UNIT 09 Similar Figures

Similarity of Polygons

Two polygons are **similar** if their corresponding angles are equal and the corresponding sides are proportional (i.e., the ratios of the lengths of corresponding sides are equal).

Identification of Similar Triangles

Case-I: If two angles in one triangle are congruent to two angles in another triangle, the third angles in each triangle must also be congruent: Since the angles are the same, the triangles are similar. Similarity symbol is "~

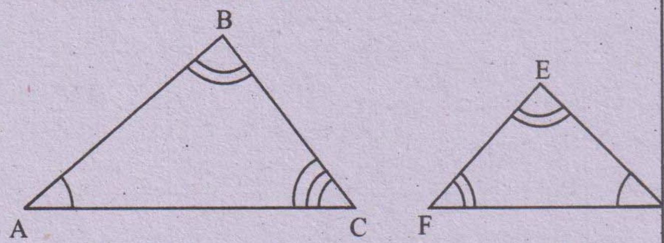
i.e., In the correspondence of the triangles *ABC* and *DEF*.

$m\angle A = m\angle D$

$m\angle B = m\angle E$

$m\angle C = m\angle F$

Hence, $\Delta ABC \sim \Delta DEF$

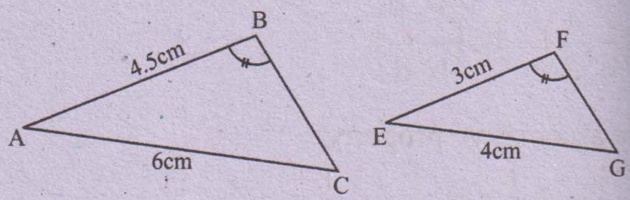


Case-II: If the ratio of two corresponding sides and their included angle are equal, then the triangles are similar. In the correspondence of the triangles *ABC* and *EFG*,

$m\angle ABC = m\angle EFG$ and the ratio of the corresponding sides are

$$\frac{m\overline{AB}}{m\overline{EF}} = \frac{4.5}{3} = \frac{3}{2}$$

and $\frac{m\overline{AC}}{m\overline{EG}} = \frac{6}{2} = \frac{3}{2}$



Hence triangles *ABC* and *EFG* are similar.

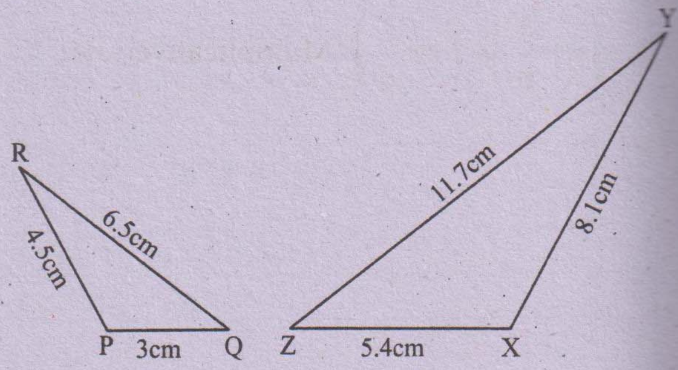
Case-III: If the ratio of all the corresponding sides are equal, then the triangles are similar.

The ratio of corresponding sides are

$$\frac{m\overline{PQ}}{m\overline{ZX}} = \frac{m\overline{QR}}{m\overline{YZ}} = \frac{m\overline{PR}}{m\overline{XY}}$$

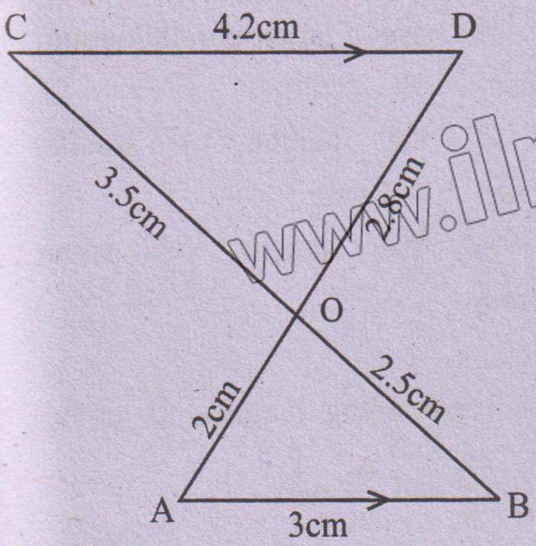
$$\frac{3}{5.4} = \frac{6.5}{11.7} = \frac{4.5}{8.1}$$

$$\frac{5}{9} = \frac{5}{9} = \frac{5}{9}$$



So, Triangle PQR and xyz are similar.

Example 1 If one pair of corresponding sides are parallel to each other, then the triangles formed as shown in the figure are similar. So, i.e. **09309001**



Solution:

In the figure

m∠AOB = m∠DOC (Vertically opposite angles)

m∠A = m∠D (Alternate angles of parallel lines)

m∠B = m∠C (Alternate angles of parallel lines)

So, all three corresponding angles are equal

so $\Delta OAB \sim \Delta ODC$

If we take the ratio of corresponding sides are equal i.e.,

$$\frac{m\overline{OA}}{m\overline{OD}} = \frac{m\overline{AB}}{m\overline{DC}} = \frac{m\overline{OB}}{m\overline{OC}}$$

$$\frac{2}{2.8} = \frac{3}{4.2} = \frac{2.5}{3.5}$$

$$\frac{5}{7} = \frac{5}{7} = \frac{5}{7}$$

So, the triangles OAB and ODC are similar.

Example 2:

The triangles XBC and XDE are similar. Find the value of x and y. **09309002**

Solution:

Since $\overline{BC}$ is parallel to $\overline{ED}$, so triangles XBC and XDE are similar. So, the ratio of corresponding sides are:

$$\frac{m\overline{XB}}{m\overline{XD}} = \frac{m\overline{BC}}{m\overline{DE}} = \frac{m\overline{XC}}{m\overline{XE}}$$

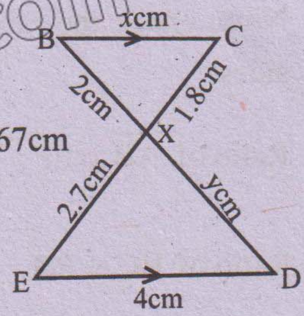
$$\frac{2}{y} = \frac{x}{4} = \frac{1.8}{2.7}$$

$$\frac{x}{4} = \frac{1.8}{2.7cm}$$

$$\Rightarrow x = \frac{1.8}{2.7} \times 4 = \frac{2}{3} \times 4 = \frac{8}{3} = 2.67cm$$

$$\frac{2}{y} = \frac{1.8}{2.7} \Rightarrow y = \frac{2.7}{1.8} \times 2$$

$$\Rightarrow y = 3cm$$

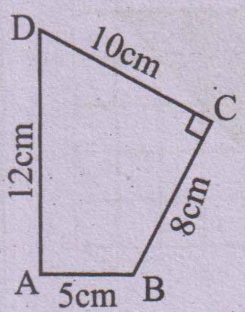


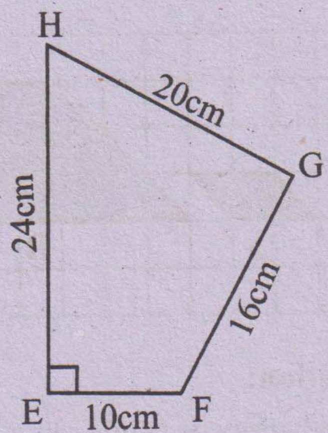
Similarly of quadrilaterals

Example 3: The Quadrilateral *ABCD* has side lengths $m\overline{AB}$ = 5cm, $m\overline{BC}$ = 8cm, $m\overline{CD}$ = 10 cm, $m\overline{AD}$ = 12cm, and its angles are m∠A = 90°, m∠B = 120°, m∠C = 90∘

Quadrilateral EFGH has side lengths $m\overline{EF}$ = 10, $m\overline{FG}$ = 16cm, $m\overline{GH}$ = 20cm, $m\overline{EH}$ = 24cm and its angles are m∠E = 90°, m∠F = 120° and m∠H = 60°.

Prove that quadrilateral ABCD is similar to quadrilateral EFGH. **09309003**





Solution:

We see that in the quadrilateral ABCD:

In the quadrilateral EFGH, m∠G = 360° –(90°+120°+60°) = 90°. Now check if the corresponding angles the quadrilaterals are congruent:

m∠D = 360° – (90°+120°+90°) = 60°.

m∠A = m∠E = 90°, m∠B = m∠F = 120°,

m∠C = m∠G = 90° and m∠D = m∠H = 60°.

Next, check the ratios of the corresponding sides:

Ratio of $\overline{AB}$ to $\overline{EF}$: $\frac{m\overline{AB}}{m\overline{EF}} = \frac{5}{10} = \frac{1}{2}$

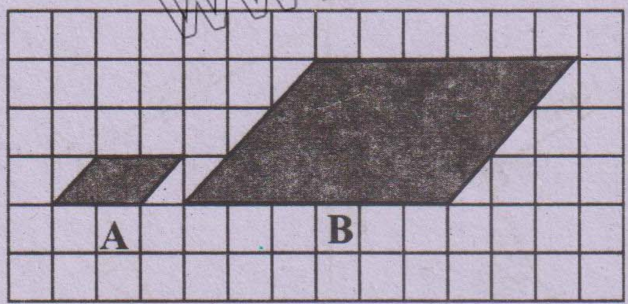
Ratio of $\overline{BC}$ to $\overline{FG}$: $\frac{m\overline{BC}}{m\overline{FG}} = \frac{8}{16} = \frac{1}{2}$

Ratio of $\overline{CD}$ to $\overline{GH}$: $\frac{m\overline{CD}}{m\overline{GH}} = \frac{10}{20} = \frac{1}{2}$

Ratio of $\overline{AD}$ to $\overline{EH}$: $\frac{m\overline{AD}}{m\overline{EH}} = \frac{12}{24} = \frac{1}{2}$

Since the corresponding angles are congruent and the corresponding sides are proportional (with a ratio of $\frac{1}{2}$), quadrilateral *ABCD* is similar to quadrilateral EFGH.

Example 4: Find whether the parallelograms are similar given that one of the angle between sides is 45° in both the parallelograms. **09309004**



Solution:

Since opposite angles in a parallelogram are equal and adjacent angles are supplementary, so the corresponding angles in both parallelograms (45°, 135°, 45°, and 135°) are equal: So, the parallelograms are similar.

Measure of the base of smaller parallelogram,

$b_1 = 2$ units

Measure of the base of larger parallelogram,

$b_2 = 6$ units.

Measure of the height of smaller parallelogram, $h_1 = 1$ unit

Measure of the height of larger parallelogram,

$h_2 = 3$ units.

Ratio of corresponding lengths are equal.

i.e., $\frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3}$ and $\frac{h_1}{h_2} = \frac{1}{3}$

Therefore, $\frac{b_1}{b_2} = \frac{h_1}{h_2}$

Example 5: The perimeter of regular octagon is 48cm. Another octagon has sides that are 1.2 times the sides of the first octagon. What is the length of side of the second octagon? **09309005**

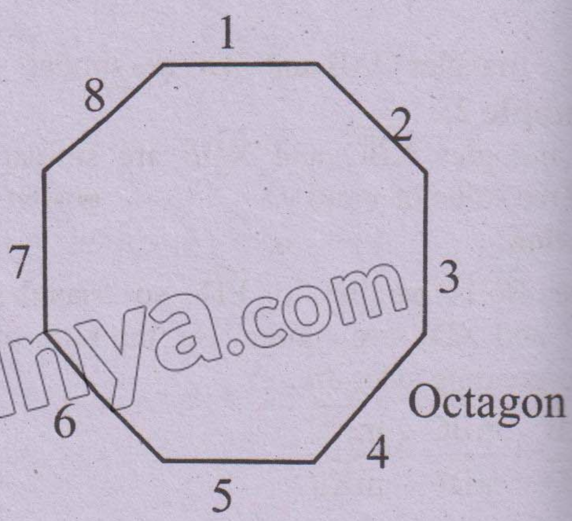
Solution:

Perimeter of first regular octagon = 48cm

Side length of first regular octagon = $\frac{48}{8}$ = 6cm.

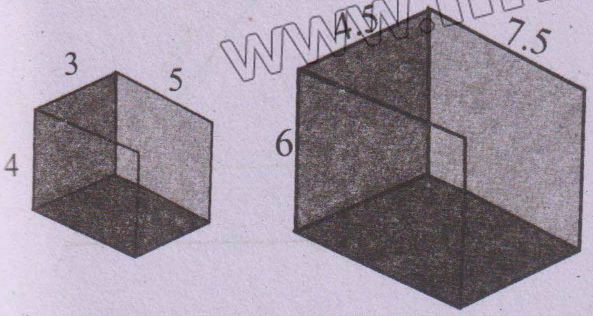
Side length of second regular octagon

= 6× 1.2 = 7.2 cm



Exercise 9.1

Q.1 Find whether the solids are similar. All lengths are in cm. 09309006



Solution:

Let L_1, W_1 and h_1 are length, width and height of smaller cuboid.

L_2, W_2 and h_2 are length, width and height of larger cuboid. The ratios of corresponding sides:

$$\frac{L_1}{L_2} = \frac{5}{7.5} = \frac{2}{3} \text{ ______ (i)}$$

Now, $\frac{W_1}{W_2} = \frac{3}{4.5} = \frac{2}{3}$ ______ (ii)

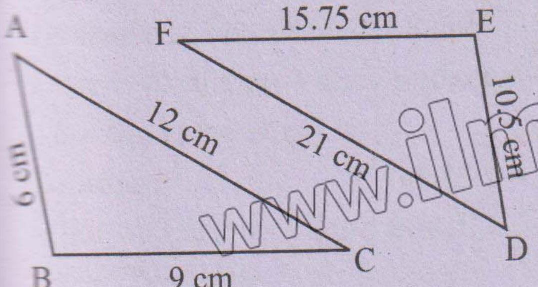
Now, $\frac{h_1}{h_2} = \frac{4}{6} = \frac{2}{3}$ ______ (iii)

From (i), (ii) and (iii)

$$\frac{L_1}{L_2} = \frac{W_1}{W_2} = \frac{h_1}{h_2}$$

Since ratios of corresponding sides are equal so the given solids (cuboid) are similar.

