Unit 11 Loci and Construction

Students' Learning Outcomes

At the end of the unit, the students will be able to:

- Construct a triangle having given two sides and the included angle.
- Construct a triangle having given one side and two of the angles.
- Construct a triangle having given two of its sides and the angle opposite to one of them.
- Draw angle bisectors, perpendicular bisectors, medians, altitudes of a given triangle and verify their concurrency.
- Draw loci and intersection of loci for set of points in two dimensions which are
  - at a given distance from a given point.
  - at a given distance from a given line
  - equidistant from two given points
  - equidistant from two given intersecting lines

  Solve real life problems using the loci and interesting loci.

INTRODUCTION

A locus plural loci is a set of points that follow a given rule. Loci are also useful for understanding and predicting patterns. For instance, consider two people walking around a room, each maintaining a fixed distance from the other. The possible locations are where each person form a specific path. By studying these loci, we can predict where each person might be relative to the other at any time. In contexts like tracking satellites orbiting Earth, we use the concept of loci to predict where they will be at given times. This helps in areas like telecommunications and GPS technology.

Loci in two dimensions are triangle, circle, parallel lines, perpendicular bisector and angle bisector.

Remember!

There are three types of triangles w.r.t. sides:
Scalene triangle: All sides are of different length.
Isosceles triangles: Two sides are of equal length.
Equilateral triangle: All sides of equal length.
There are three types of triangles w.r.t. angles:
Acute angled triangle: All angles are of measure less than 90^0.
Obtuse angled triangle: One angle is of measure greater than 90^0.
Right angled triangle: One angle is of measure equal to 90^0.

11.1 Construction of Triangles

A triangle is a closed figure having three sides and three angles. We construct triangle in the following cases:

(a) When measure of all three sides are given.

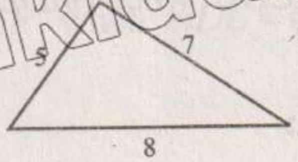
(b) When measure of two sides and their included angle are given.

(c) When measure of one side and measure of two angles are given.

(d) When measure of two sides and an angle opposite to one of them is given.

Triangle Inequality Theorem

The sum of the measure of any two sides of a triangle is always greater than the measure of the third side. For example, we can see in the figure adding any two lengths then this will be greater than the third side i.e., $5 + 7 > 8$, $5 + 8 > 7$ and $7 + 8 > 5$

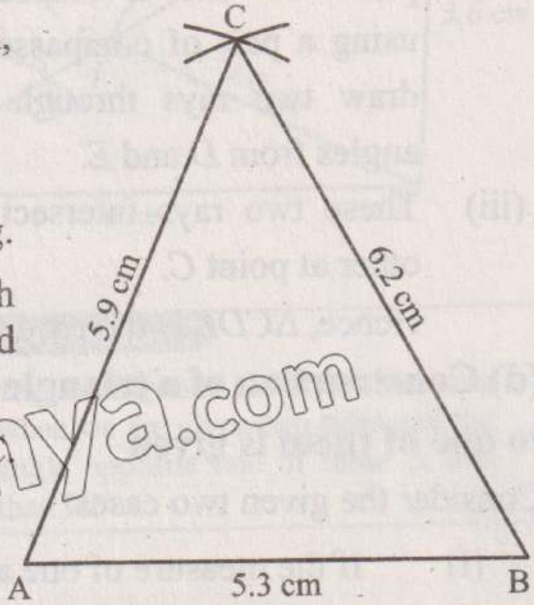


Key fact!

- An equilateral triangle is acute angled triangle.
- A right angled triangle cannot be equilateral.

(a) Construction of a triangle when measure of three sides is given

Example 1: Construct a triangle of sides 5.3 cm, 5.9 cm and 6.2 cm.



Solution: **Steps of construction:**

(i) Draw a line segment AB of length 5.3cm long.

(ii) Using a pair of compasses, draw two arcs with centres at points *A* and *B* of radii 5.9 cm and 6.2 cm respectively.

(iii) These two arcs intersect each other at point C.

(iv) Join *A* and *B* with *C*.

Hence, ΔABC is the required triangle.

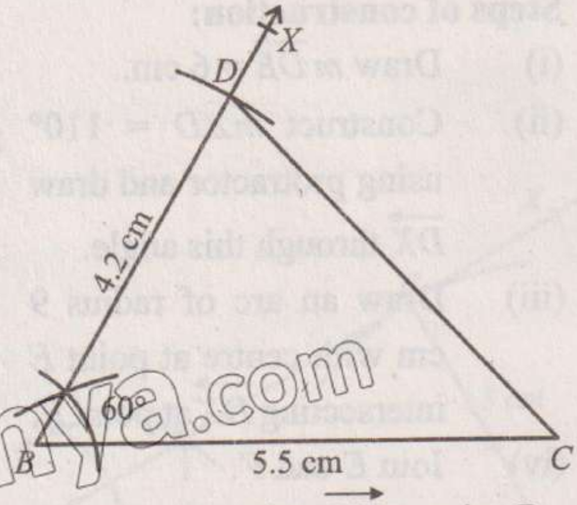
NOTE: The angles 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135° and 150° are constructed with the help a pair of compasses. Other angles are drawn using protractor.

Do you know?

When three sides are given, we can draw any length first.

(b) Construction of a triangle when the measure of two sides and their included angle are given

Example 2: Construct a triangle *BCD* in which measures of two sides are 5.5 cm and 4.2 cm and measure of their included angle is 60°.



Solution: **Steps of construction**

(i) Draw a line segment *BC* of length 5.5cm.

(ii) Draw an angle 60° at point *B* using a pair of compasses and draw a ray $\overrightarrow{BX}$ through *this angle*.

(iii) Draw an arc of radius 4.2 cm with centre at point *B* intersecting $\overrightarrow{BX}$ at point *D*.

(iv) Join *C* and *D*.

Hence, ΔBCD is the required triangle.

