Unit 2

Logarithms

Students' Learning Outcomes

At the end of the unit, the students will be able to:

- Express a number in scientific notation and vice versa.
- Describe logarithm of a number
- Differentiate between common and natural logarithm

INTRODUCTION

Logarithms are powerful mathematical tools used to simplify complex calculations, particularly those involving exponential growth or decay. They are widely applicable across various fields, including banking, science, engineering, and information technology. In chemistry, the pH scale, which measures the acidity or alkalinity of a solution, is based on logarithms. They help in transforming non-linear data into linear form for analysis, solving exponential equations and managing calculations involving very large or small numbers efficiently.

2.1 Scientific Notation

A method used to express very large or very small numbers in a more manageable form is known as Scientific notation. It is commonly used in science, engineering and mathematics to simplify complex calculations.

A number in scientific notation is written as:

$$a \times 10^n, \text{ where } 1 \leq a < 10 \text{ and } n \in Z$$

Here "a" is called the coefficient or base number.

Remember!
If the number is greater than 1 then n is positive and if the number is less than 1 then n is negative.

2.1.1 Conversion of Numbers from Ordinary Notation to Scientific Notation

Example 1: Convert 78,000,000 to scientific notation.

Solution: **Step 1:** Move the decimal to get a number between 1 and 10:

$$7.8$$

Step 2: Count the number of places you moved the decimal:

7 places

Step 3: Write in scientific notation:

$$78,000,000 = 7.8 \times 10^7$$

Since we moved the decimal to the **left**, the exponent is **positive**.

Example 2: Convert 0.0000000315 to scientific notation.

Solution:

Step 1: Move the decimal to get a number between 1 and 10:

3.15

Step 2: Count the number of places you moved the decimal:

8 places

Step 3: Write in scientific notation:

$0.0000000315 = 3.15 \times 10^{-8}$

Since we moved the decimal to the **right**, the exponent is **negative**.

Try Yourself!

Convert the following into scientific notation:

(i) 29,000,000

(ii) 0.000006

2.1.2 Conversion of Numbers from Scientific Notation to Ordinary Notation

Example 3: Convert 3.47×10^6 to ordinary notation.

Solution: **Step 1:** Identify the parts:

Coefficient: 3.47

Exponent: 10^6

Step 2: Since the exponent is **positive** 6, move the decimal point 6 places to the right.

$3.47 \times 10^6 = 3{,}470{,}000$

Remember!

If exponent is positive then the decimal will move to the right. If exponent is negative then the decimal will move to the left.

Example 4: Convert 6.23×10^{-4} to ordinary notation.

Solution: **Step 1:** Identify the parts:

Coefficient: 6.23

Exponent: 10^{-4}

Step 2: Since the exponent is **negative** 4, move the decimal point 4 places to the **left.**

$6.23 \times 10^{-4} = 0.000623$

Try Yourself!

Convert the following into ordinary notation:

(i) 5.63×10^3

(ii) 6.6×10^{-5}

EXERCISE 2.1

1. Express the following numbers in scientific notation:

(i)	2000000	(ii)	48900	(iii)	0.0042
(iv)	0.0000009	(v)	73×10^3	(vi)	0.65×10^2

2. Express the following numbers in ordinary notation:

(i)	8.04×10^2	(ii)	3×10^5	(iii)	1.5×10^{-2}
(iv)	1.77×10^7	(v)	5.5×10^{-6}	(vi)	4×10^{-5}

3. The speed of light is approximately 3×10^8 metres per second. Express it in standard form.
4. The circumference of the Earth at the equator is about 40075000 metres. Express this number in scientific notation.
5. The diameter of Mars is 6.7779×10^3 km. Express this number in standard form.
6. The diameter of Earth is about 1.2756×10^4 km. Express this number in standard form.

2.2 Logarithm

A logarithm is based on two Greek words: logos and arithmos which means ratio or proportion. John Napier, a Scottish mathematician, introduced the word logarithm. It is a way to simplify complex calculations, especially those involving multiplication and division of large numbers. Today, logarithm remain fundamental in mathematics, with applications in science, finance and technology.