Q.2 In triangle ABC, the sides are given as $m\overline{AB}$ = 6 cm, $m\overline{BC}$ = 9 cm and $m\overline{CA}$ = 12cm. In triangle DEF, the sides are given as $m\overline{DE}$ = 10.5 cm, $m\overline{EF}$ = 15.75 cm, and $m\overline{FD}$ = 21cm. Prove that the triangles are similar. 09309007



Solution:

In $\triangle ABC$,

$m\overline{AB} = 6\text{cm}$, $m\overline{BC} = 9\text{cm}$, $m\overline{CA} = 12\text{cm}$

In $\triangle DEF$,

$m\overline{DE} = 10.5\text{cm}$, $m\overline{EF} = 15.75\text{cm}$, $m\overline{FD} = 21\text{cm}$

Now, take ratios of corresponding sides:

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{6\text{cm}}{10.5\text{cm}} = \frac{60}{105} = \frac{4}{7} \text{ ______ (i)}$$

$$\frac{m\overline{BC}}{m\overline{EF}} = \frac{9\text{cm}}{15.75\text{cm}} = \frac{900}{1575} = \frac{4}{7} \text{ ______ (ii)}$$

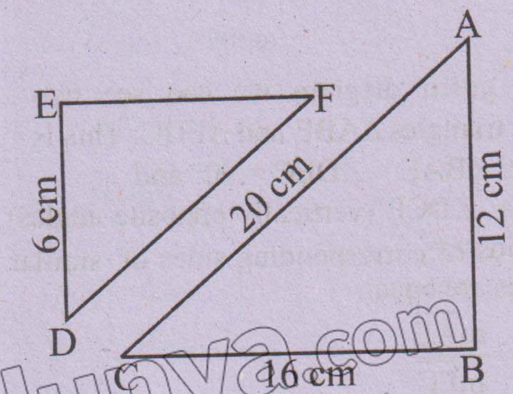
$$\frac{m\overline{AC}}{m\overline{FD}} = \frac{12\text{cm}}{21\text{cm}} = \frac{4}{7} \text{ ______ (ii)}$$

From (i), (iii) and (iii)

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{BC}}{m\overline{EF}} = \frac{m\overline{CA}}{m\overline{FD}}$$

Since, ratios of corresponding sides are equal which proves that triangles are similar.

Q.3 In the figure below, $\triangle ABC \sim \triangle DEF$. $m\overline{AB}$ = 12 cm, $m\overline{AC}$ = 20cm and $m\overline{BC}$ = 16cm. In $\triangle DEF$, $m\overline{DE}$ = 6cm. Find $m\overline{DF}$ and $m\overline{EF}$. 09309008



Solution:

In $\triangle ABC$,

$m\overline{AB} = 12\text{cm}$, $m\overline{BC} = 16\text{cm}$, $m\overline{AC} = 20\text{cm}$

In $\triangle DEF$,

$m\overline{DE} = 6cm$, $m\overline{EF} = x$, $m\overline{DF} = y$

Since $\triangle ABC \sim \triangle DEF$ (Given)

So ratios of corresponding sides are equal i.e.

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{BC}}{m\overline{EF}} = \frac{m\overline{AC}}{m\overline{DF}}$$

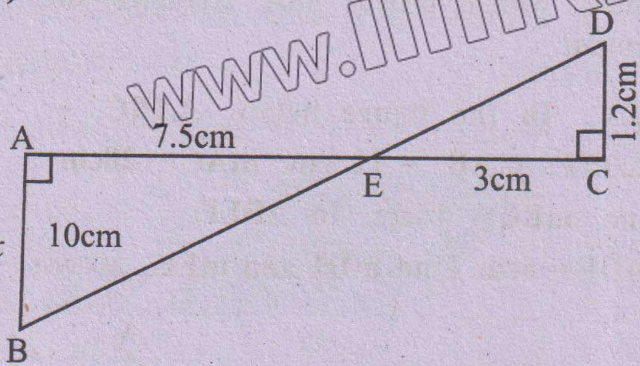
$$\frac{12}{6} = \frac{16}{x} = \frac{20}{y}$$

$\Rightarrow \frac{12}{6} = \frac{16}{x}$ and	$\frac{12}{6} = \frac{20}{y}$
$\Rightarrow 2 = \frac{16}{x}$	$2 = \frac{20}{y}$
$2x = 16$	$2y = 20$
$x = \frac{16}{2}$	$y = \frac{20}{2}$
$x = 8$	$y = 10$

Thus $m\overline{EF} = 8cm$ and $m\overline{DF} = 10cm$

Q.4 Find the value of *x in each of the following.* 09309009

(i) Solution:



In the given diagram we can see two similar triangles $\triangle ABE$ and $\triangle EDC$. This is because $\angle BAE = \angle DEC = 90°$ and $\angle AEB = \angle DCE$ (vertically opposite angles). The ratios of corresponding sides of similar triangles are equal:

$$\frac{m\overline{AB}}{m\overline{DC}} = \frac{m\overline{AE}}{m\overline{EC}}$$

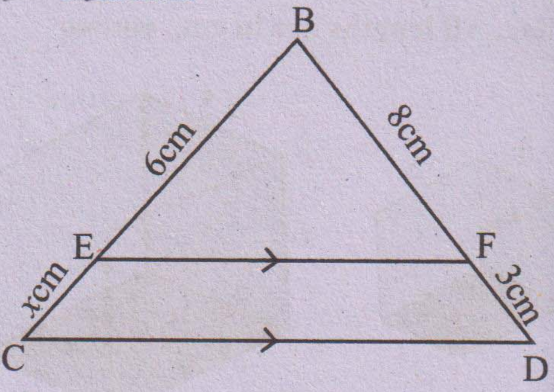
$$\frac{x}{1.2} = \frac{7.5}{3}$$

$$x = \frac{7.5}{3} \times 1.2$$

$x = 2.5 \times 1.2$

$x = 3cm$

(ii) Solution:



$m\overline{BC} = (6+x)cm$

$m\overline{BD} = (8+3)\ cm = 11cm$

Since, $\triangle BEF \sim \triangle BCD$

So corresponding sides are proportional.

$$\frac{m\overline{BE}}{m\overline{BC}} = \frac{m\overline{BF}}{m\overline{BD}}$$

$$\frac{6}{6+x} = \frac{8}{11}$$

$\Rightarrow 11 \times 6 = 8(6+x)$

$66 = 48 + 8x$

$66 - 48 = 8x$

$18 = 8x$

$\frac{18}{8} = x$

$\frac{9}{4} = x$

$\Rightarrow x = 2.25cm$

Alternate method

Since $\overline{EF} \parallel \overline{CD}$, so by proportion.

$$\frac{m\overline{BE}}{m\overline{EC}} = \frac{m\overline{BF}}{m\overline{FD}}$$

$$\frac{6}{x} = \frac{8}{3}$$

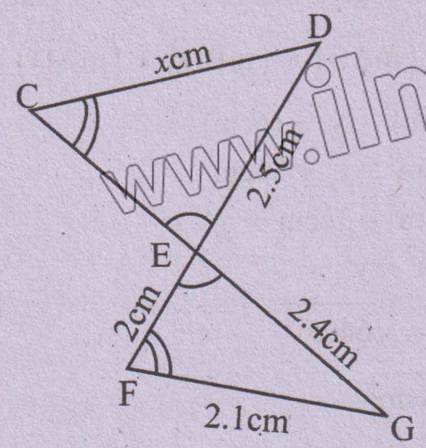
$6 \times 3 = 8x$

$\frac{18}{8} = x$

$2.25 = x$

$x = 2.25cm$

(iii) Solution:



In the given triangles,

$m\angle C = m\angle F$

$m\angle E = m\angle E$

In two triangles of two angles are equal the third angle must also be equal.

$m\angle D = m\angle G$.

So $\Delta CDE \sim \Delta FGE$

In similar triangles, the ratios of corresponding sides are equal.

$$\frac{m\overline{CD}}{m\overline{FG}} = \frac{m\overline{DE}}{m\overline{GE}}$$

$$\frac{x}{2.1} = \frac{2.5}{2.4}$$

$$x = \frac{2.5}{2.4} \times 2.1$$

$$x = \frac{5.25}{2.4}$$

$$x = 2.1875$$

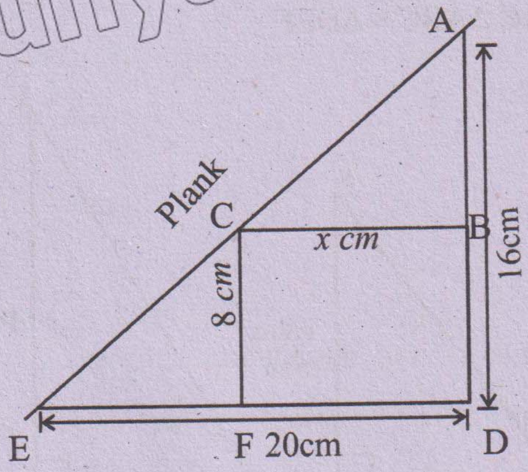
$$\Rightarrow x = 2.19\text{cm}$$

Q.5 A plank is placed straight upstairs that 20 cm wide and 16cm deep. A rectangular box of height 8 cm and width *x* cm is placed on a stair under the plank. Find the value of x. 09309010

Solution:

First we lable the figure as A,B,C,D,E, F as shown in fig.

Here $m\overline{CF} = 8\text{cm}$



In rectangle opposite sides are equal

$m\overline{BD} = m\overline{CF} = 8\text{cm}$

$\Rightarrow m\overline{BD} = 8\text{cm}$

$m\overline{AB} = m\overline{AD} - m\overline{BD}$

$m\overline{AB} = 16\text{cm} - 8\text{cm}$

$\boxed{m\overline{AB} = 8\text{cm}}$

Since, $\Delta ABC \sim \Delta ADE$, So their corresponding sides are proportional, i.e.

$$\frac{m\overline{AB}}{m\overline{AD}} = \frac{m\overline{BC}}{m\overline{DE}}$$

$$\frac{8\,\cancel{cm}}{16\,\cancel{cm}} = \frac{x}{20\text{cm}}$$

$$\frac{8}{16} \times 20\text{cm} = x$$

$$\frac{160\text{cm}}{16} = x$$

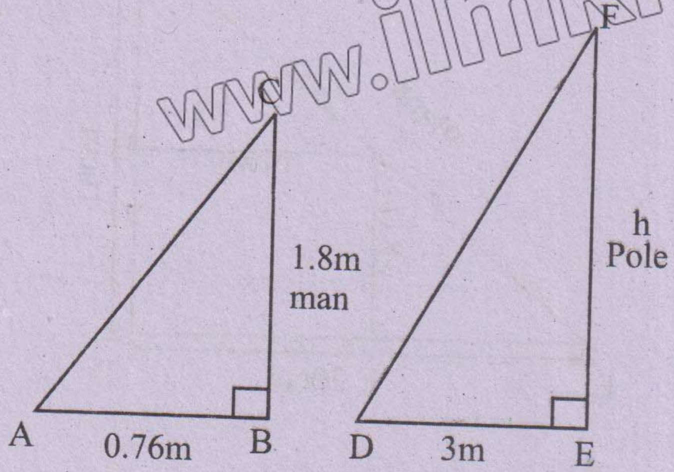
$$10\text{cm} = x$$

$$\Rightarrow x = 10\text{cm}$$

Q.6 A man who is 1.8 m tall cats a shadow of a 0.76 m in length. If at this same time a telephone pole costs a 3 m shadow, find the height of the pole. 09309011

Solution:

Since, $\Delta ABC \sim \Delta DEF$



So ratios of corresponding sides are equal.

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{BC}}{m\overline{EF}}$$

$$\frac{0.76m}{3m} = \frac{1.8m}{h}$$

$$\Rightarrow \frac{3}{0.76} = \frac{h}{1.8}$$

$$\Rightarrow \frac{3}{0.76} \times 1.8 = h$$

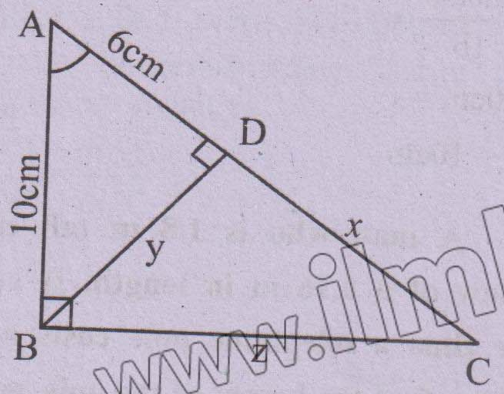
$$\Rightarrow \frac{5.4}{0.76} = h$$

$$\Rightarrow h = 7.11m$$

This height of pole is 7.11m.

Q.7 Find the values of *x*, *y* and *z* of the given figure. 09309012

Solution:



In right-angled ΔABD, by Pythagoras theorem $\left(m\overline{AB}\right)^2 = \left(m\overline{BD}\right)^2 + \left(m\overline{AD}\right)^2$

$$(10cm)^2 = y^2 + (6cm)^2$$
$$100cm^2 = y^2 + 36cm^2$$
$$100cm^2 - 36cm^2 = y^2$$
$$64cm^2 = y^2$$
$$\Rightarrow \sqrt{y^2} = \sqrt{64cm^2}$$
$$\boxed{y=8cm}$$

In right-angled ΔABC,

$$\left(m\overline{AC}\right)^2 = \left(m\overline{AB}\right)^2 + \left(m\overline{BC}\right)^2$$
$$(x+6)^2 = (10)^2 + z^2$$
$$x^2+12x+36 = 100 + z^2 \ \ldots\ldots (i)$$

Now, in right angled ΔBDC

$$z^2 = x^2+y^2 \ \ldots\ldots (ii)$$

put it in eq. (i)

$$x^2+12x+36 = 100+x^2+y^2$$
$$x^2+12x+36 = 100+x^2+64 \qquad (\because y = 8cm)$$
$$x^2+12x+36=164+x^2$$
$$x^2+12x-x^2 = 164-36$$
$$12x = 128$$
$$x = \frac{128}{12} = \frac{32}{3}$$
$$x = 10\frac{2}{3}\ cm$$

Now, putting the value of x and y in eq. (ii)

$$z^2 = \left(10\frac{2}{3}\right)^2 + (8)^2$$
$$z^2 = \left(\frac{32}{3}\right)^2 + (8)^2$$
$$z^2 = \frac{1024}{9} + 64$$
$$z^2 = \frac{1024+576}{9}$$

$\sqrt{z^2} = \sqrt{\frac{1600}{9}}$

$z = \frac{40}{3}$

$z = 13\frac{1}{3}$ cm

Q.8 Draw an isosceles trapezoid ABCD where $\overline{AB} \parallel \overline{CD}$ and $m\overline{AB} > m\overline{CD}$. Draw diagonals $\overline{AC}$ and $\overline{BD}$, intersecting at E. Prove that ΔABE is similar to ΔCDE. If $m\overline{AB}$ = 8cm, $m\overline{CD}$ = 4 cm, and $m\overline{AE}$ = 3 cm, find he length of $\overline{CE}$.

Solution:

(i) In ΔABE and ΔCDE, $\overline{AB} \parallel \overline{CD}$.

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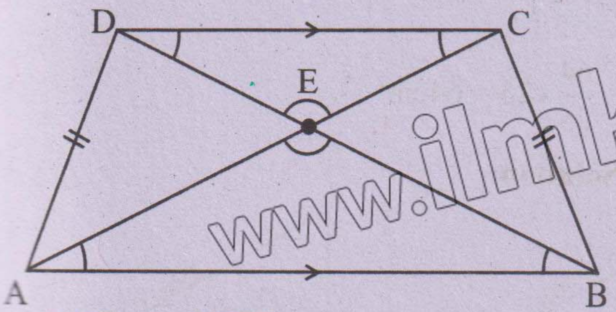


$m\angle AEB = m\angle CED$

[vertically opposite angles]

$m\angle ECD = m\angle EAB$

[alternate angles of | |lines]

$m\angle EDC = m\angle EBA$

[alternate angles of | |lines]

Since three corresponding angles are of triangles are equal which proves that given triangles are similar.

