بِسۡمِ اللّٰہِ الرَّحۡمٰنِ الرَّحِیۡمِ

(In the Name of Allah, the Most Compassionate, the Most Merciful)

MATHEMATICS

9

PUNJAB CURRICULUM AND TEXTBOOK BOARD, LAHORE

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Experimental Edition

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Unit 1 Real Numbers

Students' Learning Outcomes

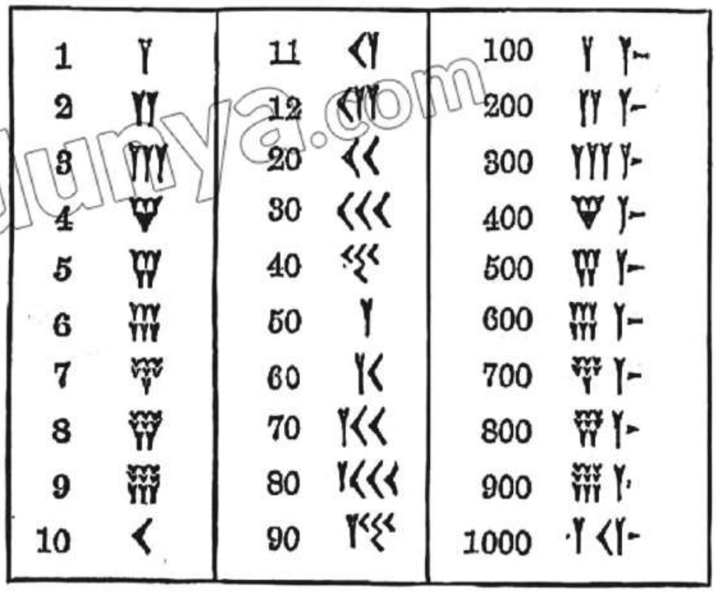
At the end of the unit, the students will be able to:

- Explain, with examples, that civilizations throughout history have systematically studied living things [e.g., the history of numbers from Sumerians and its development to the present Arabic system]
- Describe the set of real numbers as a combination of rational and irrational numbers
- Demonstrate and verify the properties of equality and inequality of real numbers
- Apply laws of indices to simplify radical expressions
- Apply concepts of real numbers to real-world problems (such as temperature, banking, measures of gain and loss, sources of income and expenditure)

1.1 Introduction to Real Numbers

The history of numbers comprises thousands of years, from ancient civilization to the modern Arabic system. Here is a brief overview:

Sumerians (4500 – 1900 BCE) used a sexagesimal (base 60) system for counting. The Sumerians used a small cone, bead, large cone, large perforated cone, sphere and perforated sphere, corresponding to 1, 10, 60 (a large unit), 600.



Egyptians (3000 – 2000 BCE) used a decimal (base 10) system for counting.

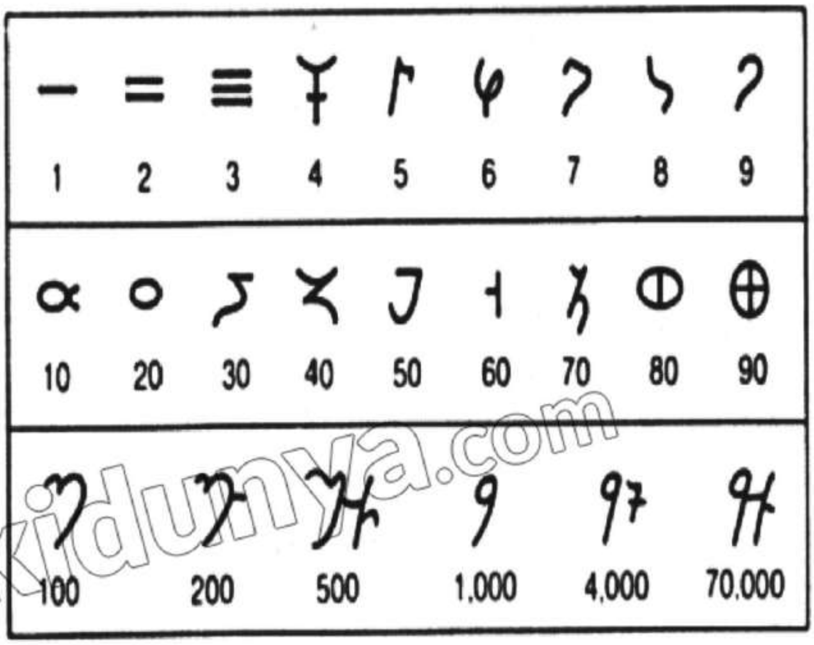
Here are some of the symbols used by the Egyptians, as shown in the figure below:

The Egyptians usually wrote numbers left to right, starting with the highest denominator. For example, 2525 would be written with 2000 first, then 500, 20, and 5.

1 | 10 | 100 | 1,000 | 10,000 | 100,000 | 1,000,000

Romans (500BCE-500CE) used the Roman numerals system for counting.
Roman numerals represent a number system that was widely used throughout Europe as the standard writing system until the late Middle Ages. The ancient Romans explained that when a number reaches 10 it is not easy to count on one's fingers. Therefore, there was a need to create a proper number system that could be used for trade and communications. Roman numerals use 7 letters to represent different numbers. These are I, V, X, L, C, D, and M which represent the numbers 1, 5, 10, 50, 100, 500 and 1000 respectively.

Indians (500 – 1200 CE) developed the concept of zero (0) and made a significant contribution to the decimal (base 10) system.



