Unit 13 Probability

Students' Learning Outcomes

At the end of the unit, the students will be able to:

- Calculate the probability of a single event and the probability of an event not occurring.
- Solve real life problems involving probability.
- Calculate relative frequency as an estimate of probability.
- Calculate expected frequencies.
- Solve real life problems involving relative and expected frequencies.

INTRODUCTION

In our daily life, we normally say that manufacturing companies give warranty on their products, there is chance that some product might not meet warranty time period. A person judges the chances of winning cricket match of a team based on previous performances etc. All the above statements have lack prediction with certainty. In such situations, what makes it easier for us to represent the chance of an event occurring numerically i.e., probability.

> **History!**
> The word "probability" is derived from the Latin word "Probabilitas". It means "probity". Girolamo Cardano is known as the father of probability. He was an Italian doctor and mathematician.

Hence, Probability is the chance of occurrence of a particular event.

Probability is calculated by using the given formula:

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

It is written as: $P(A) = \dfrac{n(A)}{n(S)}$

$P(A)$ = Probability of an event A

$n(A)$ = Number of favourable outcomes

$n(S)$ = Total number of possible outcomes

Basic Concepts of Probability

Experiment: The process which generates results e.g., tossing a coin, rolling a dice, etc. is called an experiment.

Outcomes: The results of an experiment are called outcomes e.g., the possible outcomes of tossing a coin are head or tail, the possible outcomes of rolling a dice are 1, 2, 3, 4, 5, or 6.

Favourable Outcome: An outcome which represents how many times we expect the things to be happened e.g., while tossing a coin, there is 1 favourable outcome of getting head or tail. While rolling a dice, there are 3 favourable outcomes of getting multiples of 2 i.e. $\{2, 4, 6\}$

Sample Space: The set of all possible outcomes of an experiment is called sample space. It is denoted by 'S' e.g., while tossing a coin, the sample space will be $S = \{H, T\}$.

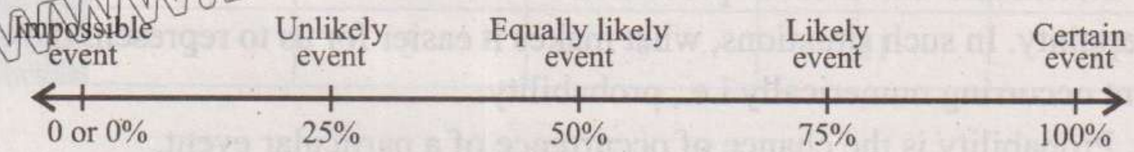
While rolling a dice, the sample space will be $S = \{1, 2, 3, 4, 5, 6\}$.

> **Remember!**
> Each element of the sample space is called sample point.

Event: The set of results of an experiment is called an event e.g., while rolling a dice getting even number is an event i.e., $A = \{2, 4, 6\}$; $n(A) = 3$.

Recall! Types of Events:

- **Certain event:** An event which is sure to occur. The probability of sure event is 1.
- **Impossible event:** An event cannot occur in any trial. The probability of this event is 0.
- **Likely event:** An event which will probably occur. It has greater chance to occur.
- **Unlikely event:** An event which will not probably occur. It has less chance to occur.
- **Equally likely events:** The events which have equal chance of occurrence. The probability of these events is 0.5.



13.1 Probability of Single Event

Example 1: Abdul Raheem rolls a fair dice, what is the probability of getting the number divisible by 3?

Solution: When a dice is rolled, the sample space will be:

$$S = \{1, 2, 3, 4, 5, 6\}\ ;\ n(S) = 6$$

Let "A" be the event of getting the number divisible by 3.

$$A = \{3, 6\};\ n(A) = 2$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

The probability of getting the number divisible by 3 is $\frac{1}{3}$.

> **Keep in mind**
> The range of probability for an event is:
> $0 \le P(A) \le 1$

> **Teachers' note:**
> Clear the concept of all the types of events by using different colours of balls or pencils etc.

Example 2: If Zeeshan rolled two fair dice, find the probability of getting:

(i) Even numbers on both dice.