(ii) $m\overline{AB}$ = 8cm **09309014**

$m\overline{CD} = 4$cm

$m\overline{AE} = 3$cm

$m\overline{CE} = x = ?$

In similar triangles ratios of corresponding sides are equals.

$\frac{m\overline{AB}}{m\overline{CD}} = \frac{m\overline{AE}}{m\overline{CE}}$

$\frac{8cm}{4cm} = \frac{3cm}{x}$

$2 = \frac{3cm}{x}$

$\Rightarrow x = \frac{3}{2}$ cm

$x = 1.5$ cm

Thus $m\overline{CE}$ is 1.5 cm.

Q.9 A regular dodecagon has its side length decreased by a factor of $\frac{1}{\sqrt{2}}$. If the perimeter of the original dodecagon is 72 cm. What is the side length of scaled dodecagon? **09309015**

Solution:

Perimeter of original dodecagon = P = 72cm

length reducing factor = $\frac{1}{\sqrt{2}}$

We know that regular dodecagon has 12 equal sides, so

Perimeter = 12 × L

$72 = 12 \times L$

$\frac{72}{12} = L$

$6 = l$

$\Rightarrow \boxed{L = 6cm}$

Since length of original dodecagon is reduced by a factor $\frac{1}{\sqrt{2}}$, so,

Length of reduced dodecagon = $\frac{1}{\sqrt{2}} \times 6cm$

$= \frac{1}{\sqrt{2}} \times 2 \times 3cm$

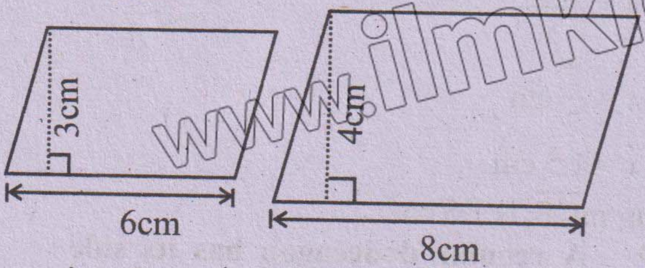
$= \frac{2}{\sqrt{2}} \times 3cm$

$= \sqrt{2} \times 3cm$

$= 3\sqrt{2}$ cm

Area of Similar Figures

There are two parallelograms with corresponding bases 6cm and 8cm and corresponding altitudes 3 cm and 4 cm respectively. The ratio between their length is 3:4 written:



$$\frac{\ell_1}{\ell_2} = \frac{3}{4}$$

The area of smaller parallelogram is:

A_1 = base × altitude

$= 6 \times 3 = 18\text{cm}^2$

The area of larger parallelogram is:

A_2 = base × altitude

$A_2 = 8 \times 4 = 32\ \text{cm}^2$

The ratio of their area is:

$$\frac{A_1}{A_2} = \frac{18}{32} = \frac{9}{16} = \left(\frac{3}{4}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Where A_1 and A_2 are areas and ℓ_1 and ℓ_2 are any two corresponding lengths of similar figure.

Hence the ratio of the areas of any two similar figures is equal to the square of the ratio of any two corresponding lengths of the figures.

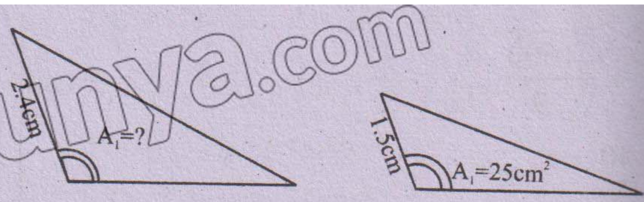
$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Since each length is k times of the other, we take $\frac{\ell_1}{\ell_2} = k$, then $\frac{A_1}{A_2} = k^2$. i.e. Area A_1 is k^2 times the area A_2 k is called scale factor.

Example 6: Find the unknown value in the following.

09309016

(i) Solution:



Since two pairs of corresponding angles are equal i.e., triangles are similar. We use the formula for ratio of areas of similar figures.

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Here

$\ell_1 = 2.4\text{cm}$, $\ell_2 = 1.5\text{cm}$, $A_2 = 25\ \text{cm}^2$

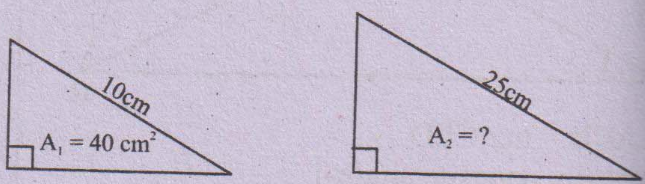
$A^1 = ?$

$$\frac{A_1}{25} = \left(\frac{2.4}{1.5}\right)^2$$

$$\frac{A_1}{25} = \left(\frac{8}{5}\right)^2$$

$$A_1 = \frac{64}{25} \times 25 = 64\text{cm}^2$$

(ii) Solution:



Apply formula:

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = 10\text{cm}$, $\ell_2 = 25\text{cm}$,

$A_1 = 40\text{cm}^2$, $A_2 = ?$

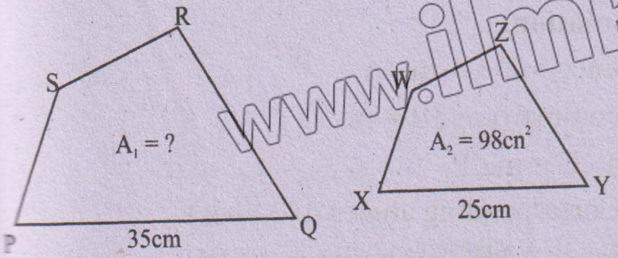
$$\frac{40}{A_2} = \left(\frac{10}{25}\right)^2$$

$$\frac{40}{A_2} = \left(\frac{2}{5}\right)^2$$

$$\frac{40}{A_2} = \frac{4}{25}$$

$$A_2 = 40 \times \frac{25}{4} = 250\text{cm}^2$$

(iii) Solution:



It is given that the quadrilateral *PQRS* is similar to quadrilateral *XYZW*.

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = 35$ cm, $\ell_2 = 25$ cm, $A_1 = ?$,

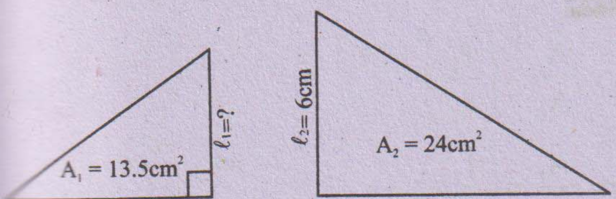
$A_2 = 98\text{ cm}^2$.

$$\frac{A_1}{98}=\left(\frac{35}{25}\right)^2$$

$$\frac{A_1}{98}=\left(\frac{7}{5}\right)^2$$

$$A_1=\frac{49}{25}\times 98=192.08\text{cm}^2$$

(iv) Solution:



Since two pairs of corresponding angles in both triangles are equal so triangles are similar.

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = ?$, $\ell_2 = 3$ cm, $A_1 = 13.5\text{ cm}^2$,

$A_2 = 24\text{ cm}^2$

$$\frac{13.5}{24}=\left(\frac{\ell_1}{3}\right)^2$$

$$\frac{135}{240}=\left(\frac{\ell_1}{3}\right)^2$$

$$\frac{9}{16}=\left(\frac{\ell_1}{3}\right)^2$$

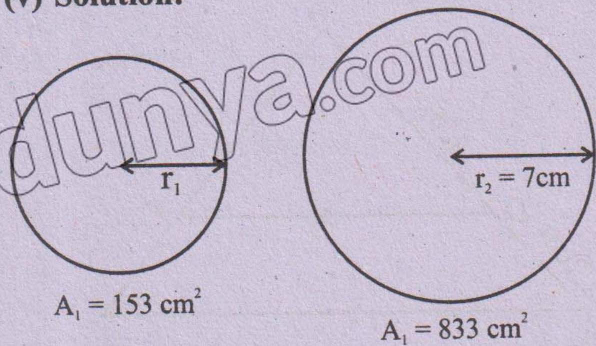
Taking square root

$$\sqrt{\left(\frac{\ell_1}{3}\right)^2}=\sqrt{\frac{9}{16}}$$

$$\frac{\ell_1}{3}=\frac{3}{4}$$

$$\ell_1=\frac{9}{4}=2.25\text{cm}$$

(v) Solution:



For similar spheres having radius r_1 and r_2.

$$\frac{A_1}{A_2}=\left(\frac{r_1}{r_2}\right)^2$$

Here $r_1 = ?$ $r_2 = 7\text{cm}^2$, $A_1 = 153\text{cm}^2$,

$A_2 = 833\text{cm}^2$

$$\frac{153}{833}=\left(\frac{r_1}{7}\right)^2$$

$$\frac{9}{49}=\left(\frac{r_1}{7}\right)^2$$

$$\sqrt{\left(\frac{r_1}{7}\right)^2}=\sqrt{\frac{9}{49}}\text{ (Taking square root)}$$

$$\frac{r_1}{7}=\frac{3}{7}\quad r_1=\frac{3}{7}\times 7\Rightarrow r_1=3\text{cm}$$

Example: 7 Two polygons are similar with a ratio of corresponding sides being $\frac{3}{5}$

the area of the smaller polygon is 54 cm^2, find the area of the larger polygon.
09309017

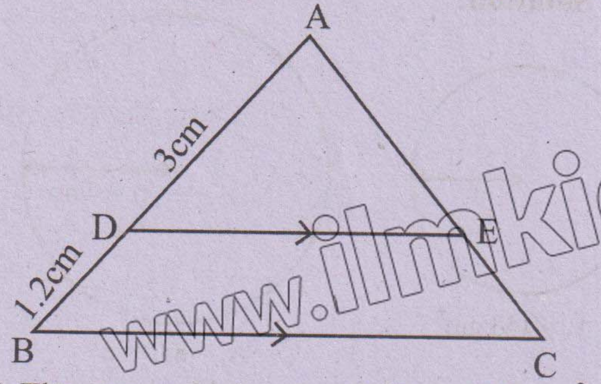
Solution: The ratio of the areas of two similar polygons is the square of the ratio of corresponding sides. So,

$$\frac{\text{Area of larger polygon}}{\text{Area of smaller polygon}} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

Therefore, Area larger $= \frac{25}{9} \times 54 = 150 \text{ cm}^2$

Example 8: Given that $\overline{BC} \parallel \overline{DE}$ prove that the triangles ABC and ADE are similar if:

(i) $m\overline{AD} = 3$ cm and $m\overline{BD} = 1.2$ cm, find the ratio of area of triangle ΔABC and ratio of area of triangle ΔADE. 09309018



(ii) The area of triangle *ADE* is 125 cm^2, find the area of triangle ΔABC and area of trapezium *BCED*.
09309019

Solution:

Since $m\angle A = m\angle A$

(common) $m\angle B = m\angle D$

$m\angle C = m\angle E$

(Corresponding angles of || lines)

Hence ΔABC is similar to ΔADE.

(i) Ratio of sides =

$$\frac{m\overline{AB}}{m\overline{AD}} = \frac{3+1.2}{3} = \frac{4.2}{3} = 1.4 = \frac{7}{5}$$

$$\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ABC} = \left(\frac{\ell_1}{\ell_2}\right)^2 = \left(\frac{7}{5}\right)^2 = \frac{49}{25}$$

(ii) Area of Δ ADE $= 125 \text{cm}^2$

$$\frac{\text{Area of } \Delta ABC}{125} = \frac{49}{25}$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{49}{25} \times 125 = 245 \text{cm}^2$$

Area of trapezium BCED

$=$ Area of $\Delta ABC -$ Area of ΔADE

$= 245 - 125 = 120 \text{cm}^2$

Exercise 9.2

Q.1 Find the ratio of the areas of similar figures if the ratio of their corresponding lengths are:

(i) 1:3 09309020

Solution

Let A_1 and A_2 be areas of smaller and larger figures with corresponding lengths ℓ_1 and ℓ_2 respectively.

$\ell_1 : \ell_2 = 1:3$

$$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{1}{3}$$

$$\because \frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\Rightarrow \frac{A_1}{A_2} = \left(\frac{1}{3}\right)^2$$

$$\frac{A_1}{A_2} = \frac{1}{9}$$

$\Rightarrow A_1 : A_2 = 1:9$

(ii) 3:4 09309021

Solution:

$\ell_1 : \ell_2 = 3:4$

$$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{3}{4}$$

$\because \frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$

$\Rightarrow \frac{A_1}{A_2} = \left(\frac{3}{4}\right)^2$

$\frac{A_1}{A_2} = \frac{9}{16} = 9:16$

(iii) 2:7 09309022

Solution:

$\ell_1 : \ell_2 = 2:7$

$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{2}{7}$

$\because \frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$

$\frac{A_1}{A_2} = \left(\frac{2}{7}\right)^2$

$\frac{A_1}{A_2} = \frac{4}{49} = 4:49$

(iv) 8:9 09309023

Solution:

$\ell_1 : \ell_2 = 8:9$

$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{8}{9}$

$\because \frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$

$\because \frac{A_1}{A_2} = \left(\frac{8}{9}\right)^2$

$\frac{A_1}{A_2} = \frac{64}{81} = 64:81$

(v) 6:5 09309024

Solution:

$\ell_2 : \ell_1 = 6:5$

$\frac{\ell_2}{\ell_1} = \frac{5}{6}$

(reciprocal)

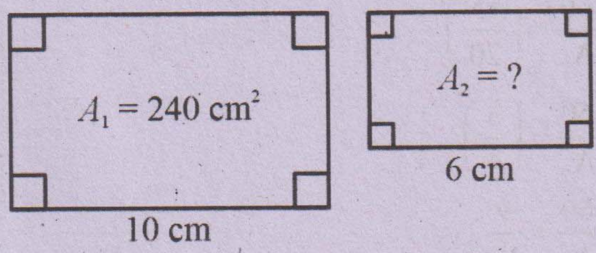
$\because \frac{A_2}{A_1} = \left(\frac{\ell_2}{\ell_1}\right)^2$

$\frac{A_2}{A_1} = \left(\frac{6}{5}\right)^2$

$\frac{A_1}{A_2} = \frac{36}{25} = 36:25$

Q.2 Find the unknowns in the following figures:

(i) 09309025



Solution:

From figures

$A_1 = 240$ cm², $A_2 = ?$

$\ell_1 = 10$cm, $\ell_2 = 6$cm

Since figures are similar,

So $\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$

$\frac{240}{A_2} = \left(\frac{10}{6}\right)^2$

$\frac{240}{A_2} = \frac{100}{36}$

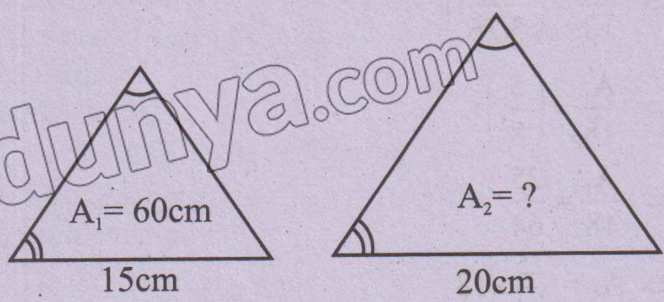
$\Rightarrow \frac{A_2}{240} = \frac{36}{100}$ (reciprocal)