(c) Construction of a triangle when measure of one side and two angles are given

Example 3: Draw a triangle CDE when $m\overline{DE} = 4.3$ cm, $m\angle D = 30°$ and $m\angle E = 120°$.

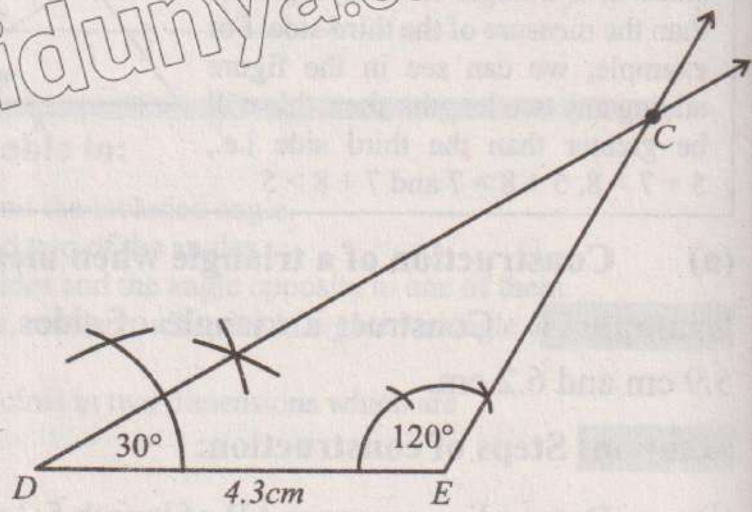
Solution: **Steps of construction:**

(i) Draw $m\,\overline{DE} = 4.3$ cm.

(ii) Draw angles 30° and 120° at points D and E respectively using a pair of compasses and draw two rays through these angles from D and E.

(iii) These two rays intersect each other at point C.

Hence, ΔCDE is the required triangle.



(d) Construction of a triangle when measure of two sides and angle opposite to one of them is given

Consider the given two cases:

(i) If the measure of one angle is greater than or equal to 90°.

(ii) If the measure of angle is less than 90°.

Example 4: Construct a triangle DEF when $m\overline{DE} = 6$ cm, $m\angle D = 110°$ and $m\,\overline{EF} = 9$ cm.

Solution:

Steps of construction:

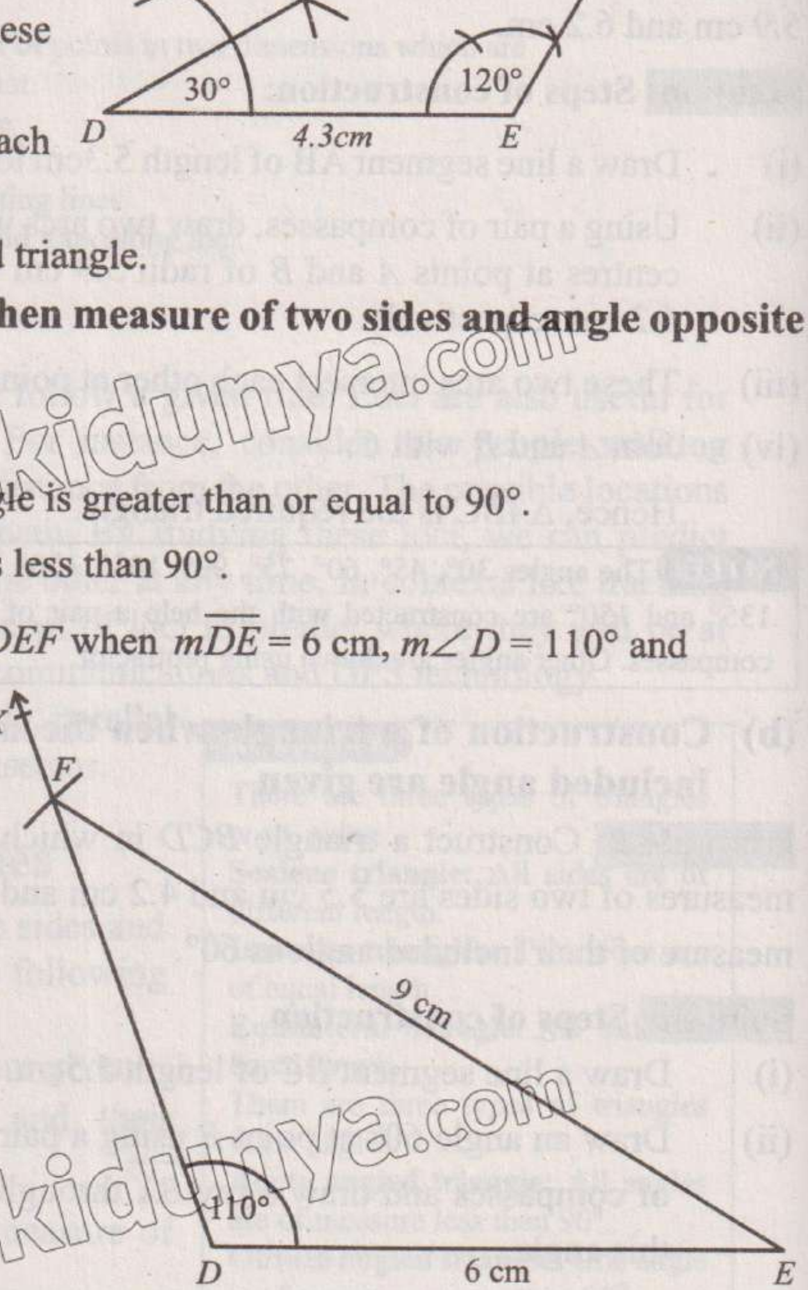
(i) Draw $m\,\overline{DE} = 6$ cm.

(ii) Construct $m\angle D = 110°$ using protractor and draw $\overrightarrow{DX}$ through this angle.

(iii) Draw an arc of radius 9 cm with centre at point E intersecting $\overrightarrow{DX}$ at point F.

(iv) Join E and F.

