using distance formula,

$$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$$

putting the values

$$|\overline{XY}| = \sqrt{(50-40)^2 + (80-100)^2}$$

$$= \sqrt{(10)^2 + (-20)^2}$$

$$= \sqrt{100+400}$$

$$= \sqrt{500}$$

$$= \sqrt{100 \times 5}$$

$$= 10\sqrt{5} \approx 22.4 \text{ units}$$

Q.10 A land surveyor is marking out a rectangular plot of land with corner at (3,1), (3,6), (8,6), and (8,1). Calculate the perimeter.

09307113

Solution.

Let coordinates of corner are A(3,1), B(3,6), C(8,6), D(8,1)

First we find length and width of rectangular plot by using the distance formula.

$$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$$

Finding length $\overline{AB}$

$$|\overline{AB}| = \sqrt{(3-3)^2 + (6-1)^2}$$

$$|\overline{AB}| = \sqrt{(0)^2 + (5)^2}$$

$$|\overline{AB}| = \sqrt{0+25} = \sqrt{25} = 5 \text{ units}$$

Finding width $\overline{BC}$

$$|\overline{BC}| = \sqrt{(8-3)^2 (6-6)^2}$$

$$|\overline{BC}| = \sqrt{(5)^2 + (0)^2} = \sqrt{25+0} = \sqrt{25} = 5$$

units

We observe that rectangular plot is a square

Finding perimeter

Perimeter = $4|\overline{AB}|$ $(\because P = 4\ell)$

$= 4(5 \text{ units})$

$= 20$ units

Q.11 A landscaper needs to install a fence around a rectangular garden. The garden has its corners at the coordinates: A(0,0), B(5,0), C(5,3), and D(0, 3). How much fencing is required?

09307114

Solution:

First we find the length and width of rectangle using distance formula.

$$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$$

Length of side $\overline{AB}$: A(0,0), B(5,0)

$$L = \overline{AB} = \sqrt{(5,0)^2 + (0-0)^2}$$

$=\sqrt{(5)^2+(0)^2}$

$=\sqrt{25+0} = \sqrt{25}$

= 5 units

Length of side $\overline{BC}$: B(5, 0), C(5, 3)

$W = |\overline{BC}| = \sqrt{(5-5)^2+(3-0)^2}$

$=\sqrt{(0)^2+(3)^2}$

$=\sqrt{0+9} = \sqrt{9} = 3$

We know that fencing required is equal to the perimeter of rectangular garden. So

Perimeter = 2(L+W)

P = 2[5+3]

P = 2 (8)

P = 16 units

Review Exercise 7

Q.1 Choose the correct option.

i. The equation of a straight line in the slope-intercept form is written as: **09307115**

(a) $y = m(x+c)$ (b) $y-y1 = m(x-x_1)$

(c) $y = c + mx$ (d) $ax + by + c = 0$

ii. The gradient of two parallel lines is:

(a) Equal (b) Zero

(c) Negative reciprocals of each other

(d) Always undefined **09307116**

iii. If the product of the gradients of two lines is (–1), then the lines are: **09307117**

(a) parallel (b) perpendicular

(c) collinear (d) coincident

iv. Distance between two points P(1, 2) and (4, 6) is: **09307118**

(a) 5 (b) 6

(c) $\sqrt{13}$ (d) 4

v. The midpoint of a line segment with endpoints (–2, 4) and (6, –2) is: **09307119**

(a) *(4, 2)* b) (2, 1)

(c) (1, 1) (d) *(0, 0)*

vi. A line passing through points (1, 2) and (4, 5) has which equation in the slope-intercept form? **09307120**

(a) $y = x + 1$ (b) $y = 2x + 3$

(c) $y = 3x - 2$ (d) $y = x + 2$

vii. The equation of a straight line in the pint slope form is written as: **09307121**