$\Rightarrow A_2 = \frac{9}{25} \times 240$

$A_2 = 9 \times 9.6$

$A_2 = 86.4\text{cm}^2$

(ii) 09309026



Solution

From figures

$A_1 = 60cm_2$, $A_2 = ?$

$\ell_1 = 15cm$, $\ell_2 = 20cm$

Since figures are similar,

So, $\dfrac{A_1}{A_2} = \left(\dfrac{\ell_1}{\ell_2}\right)^2$

$\dfrac{60}{A_2} = \left(\dfrac{15}{20}\right)^2$

$\Rightarrow \dfrac{60}{A_2} = \left(\dfrac{3}{4}\right)^2$

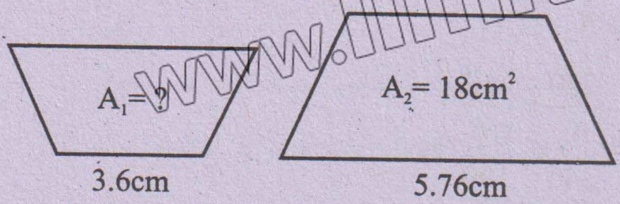
$\Rightarrow \dfrac{60}{A_2} = \dfrac{9}{16}$

$\Rightarrow \dfrac{A_2}{60} = \dfrac{16}{9}$ (reciprocal)

$\Rightarrow A_2 = \dfrac{16}{9} \times 60$

$\Rightarrow A_2 = 106.67cm^2$

(iii) **09309027**



Solution

$A_1 = ?$, $A_2 = 18\ cm^2$

$\ell_1 = 3.6cm$ $\ell_2 = 5.76cm$

Since figures are similar, therefore

$\dfrac{A_1}{A_2} = \left(\dfrac{\ell_1}{\ell_2}\right)^2$

$\Rightarrow \dfrac{A_1}{18} = \left(\dfrac{3.6}{5.76}\right)^2$

$\Rightarrow \dfrac{A_1}{18} = \left(\dfrac{5}{8}\right)^2$

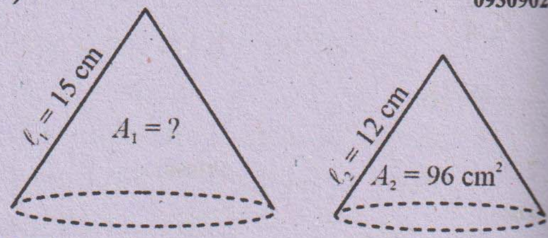
$\Rightarrow \dfrac{A_1}{18} = \dfrac{25}{64}$

$\Rightarrow A_1 = \dfrac{25}{64} \times 18$

$A_1 = \dfrac{450}{64}$

$\Rightarrow A_1 = 7.03cm^2$

(iv) **09309028**



Solution:

From figures

$A_1 = ?$, $A_2 = 96cm^2$

$\ell_1 = 15cm$, $\ell_2 = 12cm$

Since figures are similar, therefore

$\dfrac{A_1}{A_2} = \left(\dfrac{\ell_1}{\ell_2}\right)^2$

$\dfrac{A_1}{A_2} = \left(\dfrac{15cm}{12cm}\right)^2$

$\dfrac{A_1}{96} = \left(\dfrac{5}{4}\right)^2$

$\dfrac{A_1}{96} = \dfrac{25}{16}$

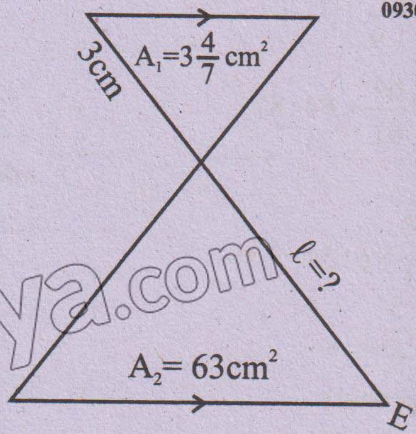
$A_1 = \dfrac{25}{16} \times 96$

$A1 = 25 \times 6$

$A_1 = 150$

$A_1 = 150cm^2$

(v) **09309029**



Solution:

$A_1 = 3\dfrac{4}{7}$, $\ell_1 = 3cm$

$A_1 = \frac{25}{7}\text{cm}^2,\ \ell_2 = ?$

$A_2 = 63\text{cm}$

Since figures are similar , therefore

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{\frac{25}{7}}{63} = \left(\frac{3}{\ell_2}\right)^2$$

$$\frac{25}{7\times 63} = \frac{9}{\ell_2^2}$$

$$\frac{25}{441} = \frac{9}{\ell_2^2}$$

$$\Rightarrow \frac{441}{25} = \frac{\ell_2^2}{9}$$

$$\frac{441}{25}\times 9 = \ell_2^2$$

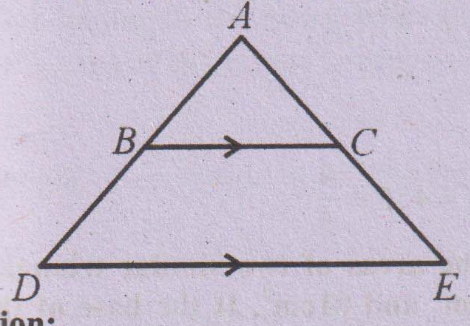
$$\frac{3969}{25} = \ell_2^2$$

$$\sqrt{158.76\text{ cm}^2} = \ell_2^2$$

$$\ell_2 = 12.6\text{cm}$$

Q.3 Given that area of Δ *ABC* = 36cm² and m$\overline{AB}$ = 6 cm , m$\overline{BD}$ = 4 cm . Find

(a) The area of *ΔADE* 09309030



Solution:

Area of ΔABC = A_1 = 36cm²

Area of ΔADE = A_2 = ?

m$\overline{AB}$ = 6cm

m$\overline{BD}$ = 4cm

m$\overline{AD}$ = (6+40) cm = 10cm

Since, ΔABC~ΔADE,

Therefore,

$$\frac{A_1}{A_2} = \left(\frac{m\overline{AB}}{m\overline{AD}}\right)^2$$

$$\frac{36}{A_2} = \left(\frac{6}{10}\right)^2$$

$$\frac{36}{A_2} = \left(\frac{3}{5}\right)^2$$

$$\frac{36}{A_2} = \frac{9}{25} \quad \text{(Reciprocal)}$$

$$\frac{A_2}{36} = \frac{25}{9}$$

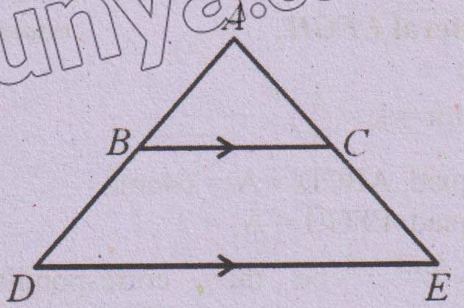
$$A_2 = \frac{25\times \cancel{36}^{4}}{\cancel{9}}$$

$A_2 = 25\times 4$

$A_2 = 100\text{cm}^2$

Thus area of ΔADE = 100 cm²

(b) The area of trapezium *BCED* 09309032



Solution:

We know that

Area of trapezium = Area of ΔADE−Area of ΔABC

= 100 cm² − 36cm²

= 64 cm²

Q.4 Given that Δ*ABC* and Δ*DEF* are similar, with a scale factor of *k* = 3. If the area of triangle Δ*ABC* is 50 cm², what is the area of triangle Δ*DEF*? 09309033

Solution:

Scale factor = k = 3

Area of ΔABC = A_1 = 50cm²

Area of ΔEF = A_2 = ?

By scale factor =

$$K = \frac{\ell_1}{\ell_2} = 3$$

Since $\Delta ABC \sim \Delta DEF$,

So, $\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$

$\frac{50}{A_2}=(3)^2$

$\frac{50}{A_2}=9$

$\Rightarrow \frac{A_2}{50}=\frac{1}{9}$ (reciprocal)

$A_2=\frac{50}{9}$

$\Rightarrow A_2=5\frac{5}{9}\text{cm}^2$

Q.5 Quadrilaterals *ABCD* and *EFGH* are similar, with a scale factor of $k=\frac{1}{4}$.If the area of quadrilateral *ABCD* is 64 cm^2, find is the area of quadrilateral *EFGH*. 09309034

Solution:

Scale factor = $k=\frac{1}{4}$

Area of quad. ABCD = A_1 = 64cm^2

Area of quad. EFGH = A_2 = ?

Let ℓ_1 and ℓ_2 be their corresponding lengths.

By scale factor = $k=\frac{\ell_1}{\ell_2}=\frac{1}{4}$

Since quadrilateral are similar.

So,

$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)=\frac{1}{4}$

$\frac{64}{A_2}=\left(\frac{1}{4}\right)^2$

$\frac{64}{A_2}=\frac{1}{16}$

$\frac{A_2}{64}=\frac{1}{16}$ (reciprocal)

$A_2=16\times 64$

$A_2=1024\text{ cm}^2$

Thus area of quad. EFGH is 1024cm^2.

Q.6 The areas of two similar triangles are 16cm^2 and 25cm^2. What is the ratio of a pair ofcorresponding sides? 09309035

Solution:

Area of smaller figure = A_1 = 16

Area of larger figure = A_2 = 25

Let ℓ_1 and ℓ_2 be the corresponding lengths of smaller and larger figure respectively.

Since triangles are similar

So,

$\because \frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$

$\Rightarrow \left(\frac{\ell_1}{\ell_2}\right)^2=\frac{A_1}{A_2}$

$\Rightarrow \left(\frac{\ell_1}{\ell_2}\right)^2=\frac{16}{25}$

$\Rightarrow \left(\frac{\ell_1}{\ell_2}\right)^2=\frac{4^2}{5^2}$

$\Rightarrow \left(\frac{\ell_1}{\ell_2}\right)^2=\left(\frac{4}{5}\right)^2 \qquad \because \frac{a^2}{b^2}=\left(\frac{a}{b}\right)^2$

Taking square roots.

$\sqrt{\left(\frac{\ell_1}{\ell_2}\right)^2}=\sqrt{\left(\frac{4}{5}\right)^2}$

$\frac{\ell_1}{\ell_2}=\frac{4}{5}$

$\Rightarrow \ell_1:\ell_2=4:5=\frac{4}{5}$

Q.7 The areas of two similar triangles are 144cm^2 and 81cm^2. If the base of the large triangleis 30cm, find the corresponding base of the smaller triangle. 09309036

Solution:

Area of larger Δ = A_1 = 144

Area of smaller Δ = A_2 = 81

Base length of larger Δ = ℓ_1 = 30

Base length of smaller Δ = ℓ_2 = ?

Since triangles are similar,

So,

$$\because \frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\Rightarrow \left(\frac{\ell_1}{\ell_2}\right)^2 = \frac{A_1}{A_2}$$

$$\Rightarrow \left(\frac{30}{\ell_2}\right)^2 = \frac{144}{81}$$

$$\Rightarrow \sqrt{\left(\frac{30}{\ell_2}\right)} = \sqrt{\frac{144}{81}}$$

$$\Rightarrow \frac{30}{\ell_2} = \frac{12}{9}$$

$$\frac{\ell_2}{30} = \frac{9}{12} \text{ (reciprocal)}$$

$$\ell_2 = \frac{9}{12} \times 30$$

$$\ell_2 = \frac{270}{12}$$

$$\ell_2 = 22.5 \text{ cm}$$

Thus base of smaller Δ is 22.5cm.

Q.8 A regular heptagon is inscribed in a larger regular heptagon and each side of the larger heptagon is 1.7 times the side of the smaller heptagon. If the area of the smaller heptagon is known to be 100cm^2, find the area of the larger heptagon.

09309037

Solution:

Let length of each side of regular heptagon be

$\ell_1 = x$

Length of each side of enlarged heptagon $= \ell_2 = 1.7x$

Area of smaller regular heptagon = A_1 = 100cm^2

Area of larger regular heptagon = A_2 = ?

Since, both regular heptagons are similar. So,

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{100}{A_2} = \left(\frac{x}{1.7x}\right)^2$$

$$\frac{100}{A_2} = \left(\frac{1}{1.7x}\right)^2$$

$$\frac{100}{A_2} = \frac{1}{2.89}$$

$$\Rightarrow 100 \times 2.89 = A_2$$

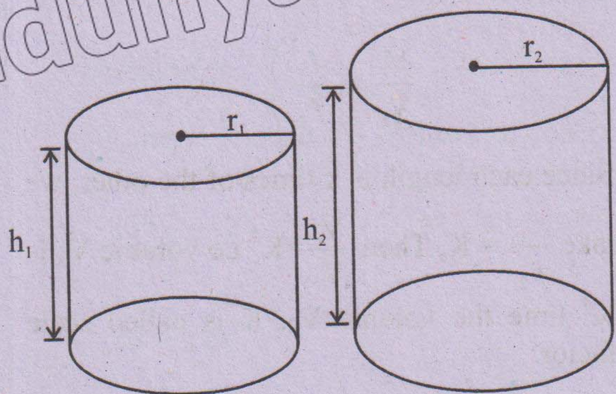
$$\Rightarrow 289 = A_2$$

$$A_2 = 289\text{cm}^2$$

Thus area of larger regular heptagon is 289cm^2.

Volume of Similar Solids

Two solids are said to be similar if lengths of the corresponding sides are proportional i.e., the ratio of the corresponding lengths are equal. e.g.



The two cylinders are similar if $\frac{r_1}{r_2} = \frac{h_1}{h_2}$

If r_1 = 4 cm, r_2 = 5 cm, h_1 = 8 cm and h_2 = 10 cm, then

We note that:

$$\frac{r_1}{r_2} = \frac{4}{5}$$

$$\frac{h_1}{h_2} = \frac{8}{10} = \frac{4}{5}$$

$$\frac{r_1}{r_2} = \frac{h_1}{h_2} \text{ \_\_\_\_\_\_ (i)}$$

Volume of smaller cylinder

$V_1 = \pi r_1^2 h_1$
$= \pi \times 4^2 \times 8$
$= 128\pi cm^2$

Volume of larger cylinder

$V_2 = \pi r_1^2 h_1$
$= \pi \times 5^2 \times 10$
$= 250\pi cm^2$

Ratio of Volumes:

$$\frac{V_1}{V_2} = \frac{128\pi}{250\pi} = \frac{64}{125} = \left(\frac{4}{5}\right)^3$$

$$\text{So, } \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\text{or } \frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 \quad \text{From (i)}$$

Hence the ratio of the volume of any two similar solids is equal to the cube of the ratio of any two corresponding lengths of the solids.

$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

Since each length is k times of the other, we take $\frac{\ell_1}{\ell_2} = K$, Then $\frac{V_1}{V_2} = K^3$ i.e volume V_1 is K^3 time the volume V_2. K is called scale factor.