Hence, ΔDEF is the required triangle



If the given angle opposite to the given side is obtuse, only one triangle is possible.

Example 5:

Construct triangles DEF and DEF' when $m\overline{DE} = 6$ cm, $m\angle D = 30°$ and $m\overline{EF} = 3.6$ cm

Solution:

Steps of construction:

(i) Draw $m\overline{DE} = 6$ cm.

(ii) Construct an angle 30° at point D using a pair of compasses and draw $\overrightarrow{DX}$ through this angle.

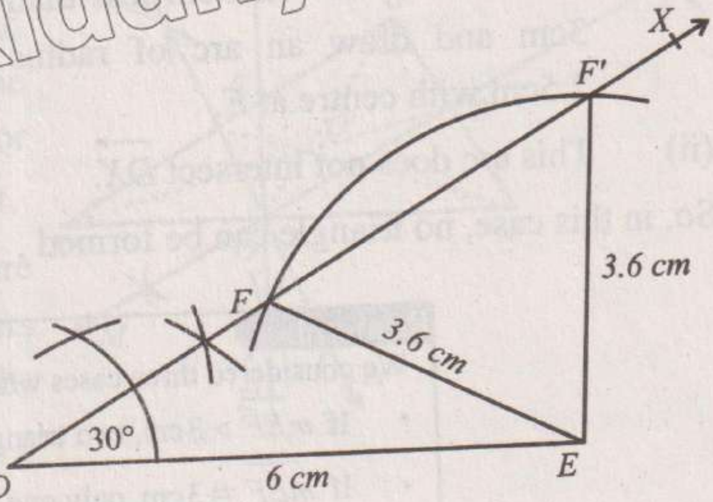
(iii) Draw an arc of radius 3.6 cm with centre at point E.

(iv) This arc intersects $\overrightarrow{DX}$ at two points F and F'.

(v) Join F and F' with E.

We get two triangles DEF and DEF'.

This is known as **ambiguous** case.



Do you know?

The **Ambiguous Case** (SSA) occurs when we are given two sides and the angle opposite one of these is less than 90°.

Example 6: In the above example if we take:

(a) $m\overline{EF} = 3$ cm

(b) $m\overline{EF} = 2.5$ cm

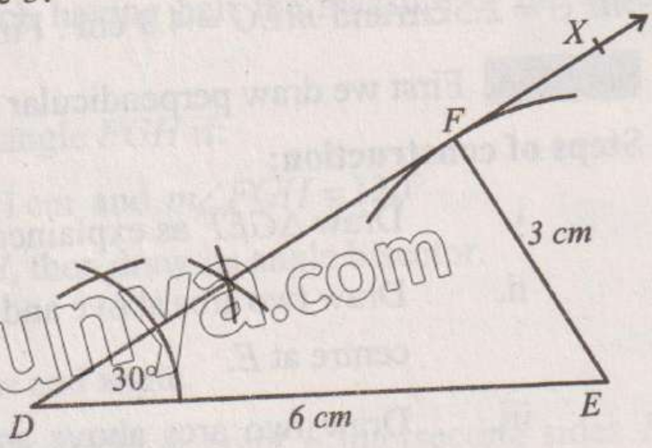
Solution: **Steps of construction:**

Follow the same steps (i) and (ii) as in Example 5.

Case (a)

(i) Draw an arc of radius 3 cm with centre at point E which touches $\overrightarrow{DX}$ at point F.

(ii) Join E with F. Here, $\overline{EF}$ will be perpendicular to $\overrightarrow{DX}$.



Hence, ΔDEF is the required triangle, which is a right angled triangle.

Case (b)

(i) If we take $m\overline{EF} = 2.5\text{cm}$ less than 3cm and draw an arc of radius 2.5cm with centre at E.

(ii) This arc does not intersect DX.

So, in this case, no triangle can be formed .

Remember:
We considered three cases when acute angle is given.

- If $m\overline{EF} > 3\,\text{cm}$, two triangles are possible.
- If $m\overline{EF} = 3\,\text{cm}$, only one triangle is possible.
- If $m\overline{EF} < 3\,\text{cm}$, no triangle is possible.

11.2 Perpendicular Bisectors and Medians of a Triangle

Perpendicular Bisector: A perpendicular bisector is a line that intersects a line segment at right angle and dividing it into two equal parts. In other words, it intersects the line segment at its midpoint and forms right angles (90°) with it.

Median: A median of a triangle is a line segment that joins a vertex to the midpoint of the side that is opposite to that vertex.

Point of concurrency: A point of concurrency is the single point where three or more lines, rays or line segments intersect or meet in a geometric figure. This concept is commonly used in triangles, where several important types of points of concurrency exist.

Example 7: Draw perpendicular bisector of the triangle EFG with $m\overline{EF} = 5\,\text{cm}$, $m\overline{FG} = 2.5\,\text{cm}$ and $m\overline{EG} = 4.3$ cm . Find the medians.

Solution: First we draw perpendicular bisectors and then medians.

Steps of construction:

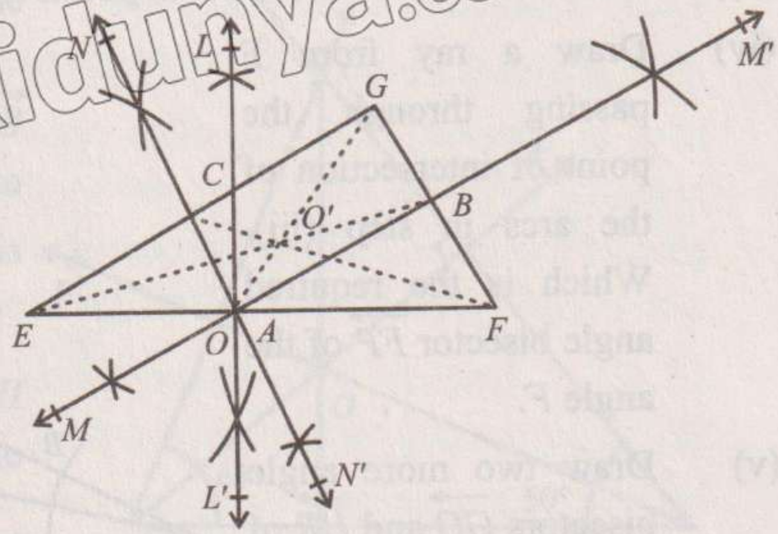
i. Draw ΔGEF as explained in the previous examples.