2.2.1 Logarithm of a Real Number

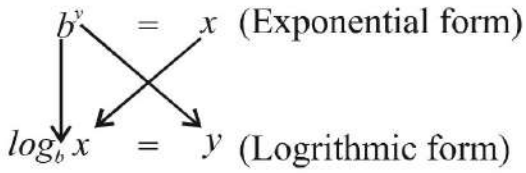
In simple words, the logarithm of a real number tells us how many times one number must be multiplied by itself to get another number.

The general form of a logarithm is: $\log_b(x) = y$

Where:
- b is the **base**,
- x is the **result** or the number whose logarithm is being taken,
- y is the **exponent** or the logarithm of x to the base b.

This means that:

$$b^y = x$$



In words, **"the logarithm of x to the base b is y,** means that when b is raised to the power y, it equals x.

The relationship between logarithmic form and exponential form is given below:

$$\log_b(x) = y \quad \Leftrightarrow \quad b^y = x \text{ where } b > 0, x > 0 \text{ and } b \neq 1$$

Example 5: Convert $\log_2 8 = 3$ to exponential form.

Solution: $\log_2 8 = 3$

Its exponential form is: $2^3 = 8$

Example 6: Convert $\log_{10}100 = 2$ to exponential form.

Solution: $\log_{10}100 = 2$

Its exponential form is: $10^2 = 100$

Example 7: Find the value of x in each case:

(i) $\log_5 25 = x$ (ii) $\log_2 x = 6$

Solution:

(i) $\log_5 25 = x$

Its exponential form is:

$5^x = 25$

$\Rightarrow 5^x = 5^2$

$\Rightarrow x = 2$

(ii) $\log_2 x = 6$

Its exponential form is:

$2^6 = x$

$\Rightarrow x = 64$

Example 8: Convert the following in logarithmic form:

(i) $3^4 = 81$ (ii) $7^0 = 1$

Solution:

(i) $3^4 = 81$

Its logarithmic form is:

$\log_3 81 = 4$

(ii) $7^0 = 1$

Its logarithmic form is:

$\log_7 1 = 0$

EXERCISE 2.2

1. Express each of the following in logarithmic form:

(i) $10^3 = 1000$ (ii) $2^8 = 256$ (iii) $3^{-3} = \frac{1}{27}$

(iv) $20^2 = 400$ (v) $16^{-\frac{1}{4}} = \frac{1}{2}$ (vi) $11^2 = 121$

(vii) $p = q^r$ (viii) $(32)^{\frac{-1}{5}} = \frac{1}{2}$

2. Express each of the following in exponential form:

(i) $\log_5 125 = 3$ (ii) $\log_2 16 = 4$ (iii) $\log_{23} 1 = 0$

(iv) $\log_5 5 = 1$ (v) $\log_2 \frac{1}{8} = -3$ (vi) $\frac{1}{2} = \log_9 3$

(vii) $5 = \log_{10} 100000$ (viii) $\log_4 \frac{1}{16} = -2$

3. Find the value of x in each of the following:

(i) $\log_x 64 = 3$ (ii) $\log_5 1 = x$ (iii) $\log_x 8 = 1$

(iv) $\log_{10} x = -3$ (v) $\log_4 x = \frac{3}{2}$ (vi) $\log_2 1024 = x$

2.3 Common Logarithm

The **common logarithm** is the logarithm with a base of 10. It is written as $\log_{10}$ or simply as log (when no base is mentioned, it is usually assumed to be base 10).

For example:

$$10^1 = 10 \Leftrightarrow \log 10 = 1$$
$$10^2 = 100 \Leftrightarrow \log 100 = 2$$
$$10^3 = 1000 \Leftrightarrow \log 1000 = 3 \text{ and so on.}$$
$$10^{-1} = \frac{1}{10} = 0.1 \Leftrightarrow \log 0.1 = -1$$
$$10^{-2} = \frac{1}{100} = 0.01 \Leftrightarrow \log 0.01 = -2$$
$$10^{-3} = \frac{1}{1000} = 0.001 \Leftrightarrow \log 0.001 = -3 \text{ and so on.}$$

History

English mathematician Henry Briggs extended Napier's work and developed the common logarithm. He also introduced logarithmic table.