Ancient Indian mathematicians have contributed immensely to the field of mathematics. The invention of zero is attributed to Indians, and this contribution outweighs all others made by any other nation since it is the basis of the decimal number system, without which no advancement in mathematics would have been possible. The number system used today was invented by Indians, and it is still called Indo-Arabic numerals because Indians invented them and the Arab merchants took them to the Western world.

Arabs (800 – 1500 CE) introduced Arabic numerals (0 – 9) to Europe. The Islamic world underwent significant developments in mathematics. Muhammad ibn Musa al-Khwārizmī played a key role in this transformation, introducing algebra as a distinct field in the 9th century. Al-Khwārizmī's approach, departing from earlier arithmetical traditions, laid the groundwork for the arithmetization of algebra, influencing mathematical thought for an extended period. Successors like Al-Karaji expanded on his work, contributing to advancements in various mathematical domains. The practicality and broad applicability of these mathematical methods facilitated the dissemination of Arabic mathematics to the West, contributing substantially to the evolution of Western mathematics.

Modern era (1700 – present): Developed modern number systems e.g., binary system (base - 2) and hexadecimal system (base - 16).

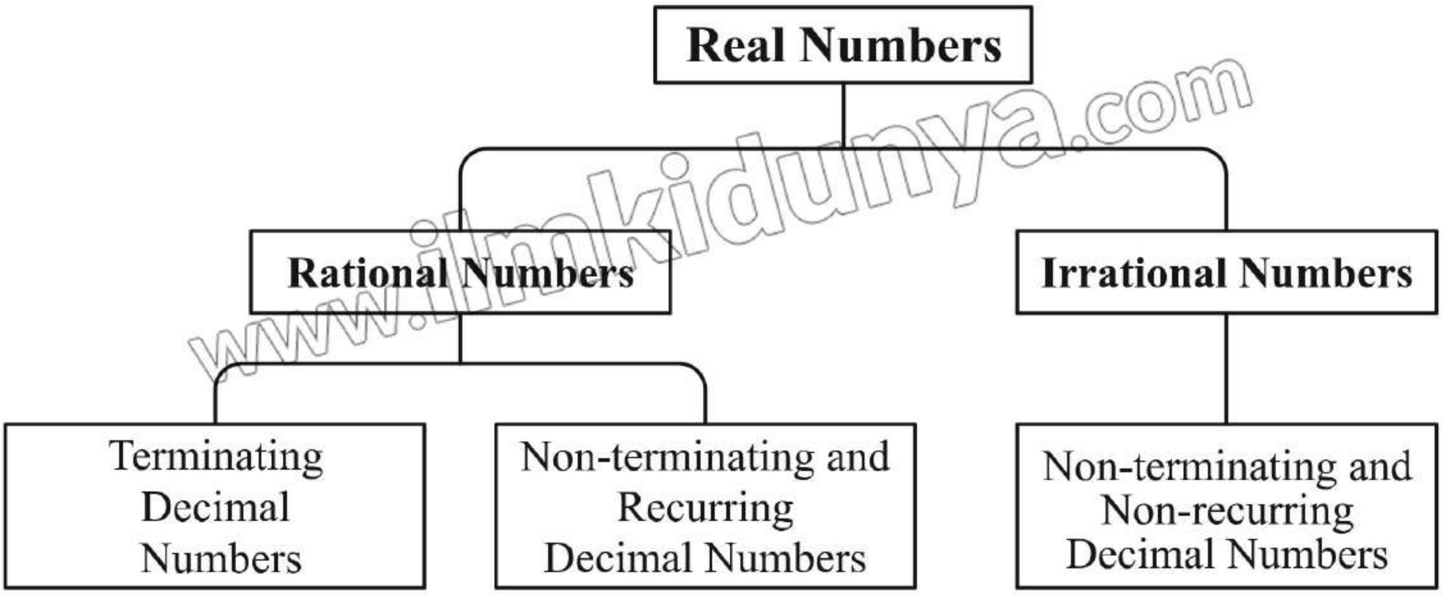
The Arabic system is the basis for modern decimal system used globally today. Its development and refinement comprise thousands of years from ancient Sumerians to modern mathematicians.

In the modern era, the set $\{1, 2, 3, ...\}$ was adopted as the counting set. This counting set represents the set of natural numbers was extended to set of real numbers which is used most frequently in everyday life.

1.1.1 Combination of Rational and Irrational Numbers

We know that the set of rational numbers is defined as $Q = \left\{ \frac{p}{q}; p, q \in Z \wedge q \neq 0 \right\}$

and set of irrational numbers (Q') contains those elements which cannot be expressed as quotient of integers. The set of Real numbers is the union of the set of rational numbers and irrational numbers i,e., $R = Q \cup Q'$



1.1.2 Decimal Representation of Rational Numbers

(i) Terminating Decimal Numbers

A decimal number with a finite number of digits after the decimal point is called a terminating decimal number.

For example, $\frac{1}{4} = 0.25$, $\frac{8}{25} = 0.32$, $\frac{3}{8} = 0.375$, $\frac{4}{5} = 0.8$ are all terminating decimal numbers.

(ii) Non-Terminating and Recurring Decimal Numbers

The decimal numbers with an infinitely repeating pattern of digits after the decimal point are called non-terminating and recurring decimal numbers.

Here are some examples.

$\frac{1}{3} = 0.333... = 0.\overline{3}$ (3 repeats infinitely)

$\frac{1}{6} = 0.1666... = 0.1\overline{6}$ (6 repeats infinitely)

$\frac{22}{7} = 3.\underline{142857142857}... = 3.\overline{142857}$ (the pattern 142857 repeats infinitely)

$\frac{4}{9} = 0.44444... = 0.\overline{4}$ (4 repeats infinitely)

Non-terminating and recurring decimal numbers are also rational numbers.