ii. Draw two arcs above and below $\overline{EF}$ with more than half of $m\overline{EF}$ with centre at E.

iii. Draw two arcs above and below $\overline{EF}$ with radius more than half of $m\overline{EF}$ with centre at F.

iv. Draw a line through the points of intersection of the arcs in steps (ii) and (iii), we get the perpendicular bisector $\overleftrightarrow{LL'}$ of the side $\overline{EF}$ at A.

v. Draw two more perpendicular bisectors $\overleftrightarrow{MM'}$ and $\overleftrightarrow{NN'}$ of the sides $\overline{FG}$ and $\overline{EG}$ at B and C respectively.



vi. Join the point G with opposite midpoint A so $\overline{GA}$ is the median.

vii. Join the point F with opposite midpoint C, we get median $\overline{FC}$ and join point E with opposite midpoint B, we get median $\overline{EB}$.

Hence, we see that the perpendicular bisector $\overleftrightarrow{LL'}$, $\overleftrightarrow{MM'}$ and $\overleftrightarrow{NN'}$ are concurrent at point O or A and the medians $\overline{GA}$, $\overline{EB}$ and $\overline{FC}$ are concurrent at point O'.

Circumcentre: The point of concurrency of perpendicular bisector of the sides of a triangle is called circumcentre.

Centroid: The point of concurrency of the medians of a triangle is called centroid of the triangle.

11.3 Angle Bisector of a Triangle

An angle bisector is a line or ray that divides an angle into two equal parts, creating two smaller angles that are congruent (each having half the measure of the original angle).

Example 8: Draw angle bisector of a triangle FGH if:

$m\overline{FG} = 5.2$ cm, $m\overline{GH} = 4.1$ cm and $m\angle FGH = 120°$

Solution: We first construct triangle FGH, then draw its angle bisector.

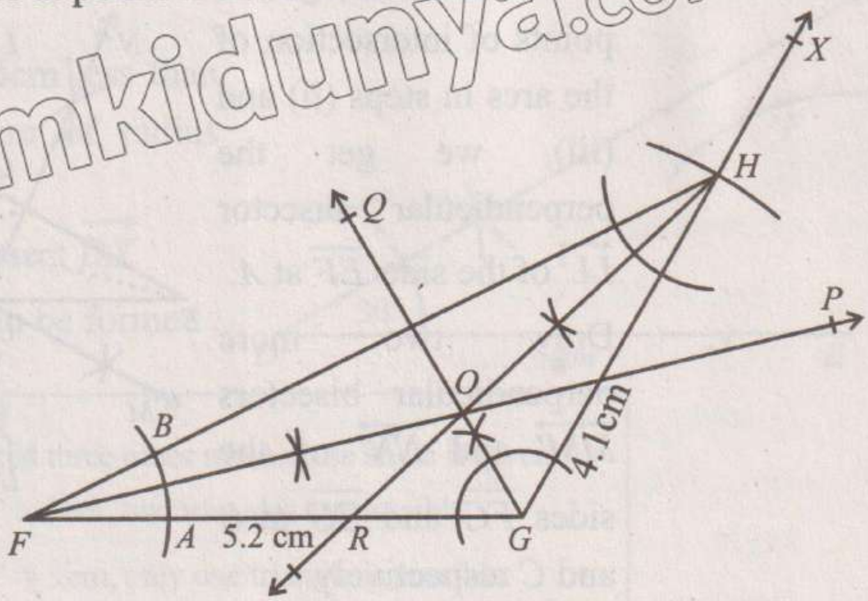
Steps of construction:

(i) construct ΔFGH with given lengths and angle.

(ii) Draw an arc of suitable radius with centre at point F intersecting sides FG and FH at points A and B.

(iii) Draw two arcs with centres at points A and B with suitable radius.

(iv) Draw a ray from F passing through the point of intersection of the arcs in step (iii). Which is the required angle bisector $\overrightarrow{FP}$ of the angle F.



(v) Draw two more angle bisectors $\overrightarrow{GQ}$ and $\overrightarrow{HR}$ of the angles G and H. respectively.

We see that all the angle bisectors $\overrightarrow{FP}$, $\overrightarrow{GQ}$ and $\overrightarrow{HR}$ intersect at one point O. i.e, the angle bisectors of the triangle are concurrent.

Incentre: The point of concurrency of the angle bisectors of a triangle is called incentre of the triangle.

11.4 Altitudes of Triangle

Altitude is a ray drawn perpendicular from a vertex to the opposite side of the triangle. There are three altitudes of the triangle which meet at a single point i,e. the altitudes of a triangle are concurrent.

Orthocentre

The point of concurrency of the altitudes of the triangle is called orthocentre of the triangle.

Example 9:

Construct a triangle GHI in which $m\overline{GH} = 5.7$ cm, $m\angle G = 68°$ and $m\angle H = 50°$. Prove that altitudes of the ΔGHI are concurrent.

Solution:

First, we construct ΔGHI using the given measurements and then draw altitudes of the triangle.