(ii) Multiples of 3 on both dice.

(iii) Even number on the first dice and the number 3 on the second dice.

(iv) At least the number 3 on the first dice and number 4 on the second dice.

Solution: When a pair of fair dice is rolled, the sample space will be:

2^{nd} \ 1^{st}	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Try Yourself!

Can you find out the sample space when 3 dice are rolled.

(i) Even numbers on both dice.

Let "A" be the event of getting even numbers on both dice.

$$A = \{(2, 2), (2, 4), (2, 6), (4, 2) (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$$

$$n(A) = 9; \; n(S) = 36$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

Thus, the probability of getting even numbers on both dice is $\frac{1}{4}$.

(ii) Multiple of 3 on both dice.

Let "B" be the event of getting multiples of 3 on both dice.

$$B = \{(3, 3), (3, 6), (6, 3), (6, 6)\}$$

$$n(B) = 4; \; n(S) = 36$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Thus, the probability of getting multiples of 3 on both dice is $\frac{1}{9}$.

(iii) Even number on the first dice and the number 3 on the second dice.

Let "C" be the event of getting even numbers on the first dice and the number 3 on the second dice.

$$C = \{(2,3), (4,3), (6,3)\}$$

$$n(C) = 3; n(S) = 36$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

Thus, the probability of getting an even number on the first dice and the number 3 on the second dice is $\frac{1}{12}$.

(iv) At least the number 3 on the first dice and number 4 on the second dice.

Let "D" be the event of getting at least the number 3 on the first dice and number 4 on the second dice.

$$D = \{(3, 4), (4, 4), (5, 4), (6, 4)\}$$

$$n(D) = 4; n(S) = 36$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Thus, the probability of getting at least the number 3 on the first dice and number 4 on the 2nd dice is $\frac{1}{9}$.

13.2 Probability of an Event Not Occurring

Sometimes, we are interested in the probability that the head will not occur while tossing a coin.

Let "A" be the event of getting head while tossing a coin, then the event "A'" be the event of not getting head while tossing a coin.

The probability of not getting head while tossing a coin is known as the complement of that event. It is written as $P(A')$ or $P(A^c)$.

The complement of an event "A" is calculated by the given formula:

$$P(A') = 1 - P(A)$$

For example, while tossing a coin, the probability of getting a head is:

$$P(A) = \frac{1}{2}$$

Teachers' note:

Give more examples to explain complement of events e.g., if the desired outcome is head on a flipping coin, the complement is tail. The compliment rule states that the sum of the probability of an event and its complement must be equal to 1.

and the probability of not getting a head is:

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Thus, the complement of the event of getting a head is $\frac{1}{2}$.

Example 3: Zubair rolls a dice, what is the probability of not getting the number 6?

Solution: Let "A" be the event of getting the number 6.

The sample space while rolling a dice is: $S = \{1, 2, 3, 4, 5, 6\}$

$$n(S) = 6$$

$$A = \{6\}; n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

Remember!

The sum of the probability of an event "A" and the probability of an event not occurring "A" is always "1"

$$P(A) + P(A') = 1$$

To find out probability of not getting the number 6, we have

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$$

Thus, the probability of not getting the number 6 is $\frac{5}{6}$.

Example 4: If two fair dice are rolled. What is the probability of getting:

(i) not a double six (ii) not the sum of both dice is 8

Solution: Sample space of two fair dice is given by:

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$$n(S) = 36$$

(i) not a double six.

Let "A" be the event that a double six occurs.

$$A = \{(6, 6)\}; n(A) = 1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$$

Let " A' " be the event that not a double six occurs

As we know that

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{1}{36} = \frac{36-1}{36} = \frac{35}{36}$$

Thus, the probability of not getting the double six is $\frac{35}{36}$.

(ii) not the sum of both dice is 8.

Let "B" be the event that the sum of both dice is 8.

$$B = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

$$n(B) = 5$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

Let " B' " be the event not sum of both dice is 8.