Since mass of a substance is proportional to its volume, the ratio of the mass of two similar solids is equal to the ratio of their volumes. If the masses of two similar solids are W_1 and W_2 and volumes are V_1 and V_2, then

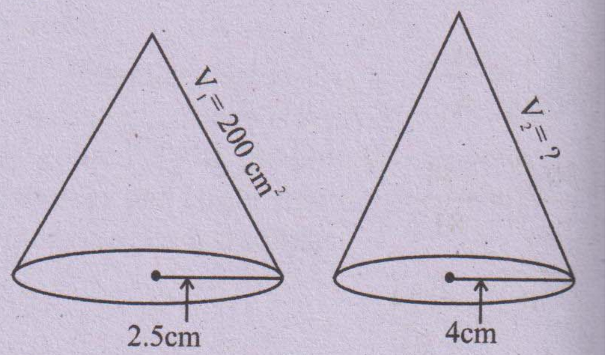
$$\frac{V_1}{V_2} = \frac{w_1}{w_2}$$

$$\text{Therefore, } \frac{w_1}{w_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

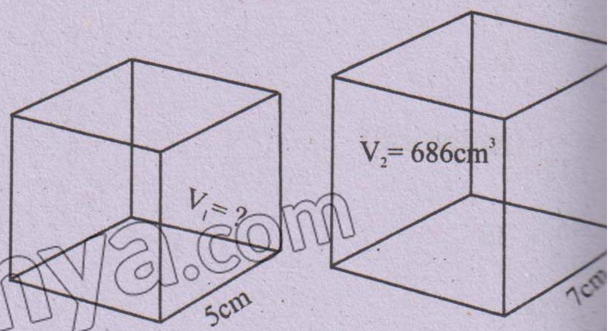
Example 9: Find the unknown volume the following similar solids. 093090

Solution:

(i)



(ii)



(i) $\frac{200}{V_2} = \left(\frac{2.5}{4}\right)^3$

$\frac{200}{V_2} = \left(\frac{5}{8}\right)^3$

$\frac{200}{V_2} = \frac{125}{512}$

$\frac{V_2}{200} = \frac{512}{125}$ (reciprocal)

$V_2 = 200 \times \frac{512}{125}$

$V_2 = 819.2cm^3$

(ii) Using formula $\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$

$\frac{V_1}{196} = \left(\frac{5}{7}\right)^3$

$\left[\begin{array}{l} \ell_1 = 5cm, \ell_2 = 7cm \\ V_1 = ?, V_2 = 686cm^3 \end{array}\right]$

$$\frac{V_1}{686} = \frac{125}{343}$$

$$V_1 = \left(\frac{125}{343}\right) \times 686$$

$$= 250 \text{ cm}^3$$

Example 10: A solid cone "C" is cut into place A and B with sloping edges 6 cm and 4cm. Find the ratio of:

(i) The diameters of the base of the cones A and C. **9309039**

(ii) The area of the bases of the cones *A* and *C*.

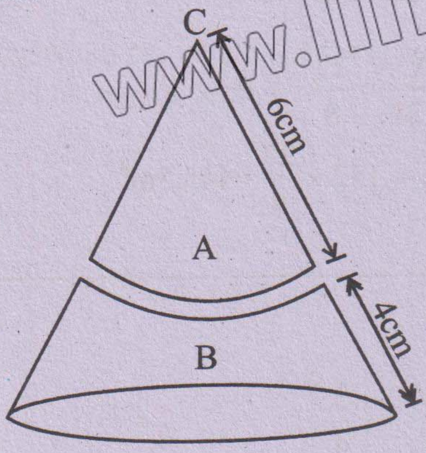
(iii) The volumes of the two cones *A* and *C*.

(iv) If volume of *A* is 72 cm^3, find the volume of solid *B*. **9309040**

Solution: Let diameter of cone $A = d_1$

Diameter of cone $C = d_2$

(i) The ratios of the corresponding lengths are equal because of similarity of the cones.



$$\therefore \quad \frac{d_1}{d_2} = \left(\frac{\ell_1}{\ell_2}\right) = \frac{6}{10} = \text{i.e. } \frac{\ell_1}{\ell_2} = \frac{3}{5}$$

$$\frac{d_1}{d_2} = \frac{3}{5}$$

(ii) $$\frac{\text{Area of cone A}}{\text{Area of cone C}} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

(iii) $$\frac{\text{Volume of cone A}}{\text{Volume of cone C}} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

(iv) V_1 = Volume of con A = 72 cm^3

V_2 = Volume of con C = ?

$$\therefore \quad \frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{72}{V_2} = \frac{27}{125}$$

$$\frac{V_2}{72} = \frac{125}{27} \quad \text{(reciprocal)}$$

$$V_2 = \frac{72 \times 125}{27}$$

$$= 333\frac{1}{3} \text{ cm}^3$$

Volume of *B*

= Volume of cone *C* – Volume of cone *A*

$$= 333\frac{1}{3} - 72$$

$$= 261\frac{1}{3} \text{ cm}^3$$

Example11: The mass of sack of rice is 50 kg and height 4m. Find the mass of thesimilar sack of rice with height of 6m. **9309041**

Solution:

Mass of the smaller sack of rice w_1 = 50kg.

Height of the smaller sack of rice $h_1 = 4$ m.

Mass of larger sack of rice w_2 = ?

Height of smaller sack of rice $h_1 = 6$ m.

Using formula $\frac{w_1}{w_2} = \left(\frac{h_1}{h_2}\right)^3$

$$\frac{50}{w_2} = \left(\frac{\cancel{4}^{2}}{\cancel{6}_{3}}\right)^3$$

$$\frac{50}{w_2} = \left(\frac{2}{3}\right)^3$$

$$\frac{50}{w_2} = \frac{8}{27}$$

$$w_2 = \frac{27\times 50}{8} = 168.75\text{kg}$$

Example 12: The ratio of the corresponding lengths of two similar cylindrical cans is 3:2.

(i) he larger cylindrical can has surface area of 67.5 square metres. Find the surface area of the smaller cylindrical can. 9309042

(ii) The smaller cylindrical can has a volume of 132 cubic metres. Find the volume of larger tin can. 09309043

Solution:

(i) Surface area of larger can = $A_1 = 67.5\text{m}^2$

Surface area of smaller can = A_2 = ?

Ratio of corresponding lengths is

$$\frac{\ell_1}{\ell_2} = \frac{3}{2}$$

Using formula for areas of the similar figures:

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{67.5}{A_2} = \left(\frac{3}{2}\right)^2$$

$$\frac{67.5}{A_2} = \frac{9}{4}$$

$$\frac{A_2}{67.5} = \frac{4}{9} \quad \text{(reciprocal)}$$

$$\Rightarrow A_2 = 67.5 \times \frac{4}{9} = 30\text{m}^2$$

(ii) Volume of smaller can = $V_2 = 132\text{m}^3$

Volume of larger can = V_1 = ?

Using formula for volume of similar figure:

$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{V_1}{132} = \left(\frac{3}{2}\right)^3$$

$$\frac{V_1}{132} = \frac{27}{8}$$

$$\Rightarrow V_1 = 132 \times \frac{27}{8} = 445.5\text{m}^3$$

Exercise 9.3

Q.1 The radii of two spheres are in the ratio 3 : 4. What is the ratio of their volumes? 09309044

Solution:

Ratio of radii of spheres = $r_1 : r_2 = 3:4$

let V_1 and V_2 be the volumes of these spheres.

Since spheres are similar,

So,

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{V_1}{V_2} = \left(\frac{3}{4}\right)^3$$

$$\frac{V_1}{V_2} = \frac{27}{64}$$

$$\Rightarrow V_1 : V_2 = 27:64$$

Q.2 Two regular tetrahedrons have volumes in the ratio 8 : 27. What is the ratio of their sides? 09309045

Solution:

Ratio of volumes = $V_1 : V_2 = 8:27$

Let ℓ_1 and ℓ_2 be the sides of these tetrahedrons respectively.

Since regular tetrahedrons are similar

So,

$$\because \quad \frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\Rightarrow \left(\frac{\ell_1}{\ell_2}\right)^3 = \frac{V_1}{V_2}$$

$$\left(\frac{\ell_1}{\ell_2}\right)^3 = \frac{8}{27}$$

Taking cube root on B.S

$$\sqrt[3]{\left(\frac{\ell_1}{\ell_2}\right)^3} = \sqrt[3]{\frac{8}{27}}$$

$$\frac{\ell_1}{\ell_2} = \frac{2}{3}$$

$$\Rightarrow \ell_1 : \ell_2 = 2:3$$

Q.3 Two right cones have volumes in the ratio 64 : 125. What is the ratio of:

(a) Their heights 09309046

Solution:

64 : 125

Let h_1 and h_2 be the heights of these cones, since right cones are similar.

So,

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\Rightarrow \left(\frac{h_1}{h_2}\right)^3 = \frac{V_1}{V_2}$$

$$\left(\frac{h_1}{h_2}\right)^3 = \frac{64}{125}$$

$$\sqrt[3]{\left(\frac{h_1}{h_2}\right)^3} = \sqrt[3]{\frac{64}{125}}$$

$$\frac{h_1}{h_2} = \frac{\sqrt[3]{64}}{\sqrt[3]{125}}$$

$$\frac{h_1}{h_2} = \frac{\sqrt{4^3}}{\sqrt{5^3}} \Rightarrow \frac{h_1}{h_2} = \frac{4}{5} \Rightarrow h_1 : h_2 = 4:5$$

(b) Their base radii? 09309047

Solution:

Let r_1 and r_2 be the radius of bases of these cones.

$$\because \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \frac{V_1}{V_2}$$

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{64}{125}$$

$$\sqrt[3]{\left(\frac{r_1}{r_2}\right)^3} = \sqrt[3]{\frac{64}{125}}$$

$$\frac{r_1}{r_2} = \frac{\sqrt[3]{64}}{\sqrt[3]{125}}$$

$$\frac{r_1}{r_2} = \frac{4}{5} \Rightarrow r_1 : r_2 = 4:5$$

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

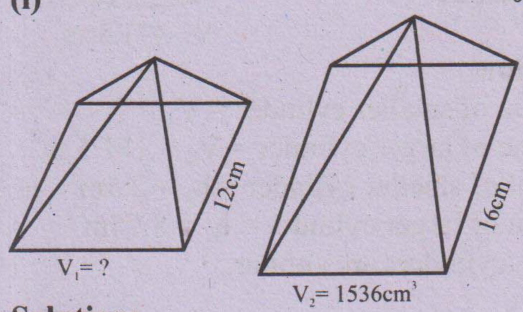
$$\frac{A_1}{A_2} = \left(\frac{4}{5}\right)^2$$

$$\frac{A_1}{A_2} = \frac{16}{25}$$

$$\Rightarrow A_1 : A_2 = 16:25$$

Q.4 Find the missing volume in the following similar figures.

(i) 09309048



Solution:

Let V_1 and V_2 be the volume of smaller and larger pyramids and ℓ_1 and ℓ_2 are their corresponding lengths.

Volume of smaller pyramid = V_1 = ?

Volume of larger pyramid = V_2 = 1536cm^3

Length of smaller pyramid = ℓ_1 = 12cm

Length of larger pyramid = ℓ_2 = 16cm

Since pyramids are similar,

so,

$$\frac{V_1}{V_2}=\left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{V_1}{V_2}=\left(\frac{12}{16}\right)^3$$

$$\frac{V_1}{V_2}=\left(\frac{\overset{3}{\cancel{12}}}{\underset{4}{\cancel{16}}}\right)^3$$

$$\frac{V_1}{1536}=\left(\frac{3}{4}\right)^3$$

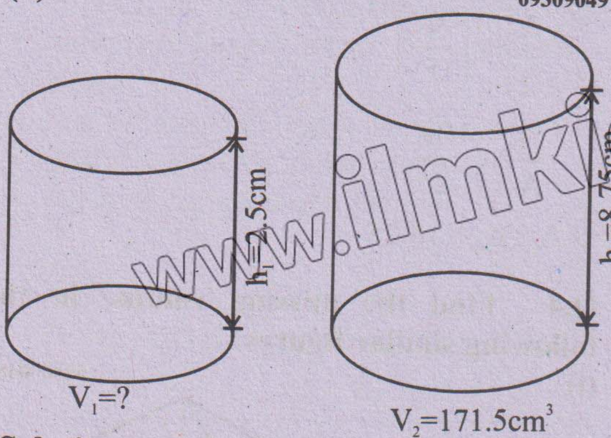
$$V_1=\frac{27}{64}\times 1536$$

$$V_1=27\times 24$$

$$V_1=648\text{cm}^3$$

Thus volume of smaller pyramid is 648cm^3.

(ii) 09309049



Solution:

volume of smaller cylinder = V_1 = ?

volume of larger cylinder = V_2 = 171.5 m^3

Height of smaller cylinder = h_1 = 2.5m

Height of larger cylinder = h_2 = 8.75m

Since, cylinders are similar,

So,

$$\frac{V_1}{V_2}=\left(\frac{h_1}{h_2}\right)^3$$

$$\frac{V_1}{171.5}=\left(\frac{2.5}{8.75}\right)^3$$

$$\frac{V_1}{171.5}=\left(\frac{10}{35}\right)^3$$

$$\frac{V_1}{171.5}=\left(\frac{2}{7}\right)^3$$

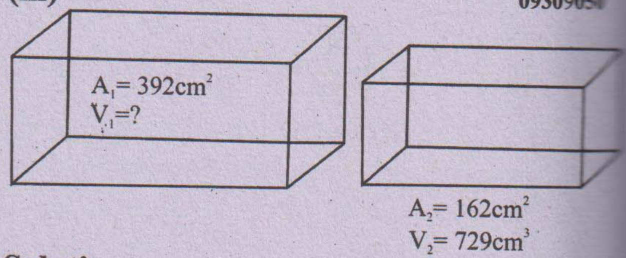
$$V_1=\frac{8}{343}\times 171.5$$

$$V_1=\frac{1372}{343}$$

$$V_1=4\text{m}^3$$

Thus volume of smaller cylinder is 4m^3.

(iii) 09309050



Solution:

Surface area of larger cuboid = A_1=392cm^2

Surface area of smaller cuboid =A_2=162cm^2

Volume of larger cuboid = V_1 = ?

Volume of smaller cuboid = V_2 = 729cm^3

Let ℓ_1 and ℓ_2 be the lengths larger and smaller cuboid respectively.

(iii)Using formula of areas of similar figures.

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\Rightarrow \sqrt{\left(\frac{\ell_1}{\ell_2}\right)^2}=\sqrt{\frac{A_1}{A_2}}$$

$$\frac{\ell_1}{\ell_2}=\sqrt{\frac{392}{162}}$$

$$\frac{\ell_1}{\ell_2}=\sqrt{\frac{196}{81}}$$

$$\frac{\ell_1}{\ell_2}=\sqrt{\frac{(14)^2}{(9)^2}}$$

$$\left(\frac{\ell_1}{\ell_2}\right)=\sqrt{\left(\frac{14}{9}\right)^2}$$

$$\boxed{\frac{\ell_1}{\ell_2} = \frac{14}{9}}$$

Now, using formula of volumes for similar figures.