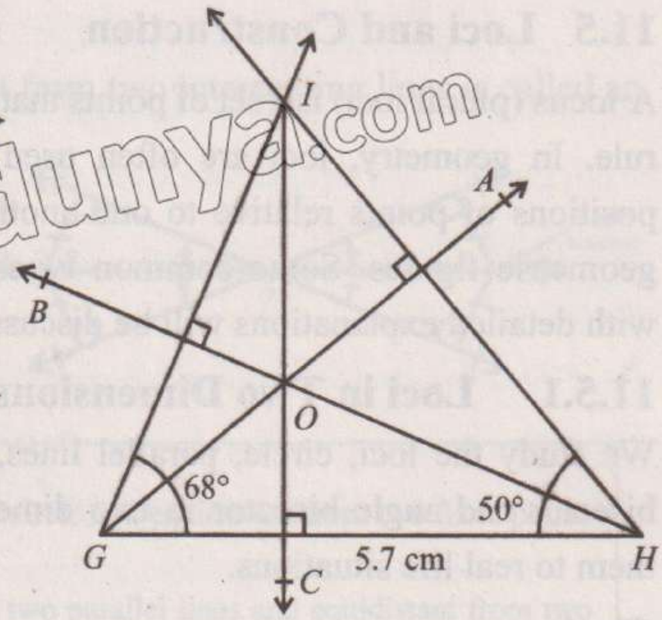
Steps of construction.

(i) Construct ΔGHI using the given measurements.

(ii) Draw perpendicular $\overrightarrow{GA}$ from G to the opposite side HI.

(iii) Draw two more perpendiculars $\overrightarrow{HB}$ and $\overrightarrow{IC}$. The first is from point H to the opposite side GI and the other is from point I to the opposite side GH.

So, $\overrightarrow{GA}$, $\overrightarrow{HB}$ and $\overrightarrow{IC}$ are the altitudes of ΔGHI and they intersect at one point O. i.e., the altitudes of ΔGHI are concurrent.



EXERCISE 11.1

1. Construct ΔABC with the given measurements and verify that the perpendicular bisectors of the triangle are concurrent.
   (i) $m\overline{AB} = 5$ cm, $m\overline{BC} = 6$ cm and $m\overline{AC} = 7$ cm
   (ii) $m\overline{AB} = 7.1$ cm, $m\angle B = 135°$ and $m\overline{BC} = 6.5$ cm
2. Construct ΔLMN of the following measurements and verify that the medians of the triangle are concurrent.
   (i) $m\overline{LM} = 4.9$ cm, $m\angle L = 51°$ and $m\angle M = 38°$
   (ii) $m\overline{MN} = 4.8$ cm, $m\angle N = 30°$ and $m\overline{LM} = 8.1$ cm
3. Verify that the angle bisectors of ΔABC are concurrent with the following measurement:
   (i) $m\overline{AB} = 4.5$ cm, $m\angle A = 45°$ and $m\overline{AC} = 5.3$ cm
   (ii) $m\overline{AB} = 6$ cm, $m\angle A = 150°$ and $m\angle B = 60°$
4. Given the measurements of ΔDEF : $m\overline{DE} = 4.8$ cm, $m\overline{EF} = 4$ cm and $m\angle E = 45°$, draw altitudes of ΔDEF and find orthocentre.
5. Construct the following triangles and find whether there exists any ambiguous case.
   (i) ΔBCD ; $m\overline{BC} = 5$ cm, $m\angle B = 62°$ and $m\overline{CD} = 4.7$ cm
   (ii) ΔKLM ; $m\overline{LM} = 6$ cm, $m\angle M = 42°$ and $m\overline{LN} = 5$ cm

11.5 Loci and Construction

Do you know? In Latin, the word *locus* is defined by the English term, location.

A locus (plural loci) is a set of points that follow a given rule. In geometry, loci are often used to define the positions of points relative to one another or to other geometric figures. Some common types of loci along with detailed explanations will be discussed.

Remember!
Equidistant: Let A be a fixed point and B be a set of points. If A is at equal distance from all points of B, then A is said to be equidistant from B.

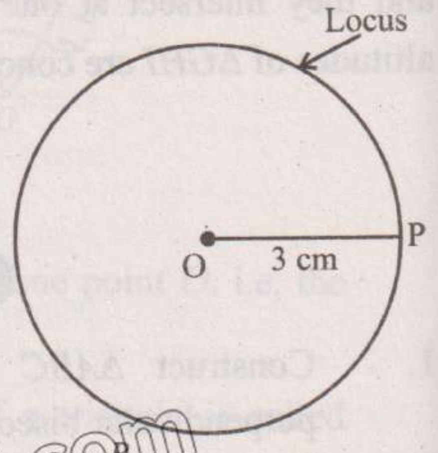
11.5.1 Loci in Two Dimensions

We study the loci, circle, parallel lines, perpendicular bisector and angle bisector in two dimensions and apply them to real life situations.

Circle

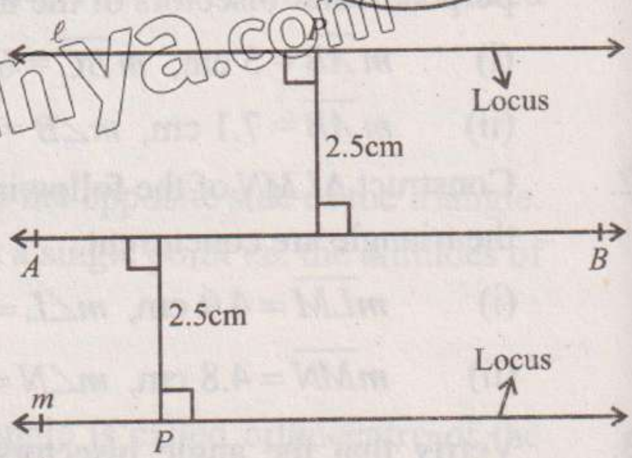
The locus of a point whose distance is constant from a fixed point is called a circle.

For example, the locus of a point P whose distance is 3 cm from a fixed-point O is a circle of radius 3 cm and centre at point O.

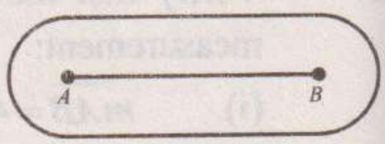


Parallel Lines

The locus of a point whose distance from a fixed line is constant are parallel lines, ℓ and m e. g. the locus of a point P whose distance is 2.5 cm from a fixed line AB are parallel lines at a distance of 2.5 cm from AB.