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{5}{36} = \frac{36-5}{36} = \frac{31}{36}$$

Thus, the probability of not the sum of both dice be 8 is $\frac{31}{36}$.

13.3 Real Life Problems Involving Probability

Example 5: Let A, B and C are three missiles and they are fired at a target. If the probabilities of hitting the target are $P(A) = \frac{1}{4}$, $P(B) = \frac{3}{7}$, $P(C) = \frac{5}{9}$, respectively. Find the probabilities of

(i) missile A does not hit the target. (ii) missile B does not hit the target.

(iii) missile C does not hit the target.

Solution: (i) missile A does not hit the target.

Since, $P(A) = \frac{1}{4}$

Let ' A' ' be the event that missile A does not hit the target

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$$

Thus, the probability of missile 'A' does not hit the target is $\frac{3}{4}$.

(ii) missile '*B*' does not hit the target.

Since, $P(B) = \frac{3}{7}$

Let 'B'' be the event missile *B* does not hit the target

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{3}{7}$$

$$= \frac{7-3}{7} = \frac{4}{7}$$

Thus, the probability of missile '*B*' does not hit the target is $\frac{4}{7}$.

(iii) missile '*C*' does not hit the target.

Since, $P(C) = \frac{5}{9}$

Let 'C'' be the event missile *C* of not hitting the target

$$P(C') = 1 - P(C)$$

$$= 1 - \frac{5}{9} = \frac{9-5}{9} = \frac{4}{9}$$

Thus, the probability of missile '*C*' does not hit the target is $\frac{4}{7}$.

Example 6: A bag contains 5 blue balls and 8 green balls. Find the probability of selecting at random:

(i) a blue ball (ii) a green ball. (iii) not a green ball.

Solution: (i) a blue ball

Let '*A*' be the event that the ball is blue

Blue balls = $n(A) = 5$

Total balls = $n(S) = 5 + 8 = 13$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{5}{13}$$

Try Yourself!

Can you find out the complement of selecting a blue ball?

Thus, the probability of selecting a blue ball is $\frac{5}{13}$.

(ii) a green ball

Let 'B' be the event that ball is green

Green balls $= n(B) = 8$

Total balls $= n(S) = 5 + 8 = 13$

$$P(B) = \frac{n(B)}{n(S)} = \frac{8}{13}$$

Thus, the probability of selecting green ball is $\frac{8}{13}$.

Think!
The probability that a person A will be alive 0.75. Can you find out the complement of that event?

(iii) not a green ball

Let 'B'' be the event that the ball is not green.

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{8}{13}$$

$$= \frac{13-8}{13} = \frac{5}{13}$$

Thus, the probability of not selecting a green ball is $\frac{5}{13}$.

Example 7: A card is drawn at random, from a pack of 52 playing cards. What is the probability of getting:

(i) a card of heart (ii) neither spade nor heart

Solution: (i) a card of heart

Total number of cards = 52 ; $n(S) = 52$

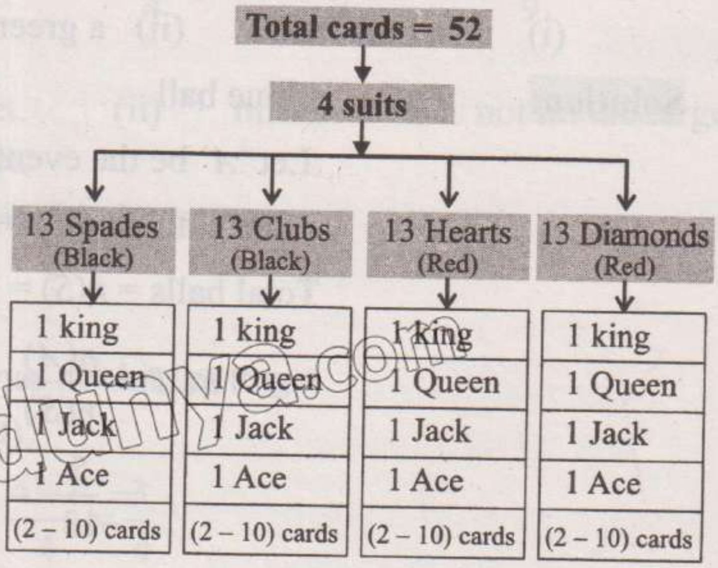
Let 'A' be the event of selecting a card of heart.