2.3.1 Characteristic and Mantissa of Logarithms

The logarithm of a number consists of two parts: **the characteristic** and **the mantissa**. Here is a simple way to understand them:

(a) Characteristic

The characteristic is the integral part of the logarithm. It tells us how big or small the number is.

Remember!

When the characteristic is negative, we write it with bar.

Rules for Finding the Characteristic

(i) For a number greater than 1:

Characteristic = number of digits to the left of the decimal point – 1

For example, in log 567 the characteristic = 3 – 1 = 2

(ii) For a number less than 1:

Characteristic = – (number of zeros between the decimal point and the first non-zero digit + 1)

For example, in log 0.0123 the characteristic = – (1 + 1) = –2 or $\bar{2}$

Example 9: Find characteristic of the followings:

(i) log 725 (ii) log 9.87

(iii) log 0.00045 (iv) log 0.54

Solution: (i) log 725

Characteristic = 3 – 1 = 2

(ii) log 9.87

Characteristic = 1 – 1 = 0

(iii) log 0.00045

Characteristic = – (3 + 1) = $\bar{4}$

(iv) log 0.54

Characteristic = – (0 + 1) = $\bar{1}$

Characteristic of the logarithm of numbers can also be find by expressing them in scientific notation. For example,

Number	Scientific Notation	Characteristic of the logarithm
725	7.25×10^2	2
9.87	9.87×10^0	0
0.00045	4.5×10^{-4}	–4
0.54	5.4×10^{-1}	–1

(b) Mantissa

The mantissa is the decimal part of the logarithm. It represents the "fractional" component and is always positive.

For example, in log 5000 = 3.698 the mantissa is 0.698

2.3.2 Finding Common Logarithm of a Number

Suppose we want to find the common logarithm of 13.45. The step-by-step procedure to find the logarithm is given below:

Step 1: Separate the integral and decimal parts.

Integral part = 13

Decimal part = 45

> **Remember!**
> *log* (Number) = Characteristic + Mantissa

Step 2: Find the characteristic of the number

Characteristic = number of digits to the left of the decimal point – 1

= 2 – 1 = 1

Step 3: In common logarithm table (Complete table is given at the end of the book), check the intersection of row number 13 and column number 4 which is 1271.

Step 4: Find mean difference: Check the intersection of row number 13 and column number 5 in the mean difference which is 16.

Logarithm Table

	0	1	2	3	4	5	6	7	8	9	Mean Difference 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27

Step 5: Add the numbers found in step 3 and step 4. i.e., 1271 + 16 = 1287 which is the mantissa of given number.

Step 6: Finally, combine the characteristic and mantissa parts found in step 2 and step 5 respectively. We get 1.1287

So, the value of log 13.45 is 1.1287

Example 10: Find logarithm of the following numbers:

(i) (ii) log 5.678 (iii) log 0.0036 (iv) log 0.0478

Solution: (i) log 345

Characteristic = 3 – 1 = 2

Mantissa = 0.5378 (Look for 34 in the row and 5 in the column of the log table)

So, log (345) = 2 + 0.5378 = 2.5378

(ii) log 5.678

Characteristic = 1 – 1 = 0

Mantissa = 0.7542 (7536 + 6 = 7542)

So, log (5.678) = 0 + 0.7542 = 0.7542

Do you know?
log (0) = undefined
log (1) = 0
$\log_a(a) = 1$

(iii) log 0.0036

Characteristic = – (2 + 1) = –3

Mantissa = 0.5563 (Look for 36 in the row and 0 in the column of the log table)

So, log (0.0036) = – 3 + 0.5563 = $\bar{2}$.4437

(iv) log 0.0478

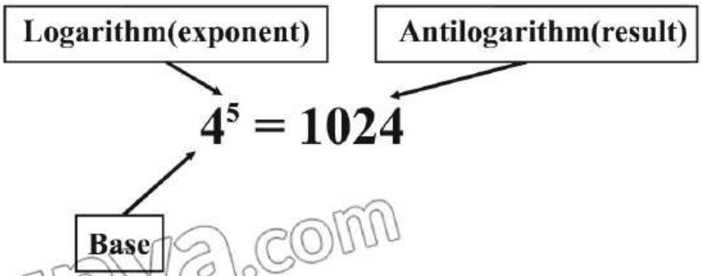
Characteristic = – (1 + 1) = – 2

Mantissa = 0.6794 (Look for 47 in the row and 8 in the column of the log table)

So, = – 2 + 0.6794 = $\bar{1}$.3206

2.3.3 Concept of Antilogarithm

An **antilogarithm** is the inverse operation of a logarithm. An antilogarithm helps to find a number whose logarithmic value is given.