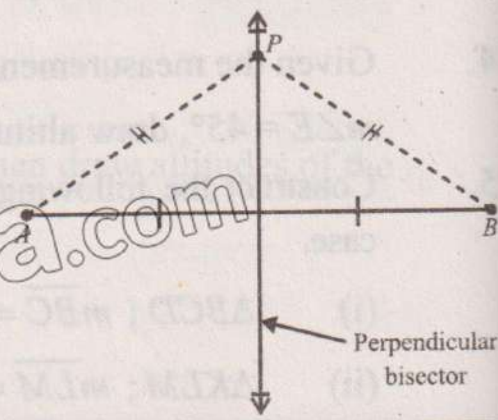
For example, a locus of points equidistant from a line segment creates a **sausage shape**. We can think of this type of locus as a track surrounding a line segment.



Perpendicular Bisector

The locus of a point whose distance from two fixed points is constant is called a perpendicular bisector.

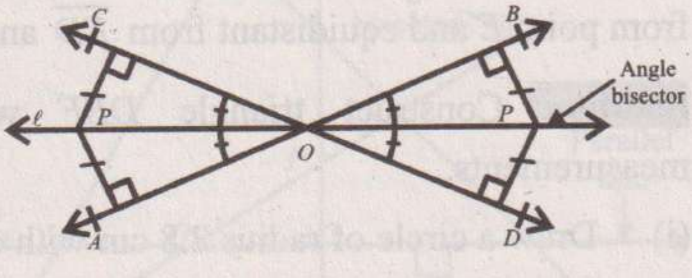
For example, the locus of a point P whose distance from fixed points A and B is constant is the perpendicular bisector of the line segment AB.



Angle Bisector

The locus of a point whose distance is constant from two intersecting lines is called an angle bisector.

For example, the locus of a point P whose distance is constant from two lines AB and CD intersecting at O is the angle bisector (ℓ) of $\angle AOC$ and $\angle BOD$.



Remember!

- Locus of points equidistant from a fixed point is a circle and equidistant from two fixed points is a perpendicular bisector.
- Locus of points equidistant from a fixed line are two parallel lines and equidistant from two fixed intersecting lines is angle bisector.

11.5.2 Intersection of Loci

If two or more loci intersect at a point P, then P satisfies all given conditions of the loci. This will be explained in the following examples:

Example 10: Construct a rectangle $ABCD$ with $mAB = 5$ cm and $mBC = 3.2$ cm. Draw the locus of all points which are:

(i) at a distance of 3.1 cm from point A. (ii) equidistant from A and B.

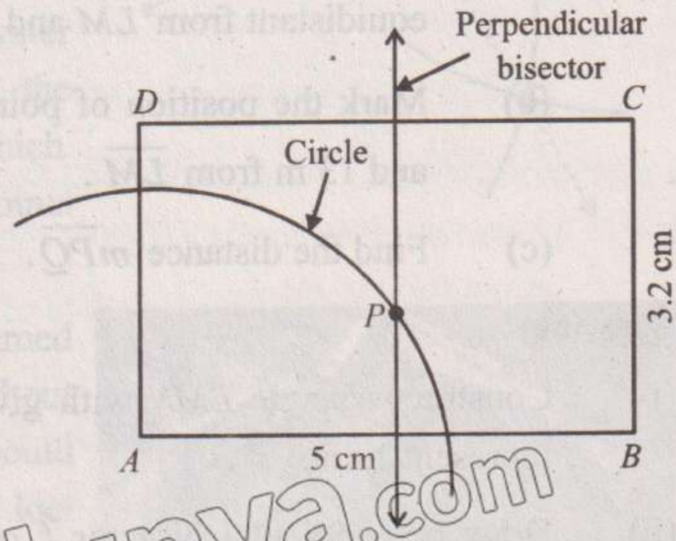
Label the point P inside the rectangle which is 3.1 cm from point A and equidistant from A and B.

Solution: Construct rectangle $ABCD$ with given lengths.

(i) Draw a circle of radius 3.1 cm with centre at A.

(ii) Draw perpendicular bisector of $\overline{AB}$.

The two loci intersect at P inside the rectangle which is 3.1 cm from point A and equidistant from A and B.



Example 11: Construct an isosceles triangle DEF with vertical angle 80° at E and $m\overline{EF} = m\overline{DE} = 4.8$ cm. Draw the locus of all points which are:

(i) at a distance of 2.8 cm from point E,

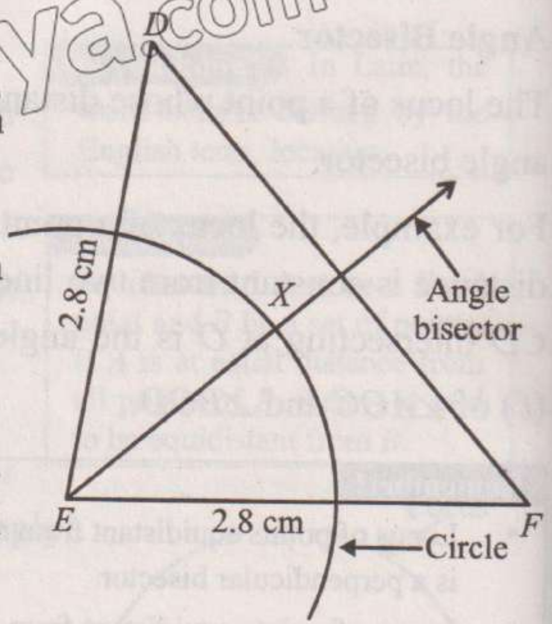
(ii) equidistant from $\overline{DE}$ and $\overline{EF}$.

Label the point X inside the triangle which is 2.8 cm from point E and equidistant from $\overline{ED}$ and $\overline{EF}$.

Solution: Construct triangle DEF with given measurements.

(i) Draw a circle of radius 2.8 cm with centre at E.