Number of heart cards = 13 ; $n(A) = 13$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{13}{52} = \frac{1}{4}$$

Thus, the probability of getting a card of heart is $\frac{1}{4}$.



(ii) neither spade nor heart

Let 'B' be the event of selecting a card of spade or heart

Number of spade and heart cards = 26 ; $n(B) = 26$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{26}{52}$$

$$= \frac{1}{2}$$

Let 'B'' be the event of selecting neither spade nor heart card.

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

Thus, the probability of getting neither spade nor heart cards is $\frac{1}{2}$.

EXERCISE 13.1

1. Arshad rolls a dice, with sides labelled L, M, N, O, P, U. What is the probability that the dice lands on consonant?

2. Shazia throws a pair of fair dice. What will be the probability of getting:
   (i) sum of dots is at least 4.
   (ii) product of both dots is between 5 to 10.
   (iii) the difference between both the dots is equal to 4.
   (iv) number at least 5 on the first dice and the number at least 4 on the second dice.

3. One alphabet is selected at random from the word "MATHEMATICS". Find the probability of getting:
   (i) vowel (ii) consonant (iii) an E
   (iv) an A (v) not M (vi) not T

4. Aslam rolled a dice. What is the probability of getting the numbers 3 or 4? Also find the probability of not getting the numbers 3 or 4.

5. Abdul Hadi labelled cards from 1 to 30 and put them in a box. He selects a card at random. What is the probability that selected card containing:
   (i) the number 25 (ii) number between 17 to 22
   (iii) number at least 20 (iv) number not 27 and 29
   (v) number not between 12 to 15
6. The probability that Ayesha will pass the examination is 0.85. What will be the probability that Ayesha will not pass the examination?
7. Taabish tossed a fair coin and rolled a fair dice once. Find the probability of the following events:
   (i) tail on coin and at least 4 on dice.
   (ii) head on coin and the number 2,3 on dice.
   (iii) head and tail on coin and the number 6 on dice.
   (iv) not tail on coin and the number 5 on dice.
   (v) not head on coin and the number 5 and 2 on dice.
8. A card is selected at random from a well shuffled pack of 52 plying cards. What will be the probability of selecting:
   (i) a queen (ii) neither a queen nor a jack
9. A card is chosen at random from a pack of 52 playing cards. Find the probability of getting:
   (i) a jack (ii) no diamond

13.4 Relative Frequency as an Estimate of Probability

Relative frequency tells us how often a specific event occurs relative to the total number of frequency event or trials. It is calculated by using the following method:

$$\text{Relative frequency} = \frac{\text{Frequency of specific event}}{\text{Total frequency}} = \frac{x}{N}, \text{ where } N = \Sigma f$$

Example 8: Find the relative frequency of the given date.

X	2	3	4	5	6	7	8
f	3	5	6	9	10	8	2

Solution:

X	f	Relative frequency
2	3	$\frac{3}{43} = 0.07$
3	5	$\frac{5}{43} = 0.12$
4	6	$\frac{6}{43} = 0.14$
5	9	$\frac{9}{43} = 0.21$
6	10	$\frac{10}{43} = 0.23$
7	8	$\frac{8}{43} = 0.19$
8	2	$\frac{2}{43} = 0.04$
Total	$\Sigma f = 43$	

13.5 Real Life Application of Relative Frequency

Keep in mind

The sum of all the relative frequencies is always equal to or approximately equal to 1.

Example 9: A survey was conducted on 80 students of Grade – IX and asked about their favourite colour. The responses are:

(i) Red colour = 23 students
(ii) Green colour = 15 students
(iii) Pink colour = 25 students
(iv) Blue colour = 10 students
(v) White colour = 7 students.