In simple terms:

If $\log_b x = y \Leftrightarrow b^y = x$ then the process of finding x is called antilogarithm of y.

Finding Antilogarithm of a Number using Tables

Let us find the antilogarithm of 2.1245.

The step-by-step procedure to find the antilogarithm is given below:

Step 1: Separate the characteristic and mantissa parts:

Characteristic = 2

Mantissa = 0.1245

> **Remember!**
> The word antilogarithm is another word for the number or result. For example, in $4^3 = 64$, the result 64 is the antilogarithm.

Step 2: Find corresponding value of mantissa from antilogarithm table (given at the end of the book):

Check the intersection of row number .12 and column number 4 which provides the number 1330.

Step 3: Find the mean difference:

Check the intersection of row number .12 and the column number 5 of the mean difference in the antilogarithm table which gives 2.

Antilogarithm Table

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3

Step 4: Add the numbers found in the step 2 and step 3, we get 1330 + 2 = 1332

Step 5: Insert the decimal point:

Since characteristic is 2, therefore the decimal point will be after 2 digits right from the reference position. So, we get 133.2.

Thus, the antilog (2.1245) = 1‸33.2

> **Remember!**
> The place between the first non-zero digit from left and its next digit is called reference position. For example, in 1332, the reference position is between 1 and 3

Example 11: Find the value of x in the followings:

(i) $\log x = 0.2568$ (ii) $\log x = -1.4567$

(iii) $\log x = -2.1234$

Solution: (i) $\log x = 0.2568$

Characteristic $= 0$ Mantissa $= 0.2568$

Table value of $0.2568 = 1803 + 3 = 1806$

So, $x = \text{antilog}\,(0.2568) = 1.806$ (Insert the decimal point at reference position because characteristic is 0.)

(ii) $\log x = -1.4567$

Since mantissa is negative, so we make it positive by adding and subtracting 2

$$\log x = -2 + 2 - 1.4567$$
$$= -2 + 0.5433 = \bar{2}.5433$$

Here characteristic $= \bar{2}$, mantissa $= 0.5433$

Table value of $0.5433 = 3491 + 2 = 3_{\wedge}493$

So, $x = \text{antilog}\,(\bar{2}.5433)$
$= 0.03493$

Since characteristic is $\bar{2}$, therefore decimal point will be before 2 digits left from the reference position.

(iii) $\log x = -2.1234$

Since mantissa is negative, so we make it positive by adding and subtracting 3

$$\log x = -3 + 3 - 2.1234$$
$$= -3 + 0.8766 = \bar{3}.8766$$

Here characteristic $= \bar{3}$, mantissa $= 0.8766$

Table value of $0.8766 = 7516 + 10 = 7_{\wedge}526$

So, $x = \text{antilog}\,(\bar{3}.8766)$
$= 0.007526$

Since characteristic $= \bar{3}$, therefore decimal point will be before 3 digits left from the reference position.

History
Swiss mathematician and physicist Leonhard Euler introduced '*e*' for the base of natural logarithm.

2.3.4 Natural Logarithm

The natural logarithm is the logarithm with base e, where e is a mathematical constant approximately equal to 2.71828. It is denoted as ln. The natural logarithm is commonly used in mathematics, particularly in calculus, to describe exponential growth, decay and many other natural phenomena.

For example, $\ln e^2 = 2$ i.e., the logarithm of e^2 to the base e is 2.