(ii) Draw angle bisector of angle DEF. The two loci intersect at X inside the triangle which is 2.8 cm from point E and equidistant from $\overline{ED}$ and $\overline{EF}$



Example 12: A field is in the form of a triangle LMN with $m\overline{LM} = 69$ m, $m\angle L = 60°$ and $m\angle M = 45°$.

(i) Construct ΔLMN with given measurements. [Scale: 10m = 1cm]

(ii) Draw the locus of all points which are equidistant from L and M, equidistant from $\overline{LM}$ and $\overline{LN}$ and at a distance of 13 m from $\overline{LM}$ inside the triangular field.

(iii) Two trees are to be planted at points P and Q inside the field.

(a) Mark the position of point P which is equidistant from L and M and equidistant from $\overline{LM}$ and $\overline{LN}$.

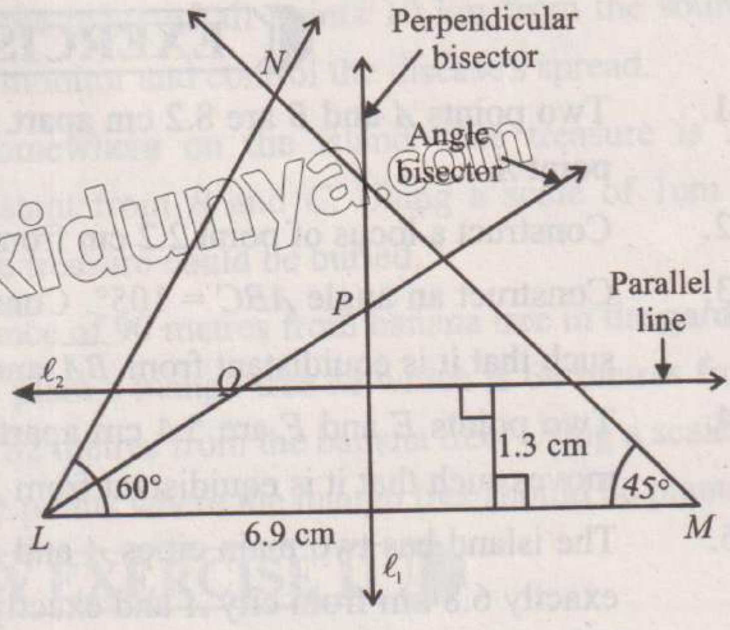
(b) Mark the position of point Q which is equidistant from $\overline{LM}$ and $\overline{LN}$ and 13 m from $\overline{LM}$.

(c) Find the distance $m\overline{PQ}$.

Solution:

(i) Construct triangle LMN with given measurements using a scale of 10 m to represent 1 cm.

(ii) Draw perpendicular bisector ℓ_1 of $\overline{LM}$. Draw angle bisector of angle MLN. Draw a parallel line ℓ_2 inside the triangle LMN, 1.3 cm from $\overline{LM}$.



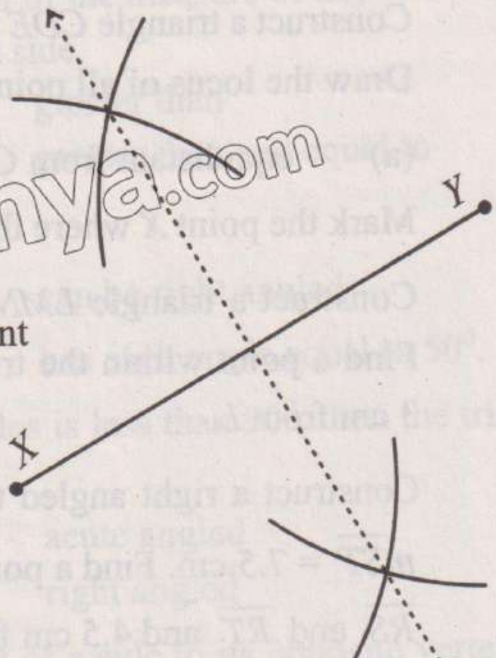
(iii) (a) Label the point P which is equidistant from L and M and equidistant from $\overline{LM}$ and $\overline{LN}$. Mark the point P inside the triangle which is equidistant from L and M.

(b) Label the point Q which is equidistant from $\overline{LM}$ and $\overline{LN}$ and 1.3 cm from $\overline{LM}$.

(c) $m\overline{PQ} = 1.2 \times 10 = 12\text{m}$

11. 6 Real Life Application of Loci

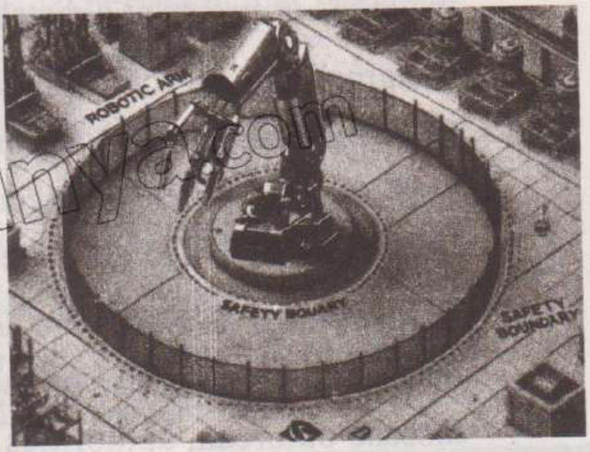
The concept of loci has many applications across fields where spatial relationships, distances, or specific constraints are important. Here, are detailed examples illustrating the use of loci in different contexts.



(i) A park has two water sources at two different points. A fire hydrant needs to be placed so it is equally accessible to both sources.

Let X and Y represent the two water sources in the park. Draw the perpendicular bisector of $\overline{XY}$ which represents the locus of all points equidistant from X and Y.

(ii) A robotic arm in a factory is programmed to work within a specific area without crossing into areas where it could interfere with other equipment. The loci of the robot's possible positions would be a defined space, such as a circle or rectangular region, ensuring it operates safely within its designated zone.