$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{V_1}{V_2} = \left(\frac{14}{9}\right)^3$$

(ii)

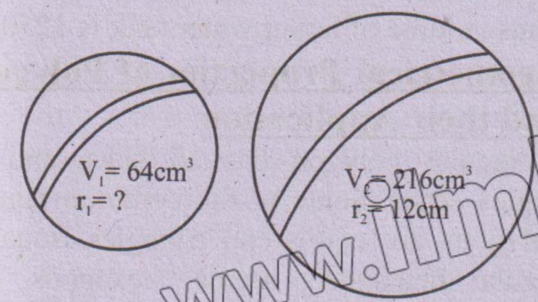
$$\frac{V_1}{729} = \frac{2744}{729}$$

$$V_1 = \frac{2744}{729} \times 729$$

$$V_1 = 2744\text{cm}^3$$

Thus volume of larger cuboid is $2744\,\text{cm}^3$.

(iv) **09309051**



Solution:

We know that

$$\left(\frac{V_1}{V_2}\right) = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{64}{216} = \left(\frac{r_1}{12}\right)^3$$

$$\frac{64}{216} = \frac{r_1^3}{(12)^3}$$

$$(12)^3 \times \frac{8}{27} = r_1^3$$

$$\Rightarrow\ r_1^3 = \frac{(12)^3 \times 2^3}{3^3}$$

$$r_1^3 = \left(\frac{\cancel{12}^{4} \times 2}{\cancel{3}_1}\right)^3$$

$$r_1^3 = (8)^3$$

Taking cube root B.S

$$\Rightarrow\ \sqrt[3]{r_1^3} = \sqrt[3]{8^3}$$

$$\Rightarrow r_1 = 8\text{cm}$$

Q.5 The ratio of corresponding lengths of two similar canonical cans is 3:2.

(i)The larger canonical can have surface area of the smaller canonical can. **09309052**

(ii) The smaller canonical can have volume of 240 m^3. Find the volume of larger canonical can. **09309053**

Solution:

(i) The ratio of corresponding lengths of similar canonical can is 3:2 i.e. $\ell_1 : \ell_2 = 3:2$.

Let A_1 an A_2 be the surface areas of larger and smaller can respectively.

Surface area of larger can = $A_1 = 96\text{m}^2$

Surface area of smaller. Can = A_2 = ?

We know that

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{96}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{96}{A_2} = \left(\frac{3}{2}\right)^2$$

$$\frac{A_2}{96} = \frac{9}{4}$$

$$\Rightarrow\ \frac{A_2}{96} = \frac{4}{9} \qquad \text{(reciprocal)}$$

$$A_2 = \frac{4}{9} \times 9.6$$

$$A_2 = \frac{384}{9}$$

$$A_2 = 42.67\text{m}^2$$

Solution (ii):

(ii) let V_1 and V_2 be the volume of larger and smaller can respectively.

Volume of smaller canonical can=V_2=240m^2

Volume of larger can V_1 = ?

We know that

$$\left(\frac{V_1}{V_2}\right)=\left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{V_1}{240}=\left(\frac{3}{2}\right)^3$$

$$V_1=\frac{27}{8}\times 240$$

$V_1 = 27\times30$

$V_1 = 810m^3$

Thus volume of larger canonical can is $810m^3$.

Q.6 The ratio of the heights of two similar cylindrical water tanks is 5:3.

(i) If the surface area of the larger tank is 250 square metres, find the surface area of the smaller tank. 09309055

Solution:

Ratio of height of water tanks $=\ell_1=\ell_2=5:3$

let A_1 and A_2 be the surface areas of larger and smaller cylindrical water tanks respectively.

Surface area of larger tank $= A_1 = 250m^2$

Surface area of smaller tank $= A_2 =?$

We know that

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{250}{A_2}=\left(\frac{5}{3}\right)^2$$

$$\frac{250}{A_2}=\frac{25}{9}$$

$$\Rightarrow \frac{A_2}{250}=\frac{9}{25} \quad \text{(reciprocal)}$$

$$A_2=\frac{9}{25}\times 250$$

$A_2 = 9\times10$

$A_2 = 90m^2$

Thus surface area of smaller water tank is $90m^2$.

(ii) If the volume of the smaller tank is 270 cubic metres, find the volume of the larger tank. 09309056

Solution

Let V_1 and V_2 be the volumes of larger and smaller tanks respectively.

Let volume of smaller tank $V_2 = 270\ m^3$

Volume of larger tank $= V_1 = ?$

We know that

$$\left(\frac{V_1}{V_2}\right)=\left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{V_1}{270}=\left(\frac{5}{3}\right)^3$$

$$\frac{V_1}{270}=\frac{125}{27}$$

$$V_1=\frac{125}{27}\times 270$$

$V1 = 125\times10$

$V_1 = 1250m^3$

Thus volume of larger water tank is $1250m^3$.

Geometrical Properties of Polygons and their Application

A regular polygon has all sides and all angles equal. Some of the common regular polygons are equilateral triangles, squares, regular pentagons, regular hexagons, etc.

Sum of Interior Angles: The formula for sum of interior angles of n-sided polygon is $(n-2)\times180°$.

Interior angle: For a regular-n-sided polygon:

$$\text{Size of each interior angles} = \frac{(n-2)\times180°}{n}$$

For instance, a regular hexagon has n = 6, so each interior angle is

$$\frac{(6-2)\times180°}{6}=\frac{720°}{6}=120°$$

Exterior angle: The sum of all exterior angles of any polygon is always 360° regardless of the number of sides. The exterior angle of each side of a regular n-sided polygon is:

$$\text{Exterior Angle} = \frac{360°}{n}$$

The interior and exterior angles are supplementary at a vertex i.e., Interior + exterior angle = 180°

Diagonals: The total number of diagonals in a regular polygon with n sides is $\frac{n(n-3)}{2}$

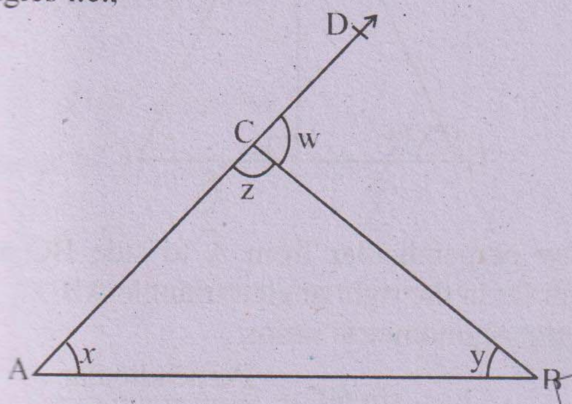
Summetry: A regular n-sided polygon has rotational symmetry and reflexive summetry both of order n. e.g., a regular hexagon has six lines of symmetry and has rotational symmetry of order . A regular n-sided polygon can be rotated by $\frac{360°}{n}$ and will look the same.

Geometrical Properties of Triangles

A triangle is a polygon with three sides and three angles. Triangles come in various types based on side length and angle measure.

Angle sum: The sum of the interior angles in any triangle is always 180°. In equilateral triangle, all sides are equal and each angle is 60°. It has three lines of symmetry and rotational symmetry of order 3. In isosceles triangle, two sides are equal, and the angles opposite to equal sides are also equal. It has one line of symmetry.

Exterior angle of a triangle: The measure of an exterior angle in a triangle is equal to sum of the measures of two opposite interior angles i.e.,



In ΔABC, $m\angle A + m\angle B = m\angle BCD$

i.e., $x + y = w$

Geometrical Properties of Parallelograms

A parallelogram is a quadrilateral whose opposite sides are parallel and equal in length and opposite angles are equal. Its adjacent angles are supplementary. The diagonals of a parallelogram bisect each other (they cross each other at the midpoint). They are not equal in length.

> **Recall:**
>
> **Rectangle:** All angles are 90° and diagonals are equal.
>
> **Rhombus:** All sides are equal, and diagonals bisect each other at right angles.
>
> **Square:** All sides are equal, all angles are 90° and diagonals are equal and bisect each of other at right angles.

Example 13: Find the measure of each interior angle of a regular pentagon. **09309057**

Solution: Interior angle $= \frac{(n-2)\times 180°}{n}$

$$= \frac{(5-2)\times 180°}{5}$$

$$= \frac{540°}{5} = 108°$$

Each exterior angle is $= \frac{360°}{5} = 72°$

Application of Polygons

Tessellation

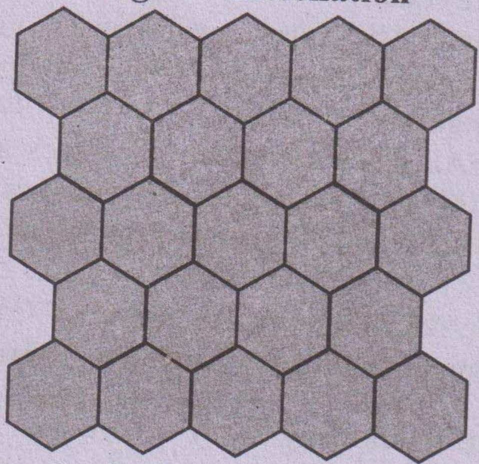
A tessellation is a pattern of shapes that fit together perfectly, without any gaps or overlaps, covering a plane. These shapes can be repeated infinitely to create a repeating pattern. Tessellations can be created using a single shape or a combination of shapes. They can be regular or irregular and they can exhibit various symmetries and patterns.

> Equilateral triangle can tessellate perfectly because the internal angle of each equilateral triangle is 60°, and six of these triangles meet at a point to form a 360° angle, allowing them to fill space seamlessly. Squares can tessellate perfectly because each square has an internal angle of 90° and four squares meet at a point to form a 360° angle.

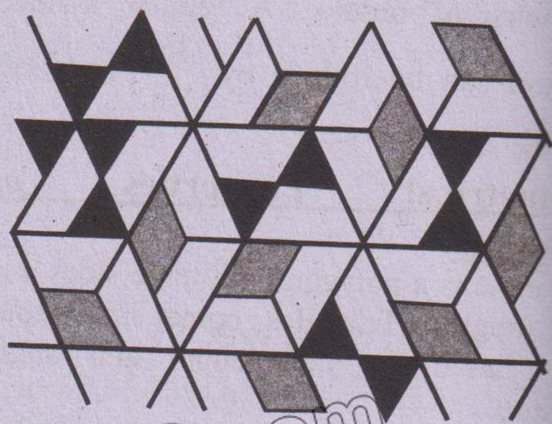
Only three regular polygons can tessellate the plane on their own: equilateral triangles, squares, and regular hexagons. They have symmetries. Hexagons (interior angel 120°) can tessellate perfectly because three hexagons meet at each vertex to form a 360° angle with no space creating a natural look inspired by honeycombs.

Regular pentagons and other polygons with angles that don't add up to 360° at each vertex cannot form gap-free patterns. i.e., Tessellation is not possible.

Regular Tessellation



Irregular Tessellation



Example 14: A tessellation is created using a combination of regular pentagons and decagons. Find the sum of the angles at a vertex where a pentagon and a decagon meet.

Solution:

Interior angle of regular decagon

$$= \frac{(n-2)\times 180°}{n}$$

$$= \frac{(10-2)\times 180°}{10} = \frac{1440°}{10} = 144°$$

Interior angle of regular pentagon = 108°

Sum of angles = 144° + 108° = 252°. Since, angle sum ≠ 360°.

Tessellation cannot be done.

Example 15: A parallelogram-shaped room has a base of 10 meters and a height of 8m. Babar wants to carpet the room using rolls cover $20m^2$ each. How many rolls of carpet does he need?

Solution:

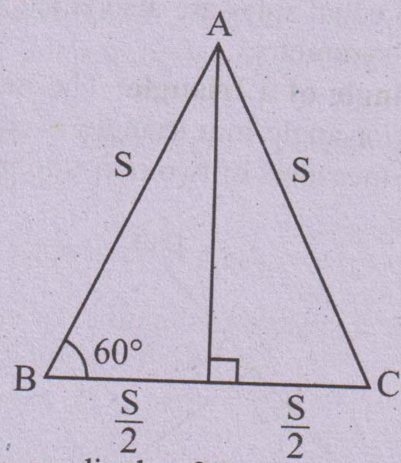
The area of the parallelogram =

$A = \text{base} \times \text{height} = 10\times 8 = 80m^2$

Number of rolls needed: $\frac{80}{20} = 4$ rolls

Example 16: Find the area of the equilateral triangle ABC of side length 's' metres.

Solution: 09309058



Draw perpendicular from A to side BC at point D. In the right angled triangle ABD.

Using trigonometric ratios:

$$\sin 60° = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\frac{\sqrt{3}}{2} = \frac{m\overline{AD}}{s}$$

$$\Rightarrow m\overline{AD} = \frac{\sqrt{3}}{2}s$$

Area of triangle ABC $= \frac{1}{2} \times s \times \frac{\sqrt{3}}{2}s$

Area of triangle ABC$= \frac{\sqrt{3}}{4}s^2$

Example 17: Ali wants to create a floor design that uses regular hexagons (each with a side length of 1 metre) and equilateral triangles (each with a side length of 1 metre) to cover a rectangular area measuring 10m by 5m. Find how many hexagons and triangles Ali will need to complete the tessellation. **09309059**

Solution

To find the area of an equilateral triangle with side length s, we can use the formula:

Area of a triangle $= \frac{\sqrt{3}}{4}.s^2$

Multiply by 6 (since there are 6 triangles)

Area of a hexagon $= \frac{6\sqrt{3}}{4}.s^2$

$$= \frac{\cancel{6}^{3}\sqrt{3}}{\cancel{4}_{2}}$$

$$= \frac{3\sqrt{3}}{2}.s^2$$

Area of a hexagon$= \frac{3\sqrt{3}}{2} \times s^2$

$\approx \frac{3\sqrt{3}}{2}.(1m)^2 \approx 2.598m^2$

Area of an equilateral triangle =

$= \frac{\sqrt{3}}{4} \times s^2 \approx \frac{\sqrt{3}}{4}(1m)^2 \Rightarrow s^2 \approx 0.433m^2$

Area of the rectangular floor = $10m \times 5m = 50m^2$

Determine the arrangement: Assume a pattern where one hexagon is surrounded by 6 triangles. The area covered by one hexagon and the 6 surrounding triangles:

Total area covered by 1 hexagon and 6 triangles

$= 2.598m^2 + 6 \times 0.433m^2 \quad 2.598m^2 + 2.598m^2$

$\approx 5.196m^2$

Calculate the total number of hexagons and triangles needed:

Number of sets $= \frac{50m^2}{5.196m^2} \approx 9.62$ sets

Rounding up, you can fit 10 sets of the pattern. Therefore, we need:

- Hexagons: 10
- Triangles: $10 \times 6 = 60$

Example 18: Falak plans to tile a square patio with an area of 100 square metres. He decides to use both square tiles and triangular tiles, each with an area of 0.25 square metres. If 60% of the tiles will be square and 40% will be triangular, how many tiles of each shape are needed? **09309060**

Solution

Total number of tiles $= \frac{\text{Patio Area}}{\text{Tile Area}} = \frac{100}{0.25}$

$= 400$ tiles

Number of square tiles $= 400 \times 0.6 = 240$

Number of triangular $= 400 \times 0.4 = 160$

Exercise 9.4

Q.1 (i) What is the sum of the interior angles of a decagon (10-sided polygon)? **09309061**

Solution:

Numbers of sides = n = 10

Sum of interior angles $= (n-2) \times 180°$

$= (10-2) \times 180°$

$= 8 \times 180°$

$= 1440°$

(ii) Calculate the measure of each interior angle of a regular hexagon. 09309062

Solution:

In regular hexagon

Numbers of sides = n = 6

Each interior angel $= \dfrac{(n-2)\times 180^\circ}{n}$

$= \dfrac{(6-2)\times 180^\circ}{6}$

$= \dfrac{4\times 180^\circ}{6}$

$= 4 \times 30^\circ$

$= 120^\circ$

(iii) What is each exterior angle of a regular pentagon? 09309063

Solution

In Regular pentagon: n = 5

Number of sides = n = 5

Each exterior angle $= \dfrac{360^\circ}{n} = \dfrac{360^\circ}{5}$

$= 72^\circ$

(iv) If the sum of the interior angles of a polygon is 1260°, how many sides does the polygon have? 09309064

Solution:

Sum = 1260°

Sum of interior angles = $(n-2) \times 180^\circ$

$1260^\circ = (n-2) \times 180^\circ$

$\dfrac{1260^\circ}{180^\circ} = n-2$

$7 = n-2$

$7+2 = n$

$9 = n$

$\Rightarrow \boxed{n = 9}$

Thus number of sides of regular polygon is 9.