Difference between Common and Natural Logarithms

Common Logarithm	Natural Logarithm
i. The base of a common logarithm is 10.	i. The base of a natural logarithm is e.
ii. It is written as $\log_{10}(x)$ or simply log (x) when no base is specified.	ii. It is written as $ln(x)$
iii. Common logarithms are widely used in everyday calculations, especially in scientific and engineering applications.	iii. Natural logarithms are commonly used in higher level mathematics particularly calculus and applications involving growth/decay processes.

EXERCISE 2.3

1. Find characteristic of the following numbers:
   (i) 5287 (ii) 59.28 (iii) 0.0567
   (iv) 234.7 (v) 0.000049 (vi) 145000
2. Find logarithm of the following numbers:
   (i) 43 (ii) 579 (iii) 1.982
   (iv) 0.0876 (v) 0.047 (vi) 0.000354
3. If log 3.177 = 0.5019, then find:
   (i) log 3177 (ii) log 31.77 (iii) log 0.03177
4. Find the value of x.
   (i) $\log x = 0.0065$ (ii) $\log x = 1.192$ (iii) $\log x = -3.434$
   (iv) $\log x = -1.5726$ (v) $\log x = 4.3561$ (vi) $\log x = -2.0184$

2.4 Laws of Logarithm

Laws of logarithm are also known as rules or properties of logarithm. These laws help to simplify logarithmic expressions and solve logarithmic equations.

1. Product Law

$$\log_b xy = \log_b x + \log_b y$$

The logarithm of the product is the sum of the logarithms of the factors.

Proof: Let $m = \log_b x$ ...(i)

and $n = \log_b y$ ...(ii)

Express (i) and (ii) in exponential form:

$$x = b^m \quad \text{and} \quad y = b^n$$

Multiply x and y, we get

$$x.y = b^m.\, b^n = b^{m+n}$$

Its logarithmic form is:

$$\log_b xy = m + n$$

$$\log_b xy = \log_b x + \log_b y \qquad \text{[From (i) and (ii)]}$$

2. Quotient Law

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

The logarithm of a quotient is the difference between the logarithms of the numerator and the denominator.

Proof:

Let $m = \log_b x$ ...(i)

and $n = \log_b y$ ...(ii)

Express (i) and (ii) in exponential form:

$$x = b^m \text{ and } y = b^n$$

Divide x by y, we get

$$\frac{x}{y} = \frac{b^m}{b^n} = b^{m-n}$$

Its logarithmic form is:

$$\log_b\left(\frac{x}{y}\right) = m - n$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

Activity

- Divide the students into small groups.
- Distribute the logarithmic expression cards randomly among the groups.
- Each group will work together to identify which logarithmic law applies to each expression.
- After completing the task, each group will present its findings.

3. Power Law

$$\log_b x^n = n.\log_b x$$

The logarithm of a number raised to a power is the product of the power and the logarithm of the base number.

Proof:

Let $m = \log_b x$ ...(i)

Its exponential form is:

$$x = b^m$$

Raise both sides to the power n

$$x^n = (b^m)^n = b^{nm}$$

Its logarithmic form is:

$$\log_b x^n = nm$$

$$\log_b x^n = n.\log_b x \qquad \text{[From (i)]}$$

4. Change of Base Law

$$\log_b x = \frac{\log_a x}{\log_a b}$$

This law allows to change the base of a logarithm from "b" to any other base "a".

Proof: Let

$m = \log_b x$ ...(i)

Its exponential form is:

$$b^m = x$$

Taking log with base "a" on both sides, we get

$$\log_a b^m = \log_a x$$

$$m \log_a b = \log_a x$$

$$m = \frac{\log_a x}{\log_a b}$$

$$\log_b x = \frac{\log_a x}{\log_a b} \qquad \text{[From (i)]}$$

2.4.1 Applications of Logarithm

Logarithms have a wide range of applications in many fields. Here some examples are given about the applications of logarithms.