1.1.3 Decimal Representation of Irrational Numbers

Decimal numbers that do not repeat a pattern of digits after the decimal point continue indefinitely without terminating.

Non-terminating and non-recurring decimal numbers are known as irrational numbers.

For examples,

- $\pi = 3.14159265358979 32...$
- $e = 2.71828182845904...$
- $\sqrt{2} = 1.41421356237309...$

Remember!
$e = 2.7182...$ is called Euler's Number.

Example 1: Identify the following decimal numbers as rational or irrational numbers:

(i) 0.35 (ii) 0.444… (iii) $3.\overline{5}$

(iv) 3.36788542… (v) 1.709975947…

Solution: (i) 0.35 is a terminating decimal number, therefore it is a rational number.

(ii) 0.444… is a non-terminating and recurring decimal number, therefore it is a rational number.

(iii) $3.\overline{5} = 3.5555...$ is a non-terminating and recurring decimal number, therefore it is a rational number.

(iv) 3.36788542… is a non-terminating and non-recurring decimal number, therefore it is an irrational number.

(v) 1.709975947… is a non-terminating and non-recurring decimal number, therefore it is an irrational number.

1.1.4 Representation of Rational and Irrational Numbers on Number Line

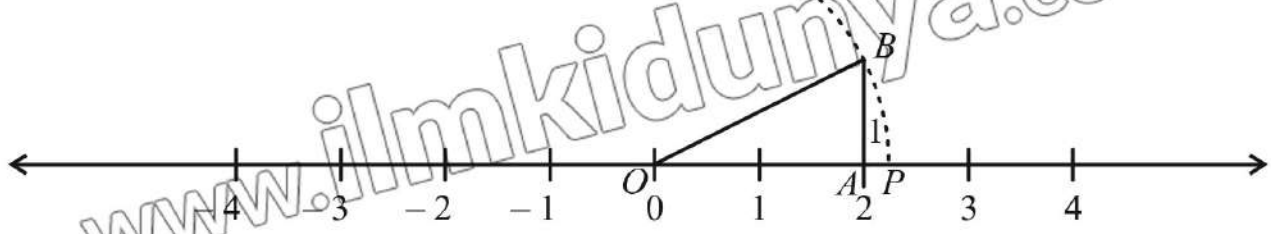
In previous grades, we have learnt to represent rational numbers on a number line. Now, we move to the next step and learn how to represent irrational numbers on a number line.

Example 2: Represent $\sqrt{5}$ on a number line.

Solution: $\sqrt{5}$ can be located on the number line by geometric construction. As, $\sqrt{5} = 2.236...$ which is near to 2. Draw a line of $m\overline{AB} = 1$ unit at point A, where $m\overline{OA} = 2$ units, and we have a right-angled triangle OAB. By using Pythagoras theorem

$$(m\overline{OB})^2 = (m\overline{OA})^2 + (m\overline{AB})^2$$

$$= (2)^2 + (1)^2 = 4 + 1 = 5 \quad \Rightarrow \quad m\overline{OB} = \sqrt{5}$$



Draw an arc of radius $m\overline{OB} = \sqrt{5}$ taking O as centre, we got point "P" representing $\sqrt{5}$ on the number line. So, $|\overline{OP}| = \sqrt{5}$

Remember!

(i) Rational no. + Irrational no. = Irrational no.

(ii) Rational no. ($\neq 0$) × Irrational no. = Irrational no.

Example 3: Express the following recurring decimals as the rational number $\frac{p}{q}$, where p and q are integers.

(i) $0.\overline{5}$ (ii) $0.\overline{93}$

Solution: (i) $0.\overline{5}$

Let $x = 0.\overline{5}$

$x = 0.55555\ldots \quad \ldots(i)$

Multiply both sides by 10

$10x = 10(0.5555\ldots)$

$10x = 5.55555\ldots \quad \ldots(ii)$

Subtracting (i) from (ii)

$$10x - x = (5.55555\ldots) - (0.55555\ldots)$$
$$9x = 5$$
$$\Rightarrow \quad x = \frac{5}{9}$$

Which shows the decimal number in the form of $\frac{p}{q}$.

(ii) Let $x = 0.\overline{93}$

$x = 0.939393\ldots \quad \ldots(i)$

Multiply by 100 on both sides

$100x = 100\,(0.939393\ldots)$

$100x = 93.939393\ldots \quad \ldots(ii)$

Subtracting (i) from (ii)

$$100x - x = 93.939393\ldots - 0.939393\ldots$$
$$99x = 93$$
$$x = \frac{93}{99} \text{ which is in the form of } \frac{p}{q}.$$

Example 4 : Insert two rational numbers between 2 and 3.

Solution: There are infinite rational numbers between 2 and 3.
We have to find any two of them.

For this, find the average of 2 and 3 as $\frac{2+3}{2} = \frac{5}{2}$

So, $\frac{5}{2}$ is a rational number between 2 and 3, to find another rational number between 2 and 3 we will again find average of $\frac{5}{2}$ and 3

i.e,. $\dfrac{\frac{5}{2}+3}{2} = \dfrac{\frac{5+6}{2}}{2} = \dfrac{\frac{11}{2}}{2} = \dfrac{11}{4}$

Try Yourself!
What will be the product of two irrational numbers?

Hence, two rational numbers between 2 and 3 are $\frac{5}{2}$ and $\frac{11}{4}$.

1.1.5 Properties of Real Numbers

All calculations involving addition, subtraction, multiplication, and division of real numbers are based on their properties. In this section, we shall discuss these properties.