(a) $y = m(x + c)$ (b) $y - y_1 = m(x - x_1)$

(c) $y = c + mx$ (d) $ax + by + c = 0$

viii. 2x+3y–6 = 0 in he slope-intercept form is: **09307122**

(a) $y = \frac{-2}{3}x + 2$ (b) $y = \frac{2}{3}x - 2$

(c) $y = \frac{2}{3}x + 1$ (d) $y = \frac{-2}{3}x - 2$

ix. The equation of a line in symmetric form is: **09307123**

(a) $\frac{x}{a} + \frac{y}{b} = 1$

(b) $\frac{x-x_1}{1} + \frac{y-y_1}{m} = \frac{z-z_1}{1}$

(c) $ax + by + c = 0$

(d) $y-y_1 = m(x-x_1)$

x. The equation of a line in normal form is: **09307124**

(a) $y = mx+c$ (b) $\frac{x}{a} = \frac{y}{b} = 1$

(c) $\frac{x-x_1}{cos\alpha} = \frac{y-y_1}{\sin\alpha}$ (d) $y-y_1 = m(x-x_1)$

Answers Key

i	c	ii	a	iii	b	iv	a	v	b
vi	a	vii	b	viii	a	ix	c	x	d

Multiple Choice Questions (Additional)

Coordinate plane

1. The first component of each ordered pair (x,y) is called: **09307125**
(a) ordinate (b) Coordinate
(c) orgin (d) Abscissa

2. All points (x,y) with x>0,y>0 lie in quadrant: **09307126**
(a) I (b) II
(c) III (d) Iv

3. All points (x,y) with x<0,y<0 lie in quadrant: **09307127**
(a) I (b) II
(c) III (d) Iv

4. All points (x,y) with x>0,y<0 lie in quadrant: **09307128**
(a) I (b) II
(c) III (d) Iv

5. All points (x,y) with x<0,y>0 lie in quadrant: **09307129**
(a) I (b) II
(c) III (d) Iv

6. Which of the following is not on the x-axis: **09307130**
(a) (00) (b)(a,0)
(c) (b,0) (d) (0,c)

7. Which of the following is not on the y-axis: **09307131**
(a) (00) (b)(0,e)
(c) (0,*f*) (d) (*g*,0)

8. The line of which equation bisect the 1st and 3rd quadrant? **09307132**
(a) $x-y=0$ (b) $x+y=0$
(c) y=2x (d) y=5x

9. The line of which equation bisect the 2nd and 4th quadrant? **09307133**
(a) $x-y=0$ (b) $x+y=0$
(c) $y=-4x$ (d) $y=-6x$

Slope of lines

10. The slope of the line is: **09307134**
(a) $m=\dfrac{x_2-x_1}{y_2-y_1}$ (b) $m=\dfrac{y_2-y_1}{x_2-x_1}$
(c) $m=\dfrac{x_1-x_2}{y_1-y_2}$ (d) $m=\dfrac{y_1+y_2}{x_1-x_2}$

11. If m_1 and m_2 are slopes of two parallel lines then: **09307135**
(a) $m_1\times m_2=0$ (b) $m_1+m_2=0$
(c) $m_1-m_2=0$ (d) $m_1\times m_2=-1$

12. If m_1 and m_2 are slopes of two perpendicular lines then: **09307136**
(a) $m_1\times m_2=0$ (b) $m_1+m_2=0$
(c) $m_1-m_2=0$ (d) $m_1\times m_2=-1$

13. The slope line $\dfrac{x}{3}+\dfrac{y}{2}=1$ is: **09307137**
(a) $\dfrac{2}{3}$ (b) $-\dfrac{2}{3}$
(c) $-\dfrac{3}{2}$ (d) $\dfrac{3}{2}$

14. The line of which equation has slope 2 and passes through the origin? **09307138**
(a) $y=x+2$ (b) $y=2x+2$
(c) $y=2x-2$ (d) $y=2x$