Example 12: Expand the following using laws of logarithms:

(i) $\log_3(20)$ (ii) $\log_2 (9)^5$ (iii) $\log_{32} 27$

Solution: (i) $\log_3(20)$

$= \log_3(2\times 2\times 5)$

$= \log_3(2^2\times 5)$

$= \log_3(2)^2 + \log_3 5$

$= 2\log_3 2 + \log_3 5$

(ii) $\log_2(9)^5$

$= \log_2(3^2)^5$

$= \log_2(3)^{10}$

$= 10\log_2 3$

(iii) $\log_{32} 27$

$= \dfrac{\log 27}{\log 32}$

$= \dfrac{\log 3^3}{\log 2^5}$

$= \dfrac{3\log 3}{5\log 2}$

$= \dfrac{3}{5}\log_2 3$

Example 13: Expand the following using laws of logarithms:

(i) $\log_2\left(\dfrac{x-y}{z}\right)^3$ (ii) $\log_5\left(\dfrac{xy}{z}\right)^8$

Solution: (i) $\log_2\left(\dfrac{x-y}{z}\right)^3 = 3\log_2\left(\dfrac{x-y}{z}\right)$

$= 3[\log_2(x-y) - \log_2 z]$

(ii) $\log_5\left(\dfrac{xy}{z}\right)^8 = 8\log_5\left(\dfrac{xy}{z}\right)$

$= 8[\log_5(xy) - \log_5 z]$

$= 8[\log_5 x + \log_5 y - \log_5 z]$

Example 14: Write the following as a single logarithm:

(i) $2\log_3 10 - \log_3 4$ (ii) $6\log_3 x + 2\log_3 11$

Solution: (i) $2\log_3 10 - \log_3 4$

$= \log_3(10)^2 - \log_3 4$

$= \log_3 100 - \log_3 4$

$= \log_3\left(\dfrac{100}{4}\right)$

$= \log_3 25$

(ii) $6\log_3 x + 2\log_3 11$

$= \log_3 x^6 + \log_3(11)^2$

$= \log_3 x^6 + \log_3(121)$

$= \log_3(121x^6)$

Example 15: The decibel scale measures sound intensity using the formula $L = 40\log_{10}\left(\dfrac{I}{I_o}\right)$. If a sound has an intensity (I) of 10^6 times the reference intensity

(I_o). What is the sound level in decibels?

Do you know?

$ln\,(0)$ = undefined

$ln\,(1) = 0$

$ln\,(e) = 1$

Solution: $L = 40\log_{10}\left(\dfrac{I}{I_o}\right)$

Put $I = 10^6 I_o$, we get

$$L = 40\log_{10}\left(\frac{10^6 I_o}{I_o}\right)$$

$$L = 40\log_{10}(10)^6$$

$$L = 40\times 6\log_{10}10$$

$$L = 40\times 6 \qquad (\because \log_{10}10 = 1)$$

$$L = 240 \text{ decibels}$$

EXERCISE 2.4

1. Without using calculator, evaluate the following:

(i) $\log_2 18 - \log_2 9$ (ii) $\log_2 64 + \log_2 2$ (iii) $\frac{1}{3}\log_3 8 - \log_3 18$

(iv) $2\log 2 + \log 25$ (v) $\frac{1}{3}\log_4 64 + 2\log_5 25$ (vi) $\log_3 12 + \log_3 0.25$

2. Write the following as a single logarithm:

(i) $\frac{1}{2}\log 25 + 2\log 3$ (ii) $\log 9 - \log\frac{1}{3}$

(iii) $\log_5 b^2.\log_a 5^3$ (iv) $2\log_3 x + \log_3 y$

(v) $4\log_5 x - \log_5 y + \log_5 z$ (vi) $2\ln a + 3\ln b - 4\ln c$

3. Expand the following using laws of logarithms:

(i) $\log\left(\dfrac{11}{5}\right)$ (ii) $\log_5\sqrt{8a^6}$ (iii) $\ln\left(\dfrac{a^2b}{c}\right)$

(iv) $\log\left(\dfrac{xy}{z}\right)^{\frac{1}{9}}$ (v) $\ln\sqrt[3]{16x^3}$ (vi) $\log_2\left(\dfrac{1-a}{b}\right)^5$