Additive properties

Name of the property	$\forall$ *a, b, c* $\in$ *R*	Examples
Closure	$a+b\in R$	$2+3=5\in R$
Commutative	$a+b=b+a$	$2+5=5+2$ $7=7$
Associative	$a+(b+c)=(a+b)+c$	$2+(3+5)=(2+3)+5$ $2+8=5+5$ $10=10$
Identity	$a+0=a=0+a$	$5+0=5=0+5$
Inverse	$a+(-a)=-a+a=0$	$6+(-6)=(-6)+6=0$

Multiplicative properties

Name of the property	$\forall$ *a, b, c* $\in$ *R*	Examples
Closure	$ab\in R$	$2\times 5=10\in R$
Commutative	$ab=ba$	$2\times 3=3\times 2=6\in R$
Associative	$a(bc)=(ab)c$	$2\times(3\times 5)=(2\times 3)\times 5$ $2\times 15=6\times 5$ $30=30$
Identity	$a\times 1=1\times a=a$	$5\times 1=1\times 5=5$
Inverse	$a\times\frac{1}{a}=\frac{1}{a}\times a=1$	$7\times\frac{1}{7}=\frac{1}{7}\times 7=1$

Distributive Properties

For all real numbers *a, b, c*

(i) $a(b+c)=ab+ac$ is called left distributive property of multiplication over addition.

(ii) $a(b-c)=ab-ac$ is called left distributive property of multiplication over subtraction.

(iii) $(a+b)c=ac+bc$ is called right distributive property of multiplication over addition.

(iv) $(a-b)c=ac-bc$ is called right distributive property of multiplication over subtraction.

Do you know?

0 and 1 are the additive and multiplicative identities of real numbers respectively.

Remember!

$0\in R$ has no multiplicative inverse.

Properties of Equality of Real Numbers

i	Reflexive property	$\forall a \in R,\ a = a$
ii	Symmetric property	$\forall a, b \in R,\ a = b \Rightarrow b = a$
iii	Transitive property	$\forall a, b, c \in R,\ a = b \wedge b = c \Rightarrow a = c$
iv	Additive property	$\forall a, b, c \in R,\ a = b \Rightarrow a + c = b + c$
v	Multiplicative property	$\forall a, b, c \in R,\ a = b \Rightarrow ac = bc$
vi	Cancellation property w.r.t addition	$\forall a, b, c \in R,\ a + c = b + c \Rightarrow a = b$
vii	Cancellation property w.r.t multiplication	$\forall a, b, c \in R$ and $c \neq 0,\ ac = bc \Rightarrow a = b$

Order Properties

i	Trichotomy property	$\forall a, b \in R$, either $a = b$ or $a > b$ or $a < b$
ii	Transitive Property	$\forall a, b, c \in R$ • $a > b \wedge b > c \Rightarrow a > c$ • $a < b \wedge b < c \Rightarrow a < c$
iii	Additive property	$\forall a, b, c \in R$ • $a > b \Rightarrow a + c > b + c$ • $a < b \Rightarrow a + c < b + c$
iv	Multiplicative property	$\forall a, b, c \in R$ • $a > b \Rightarrow ac > bc$ if $c > 0$ • $a < b \Rightarrow ac < bc$ if $c > 0$ • $a > b \Rightarrow ac < bc$ if $c < 0$ • $a < b \Rightarrow ac > bc$ if $c < 0$ • $a > b \wedge c > d \Rightarrow ac > bd$ • $a < b \wedge c < d \Rightarrow ac < bd$
v	Division property	$\forall a, b, c \in R$ • $a < b \Rightarrow \frac{a}{c} < \frac{b}{c}$ if $c > 0$ • $a < b \Rightarrow \frac{a}{c} > \frac{b}{c}$ if $c < 0$ • $a > b \Rightarrow \frac{a}{c} > \frac{b}{c}$ if $c > 0$ • $a > b \Rightarrow \frac{a}{c} < \frac{b}{c}$ if $c < 0$

vi	Reciprocal property	$\forall\, a,b \in R$ and have same sign • $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$ • $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$

Example 5 : If $a = \frac{2}{3}, b = \frac{3}{2}, c = \frac{5}{3}$ then verify the distributive properties over addition.

Solution: (i) Left distributive property

$$a(b+c) = ab + ac$$

LHS $= a(b+c)$

$$= \frac{2}{3}\left(\frac{3}{2}+\frac{5}{3}\right) = \frac{2}{3}\left(\frac{9+10}{6}\right)$$

$$= \frac{2}{3}\left(\frac{19}{6}\right) = \frac{19}{9}$$

RHS $= ab + ac$

$$= \left(\frac{2}{3}\right)\left(\frac{3}{2}\right) + \left(\frac{2}{3}\right)\left(\frac{5}{3}\right) = 1 + \frac{10}{9}$$

$$= \frac{9+10}{9} = \frac{19}{9}$$

LHS = RHS

Hence, it is verified that $a(b + c) = ab + ac$

(ii) Right distributive property

$$(a+b)c = ac + bc$$

LHS $= (a+b)c$

$$= \left(\frac{2}{3}+\frac{3}{2}\right)\frac{5}{3} = \left(\frac{4+9}{6}\right)\frac{5}{3}$$

$$= \left(\frac{13}{6}\right)\left(\frac{5}{3}\right) = \frac{65}{18}$$

RHS $= ac + bc$

$$= \left(\frac{2}{3}\right)\left(\frac{5}{3}\right) + \left(\frac{3}{2}\right)\left(\frac{5}{3}\right) = \frac{10}{9} + \frac{15}{6}$$

$$= \frac{20+45}{18} = \frac{65}{18}$$

LHS = RHS

Hence, it is verified that $(a + b)c = ac + bc$

Example 6: Identify the property that justifies the statement

(i) If $a > 13$ then $a + 2 > 15$

(ii) If $3 < 9$ and $6 < 12$ then $9 < 21$

(iii) If $7 > 4$ and $5 > 3$ then $35 > 12$

(iv) If $-5 < -4 \Rightarrow 20 > 16$

Solution:

(i) $a > 13$

Add 2 on both sides

$a + 2 > 13 + 2$

$a + 2 > 15$ (order property w.r.t addition)

$a + 2 > 13 + 2$

$a + 2 > 15$

(ii) As $3 < 9$ and $6 < 12$

$\Rightarrow 3 + 6 < 9 + 12$

$9 < 21$ (order property w.r.t addition)

(iii) $7 > 4$ and $5 > 3$

$\Rightarrow 7 \times 5 > 4 \times 3$

$\Rightarrow 35 > 12$ (order property w.r.t multiplication)

(iv) As $-5 < -4$

Multiply on both sides by -4

$(-5) \times (-4) > (-4) \times (-4)$

$\Rightarrow 20 > 16$ (order property w.r.t multiplication)

EXERCISE 1.1

1. Identify each of the following as a rational or irrational number:

(i) 2.353535 (ii) $0.\bar{6}$ (iii) $2.236067\ldots$ (iv) $\sqrt{7}$

(v) e (vi) π (vii) $5+\sqrt{11}$ (viii) $\sqrt{3}+\sqrt{13}$

(ix) $\frac{15}{4}$ (x) $(2-\sqrt{2})(2+\sqrt{2})$

2. Represent the following numbers on number line:

(i) $\sqrt{2}$ (ii) $\sqrt{3}$ (iii) $4\frac{1}{3}$

(iv) $-2\frac{1}{7}$ (v) $\frac{5}{8}$ (vi) $2\frac{3}{4}$

3. Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$:

(i) $0.\bar{4}$ (ii) $0.\overline{37}$ (iii) $0.\overline{21}$

4. Name the property used in the following:

(i) $(a+4)+b=a+(4+b)$ (ii) $\sqrt{2}+\sqrt{3}=\sqrt{3}+\sqrt{2}$

(iii) $x-x=0$ (iv) $a(b+c)=ab+ac$

(v) $16+0=16$ (vi) $100\times 1=100$

(vii) $4\times(5\times 8)=(4\times 5)\times 8$ (viii) $ab=ba$

5. Name the property used in the following:

(i) $-3<-1\Rightarrow 0<2$ (ii) If $a<b$ then $\frac{1}{a}>\frac{1}{b}$

(iii) If $a<b$ then $a+c<b+c$ (iv) If $ac<bc$ and $c>0$ then $a<b$

(v) If $ac<bc$ and $c<0$ then $a>b$ (vi) Either $a>b$ or $a=b$ or $a<b$

6. Insert two rational numbers between:

(i) $\frac{1}{3}$ and $\frac{1}{4}$ (ii) 3 and 4 (iii) $\frac{3}{5}$ and $\frac{4}{5}$

1.2 Radical Expressions

If n is a positive integer greater than 1 and a is a real number, then any real number x such that $x=\sqrt[n]{a}$ is called n^{th} root of a.

Here, $\sqrt{\ }$ is called radical and n is the index of radical. A real number under the radical sign is called a radicand. $\sqrt[3]{5},\sqrt[5]{7}$ are the examples of radical form.

Exponential form of $x=\sqrt[n]{a}$ is $x=(a)^{\frac{1}{n}}$.

1.2.1 Laws of Radicals and Indices

Laws of Radical	Laws of Indices
(i) $\sqrt[n]{ab}=\sqrt[n]{a}\cdot\sqrt[n]{b}$ (ii) $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	(i) $a^m.a^n=a^{m+n}$ (ii) $(a^m)^n=a^{mn}$
(iii) $\sqrt[n]{a^m}=(\sqrt[n]{a})^m$	(iii) $(ab)^n=a^nb^n$ (iv) $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$
(iv) $\left(\sqrt[n]{a}\right)^n=(a^{\frac{1}{n}})^n=a$	(v) $\frac{a^m}{a^n}=a^{m-n}$ (vi) $a^0=1$

Example 7: Simplify the following:

(i) $\sqrt[4]{16x^4y^8}$ (ii) $\sqrt[3]{27x^6y^9z^3}$ (iii) $(64)^{-\frac{4}{3}}$

Solution: (i) $\sqrt[4]{16x^4y^8} = (16x^4y^8)^{\frac{1}{4}}$ $\qquad \because \sqrt[n]{a} \quad a^{\frac{1}{n}}$

$= (16)^{\frac{1}{4}}(x^4)^{\frac{1}{4}}(y^8)^{\frac{1}{4}}$ $\qquad \because (ab)^m = a^m b^m$

$= 2^{4\cdot\frac{1}{4}} \times x^{4\cdot\frac{1}{4}} \times y^{8\cdot\frac{1}{4}}$ $\qquad \because (a^m)^n = a^{mn}$

$= 2xy^2$

(ii) $\sqrt[3]{27x^6y^9z^3} = \left(27x^6y^9z^3\right)^{\frac{1}{3}}$ $\qquad \because \sqrt[n]{a} = a^{\frac{1}{n}}$

$= \left(27\right)^{\frac{1}{3}}\left(x^6\right)^{\frac{1}{3}}\left(y^9\right)^{\frac{1}{3}}\left(z^3\right)^{\frac{1}{3}}$ $\qquad \because (ab)^m = a^m b^m$

$= \left(3^3\right)^{\frac{1}{3}}\left(x^6\right)^{\frac{1}{3}}\left(y^9\right)^{\frac{1}{3}}\left(z^3\right)^{\frac{1}{3}}$ $\qquad \because (a^m)^n = a^{mn}$

$= 3^{3\times\frac{1}{3}} \cdot x^{6\times\frac{1}{3}} \cdot y^{9\times\frac{1}{3}} . z^{3\times\frac{1}{3}}$

$= 3x^2y^3z$

(iii) $(64)^{-\frac{4}{3}} = \dfrac{1}{(64)^{\frac{4}{3}}}$

$= \dfrac{1}{4^{3\times\frac{4}{3}}} = \dfrac{1}{4^4}$

$= \dfrac{1}{256}$

1.2.2 Surds and their Applications

An irrational radical with rational radicand is called a surd.