Find the relative frequency for each colour.

Remember!

Relative frequency is an estimated probability of an event occurring when an experiment is repeated a fixed number of times.

Solution: Total number of students = 80

(i) Relative frequency for red colour $= \frac{23}{80} = 0.29$

It means that 29% students prefer red colour.

(ii) Relative frequency for green colour $= \frac{15}{80} = 0.19$

It means that 19% students prefer green colour.

(iii) Relative frequency for pink colour = $\frac{25}{80} = 0.31$

It means that 31% students prefer pink colour.

(iv) Relative frequency for blue colour $\frac{10}{80} = 0.12$

It means that 12% students prefer blue colour.

(v) Relative frequency for white colour = $\frac{7}{80} = 0.09$

It means that 9% students prefer white colour.

Try Yourself!

Out of 200 students in a school, 80 play cricket, 50 play football, 25 play volleyball and 45 do not play any game. Can you find out the probability of the students who do not play any game and relative frequency of the students who play cricket?

Example 10: Abdul Rehman obtained different marks in different subjects out of 100 marks. The detail is as under:

Subject	Urdu	English	Islamiyat	Mathematics	Science	Computer Science
Marks Obtained	75	80	72	95	81	85

Find the relative frequency of above given data.

Solution:

Subject	Marks obtained	Relative frequency
Urdu	75	$\frac{75}{488} = 0.15$
English	80	$\frac{80}{488} = 0.16$
Islamiyat	72	$\frac{72}{488} = 0.15$
Mathematics	95	$\frac{95}{488} = 0.19$
Science	81	$\frac{81}{488} = 0.17$
Computer Science	85	$\frac{85}{488} = 0.17$
Total	$\Sigma f = 488$	

13.6 Expected Frequency

Expected frequency is a measure that estimate how often an event should be occurred depended on probability. Expected frequency is found by using the following method:

Teachers' note

Clear the concept to the students that relative frequency as an estimate of probability by using different real life problems.

Expected frequency = Total number of trials × Probability of the event.

$$= N \times P(A)$$

Example 11: Six fair dice are rolled 50 times. The probability of occurrence of different number of sixes are given below. Find the expected frequency of the following data:

x	0	1	2	3	4	5	6
$P(x)$	0.09	0.10	0.12	0.24	0.10	0.20	0.15

Find the expected frequency of occurrence of each six.

Solution:

No. of Sixes (x)	$P(x)$	Expected frequency $= N \times P(x) = 50 \times P(x)$
0	0.09	$50 \times 0.09 = 4.5$
1	0.10	$50 \times 0.10 = 5$
2	0.12	$50 \times 0.12 = 6$
3	0.24	$50 \times 0.24 = 12$
4	0.10	$50 \times 0.10 = 5$
5	0.20	$50 \times 0.20 = 10$
6	0.15	$50 \times 0.15 = 7.5$

13.7 Real Life Application on Expected Frequency

Example 12: Find the average number of times getting 1 or 6, when a fair dice is rolled 300 times.

Solution: Let "S" be the sample space when a fair dice is rolled:

Remember!

Sum of all expected frequencies is always equal to or approximately equal to a fixed number of trials.

$S = \{1, 2, 3, 4, 5, 6\}$; $n(S) = 6$

Let "B" be the event that 1 or 6 comes up.

$B = \{1, 6\}$; $n(B) = 2$

So, $P(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Therefore, $E(B) = N \times P(B)$

$= 300 \times \frac{1}{3} = 100$

Thus, the average number of times 1 or 6 comes up is 100.

Example 13: If the probability of a defective bolt is 0.3. Find the number of non-defective bolts in a total to 800.

Solution: The probability of defective bolt is = 0.3

Probability of non-defective bolt = 1 − 0.3 = 0.7

Number of non-defective bolts = 0.7 × 800 = 560

Thus, the non-defective bolts will be 560.

EXERCISE 13.2

1. A researcher collected data on number of deaths from Horse-Ricks in Russian Army crops over to years. The table is as follows:

No. of death	0	1	2	3	4	5	6
Frequency	60	50	87	40	32	15	10

Find the relative frequency of the given data.