EXERCISE 11.2

1. Two points A and B are 8.2 cm apart. Construct the locus of points 5 cm from point A.
2. Construct a locus of point 2.2 cm from line segment CD of measure 5.7cm.
3. Construct an angle $ABC = 105°$. Construct a locus of a point P which moves such that it is equidistant from $\overline{BA}$ and $\overline{BC}$.
4. Two points E and F are 5.4 cm apart. Construct a locus of a point P which moves such that it is equidistant from E and F.
5. The island has two main cities A and B 8 km apart. Kashif lives on the island exactly 6.8 km from city A and exactly 7.3 km from city B. Mark with a cross the points on the island where Kashif could live.
6. Construct a triangle CDE with $m\overline{CD} = 7.6$ cm, $m\angle D = 45°$ and $m\overline{DE} = 5.9$ cm. Draw the locus of all points which are:

   (a) equidistant from C and D (b) equidistant from $\overline{CD}$ and $\overline{CE}$

   Mark the point X where the two loci intersect.
7. Construct a triangle LMN with $m\overline{LM} = 7$ cm, $m\angle L = 70°$ and $m\angle M = 45°$. Find a point within the triangle LMN which is equidistant from L and M and 3 cm from L.
8. Construct a right angled triangle RST with $m\overline{RS} = 6.8$ cm, $m\angle S = 90°$ and $m\overline{ST} = 7.5$ cm. Find a point within the triangle RST which is equidistant from $\overline{RS}$ and $\overline{RT}$ and 4.5 cm from R.
9. Construct a rectangle $UVWX$ with $m\overline{UV} = 7.2$ cm and $m\overline{VW} = 5.6$ cm. Draw the locus of points at a distance of 2 cm from $\overline{UV}$ and 3.5 cm from W.
10. Imagine two cell towers located at points A and B on a coordinate plane. The GPS-enabled device, positioned somewhere on the plane, receives signals from both towers. To ensure accurate navigation, the device is placed equidistant from both towers to estimate its position. Draw this locus of navigation.
11. Epidemiologists use loci to determine infection zones, especially for contagious diseases, to predict the spread and take containment measures. In the case of a disease outbreak, authorities might determine a quarantine zone within 10 km

of the infection source. Draw the locus of all points 10 km from the source defining the quarantine area to monitor and control the disease's spread.

12. There is a treasure buried somewhere on the island. The treasure is 24 kilometres from A and equidistant from B and C. Using a scale of 1cm to represent 10 km, find where the treasure could be buried.

13. There is an apple tree at a distance of 90 metres from banana tree in the garden of Sara's house. Sara wants to plant a mango tree M which is 64 metres from apple tree and between 54 and 82 metres from the banana tree. Using a scale of 1cm to represent 10m, Find the points where the mango tree should be planted.

REVIEW EXERCISE 11

1. Four options are given against each statement. Encircle the correct option.

(i) A triangle can be constructed if the sum of the measure of any two sides is ________ the measure of the third side.

(a) less than (b) greater than
(c) equal to (d) greater than and equal to

(ii) An equilateral triangle ________.

(a) can be isosceles (b) can be right angled
(c) can be obtuse angled (d) has each angle equal to 50^0.

(iii) If the sum of the measures of two angles is less than 90^0, then the triangle is ________.

(a) equilateral (b) acute angled
(c) obtuse angled (d) right angled

(iv) The line segment joining the midpoint of a side to its opposite vertex in a triangle is called ________.

(a) median (b) perpendicular bisector
(c) angle bisector (d) circle

(v) The angle bisectors of a triangle intersect at ________.

(a) one point (b) two points
(c) three points (d) four points

(vi) Locus of all points equidistant from a fixed point is ________.

(a) circle (b) perpendicular bisector
(c) angle bisector (d) parallel lines

(vii) Locus of points equidistant from two fixed points is ----------------
(a) circle (b) perpendicular bisector
(c) angle bisector (d) parallel lines

(viii) Locus of points equidistant from a fixed line is/are ----------------
(a) circle (b) perpendicular bisector
(c) angle bisector (d) parallel lines

(ix) Locus of points equidistant from two intersecting lines is ________.
(a) circle (b) perpendicular bisector
(c) angle bisector (d) parallel lines

(x) The set of all points which is farther than 2 km from a fixed point B is a region outside a circle of radius ________ and centre at B.
(a) 1 km (b) 1.9 km
(c) 2 km (d) 2.1 km

2. Construct a right angled triangle with measures of sides 6 cm, 8 cm and 10 cm.

3. Construct a triangle ABC with $m\overline{AB} = 5.3$ cm, $m\angle A = 30°$ and $m\angle B = 120°$. Draw the locus of all points which are equidistant from A and B.

4. Construct a triangle with $m\overline{DE} = 7.3$ cm, $m\angle D = 42°$ and $m\overline{EF} = 5.4$ cm.

5. Construct a triangle XYZ with $m\overline{YX} = 8$ cm, $m\overline{YZ} = 7$ cm and $m\overline{XZ} = 6.5$ cm. Draw the locus of all points which are equidistant from $\overline{XY}$ and $\overline{XZ}$.

6. Construct a triangle FGH such that $m\overline{FG} = m\overline{GH} = 6.4$ cm, $m\angle G = 122°$. Draw the locus of all points which are:
(a) equidistant from F and G,
(b) equidistant from $\overline{FG}$ and $\overline{GH}$.
(c) Mark the point where the two loci intersect.

7. Two houses Q and R are 73 metres apart. Using a scale of 1 cm to represent 10 m, construct the locus of a point P which moves such that it is:
(i) at a distance of 32 metres from Q
(ii) at a distance of 48 metres from the line joining Q and R.

8. The field is in the form of a rectangle $ABCD$ with $m\overline{AB} = 70$m and $m\overline{BC} = 60$ m. Construct the rectangle $ABCD$ using a scale of 1cm to represent 10 m. Show the region inside the field which is less than 30 m from C and farther than 25 m from $\overline{AB}$.