For example, if we take the n^{th} root of any rational number a then $\sqrt[n]{a}$ is a surd. $\sqrt{5}$ is a surd because the square root of 5 does not give a whole number but $\sqrt{9}$ is not a surd because it simplifies to a whole number 3 and our result is not an irrational number. Therefore, the radical $\sqrt[n]{a}$ is irrational $\sqrt{7}, \sqrt{2}, \sqrt[3]{11}$ are surds but $\sqrt{\pi}, \sqrt{e}$ are not surds.

> **Remember!**
> Every surd is an irrational number but every irrational number is not a surd e.g., $\sqrt{\pi}$ is not a surd.

The different types of surds are as follow:

(i) A surd that contains a single term is called a monomial e.g., $\sqrt{5}, \sqrt{7}$ etc.

(ii) A surd that contains the sum of two monomial surds is called a binomial surd e.g., $\sqrt{3}+\sqrt{5}, \sqrt{2}+\sqrt{7}$ etc.

(iii) $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are called conjugate surds of each other.

Remember!

The product of two conjugate surds is a rational number.

1.2.3 Rationalization of Denominator

To rationalize a denominator of the form $a + b\sqrt{x}$ or $a - b\ \sqrt{x}$, we multiply both the numerator and denominator by the conjugate factor.

Example 8: Rationalize the denominator of:

(i) $\dfrac{3}{\sqrt{5}+\sqrt{2}}$ (ii) $\dfrac{3}{\sqrt{5}-\sqrt{3}}$

Solution (i):

$$\frac{3}{\sqrt{5}+\sqrt{2}} = \frac{3}{\sqrt{5}\quad\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{3(\sqrt{5}-\sqrt{2})}{5-2}$$

$$= \frac{3(\sqrt{5}-\sqrt{2})}{3} = \sqrt{5}-\sqrt{2}$$

(ii)

$$\frac{3}{\sqrt{5}-\sqrt{3}} = \frac{3}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{3\left(\sqrt{5}+\sqrt{3}\right)}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{3\left(\sqrt{5}+\sqrt{3}\right)}{5-3}$$

$$= \frac{3(\sqrt{5}+\sqrt{3})}{2}$$

EXERCISE 1.2

1. Rationalize the denominator of following:

(i) $\dfrac{13}{4+\sqrt{3}}$ (ii) $\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$ (iii) $\dfrac{\sqrt{2}-1}{\sqrt{5}}$

(iv) $\dfrac{6-4\sqrt{2}}{6+4\sqrt{2}}$ (v) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (vi) $\dfrac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

2. Simplify the following:

(i) $\left(\dfrac{81}{16}\right)^{-\frac{3}{4}}$ (ii) $\left(\dfrac{3}{4}\right)^{-2} \div \left(\dfrac{4}{9}\right)^{3} \times \dfrac{16}{27}$ (iii) $(0.027)^{-\frac{1}{3}}$

(iv) $\sqrt[7]{\dfrac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^{7}}}$ (v) $\dfrac{5\cdot(25)^{n+1} - 25\cdot(5)^{2n}}{5\cdot(5)^{2n+3} - (25)^{n+1}}$

(vi) $\dfrac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$ (vii) $(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

(viii) $\dfrac{3^{n} \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$ (ix) $\dfrac{5^{n+3} - 6.5^{n+1}}{9 \times 5^{n} - 2^{n} \times 5^{n}}$

3. If $x = 3 + \sqrt{8}$ then find the value of:

(i) $x + \dfrac{1}{x}$ (ii) $x - \dfrac{1}{x}$ (iii) $x^2 + \dfrac{1}{x^2}$

(iv) $x^2 - \dfrac{1}{x^2}$ (v) $x^4 + \dfrac{1}{x^4}$ (vi) $\left(x - \dfrac{1}{x}\right)^2$

4. Find the rational numbers p and q such that $\dfrac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$

5. Simplify the following:

(i) $\dfrac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$ (ii) $\dfrac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

(iii) $\sqrt{\dfrac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$ (iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}} b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

1.3 Applications of Real Numbers in Daily Life.

Real numbers are extremely useful in our daily life. That is probably one of the main reasons we learn how to count, add and subtract from a very young age. We cannot imagine life without numbers.

Real numbers are used in various fields including

- Science and engineering (physics, mechanical systems, electrical circuits)
- Medicine and Health
- Environmental science (climate modding, pollution monitoring etc.)
- Computer science (algorithm design, data compression, graphic rendering)
- Navigation and transportation (GPS, flight planning)
- Surveying and architecture
- Statistics and data

Example 9: The sum of two real numbers is 8, and their difference is 2. Find the numbers.