2. The frequency of defective products in 750 samples are shown in the following table. Find the relative frequency for the given table.

No. of defectives per sample	0	1	2	3	4	5	6	7	8
No. of sample	120	140	94	85	105	50	40	66	50

3. A quiz competition on general knowledge is conducted. The number of corrected answers out of 5 questions for 100 sets of questions is given below.

X	0	1	2	3	4	5
f	10	23	15	25	18	9

Find the relative frequencies for the given data.

4. A survey was conducted from the 50 students of a class and asked about their favourite food. The responses are as under:

Name of food item	Biryani	Fresh Juice	Chicken	Bar. B.Q	Sweets
No. of students	40	07	21	15	25

(i) how many percentages of students like biryani?

(ii) how many percentages of students like chicken?

(iii) which food is the least like by the students?

(iv) which food is the most prefer by the students?

5. In 500 trials of a thrown of two dice, what is expected frequency that the sum will be greater than 8?

6. What is the expectation of a person who is to get Rs. 120 if he obtains at least 2 heads in single toss of three coins?

7. Find the expected frequencies of the given data if the experiment is repeated 200 times.

x	0	1	2	3	4	5	6
$P(x)$	0.11	0.21	0.17	0.18	0.09	0.17	0.07

8. The probability of getting 5 sixes while tossing six dice is $\frac{2}{5}$, the dice is rolled 200 times. How many times would you expect it to show 5 sixes?

REVIEW EXERCISE 13

1. Four options are given against each statement. Encircle the correct option.

(i) Each element of the sample space is called:

(a) event (b) experiment

(c) sample point (d) outcomes

(ii) An outcome which represents how many times we expect the things to be happened is called:

(a) outcomes (b) favourable outcome

(c) sample space (d) sample point

(iii) Which one tells us how often a specific event occurs relative to the total number of frequency event or trials?
(a) expected frequency (b) sum of relative frequency
(c) relative frequency (d) frequency

(iv) Estimated probability of an event occurring is also known as:
(a) relative frequency (b) expected frequency
(c) class boundaries (d) sum of expected frequency

(v) The sum of all expected frequencies is equal to the fixed number of:
(a) trials (b) relative frequencies
(c) outcomes (d) events

(vi) The chance of occurrence of a particular event is called:
(a) sample space (b) estimated probability
(c) probability (d) expected frequency

(vii) An event which will probably occur. It has greater chance to occur is called:
(a) equally likely event (b) likely event
(c) unlikely event (d) certain event

(viii) Find out the total number of possible sample space when 4 dice are rolled.:
(a) 6^2 (b) 6^3 (c) 6^4 (d) 6^6

(ix) While rolling a pair of dice, what will be the probability of double 2?
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{36}$

(x) A card is chosen from a pack of 52 playing cards, find the probability of getting no jack and king:
(a) $\frac{2}{13}$ (b) $\frac{11}{13}$ (c) $\frac{2}{52}$ (d) $\frac{11}{52}$

2. Define the following:
(i) relative frequency (ii) expected frequency

3. An urn contains 10 red balls, 5 green balls and 8 blue balls. Find the probability of selecting at random.
(i) a green ball (ii) a red ball (iii) a blue ball
(iv) not a red ball (v) not a green ball

4. Three coins are tossed together. what is the probability of getting:
   (i) exactly three heads
   (ii) at least two tails
   (iii) not at least two heads
   (iv) not exactly two heads
5. A card is drawn from a well shuffled pack of 52 playing cards. What will be the probability of getting:
   (i) king or jack of red colour
   (ii) not "2" of club and spade
6. Six coins are tossed 600 times. The number of occurrence of tails are recorded and shown in the table given below:

No. of tails	0	1	2	3	4	5	6
Frequency	110	90	105	80	76	123	16

Find the relative frequency of given table.

7. From a lot containing 25 items, 8 items are defective. Find the relative frequency of non-defective items, also find the expected frequency of non-defective items.