15. If a line of slope−3 passes through origin and P(3,*k*) then value of *k* is: **09307139**
(a) 3 (b) −3
(c) 9 (d) −9

16. For what value of *k*, a line passing through the points (−3,−7) and (4,*k*) has gradient $\dfrac{3}{7}$?: **09307140**
(a) 4 (b) −4
(c) −3 (d) −7

17. If x-coordinates of two points are same then line passing through them is parallel to: **09307141**
(a) x-axis (b) y-axis
(c) origin (d) any line

18. If x-coordinates of two points are same then line passing through them is perpendicular to: **09307142**
(a) x-axis (b) y-axis
(c) origin (d) any line

19. If y-coordinates of two points are same then line passing through them is parallel to: 09307143

(a) x-axis (b) y-axis
(c) origin (d) any line

20. If y-coordinates of two points are same then line passing through them is perpendicular to: 09307144

(a) x-axis (b) y-axis
(c) origin (d) any line

Answer Key

1	d	2	a	3	c	4	d	5	b	6	d	7	d	8	a	9	b	10	b
11	c	12	d	13	c	14	d	15	d	16	b	17	b	18	a	19	a	20	b

Q.2 Find the distance between two points $A(2, 3)$ and $B(7, 8)$ on a coordinate plane. 09307145

Solution

A(2,3), B(7,8)

Using distance formula

$$\mathbf{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y)^2}$$

putting values

$$|\overline{AB}| = \sqrt{(7-2)^2 + (8-3)^2}$$

$$|\overline{AB}| = \sqrt{(5)^2 + (5)^2}$$

$$= \sqrt{25+25}$$

$$= \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

Q.3 Find the midpoint of the line segment joining the points (4, –2) and (–6, 3). 09307146

Solution

(4, –2) and (–6, 3).

Let $M(x_m, y_m)$ be midpoint of A and B.

$$M(x_m, y_m) = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Putting values

$$= M\left(\frac{4+(-6)}{2}, \frac{-2+3}{2}\right)$$

$$= M\left(\frac{4-6}{2}, \frac{1}{2}\right)$$

$$= M\left(\frac{-2}{2}, \frac{1}{2}\right)$$

$$= M(-1, 0.5)$$

Thus required midpoint is M(–1, 0.5)

Q.4 Calculate the gradient (slope) of the line passing through the points (1, 2) and (4, 6). 09307147

Solution

(1, 2), (4, 6)

$A(1, 2) = (x_1, y_1)$

$B(4, 6) = (x_2, y_2)$

$$\text{Slope of line AB} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{4-1} = \frac{4}{3}$$

Q.5 Find the equation of the line in the form $y = mx + c$ that passes through the points (3, 7) and (5, 11). 09307148

Solution

A (3, 7) , B (5, 11)

Let $A(3,7) = (x_1, y_1)$

$B(5,11) = (x_2, y_2)$

$$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-7}{5-3} = \frac{4}{2} = 2$$

The equation of line in point slope form.

$y - y_1 = m(x - x_1)$

$y - 7 = 2(x-3)$ $\quad (\because A(3, 7) = (x_1, y_1))$

$y - 7 = 2x - 6$

$y = 2x - 6 + 7$

$y = 2x + 1$

Q.6 If two lines are parallel, and one line has a gradient of $\frac{2}{3}$, what is the gradient of the other lines? 09307149

Solution

Let gradient of one line $= m_1 = \frac{2}{3}$

Since parallel lines have same gradie

(slope) so, gradient of other line, $m_2 = \frac{2}{3}$ i.e.