Solution: Let a and b be two real numbers then

$$a + b = 8 \quad \ldots(i)$$
$$a - b = 2 \quad \ldots(ii)$$

Add (i) and (ii)

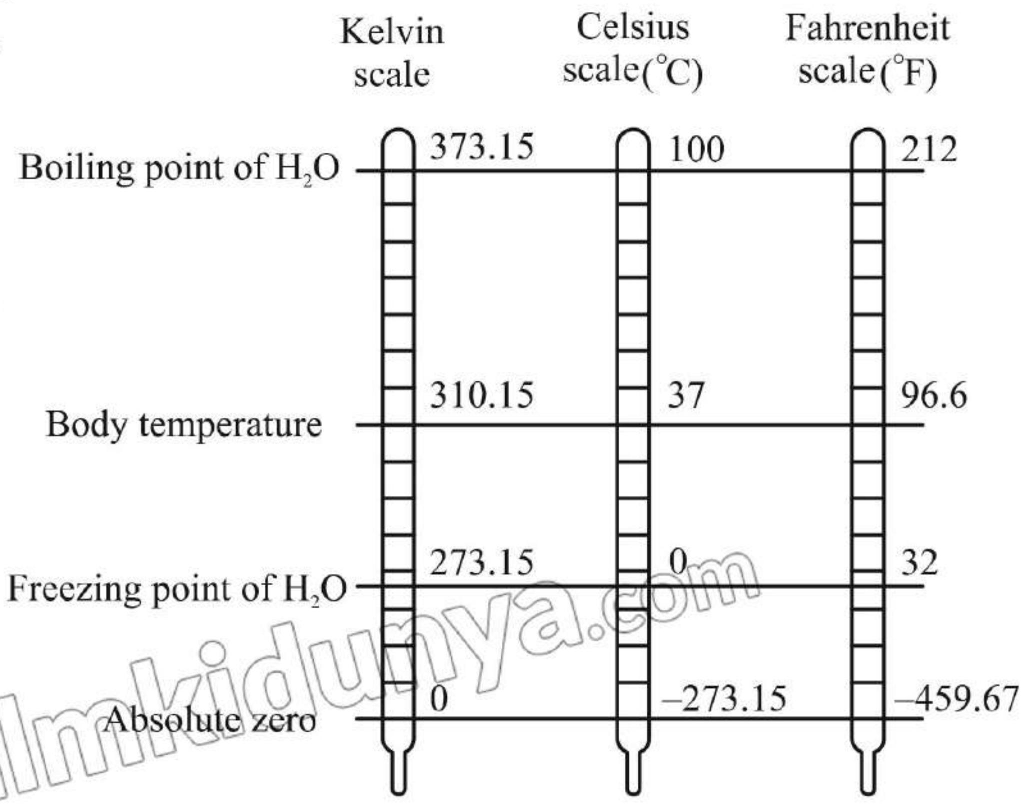
$$2a = 10 \quad \Rightarrow a = 5 \quad \text{put in (ii)}$$

$$\Rightarrow 5 - b = 2 \quad \Rightarrow -b = 2 - 5 \Rightarrow -b = -3 \Rightarrow b = 3$$

So, 5 and 3 are the required real numbers.

1.3.1 Temperature Conversions

In the figure, three types of thermometers are shown. We can convert three temperature scales, Celsius, Fahrenheit, and Kelvin, with each other.



Conversion formulae are given below:

(i) $K = °C + 273$

(ii) $°C = \frac{5}{9}(F - 32)°$

(iii) $°F = \frac{9°C}{5} + 32$

Where K, $°C$, and $°F$ show the Kelvin, Celsius, and Fahrenheit scales respectively.

Example 10: Normal human body temperature is 98.6 $°F$. Convert it into Celsius and Kelvin scale.

Solution: Given that $°F = 98.6$

So, to convert it into Celsius scale, we use

$$°C = \frac{5}{9}(F-32)°$$

$$°C = \frac{5}{9}(98.6-32)$$

$$= \frac{5}{9}(66.6)$$

$$= (0.55)(66.6)$$

$$°C = 37°$$

Hence, normal human body temperature at Celsius scale is 37°.

Now, we convert it into Kelvin scale.

$$K = C + 273°$$

$$K = 37° + 273°$$

$$K = 310 \text{ kelvin}$$

1.3.2 Profit and Loss

The traders may earn profit or incur losses. Profit and loss are a part of business. Profit and loss can be calculated by the following formula:

(i) Profit = selling Price – cost price

$$P = SP - CP$$

$$\text{Profit \%} = \left(\frac{\text{profit}}{CP} \times 100\right)\%$$

(ii) Loss = cost price – selling price

$$\text{Loss} = CP - SP$$

$$\text{Loss \%} = \left(\frac{\text{loss}}{CP} \times 100\right)\%$$

Example 11: Hamail purchased a bicycle for Rs. 6590 and sold it for Rs.6850. Find the profit percentage.

Solution: Cost Price $= CP = \text{Rs. } 6590$

Selling Price $= SP = \text{Rs. } 6850$

Profit $= SP - CP$

$= 6850 - 6590$

$= \text{Rs. } 260$

Now, we find the profit percentage.

$$\text{Profit \%} = \left(\frac{profit}{CP} \times 100\right)\%$$

$$= \left(\frac{260 \times 100}{6590}\right)\%$$

$$= 3.94\%$$

$$\approx \ 4\%$$

Example 12: Umair bought a book for Rs. 850 and sold it for Rs. 720. What was his loss percentage?