4. Find the value of x in the following equations:

(i) $\log 2 + \log x = 1$ (ii) $\log_2 x + \log_2 8 = 5$

(iii) $(81)^x = (243)^{x+2}$ (iv) $\left(\dfrac{1}{27}\right)^{x-6} = 27$

(v) $\log(5x-10)=2$ (vi) $\log_2(x+1)-\log_2(x-4)=2$

5. Find the values of the following with the help of logarithm table:

(i) $\dfrac{3.68\times 4.21}{5.234}$ (ii) $4.67\times 2.11\times 2.397$

(iii) $\dfrac{(20.46)^2\times(2.4122)}{754.3}$ (iv) $\dfrac{\sqrt[3]{9.364}\times 21.64}{3.21}$

6. The formula to measure the magnitude of earthquakes is given by $M=\log_{10}\left(\dfrac{A}{A_o}\right)$. If amplitude ($A$) is 10,000 and reference amplitude (A_o) is 10. What is the magnitude of the earthquake?

7. Abdullah invested Rs. 100,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is Rs y. This is modelled by an equation $y = 100,000\,(1.05)^t$, $t \geq 0$. Find after how many years the investment will be double.

8. Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 metres gained in altitude. The initial temperature (T_i) at sea level is 20°C. Using the formula $T = T_i \times 0.97^{\frac{h}{100}}$, calculate the temperature at an altitude (h) of 500 metres.

REVIEW EXERCISE 2

1. Four options are given against each statement. Encircle the correct option.

(i) The standard form of 5.2×10^6 is:

(a) 52,000 (b) 520,000 (c) 5,200,000 (d) 52,000,000

(ii) Scientific notation of 0.00034 is:

(a) 3.4×10^3 (b) 3.4×10^{-4} (c) 3.4×10^4 (d) 3.4×10^{-3}

(iii) The base of common logarithm is:

(a) 2 (b) 10 (c) 5 (d) e

(iv) $\log_2 2^3 =$ __________.

(a) 1 (b) 2 (c) 5 (d) 3

(v) $\log 100 =$ __________.

(a) 2 (b) 3 (c) 10 (d) 1

(vi) If $\log 2 = 0.3010$, then $\log 200$ is:

(a) 1.3010 (b) 0.6010 (c) 2.3010 (d) 2.6010

(vii) log (0) = __________.

(a) positive (b) negative (c) zero (d) undefined

(viii) log 10,000 =

(a) 2 (b) 3 (c) 4 (d) 5

(ix) log 5 + log 3 = __________.

(a) log 0 (b) log 2 (c) $\log\left(\frac{5}{3}\right)$ (d) log 15

(x) $3^4 = 81$ in logarithmic form is:

(a) $\log_3 4 = 81$ (b) $\log_4 3 = 81$

(c) $\log_3 81 = 4$ (d) $\log_4 81 = 3$

2. Express the following numbers in scientific notation:

(i) 0.000567 (ii) 734 (iii) 0.33×10^3

3. Express the following numbers in ordinary notation:

(i) 2.6×10^3 (ii) 8.794×10^{-4} (iii) 6×10^{-6}

4. Express each of the following in logarithmic form:

(i) $3^7 = 2187$ (ii) $a^b = c$ (iii) $(12)^2 = 144$

5. Express each of the following in exponential form:

(i) $\log_4 8 = x$ (ii) $\log_9 729 = 3$ (iii) $\log_4 1024 = 5$

6. Find value of x in the following:

(i) $\log_9 x = 0.5$ (ii) $\left(\frac{1}{9}\right)^{3x} = 27$ (iii) $\left(\frac{1}{32}\right)^{2x} = 64$

7. Write the following as a single logarithm:

(i) $7\log x - 3\log y^2$ (ii) $3\log 4 - \log 32$

(iii) $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

8. Expand the following using laws of logarithms:

(i) $\log(x\,y\,z^6)$ (ii) $\log_3 \sqrt[6]{m^5 n^3}$ (iii) $\log\sqrt{8x^3}$

9. Find the values of the following with the help of logarithm table:

(i) $\sqrt[3]{68.24}$ (ii) 319.8×3.543 (iii) $\frac{36.12 \times 750.9}{113.2 \times 9.98}$

10. In the year 2016, the population of a city was 22 millions and was growing at a rate of 2.5% per year. The function $p(t) = 22(1.025)^t$ gives the population in millions, t years after 2016. Use the model to determine in which year the population will reach 35 millions. Round the answer to the nearest year.