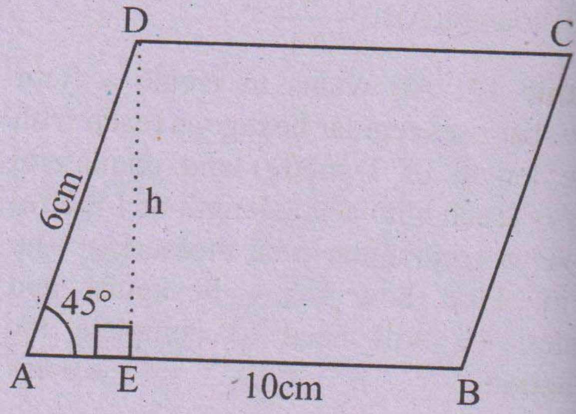
Q.2 In a parallelogram ABCD, $m\overline{AB}$ = 10cm, $m\overline{AD}$ = 6cm and $m\angle BAD$ = 45°. Calculate area of ABCD 09309065

Solution:

Given:

$m\overline{AB} = 10cm$

$m\overline{AD} = 6cm$



We know that

Area of || gm = base × height

Area of || gm ABCD = $m\overline{AB} \times m\overline{DE}$ ___(i)

In right angled ΔAED,

$\sin m\angle A = \dfrac{m\overline{DE}}{mAD}$

$\sin 45^\circ = \dfrac{h}{6}$

$\dfrac{1}{\sqrt{2}} = \dfrac{h}{6}$ $\left(\because \sin 45^\circ = \dfrac{1}{\sqrt{2}}\right)$

$6 \times \dfrac{1}{\sqrt{2}} = h$

$3\times 2\times \dfrac{1}{\sqrt{2}} = h$

$3\sqrt{2} = h$ $\left(\because \dfrac{a}{\sqrt{a}} = \sqrt{a}\right)$

$\Rightarrow h = 3\sqrt{2} \Rightarrow m\overline{DE} = 3\sqrt{2}$ cm

Now, using formula (i)

Area of parallelogram ABCD

$= 10cm \times 3\sqrt{2}$ cm

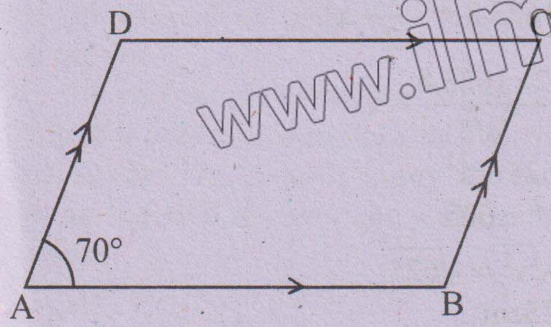
$= 30\sqrt{2}\ cm^2 = 42.43cm^2$

Q.3 In a parallelogram ABCD if $m\angle DAB$ = 70°, find the measures of all other angles in the parallelogram. 09309066

Solution:

$m\angle BAD = 70°$ _________ (i) (given)

(i) In || gm ABCD,



$m\angle A = m\angle C$ [opposite angles of || gm]

$m\angle C = 70°$

(ii) In parallelogram ABCD,

$m\angle A + m\angle B = 180°$ [anlges on end points of a side]

$70° + m\angle B = 180°$

$m\angle B = 180° - 70°$

$m\angle B = 110°$

(iii) In parallelogram ABCD,

$m\angle D = m\angle B$ (opposite angles)

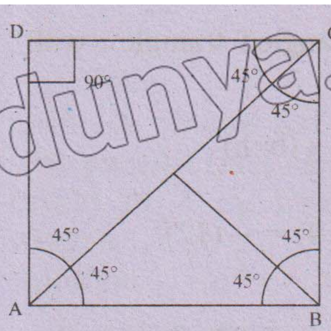
$m\angle D = 110°$

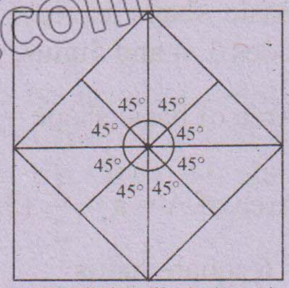
Q.4 A shape is created by cutting a square in half diagonally and then attaching a right-angled triangle to the hypotenuse of each half. Explain why this shape can tessellate and calculate the interior angles of the new shape. 09309067

Solution:

Tessellation of the New Shape:

By cutting a square in half diagonally we get two right-angled isosceles triangles, each with angles of 90°, 45°, and 45. When two right-angled triangles are attached to the hypotenuses they form a new shape that is a square. Squares can tessellate perfectly because each square has an internal angle of 90^{o} and four squares meet at a point to form a 360^{o} angle.





Tessellation of basic shape

Angle at A:

The 45° angle of the original triangle and the 45° angle of the attached triangle at the point of attachment sum to 90°.

Angle at B:

The sum of the two 45° angles at the attachment point from two attached triangles is 90^{o}.

Angle at C:

The 45° angle of the original triangle and the 45° angle of the attached triangle at the point of attachment sum to 90°.

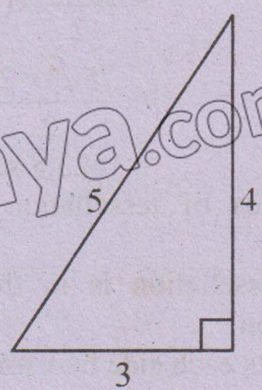
Angle at D:

One 90^{o} angle from original ΔADC

Since all interior angles are 90° and their sum is 360 °.The new shape is a square which naturally tessellates the plane and it can cover the plane without gaps or overlaps.

Q.5 A tessellation is created by repeatedly reflecting a basic shape. The basic shape is a right angled triangle with sides of length 3,4, and 5 units. Find: The minimum number of reflections needed to create a tessellation that covers a square with an area of 3600 square units. 09309068

Solution:



Basic shape is right angled triangle with sides 3, 4 and 5units.

Area of basic shape = $\frac{1}{2}(b \times h)$

Area of A = a = $\frac{1}{2}(3 \times 4) = \frac{1}{2}(12)$

= 6 square units

Area to be covered = A = 3600 square units

Number of reflections = $\frac{\text{Total area}}{\text{Area of } \Delta}$

$= \frac{3600}{6} = 600$

Thus minimum 600 reflections are needed to cover the area of 3600 square units.

Q.6 A tessellation is created using regular hexagons. Each hexagon has a side length of 5cm. Find the total area of the tessellation if it consists of 25 hexagons and total perimeter of the outer edge of the tessellation, assuming it's a perfect hexagon. 09309069

Solution:

Each side of hexagon = s = 5cm

Area of an equilateral $\Delta = \frac{\sqrt{3}}{4}.s^2$

Area of 1 regular hexagon = $6 \times \frac{\sqrt{3}}{4}.s^2$

$= \frac{3\sqrt{3}}{2}.s^2$

Area of 25 regular hexagon

$= 25 \times \frac{3\sqrt{3}}{2} \times s^2$

$= \frac{75\sqrt{3}}{2} \times (5)^2$

$= \frac{75\sqrt{3} \times 25}{2}$

= 1623.8 sq.unit.

Thus total area of tessellation is 1623.8 square unit.

Given that tessellation is in the form of regular hexagon.

Let length of its each side be x units.

Area of tessellation (hexagon)= $\frac{3\sqrt{3}}{2} \times x^2$

$1623.8 = \frac{3\sqrt{3}}{2}.x^2$

$\frac{2 \times 1623.8}{3\sqrt{3}} = x^2$

$625 = x^2$

$\Rightarrow x^2 = 625$

$\sqrt{x^2} = \sqrt{625}$

$x = 25$cm

The length of each side of tessellation

$= x = 25$ cm

Perimeter of tessellation = $6 \times x$

$= 6 \times 25 = 150$cm

Q.7 A rectangular floor is 12 m by 15m. How many square tiles, each 1m by 1m, are needed to cover the floor? 09309070

Solution:

Dimension of floor = 12m by 15m

Area of floor = $12 \times 15 = 180m^2$

Dimension of tile = 1m by 1m

Area of 1 tile = $1m \times 1m = 1m^2$

Number of tiles to cover floor =

$= \frac{\text{total area of floor}}{\text{Area of 1 tile}}$

$= \frac{180m^2}{1m^2} = 180$

Thus 180 tiles are needed to cover the floor.

Q.8 A rectangular wall is 10m tall and 120m wide. How many gallons of paint are needed to cover the wall, if one gallon covers $35m^2$? 09309071

Solution:

Length of wall = L = 10m

Width of wall = W = 120m

Area of wall = L × W

$= 10m \times 120m = 1200m^2$

Given that 1 gallon covers = $35m^2$

No. of gallons to cover $35m^2 = 1$

No. of gallons to cover $1m^2 = \frac{1}{35}$

No. of gallones cover $1200\,m^2 = \frac{1}{35} \times 1200 = 34.29$

≈ 35

Thus 35 gallons of paint are required to paint the wall.

Q.9 A rectangular wall has a length of 10m and a width of 4 meters. If 1 litre of paint covers 7m², how many liters of paint are needed to cover the wall? 09309072

Solution:

Length of wall = L = 10m

Width of wall = W = 4m

Area of wall = L × W

$= 10m \times 4m = 40m^2$

Given that 1 liter of paint covers = $7m^2$

No. of liters to cover $7m^2$ = 1litre

No. of liters of paint to cover $1m^2$

$= \frac{1}{7}$ litres

No. of liters of paint to cover $40m^2$

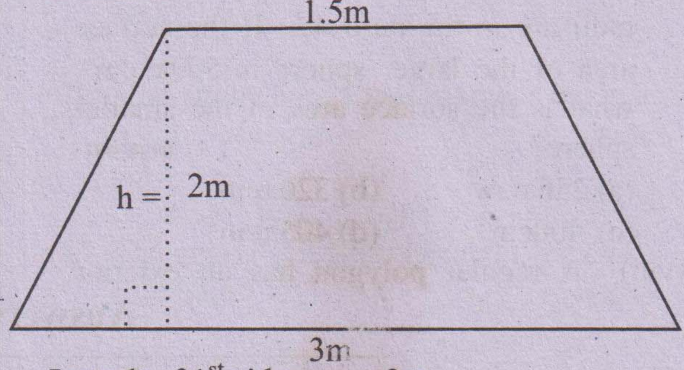
$= \frac{1}{7} \times 40$ litres

$= 5.71 \approx 6$ litres

Thus 6 liters of paint are required to cover area of $40m^2$.

Q.10 A window has a trapezoidal shape with parallel sides of 3 m and 1.5 m and a height of 2 m. Find the area of the window. 09309073

Solution:



Length of 1st side = s_1 = 3m

Length of 2nd side = s_2 = 1.5m

Height of trapezium h = 2m

Area of trapezoidal window $= \left(\frac{s_1 + s_2}{2}\right) \times h$

$= \left(\frac{3+1.5}{2}\right) \times 2 = \frac{(4.5)}{\cancel{2}} \times \cancel{2} = 4.5m^2$

Review Exercise 9

Q.1 Choose the correct option.

(i) If two polygons are similar, then: 09309075

(a) Their corresponding angles are equal.

(b) Their areas are equal.

(c) Their volumes are equal.

(d) Their corresponding sides are equal.

(ii) The ratio of the areas of two similar polygons is: 09309076

(a) Equal to the ratio of their perimeters.

(b) Equal to the square of the ratio of their corresponding sides.

(c) Equal to the cube of the ratio of their corresponding sides.

(d) Equal to the sum of their corresponding sides.