Solution: Cost price of book $= CP = \text{Rs. } 850$

Selling price of book $= SP = \text{Rs. } 720$

Loss $= CP - SP$

$= 850 - 720$

$= \text{Rs. } 130$

$$\text{Loss percentage} = \left(\frac{Loss}{CP} \times 100\right)\%$$

$$= \left(\frac{130}{850} \times 100\right)\%$$

$$= 15.29\%$$

Example 13: Saleem, Nadeem, and Tanveer earned a profit of Rs. 4,50,000 from a business. If their investments in the business are in the ratio 4: 7: 14, find each person's profit.

Solution: Profit earned $= \text{Rs. } 4,50,000$

Given ratio $= 4 : 7 : 14$

Sum of ratios $= 4 + 7 + 14$

$= 25$

Saleem earned profit $= \frac{4}{25} \times 4,50,000$ = Rs. 72,000

Nadeem earned profit $= \frac{7}{25} \times 4,50,000$ = Rs. 126,000

Tanveer earned profit $= \frac{14}{25} \times 4,50,000$ = Rs. 252,000

Example 14: If the simple profit on Rs. 6400 for 12 years is Rs. 3840. Find the rate of profit.

Solution: Principal = Rs. 6400

Simple profit = Rs. 3840

Time = 12 years

To find the rate we use the following formula:

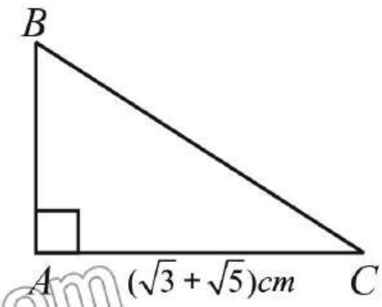
$$\text{Rate} = \frac{\text{amount of profit} \times 100}{\text{time} \times \text{principal}}$$

$$= \frac{3840 \times 100}{12 \times 6400} = 5\%$$

Thus, rate of profit is 5%.

EXERCISE 1.3

1. The sum of three consecutive integers is forty-two, find the three integers.
2. The diagram shows right angled ΔABC in which the length of $\overline{AC}$ is $(\sqrt{3}+\sqrt{5})$cm. The area of ΔABC is $(1+\sqrt{15})$ cm^2. Find the length $\overline{AB}$ in the form $(a\sqrt{3}+b\sqrt{5})$ cm, where a and b are integers.



3. A rectangle has sides of length $2+\sqrt{18}$ m and $\left(5-\frac{4}{\sqrt{2}}\right)$ m. Express the area of the rectangle in the form $a+b\sqrt{2}$, where a and b are integers.
4. Find two numbers whose sum is 68 and difference is 22.
5. The weather in Lahore was unusually warm during the summer of 2024. The

TV news reported temperature as high as 48°C. By using the formula, $(°F = \frac{9}{5}°C + 32)$ find the temperature as Fahrenheit scale.

6. The sum of the ages of the father and son is 72 years. Six years ago, the father's age was 2 times the age of the son. What was son's age six years ago?
7. Mirha sells a toy for Rs. 1520. What will the selling price be to get a 15% profit?
8. The annual income of Tayyab is Rs. 9,60,000, while the exempted amount is Rs. 1,30,000. How much tax would he have to pay at the rate of 0.75%?
9. Find the compound markup on Rs. 3,75,000 for one year at the rate of 14% compounded annually.

REVIEW EXERCISE 1

1. Four options are given against each statement. Encircle the correct option.

(i) $\sqrt{7}$ is:
(a) integer (b) rational number
(c) irrational number (d) natural number

(ii) π and e are:
(a) natural numbers (b) integers
(c) rational numbers (d) irrational numbers

(iii) If n is not a perfect square, then $\sqrt{n}$ is:
(a) rational number (b) natural number
(c) integer (d) irrational number

(iv) $\sqrt{3}+\sqrt{5}$ is:
(a) whole number (b) integer
(c) rational number (d) irrational number

(v) For all $x \in R$, $x = x$ is called:
(a) reflexive property (b) transitive number
(c) symmetric property (d) trichotomy property

(vi) Let $a, b, c \in R$, then $a > b$ and $b > c \Rightarrow a > c$ is called _________ property.
(a) trichotomy (b) transitive
(c) additive (d) multiplicative

(vii) $2^x \times 8^x = 64$ then $x =$

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$

(viii) Let $a, b \in R$, then $a = b$ and $b = a$ is called _________ property.

(a) reflexive (b) symmetric

(c) transitive (d) additive

(ix) $\sqrt{75} + \sqrt{27} =$

(a) $\sqrt{102}$ (b) $9\sqrt{3}$ (c) $5\sqrt{3}$ (d) $8\sqrt{3}$

(x) The product of $(3 + \sqrt{5})(3 - \sqrt{5})$ is:

(a) prime number (b) odd number

(c) irrational number (d) rational number

2. If $a = \frac{3}{2}$, $b = \frac{5}{3}$ and $c = \frac{7}{5}$, then verify that

(i) $a(b + c) = ab + ac$ (ii) $(a + b)c = ac + bc$

3. If $a = \frac{4}{3}$, $b = \frac{5}{2}$, $c = \frac{7}{4}$, then verify the associative property of real numbers w.r.t addition and multiplication.

4. Is 0 a rational number? Explain.

5. State trichotomy property of real numbers.

6. Find two rational numbers between 4 and 5.

7. Simplify the following:

(i) $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$ (ii) $\sqrt[3]{(27)^{2x}}$ (iii) $\frac{6(3)^{n+2}}{3^{n+1} - 3^n}$

8. The sum of three consecutive odd integers is 51. Find the three integers.

9. Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls were in each bucket?

10. Salma invested Rs. 3,50,000 in a bank, which paid simple profit at the rate of $7\frac{1}{4}\%$ per annum. After 2 years, the rate was increased to 8% per annum. Find the amount she had at the end of 7 years.