$m_1 = m_2$

Q.7 An airplane needs to fly from city A to coordinates (12,5) to city B at coordinates (8, –4). Calculate the straight-line distance between these two cities. **09307150**

Solution

City A = A(12, 5)

City B = A(8, –4)

Using distance formula,

$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$

putting values

$|\overline{AB}| = \sqrt{(8-12)^2 + (-4-5)^2}$

$|\overline{AB}| = \sqrt{(-4)^2 + (-9)^2}$

$|\overline{AB}| = \sqrt{16+81}$

$|\overline{AB}| = \sqrt{97}$ units

Q.8 In a landscaping project, the path starts at (2, 3) and ends at (10, 7). Find the midpoint. **09307151**

Solution:

Let start point A(2, 3)

end point B(10,7)

Let $M(x_m, y_m) = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= M\left(\frac{2+10}{2}, \frac{3+7}{2}\right)$

$= M\left(\frac{12}{2}, \frac{10}{2}\right)$

$= M(6, 5)$

This required midpoint is M(6, 5).

Q.9 A drone is flying from point (2, 3) to point (10, 15) on the grid. Calculate the gradient of the line along which the drone is flying and the total distance traveled. **09307152**

Solution

Let $A(2,3) = (x_1, y_1)$

$B(10, 15) = (x_2, y_2)$

(a) Gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{15-3}{10-2} = \frac{12}{8} = \frac{3}{2}$

(b) By distance formula,

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|\overline{AB}| = \sqrt{(10-2)^2 + (15-3)^2}$

$= \sqrt{(8)^2 + (12)^2}$

$= \sqrt{64+144}$

$= \sqrt{208}$

$= \sqrt{16 \times 13}$

$= 4\sqrt{13}$ units

Q.10 For a line with a gradient of (–3) and a y-intercept of (2), write the equation of the line in:

(a) Slope-intercept form **09307153**

(b) Point-slope form (using the point (1, 2) **09307154**

(c) Two-point form (using the points (1, 2) and (4, -7) **09307155**

(d) Intercepts form **09307156**

(e) Symmetric form **09307157**

(f) Normal form **09307158**

Solution:

Slope /gradient = m = –3

y – intercept = c = 2

$y = mx + c$

$y = -3x + 2$

$\Rightarrow 3x + y - 2 = 0$ ... (i)

(a) Slope intercept form:

$y = -3x + 2$

(b) Point slope form:

$y - y_1 = m(x - x_1)$

Put $m = -3$, $x_1 = 1$, $y_1 = 2$

$y - 2 = -3(x - 1)$

(c) Two-point form using (1, 2), and (4, –7)

Let $(x_1, y_1) = (1, 2) \Rightarrow x_1 = 1, y_1 = 2$

$(x_2, y_2) = (4, -7) \Rightarrow x_2 = 4, y_2 = -7$

$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

$\frac{y-2}{-7-2} = \frac{x-1}{4-1}$

(d) Intercepts form:

From (i) $3x+y-2=0$

$3x+y=02$

Dividing "2, " we get

$$\frac{3x}{2}+\frac{y}{2}=\frac{2}{2}$$

$$\frac{x}{\frac{2}{3}}+\frac{y}{2}=1$$

(e) Symmetric Form

From (i) $3x+y=2$ $\left(\because \sqrt{3^2+1^2}=\sqrt{9+1}=\sqrt{10}\right)$

Dividing B.S by $\sqrt{10}$

$$\frac{3x}{\sqrt{10}}+\frac{y}{\sqrt{10}}=\frac{2}{\sqrt{10}}$$

(f) Normal form

From (i)

$3x+y=2$ $\left(\because \sqrt{3^2+1}=\sqrt{9+1}=\sqrt{10}\right)$

Dividing B.S by $\sqrt{10}$

$$\frac{3x}{\sqrt{10}}+\frac{y}{\sqrt{10}}=\frac{2}{\sqrt{10}}$$

Comparing with

$x\cos\alpha+y\sin\alpha=p$

$\cos\alpha=\frac{3}{\sqrt{10}}>0$, $\sin\alpha=\frac{1}{\sqrt{10}}>0 \Rightarrow \alpha$ lies in Q.I.

$\alpha=18.43$

$$\Rightarrow x\cos 18.43^\circ+y\sin 18.43^\circ=\frac{2}{\sqrt{10}}$$