(iii) If the volume of two similar solids is 125 cm³ and 27 cm³, the ratio of their corresponding heights is: 09309077

(a) 3:5 (b) 5:3

(c) 25:9 (d) 9:25

(iv) The exterior angle of regular pentagon is:

(a) 40° (b) 45° 09309078

(c) 60° (d) 72°

(v) A parallelogram has an area of 64 cm² and a similar parallelogram has an area of 144 cm². If a side of the smaller parallelogram is 8 cm, what is the corresponding side of the larger parallelogram? 09309079

(a) 10 cm (b) 12 cm
(c) 18 cm (d) 16 cm

(vi) The total number of diagonals in a polygon with 9 sides is: 09309080
(a) 18 (b) 21
(c) 25 (d) 27

(vii) Two spheres are similar, and their radii are in the ratio 4:5. If the surface area of the larger sphere is 500π cm^2, what is the surface area of the smaller sphere? 09309081
(a) $256\pi cm^2$ (b) $320\pi cm^2$
(c) $400 cm^2$ (d) $405\pi cm^2$

(viii) A regular polygon has an exterior angle of 30°. How many diagonals does the Polygon have? 09309082
(a) 54 (b) 90
(c) 72 (d) 108

(ix) In a regular hexagon, the ratio of the length of a diagonal to the side length is: 09309083
(a) $\sqrt{3}:1$ (b) 2:1
(c) 3:2 (d) 2:3

(x) A regular polygon has an interior angle of 165°. How many sides does it have? 09309084
(a) 15 (b) 16
(c) 20 (d) 24

Answers Key

i	**a**	ii	**b**	iii	**b**	iv	**d**	v	**b**
vi	**d**	vii	**b**	viii	**a**	ix	**b**	x	**d**

Multiple Choice Questions (Additional)

Similarity of Polygons

1. Which of the following is the symbol of similarity? 09309085
(a) ≃ (b) ≈
(c) ~ (d) ≅

2. In similar figures corresponding angles are congruent and corresponding sides are: 09309087
(a) congruent (b) parallel
(c) perpendicular (d) proportional

3. Ratio has: 09309088
(a) fixed value (b) no symbol
(c) no unit (d) no importance

4. The ratio of corresponding sides of similar figures is called: 09309089
(a) common factor (b) scale factor
(c) grading factor (d) proportion

5. The equality of two ratios is called: 09309090
(a) proper factor (b) scale factor
(c) area factor (d) proportion

6. Which of the following is scale factor of area? 09309091
(a) K (b) K^2
(c) 2K (d) K^3

7. If radii of two circles are in the ratio 2:3 then their surface areas are in the ratio: 09309092
(a) 2:3 (b) 4:9
(c) 8:18 (d) 8:27

8. If two Spheres have volumes in the ratio 8:27then their corresponding lengths are in the ratio: 09309093
(a) 2:3 (b) 4:9
(c) 8:18 (d) 8:27

Geometrical Properties of Regular Polygons

9. The sum of interior angles of a n-sided polygon is: 09309094
(a) $n\times180°$ (b) $(n-1)180°$
(c) $(n-2)+180°$ (d) $(n-2)180°$

10. The each interior angle of a n-sided polygon is: 09309095
(a) $\frac{n\times180°}{n}$ (b) $\frac{(n-1)180°}{n}$

(c) $\frac{(n-2)+180°}{n}$ (d) $\frac{(n-2)180°}{n}$

11. The total number of diagonals from a vertex of a n-sided polygon is: 09309096

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n-2)}{2}$

(c) $\frac{n(n-3)}{2}$ (d) $\frac{n(n-4)}{2}$

12. The sum of interior angles of which regular polygon is 1080^o: 09309097

(a) pentagon (b) hexagon

(c) heptagon (d) octagon

13. The each interior angle of which regular polygon is 108^o: 09309098

(a) Square (b) pentagon

(c) hexagon (d) heptagon

14. The each interior and exterior angle of which regular polygon is equal: 09309099

(a) Square (b) pentagon

(c) hexagon (d) heptagon

15. The interior and exterior angles of regular hexagon are in the ratio: 09309100

(a) 1:2 (b) 2:1

(c) 1:6 (d) 2:3

16. If the sum of interior angles of a regular polygon is 1440^o then number of sides are: 09309101

(a) 8 (b) 10

(c) 12 (d) 14

17. The each interior angle of regular pentagon is: 09309102

(a) 20^o (b) 108^o

(c) 36^o (d) 72^o

18. How many equilateral triangles are in a regular hexagon? 09309103

(a) 4 (b) 5

(c) 6 (d) 8

19. The area of a regular hexagon can be calculated by: 09309104

(a) $A=\frac{6\sqrt{3}}{2}S^2$ (b) $A=\frac{3\sqrt{3}}{4}S^2$

(c) $A=\frac{3\sqrt{3}}{2}S^2$ (d) $A=\frac{6\sqrt{2}}{4}S^2$

20. The repeating pattern of regular shapes is called: 09309105

(a) tessellation (b) oscillation

(c) rotation d) citation

Answer Key

1	c	2	d	3	c	4	b	5	d	6	b	7	b	8	a	9	d	10	d
11	c	12	d	13	b	14	a	15	b	16	b	17	b	18	c	19	c	20	a

Q.2 If the sum of their interior angles of polygon is 1080°, how many sides does the polygon has? 09309106

Solution:

Sum of interior angles = 1080°

Number of sides = n = ?

We know that

Sum of interior angles = (n–2) × 180°

$1080° = (n-2)\times 180°$

$\frac{1080°}{180} = n-2$

$6 = n-2$

$6+2 = n$

$8 = n$

$\Rightarrow n = 8$

Thus the polygon has 8 sides.

Q.3 Two similar bottles are such that one is twice as high as the other. What is the ratio of their surface areas and their capacities? 09309107

Solution:

If height of 2^{nd} bottle = h

Then height of 1^{st} bottle = 2h

(i) If A_1 and A_2 be the surface areas of 1^{st} and 2^{nd} bottles respectively.

Using formula of area for similar solids

$$\frac{A_1}{A_2}=\left(\frac{2\cancel{h}}{\cancel{h}}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{2}{1}\right)^2$$

$$\frac{A_1}{A_2} = \frac{4}{1}$$

$A_1:A_2 = 4:1$

(ii) Let C_1, and C_2 be the capacities (volumes) of these bottles,
Using formula of volumes for similar solids.

$$\frac{C_1}{C_2} = \left(\frac{2h}{h}\right)^3$$

$$\frac{C_1}{C_2} = \left(\frac{2}{1}\right)^3$$

$$\frac{C_1}{C_2} = \frac{8}{1}$$

$C_1:C_2 = 8:1$

Q.4 Each dimension of a model car is $\frac{1}{10}$ of the corresponding car dimension.

Find the ratio of: 09309109

(a) the areas of their windscreens

(b) the capacities of their boots

(c) the widths of the cars

(d) the number of wheels they have

Solution:

As each dimension of a model car is $\frac{1}{10}$ of the corresponding car dimensions. So

Scale factor = $k = \frac{1}{10}$

(a) the areas of their windscreens 09309110

Solution:

Let A_1 and A_2 be the areas of model and actual car's wind screens.

Using formula of areas for similar solids.

$$\frac{A_1}{A_2} = (k)^2$$

$$\frac{A_1}{A_2} = \left(\frac{1}{10}\right)^2$$

$$\frac{A_1}{A_2} = \frac{1}{100}$$

$\Rightarrow A_1:A_2 = 1:100$

(b) the capacities of their boots 09309111

Solution:

Let C_1 and C_2 be the capacities of their boots.

Using formula of volumes (capacities) for similar solid.

$$\frac{C_1}{C_2} = (k)^3$$

$$\frac{C_1}{C_2} = \left(\frac{1}{10}\right)^3$$

$$\frac{C_1}{C_2} = \frac{1}{1000}$$

$\Rightarrow C_1:C_2 = 1:1000$

(c) the widths of the cars 09309112

Solution:

Let W_1 and W_2 be width of model and actual cars respectively. $\frac{W_1}{W_2} = \frac{1}{10}$

$\Rightarrow W_1:W_2 = 1:10$

(d) the number of wheels they have. 09309113

Solution:

No. of wheels of model car = 4

No. of wheels of actual car = 4

The ratio of number of wheels = 4:4 = 1:1

Q.5 Three similar jug have heights 8 cm, 12 cm and 16cm. If the smallest jug holds $\frac{1}{2}$ liters, find the capacities of the other two. 09309114

Solution:

Heights of three jugs are h_1 = 8cm, h_2 = 12cm

h_3 = 16cm.

Let V_1, V_2 and V_3 be the capacities of three jugs.

Given that volume (capacity) of smallest jug $= V_1 = \frac{1}{2}$ litre.

(a) Finding capacity of 2nd jug:

We know that

$\because \frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$

$\frac{\frac{1}{2}}{V_2} = \left(\frac{8}{12}\right)^3$

$\frac{1}{2V_2} = \left(\frac{2}{3}\right)^3$

$\frac{1}{2V_2} = \frac{8}{27}$

$\frac{1}{V_2} = \frac{8}{27} \times 2$

$\frac{1}{V_2} = \frac{16}{27}$

$\Rightarrow V_2 = \frac{27}{16}$

$V_2 = 1.6875$

$V_2 = 1.69$ litres

(b) Finding capacity of 3rd Jug

We know that

$\because \frac{V_1}{V_3} = \left(\frac{h_1}{h_2}\right)^3$

$\frac{\frac{1}{2}}{V_3} = \left(\frac{8}{16}\right)^3$

$\frac{1}{2V_3} = \left(\frac{1}{2}\right)^3$

$\frac{1}{2V_3} = \frac{1}{8}$

$\frac{1}{V_3} = \frac{1}{8} \times 2$

$\frac{1}{V_3} = \frac{1}{4}$

$\Rightarrow V_3 = 4$ litres (reciprocal)

Thus capacity of 3rd jug is 4 litres.

Q.6 Three similar drinking glasses have heights 7.5 cm, 9 cm and 10.5 cm. If the tallest glass holds 343 millilitres, find the capacities of the other two. 09309115

Solution:

The heights of glasses are $h_1 = 7.5$cm, $h_2 = 9$cm, and $h_3 = 10.5$cm.

Let V_1, V_2 and V_3 be the capacities of three glasses respectively.

Given that

$V_3 = 343$ml.

Finding capacity of 1st glass

We know that

$\because \frac{V_1}{V_3} = \left(\frac{h_1}{h_3}\right)^3$

$\frac{V_1}{343} = \left(\frac{7.5}{10.5}\right)^3$

$\frac{V_1}{343} = 0.3644315$

$V_1 = 343 \times 0.3644315$

$V_1 = 125ml$

Thus capacity of 2nd glass is125ml.

Finding capacity of 2nd glass

We know that

$\because \frac{V_1}{V_3} = \left(\frac{h_1}{h_3}\right)^3$

$\frac{V_1}{343} = \left(\frac{9}{10.5}\right)^3$

$\frac{V_2}{343} = \frac{729}{1157.625}$

$V_2 = 343 \times \frac{729}{1157.625}$

$V_2 = 216\ ml$

Thus capacity of 2nd glass is 216 ml.

Q.7 A toy manufacturer produces model cars which are similar in every way to the actual cars. If the ratio of the door area of the model to the door area of the car is 1 cm to 2500 cm, find:

(a) the ratio of their lengths 09309116

(b) the ratio of the capacities of their petrol 09309117

(c) the width of the model, if the actual car is 150cm wide 09309118

(d) the area of the rear window of the

actual car if the area of the rear window of the model is 3cm². 09309119

Solution:

Ratio of areas areas of doors of model and actual cars = $A_1:A_2=1:2500$

(a) the ratio of their lengths

Let ℓ_1 and ℓ_2 be the lengths of model and actual cars respectively.

We know that

$$\left(\frac{\ell_1}{\ell_2}\right)^2 = \frac{A_1}{A_2}$$

$$\left(\frac{\ell_1}{\ell_2}\right)^2 = \frac{1}{2500}$$

$$\sqrt{\left(\frac{\ell_1}{\ell_2}\right)^2} = \sqrt{\frac{1}{2500}}$$

$$\frac{\ell_1}{\ell_2} = \frac{1}{50}$$

$\Rightarrow \ell_1 : \ell_2 = 1:50$

(b) the ratio of the capacities of their petrol tanks

Let V_1 and V_2 be the volumes capacities of model and actual petrol tank respectively.

We know that

$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{V_1}{V_2} = \left(\frac{1}{50}\right)^3$$

$$\frac{V_1}{V_2} = \frac{1}{125,000}$$

$V_1:V_2 = 1:125,000.$

(c) the width of the model, if the actual car is 150 cm wide

Solution:

Let width of model car = w_1=?

Width of actual car = w_2 = 150cm

We know that

$$\frac{w_1}{w_2} = \frac{\ell_1}{\ell_2}$$

$$\frac{w_1}{150} = \frac{1}{50}$$

$$\Rightarrow w_1 = \frac{1}{50} \times 150$$

w_1 = 3cm

Thus width of model car is 3cm.

(d) the area of the rear window of the actual car if the area of the rear window of the model is 3 cm².

Solution:

Let area of window of model car=A_1= 3cm²

area of window of actual car = A_2 = ?

We know that

$$\because \frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{3cm^2}{A_2} = \left(\frac{1}{50}\right)^2$$

$$\frac{3}{A_2} = \frac{1}{2500}$$

$$\Rightarrow \frac{A_2}{3} = 2500 \qquad \text{(reciprocal)}$$

$A_2 = 3\times2500$

A_2 = 7,500

Thus area of rear window of actual car is 7,500 cm².

Q.8 The ratio of the areas of two similar labels on two similar jars of coffee is144 : 169. Find the ratio of

(a) the heights of the two jars 09309120

(b) their capacities 09309121

Solution:

Ratio of areas of two lables

= $A_1:A_2$ = 144:169

$$\Rightarrow \frac{A_1}{A_2} = \frac{144}{169}$$

(a) the heights of the two jars

Solution:

Let h_1 and h_2 be the heights of two jars respectively.

We know that

$$\because \frac{A_1}{A_2} = \left(\frac{h_1}{h_2}\right)^2$$

$$\Rightarrow \sqrt{\left(\frac{h_1}{h_2}\right)^2} = \sqrt{\frac{A_1}{A_2}}$$

$$\frac{h_1}{h_2} = \sqrt{\frac{144}{169}}$$

$$\frac{h_1}{h_2} = \frac{12}{13}$$

$\Rightarrow$ $h_1:h_2 = 12:13$

(b) their capacities.

Solution:

Let V_1 and V_2 be volumes (capacities) of two jars respectively.

We know that

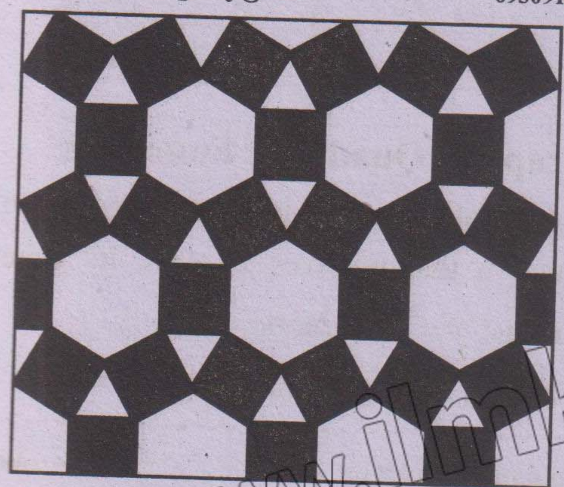
$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{V_1}{V_2} = \left(\frac{12}{13}\right)^3$$

$$\frac{V_1}{V_2} = \frac{1728}{2197}$$

$\Rightarrow$ $V_1:V_2 = 1728:2197$

Q.9 A tessellation of tiles on a floor has been made using a repeating pattern of a regular hexagon, six squares and six equilateral triangles. Find the total area of a single pattern with side length $\frac{1}{2}$ metre of each polygon. **09309122**



Solution:

Length of each side $= \frac{1}{2}$ m $= 0.5$m

No. of squares = 6

No. of equilateral triangles = 6

No. of hexagon = one

(i) Finding areas of 6 squares:

Area of 1 square $= \ell^2 = (0.5)^2 = 0.25\text{m}^2$

Area of 6 squares $= 6 \times 0.25 = 1.5\text{m}^2$

(ii) Finding area of 6 equilateral Δs:

Area of 1 equilateral $\Delta = \frac{\sqrt{3}}{4}.s^2$

Area of 6 equilateral $\Delta\text{s} = 6 \times \frac{\sqrt{3}}{4} \times (0.5)^2$

$$= 6 \times \frac{\sqrt{3}}{4} \times 0.25$$

$$= \frac{1.5\sqrt{3}}{4} = 0.6495 \approx 0.65\text{m}^2$$

(iii) Finding area of a hexagon:

Area of regular hexagon $= \frac{6\sqrt{3}}{4} \times .s^2$

$$= \frac{6\sqrt{3}}{4} \times (0.5)^2$$

$$= \frac{6\sqrt{3}(0.25)}{4}$$

$$= \frac{1.5\sqrt{3}}{4}$$

$$= 0.65\text{m}^2$$

Total area of tessellation

$$= (1.5+0.65+0.65)\ \text{m}^2$$

$$= 2.8\text{m}^2$$